1.1

1.1 The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

$$F = C_D \rho V^2 A/2$$

or

$$C_D = 2F/\rho V^2 A \text{ , where } \begin{aligned} F &\doteq MLT^{-2} \\ \rho &\doteq ML^{-3} \\ V &\doteq LT^{-1} \\ A &\doteq L^2 \end{aligned}$$

Thus,

$$C_D \doteq (MLT^{-2})/[(ML^{-3})(LT^{-1})^2(L^2)] = M^0L^0T^0$$

Hence, C_D is <u>dimensionless.</u>

1.2

1.2 Verify the dimensions, in both the *FLT* and *MLT* systems, of the following quantities which appear in Table 1.1: **(a)** volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

(a) volume $\doteq \underline{\underline{L^3}}$

(b) acceleration = time rate of change of velocity

$$\doteq \frac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$$

(c) mass $\doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

mass $\doteq \underline{\underline{FL^{-1}T^{2}}}$

(d) moment of inertia (area) = second moment of area

$$\doteq (L^2)(L^2) \doteq \underline{\underline{L^4}}$$

(e) work = force × distance

$$\doteq \underline{\underline{FL}}$$

or with $F \doteq MLT^{-2}$

work $\doteq \underline{\underline{ML^2T^{-2}}}$

1.3

1.3 Determine the dimensions, in both the *FLT* system and the *MLT* system, for **(a)** the product of force times acceleration, **(b)** the product of force times velocity divided by area, and **(c)** momentum divided by volume.

(a) $\text{force} \times \text{acceleration} \doteq (F)(LT^{-2}) \doteq \underline{\underline{FLT^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{force} \times \text{acceleration} \doteq (MLT^{-2})(LT^{-2}) \doteq \underline{\underline{ML^2T^{-4}}}$

(b)
$$\frac{\text{force} \times \text{velocity}}{\text{area}} \doteq \frac{(F)(LT^{-1})}{L^2} \doteq \underline{\underline{FL^{-1}T^{-1}}}$$

$$\doteq \frac{(MLT^{-2})(LT^{-1})}{L^2} \doteq \underline{\underline{MT^{-3}}}$$

(c)
$$\frac{\text{momentum}}{\text{volume}} = \frac{\text{mass} \times \text{velocity}}{\text{volume}}$$

$$\doteq \frac{(FT^2L^{-1})(LT^{-1})}{L^3} \doteq \underline{\underline{FL^{-3}T}}$$

$$\doteq \frac{(M)(LT^{-1})}{L^3} \doteq \underline{\underline{ML^{-2}T^{-1}}}$$

1.4

1.4 Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table 1.1: **(a)** frequency, **(b)** stress, **(c)** strain, **(d)** torque, and **(e)** work.

(a) $\text{frequency} = \dfrac{\text{cycles}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

(b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$$\text{stress} \doteq \frac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

(c) $\text{strain} = \dfrac{\text{change in length}}{\text{length}} \doteq \dfrac{L}{L} \doteq \underline{\underline{L^0}}$ (dimensionless)

(d) $\text{torque} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

$$\doteq (MLT^{-2})(L) \doteq \underline{\underline{ML^2T^{-2}}}$$

(e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

$$\doteq (MLT^{-2})(L) \doteq \underline{\underline{ML^2T^{-2}}}$$

1.5

1.5 If u is a velocity, x a length, and t a time, what are the dimensions (in the MLT system) of **(a)** $\partial u/\partial t$, **(b)** $\partial^2 u/\partial x \partial t$, and **(c)** $\int (\partial u/\partial t)\, dx$?

(a) $\frac{\partial u}{\partial t} \doteq \frac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

(b) $\frac{\partial^2 u}{\partial x \partial t} \doteq \frac{LT^{-1}}{(L)(T)} \doteq \underline{\underline{T^{-2}}}$

(c) $\int \frac{\partial u}{\partial t}\, dx \doteq \frac{(LT^{-1})}{T}(L) \doteq \underline{\underline{L^2T^{-2}}}$

1.6

1.6 If p is a pressure, V a velocity, and ρ a fluid density, what are the dimensions (in the MLT system) of (a) p/ρ, (b) $pV\rho$, and (c) $p/\rho V^2$?

(a) $$\frac{p}{\rho} \doteq \frac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$$

(b) $$pV\rho \doteq (ML^{-1}T^{-2})(LT^{-1})(ML^{-3}) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$$

(c) $$\frac{p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \ (\underline{\underline{\text{dimensionless}}})$$

1.7

1.7 If V is a velocity, ℓ a length, and ν a fluid property (the kinematic viscosity) having dimensions of L^2T^{-1}, which of the following combinations are dimensionless: (a) $V\ell\nu$, (b) $V\ell/\nu$, (c) $V^2\nu$, (d) $V/\ell\nu$?

(a) $V\ell\nu \doteq (LT^{-1})(L)(L^2T^{-1}) \doteq L^4T^{-2}$ (not dimensionless)

(b) $\dfrac{V\ell}{\nu} \doteq \dfrac{(LT^{-1})(L)}{(L^2T^{-1})} \doteq L^0T^0$ (dimensionless)

(c) $V^2\nu \doteq (LT^{-1})^2(L^2T^{-1}) \doteq L^4T^{-3}$ (not dimensionless)

(d) $\dfrac{V}{\ell\nu} \doteq \dfrac{(LT^{-1})}{(L)(L^2T^{-1})} \doteq L^{-2}$ (not dimensionless)

1.8

1.8 If V is a velocity, determine the dimensions of Z, α, and G, which appear in the dimensionally homogeneous equation

$$V = Z(\alpha - 1) + G$$

$$V = Z(\alpha - 1) + G$$

$$[LT^{-1}] \doteq [Z][\alpha - 1] + [G]$$

Since each term in the equation must have the same dimensions, it follows that

$$Z \doteq \underline{\underline{LT^{-1}}}$$

$$\alpha \doteq \underline{\underline{F^0L^0T^0}}$$ (dimensionless since combined with a number)

$$G \doteq \underline{\underline{LT^{-1}}}$$

1.9

1.9 The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8\mu\ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $(FL^{-2}T)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] \frac{[L^4][FL^{-2}]}{[FL^{-2}T][L]}$$

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] [L^3 T^{-1}]$$

The constant $\pi/8$ is dimensionless, and the equation is a general homogeneous equation that is valid in any consistent unit system. Yes.

1.10

1.10 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

$$h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$\left[\frac{FL}{F}\right] \doteq \left[0.04 \text{ to } 0.09\right]\left[\frac{L^4}{L^4}\right]\left[\frac{1}{2}\right]\left[\frac{L^2}{T^2}\right]\left[\frac{T^2}{L}\right]$$

$$[L] \doteq [0.04 \text{ to } 0.09][L]$$

Since each term in the equation must have the same dimensions, the constant term (0.04 to 0.09) must be dimensionless. Thus, the equation is a general homogeneous equation that is valid in any system of units. Yes.

1.11

1.11 The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity ($FL^{-2}T$), ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left[\frac{A_0}{A_1} - 1\right]^2 \rho V^2$$

$$\left[FL^{-2}\right] \doteq \left[K_v\right]\left[\left(\frac{FT}{L^2}\right)\left(\frac{L}{T}\right)\left(\frac{1}{L}\right)\right] + \left[K_u\right]\left[\frac{(L^2)}{(L^2)} - 1\right]^2 \left[\frac{FT^2}{L^4}\right]\left[\frac{L}{T}\right]^2$$

$$\left[FL^{-2}\right] \doteq \left[K_v\right]\left[FL^{-2}\right] + \left[K_u\right]\left[FL^{-2}\right]$$

Since each term must have the same dimensions, K_v and K_u are dimensionless. Thus, the equation is a general homogeneous equation that would be valid in any consistent system of units. Yes.

1.12

1.12 Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_v, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_v)^a(\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See Eq. 1.19.)

$$c = (E_v)^a(\rho)^b$$

Since $c \doteq LT^{-1}$ $\quad E_v \doteq FL^{-2}$ $\quad \rho = FL^{-4}T^2$

$$\left[\frac{L}{T}\right] \doteq \left[\frac{F^a}{L^{-2a}}\right]\left[\frac{F^b T^{2b}}{L^{-4b}}\right] \qquad (1)$$

For a dimensionally homogeneous equation each term in the equation must have the same dimensions. Thus, the right hand side of Eq. (1) must have the dimensions of LT^{-1}. Therefore,

$$a + b = 0 \quad \text{(to eliminate } F\text{)}$$
$$2b = -1 \quad \text{(to satisfy condition on } T\text{)}$$
$$2a + 4b = -1 \quad \text{(to satisfy condition on } L\text{)}$$

It follows that $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

So that

$$c = \sqrt{\frac{E_v}{\rho}}$$

This result is consistent with the standard formula for the speed of sound. Yes.

1.13

1.13 A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09BH^{3/2}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in ft^3/s when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

$$Q = 3.09\ B\ H^{3/2}$$

$$[L^3T^{-1}] \doteq [3.09][L][L]^{3/2}$$

$$[L^3T^{-1}] \doteq [3.09][L]^{5/2}$$

Since each term in the equation must have the same dimensions the constant 3.09 must have dimensions of $L^{1/2}T^{-1}$ and is therefore not dimensionless. No.

Since the constant has dimensions its value will change with a change in units. No.

1.14

1.15

1.15 Make use of Table 1.3 to express the following quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s², **(e)** 0.0234 lb·s/ft².

(a) $$10.2\ \frac{\text{in.}}{\text{min}} = \left(10.2\ \frac{\text{in.}}{\text{min}}\right)\left(2.540\times10^{-2}\ \frac{\text{m}}{\text{in.}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= 4.32\times10^{-3}\ \frac{\text{m}}{\text{s}} = \underline{\underline{4.32\ \frac{\text{mm}}{\text{s}}}}$$

(b) $$4.81\ \text{slugs} = (4.81\ \text{slugs})\left(1.459\times10\ \frac{\text{kg}}{\text{slug}}\right) = \underline{\underline{70.2\ \text{kg}}}$$

(c) $$3.02\ \text{lb} = (3.02\ \text{lb})\left(4.448\ \frac{\text{N}}{\text{lb}}\right) = \underline{\underline{13.4\ \text{N}}}$$

(d) $$73.1\ \frac{\text{ft}}{\text{s}^2} = \left(73.1\ \frac{\text{ft}}{\text{s}^2}\right)\left(3.048\times10^{-1}\ \frac{\frac{\text{m}}{\text{s}^2}}{\frac{\text{ft}}{\text{s}^2}}\right) = \underline{\underline{22.3\ \frac{\text{m}}{\text{s}^2}}}$$

(e) $$0.0234\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2} = \left(0.0234\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(4.788\times10\ \frac{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}\right)$$

$$= \underline{\underline{1.12\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

1.15.

1.16

1.16 Make use of Table 1.4 to express the following quantities in BG units: **(a)** 14.2 km, **(b)** 8.14 N/m³, **(c)** 1.61 kg/m³, **(d)** 0.0320 N·m/s, **(e)** 5.67 mm/hr.

(a) $14.2\ km = (14.2 \times 10^3\ m)\left(3.281\ \frac{ft}{m}\right) = \underline{\underline{4.66 \times 10^4\ ft}}$

(b) $8.14\ \frac{N}{m^3} = \left(8.14\ \frac{N}{m^3}\right)\left(6.366 \times 10^{-3}\ \frac{\frac{lb}{ft^3}}{\frac{N}{m^3}}\right) = \underline{\underline{5.18 \times 10^{-2}\ \frac{lb}{ft^3}}}$

(c) $1.61\ \frac{kg}{m^3} = \left(1.61\ \frac{kg}{m^3}\right)\left(1.940 \times 10^{-3}\ \frac{\frac{slugs}{ft^3}}{\frac{kg}{m^3}}\right) = \underline{\underline{3.12 \times 10^{-3}\ \frac{slugs}{ft^3}}}$

(d) $0.0320\ \frac{N \cdot m}{s} = \left(0.0320\ \frac{N \cdot m}{s}\right)\left(7.376 \times 10^{-1}\ \frac{\frac{ft \cdot lb}{s}}{\frac{N \cdot m}{s}}\right)$

$= \underline{\underline{2.36 \times 10^{-2}\ \frac{ft \cdot lb}{s}}}$

(e) $5.67\ \frac{mm}{hr} = \left(5.67 \times 10^{-3}\ \frac{m}{hr}\right)\left(3.281\ \frac{ft}{m}\right)\left(\frac{1\ hr}{3600\ s}\right)$

$= \underline{\underline{5.17 \times 10^{-6}\ \frac{ft}{s}}}$

1.17

1.17 Express the following quantities in SI units: (a) 160 acre, (b) 15 gallons (U.S.), (c) 240 miles, (d) 79.1 hp, (e) 60.3 °F.

(a) $160 \text{ acre} = (160 \text{ acre})\left(4.356 \times 10^4 \frac{ft^2}{acre}\right)\left(9.290 \times 10^{-2} \frac{m^2}{ft^2}\right)$

$= \underline{\underline{6.47 \times 10^5 \, m^2}}$

(b) $15 \text{ gallons} = (15 \text{ gallons})\left(3.785 \frac{liters}{gallon}\right)\left(10^{-3} \frac{m^3}{liter}\right) = \underline{\underline{56.8 \times 10^{-2} \, m^3}}$

(c) $240 \, mi = (240 \, mi)\left(5280 \frac{ft}{mi}\right)\left(3.048 \times 10^{-1} \frac{m}{ft}\right) = \underline{\underline{3.86 \times 10^5 \, m}}$

(d) $79.1 \, hp = (79.1 \, hp)\left(550 \frac{ft \cdot lb/s}{hp}\right)\left(1.356 \frac{J}{ft \cdot lb}\right) = 5.90 \times 10^4 \, \frac{J}{s}$

and $1 \frac{J}{s} = 1 \, W$ so that

$79.1 \, hp = \underline{\underline{5.90 \times 10^4 \, W}}$

(e) $T_c = \frac{5}{9}(60.3^\circ F - 32) = 15.7^\circ C$

$= 15.7^\circ C + 273 = \underline{\underline{289 \, K}}$

1,18

1.18

1.18 For Table 1.3 verify the conversion relationships for: **(a)** area, **(b)** density, **(c)** velocity, and **(d)** specific weight. Use the basic conversion relationships: 1 ft = 0.3048 m; 1 lb = 4.4482 N; and 1 slug = 14.594 kg.

(a) $$1\ ft^2 = (1\ ft^2)\left[(0.3048)^2 \frac{m^2}{ft^2}\right] = 0.09290\ m^2$$

Thus, multiply ft^2 by $\underline{\underline{9.290\ E-2}}$ to convert to m^2.

(b) $$1\ \frac{slug}{ft^3} = \left(1\ \frac{slug}{ft^3}\right)\left(14.594\ \frac{kg}{slug}\right)\left[\frac{1\ ft^3}{(0.3048)^3 m^3}\right]$$

$$= 515.4\ \frac{kg}{m^3}$$

Thus, multiply $slugs/ft^3$ by $\underline{\underline{5.154\ E+2}}$ to convert to kg/m^3.

(c) $$1\ \frac{ft}{s} = \left(1\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = 0.3048\ \frac{m}{s}$$

Thus, multiply ft/s by $\underline{\underline{3.048\ E-1}}$ to convert to m/s.

(d) $$1\ \frac{lb}{ft^3} = \left(1\ \frac{lb}{ft^3}\right)\left(4.4482\ \frac{N}{lb}\right)\left[\frac{1\ ft^3}{(0.3048)^3 m^3}\right]$$

$$= 157.1\ \frac{N}{m^3}$$

Thus, multiply lb/ft^3 by $\underline{\underline{1.571\ E+2}}$ to convert to N/m^3.

1.19

1.19

1.19 For Table 1.4 verify the conversion relationships for: **(a)** acceleration, **(b)** density, **(c)** pressure, and **(d)** volume flowrate. Use the basic conversion relationships: 1 m = 3.2808 ft; 1 N = 0.22481 lb; and 1 kg = 0.068521 slug.

(a) $$1\ \frac{m}{s^2} = \left(1\ \frac{m}{s^2}\right)\left(3.2808\ \frac{ft}{m}\right) = 3.281\ \frac{ft}{s^2}$$

Thus, multiply m/s^2 by $\underline{\underline{3.281}}$ to convert to ft/s^2.

(b) $$1\ \frac{kg}{m^3} = \left(1\ \frac{kg}{m^3}\right)\left(0.068521\ \frac{slugs}{kg}\right)\left[\frac{1\ m^3}{(3.2808)^3\ ft^3}\right]$$

$$= 1.940 \times 10^{-3}\ \frac{slugs}{ft^3}$$

Thus, multiply kg/m^3 by $\underline{\underline{1.940\ E{-3}}}$ to convert to $slugs/ft^3$.

(c) $$1\ \frac{N}{m^2} = \left(1\ \frac{N}{m^2}\right)\left(0.22481\ \frac{lb}{N}\right)\left[\frac{1\ m^2}{(3.2808)^2\ ft^2}\right]$$

$$= 2.089 \times 10^{-2}\ \frac{lb}{ft^2}$$

Thus, multiply N/m^2 by $\underline{\underline{2.089\ E{-2}}}$ to convert to lb/ft^2.

(d) $$1\ \frac{m^3}{s} = \left(1\ \frac{m^3}{s}\right)\left[(3.2808)^3\ \frac{ft^3}{m^3}\right] = 35.31\ \frac{ft^3}{s}$$

Thus, multiply m^3/s by $\underline{\underline{3.531\ E{+1}}}$ to convert to ft^3/s.

1.20

1.20 Water flows from a large drainage pipe at a rate of 1200 gal/min. What is this volume rate of flow in (a) m^3/s, (b) liters/min, and (c) ft^3/s?

(a)

$$\text{flowrate} = \left(1200\ \frac{gal}{min}\right)\left(6.309\times10^{-5}\ \frac{\frac{m^3}{s}}{\frac{gal}{min}}\right)$$

$$= \underline{\underline{7.57\times10^{-2}\ \frac{m^3}{s}}}$$

(b) Since $1\ \text{liter} = 10^{-3}\ m^3$,

$$\text{flowrate} = \left(7.57\times10^{-2}\ \frac{m^3}{s}\right)\left(\frac{10^3\ \text{liters}}{m^3}\right)\left(\frac{60\ s}{min}\right)$$

$$= \underline{\underline{4540\ \frac{\text{liters}}{min}}}$$

(c) $$\text{flowrate} = \left(7.57\times10^{-2}\ \frac{m^3}{s}\right)\left(3.531\times10\ \frac{\frac{ft^3}{s}}{\frac{m^3}{s}}\right)$$

$$= \underline{\underline{2.67\ \frac{ft^3}{s}}}$$

1,23

1.21

1.21 An important dimensionless parameter in certain types of fluid flow problems is the *Froude number* defined as $V/\sqrt{g\ell}$, where V is a velocity, g the acceleration of gravity, and ℓ a length. Determine the value of the Froude number for V = 10 ft/s, g = 32.2 ft/s², and ℓ = 2 ft. Recalculate the Froude number using SI units for V, g, and ℓ. Explain the significance of the results of these calculations.

In BG units,

$$\frac{V}{\sqrt{g\ell}} = \frac{10\ \frac{ft}{s}}{\sqrt{\left(32.2\ \frac{ft}{s^2}\right)(2\ ft)}} = \underline{\underline{1.25}}$$

In SI units:

$$V = \left(10\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = 3.05\ \frac{m}{s}$$

$$g = 9.81\ \frac{m}{s^2}$$

$$\ell = (2\ ft)\left(0.3048\ \frac{m}{ft}\right) = 0.610\ m$$

Thus,

$$\frac{V}{\sqrt{g\ell}} = \frac{3.05\ \frac{m}{s}}{\sqrt{\left(9.81\ \frac{m}{s^2}\right)(0.610\ m)}} = \underline{\underline{1.25}}$$

The value of a dimensionless parameter is independent of the unit system.

1.23

1.23 A tank contains 500 kg of a liquid whose specific gravity is 2. Determine the volume of the liquid in the tank.

$$m = \rho V = SG\, \rho_{H_2O} V$$

Thus,

$$V = m/(SG\, \rho_{H_2O}) = 500\,kg / ((2)(999\,\frac{kg}{m^3}))$$

$$= \underline{\underline{0.250\, m^3}}$$

1.24

1.24 Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter (g/m^3). Assume that a cumulus cloud occupies a volume of one cubic kilometer, and its liquid water content is 0.2 g/m^3. **(a)** What is the volume of this cloud in cubic miles? **(b)** How much does the water in the cloud weigh in pounds?

(a) $\Psi olume = 1\,(km)^3 = 10^9\, m^3$

Since $1\,m = 3.281\, ft$

$$\Psi olume = \frac{(10^9 m^3)(3.281\,\frac{ft}{m})^3}{(5.280 \times 10^3\, \frac{ft}{mi})^3}$$

$$= \underline{\underline{0.240\, mi^3}}$$

(b) $\mathcal{W} = \gamma \times \Psi olume$

$$\gamma = \rho g = (0.2\,\frac{g}{m^3})(10^{-3}\,\frac{kg}{g})(9.81\,\frac{m}{s^2}) = 1.962 \times 10^{-3}\,\frac{N}{m^3}$$

$$\mathcal{W} = (1.962 \times 10^{-3}\,\frac{N}{m^3})(10^9 m^3) = 1.962 \times 10^6\, N$$

$$= (1.962 \times 10^6\, N)(2.248 \times 10^{-1}\,\frac{lb}{N}) = \underline{\underline{4.41 \times 10^5\, lb}}$$

1.21

1.25

1.25 A tank of oil has a mass of 25 slugs. **(a)** Determine its weight in pounds and in newtons at the earth's surface. **(b)** What would be its mass (in slugs) and its weight (in pounds) if located on the moon's surface where the gravitational attraction is approximately one-sixth that at the earth's surface?

(a) $\text{weight} = \text{mass} \times g$

$$= (25 \text{ slugs})\left(32.2 \frac{ft}{s^2}\right) = \underline{\underline{805 \text{ lb}}}$$

$$= (25 \text{ slugs})\left(14.59 \frac{kg}{slug}\right)\left(9.81 \frac{m}{s^2}\right) = \underline{\underline{3580 \text{ N}}}$$

(b) $\text{mass} = \underline{\underline{25 \text{ slugs}}}$ (mass does not depend on gravitational attraction)

$$\text{weight} = (25 \text{ slugs})\left(\frac{32.2 \frac{ft}{s^2}}{6}\right) = \underline{\underline{134 \text{ lb}}}$$

1.22

1.26

1.26 A certain object weighs 300 N at the earth's surface. Determine the mass of the object (in kilograms) and its weight (in newtons) when located on a planet with an acceleration of gravity equal to 4.0 ft/s^2.

$$\text{mass} = \frac{\text{weight}}{g}$$

$$= \frac{300 \text{ N}}{9.81 \frac{m}{s^2}} = \underline{\underline{30.6 \text{ kg}}}$$

For $g = 4.0 \text{ ft/s}^2$,

$$\text{weight} = (30.6 \text{ kg})\left(4.0 \frac{ft}{s^2}\right)\left(0.3048 \frac{m}{ft}\right)$$

$$= \underline{\underline{37.3 \text{ N}}}$$

1.27

1.27 The density of a certain type of jet fuel is 775 kg/m³. Determine its specific gravity and specific weight.

$$SG = \frac{\rho}{\rho_{H_2O} @ 4°C} = \frac{775 \frac{kg}{m^3}}{1000 \frac{kg}{m^3}} = \underline{\underline{0.775}}$$

$$\gamma = \rho g = \left(775 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) = \underline{\underline{7.60 \frac{kN}{m^3}}}$$

1.28

1.28 A *hydrometer* is used to measure the specific gravity of liquids. (See Video V2.8.) For a certain liquid a hydrometer reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight? Express your answer in SI units.

$$SG = \frac{\rho}{\rho_{H_2O} @ 4°C}$$

$$1.15 = \frac{\rho}{1000 \frac{kg}{m^3}}$$

$$\rho = (1.15)\left(1000 \frac{kg}{m^3}\right) = \underline{\underline{1150 \frac{kg}{m^3}}}$$

$$\gamma = \rho g = \left(1150 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) = \underline{\underline{11.3 \frac{kN}{m^3}}}$$

1.26

1.29

1.29 An open, rigid-walled, cylindrical tank contains 4 ft^3 of water at 40 °F. Over a 24-hour period of time the water temperature varies from 40 °F to 90 °F. Make use of the data in Appendix B to determine how much the volume of water will change. For a tank diameter of 2 ft, would the corresponding change in water depth be very noticeable? Explain.

mass of water $= \forall \times \rho$

where $\forall$ is the volume and ρ the density. Since the mass must remain constant as the temperature changes

$$\forall_{40°} \times \rho_{40°} = \forall_{90°} \times \rho_{90°} \qquad (1)$$

From Table B.1 $\rho_{H_2O\,@\,40°F} = 1.940 \frac{slugs}{ft^3}$

$\rho_{H_2O\,@\,90°F} = 1.931 \frac{slugs}{ft^3}$

Therefore, from Eq. (1)

$$\forall_{90°} = \frac{(4\,ft^3)(1.940\,\frac{slugs}{ft^3})}{1.931\,\frac{slugs}{ft^3}} = 4.0186\,ft^3$$

Thus, the increase in volume is

$$4.0186 - 4.000 = \underline{\underline{0.0186\,ft^3}}$$

The change in water depth, Δl, is equal to

$$\Delta l = \frac{\Delta \forall}{area} = \frac{0.0186\,ft^3}{\frac{\pi}{4}(2ft)^2} = 5.92 \times 10^{-3}\,ft = 0.0710\,in.$$

This small change in depth would not be very noticeable. $\underline{\underline{No}}$.

Note: A slightly different value for Δl will be obtained if specific weight of water is used rather than density. This is due to the fact that there is some uncertainty in the fourth significant figure of these two values, and the solution is sensitive to this uncertainty.

1.31

1.31 A mountain climber's oxygen tank contains 1 lb of oxygen when he begins his trip at sea level where the acceleration of gravity is 32.174 ft/s^2. What is the weight of the oxygen in the tank when he reaches to top of Mt. Everest where the acceleration of gravity is 32.082 ft/s^2? Assume that no oxygen has been removed from the tank; it will be used on the descent portion of the climb.

$$\mathcal{W} = mg$$

Let $(\ )_{sl}$ denote sea level and $(\ )_{MtE}$ denote the top of Mt. Everest

Thus,

$$\mathcal{W}_{sl} = 1\,lb = m_{sl}\, g_{sl} \quad \text{and}$$

$$\mathcal{W}_{MtE} = m_{MtE}\, g_{MtE}$$

However $m_{sl} = m_{MtE}$ so that since $m = \frac{\mathcal{W}}{g}$,

$$m_{sl} = \frac{\mathcal{W}_{sl}}{g_{sl}} = m_{MtE} = \frac{\mathcal{W}_{MtE}}{g_{MtE}}$$

or

$$\mathcal{W}_{MtE} = \mathcal{W}_{sl}\,\frac{g_{MtE}}{g_{sl}} = 1\,lb\,\frac{32.082\ ft/s^2}{32.174\ ft/s^2} = \underline{\underline{0.9971\ lb}}$$

1.29

1.32

1.32 The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}} \qquad (1)$$

total weight = mass x g = $(0.369\,kg)(9.81\,\frac{m}{s^2}) = 3.62\,N$

weight of can = $0.153\,N$

volume of fluid = $(355 \times 10^{-3}\,L)(10^{-3}\,\frac{m^3}{L}) = 355 \times 10^{-6}\,m^3$

Thus, from Eq. (1)

$$\gamma = \frac{3.62\,N - 0.153\,N}{355 \times 10^{-6}\,m^3} = \underline{\underline{9770\,\frac{N}{m^3}}}$$

$$\rho = \frac{\gamma}{g} = \frac{9770\,\frac{N}{m^3}}{9.81\,\frac{m}{s^2}} = 996\,\frac{N \cdot s^2}{m^4} = \underline{\underline{996\,\frac{kg}{m^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O @ 4°C}} = \frac{996\,\frac{kg}{m^3}}{1000\,\frac{kg}{m^3}} = \underline{\underline{0.996}}$$

For water at 20°C (see Table B.2 in Appendix B)

$$\gamma_{H_2O} = 9789\,\frac{N}{m^3}\,; \quad \rho_{H_2O} = 998.2\,\frac{kg}{m^3}\,; \quad SG = 0.9982$$

A comparison of these values for water with those for the pop shows that the specific weight, density, and specific gravity of the pop are all slightly lower than the corresponding values for water.

*1.33

*1.33 The variation in the density of water, ρ, with temperature, T, in the range $20\,°C \le T \le 50\,°C$, is given in the following table.

Density (kg/m³)	998.2	997.1	995.7	994.1	992.2	990.2	988.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form $\rho = c_1 + c_2T + c_3T^2$ which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at 42.1 °C?

Fit the data to a second order polynomial using a standard curve-fitting program such as found in EXCEL. Thus,

$$\rho = \underline{\underline{1001 - 0.0533\,T - 0.0041\,T^2}} \qquad (1)$$

As shown in the table below, ρ (predicted) from Eq. (1) is in good agreement with ρ (given).

T, °C	ρ, kg/m^3	ρ, Predicted
20	998.2	998.3
25	997.1	997.1
30	995.7	995.7
35	994.1	994.1
40	992.2	992.3
45	990.2	990.3
50	988.1	988.1

At $T = 42.1\ °C$

$$\rho = 1001 - 0.0533\,(42.1\,°C) - 0.0041\,(42.1\,°C)^2 = \underline{991.5}\ \frac{kg}{m^3}$$

1.34

1.34 If 1 cup of cream having a density of 1005 kg/m^3 is turned into 3 cups of whipped cream, determine the specific gravity and specific weight of the whipped cream.

Mass of cream, $m = \left(1005 \frac{kg}{m^3}\right) \times \left(\forall_{cup}\right)$

where $\forall \sim$ volume.

Since $m_{cream} = m_{\substack{whipped \\ cream}}$

$$\rho_{\substack{whipped \\ cream}} = \frac{m_{\substack{whipped \\ cream}}}{\forall_{3\ cups}} = \frac{\left(1005 \frac{kg}{m^3}\right)\forall_{cup}}{\forall_{3\ cups}}$$

$$= \frac{1005 \frac{kg}{m^3}}{3} = 335 \frac{kg}{m^3}$$

$$SG = \frac{\rho_{\substack{whipped \\ cream}}}{\rho_{H_2O\ @\ 4°C}} = \frac{335 \frac{kg}{m^3}}{1000 \frac{kg}{m^3}} = \underline{\underline{0.335}}$$

$$\gamma_{\substack{whipped \\ cream}} = \rho_{\substack{whipped \\ cream}} \times g = \left(335 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)$$

$$= \underline{\underline{3290 \frac{N}{m^3}}}$$

1.36

1.36 Determine the mass of air in a 2 m^3 tank if the air is at room temperature, 20 °C, and the absolute pressure within the tank is 200 kPa (abs).

$m = \rho \mathcal{V}$ where $\mathcal{V} = 2\, m^3$ and

$\rho = p/RT$ with $T = 20°C = (20 + 273)\,K = 293\,K$

and $p = 200\,kPa = 200 \times 10^3 \frac{N}{m^2}$

Thus,

$$\rho = \left(200 \times 10^3 \frac{N}{m^2}\right) \Big/ \left[\left(2.869 \times 10^2 \frac{N \cdot m}{kg \cdot K}\right)(293\,K)\right]$$

$$= 2.38 \frac{kg}{m^3}$$

Hence,

$$m = \rho \mathcal{V} = 2.38 \frac{kg}{m^3}(2\,m^3) = \underline{\underline{4.76\,kg}}$$

1.37

1.37 Nitrogen is compressed to a density of 4 kg/m³ under an absolute pressure of 400 kPa. Determine the temperature in degrees Celsius.

$$T = \frac{p}{\rho R} = \frac{400 \times 10^3 \ \frac{N}{m^2}}{\left(4 \ \frac{kg}{m^3}\right)\left(296.8 \ \frac{J}{kg \cdot K}\right)} = 337 \ K$$

$$T_c = T_K - 273 = 337\,K - 273 = \underline{\underline{64\ °C}}$$

1.38

1.38 The temperature and pressure at the surface of Mars during a Martian spring day were determined to be −50 °C and 900 Pa, respectively. (a) Determine the density of the Martian atmosphere for these conditions if the gas constant for the Martian atmosphere is assumed to be equivalent to that of carbon dioxide. (b) Compare the answer from part (a) with the density of the earth's atmosphere during a spring day when the temperature is 18 °C and the pressure 101.6 kPa (abs).

(a) $$\rho_{Mars} = \frac{p}{RT} = \frac{900 \ \frac{N}{m^2}}{\left(188.9 \ \frac{J}{kg \cdot K}\right)\left[(-50°C + 273)K\right]} = \underline{\underline{0.0214 \ \frac{kg}{m^3}}}$$

(b) $$\rho_{earth} = \frac{p}{RT} = \frac{101.6 \times 10^3 \ \frac{N}{m^2}}{\left(286.9 \ \frac{J}{kg \cdot K}\right)\left[(18°C + 273)K\right]} = 1.22 \ \frac{kg}{m^3}$$

Thus,

$$\frac{\rho_{Mars}}{\rho_{earth}} = \frac{0.0214 \ \frac{kg}{m^3}}{1.22 \ \frac{kg}{m^3}} = 0.0175 = \underline{\underline{1.75\%}}$$

1.34

1.39

1.39 A closed tank having a volume of 2 ft^3 is filled with 0.30 lb of a gas. A pressure gage attached to the tank reads 12 psi when the gas temperature is 80 °F. There is some question as to whether the gas in the tank is oxygen or helium. Which do you think it is? Explain how you arrived at your answer.

Density of gas in tank $\rho = \dfrac{\text{weight}}{g \times \text{volume}} = \dfrac{0.30\ lb}{\left(32.2\ \frac{ft}{s^2}\right)(2\ ft^3)}$

$$= 4.66 \times 10^{-3}\ \frac{slugs}{ft^3}$$

Since $\rho = \dfrac{p}{RT}$ with $p = (12 + 14.7)\ psia$

(atmospheric pressure assumed to be $\approx$ 14.7 psia)

and with $T = (80°F + 460)\ °R$ it follows that

$$\rho = \frac{\left(26.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{R\ (540°R)} = \frac{7.12}{R}\ \frac{slugs}{ft^3} \qquad (1)$$

From Table 1.7 $R = 1.554 \times 10^3$ for oxygen

and $R = 1.242 \times 10^4\ \dfrac{ft \cdot lb}{slug \cdot °R}$ for helium.

Thus, from Eq. (1) if the gas is oxygen

$$\rho = \frac{7.12}{1.554 \times 10^3}\ \frac{slugs}{ft^3} = 4.58 \times 10^{-3}\ \frac{slugs}{ft^3}$$

and for helium

$$\rho = \frac{7.12}{1.242 \times 10^4} = 5.73 \times 10^{-4}\ \frac{slugs}{ft^3}$$

A comparison of these values with the actual density of the gas in the tank indicates that the gas must be <u>oxygen</u>.

1.40

1.40 A compressed air tank contains 5 kg of air at a temperature of 80 °C. A gage on the tank reads 300 kPa. Determine the volume of the tank.

$$\text{volume} = \frac{\text{mass}}{\rho}$$

$$\rho = \frac{p}{RT} = \frac{(300 + 101) \times 10^3 \ \frac{N}{m^2}}{\left(286.9 \ \frac{J}{kg \cdot K}\right)\left[(80°C + 273)K\right]} = 3.96 \ \frac{kg}{m^3}$$

$$\text{volume} = \frac{5 \ kg}{3.96 \ \frac{kg}{m^3}} = \underline{\underline{1.26 \ m^3}}$$

1.41

1.41 A rigid tank contains air at a pressure of 90 psia and a temperature of 60 °F. By how much will the pressure increase as the temperature is increased to 110 °F?

$$p = \rho R T \qquad (\text{Eq. } 1.8)$$

For a rigid closed tank the air mass and volume are constant so ρ = constant. Thus, from Eq. 1.8 (with R constant)

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad (1)$$

where $p_1 = 90$ psia, $T_1 = 60°F + 460 = 520°R$, and $T_2 = 110°F + 460 = 570°R$. From Eq. (1)

$$p_2 = \frac{T_2}{T_1} p_1 = \left(\frac{570°R}{520°R}\right)(90 \ psia) = \underline{\underline{98.7 \ psia}}$$

1.42

1.42 The helium-filled blimp shown in Fig. P1.42 is used at various athletic events. Determine the number of pounds of helium within it if its volume is 68,000 ft^3 and the temperature and pressure are 80 °F and 14.2 psia, respectively.

FIGURE P1.42

$\mathcal{W} = \gamma \mathcal{V}$ where $\mathcal{V} = 68{,}000\,ft^3$ and $\gamma = \rho g = (p/RT)g$

Thus,

$$\gamma = \left[14.2\,\frac{lb}{in.^2}\left(144\,\frac{in.^2}{ft^2}\right) \Big/ \left(\left(1.242\times10^4\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)(80+460)\,{}^\circ R\right)\right]\left(32.2\,\frac{ft}{s^2}\right)$$

$$= 9.82\times10^{-3}\,\frac{slug}{ft^2\cdot s^2}\left(1\,lb/(slug\,ft/s^2)\right) = 9.82\times10^{-3}\,\frac{lb}{ft^3}$$

Hence,

$$\mathcal{W} = 9.82\times10^{-3}\,\frac{lb}{ft^3}\left(68{,}000\,ft^3\right) = \underline{\underline{668\,lb}}$$

*1.43

*1.43 Develop a computer program for calculating the density of an ideal gas when the gas pressure in pascals (abs), the temperature in degrees Celsius, and the gas constant in J/kg · K are specified. Plot the density of helium as a function of temperature from 0 °C to 200 °C and pressures of 50, 100, 150, and 200 kPa (abs).

For an ideal gas

$$p = \rho R T$$

so that

$$\rho = \frac{p}{RT}$$

where p is absolute pressure, R the gas constant, and T is absolute temperature. Thus, if the temperature is in °C then

$$T = °C + 273.15$$

A spreadsheet (EXCEL) program for calculating ρ follows.

This program calculates the density of an ideal gas
when the absolute pressure in Pascals, the temperature
in degrees C, and the gas constant in J/kg-K are specified.
To use, replace current values with desired values of
temperature, pressure, and gas constant.

A	B	C	D	
Pressure,	Temperature,	Gas constant,	Density,	
Pa	°C	J/kg-K	kg/m³	
1.01E+05	15	286.9	1.23	Row 10

Formula:
=A10/((B10+273.15)*C10)

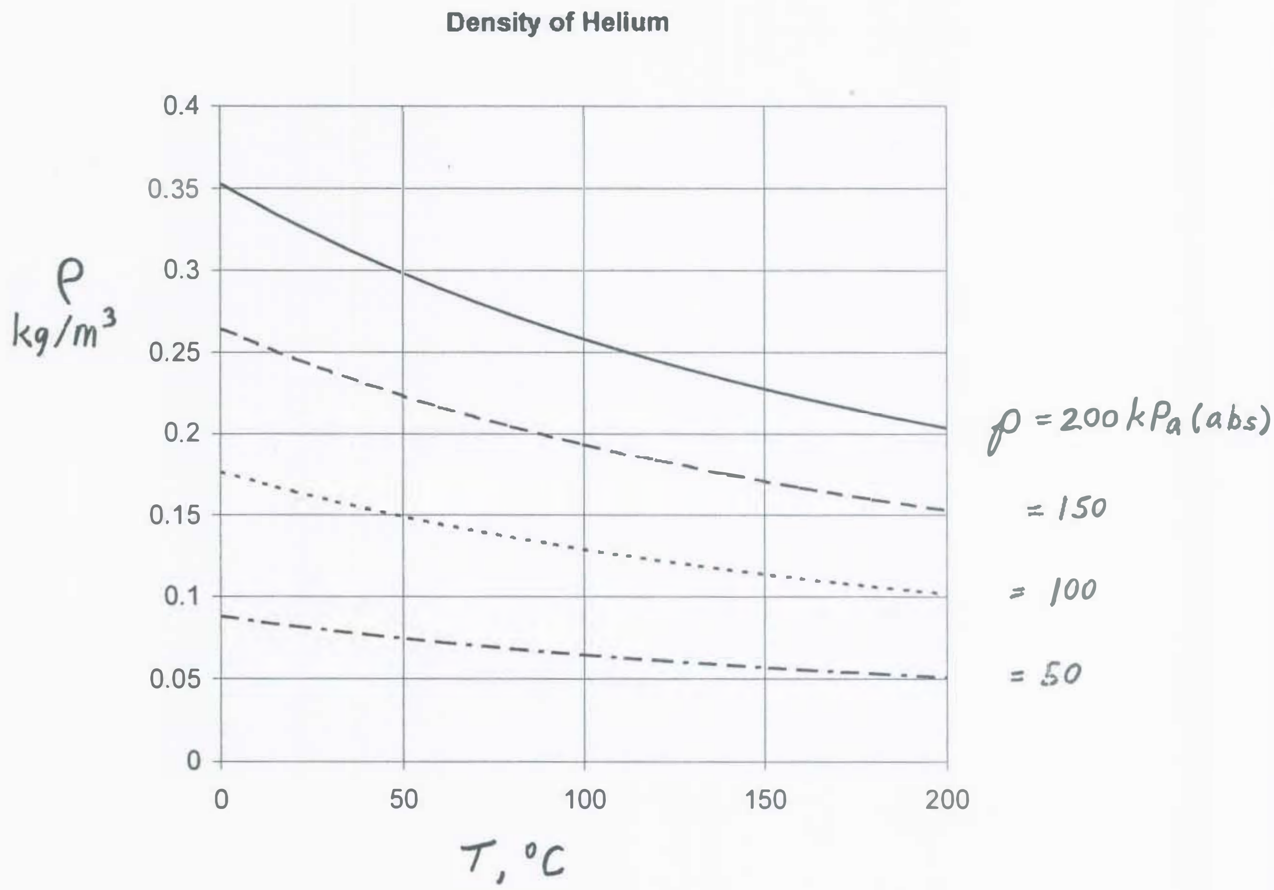
Example: Calculate ρ for $p = 200\ kPa$, temperature = 20°C, and $R = 287\ J/kg \cdot K$.

A	B	C	D	
Pressure,	Temperature,	Gas constant,	Density,	
Pa	°C	J/kg-K	kg/m³	
2.00E+05	20	287	2.38	Row 10

(con't)

*1.43 (con't)

The density of helium is plotted in the graph below.



1.45

1.45 For flowing water, what is the magnitude of the velocity gradient needed to produce a shear stress of 1.0 N/m^2?

$$\tau = \mu \frac{du}{dy} \quad \text{where } \mu = 1.12 \times 10^{-3} \frac{N \cdot s}{m^2} \text{ and } \tau = 1.0 \frac{N}{m^2}$$

Thus,

$$\frac{du}{dy} = \frac{\tau}{\mu} = \frac{1.0 \frac{N}{m^2}}{1.12 \times 10^{-3} \frac{N \cdot s}{m^2}} = \underline{\underline{893 \frac{1}{s}}}$$

1.46

1.46 Make use of the data in Appendix B to determine the dynamic viscosity of glycerin at 85 °F. Express your answer in both SI and BG units.

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(85°F - 32) = 29.4\,°C$$

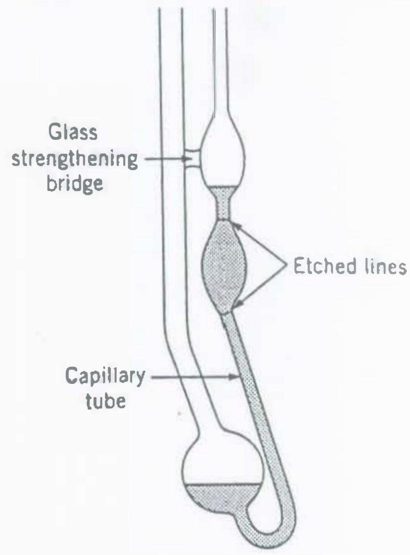
From Fig. B.1 in Appendix B:

$$\mu \text{ (glycerin at } 85°F\ (29.4°C)) \approx \underline{\underline{0.6 \frac{N \cdot s}{m^2}}} \quad \text{(SI units)}$$

$$\mu \approx \left(0.6 \frac{N \cdot s}{m^2}\right)\left(2.089 \times 10^{-2} \frac{\frac{lb \cdot s}{ft^2}}{\frac{N \cdot s}{m^2}}\right) \approx \underline{\underline{1.3 \times 10^{-2} \frac{lb \cdot s}{ft^2}}} \quad \text{(BG units)}$$

1.47

1.47 One type of *capillary-tube viscometer* is shown in Video V1.5 and in Fig. P1.47. For this device the liquid to be tested is drawn into the tube to a level above the top etched line. The time is then obtained for the liquid to drain to the bottom etched line. The kinematic viscosity, ν, in m^2/s is then obtained from the equation $\nu = KR^4t$ where K is a constant, R is the radius of the capillary tube in mm, and t is the drain time in seconds. When glycerin at 20°C is used as a calibration fluid in a particular viscometer the drain time is 1,430 s. When a liquid having a density of 970 kg/m^3 is tested in the same viscometer the drain time is 900 s. What is the dynamic viscosity of this liquid?



■ FIGURE P1.47

$$\nu = KR^4 t$$

For glycerin @ 20°C $\nu = 1.19 \times 10^{-3}\ m^2/s$

$$\therefore\ 1.19 \times 10^{-3}\ m^2/s = (KR^4)(1{,}430\ s)$$

$$KR^4 = 8.32 \times 10^{-7}\ m^2/s^2$$

For unknown liquid with $t = 900s$

$$\nu = (8.32 \times 10^{-7}\ m^2/s^2)(900\ s)$$

$$= 7.49 \times 10^{-4}\ m^2/s$$

Since $\mu = \rho\nu$

$$= (970\ kg/m^3)(7.49 \times 10^{-4}\ m^2/s)$$

$$= 0.727\ \frac{kg}{m \cdot s} = \underline{\underline{0.727\ \frac{N \cdot s}{m^2}}}$$

1.48

1.48 The viscosity of a soft drink was determined by using a capillary tube viscometer similar to that shown in Fig. P1.47 and Video V1.5. For this device the kinematic viscosity, ν, is directly proportional to the time, t, that it takes for a given amount of liquid to flow through a small capillary tube. That is, $\nu = Kt$. The following data were obtained from regular pop and diet pop. The corresponding measured specific gravities are also given. Based on these data, by what percent is the absolute viscosity, μ, of regular pop greater than that of diet pop?

	Regular pop	Diet pop
t(s)	377.8	300.3
SG	1.044	1.003

$$\% \text{ greater} = \left[\frac{\mu_{reg} - \mu_{diet}}{\mu_{diet}}\right] \times 100 = \left[\frac{\mu_{reg}}{\mu_{diet}} - 1\right] \times 100$$

Since $\nu = \mu/\rho$, $\nu = Kt$, and $\rho = (SG)\rho_{H_2O@4°C}$ it follows that

$$\% \text{ greater} = \left[\frac{(\nu\rho)_{reg}}{(\nu\rho)_{diet}} - 1\right] \times 100$$

$$= \left[\frac{(t \times SG)_{reg}}{(t \times SG)_{diet}} - 1\right] \times 100$$

$$= \left[\frac{(377.8\text{ s})(1.044)}{(300.3\text{ s})(1.003)} - 1\right] \times 100$$

$$= \underline{\underline{31.0\%}}$$

1.49

1.49 Determine the ratio of the dynamic viscosity of water to air at a temperature of 60 °C. Compare this value with the corresponding ratio of kinematic viscosities. Assume the air is at standard atmospheric pressure.

From Table B.2 in Appendix B:

(for water at 60°C) $\mu = 4.665 \times 10^{-4}\ \frac{N \cdot s}{m^2}$; $\nu = 4.745 \times 10^{-7}\ \frac{m^2}{s}$

From Table B.4 in Appendix B:

(for air at 60°C) $\mu = 1.97 \times 10^{-5}\ \frac{N \cdot s}{m^2}$; $\nu = 1.86 \times 10^{-5}\ \frac{m^2}{s}$

Thus,

$$\frac{\mu_{H_2O}}{\mu_{air}} = \frac{4.665 \times 10^{-4}}{1.97 \times 10^{-5}} = \underline{\underline{23.7}}$$

$$\frac{\nu_{H_2O}}{\nu_{air}} = \frac{4.745 \times 10^{-7}}{1.86 \times 10^{-5}} = \underline{\underline{2.55 \times 10^{-2}}}$$

1.50

1.50 The viscosity of a certain fluid is 5×10^{-4} poise. Determine its viscosity in both SI and BG units.

From Appendix E, $10^{-1}\ \frac{N\cdot s}{m^2} = 1$ poise. Thus,

$$\mu = (5\times10^{-4}\,\text{poise})\left(10^{-1}\ \frac{\frac{N\cdot s}{m^2}}{\text{poise}}\right) = \underline{\underline{5\times10^{-5}\ \frac{N\cdot s}{m^2}}}$$

and From Table 1.4

$$\mu = \left(5\times10^{-5}\ \frac{N\cdot s}{m^2}\right)\left(2.089\times10^{-2}\ \frac{\frac{lb\cdot s}{ft^2}}{\frac{N\cdot s}{m^2}}\right) = \underline{\underline{10.4\times10^{-7}\ \frac{lb\cdot s}{ft^2}}}$$

1.51

1.51 The kinematic viscosity of oxygen at 20 °C and a pressure of 150 kPa (abs) is 0.104 stokes. Determine the dynamic viscosity of oxygen at this temperature and pressure.

$$\mu = \nu\rho$$

$$\rho = \frac{p}{RT} = \frac{150\times10^3\ \frac{N}{m^2}}{\left(259.8\ \frac{J}{kg\cdot K}\right)\left[(20^\circ C + 273)K\right]} = 1.97\ \frac{kg}{m^3}$$

$$\nu = 0.104\ \text{stokes} = 0.104\ \frac{cm^2}{s}$$

$$\mu = \left(0.104\ \frac{cm^2}{s}\right)\left(10^{-4}\ \frac{m^2}{cm^2}\right)\left(1.97\ \frac{kg}{m^3}\right)$$

$$= 2.05\times10^{-5}\ \frac{kg}{m\cdot s} = \underline{\underline{2.05\times10^{-5}\ \frac{N\cdot s}{m^2}}}$$

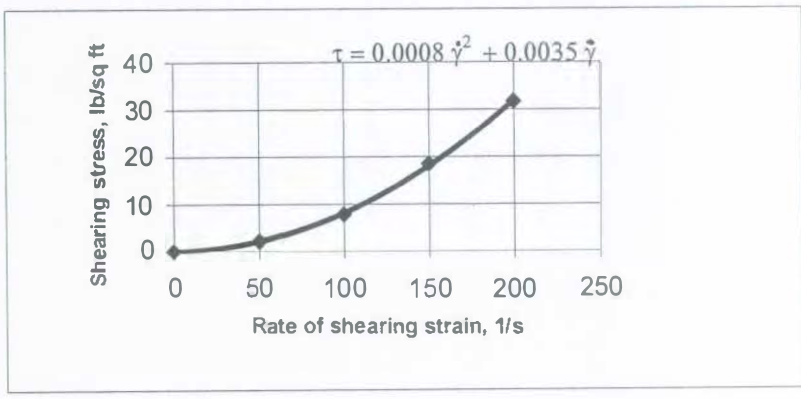
1.52

*1.52 Fluids for which the shearing stress, τ, is not linearly related to the rate of shearing strain, $\dot{\gamma}$, are designated as non-Newtonian fluids. Such fluids are commonplace and can exhibit unusual behavior as shown in Video V1.6. Some experimental data obtained for a particular non-Newtonian fluid at 80 °F are shown below.

τ (lb/ft^2)	0	2.11	7.82	18.5	31.7
$\dot{\gamma}$ (s^{-1})	0	50	100	150	200

Plot these data and fit a second-order polynomial to the data using a suitable graphing program. What is the apparent viscosity of this fluid when the rate of shearing strain is 70 s^{-1}? Is this apparent viscosity larger or smaller than that for water at the same temperature?

Rate of shearing strain, 1/s	Shearing stress, lb/sq ft
0	0
50	2.11
100	7.82
150	18.5
200	31.7



From the graph $\tau = 0.0008\,\dot{\gamma}^2 + 0.0035\,\dot{\gamma}$ where τ is the shearing stress in lb/ft^2 and $\dot{\gamma}$ is the rate of shearing strain in s^{-1}.

$$\mu_{apparent} = \frac{d\tau}{d\dot{\gamma}} = (2)(0.0008)\dot{\gamma} + 0.0035$$

At $\dot{\gamma} = 70\ s^{-1}$

$$\mu_{apparent} = (2)\left(0.0008\ \frac{lb\cdot s^2}{ft^2}\right)(70\ s^{-1}) + 0.0035\ \frac{lb\cdot s}{ft^2}$$

$$= \underline{\underline{0.116\ \frac{lb\cdot s}{ft^2}}}$$

From Table B.1 in Appendix B, $\mu_{H_2O\,@\,80°F} = 1.791\times10^{-5}\ \frac{lb\cdot s}{ft^2}$, and since water is a Newtonian fluid this value is independent of $\dot{\gamma}$. Thus, the unknown non-Newtonian fluid has a much larger value.

1.53

1.53 Water flows near a flat surface and some measurements of the water velocity, u, parallel to the surface, at different heights, y, above the surface are obtained. At the surface $y = 0$. After an analysis of the data, the lab technician reports that the velocity distribution in the range $0 < y < 0.1$ ft is given by the equation

$$u = 0.81 + 9.2y + 4.1 \times 10^3 y^3$$

with u in ft/s when y is in ft. (**a**) Do you think that this equation would be valid in any system of units? Explain. (**b**) Do you think this equation is correct? Explain. You may want to look at Video 1.4 to help you arrive at your answer.

(a)
$$u = 0.81 + 9.2\,y + 4.1 \times 10^3\, y^3$$
$$[LT^{-1}] \doteq [0.81] + [9.2][L] + [4.1 \times 10^3][L^3]$$

Each term in the equation must have the same dimensions. Thus, the constant 0.81 must have dimensions of LT^{-1}, 9.2 dimensions of T^{-1}, and 4.1×10^3 dimensions of $L^{-2}T^{-1}$. Since the constants in the equation have dimensions their values will change with a change in units. <u>No.</u>

(b) Equation cannot be correct since at $y = 0$ $u = 0.81$ ft/s, a non-zero value which would violate the "no-slip" condition. <u>Not correct.</u>

1.54

1.54 Calculate the Reynolds numbers for the flow of water and for air through a 4-mm-diameter tube, if the mean velocity is 3 m/s and the temperature is 30 °C in both cases (see Example 1.4). Assume the air is at standard atmospheric pressure.

For water at 30°C (from Table B.2 in Appendix B):

$$\rho = 995.7 \ \frac{kg}{m^3} \qquad \mu = 7.975 \times 10^{-4} \ \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho V D}{\mu} = \frac{\left(995.7 \ \frac{kg}{m^3}\right)\left(3 \ \frac{m}{s}\right)(0.004 \ m)}{7.975 \times 10^{-4} \ \frac{N \cdot s}{m^2}} = \underline{\underline{15{,}000}}$$

For air at 30°C (from Table B.4 in Appendix B):

$$\rho = 1.165 \ \frac{kg}{m^3} \qquad \mu = 1.86 \times 10^{-5} \ \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho V D}{\mu} = \frac{\left(1.165 \ \frac{kg}{m^3}\right)\left(3 \ \frac{m}{s}\right)(0.004 \ m)}{1.86 \times 10^{-5} \ \frac{N \cdot s}{m^2}} = \underline{\underline{752}}$$

1.49

1.55

1.55 For air at standard atmospheric pressure the values of the constants that appear in the Sutherland equation (Eq. 1.10) are $C = 1.458 \times 10^{-6}$ kg/(m·s·K$^{1/2}$) and $S = 110.4$ K. Use these values to predict the viscosity of air at 10 °C and 90 °C and compare with values given in Table B.4 in Appendix B.

$$\mu = \frac{C\,T^{\frac{3}{2}}}{T+S} = \frac{\left(1.458\times10^{-6}\ \frac{kg}{m\cdot s\cdot K^{1/2}}\right)T^{\frac{3}{2}}}{T + 110.4\ K}$$

For $T = 10°C = 10°C + 273.15 = 283.15\ K$,

$$\mu = \frac{(1.458\times10^{-6})(283.15\ K)^{3/2}}{283.15\ K + 110.4} = \underline{\underline{1.765\times10^{-5}\ \frac{N\cdot s}{m^2}}}$$

From Table B.4, $\mu = 1.76\times10^{-5}\ \frac{N\cdot s}{m^2}$

For $T = 90°C = 90°C + 273.15 = 363.15\ K$,

$$\mu = \frac{(1.458\times10^{-6})(363.15\ K)^{3/2}}{363.15\ K + 110.4} = \underline{\underline{2.13\times10^{-5}\ \frac{N\cdot s}{m^2}}}$$

From Table B.4, $\mu = 2.14\times10^{-5}\ \frac{N\cdot s}{m^2}$

1.50

1.56*

1.56* Use the values of viscosity of air given in Table B.4 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants C and S which appear in the Sutherland equation (Eq. 1.10). Compare your results with the values given in Problem 1.55. (*Hint*: Rewrite the equation in the form

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{C}\right)T + \frac{S}{C}$$

and plot $T^{3/2}/\mu$ versus T. From the slope and intercept of this curve C and S can be obtained.)

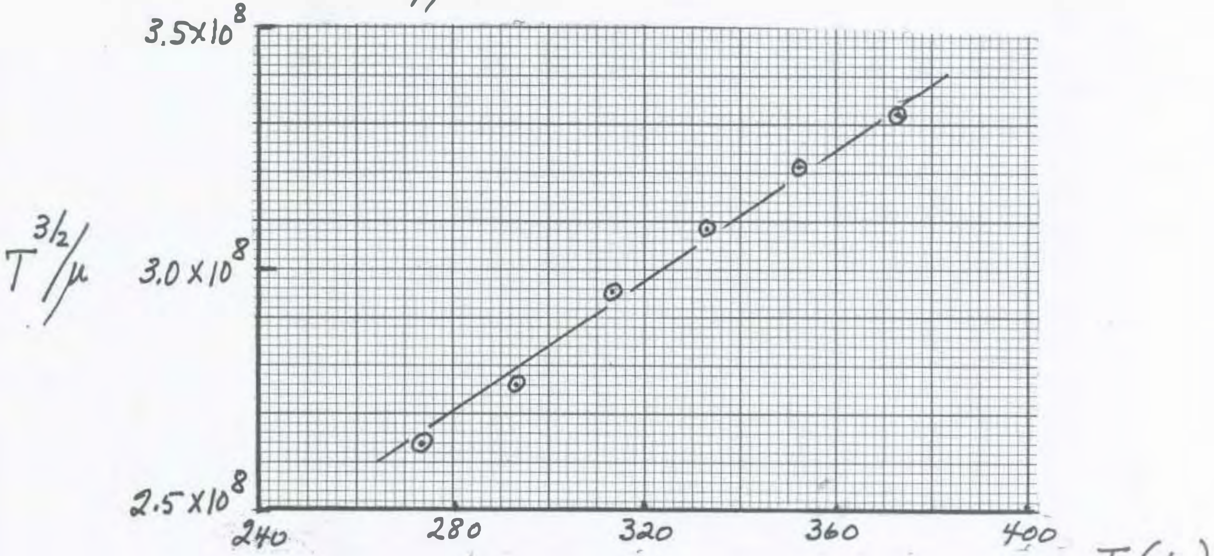
Equation 1.10 can be written in the form

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{C}\right)T + \frac{S}{C} \qquad (1)$$

and with the data from Table B.4:

$T(°C)$	$T(K)$	$\mu\ (N\cdot s/m^2)$	$T^{3/2}/\mu\ [K^{3/2}/(kg/m\cdot s)]$
0	273.15	1.71×10^{-5}	2.640×10^{8}
20	293.15	1.82×10^{-5}	2.758×10^{8}
40	313.15	1.87×10^{-5}	2.963×10^{8}
60	333.15	1.97×10^{-5}	3.087×10^{8}
80	353.15	2.07×10^{-5}	3.206×10^{8}
100	373.15	2.17×10^{-5}	3.322×10^{8}

A plot of $T^{3/2}/\mu$ vs. T is shown below:



(con't)

1.56* (Con't)

Since the data plot as an approximate straight line, Eq. (1) can be represented by an equation of the form

$$y = bx + a$$

where $y \sim T^{3/2}/\mu$, $x \sim T$, $b \sim 1/C$, and $a \sim S/C$.

Fit the data to a linear equation using a standard curve-fitting program such as found in EXCEL. Thus,

$$y = 6.969 \times 10^5 x + 7.441 \times 10^7$$

and

$$\frac{1}{C} = b = 6.969 \times 10^5$$

so that

$$\underline{\underline{C = 1.43 \times 10^{-6} \ kg/(m \cdot s \cdot K^{1/2})}}$$

and

$$\frac{S}{C} = a = 7.441 \times 10^7$$

and therefore

$$\underline{\underline{S = 107 \ K}}$$

These values for C and S are in good agreement with values given in Problem 1.55.

1.57

1.57 The viscosity of a fluid plays a very important role in determining how a fluid flows. (See Video V1.3) The value of the viscosity depends not only on the specific fluid but also on the fluid temperature. Some experiments show that when a liquid, under the action of a constant driving pressure, is forced with a low velocity, V, through a small horizontal tube, the velocity is given by the equation $V = K/\mu$. In this equation K is a constant for a given tube and pressure, and μ is the dynamic viscosity. For a particular liquid of interest, the viscosity is given by Andrade's equation (Eq. 1.11) with $D = 5 \times 10^{-7}\ \text{lb}\cdot\text{s/ft}^2$ and $B = 4000\ °R$. By what percentage will the velocity increase as the liquid temperature is increased from 40 °F to 100 °F? Assume all other factors remain constant.

$$V_{40°} = \frac{K}{\mu_{40°}} \quad (1)$$

$$V_{100°} = \frac{K}{\mu_{100°}} \quad (2)$$

$$\%\ \text{increase in } V = \left[\frac{V_{100°} - V_{40°}}{V_{40°}}\right] \times 100 = \left[\frac{V_{100°}}{V_{40°}} - 1\right] \times 100$$

and from Eq. (1) & (2)

$$\%\ \text{increase in } V = \left[\frac{K/\mu_{100°}}{K/\mu_{40°}} - 1\right] \times 100 = \left[\frac{\mu_{40°}}{\mu_{100°}} - 1\right] \times 100 \quad (3)$$

From Andrade's equation

$$\mu_{40°} = 5 \times 10^{-7}\, e^{\frac{4000}{(40°F + 460)}}$$

and

$$\mu_{100°} = 5 \times 10^{-7}\, e^{\frac{4000}{(100°F + 460)}}$$

Thus, from Eq. (3)

$$\%\ \text{increase in } V = \left[\frac{5 \times 10^{-7}\, e^{\frac{4000}{500}}}{5 \times 10^{-7}\, e^{\frac{4000}{560}}} - 1\right] \times 100$$

$$= \underline{\underline{136\%}}$$

1.52

*1.58

*1.58 Use the value of the viscosity of water given in Table B.2 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants D and B which appear in Andrade's equation (Eq. 1.11). Calculate the value of the viscosity at 50 °C and compare with the value given in Table B.2. (*Hint*: Rewrite the equation in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D$$

and plot $\ln \mu$ versus $1/T$. From the slope and intercept of this curve B and D can be obtained. If a nonlinear curve fitting program is available the constants can be obtained directly from Eq. 1.11 without rewriting the equation.)

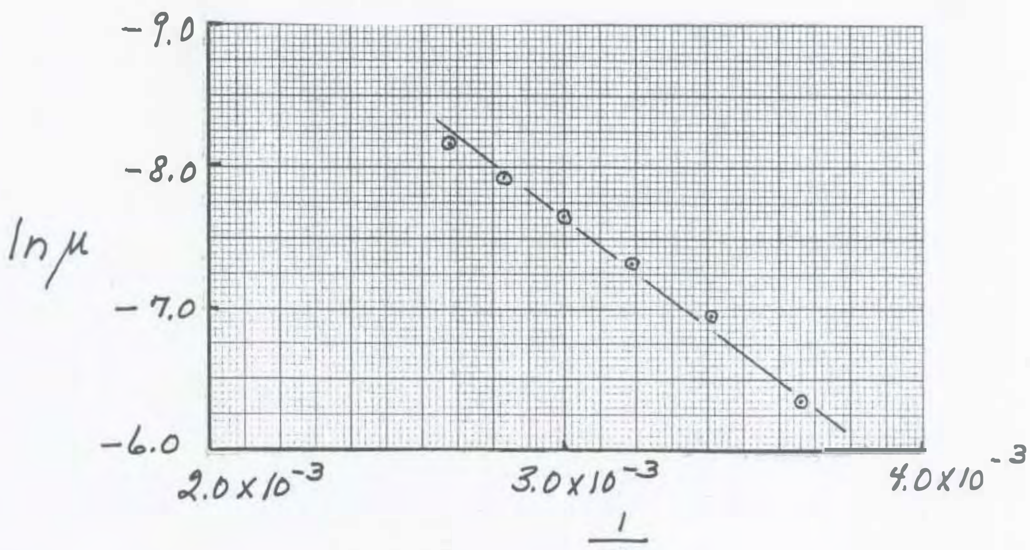
Equation 1.11 can be written in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D \qquad (1)$$

and with the data from Table B.2:

$T\,(°C)$	$T\,(K)$	$1/T\,(K)$	$\mu\ (N\cdot s/m^2)$	$\ln \mu$
0	273.15	3.661×10^{-3}	1.787×10^{-3}	-6.327
20	293.15	3.411×10^{-3}	1.002×10^{-3}	-6.906
40	313.15	3.193×10^{-3}	6.529×10^{-4}	-7.334
60	333.15	3.002×10^{-3}	4.665×10^{-4}	-7.670
80	353.15	2.832×10^{-3}	3.547×10^{-4}	-7.944
100	373.15	2.680×10^{-3}	2.818×10^{-4}	-8.174

A plot of $\ln \mu$ vs. $1/T$ is shown below:



(con't)

*1.58 (con't)

Since the data plot as an approximate straight line, Eq. (1) can be used to represent these data. To obtain B and D, fit the data to an exponential equation of the form $y = ae^{bx}$ such as found in EXCEL.

Thus,

$$\underline{\underline{D = a = 1.767 \times 10^{-6} \; N\cdot s/m^2}}$$

and

$$\underline{\underline{B = b = 1.870 \times 10^{3} \; K}}$$

so that

$$\mu = 1.767 \times 10^{-6} \, e^{\frac{1870}{T}}$$

At $50°C$ $(323.15K)$,

$$\mu = 1.767 \times 10^{-6} \, e^{\frac{1870}{323.15}} = \underline{\underline{5.76 \times 10^{-4} \; N\cdot s/m^2}}$$

From Table B.2, $\mu = 5.468 \times 10^{-4} \; N\cdot s/m^2$.

1.59

1.59 For a parallel plate arrangement of the type shown in Fig. 1.5 it is found that when the distance between plates is 2 mm, a shearing stress of 150 Pa develops at the upper plate when it is pulled at a velocity of 1 m/s. Determine the viscosity of the fluid between the plates. Express your answer in SI units.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{U}{b}$$

$$\mu = \frac{\tau}{\left(\frac{U}{b}\right)} = \frac{150 \frac{N}{m^2}}{\left(\frac{1 \frac{m}{s}}{0.002 m}\right)} = \underline{\underline{0.300 \frac{N \cdot s}{m^2}}}$$

1.60

1.60 Two flat plates are oriented parallel above a fixed lower plate as shown in Fig. P1.60. The top plate, located a distance b above the fixed plate, is pulled along with speed V. The other thin plate is located a distance cb, where $0 < c < 1$, above the fixed plate. This plate moves with speed V_1, which is determined by the viscous shear forces imposed on it by the fluids on its top and bottom. The fluid on the top is twice as viscous as that on the bottom. Plot the ratio V_1/V as a function of c for $0 < c < 1$.

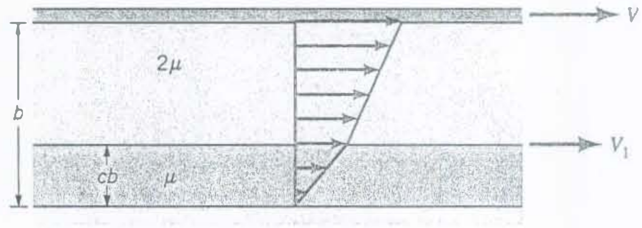


FIGURE P1.60

For constant speed, V_1, of the middle plate, the net force on the plate is 0. Hence, $F_{top} = F_{bottom}$, where $F = \tau A$.

Thus, the shear stress on the top and bottom of the plate must be equal.

$$\tau_{top} = \tau_{bottom} \quad \text{where} \quad \tau = \mu \frac{du}{dy} \qquad (1)$$

For the bottom fluid $\frac{du}{dy} = \frac{V_1}{cb}$, while for the top fluid $\frac{du}{dy} = \frac{(V - V_1)}{b - cb}$

Hence, from Eqn. (1),

$$(2\mu)\frac{(V - V_1)}{b(1-c)} = (\mu)\frac{V_1}{cb}$$, which can be written as:

$$2cV - 2cV_1 = V_1 - cV_1$$

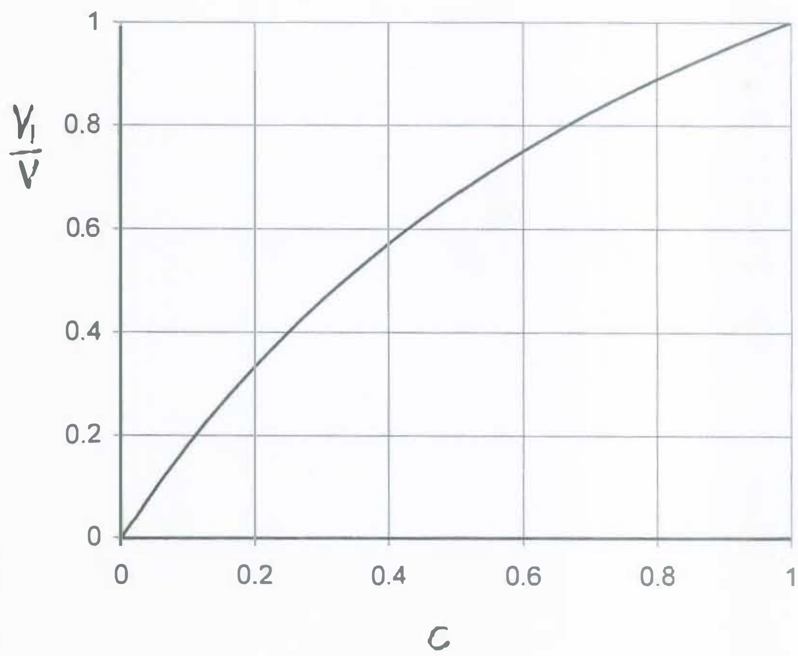
or

$$\frac{V_1}{V} = \underline{\underline{\frac{2c}{c+1}}}$$

Note: If $c = 0$, $\frac{V_1}{V} = 0$

If $c = \frac{1}{2}$, $\frac{V_1}{V} = \frac{2}{3}$

If $c = 1$, $\frac{V_1}{V} = 1$



1.61

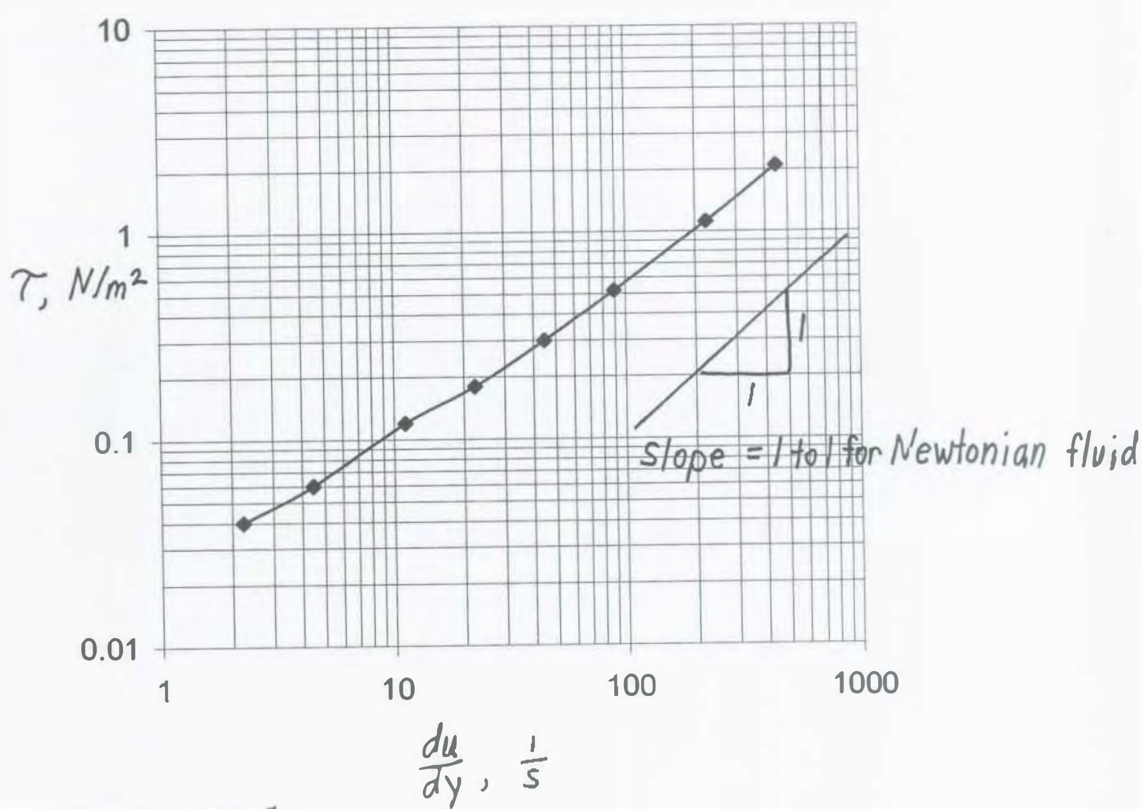
1.61 There are many fluids that exhibit non-Newtonian behavior (see, for example, **Video V1.6**). For a given fluid the distinction between Newtonian and non-Newtonian behavior is usually based on measurements of shear stress and rate of shearing strain. Assume that the viscosity of blood is to be determined by measurements of shear stress, τ, and rate of shearing strain, du/dy, obtained from a small blood sample tested in a suitable viscometer. Based on the data given below determine if the blood is a Newtonian or non-Newtonian fluid. Explain how you arrived at your answer.

τ (N/m^2)	0.04	0.06	0.12	0.18	0.30	0.52	1.12	2.10
du/dy (s^{-1})	2.25	4.50	11.25	22.5	45.0	90.0	225	450

For a Newtonian fluid the ratio of τ to du/dy is a constant. For the data given:

$\dfrac{\tau}{du/dy}$ $(N\cdot s/m^2)$	0.0178	0.0133	0.0107	0.0080	0.0067	0.0058	0.0050	0.0047

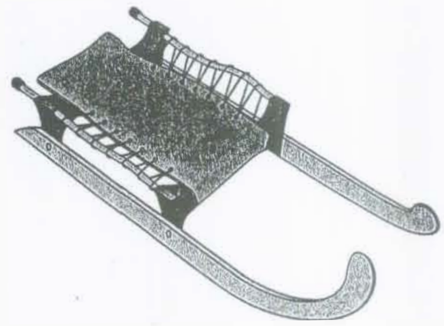
The ratio is not a constant but decreases as the rate of shearing strain increases. Thus, this fluid (blood) is a non-Newtonian fluid. A plot of the data is shown below. For a Newtonian fluid the curve would be a straight line with a slope of 1 to 1.



Note: $\tau = \mu\left(\frac{du}{dy}\right)^a$, where $a = 1$ for a Newtonian fluid.

1.62

1.62 The sled shown in Fig. P1.62 slides along on a thin horizontal layer of water between the ice and the runners. The horizontal force that the water puts on the runners is equal to 1.2 lb when the sled's speed is 50 ft/s. The total area of both runners in contact with the water is 0.08 ft^2, and the viscosity of the water is 3.5×10^{-5} lb s/ft^2. Determine the thickness of the water layer under the runners. Assume a linear velocity distribution in the water layer.



■ FIGURE P1.62

$$F \text{ (force)} = \tau A$$

$$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{d}$$ where d = thickness of water layer

Thus,

$$F = \mu \frac{V}{d} A$$

and

$$d = \frac{\mu V A}{F} = \frac{\left(3.5 \times 10^{-5} \frac{lb \cdot s}{ft^2}\right)\left(50 \frac{ft}{s}\right)\left(0.08\ ft^2\right)}{1.2\ lb}$$

$$= \underline{\underline{11.7 \times 10^{-4}\ ft}}$$

1.63

1.63 A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Fig. P1.63 The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of 8.0×10^{-4} m²/s and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.

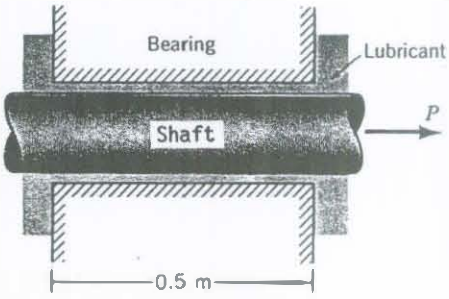
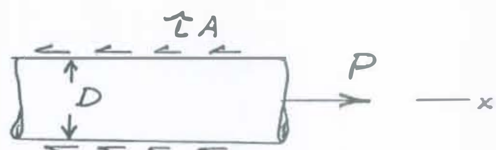


FIGURE P1.63



$\sum F_x = 0$

Thus, $P = \tau A$

where $A = \pi D \times (\text{shaft length in bearing}) = \pi D \ell$

and $\tau = \mu \dfrac{(\text{velocity of shaft})}{(\text{gap width})} = \mu \dfrac{V}{b}$

so that

$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell)$$

Since $\mu = \nu \rho = \nu (SG)(\rho_{H_2O @ 4°C})$,

$$P = \frac{\left(8.0 \times 10^{-4} \frac{m^2}{s}\right)\left(0.91 \times 10^3 \frac{kg}{m^3}\right)\left(3 \frac{m}{s}\right)(\pi)(0.025m)(0.5m)}{(0.0003m)}$$

$= \underline{\underline{286\ N}}$

1.64

1.64 A 10-kg block slides down a smooth inclined surface as shown in Fig. P1.64. Determine the terminal velocity of the block if the 0.1-mm gap between the block and the surface contains SAE 30 oil at 60 °F. Assume the velocity distribution in the gap is linear, and the area of the block in contact with the oil is 0.1 m².

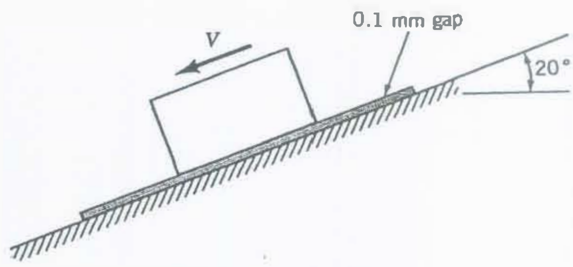
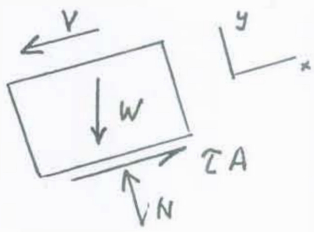


FIGURE P1.64



$$\sum F_x = 0$$

Thus,

$$W \sin 20° = \tau A$$

Since

$$\tau = \mu \frac{V}{b} \quad \text{, where } b \text{ is film thickness,}$$

$$W \sin 20° = \mu \frac{V}{b} A$$

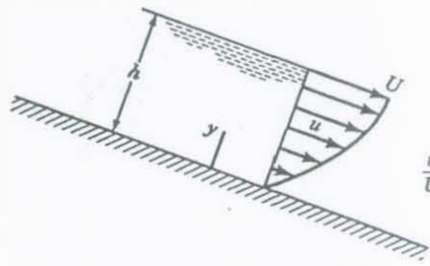
Thus, (with $W = mg$)

$$V = \frac{b\, W \sin 20°}{\mu A} = \frac{(0.0001\,m)(10\,kg)(9.81\,\frac{m}{s^2})(\sin 20°)}{(0.38\,\frac{N \cdot s}{m^2})(0.1\,m^2)}$$

$$= \underline{\underline{0.0883\,\frac{m}{s}}}$$

1.65

1.65 A layer of water flows down an inclined fixed surface with the velocity profile shown in Fig. P1.65. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for $U = 2$ m/s and $h = 0.1$ m.



$$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$$

FIGURE P1.65

$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = U\left(\frac{2}{h} - \frac{y^2}{h^2}\right)$$

Thus, at the fixed surface $(y=0)$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{h}$$

so that

$$\tau = \mu\left(\frac{2U}{h}\right) = \left(1.12\times10^{-3}\ \frac{N\cdot s}{m^2}\right)(2)\frac{\left(2\ \frac{m}{s}\right)}{(0.1\,m)}$$

$$= 4.48\times10^{-2}\ \frac{N}{m^2}$$ acting in direction of flow

*1.66

***1.66** Standard air flows past a flat surface and velocity measurements near the surface indicate the following distribution:

y (ft)	0.005	0.01	0.02	0.04	0.06	0.08
u (ft/s)	0.74	1.51	3.03	6.37	10.21	14.43

The coordinate y is measured normal to the surface and u is the velocity parallel to the surface. **(a)** Assume the velocity distribution is of the form

$$u = C_1 y + C_2 y^3$$

and use a standard curve-fitting technique to determine the constants C_1 and C_2. **(b)** Make use of the results of part (a) to determine the magnitude of the shearing stress at the wall ($y = 0$) and at $y = 0.05$ ft.

(a) Use nonlinear regression program to obtain coefficients C_1 and C_2. The program produces least squares estimates of the parameters of a nonlinear model. For the data given,

$$\underline{\underline{C_1 = 153\ s^{-1}}} \quad \text{and} \quad \underline{\underline{C_2 = 4350\ ft^{-2}s^{-1}}}$$

(b) Since,

$$\tau = \mu \frac{du}{dy}$$

it follows that

$$\tau = \mu\left(C_1 + 3C_2 y^2\right)$$

Thus, at the wall ($y=0$)

$$\tau = \mu C_1 = \left(3.74 \times 10^{-7}\ \frac{lb \cdot s}{ft^2}\right)\left(153\ \frac{1}{s}\right) = \underline{\underline{5.72 \times 10^{-5}\ \frac{lb}{ft^2}}}$$

At $y = 0.05$ ft

$$\tau = \left(3.74 \times 10^{-7}\ \frac{lb \cdot s}{ft^2}\right)\left[153\ \frac{1}{s} + 3\left(4350\ \frac{1}{ft^2 \cdot s}\right)(0.05\ ft)^2\right]$$

$$= \underline{\underline{6.94 \times 10^{-5}\ \frac{lb}{ft^2}}}$$

1.67

1.67 A new computer drive is proposed to have a disc, as shown in Fig. P1.67. The disc is to rotate at 10,000 rpm, and the reader head is to be positioned 0.0005 in. above the surface of the disc. Estimate the shearing force on the reader head as result of the air between the disc and the head.

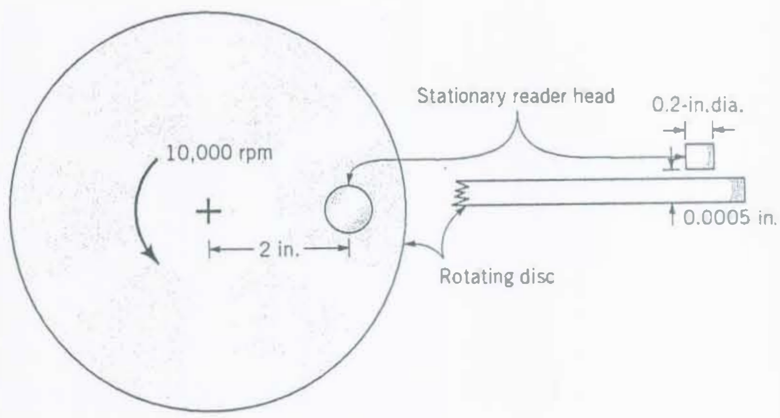


FIGURE P1.67

F = shear force on head $= \tau A$, where, if the velocity profile in the gap between the disc and head is linear and uniform across the head, then

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{b}, \text{ where}$$

$$U = \omega R = 10{,}000 \frac{rev}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{rev}\right)\left(\frac{2}{12}\,ft\right) = 175\,\frac{ft}{s}$$

Thus,

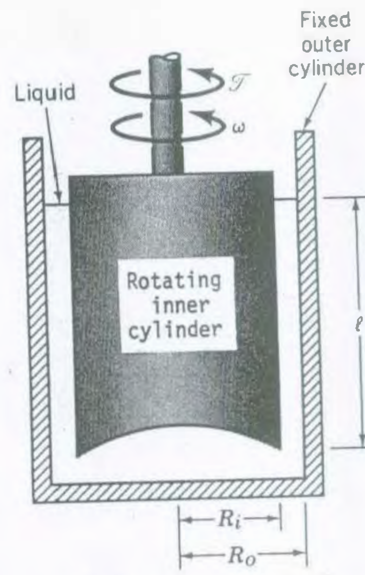
$$\tau = \left(3.74\times10^{-7}\,\frac{lb\cdot s}{ft^2}\right)\frac{175\,\frac{ft}{s}}{\left(\frac{0.005}{12}\,ft\right)} = 1.57\,\frac{lb}{ft^2}$$

so that

$$F = \tau A = \left(1.57\,\frac{lb}{ft^2}\right)\frac{\pi}{4}\left(\frac{0.2}{12}\,ft\right)^2 = \underline{\underline{3.43\times10^{-4}\,lb}}$$

1.68

1.68 The space between two 6-in.-long concentric cylinders is filled with glycerin (viscosity = 8.5×10^{-3} lb · s/ft²). The inner cylinder has a radius of 3 in. and the gap width between cylinders is 0.1 in. Determine the torque and the power required to rotate the inner cylinder at 180 rev/min. The outer cylinder is fixed. Assume the velocity distribution in the gap to be linear.



Torque, $d\mathcal{T}$, due to shearing stress on inner cylinder is equal to

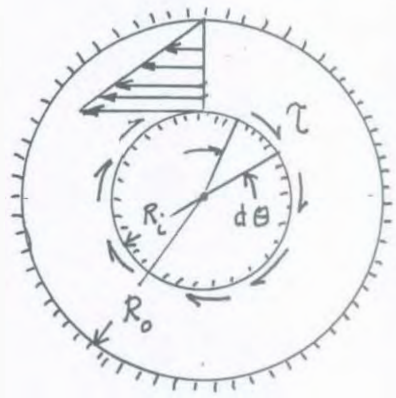
$$d\mathcal{T} = R_i \, \tau \, dA$$

where $dA = (R_i \, d\theta)\ell$. Thus,

$$d\mathcal{T} = R_i^2 \ell \, \tau \, d\theta$$

and torque required to rotate inner cylinder is

$$\mathcal{T} = R_i^2 \ell \, \tau \int_0^{2\pi} d\theta = 2\pi R_i^2 \ell \, \tau$$



top view

(ℓ ~ cylinder length)

For a linear velocity distribution in the gap

$$\tau = \mu \frac{R_i \omega}{R_o - R_i} \quad \text{so that}$$

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu \omega}{R_o - R_i}$$

and with $\omega = \left(180 \frac{\text{rev}}{\text{min}}\right)\left(2\pi \frac{\text{rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 6\pi \frac{\text{rad}}{\text{s}}$

then

$$\mathcal{T} = \frac{2\pi \left(\frac{3}{12}\text{ft}\right)^3 \left(\frac{6}{12}\text{ft}\right)\left(8.5\times10^{-3}\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(6\pi \frac{\text{rad}}{\text{s}}\right)}{\left(\frac{0.1}{12}\text{ft}\right)} = \underline{\underline{0.944 \text{ ft}\cdot\text{lb}}}$$

Since $\text{power} = \mathcal{T} \times \omega$ it follows that

$$\text{power} = (0.944 \text{ ft}\cdot\text{lb})\left(6\pi \frac{\text{rad}}{\text{s}}\right) = \underline{\underline{17.8 \frac{\text{ft}\cdot\text{lb}}{\text{s}}}}$$

1.69

1.69 A pivot bearing used on the shaft of an electrical instrument is shown in Fig. P1.69. An oil with a viscosity of $\mu = 0.010 \text{ lb}\cdot\text{s/ft}^2$ fills the 0.001-in. gap between the rotating shaft and the stationary base. Determine the frictional torque on the shaft when it rotates at 5,000 rpm.

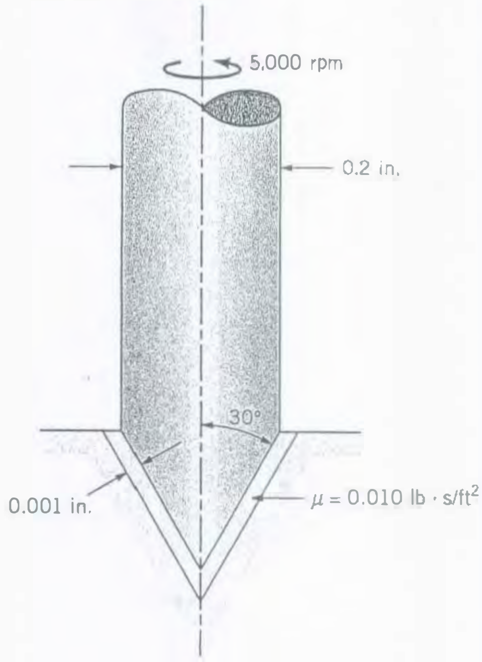
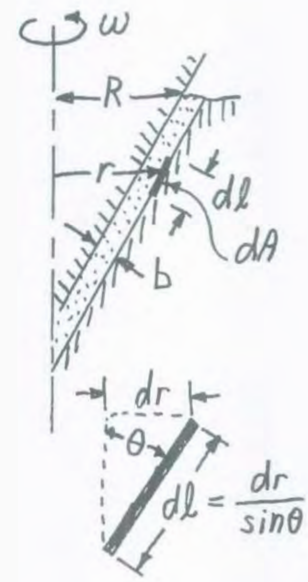


FIGURE P1.69



Let $d\mathcal{T}$ = torque on area element dA,
where $dA = 2\pi r\, d\ell = 2\pi r\, dr/\sin\theta$

Thus,

$$d\mathcal{T} = r\, dF = r\,\tau\, dA \quad \text{where} \quad \tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{b}$$

so that,

$$d\mathcal{T} = r\left(\mu \frac{\omega r}{b}\right)\left(2\pi r\, dr/\sin\theta\right)$$

$$= \frac{2\pi \mu \omega}{b \sin\theta}\, r^3 dr$$

Hence,

$$\mathcal{T} = \int d\mathcal{T} = \frac{2\pi\mu\omega}{b\sin\theta}\int_{r=0}^{r=R} r^3 dr = \frac{\pi\mu\omega}{2 b \sin\theta} R^4 \qquad (1)$$

Now,

$R = 0.1 \text{ in.}$, $b = 0.001 \text{ in.}$, $\mu = 0.010 \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}$, $\theta = 30 \text{deg}$, and

$$\omega = 5{,}000 \frac{\text{rev}}{\text{min}}\left(\frac{\text{min}}{60\text{ s}}\right)\left(2\pi \frac{\text{rad}}{\text{rev}}\right) = 524 \frac{\text{rad}}{\text{s}}$$

Thus, from Eq. (1),

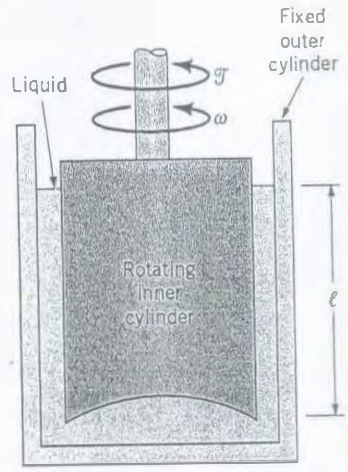
$$\mathcal{T} = \frac{\pi\left(0.010 \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(524 \frac{\text{rad}}{\text{s}}\right)}{2\left(\frac{0.001}{12}\text{ft}\right)\sin 30^\circ}\left(\frac{0.1}{12}\text{ft}\right)^4 = \underline{\underline{9.53\times10^{-4}\ \text{ft}\cdot\text{lb}}}$$

1.70

1.70 The viscosity of liquids can be measured through the use of a *rotating cylinder viscometer* of the type illustrated in Fig. P1.70. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The torque $\mathcal{T}$ required to develop ω is measured and the viscosity is calculated from these two measurements. **(a)** Develop an equation relating μ, ω, $\mathcal{T}$, ℓ, R_o, and R_i. Neglect end effects and assume the velocity distribution in the gap is linear. **(b)** The following torque-angular velocity data were obtained with a rotating cylinder viscometer of the type discussed in part (a).

Torque (ft · lb)	13.1	26.0	39.5	52.7	64.9	78.6
Angular velocity (rad/s)	1.0	2.0	3.0	4.0	5.0	6.0

For this viscometer $R_o = 2.50$ in., $R_i = 2.45$ in., and $\ell = 5.00$ in. Make use of these data and a standard curve-fitting program to determine the viscosity of the liquid contained in the viscometer.



■ FIGURE P1.70

(a) Torque, $d\mathcal{T}$, due to shearing stress on inner cylinder is equal to

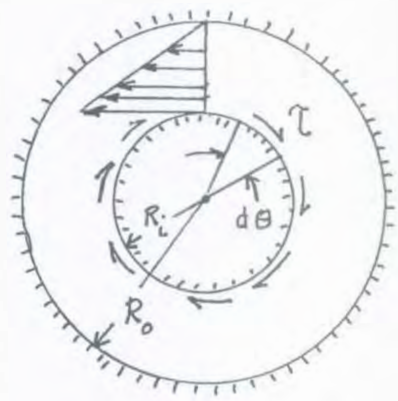
$$d\mathcal{T} = R_i \, \tau \, dA$$

where $dA = (R_i \, d\theta)\,\ell$. Thus,

$$d\mathcal{T} = R_i^2 \ell \, \tau \, d\theta$$

and torque required to rotate inner cylinder is

$$\mathcal{T} = R_i^2 \ell \, \tau \int_0^{2\pi} d\theta = 2\pi R_i^2 \ell \, \tau$$



top view

(ℓ ~ cylinder length)

For a linear velocity distribution in the gap

$$\tau = \mu \frac{R_i \omega}{R_o - R_i} \quad \text{so that}$$

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu \omega}{R_o - R_i} \qquad (1)$$

(b) Thus, for a fixed geometry and a given viscosity, Eq. (1) is of the form

$$y = bx \qquad (y \sim \mathcal{T} \text{ and } x \sim \omega)$$

where b is a constant equal to

(con't)

1.70 (con't)

$$b = \frac{2\pi R_i^3 \ell \mu}{R_o - R_i} \qquad (2)$$

To obtain b fit the data to a linear equation of the form $y = bx$ using a standard curve-fitting program such as found in EXCEL.

Thus, from Eq. (2)

$$\mu = \frac{(b)(R_o - R_i)}{2\pi R_i^3 \ell}$$

and with the data given, $b = 13.08 \text{ ft·lb·s}$, so that

$$\mu = \frac{(13.08 \text{ ft·lb·s})\left(\frac{2.50 - 2.45}{12} \text{ ft}\right)}{2\pi \left(\frac{2.45}{12} \text{ ft}\right)^3 \left(\frac{5.00}{12} \text{ ft}\right)} = \underline{\underline{2.45 \ \frac{\text{lb·s}}{\text{ft}^2}}}$$

1.71

1.71 A 12-in.-diameter circular plate is placed over a fixed bottom plate with a 0.1-in. gap between the two plates filled with glycerin as shown in Fig. P1.71. Determine the torque required to rotate the circular plate slowly at 2 rpm. Assume that the velocity distribution in the gap is linear and that the shear stress on the edge of the rotating plate is negligible.

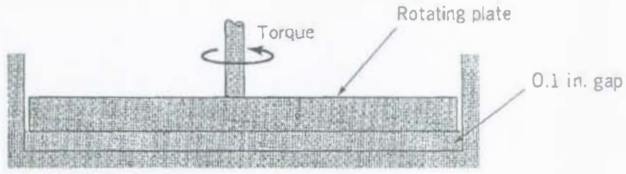


FIGURE P1.71

Torque, $d\mathcal{T}$, due to shearing stresses on plate is equal to

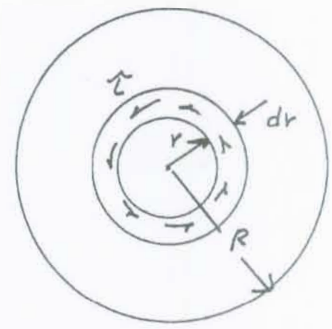
$$d\mathcal{T} = r\,\tau\,dA$$

where $dA = 2\pi r\,dr$. Thus,

$$d\mathcal{T} = r\,\tau\,2\pi r\,dr$$

and

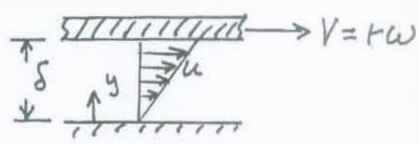
$$\mathcal{T} = 2\pi\int_0^R r^2\tau\,dr$$



stresses acting on bottom of plate

Since $\tau = \mu\dfrac{du}{dy}$, and for a linear velocity distribution (see figure)

$$\tau = \mu\frac{r\omega}{\delta}$$



$$\frac{du}{dy} = \frac{V}{\delta} = \frac{r\omega}{\delta}$$

velocity distribution

Thus,

$$\mathcal{T} = \frac{2\pi\mu\omega}{\delta}\int_0^R r^3\,dr = \frac{2\pi\mu\omega}{\delta}\left(\frac{R^4}{4}\right)$$

and with the data given

$$\mathcal{T} = \frac{2\pi\left(0.0313\,\frac{lb\cdot s}{ft^2}\right)\left(2\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{6}{12}\,ft\right)^4}{\left(\frac{0.1}{12}\,ft\right)(4)}$$

$$= \underline{\underline{0.0772\ ft\cdot lb}}$$

1.73

1.73 Some measurements on a blood sample at 37 °C (98.6 °F) indicate a shearing stress of 0.52 N/m^2 for a corresponding rate of shearing strain of 200 s^{-1}. Determine the apparent viscosity of the blood and compare it with the viscosity of water at the same temperature.

$$\tau = \mu \frac{du}{dy} = \mu \dot{\gamma}$$

$$\mu_{blood} = \frac{\tau}{\dot{\gamma}} = \frac{0.52 \frac{N}{m^2}}{200 \frac{1}{s}} = \underline{\underline{26.0 \times 10^{-4} \frac{N \cdot s}{m^2}}}$$

From Table B.2 in Appendix B:

@ 30°C $\mu_{H_2O} = 7.975 \times 10^{-4} \frac{N \cdot s}{m^2}$

@ 40°C $\mu_{H_2O} = 6.529 \times 10^{-4} \frac{N \cdot s}{m^2}$

Thus, with linear interpolation, $\mu_{H_2O}(37°C) = 6.96 \times 10^{-4} \frac{N \cdot s}{m^2}$

and

$$\frac{\mu_{blood}}{\mu_{H_2O}} = \frac{26.0 \times 10^{-4} \frac{N \cdot s}{m^2}}{6.96 \times 10^{-4} \frac{N \cdot s}{m^2}} = \underline{\underline{3.74}}$$

1.75

1.75 A sound wave is observed to travel through a liquid with a speed of 1500 m/s. The specific gravity of the liquid is 1.5. Determine the bulk modulus for this fluid.

$c = \sqrt{\frac{E_v}{\rho}}$, where $\rho = SG\,\rho_{H_2O}$ and $SG = 1.5$

Thus,

$$E_v = c^2 \rho = c^2\, SG\, \rho_{H_2O}$$

$$= \left(1500\,\frac{m}{s}\right)^2 (1.5)\left(999\,\frac{kg}{m^3}\right)$$

$$= 3.37 \times 10^9\ \frac{kg \cdot m}{s^2\, m^2}$$

or

$$\underline{\underline{E_v = 3.37 \times 10^9\ \frac{N}{m^2}}}$$

1.76

1.76 Estimate the increase in pressure (in psi) required to decrease a unit volume of mercury by 0.1%.

$$E_v = -\frac{dp}{d\forall/\forall} \qquad (\text{Eq. } 1.12)$$

Thus,

$$\Delta p \approx -\frac{E_v \, \Delta\forall}{\forall} = -\left(4.14 \times 10^6 \ \frac{\text{lb}}{\text{in.}^2}\right)(-0.001)$$

$$\Delta p \approx \underline{\underline{4.14 \times 10^3 \ \text{psi}}}$$

1.77

1.77 A 1-m^3 volume of water is contained in a rigid container. Estimate the change in the volume of the water when a piston applies a pressure of 35 MPa.

$$E_v = -\frac{dp}{d\forall/\forall} \qquad (\text{Eq. } 1.12)$$

Thus,

$$\Delta\forall \approx -\frac{\forall \, \Delta p}{E_v} = -\frac{(1 \ \text{m}^3)\left(35 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)}{2.15 \times 10^9 \ \frac{\text{N}}{\text{m}^2}} = -0.0163 \ \text{m}^3$$

or

$$\underline{\underline{\text{decrease}}} \text{ in volume} \approx \underline{\underline{0.0163 \ \text{m}^3}}$$

1.78

1.78 Determine the speed of sound at 20 °C in **(a)** air, **(b)** helium, and **(c)** natural gas. Express your answer in m/s.

$$c = \sqrt{kRT} \qquad (\text{Eq. } 1.20)$$

With $T = 20°C + 273 = 293\,K$:

(a) For air, $c = \sqrt{(1.40)\left(286.9 \frac{J}{kg \cdot K}\right)(293\,K)} = \underline{\underline{343 \frac{m}{s}}}$

(b) For helium, $c = \sqrt{(1.66)\left(2077 \frac{J}{kg \cdot K}\right)(293\,K)} = \underline{\underline{1010 \frac{m}{s}}}$

(c) For natural gas, $c = \sqrt{(1.31)\left(518.3 \frac{J}{kg \cdot K}\right)(293\,K)} = \underline{\underline{446 \frac{m}{s}}}$

1.70

1.79

1.79 Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 25 psi. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process to be isothermal and the local atmospheric pressure to be 14.7 psi.

For isothermal compression, $\frac{p}{\rho} = \text{constant}$ so that

$$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus, $$p_f = \frac{\rho_f}{\rho_i} p_i$$

Since $\rho = \frac{\text{mass}}{\text{volume}}$, $\frac{\rho_f}{\rho_i} = \frac{\text{initial volume}}{\text{final volume}} = 3$ (for constant mass)

and therefore

$$p_f = (3)\left[(25 + 14.7)\ \text{psi (abs)}\right] = 119 \quad \text{psi (abs)}$$

or

$$p_f\ (\text{gage}) = (119 - 14.7)\ \text{psi} = \underline{\underline{104\ \text{psi (gage)}}}$$

1.80

1.80 Repeat Problem 1.79 if the compression process takes place without friction and without heat transfer (isentropic process).

For isentropic compression, $\frac{p}{\rho^k} = \text{constant}$ so that

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus,

$$p_f = \left(\frac{\rho_f}{\rho_i}\right)^k p_i$$

Since $\rho = \frac{\text{mass}}{\text{volume}}$, $\frac{\rho_f}{\rho_i} = \frac{\text{initial volume}}{\text{final volume}} = 3$ (for constant mass)

and therefore

$$p_f = (3)^{1.40}\left[(25 + 14.7)\text{ psi (abs)}\right] = 184.8 \text{ psi (abs)}$$

or

$$p_f\text{ (gage)} = 184.8 - 14.7 = \underline{\underline{170 \text{ psi (gage)}}}$$

1.81

1.81 Carbon dioxide at 30 °C and 300 kPa absolute pressure expands isothermally to an absolute pressure of 165 kPa. Determine the final density of the gas.

For isothermal expansion, $\frac{p}{\rho} = \text{constant}$ so that

$$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus,

$$\rho_f = \frac{p_f}{p_i}\rho_i$$

Also,

$$\rho_i = \frac{p_i}{RT_i} = \frac{300 \times 10^3 \frac{N}{m^2}}{\left(188.9 \frac{J}{kg \cdot K}\right)\left[(30°C + 273)K\right]} = 5.24 \frac{kg}{m^3}$$

so that

$$\rho_f = \left(\frac{165 \text{ kPa}}{300 \text{ kPa}}\right)\left(5.24 \frac{kg}{m^3}\right) = \underline{\underline{2.88 \frac{kg}{m^3}}}$$

1.73

1.82

1.82 Natural gas at 70 °F and standard atmospheric pressure of 14.7 psi (abs) is compressed isentropically to a new absolute pressure of 70 psi. Determine the final density and temperature of the gas.

For isentropic compression, $\frac{p}{\rho^k} = \text{constant}$ so that

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus,

$$\rho_f^k = \frac{p_f}{p_i}\rho_i^k$$

or

$$\rho_f = \left(\frac{p_f}{p_i}\right)^{\frac{1}{k}} \rho_i$$

Also,

$$\rho_i = \frac{p_i}{RT_i} = \frac{\left(14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(3.099\times 10^3\,\frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)\left[(70^\circ F + 460)^\circ R\right]} = 1.29\times 10^{-3}\,\frac{slugs}{ft^3}$$

so that

$$\rho_f = \left[\frac{70\text{ psi (abs)}}{14.7\text{ psi (abs)}}\right]^{\frac{1}{1.31}}\left(1.29\times 10^{-3}\,\frac{slugs}{ft^3}\right) = \underline{\underline{4.25\times 10^{-3}\,\frac{slugs}{ft^3}}}$$

and

$$T_f = \frac{p_f}{\rho_f R} = \frac{\left(70\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(4.25\times 10^{-3}\,\frac{slugs}{ft^3}\right)\left(3.099\times 10^3\,\frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)}$$

$$= 765\ {}^\circ R$$

or

$$T_f = 765\ {}^\circ R - 460 = \underline{\underline{305\ {}^\circ F}}$$

1.83

1.83 Compare the isentropic bulk modulus of air at 101 kPa (abs) with that of water at the same pressure.

For air (Eq. 1.17),

$$E_v = kp = (1.40)(101 \times 10^3 \, Pa) = 1.41 \times 10^5 \, Pa$$

For water (Table 1.6)

$$E_v = 2.15 \times 10^9 \, Pa$$

Thus,

$$\frac{E_v \, (water)}{E_v \, (air)} = \frac{2.15 \times 10^9 \, Pa}{1.41 \times 10^5 \, Pa} = \underline{\underline{1.52 \times 10^4}}$$

1.75

*1.84

*1.84 Develop a computer program for calculating the final gage pressure of gas when the initial gage pressure, initial and final volumes, atmospheric pressure, and the type of process (isothermal or isentropic) are specified. Use BG units. Check your program against the results obtained for Problem 1.79.

1.70

For compression or expansion,

$$\frac{p}{\rho^k} = \text{constant}$$

where $k=1$ for isothermal process, and $k=$ specific heat ratio for isentropic process. Thus,

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$

where $i \sim$ initial state, $f \sim$ final state, so that

$$p_f = \left(\frac{\rho_f}{\rho_i}\right)^k p_i \qquad (1)$$

Since

$$\rho = \frac{\text{mass}}{\text{volume}}$$

then

$$\frac{\rho_f}{\rho_i} = \frac{V_i}{V_f}$$

where V_i, V_f, are the initial and final volumes, respectively. Thus, from Eq. (1)

$$p_{fg} + p_{atm} = \left(\frac{V_i}{V_f}\right)^k (p_{ig} + p_{atm}) \qquad (2)$$

where the subscript g refers to gage pressure. Equation (2) can be written as

$$p_{fg} = \left(\frac{V_i}{V_f}\right)^k (p_{ig} + p_{atm}) - p_{atm} \qquad (3)$$

A spreadsheet (EXCEL) program for calculating the final gage pressure follows.

(con't)

*1.84 (con't)

This program calculates the final gage pressure of an ideal gas when the initial gage pressure in psi, the initial volume, the final volume, the atmospheric pressure in psia, and the type of process (isothermal or isentropic) is specified. To use, replace current values and let k = 1 for isothermal process or k = specific heat for isentropic process.						
A	B	C	D	E	F	
Initial gage pressure	Initial volume	Final volume	Atmospheric pressure		Final gage pressure	
p_{ig}(psi)	V_i	V_f	p_{atm}(psia)	k	p_{fg}(psi)	
25	1	0.3333	14.7	1	104.4	Row 10

Formula:
=((B10/C10)^E10)*(A10+D10)-D10

Data from Problem 1.79 are included in the above table, giving a final gage pressure of 104.4 psi.

1.85

1.85 An important dimensionless parameter concerned with very high speed flow is the *Mach number*, defined as V/c, where V is the speed of the object such as an airplane or projectile, and c is the speed of sound in the fluid surrounding the object. For a projectile traveling at 800 mph through air at 50 °F and standard atmospheric pressure, what is the value of the Mach number?

$$\text{Mach number} = \frac{V}{c}$$

From Table B.3 in Appendix B

$$c_{air\,@\,50^\circ F} = 1106\ \frac{ft}{s}$$

Thus

$$\text{Mach number} = \frac{(800\ mph)\left(5280\ \frac{ft}{mi}\right)\left(\frac{1\ hr}{3600\ s}\right)}{1106\ \frac{ft}{s}}$$

$$= \underline{\underline{1.06}}$$

1.86

1.86 Jet airliners typically fly at altitudes between approximately 0 to 40,000 ft. Make use of the data in Appendix C to show on a graph how the speed of sound varies over this range.

$$c = \sqrt{kRT} \qquad \text{(Eq. 1.20)}$$

For $k = 1.40$ and $R = 1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}$

$$c = 49.0\sqrt{T({}^\circ R)}$$

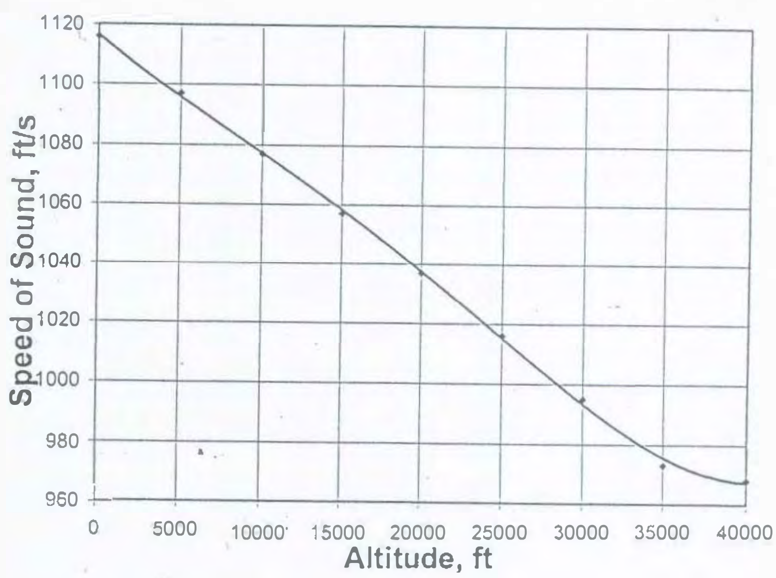
From Table C.1 in Appendix C at an altitude of 0 ft

$T = 59.00 + 460 = 519\,{}^\circ R$ so that

$$c = 49.0\sqrt{519\,{}^\circ R} = 1116 \frac{ft}{s}$$

Similar calculations can be made for other altitudes and the resulting graph is shown below.

Altitude, ft	Temp., °F	Temp., °R	c, ft/s
0	59	519	1116
5000	41.17	501.17	1097
10000	23.36	483.36	1077
15000	5.55	465.55	1057
20000	-12.26	447.74	1037
25000	-30.05	429.95	1016
30000	-47.83	412.17	995
35000	-65.61	394.39	973
40000	-69.7	390.3	968



1.87

1.87 (See Fluids in the News article titled "This water jet is a blast." Section 1.7.1) By what percent is the volume of water decreased if its pressure is increased to an equivalent to 3000 atmospheres (44,100 psi)?

$$E_v = -\frac{dp}{d\forall/\forall} \approx -\frac{\Delta p}{\Delta\forall/\forall} \qquad (\text{Eq. } 1.12)$$

$$\frac{\Delta\forall}{\forall} = -\frac{\Delta p}{E_v} = -\frac{44{,}100 \text{ psia} - 14.7 \text{ psia}}{3.12 \times 10^5 \text{ psia}} = -0.141$$

Thus,

% decrease in volume = $\underline{\underline{14.1\,\%}}$

1.88

1.88 During a mountain climbing trip it is observed that the water used to cook a meal boils at 90 °C rather than the standard 100 °C at sea level. At what altitude are the climbers preparing their meal? (See Tables B.2 and C.2 for data needed to solve this problem.)

When the water boils,

$p_{boil} = p_v$, where from Table B.2, at $T = 90°C$

$p_v = 7.01 \times 10^4 \frac{N}{m^2}$ (abs)

Also, from Table C.2, for a standard atmosphere

$p = 7.01 \times 10^4 \frac{N}{m^2}$ (abs) at an altitude of $\underline{\underline{3{,}000 \text{ m}}}$

1.89

1.89 When a fluid flows through a sharp bend, low pressures may develop in localized regions of the bend. Estimate the minimum absolute pressure (in psi) that can develop without causing cavitation if the fluid is water at 160 °F.

Cavitation may occur when the local pressure equals the vapor pressure. For water at 160 °F (from Table B.1 in Appendix B)

$$p_v = 4.74 \text{ psi (abs)}$$

Thus, minimum pressure = $\underline{\underline{4.74 \text{ psi (abs)}}}$

1.90

1.90 Estimate the minimum absolute pressure (in pascals) that can be developed at the inlet of a pump to avoid cavitation if the fluid is carbon tetrachloride at 20°C.

Cavitation may occur when the suction pressure at the pump inlet equals the vapor pressure.

For carbon tetrachloride at 20°C $p_v = 13 \text{ kPa (abs)}$.

Thus, minimum pressure = $\underline{\underline{13 \text{ kPa (abs)}}}$

1.91

1.91 When water at 70 °C flows through a converging section of pipe, the pressure decreases in the direction of flow. Estimate the minimum absolute pressure that can develop without causing cavitation. Express your answer in both BG and SI units.

Cavitation may occur in the converging section of pipe when the pressure equals the vapor pressure. From Table B.2 in Appendix B for water at 70°C, $p_v = 31.2$ kPa (abs). Thus,

$$\text{minimum pressure} = \underline{\underline{31.2 \text{ kPa (abs)}}} \text{ in SI units.}$$

In BG units

$$\text{minimum pressure} = \left(31.2 \times 10^{3} \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{\text{psi}}{\frac{N}{m^2}}\right)$$

$$= \underline{\underline{4.52 \text{ psia}}}$$

1.92

1.92 At what atmospheric pressure will water boil at 35 °C? Express your answer in both SI and BG units.

The vapor pressure of water at 35°C is 5.81 kPa (abs) (from Table B.2 in Appendix B using linear interpolation). Thus, if water boils at this temperature the atmospheric pressure must be equal to $\underline{\underline{5.81 \text{ kPa (abs)}}}$ in SI units. In BG units,

$$\left(5.81 \times 10^{3} \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{\frac{N}{m^2}}{\frac{lb}{in^2}}\right) = \underline{\underline{0.842 \text{ psi (abs)}}}$$

1.94

1.94 When a 2-mm-diameter tube is inserted into a liquid in an open tank, the liquid is observed to rise 10 mm above the free surface of the liquid. the contact angle between the liquid and the tube is zero, and the specific weight of the liquid is 1.2×10^4 N/m^3. Determine the value of the surface tension for this liquid.

$$h = \frac{2\sigma\cos\theta}{\gamma R}, \quad \text{where } \theta = 0$$

Thus,

$$\sigma = \frac{\gamma h R}{2\cos\theta} = \frac{\left(1.2\times10^4 \frac{N}{m^3}\right)(10\times10^{-3}\,m)(2\times10^{-3}\,m/2)}{2\cos 0}$$

$$= \underline{\underline{0.060\ \frac{N}{m}}}$$

1.95

1.95 Small droplets of carbon tetrachloride at 68 °F are formed with a spray nozzle. If the average diameter of the droplets is 200 μm what is the difference in pressure between the inside and outside of the droplets?

$$p = \frac{2\sigma}{R} \qquad \text{(Eq. 1.21)}$$

Since $\sigma = 2.69 \times 10^{-2}\ \frac{N}{m}$ at $68°F (=20°C)$,

$$p = \frac{2\left(2.69 \times 10^{-2}\ \frac{N}{m}\right)}{100 \times 10^{-6}\ m} = \underline{\underline{538\ Pa}}$$

1.96

1.96 A 12-mm diameter jet of water discharges vertically into the atmosphere. Due to surface tension the pressure inside the jet will be slightly higher than the surrounding atmospheric pressure. Determine this difference in pressure.

For equilibrium (see figure),

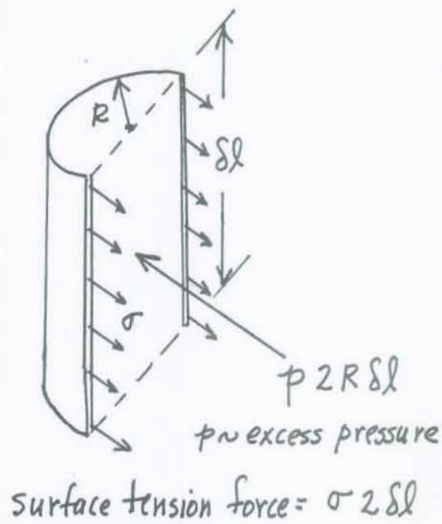
$$p(2R\,\delta\ell) = \sigma(2\,\delta\ell)$$

so that

$$p = \frac{\sigma}{R}$$

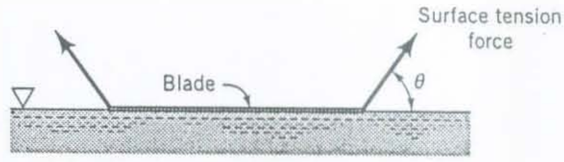
$$= \frac{7.34 \times 10^{-2}\ \frac{N}{m}}{\frac{12}{2} \times 10^{-3}\ m}$$

$$= \underline{\underline{12.2\ Pa}}$$

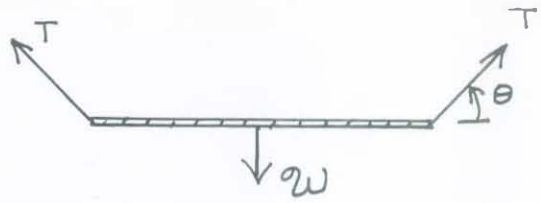


1.97

1.97 As shown in **Video V1.9**, surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water, but a single-edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in Fig. P1.97. **(a)** The mass of the double-edge blade is 0.64×10^{-3} kg, and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. **(b)** The mass of the single-edge blade is 2.61×10^{-3} kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.



■ FIGURE P1.97



(a) $\Sigma F_{vertical} = 0$

$$\mathcal{W} = T \sin\theta$$

where $\mathcal{W} = m_{blade} \times g$ and $T = \sigma \times$ length of sides.

$$\therefore \; (0.64 \times 10^{-3}\,kg)(9.81\,m/s^2) = (7.34 \times 10^{-2}\,\tfrac{N}{m})(0.206\,m)\sin\theta$$

$$\sin\theta = 0.415$$

$$\underline{\underline{\theta = 24.5^\circ}}$$

(b) For single-edge blade

$$\mathcal{W} = m_{blade} \times g = (2.61 \times 10^{-3}\,kg)(9.81\,m/s^2)$$
$$= 0.0256\,N$$

and

$$T\sin\theta = (\sigma \times \text{length of blade})\sin\theta$$
$$= (7.34 \times 10^{-2}\,N/m)(0.154\,m)\sin\theta$$
$$= 0.0113\sin\theta$$

In order for blade to "float" $\mathcal{W} < T\sin\theta$. Since maximum value for $\sin\theta$ is 1, it follows that $\mathcal{W} > T\sin\theta$ and single-edge blade will sink.

1.98

1.98 To measure the water depth in a large open tank with opaque walls, an open vertical glass tube is attached to the side of the tank. The height of the water column in the tube is then used as a measure of the depth of water in the tank. (a) For a true water depth in the tank of 3 ft, make use of Eq. 1.22 (with $\theta \simeq 0°$) to determine the percent error due to capillarity as the diameter of the glass tube is changed. Assume a water temperature of 80 °F. Show your results on a graph of percent error versus tube diameter, D, in the range 0.1 in. $< D <$ 1.0 in. (b) If you want the error to be less than 1%, what is the smallest tube diameter allowed?

(a) The excess height, h, caused be the surface tension is

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad \text{(Eq. 1.22)}$$

For $\theta \cong 0°$ with $D = 2R$

$$h = \frac{4\sigma}{\gamma D} \qquad (1)$$

From Table B.1 in Appendix B for water at 80°F
$\sigma = 4.91 \times 10^{-3}$ lb/ft and $\gamma = 62.22$ lb/ft^3.
Thus, from Eq. (1)

$$h\,(\text{ft}) = \frac{4\left(4.91\times10^{-3}\,\frac{\text{lb}}{\text{ft}}\right)}{\left(62.22\,\frac{\text{lb}}{\text{ft}^3}\right)\frac{D\,(\text{in.})}{12\ \text{in./ft}}} = \frac{3.79\times10^{-3}}{D\,(\text{in.})} \qquad (2)$$

Since

$$\%\ \text{error} = \frac{h\,(\text{ft})}{3\ \text{ft}} \times 100 \quad \text{(with the true depth} = 3\ \text{ft)}$$

it follows from Eq. (2) that

$$\%\ \text{error} = \frac{3.79\times10^{-3}}{3\,D\,(\text{in.})} \times 100$$

$$= \frac{0.126}{D\,(\text{in.})} \qquad (3)$$

A plot of % error versus tube diameter is shown on the next page.

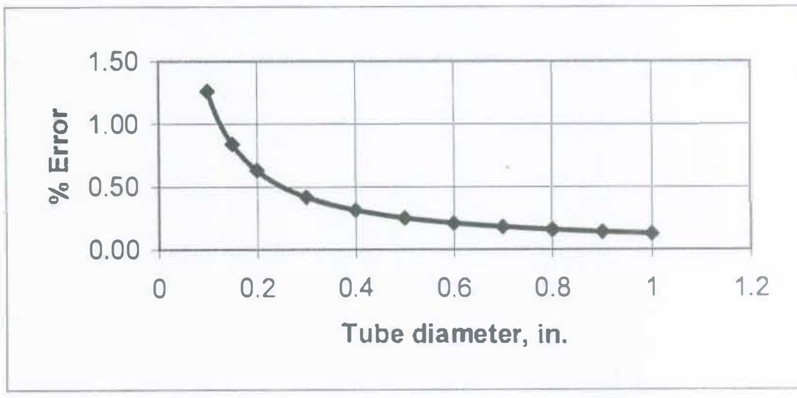
(con't)

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

1.98 (con't)

Diameter of tube, in.	% Error
0.1	1.26
0.15	0.84
0.2	0.63
0.3	0.42
0.4	0.32
0.5	0.25
0.6	0.21
0.7	0.18
0.8	0.16
0.9	0.14
1	0.13

Values obtained from Eq. (3)



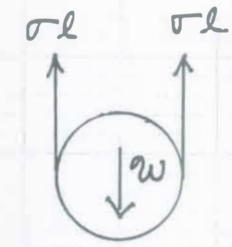
(b) For 1% error from Eq. (3)

$$1 = \frac{0.126}{D(\text{in.})}$$

$$D = \underline{\underline{0.126 \text{ in.}}}$$

1.99

1.99 Under the right conditions, it is possible, due to surface tension, to have metal objects float on water. (See Video V1.9.) Consider placing a short length of a small diameter steel (sp. wt. = 490 lb/ft^3) rod on a surface of water. What is the maximum diameter that the rod can have before it will sink? Assume that the surface tension forces act vertically upward. *Note:* A standard paper clip has a diameter of 0.036 in. Partially unfold a paper clip and see if you can get it to float on water. Do the results of this experiment support your analysis?



ℓ = rod length

In order for rod to float (see figure) it follows that

$$2\sigma\ell \geq \mathcal{W} \geq \left(\frac{\pi}{4}\right)(D^2)\,\ell\,\gamma_{steel}$$

Thus, for the limiting case

$$D_{max}^2 = \frac{2\sigma\ell}{\left(\frac{\pi}{4}\right)\ell\,\gamma_{steel}} = \frac{8\sigma}{\pi\,\gamma_{steel}}$$

so that

$$D_{max} = \left[\frac{8\left(5.03\times10^{-3}\ \frac{lb}{ft}\right)}{\pi\left(490\ \frac{lb}{ft^3}\right)}\right]^{1/2} = 5.11\times10^{-3}\ ft$$

$$= \underline{\underline{0.0614\ in.}}$$

Since a standard steel paper clip has a diameter of 0.036 in., which is less than 0.0614 in., it should float. A simple experiment will verify this. $\underline{\underline{\text{Yes}}}$.

1.100

1.100 An open, clean glass tube, having a diameter of 3 mm, is inserted vertically into a dish of mercury at 20 °C. How far will the column of mercury in the tube be depressed?

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad \text{(Eq. 1.22)}$$

For $\theta = 130°$,

$$h = \frac{2\left(4.66\times10^{-1}\ \frac{N}{m}\right)\cos 130°}{\left(133\times10^{3}\ \frac{N}{m^3}\right)(0.0015\ m)} = -3.00\times10^{-3}\ m$$

Thus, column will be depressed $\underline{\underline{3.00\ mm}}$

1.101

1.101 An open, clean glass tube ($\theta = 0°$) is inserted vertically into a pan of water. What tube diameter is needed if the water level in the tube is to rise one tube diameter (due to surface tension)?

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad \text{(Eq. 1.22)}$$

For $h = 2R$ and $\theta = 0°$

$$2R = \frac{2\sigma(1)}{\gamma R}$$

and

$$R^2 = \frac{\sigma}{\gamma} = \frac{5.03\times10^{-3}\ \frac{lb}{ft}}{62.4\ \frac{lb}{ft^3}}$$

$$R = 8.98\times10^{-3}\ ft$$

$$\text{diameter} = 2R = \underline{\underline{1.80\times10^{-2}\ ft}}$$

1.102

1.102 Determine the height water at 60 °F will rise due to capillary action in a clean, $\frac{1}{4}$-in.-diameter tube. What will be the height if the diameter is reduced to 0.01 in.?

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad \text{(Eq. 1.22)}$$

For water at 60° F (from Table B.1 in Appendix B),

$\sigma = 5.03 \times 10^{-3}\ \frac{lb}{ft}$ and $\gamma = 62.37\ \frac{lb}{ft^3}$. Thus, with $\theta = 0$,

(for $R = 0.125$ in.)

$$h = \frac{2\left(5.03\times10^{-3}\ \frac{lb}{ft}\right)(1)}{\left(62.37\ \frac{lb}{ft^3}\right)\left(\frac{0.125}{12}\ ft\right)} = 1.55\times10^{-2}\ ft$$

or

$$h = \left(1.55\times10^{-2}\ ft\right)\left(\frac{12\ in.}{ft}\right) = \underline{\underline{0.186\ in.}}$$

Similarly,

(for $R = 0.005$ in.)

$$h = (0.186\ in.)\left(\frac{0.125\ in.}{0.005\ in.}\right) = \underline{\underline{4.65\ in.}}$$

1.103

1.103 (See Fluids in the News article titled "Walking on water," Section 1.9.) (a) The water strider bug shown in Fig. P1.103 is supported on the surface of a pond by surface tension acting along the interface between the water and the bug's legs. Determine the minimum length of this interface needed to support the bug. Assume the bug weighs 10^{-4} N and the surface tension force acts vertically upwards. (b) Repeat part (a) if surface tension were to support a person weighing 750 N.

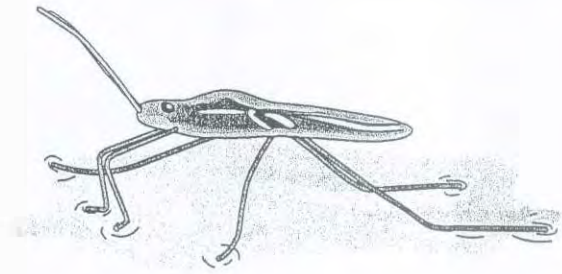
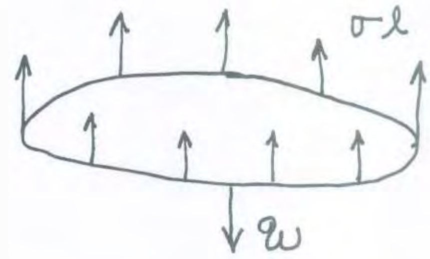


FIGURE P1.103



$\mathcal{W} \sim$ weight
$\sigma \sim$ surface tension
$\ell \sim$ length of interface

For equilibrium,

$$\mathcal{W} = \sigma \ell$$

(a)
$$\ell = \frac{\mathcal{W}}{\sigma} = \frac{10^{-4}\ N}{7.34 \times 10^{-2}\ \frac{N}{m}}$$
$$= 1.36 \times 10^{-3}\ m$$
$$= (1.36 \times 10^{-3}\ m)\left(10^{3}\ \frac{mm}{m}\right) = \underline{\underline{1.36\ mm}}$$

(b)
$$\ell = \frac{750\ N}{7.34 \times 10^{-2}\ \frac{N}{m}} = \underline{\underline{1.02 \times 10^{4}\ m}} \quad (6.34\ mi\ !!)$$

1.104

1.104 Fluid Characterization by Use of a Stormer Viscometer

Objective: As discussed in Section 1.6, some fluids can be classified as Newtonian fluids; others are non-Newtonian. The purpose of this experiment is to determine the shearing stress versus rate of strain characteristics of various liquids and, thus, to classify them as Newtonian or non-Newtonian fluids.

Equipment: Stormer viscometer containing a stationary outer cylinder and a rotating, concentric inner cylinder (see Fig. P1.104); stop watch; drive weights for the viscometer; three different liquids (silicone oil, Latex paint, and corn syrup).

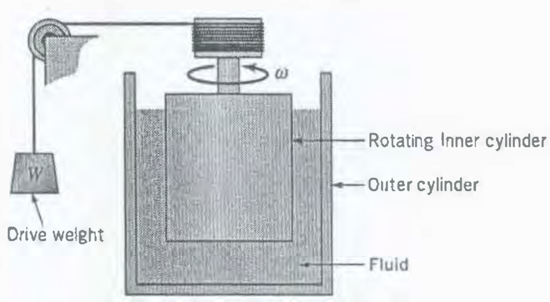
Experimental Procedure: Fill the gap between the inner and outer cylinders with one of the three fluids to be tested. Select an appropriate drive weight (of mass m) and attach it to the end of the cord that wraps around the drum to which the inner cylinder is fastened. Release the brake mechanism to allow the inner cylinder to start to rotate. (The outer cylinder remains stationary.) After the cylinder has reached its steady-state angular velocity, measure the amount of time, t, that it takes the inner cylinder to rotate N revolutions. Repeat the measurements using various drive weights. Repeat the entire procedure for the other fluids to be tested.

Calculations: For each of the three fluids tested, convert the mass, m, of the drive weight to its weight, $W = mg$, where g is the acceleration of gravity. Also determine the angular velocity of the inner cylinder, $\omega = N/t$.

Graph: For each fluid tested, plot the drive weight, W, as ordinates and angular velocity, ω, as abscissas. Draw a best fit curve through the data.

Results: Note that for the flow geometry of this experiment, the weight, W, is proportional to the shearing stress, τ, on the inner cylinder. This is true because with constant angular velocity, the torque produced by the viscous shear stress on the cylinder is equal to the torque produced by the weight (weight times the appropriate moment arm). Also, the angular velocity, ω, is proportional to the rate of strain, du/dy. This is true because the velocity gradient in the fluid is proportional to the inner cylinder surface speed (which is proportional to its angular velocity) divided by the width of the gap between the cylinders. Based on your graphs, classify each of the three fluids as to whether they are Newtonian, shear thickening, or shear thinning (see Fig. 1.7).

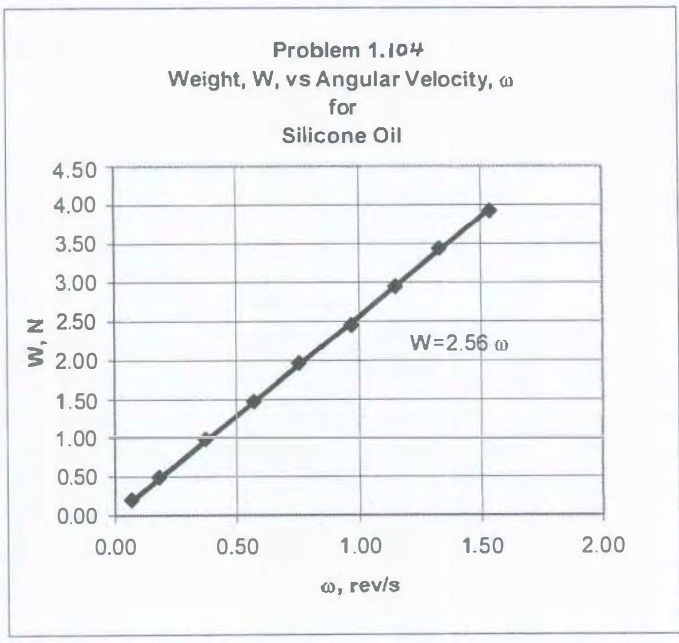
Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



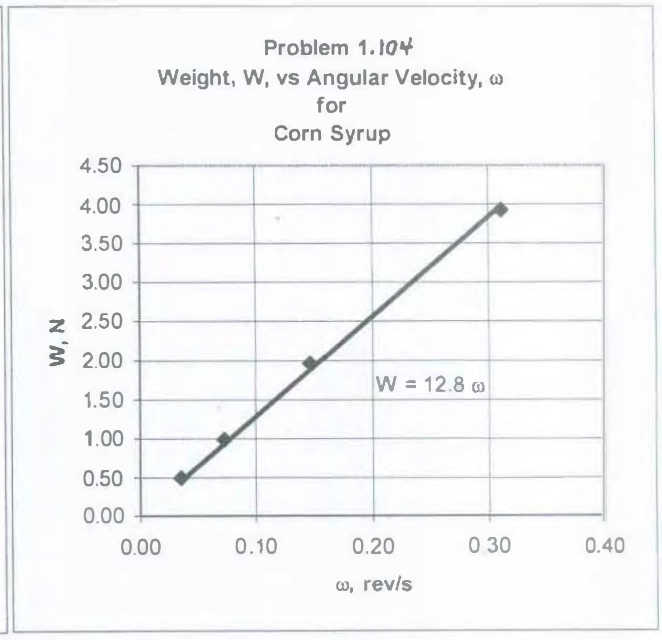
■ FIGURE P1.104

(con't)

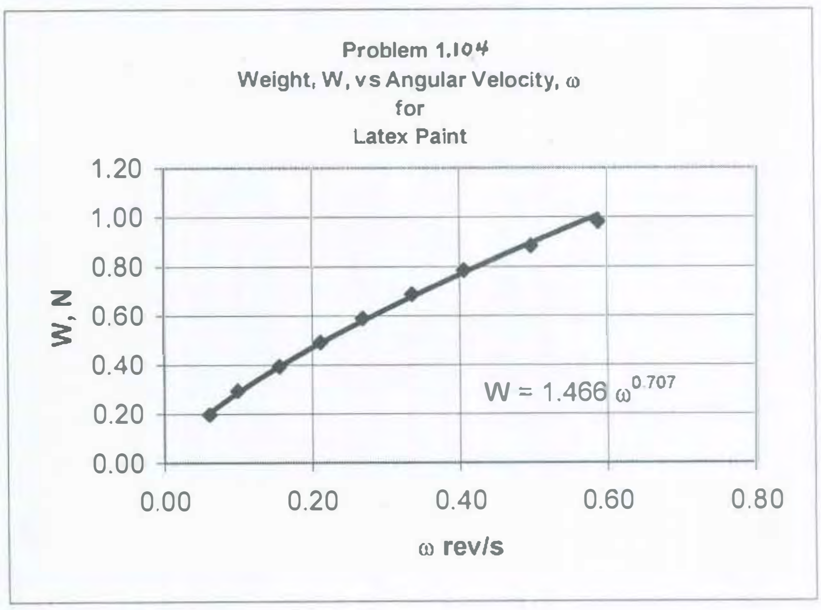
1.104 (Con't)



Problem 1.104
Weight, W, vs Angular Velocity, ω
for
Silicone Oil
W=2.56 ω
W, N
ω, rev/s



Problem 1.104
Weight, W, vs Angular Velocity, ω
for
Corn Syrup
W = 12.8 ω
W, N
ω, rev/s



Problem 1.104
Weight, W, vs Angular Velocity, ω
for
Latex Paint
W = 1.466 ω^0.707
W, N
ω rev/s

1.105

1.105 Capillary Tube Viscometer

Objective: The flowrate of a viscous fluid through a small diameter (capillary) tube is a function of the viscosity of the fluid. For the flow geometry shown in Fig. P1.105, the kinematic viscosity, ν, is inversely proportional to the flowrate, Q. That is, $\nu = K/Q$, where K is the calibration constant for the particular device. The purpose of this experiment is to determine the value of K and to use it to determine the kinematic viscosity of water as a function of temperature.

Equipment: Constant temperature water tank, capillary tube, thermometer, stop watch, graduated cylinder.

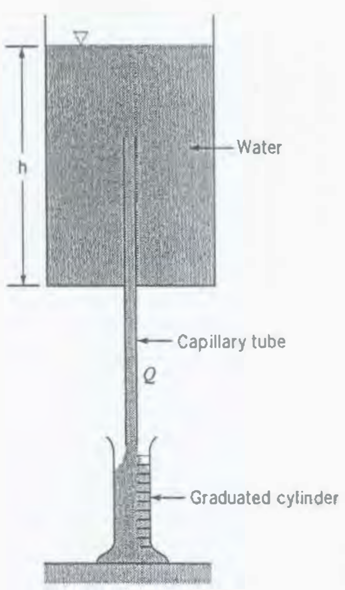
Experimental Procedure: Adjust the water temperature to 15.6°C and determine the flowrate through the capillary tube by measuring the time, t, it takes to collect a volume, V, of water in a small graduated cylinder. Repeat the measurements for various water temperatures, T. Be sure that the water depth, h, in the tank is the same for each trial. Since the flowrate is a function of the depth (as well as viscosity), the value of K obtained will be valid for only that value of h.

Calculations: For each temperature tested, determine the flowrate, $Q = V/t$. Use the data for the 15.6°C water to determine the calibration constant, K, for this device. That is, $K = \nu Q$, where the kinematic viscosity for 15.6°C water is given in Table 1.5 and Q is the measured flowrate at this temperature. Use this value of K and your other data to determine the viscosity of water as a function of temperature.

Graph: Plot the experimentally determined kinematic viscosity, ν, as ordinates and temperature, T, as abscissas.

Results: On the same graph, plot the standard viscosity-temperature data obtained from Table B.2.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P1.105

(con't)

1.105 (Con't)

Solution for Problem 1.105 Capillary Tube Viscometer

V, ml	t, s	T, deg C	Q, ml/s	ν, m^2/s
9.2	19.8	15.6	0.465	1.12E-06
9.7	15.8	26.3	0.614	8.49E-07
9.2	16.8	21.3	0.548	9.51E-07
9.1	21.3	12.3	0.427	1.22E-06
9.2	13.1	34.3	0.702	7.42E-07
9.4	10.1	50.4	0.931	5.60E-07
9.1	8.9	58.1	1.022	5.10E-07

From Table B.2

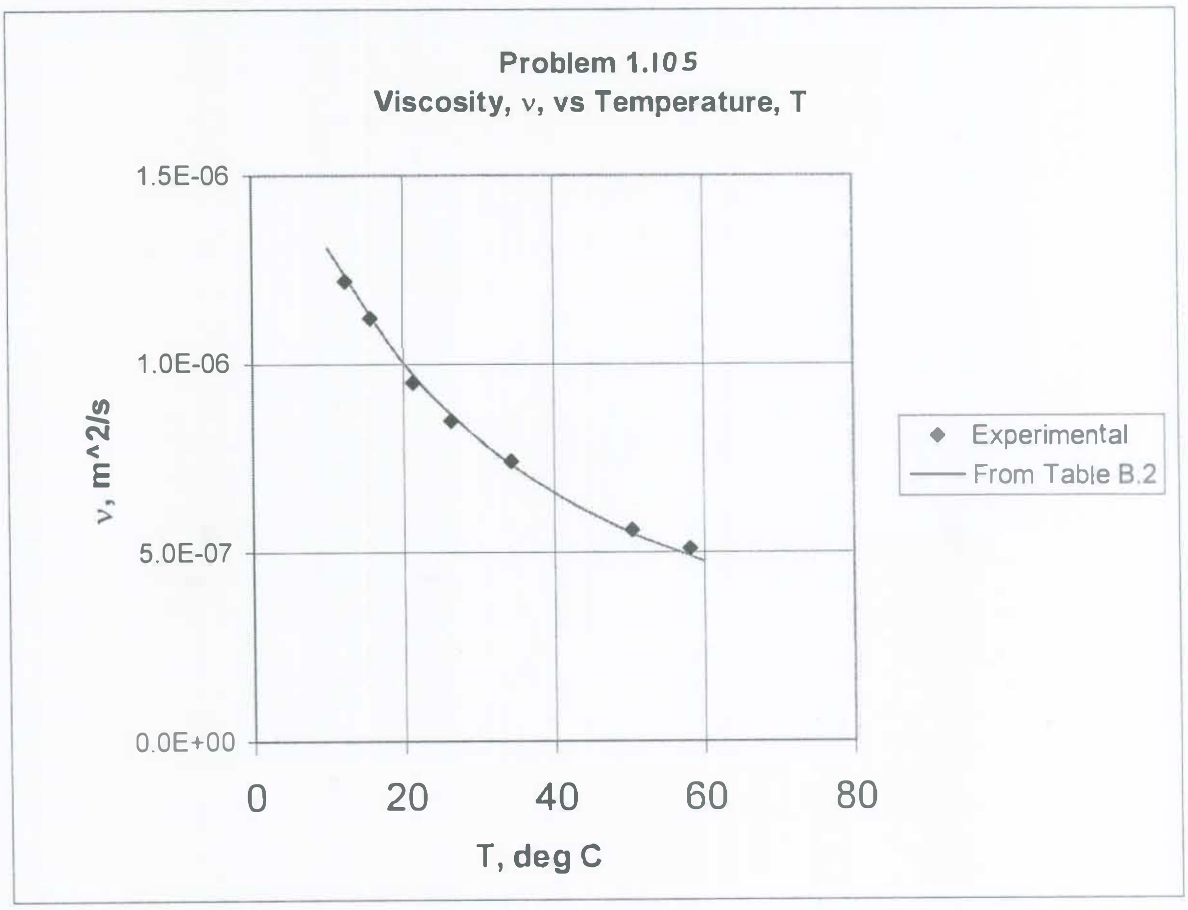
T, deg C	ν, m^2/s
10	1.31E-06
20	1.00E-06
30	8.01E-07
40	6.58E-07
50	5.53E-07
60	4.75E-07

ν = K/Q

K, m^2 ml/s^2
5.21E-07

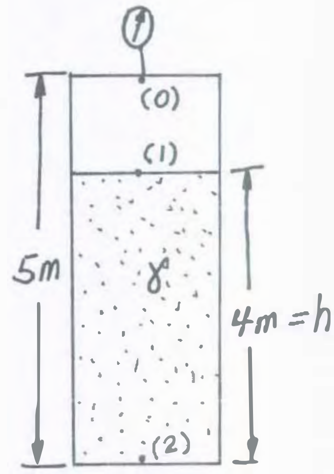
ν (at 15.6 deg C), m^2/s
1.12E-06

K = ν Q = 1.12E-6 m^2/s * 0.465 ml/s = 5.21E-7 m^2 ml/s^2



2.2

2.2 A closed, 5-m-tall tank is filled with water to a depth of 4 m. The top portion of the tank is filled with air which, as indicated by a pressure gage at the top of the tank, is at a pressure of 20 kPa. Determine the pressure that the water exerts on the bottom of the tank.



$$p_0 = 20 \times 10^3 \frac{N}{m^2} = p_1$$

$$p_2 = p_1 + \gamma h = 20 \times 10^3 \frac{N}{m^2} + 9.80 \times 10^3 \frac{N}{m^3}(4m)$$

$$= 59.2 \times 10^3 \frac{N}{m^2} = \underline{\underline{59.2\ kPa}}$$

2.3

2.3 A closed tank is partially filled with glycerin. If the air pressure in the tank is 6 lb/in.^{2} and the depth of glycerin is 10 ft, what is the pressure in lb/ft^2 at the bottom of the tank?

$$p = \gamma h + p_o = \left(78.6 \frac{lb}{ft^3}\right)(10\ ft) + \left(6 \frac{lb}{in.^2}\right)\left(\frac{144\ in.^2}{ft^2}\right)$$

$$= \underline{\underline{1650 \frac{lb}{ft^2}}}$$

2.4

2.4 Blood pressure is usually given as a ratio of the maximum pressure (systolic pressure) to the minimum pressure (diastolic pressure). As shown in Video V2.2 such pressures are commonly measured with a mercury manometer. A typical value for this ratio for a human would be 120/70, where the pressures are in mm Hg. (a) What would these pressures be in pascals? (b) If your car tire was inflated to 120 mm Hg, would it be sufficient for normal driving?

$$p = \gamma h$$

(a) For 120 mm Hg: $p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.120\ m) = \underline{\underline{16.0\ kPa}}$

For 70 mm Hg: $p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.070\ m) = \underline{\underline{9.31\ kPa}}$

(b) For 120 mm Hg: $p = \left(16.0 \times 10^3 \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{lb/in.^2}{N/m^2}\right)$

$= 2.32\ psi$

Since a typical tire pressure is 30-35 psi, 120 mm Hg is <u>not sufficient</u> for normal driving.

2.5

2.5 An unknown immiscible liquid seeps into the bottom of an open oil tank. Some measurements indicate that the depth of the unknown liquid is 1.5 m and the depth of the oil (specific weight = 8.5 kN/m³) floating on top is 5.0 m. A pressure gage connected to the bottom of the tank reads 65 kPa. What is the specific gravity of the unknown liquid?

$$p_{bottom} = (\gamma_{oil})(5\,m) + (\gamma_u)(1.5\,m) \quad \text{where } \gamma_u \sim \text{unknown liquid } \gamma$$

$$\gamma_u = \frac{p_{bottom} - (\gamma_{oil})(5\,m)}{1.5\,m} = \frac{65 \times 10^3 \frac{N}{m^2} - \left(8.5 \times 10^3 \frac{N}{m^3}\right)(5\,m)}{1.5\,m}$$

$$= 15 \times 10^3 \frac{N}{m^3}$$

$$SG = \frac{\gamma_u}{\gamma_{H_2O}\,@\,4°C} = \frac{15 \times 10^3 \frac{N}{m^3}}{9.81 \times 10^3 \frac{N}{m^3}} = \underline{\underline{1.53}}$$

2.6

2.6 Bathyscaphes are capable of submerging to great depths in the ocean. What is the pressure at a depth of 5 km, assuming that seawater has a constant specific weight of 10.1 kN/m³? Express your answer in pascals and psi.

$$p = \gamma h + p_0$$

At the surface $p_0 = 0$ so that

$$p = \left(10.1 \times 10^3 \frac{N}{m^3}\right)(5 \times 10^3\,m) = 50.5 \times 10^6 \frac{N}{m^2} = \underline{\underline{50.5\ MPa}}$$

Also,

$$p = \left(50.5 \times 10^6 \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{\frac{lb}{in.^2}}{\frac{N}{m^2}}\right) = \underline{\underline{7320\ psi}}$$

2.7

2.7 For the great depths that may be encountered in the ocean the compressibility of seawater may become an important consideration. **(a)** Assume that the bulk modulus for seawater is constant and derive a relationship between pressure and depth which takes into account the change in fluid density with depth. **(b)** Make use of part (a) to determine the pressure at a depth of 6 km assuming seawater has a bulk modulus of 2.3×10^9 Pa, and a density of 1030 kg/m^3 at the surface. Compare this result with that obtained by assuming a constant density of 1030 kg/m^3.

(a)

$$\frac{dp}{dz} = -\gamma = -\rho g \qquad (\text{Eq. } 2.4)$$

Thus,
$$\frac{dp}{\rho} = -g\, dz \qquad (1)$$

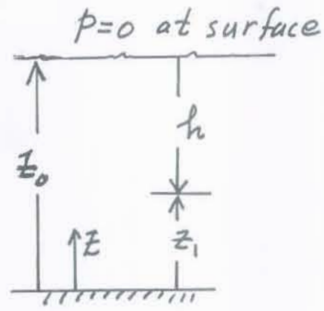
If ρ is a function of p, we must determine $\rho = f(p)$ before integrating Eq.(1). Since,

$$E_v = \frac{dp}{d\rho/\rho} \qquad (\text{Eq. } 1.13)$$

then
$$\int_0^p dp = E_v \int_{\rho_0}^{\rho} \frac{d\rho}{\rho}$$

so that
$$p = E_v \ln \frac{\rho}{\rho_0}$$

Thus,
$$\rho = \rho_0 e^{\frac{p}{E_v}} \qquad \text{where } \rho = \rho_0 \text{ at } p = 0$$



From Eq.(1)

$$\int_{p_1}^{0} \frac{dp}{\rho_0 e^{\frac{p}{E_v}}} = -g \int_{z_1}^{z_0} dz$$

or

$$\int_{p_1}^{0} e^{-\frac{p}{E_v}}\, dp = -\rho_0 g \int_{z_1}^{z_0} dz$$

so that

$$\underline{\underline{p = -E_v \ln \left(1 - \frac{\rho_0 g h}{E_v}\right)}} \qquad \text{where, } h = z_0 - z_1 \text{, the depth below surface}$$

(con't)

2.7	(con't)

(b) From part (a),

$$p = -E_v \ln\left(1 - \frac{\rho_0 g h}{E_v}\right)$$

so that at $h = 6\,km$

$$p = -\left(2.3\times10^9\,\frac{N}{m^2}\right)\ln\left[1 - \frac{\left(1.03\times10^3\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(6\times10^3\,m\right)}{2.3\times10^9\,\frac{N}{m^2}}\right]$$

$$= 6.14\times10^7\,\frac{N}{m^2} = \underline{\underline{61.4\,MPa}}$$

(c) For constant density

$$p = \gamma h = \rho g h = \left(1.03\times10^3\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(6\times10^3\,m\right)$$

$$= \underline{\underline{60.6\,MPa}}$$

2.8

2.8 Sometimes when riding an elevator or driving up or down a hilly road a person's ears "pop" as the pressure difference between the inside and outside of the ear is equalized. Determine the pressure difference (in psi) associated with this phenomenon if it occurs during a 150 ft elevation change.

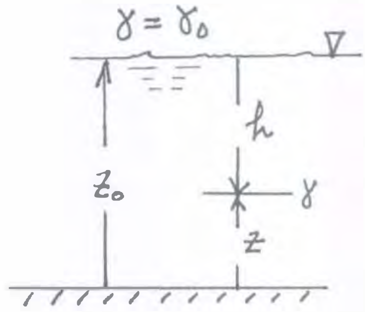
$$\Delta p = \gamma \Delta h = 0.0765 \frac{lb}{ft^3}(150 ft)$$

$$= 11.5 \frac{lb}{ft^2}\left(\frac{1 ft^2}{144 in.^2}\right)$$

$$= \underline{\underline{0.0797 psi}}$$

2.9

2.9 Develop an expression for the pressure variation in a liquid in which the specific weight increases with depth, h, as $\gamma = Kh + \gamma_0$, where K is a constant and γ_0 is the specific weight at the free surface.



$$\frac{dp}{dz} = -\gamma \qquad (Eq.\ 2.4)$$

Let $h = z_0 - z$

so that $dh = -dz$

Thus,

$$dp = \gamma\, dh$$

and

$$\int_0^p dp = \int_0^h \gamma\, dh$$

For $\gamma = Kh + \gamma_0$,

$$\int_0^p dp = \int_0^h (Kh + \gamma_0)\, dh$$

and

$$\underline{\underline{p = \frac{Kh^2}{2} + \gamma_0 h}}$$

2.10

(con't)

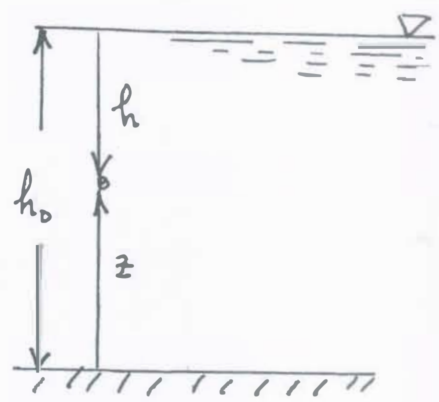
*2.10 In a certain liquid at rest, measurements of the specific weight at various depths show the following variation:

h (ft)	γ (lb/ft^3)
0	70
10	76
20	84
30	91
40	97
50	102
60	107
70	110
80	112
90	114
100	115

The depth $h = 0$ corresponds to a free surface at atmospheric pressure. Determine, through numerical integration of Eq. 2.4, the corresponding variation in pressure and show the results on a plot of pressure (in psf) versus depth (in feet).

$$\frac{dp}{dz} = -\gamma \qquad \text{(Eq. 2.4)}$$

Let $z = h_o - h$ (see figure) so that $dz = -dh$ and therefore



$$dp = -\gamma\, dz = \gamma\, dh$$

Thus,
$$\int_0^{p_i} dp = \int_0^{h_i} \gamma\, dh$$

or
$$p_i = \int_0^{h_i} \gamma\, dh \qquad (1)$$

where p_i is the pressure at depth h_i.

Equation (1) can be integrated numerically using the trapezoidal rule, i.e.,

$$I = \frac{1}{2}\sum_{i=1}^{n-1} (y_i + y_{i+1})(x_{i+1} - x_i)$$

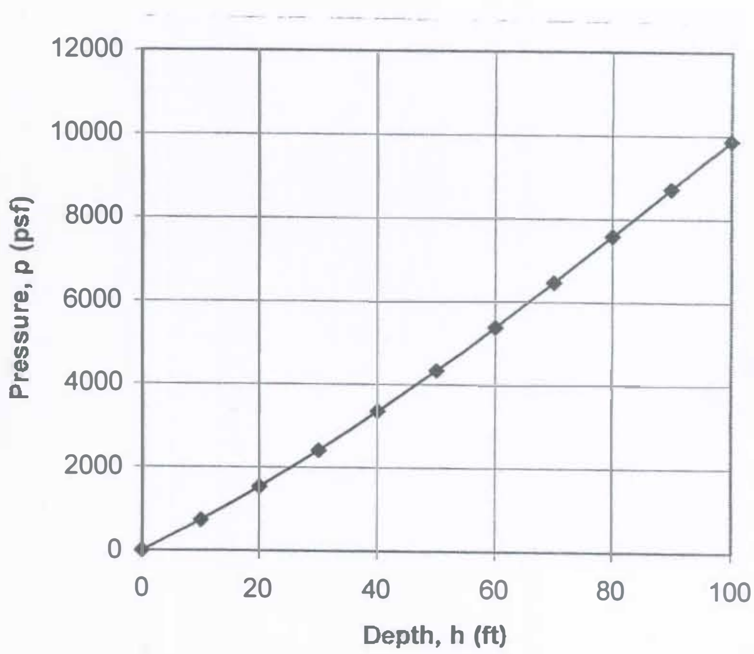
Where $y \sim \gamma$, $x \sim h$, and n = number of data points.

(con't)

*2.10 (Con't)

The tabulated results are given below, along with the corresponding plot of pressure vs. depth.

h(ft)	γ, lb/ft^3	Pressure,psf
0	70	0
10	76	730
20	84	1530
30	91	2405
40	97	3345
50	102	4340
60	107	5385
70	110	6470
80	112	7580
90	114	8710
100	115	9855



*2.12

***2.12** Under normal conditions the temperature of the atmosphere decreases with increasing elevation. In some situations, however, a temperature inversion may exist so that the air temperature increases with elevation. A series of temperature probes on a mountain give the elevation–temperature data shown in Table P2.12. If the barometric pressure at the base of the mountain is 12.1 psia, determine by means of numerical integration the pressure at the top of the mountain.

Elevation (ft)	Temperature (°F)
5000	50.1 (base)
5500	55.2
6000	60.3
6400	62.6
7100	67.0
7400	68.4
8200	70.0
8600	69.5
9200	68.0
9900	67.1 (top)

TABLE P2.12

From Eq. 2.9,

$$\ln \frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T}$$

In the table below the temperature in °R is given and the integrand $1/T(°R)$ tabulated.

Elevation, ft	T, °F	T, °R	1/ T(°R)
5000	50.1	509.8	0.001962
5500	55.2	514.9	0.001942
6000	60.3	520.0	0.001923
6400	62.6	522.3	0.001915
7100	67.0	526.7	0.001899
7400	68.4	528.1	0.001894
8200	70.0	529.7	0.001888
8600	69.5	529.2	0.00189
9200	68.0	527.7	0.001895
9900	67.1	526.8	0.001898

The approximate value of the integral in Eq. 2.9 is 9.34 obtained using the trapezoidal rule, i.e.,

$I = \frac{1}{2}\sum_{i=1}^{n-1}(y_i + y_{i+1})(x_{i+1} - x_i)$ where $y \sim 1/T$, $x \sim$ elevation, and $n =$ number of data points. Thus,

$$\int_{5000\,ft}^{9900\,ft} \left(\frac{1}{T}\right) dz = 9.34 \ \frac{ft}{°R}$$

so that (with $g = 32.2\ ft/s^2$ and $R = 1716\ ft \cdot lb/slug \cdot °R$

$$\ln \frac{p_2}{p_1} = -\frac{(32.2\ \frac{ft}{s^2})(9.34\ \frac{ft}{°R})}{1716\ ft \cdot lb/slug \cdot °R} = -0.1753 \qquad (1)$$

(con't)

*2.12 (con't)

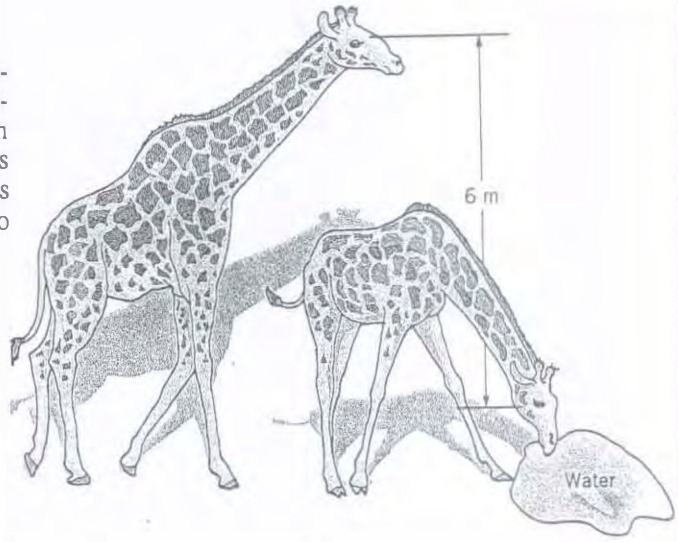
It follows from Eq.(1) with $p_1 = 12.1$ psia that

$$p_2 = (12.1 \text{ psia})\, e^{-0.1753} = \underline{\underline{10.2 \text{ psia}}}$$

(Note: Since the temperature variation is not very large, it would be expected that the assumption of a constant temperature would give good results. If the temperature is assumed to be constant at the base temperature (50.1 °F), $p_2 = 10.1$ psia, which is only slightly different from the result given above.)

2.14

2.14 (See Fluids in the News article titled "**Giraffe's blood pressure,**" Section 2.3.1.) (**a**) Determine the change in hydrostatic pressure in a giraffe's head as it lowers its head from eating leaves 6 m above the ground to getting a drink of water at ground level as shown in Fig. P2.14. Assume the specific gravity of blood is $SG = 1$. (**b**) Compare the pressure change calculated in part (**a**) to the normal 120 mm of mercury pressure in a human's heart.



(a) For hydrostatic pressure change,

$$\Delta p = \gamma h = \left(9.80 \frac{kN}{m^2}\right)(6\,m) = 58.8 \frac{kN}{m^2} = \underline{\underline{58.8\,kPa}}$$

(b) To compare with pressure in human heart convert pressure in part (a) to mm Hg:

$$58.8 \frac{kN}{m^2} = \gamma_{Hg}\, h_{Hg} = \left(133 \frac{kN}{m^3}\right) h_{Hg}$$

$$h_{Hg} = (0.442\,m)\left(10^3 \frac{mm}{m}\right) = 442\,mm\,Hg$$

Thus, the pressure change in the giraffes head is $\underline{\underline{442\,mm\,Hg}}$ compared with $\underline{\underline{120\,mm\,Hg}}$ in the human heart.

2.15

2.15 Assume that a person skiing high in the mountains at an altitude of 15,000 ft takes in the same volume of air with each breath as she does while walking at sea level. Determine the ratio of the mass of oxygen inhaled for each breath at this high altitude compared to that at sea level.

Let $(\ )_0$ denote sea level and $(\ )_{15}$ denote 15,000 ft altitude.
Thus, since m = mass $= \rho \mathcal{V}$, where $\mathcal{V}$ = volume,
$m_0 = \rho_0 \mathcal{V}_0$ and $m_{15} = \rho_{15} \mathcal{V}_{15}$, where $\mathcal{V}_0 = \mathcal{V}_{15}$.
Hence,

$$\frac{m_{15}}{m_0} = \frac{\rho_{15} \mathcal{V}_{15}}{\rho_0 \mathcal{V}_0} = \frac{\rho_{15}}{\rho_0}$$

If it is assumed that the air composition (e.g., % of air that is oxygen) is the same at sea level as it is at 15,000 ft, then we can use the ρ values from Table C.1:

$$\rho_0 = 2.377 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3} \text{ and } \rho_{15} = 1.496 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3} \text{ so that}$$

$$\frac{m_{15}}{m_0} = \frac{1.496 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}}{2.377 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}} = 0.629 = \underline{\underline{62.9\%}}$$

2.16

2.16 Pikes Peak near Denver, Colorado has an elevation of 14,110 ft. **(a)** Determine the pressure at this elevation, based on Eq. 2.12. **(b)** If the air is assumed to have a constant specific weight of 0.07647 lb/ft³, what would the pressure be at this altitude? **(c)** If the air is assumed to have a constant temperature of 59 °F what would the pressure be at this elevation? For all three cases assume standard atmospheric conditions at sea level (see Table 2.1).

(a)
$$p = p_a \left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}} \qquad \text{(Eq. 2.12)}$$

For $p_a = 2116.2 \, \frac{lb}{ft^2}$, $\beta = 0.00357 \, \frac{°R}{ft}$, $g = 32.174 \, \frac{ft}{s^2}$,
$T_a = 518.67°R$, $R = 1716 \, \frac{ft \cdot lb}{slug \cdot °R}$, and

$$\frac{g}{R\beta} = \frac{32.174 \, \frac{ft}{s^2}}{\left(1716 \, \frac{ft \cdot lb}{slug \cdot °R}\right)\left(0.00357 \, \frac{°R}{ft}\right)} = 5.252$$

then

$$p = \left(2116.2 \, \frac{lb}{ft^2}\right)\left[1 - \frac{\left(0.00357 \, \frac{°R}{ft}\right)(14{,}110 \, ft)}{518.67 \, °R}\right]^{5.252}$$

$$= \underline{\underline{1240 \, \frac{lb}{ft^2} \text{ (abs)}}}$$

(b)
$$p = p_a - \gamma h$$

$$= 2116.2 \, \frac{lb}{ft^2} - \left(0.07647 \, \frac{lb}{ft^3}\right)(14{,}110 \, ft)$$

$$= \underline{\underline{1040 \, \frac{lb}{ft^2} \text{ (abs)}}}$$

(c)
$$p = p_a \, e^{-\frac{gh}{RT_a}} \qquad \text{(Eq. 2.10)}$$

$$= \left(2116.2 \, \frac{lb}{ft^2}\right) e^{-\left[\frac{\left(32.174 \, \frac{ft}{s^2}\right)(14{,}110 \, ft)}{\left(1716 \, \frac{ft \cdot lb}{slug \cdot °R}\right)(518.67°R)}\right]}$$

$$= \underline{\underline{1270 \, \frac{lb}{ft^2} \text{ (abs)}}}$$

2.17

2.17 Equation 2.12 provides the relationship between pressure and elevation in the atmosphere for those regions in which the temperature varies linearly with elevation. Derive this equation and verify the value of the pressure given in Table C.2 in Appendix C for an elevation of 5 km.

$$\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T} \qquad (\text{Eq. } 2.9)$$

Let $p_1 \sim p_a$ for $z_1 = 0$, $p_2 \sim p$ for $z_2 = z$, and $T = T_a - \beta z$.

Thus,

$$\int_{p_a}^{p} \frac{dp}{p} = -\frac{g}{R}\int_{0}^{z} \frac{dz}{T_a - \beta z}$$

or

$$\ln \frac{p}{p_a} = -\frac{g}{R}\left[-\frac{1}{\beta}\ln(T_a - \beta z)\right]_0^z = \frac{g}{R\beta}\left[\ln(T_a - \beta z) - \ln T_a\right]$$

$$= \frac{g}{R\beta}\ln\left(1 - \frac{\beta z}{T_a}\right)$$

and taking logarithm of both sides of equation yields

$$\underline{\underline{p = p_a\left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}}}} \qquad (\text{Eq. } 2.12)$$

For $z = 5\,km$ with $p_a = 101.33\,kPa$, $T_a = 288.15\,K$, $g = 9.807\,\frac{m}{s^2}$, $\beta = 0.00650\,\frac{K}{m}$, $R = 287\,\frac{J}{kg\cdot K}$,

$$p = (101.33\,kPa)\left[1 - \frac{(0.0065\,\frac{K}{m})(5\times10^3\,m)}{288.15\,K}\right]^{\frac{9.807\,\frac{m}{s^2}}{(287\,\frac{J}{kg\cdot K})(0.0065\,\frac{K}{m})}}$$

$$= \underline{\underline{5.40 \times 10^4\,\frac{N}{m^2}}}$$

(From Table C.2 in Appendix C, $p = 5.405\times10^4\,\frac{N}{m^2}$.)

2.18

2.18 As shown in Fig. 2.6 for the U.S. standard atmosphere, the troposphere extends to an altitude of 11 km where the pressure is 22.6 kPa (abs). In the next layer, called the stratosphere, the temperature remains constant at −56.5 °C. Determine the pressure and density in this layer at an altitude of 15 km. Assume $g = 9.77\ \text{m/s}^2$ in your calculations. Compare your results with those given in Table C.2 in Appendix C.

For isothermal conditions,

$$p_2 = p_1\, e^{-\frac{g(z_2 - z_1)}{R T_0}} \qquad \text{(Eq. 2.10)}$$

Let $z_1 = 11\ km$, $p_1 = 22.6\ kPa$, $R = 287\ \frac{J}{kg \cdot K}$, $g = 9.77\ \frac{m}{s^2}$,

and $T_0 = -56.5\,°C + 273.15 = 216.65\ K$.

Thus,

$$p_2 = (22.6\ kPa)\, e^{-\left[\frac{\left(9.77\ \frac{m}{s^2}\right)\left(15 \times 10^3\ m - 11 \times 10^3\ m\right)}{\left(287\ \frac{J}{kg \cdot K}\right)(216.65\ K)}\right]}$$

$$= \underline{\underline{12.1\ kPa}}$$

Also,

$$\rho_2 = \frac{p}{RT} = \frac{12.1 \times 10^3\ \frac{N}{m^2}}{\left(287\ \frac{J}{kg \cdot K}\right)(216.65\ K)} = \underline{\underline{0.195\ \frac{kg}{m^3}}}$$

(From Table C.2 in Appendix C, $p_2 = 12.11\ kPa$ and $\rho_2 = 0.1948\ \frac{kg}{m^3}$.)

2.19

2.19 (See Fluids in the News article titled "**Weather, barometers, and bars,**" Section 2.5.) The record low sea-level barometric pressure ever recorded is 25.8 in. of mercury. At what altitude in the standard atmosphere is the pressure equal to this value?

For record low pressure,

$$p = \gamma_{Hg} h_{Hg} = \left(847 \frac{lb}{ft^3}\right)\left(\frac{25.8 \text{ in.}}{12 \frac{in.}{ft}}\right)\left(\frac{ft^2}{144 \text{ in.}^2}\right) = 12.6 \frac{lb}{in.^2}$$

From Table C.1 in Appendix C

@ 0 ft altitude $\quad p = 14.696 \frac{lb}{in.^2}$

@ 5000 ft altitude $\quad p = 12.228 \frac{lb}{in.^2}$

Assume linear variation change in pressure per foot. Thus,

$$\text{pressure change per foot} = \frac{14.696 \frac{lb}{in.^2} - 12.228 \frac{lb}{in.^2}}{5000 \text{ ft}}$$

$$= 4.936 \times 10^{-4} \frac{lb}{in.^2} \text{ per ft}$$

and

$$14.696 \frac{lb}{in.^2} - d\,(ft)\left[4.936 \times 10^{-4} \frac{\frac{lb}{in.^2}}{ft}\right] = 12.6 \frac{lb}{in.^2}$$

So that $\qquad d = \underline{\underline{4,250 \text{ ft}}}$

2.20

2.20 On a given day, a barometer at the base of the Washington Monument reads 29.97 in. of mercury. What would the barometer reading be when you carry it up to the observation deck 500 ft above the base of the monument?

Let $(\ )_b$ and $(\ )_{od}$ correspond to the base and observation deck, respectively.

Thus, with H = height of the monument,

$$p_b - p_{od} = \gamma_{air} H = 7.65\times10^{-2}\,\frac{lb}{ft^3}\,(500\,ft) = 38.5\,\frac{lb}{ft^2}$$

But

$p = \gamma_{Hg}\, h$, where $\gamma_{Hg} = 847\,\frac{lb}{ft^3}$ and h = barometer reading.

Thus,

$$\gamma_{Hg}\left(\frac{29.97}{12}\,ft\right) - \gamma_{Hg}\, h_{od} = 38.5\,\frac{lb}{ft^2}$$

or

$$h_{od} = \left(\frac{29.97}{12}\,ft\right) - \frac{38.5\,\frac{lb}{ft^2}}{847\,\frac{lb}{ft^3}} = \left[\left(\frac{29.97}{12}\,ft\right) - 0.0455\,ft\right]\left(12\,\frac{in.}{ft}\right)$$

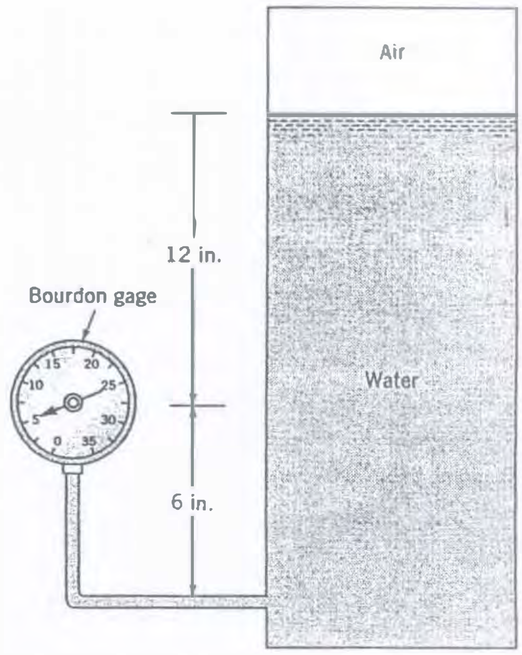
$$= (29.97 - 0.545)\,in.$$

or

$$h_{od} = \underline{\underline{29.43\ in.}}$$

2.21

2.21 Bourdon gages (see Video V2.3 and Fig. 2.13) are commonly used to measure pressure. When such a gage is attached to the closed water tank of Fig. P2.21 the gage reads 5 psi. What is the absolute air pressure in the tank? Assume standard atmospheric pressure of 14.7 psi.



■ FIGURE P2.21

$$p = \gamma h + p_o$$

$$p_{gage} - \left(\frac{12}{12}\ ft\right)\gamma_{H_2O} = p_{air}$$

$$p_{air} = \left(5\ \frac{lb}{in.^2} + 14.7\ \frac{lb}{in.^2}\right) - \frac{(1\ ft)\left(62.4\ \frac{lb}{ft^3}\right)}{144\ \frac{in.^2}{ft^2}}$$

$$p_{air} = \underline{\underline{19.3\ psia}}$$

2.22

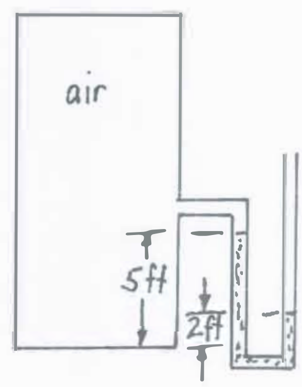
2.22 On the suction side of a pump a Bourdon pressure gage reads 40-kPa vacuum. What is the corresponding absolute pressure if the local atmospheric pressure is 100 kPa (abs)?

$$p(abs) = p(gage) + p(atm)$$

$$= -40\ kPa + 100\ kPa = \underline{\underline{60\ kPa}}$$

2.24

2.24 A water-filled U-tube manometer is used to measure the pressure inside a tank that contains air. The water level in the U-tube on the side that connects to the tank is 5 ft above the base of the tank. The water level in the other side of the U-tube (which is open to the atmosphere) is 2 ft above the base. Determine the pressure within the tank.



$$p_{air} + \gamma_{H_2O}(5\ ft) - \gamma_{H_2O}(2\ ft) = 0$$

or

$$p_{air} = -(3\ ft)\gamma_{H_2O} = -(3\ ft)(62.4\ \frac{lb}{ft^3})$$

$$= \underline{\underline{-187\ \frac{lb}{ft^2}}}$$

2.25

2.25 A barometric pressure of 29.4 in. Hg corresponds to what value of atmospheric pressure in psia, and in pascals?

(In psi) $p = \gamma h = \left(847 \frac{lb}{ft^3}\right)\left(\frac{29.4}{12}\ ft\right)\left(\frac{1\ ft^2}{144\ in.^2}\right) = \underline{\underline{14.4\ psia}}$

(In Pa) $p = \gamma h = \left(133 \times 10^3 \frac{N}{m^3}\right)\left(29.4\ in.\right)\left(2.540 \times 10^{-2} \frac{m}{in.}\right) = \underline{\underline{99.3\ kPa\ (abs)}}$

2.26

2.26 For an atmospheric pressure of 101 kPa (abs) determine the heights of the fluid columns in barometers containing one of the following liquids: **(a)** mercury, **(b)** water, and **(c)** ethyl alcohol. Calculate the heights including the effect of vapor pressure, and compare the results with those obtained neglecting vapor pressure. Do these results support the widespread use of mercury for barometers? Why?

(Including vapor pressure)

$$p(atm) = \gamma h + p_v$$

where $p_v \sim$ vapor pressure

Thus, $$h = \frac{p(atm) - p_v}{\gamma}$$

(a) For mercury: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.6 \times 10^{-1} \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.759\ m}}$$

(b) For water: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.77 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{10.1\ m}}$$

(c) For ethyl alcohol: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 5.9 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{12.3\ m}}$$

(Without vapor pressure)

$$p(atm) = \gamma h$$

$$h = \frac{p(atm)}{\gamma}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.759\ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{10.3\ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{13.0\ m}}$$

<u>Yes</u>. For mercury barameters the effect of vapor pressure is negligible, and the required height of the mercury column is reasonable.

2.27

2.27 A mercury manometer is connected to a large reservoir of water as shown in Fig. P2.27. Determine the ratio, h_w/h_m, of the distances h_w and h_m indicated in the figure.

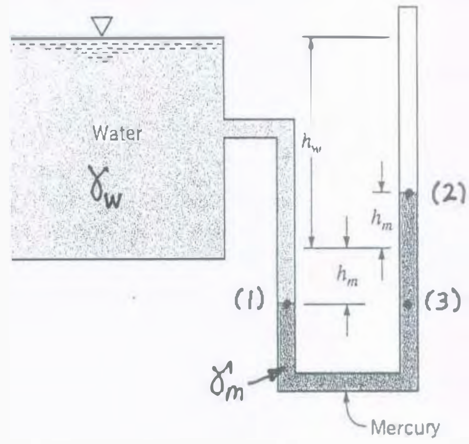


FIGURE P2.27

$p_1 = \gamma_w h_w + \gamma_w h_m$

but $p_1 = p_3 = \gamma_m (2h_m)$

Thus,

$\gamma_w h_w + \gamma_w h_m = 2\gamma_m h_m$

or

$(\gamma_w) h_w = (2\gamma_m - \gamma_w) h_m$

so that

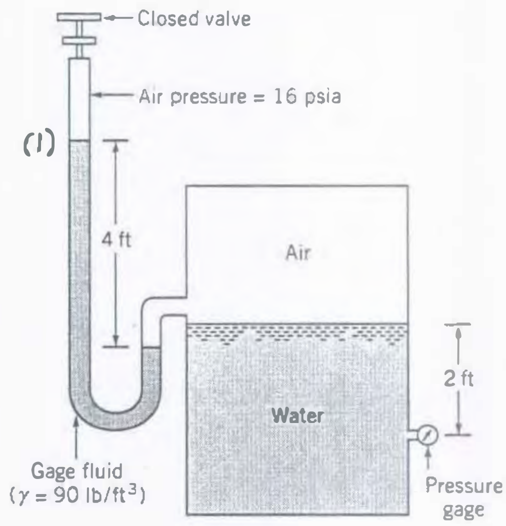
$$\frac{h_w}{h_m} = \frac{(2\gamma_m - \gamma_w)}{\gamma_w} = 2\,SG_m - 1 \text{ , where } SG_m = \frac{\gamma_m}{\gamma_w} = 13.56$$

Thus,

$$\frac{h_w}{h_m} = 2(13.56) - 1 = \underline{\underline{26.1}}$$

2.28

2.28 A U-tube manometer is connected to a closed tank containing air and water as shown in Fig. P2.28. At the closed end of the manometer the air pressure is 16 psia. Determine the reading on the pressure gage for a differential reading of 4 ft on the manometer. Express your answer in psi (gage). Assume standard atmospheric pressure, and neglect the weight of the air columns in the manometer.



■ FIGURE P2.28

$$p_1 + \gamma_{gf}\,(4\text{ ft}) + \gamma_{H_2O}\,(2\text{ ft}) = p_{gage}$$

Thus,

$$p_{gage} = \left(16\,\frac{\text{lb}}{\text{in.}^2} - 14.7\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) + \left(90\,\frac{\text{lb}}{\text{ft}^3}\right)(4\text{ ft})$$

$$+ \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(2\text{ ft})$$

$$= 672\,\frac{\text{lb}}{\text{ft}^2} = \left(672\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}^2}{144\text{ in.}^2}\right) = \underline{\underline{4.67\text{ psi}}}$$

2.29

2.29 A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in Fig. P2.29. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure gage reading at A is 60 kPa, determine: **(a)** the pressure in pipe B, and **(b)** the pressure head, in millimeters of mercury, at the top of the dome (point C).

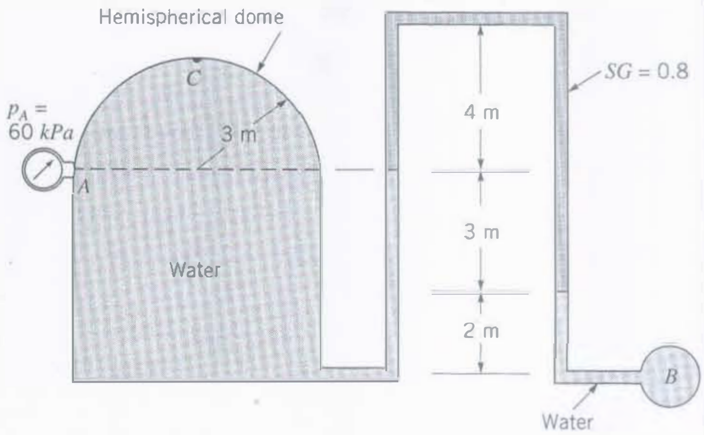


FIGURE P2.29

(a)

$$p_A + (SG)(\gamma_{H_2O})(3\,m) + \gamma_{H_2O}(2\,m) = p_B$$

$$p_B = 60\,kPa + (0.8)\left(9.81 \times 10^3 \frac{N}{m^3}\right)(3\,m) + \left(9.80 \times 10^3 \frac{N}{m^3}\right)(2\,m)$$

$$= \underline{\underline{103\,kPa}}$$

(b)

$$p_C = p_A - \gamma_{H_2O}(3\,m)$$

$$= 60\,kPa - \left(9.80 \times 10^3 \frac{N}{m^3}\right)(3\,m)$$

$$= 30.6 \times 10^3 \frac{N}{m^2}$$

$$h = \frac{p_c}{\gamma_{Hg}} = \frac{30.6 \times 10^3 \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}} = 0.230\,m$$

$$= 0.230\,m\left(\frac{10^3\,mm}{m}\right) = \underline{\underline{230\,mm}}$$

2.30

2.30 Two pipes are connected by a manometer as shown in Fig. P2.30. Determine the pressure difference, $p_A - p_B$, between the pipes.

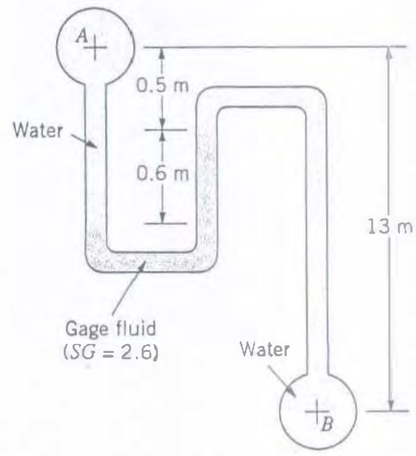


FIGURE P2.30

$$p_A + \gamma_{H_2O}(0.5m + 0.6m) - \gamma_{gf}(0.6m) + \gamma_{H_2O}(1.3m - 0.5m) = p_B$$

Thus,

$$p_A - p_B = \gamma_{gf}(0.6m) - \gamma_{H_2O}(0.5m + 0.6m + 1.3m - 0.5m)$$

$$= (2.6)\left(9.81\frac{kN}{m^3}\right)(0.6m) - \left(9.80\frac{kN}{m^3}\right)(1.9m)$$

$$= -\underline{\underline{3.32\ kPa}}$$

2.31

2.31 A U-tube manometer is connected to a closed tank as shown in Fig. P2.31. The air pressure in the tank is 0.50 psi and the liquid in the tank is oil ($\gamma = 54.0\ \text{lb/ft}^3$). The pressure at point A is 2.00 psi. Determine: **(a)** the depth of oil, z, and **(b)** the differential reading, h, on the manometer.

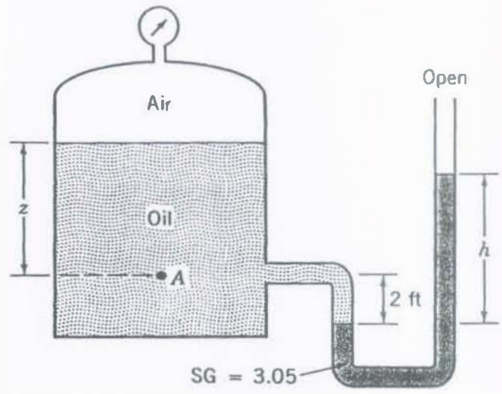


FIGURE P2.31

(a) $p_A = \gamma_{oil}\, z + p_{air}$

Thus,

$$z = \frac{p_A - p_{air}}{\gamma_{oil}} = \frac{\left(2\ \frac{\text{lb}}{\text{in.}^2} - 0.5\ \frac{\text{lb}}{\text{in.}^2}\right)\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right)}{54.0\ \frac{\text{lb}}{\text{ft}^3}} = \underline{\underline{4.00\ \text{ft}}}$$

(b) $p_A + \gamma_{oil}\,(2\ \text{ft}) - (SG)(\gamma_{H_2O})\, h = 0$

Thus,

$$h = \frac{p_A + \gamma_{oil}\,(2\ \text{ft})}{(SG)(\gamma_{H_2O})}$$

$$= \frac{\left(2\ \frac{\text{lb}}{\text{in.}^2}\right)\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right) + \left(54.0\ \frac{\text{lb}}{\text{ft}^3}\right)(2\ \text{ft})}{(3.05)\left(62.4\ \frac{\text{lb}}{\text{ft}^3}\right)}$$

$$= \underline{\underline{2.08\ \text{ft}}}$$

2.32

2.32 For the inclined-tube manometer of Fig. P2.32 the pressure in pipe A is 0.6 psi. The fluid in both pipes A and B is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe B corresponding to the differential reading shown?

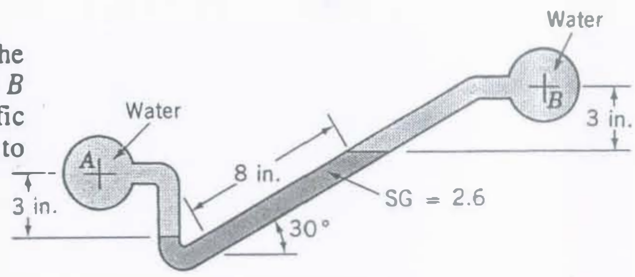


FIGURE P2.32

$$p_A + \gamma_{H_2O}\left(\tfrac{3}{12}\,ft\right) - \gamma_{gf}\left(\tfrac{8}{12}\,ft\right)\sin 30° - \gamma_{H_2O}\left(\tfrac{3}{12}\,ft\right) = p_B$$

(where γ_{gf} is the specific weight of the gage fluid)

Thus,

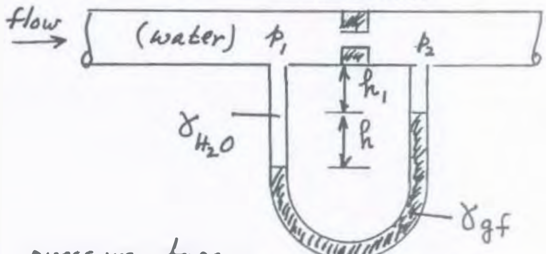
$$p_B = p_A - \gamma_{gf}\left(\tfrac{8}{12}\,ft\right)\sin 30°$$

$$= \left(0.6\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) - (2.6)\left(62.4\,\frac{lb}{ft^3}\right)\left(\tfrac{8}{12}\,ft\right)(0.5) = 32.3\,\frac{lb}{ft^2}$$

$$= 32.3\,lb/ft^2 / 144\,in.^2/ft^2 = \underline{\underline{0.224\ psi}}$$

2.33

2.33 A flowrate measuring device is installed in a horizontal pipe through which water is flowing. A U-tube manometer is connected to the pipe through pressure taps located 3 in. on either side of the device. The gage fluid in the manometer has a specific weight of 112 lb/ft³. Determine the differential reading of the manometer corresponding to a pressure drop between the taps of 0.5 lb/in.².



Let p_1 and p_2 be pressures at pressure taps.
Write manometer equation between p_1 and p_2. Thus,

$$p_1 + \gamma_{H_2O}(h_1 + h) - \gamma_{gf}\,h - \gamma_{H_2O}\,h_1 = p_2$$

so that

$$h = \frac{p_1 - p_2}{\gamma_{gf} - \gamma_{H_2O}} = \frac{\left(0.5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{112\,\frac{lb}{ft^3} - 62.4\,\frac{lb}{ft^3}}$$

$$= \underline{\underline{1.45\ ft}}$$

2.34

2.34 Small differences in gas pressures are commonly measured with a *micromanometer* of the type illustrated in Fig. P2.34. This device consists of two large reservoirs each having a cross-sectional area, A_r, which are filled with a liquid having a specific weight, γ_1, and connected by a U-tube of cross-sectional area, A_t, containing a liquid of specific weight, γ_2. When a differential gas pressure, $p_1 - p_2$, is applied a differential reading, h, develops. It is desired to have this reading sufficiently large (so that it can be easily read) for small pressure differentials. Determine the relationship between h and $p_1 - p_2$ when the area ratio A_t/A_r is small, and show that the differential reading, h, can be magnified by making the difference in specific weights, $\gamma_2 - \gamma_1$, small. Assume that initially (with $p_1 = p_2$) the fluid levels in the two reservoirs are equal.

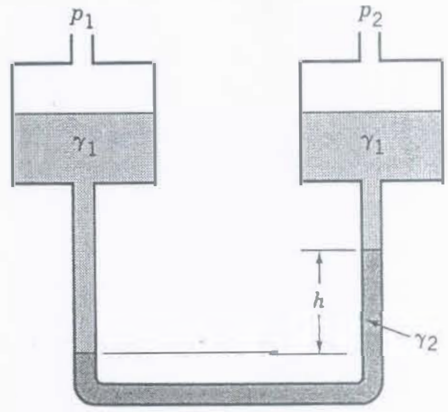
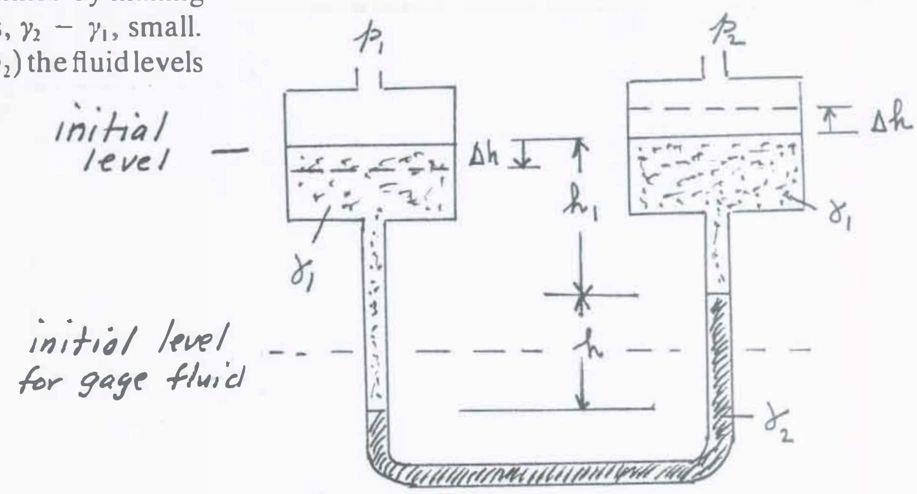


FIGURE P2.34



When a differential pressure, $p_1 - p_2$, is applied we assume that level in left reservoir drops by a distance, Δh, and right level rises by Δh. Thus, the manometer equation becomes

$$p_1 + \gamma_1 (h_1 + h - \Delta h) - \gamma_2 h - \gamma_1 (h_1 + \Delta h) = p_2$$

or

$$p_1 - p_2 = \gamma_2 h - \gamma_1 h + \gamma_1 (2 \Delta h) \qquad (1)$$

Since the liquids in the manometer are incompressible,

$$\Delta h \, A_r = \frac{h}{2} A_t \qquad \text{or} \qquad \frac{2 \Delta h}{h} = \frac{A_t}{A_r}$$

and if $\frac{A_t}{A_r}$ is small then $2\Delta h << h$ and last term in Eq.(1) can be neglected. Thus,

$$p_1 - p_2 = (\gamma_2 - \gamma_1) h$$

or

$$\underline{h = \frac{p_1 - p_2}{\gamma_2 - \gamma_1}}$$

and large values of h can be obtained for small pressure differentials if $\gamma_2 - \gamma_1$ is small.

2.35

2.35 The cyclindrical tank with hemispherical ends shown in Fig. P2.35 contains a volatile liquid and its vapor. The liquid density is 800 kg/m³, and its vapor density is negligible. The pressure in the vapor is 120 kPa (abs), and the atmospheric pressure is 101 kPa (abs). Determine: (a) the gage pressure reading on the pressure gage; and (b) the height, h, of the mercury manometer.

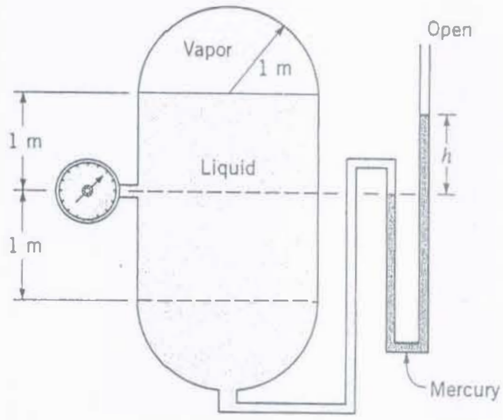


FIGURE P2.35

(a) Let γ_ℓ = sp. wt. of liquid $= \left(800 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) = 7850 \frac{N}{m^3}$

and

$$p_{vapor} \text{ (gage)} = 120 \text{ kPa (abs)} - 101 \text{ kPa (abs)} = 19 \text{ kPa}$$

Thus,

$$p_{gage} = p_{vapor} + \gamma_\ell \,(1\,m)$$

$$= 19 \times 10^3 \frac{N}{m^2} + \left(7850 \frac{N}{m^3}\right)(1\,m)$$

$$= \underline{\underline{26.9 \text{ kPa}}}$$

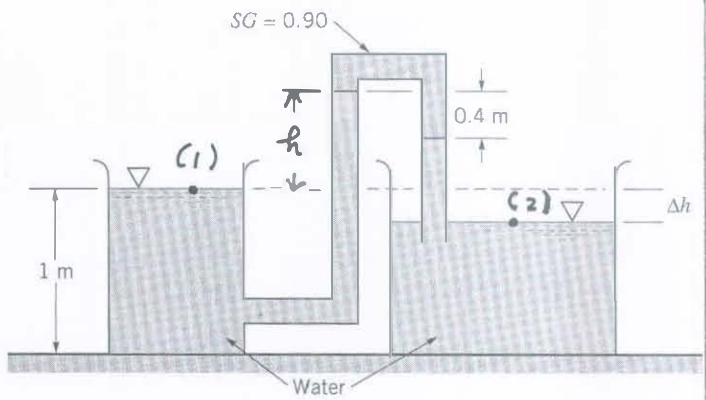
(b) $p_{vapor} \text{ (gage)} + \gamma_\ell \,(1\,m) - \gamma_{Hg}\,(h) = 0$

$$19 \times 10^3 \frac{N}{m^2} + \left(7850 \frac{N}{m^3}\right)(1\,m) - \left(133 \times 10^3 \frac{N}{m^3}\right)(h) = 0$$

$$h = \underline{\underline{0.202 \text{ m}}}$$

2.36

2.36 Determine the elevation difference, Δh, between the water levels in the two open tanks shown in Fig. P2.36.



■ FIGURE P2.36

$$p_1 - \gamma_{H_2O}\, h + (SG)\gamma_{H_2O}(0.4m) + \gamma_{H_2O}(h - 0.4m) + \gamma_{H_2O}(\Delta h) = p_2$$

Since $p_1 = p_2 = 0$

$$\Delta h = 0.4m - (0.9)(0.4m) = \underline{\underline{0.040\,m}}$$

2.37

2.37 For the configuration shown in Fig. P2.37 what must be the value of the specific weight of the unknown fluid? Express your answer in lb/ft³.

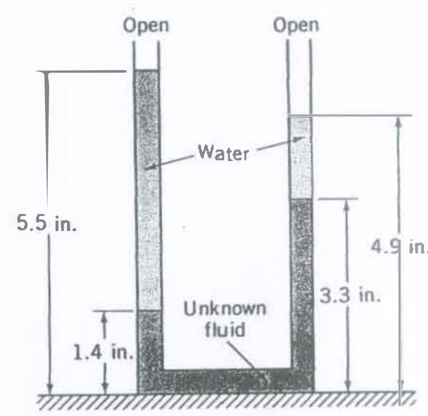


FIGURE P2.37

Let γ be specific weight of unknown fluid. Then,

$$\gamma_{H_2O}\left[\frac{(5.5-1.4)}{12}\,ft\right] - \gamma\left[\frac{(3.3-1.4)}{12}\,ft\right] - \gamma_{H_2O}\left[\frac{(4.9-3.3)}{12}\,ft\right] = 0$$

and

$$\gamma = \frac{\gamma_{H_2O}\left[(5.5-1.4)-(4.9-3.3)\right]in.}{(3.3-1.4)\,in.} = \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{4.1-1.6}{1.9}\right)$$

$$= \underline{\underline{82.1\,\frac{lb}{ft^3}}}$$

2.38

2.38 An air-filled, hemispherical shell is attached to the ocean floor at a depth of 10 m as shown in Fig. P2.38. A mercury barometer located inside the shell reads 765 mm Hg, and a mercury U-tube manometer designed to give the outside water pressure indicates a differential reading of 735 mm Hg as illustrated. Based on these data what is the atmospheric pressure at the ocean surface?

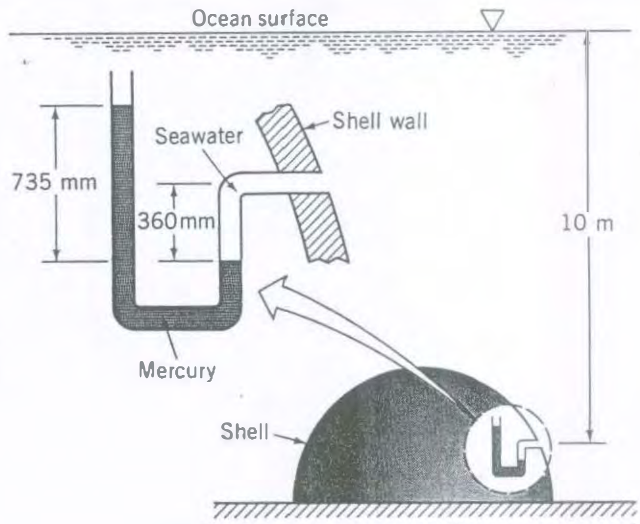


FIGURE P2.38

Let: $p_a \sim$ absolute air pressure inside shell $= \gamma_{Hg}\,(0.765\,m)$

$p_{atm} \sim$ surface atmospheric pressure

$\gamma_{sw} \sim$ specific weight of seawater

Thus, manometer equation can be written as

$$p_{atm} + \gamma_{sw}\,(10\,m) + \gamma_{sw}\,(0.360\,m) - \gamma_{Hg}\,(0.735\,m) = p_a$$

so that

$$p_{atm} = p_a - \gamma_{sw}\,(10.36\,m) + \gamma_{Hg}\,(0.735\,m)$$

$$= \left(133\,\frac{kN}{m^3}\right)(0.765\,m) - \left(10.1\,\frac{kN}{m^3}\right)(10.36\,m) + \left(133\,\frac{kN}{m^3}\right)(0.735\,m)$$

$$= \underline{\underline{94.9\ kPa}}$$

*2.39

*2.39 Both ends of the U-tube mercury manometer of Fig. P2.39 are initially open to the atmosphere and under standard atmospheric pressure. When the valve at the top of the right leg is open the level of mercury below the valve is h_i. After the valve is closed, air pressure is applied to the left leg. Determine the relationship between the differential reading on the manometer and the applied gage pressure, p_g. Show on a plot how the differential reading varies with p_g for h_i = 25, 50, 75, and 100 mm over the range $0 \le p_g \le 300$ kPa. Assume that the temperature of the trapped air remains constant.

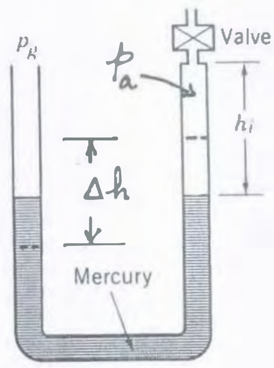


FIGURE P2.39

With the valve closed and a pressure, p_g, applied,

$$p_g - \gamma_{Hg}\,\Delta h = p_a$$

or

$$\Delta h = \frac{p_g - p_a}{\gamma_{Hg}} \qquad (1)$$

where p_g and p_a are gage pressures. For isothermal compression of trapped air

$$\frac{p}{\rho} = \text{Constant}$$

so that for constant air mass

$$p_i \forall_i = p_f \forall_f$$

where $\forall$ is air volume, p is absolute pressure, and i and f refer to initial and final states, respectively. Thus,

$$p_{atm} \forall_i = (p_a + p_{atm}) \forall_f \qquad (2)$$

For air trapped in right leg, $\forall_i = h_i$ (Area of tube) so that Eq. (2) can be written as

$$p_a = p_{atm}\left[\frac{h_i}{h_i - \frac{\Delta h}{2}} - 1\right] \qquad (3)$$

Substitute Eq. (3) into Eq. (1) to obtain

$$\Delta h = \frac{1}{\gamma_{Hg}}\left[p_g + p_{atm}\left(1 - \frac{h_i}{h_i - \frac{\Delta h}{2}}\right)\right] \qquad (4)$$

(con't)

*2.39 (con't)

Equation (4) can be expressed in the form

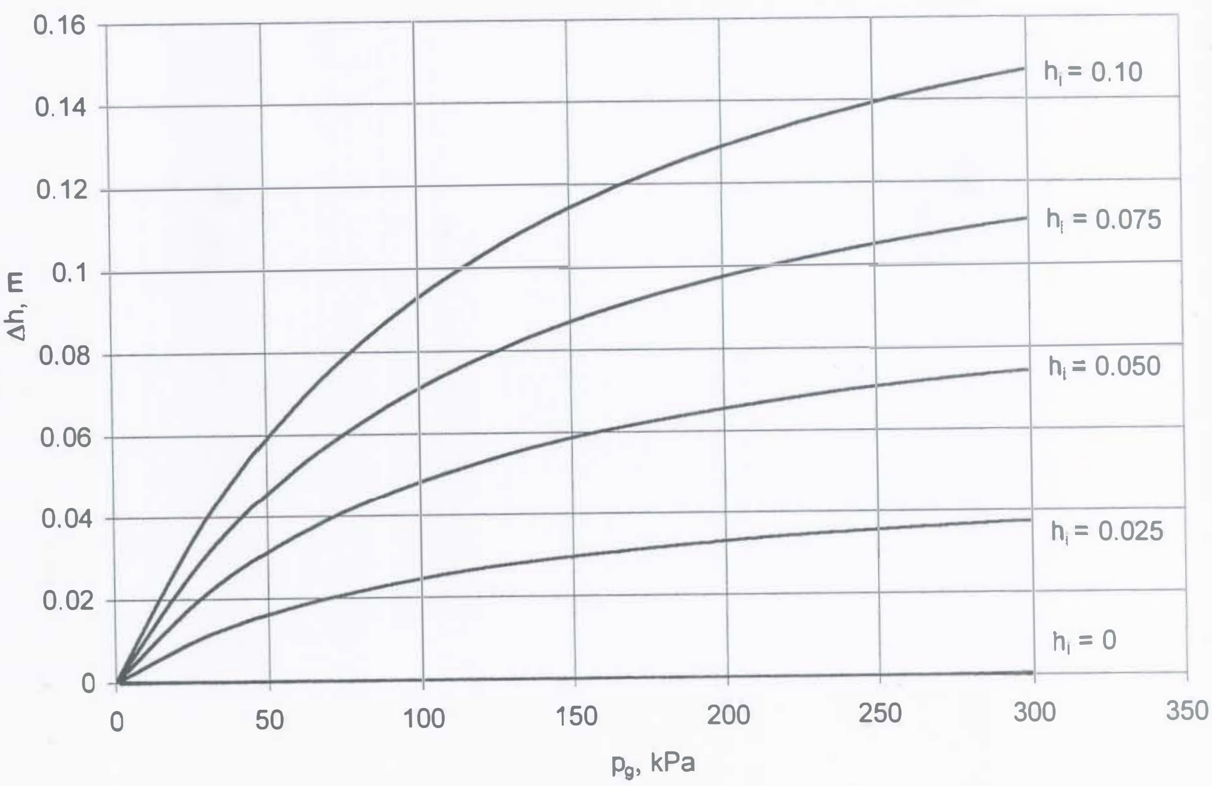
$$(\Delta h)^2 - \left(2h_i + \frac{p_g + p_{atm}}{\gamma_{Hg}}\right)\Delta h + \frac{2p_g h_i}{\gamma_{Hg}} = 0$$

and the roots of this quadratic equation are

$$\Delta h = \left(h_i + \frac{p_g + p_{atm}}{2\gamma_{Hg}}\right) \pm \sqrt{\left(h_i + \frac{p_g + p_{atm}}{2\gamma_{Hg}}\right)^2 - \frac{2p_g h_i}{\gamma_{Hg}}} \quad (5)$$

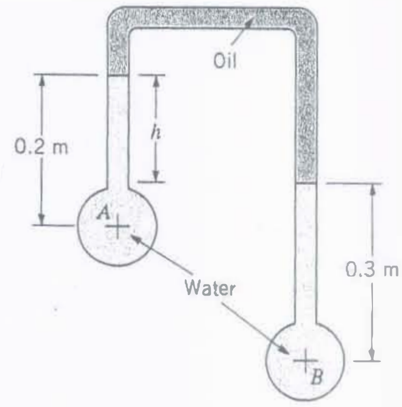
To evaluate Δh the negative sign is used since $\Delta h = 0$ for $p_g = 0$. Tabulated values of Δh for various values of p_g are given in the following table for different values of h_i (with $p_{atm} = 101$ kPa and $\gamma_{Hg} = 133$ kN/m³). A plot of the data follows.

h_i (m)	patm (kPa)	γ hg (kN/m3)	p_g (kPa)	$\Delta h(h_i = 0)$ (m)	$\Delta h(h_i=0.025)$ (m)	$\Delta h(h_i=0.05)$ (m)	$\Delta h(h_i=0.075)$ (m)	$\Delta h(h_i=0.1)$ (m)
0.025	101	133	0	0	0	0	0	0
0.05	101	133	30	0	0.0110	0.0212	0.0306	0.0394
0.075	101	133	60	0	0.0182	0.0354	0.0517	0.0672
0.1	101	133	90	0	0.0231	0.0454	0.0668	0.0874
	101	133	120	0	0.0268	0.0528	0.0781	0.1026
	101	133	150	0	0.0296	0.0585	0.0867	0.1143
	101	133	180	0	0.0318	0.0630	0.0936	0.1236
	101	133	210	0	0.0335	0.0666	0.0991	0.1312
	101	133	240	0	0.0350	0.0696	0.1037	0.1374
	101	133	270	0	0.0362	0.0721	0.1075	0.1426
	101	133	300	0	0.0372	0.0742	0.1108	0.1470



2.40

2.40 The inverted U-tube manometer of Fig. P2.40 contains oil ($SG = 0.9$) and water as shown. The pressure differential between pipes A and B, $p_A - p_B$, is -5 kPa. Determine the differential reading, h.



■ FIGURE P2.40

$$p_A - \gamma_{H_2O}(0.2\,m) + \gamma_{oil}(h) + \gamma_{H_2O}(0.3\,m) = p_B$$

Thus,

$$h = \frac{(p_B - p_A) + \gamma_{H_2O}(0.2\,m) - \gamma_{H_2O}(0.3\,m)}{\gamma_{oil}}$$

$$= \frac{5\times10^3\,\frac{N}{m^2} - \left(9.80\times10^3\,\frac{N}{m^3}\right)(0.1\,m)}{8.95\times10^3\,\frac{N}{m^3}} = \underline{\underline{0.449\,m}}$$

2.41

2.41 An inverted U-tube manometer containing oil ($SG = 0.8$) is located between two reservoirs as shown in Fig. P2.41. The reservoir on the left, which contains carbon tetrachloride, is closed and pressurized to 8 psi. The reservoir on the right contains water and is open to the atmosphere. With the given data, determine the depth of water, h, in the right reservoir.

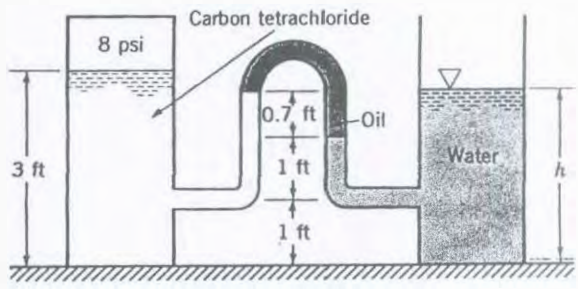


FIGURE P2.41

Let p_A be the air pressure in left reservoir. Manometer equation can be written as

$$p_A + \gamma_{CCl_4}(3\text{ ft} - 1\text{ ft} - 1\text{ ft} - 0.7\text{ ft}) + \gamma_{oil}(0.7\text{ ft}) - \gamma_{H_2O}(h - 1\text{ ft} - 1\text{ ft}) = 0$$

so that

$$h = \frac{p_A + \gamma_{CCl_4}(0.3\text{ ft}) + \gamma_{oil}(0.7\text{ ft})}{\gamma_{H_2O}} + 2\text{ ft}$$

$$= \frac{\left(8\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right) + \left(99.5\frac{lb}{ft^3}\right)(0.3\text{ ft}) + \left(57.0\frac{lb}{ft^3}\right)(0.7\text{ ft})}{62.4\frac{lb}{ft^3}} + 2\text{ ft}$$

$$= \underline{\underline{21.6\text{ ft}}}$$

2.42

2.42 Determine the pressure of the water in pipe A shown in Fig. P2.42 if the gage pressure of the air in the tank is 2 psi.

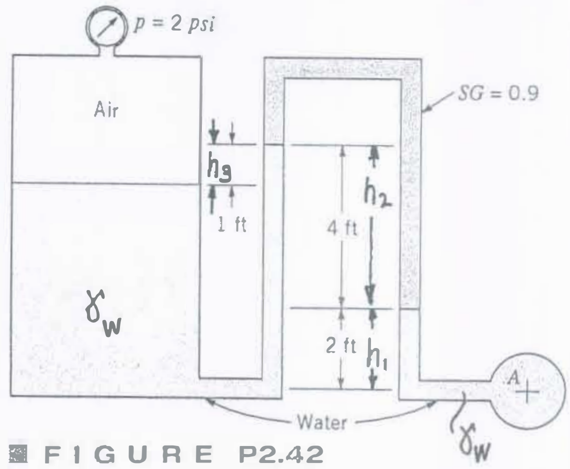


FIGURE P2.42

$$p_A - \gamma_w h_1 - (0.9\gamma_w) h_2 + \gamma_w h_3 = p_{air}$$

or

$$p_A = p_{air} + \gamma_w (h_1 + 0.9 h_2 - h_3)$$

$$= 2\,\frac{\text{lb}}{\text{in.}^2}\left(\frac{144\,\text{in.}^2}{\text{ft}^2}\right) + 62.4\,\frac{\text{lb}}{\text{ft}^3}\left(\cdot\ \text{ft} + 0.9(4\,\text{ft}) - 1\,\text{ft}\right)$$

$$= \underline{\underline{575\,\frac{\text{lb}}{\text{ft}^2}}}$$

2.43

2.43 In Fig. P2.43 pipe A contains gasoline ($SG = 0.7$), pipe B contains oil ($SG = 0.9$), and the manometer fluid is mercury. Determine the new differential reading if the pressure in pipe A is decreased 25 kPa, and the pressure in pipe B remains constant. The initial differential reading is 0.30 m as shown.

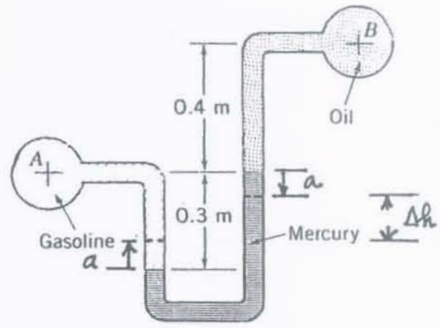


FIGURE P2.43

For the initial configuration:

$$p_A + \gamma_{gas}(0.3\,m) - \gamma_{Hg}(0.3\,m) - \gamma_{oil}(0.4\,m) = p_B \qquad (1)$$

With a decrease in p_A to p_A' gage fluid levels change as shown on figure. Thus, for final configuration:

$$p_A' + \gamma_{gas}(0.3-a) - \gamma_{Hg}(\Delta h) - \gamma_{oil}(0.4+a) = p_B \qquad (2)$$

where all lengths are in m. Subtract Eq.(2) from Eq.(1) to obtain,

$$p_A - p_A' + \gamma_{gas}(a) - \gamma_{Hg}(0.3-\Delta h) + \gamma_{oil}(a) = 0 \qquad (3)$$

Since $2a + \Delta h = 0.3$ (see figure) then

$$a = \frac{0.3-\Delta h}{2}$$

and from Eq.(3)

$$p_A - p_A' + \gamma_{gas}\left(\frac{0.3-\Delta h}{2}\right) - \gamma_{Hg}(0.3-\Delta h) + \gamma_{oil}\left(\frac{0.3-\Delta h}{2}\right) = 0$$

Thus,

$$\Delta h = \frac{p_A - p_A' + \gamma_{gas}(0.15) - \gamma_{Hg}(0.3) + \gamma_{oil}(0.15)}{-\gamma_{Hg} + \frac{\gamma_{gas}}{2} + \frac{\gamma_{oil}}{2}}$$

and with $p_A - p_A' = 25\,kPa$

$$\Delta h = \frac{25\frac{kN}{m^2} + (0.7)(9.81\frac{kN}{m^3})(0.15\,m) - (133\frac{kN}{m^3})(0.3\,m) + (0.9)(9.81\frac{kN}{m^3})(0.15\,m)}{-133\frac{kN}{m^3} + \frac{(0.7)(9.81\frac{kN}{m^3})}{2} + \frac{(0.9)(9.81\frac{kN}{m^3})}{2}}$$

$$= \underline{\underline{0.100\,m}}$$

2.44

2.44 The inclined differential manometer of Fig. P2.44 contains carbon tetrachloride. Initially the pressure differential between pipes A and B, which contain a brine ($SG = 1.1$), is zero as illustrated in the figure. It is desired that the manometer give a differential reading of 12 in. (measured along the inclined tube) for a pressure differential of 0.1 psi. Determine the required angle of inclination, θ.

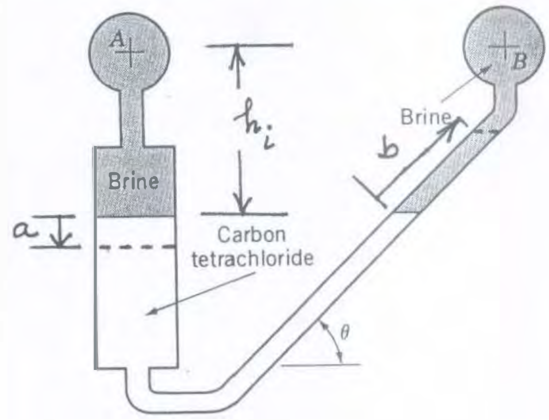


FIGURE P2.44

When $p_A - p_B$ is increased to $p_A' - p_B'$ the left column falls a distance, a, and the right column rises a distance b along the inclined tube as shown in figure. For this final configuration:

$$p_A' + \gamma_{br}(h_i + a) - \gamma_{CCl_4}(a + b\sin\theta) - \gamma_{br}(h_i - b\sin\theta) = p_B'$$

or

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(a + b\sin\theta) = 0 \qquad (1)$$

The differential reading, Δh, along the tube is

$$\Delta h = \frac{a}{\sin\theta} + b$$

Thus, from Eq. (1)

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(\Delta h \sin\theta) = 0$$

or

$$\sin\theta = \frac{-(p_A' - p_B')}{(\gamma_{br} - \gamma_{CCl_4})(\Delta h)}$$

and with $p_A' - p_B' = 0.1\,psi$

$$\sin\theta = \frac{-\left(0.1\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left[(1.1)\left(62.4\,\frac{lb}{ft^3}\right) - 99.5\,\frac{lb}{ft^3}\right]\left(\frac{12}{12}\,ft\right)} = 0.466$$

for $\Delta h = 12$ in.

Thus,

$$\underline{\underline{\theta = 27.8^\circ}}$$

2.45

2.45 Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.45, if the pressure in pipe A is decreased 10 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.9 and the fluid in B is water.

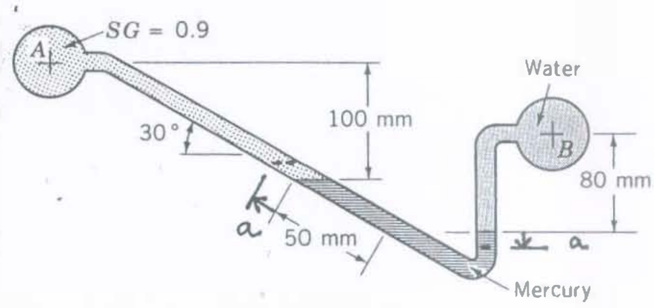


FIGURE P2.45

For the initial configuration:

$$p_A + \gamma_A(0.1) + \gamma_{Hg}(0.05\sin 30°) - \gamma_{H_2O}(0.08) = p_B \qquad (1)$$

where all lengths are in m. When p_A decreases left column moves up a distance, a, and right column moves down a distance, a, as shown in figure. For the final configuration:

$$p_A' + \gamma_A(0.1 - a\sin 30°) + \gamma_{Hg}(a\sin 30° + 0.05\sin 30° + a) - \gamma_{H_2O}(0.08 + a) = p_B \qquad (2)$$

where p_A' is the new pressure in pipe A.
Subtract Eq. (2) from Eq. (1) to obtain

$$p_A - p_A' + \gamma_A(a\sin 30°) - \gamma_{Hg}\,a(\sin 30° + 1) + \gamma_{H_2O}(a) = 0$$

Thus,

$$a = \frac{-(p_A - p_A')}{\gamma_A\sin 30° - \gamma_{Hg}(\sin 30° + 1) + \gamma_{H_2O}}$$

For $p_A - p_A' = 10\ kPa$

$$a = \frac{-10\ \frac{kN}{m^2}}{(0.9)(9.81\ \frac{kN}{m^3})(0.5) - (133\ \frac{kN}{m^3})(0.5+1) + 9.80\ \frac{kN}{m^3}}$$

$$= 0.0540\ m$$

New differential reading, Δh, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a$$

$$= \frac{0.0540\ m}{0.5} + 0.05\,m + 0.0540\,m = \underline{\underline{0.212\ m}}$$

2.46

2.46 Determine the change in the elevation of the mercury in the left leg of the manometer of Fig. P2.46 as a result of an increase in pressure of 5 psi in pipe A while the pressure in pipe B remains constant.

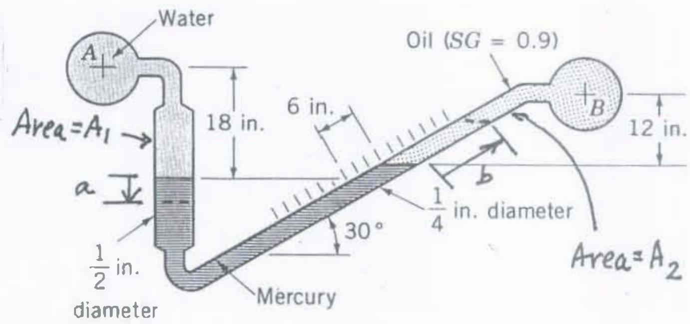


FIGURE P2.46

For the initial configuration:

$$p_A + \gamma_{H_2O}\left(\frac{18}{12}\right) - \gamma_{Hg}\left(\frac{6}{12}\sin 30°\right) - \gamma_{oil}\left(\frac{12}{12}\right) = p_B \qquad (1)$$

where all lengths are in ft. When p_A increases to p_A' the left column falls by the distance, a, and the right column moves up the distance, b, as shown in the figure. For the final configuration:

$$p_A' + \gamma_{H_2O}\left(\frac{18}{12}+a\right) - \gamma_{Hg}\left(a + \frac{6}{12}\sin 30° + b\sin 30°\right) -$$
$$\gamma_{oil}\left(\frac{12}{12} - b\sin 30°\right) = p_B \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + b\sin 30°) + \gamma_{oil}(b\sin 30°) = 0 \qquad (3)$$

Since the volume of liquid must be constant $A_1 a = A_2 b$,

or $$\left(\tfrac{1}{2}\text{ in.}\right)^2 a = \left(\tfrac{1}{4}\text{ in.}\right)^2 b$$

so that $$b = 4a$$

Thus, Eq. (3) can be written as

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + 4a\sin 30°) + \gamma_{oil}(4a\sin 30°) = 0$$

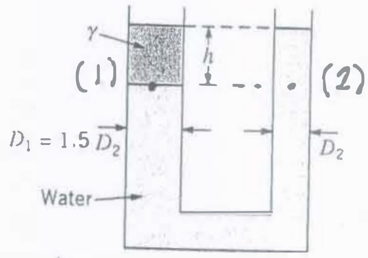
and

$$a = \frac{-(p_A' - p_A)}{\gamma_{H_2O} - \gamma_{Hg}(3) + \gamma_{oil}(2)} = \frac{-\left(5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{62.4\,\frac{lb}{ft^3} - \left(847\,\frac{lb}{ft^3}\right)(3) + (0.9)\left(62.4\,\frac{lb}{ft^3}\right)(2)}$$

$$= \underline{\underline{0.304\text{ ft (down)}}}$$

2.47

2.47 The U-shaped tube shown in Fig. P2.47 initially contains water only. A second liquid with specific weight, γ, less than water is placed on top of the water with no mixing occurring. Can the height, h, of the second liquid be adjusted so that the left and right levels are at the same height? Provide proof of your answer.



■ FIGURE P2.47

The pressure at point (1) must be equal to the pressure at point (2) since the pressures at equal elevations in a continuous mass of fluid must be the same. Since,

$$p_1 = \gamma h$$

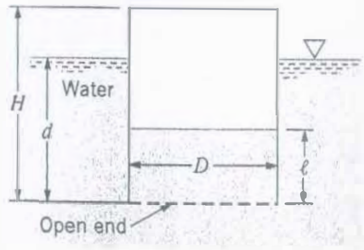
and

$$p_2 = \gamma_{H_2O} h$$

these two pressures can only be equal if $\gamma = \gamma_{H_2O}$. Since $\gamma \neq \gamma_{H_2O}$ the configuration shown in the figure is not possible. <u>No.</u>

*2.48

*2.48 An inverted hollow cylinder is pushed into the water as is shown in Fig. P2.48. Determine the distance, ℓ, that the water rises in the cylinder as a function of the depth, d, of the lower edge of the cylinder. Plot the results for $0 \le d \le H$, when H is equal to 1 m. Assume the temperature of the air within the cylinder remains constant.



■ FIGURE P2.48

For constant temperature compression within the cylinder,

$$p_i \forall_i = p_f \forall_f \qquad (1)$$

where $\forall$ is the air volume, and i and f refer to the initial and final states, respectively. It follows that (see figure)

$$p_i = p_{atm} \qquad p_f = \gamma(d-\ell) + p_{atm}$$

$$\forall_i = \frac{\pi}{4}D^2 H \qquad \forall_f = \frac{\pi}{4}D^2(H-\ell)$$

Thus, from Eq. (1)

$$p_{atm}\left(\frac{\pi}{4}D^2 H\right) = \left(\gamma(d-\ell) + p_{atm}\right)\frac{\pi}{4}D^2(H-\ell) \qquad (2)$$

and with

$$p_{atm} = 101\ kPa, \quad \gamma = 9.80\ \frac{kN}{m^3}, \text{ and } \quad H = 1\ m$$

Eq. (2) simplifies to

$$\ell^2 - (d+11.31)\ell + d\,(1m) = 0$$

so that (using the quadratic formula)

$$\ell = \frac{(d+11.31) \pm \sqrt{d^2 + 18.61\,d + 128}}{2}$$

Since for $d=0$, $\ell=0$, the negative sign should be used and

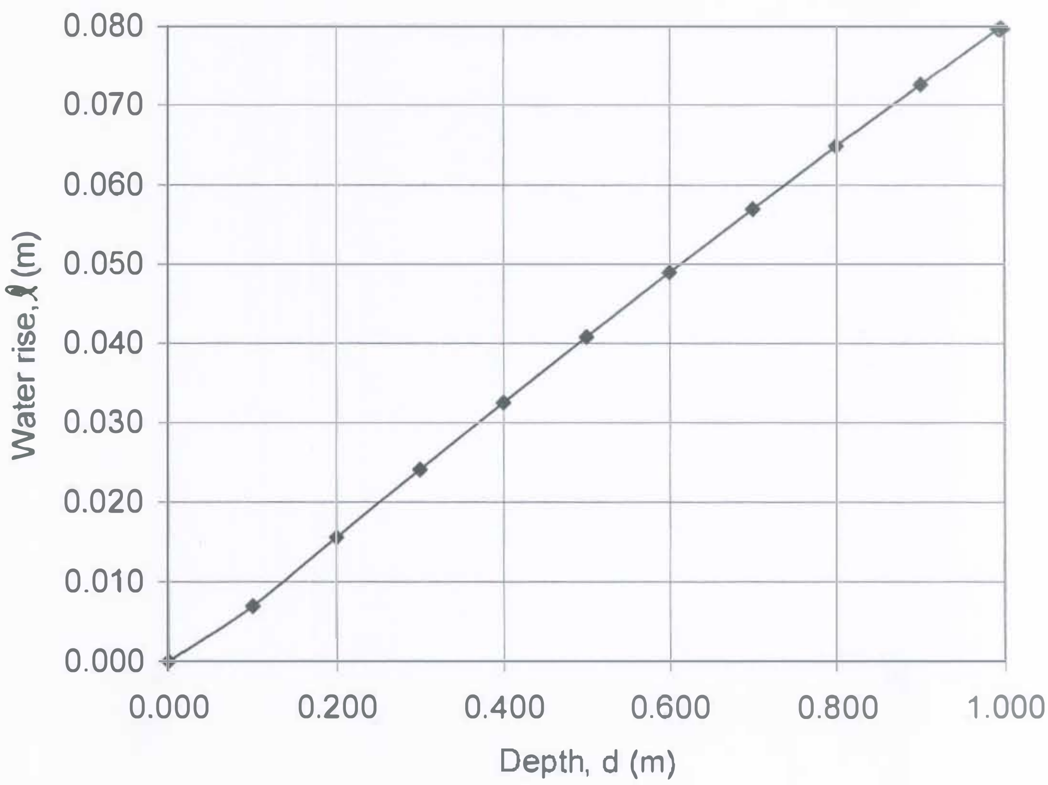
$$\ell = \frac{(d+11.31) - \sqrt{d^2 + 18.61\,d + 128}}{2}$$

Tabulated data with the corresponding plot are shown on the following page.

(con't)

*2.48 (con't)

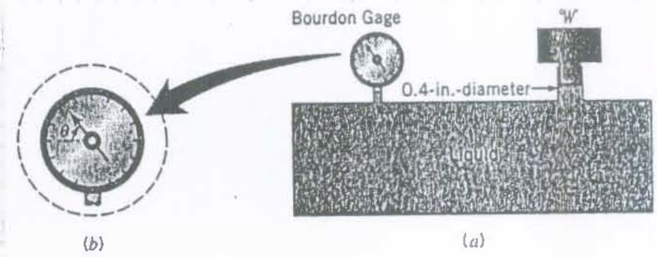
Depth, d (m)	Water rise, ℓ, (m)
0.000	0.000
0.100	0.007
0.200	0.016
0.300	0.024
0.400	0.033
0.500	0.041
0.600	0.049
0.700	0.057
0.800	0.065
0.900	0.073
1.000	0.080



*2.50

*2.50 A Bourdon gage (see Fig. 2.13 and Video V2.3) is often used to measure pressure. One way to calibrate this type of gage is to use the arangement shown in Fig. P2.50a. The container is filled with a liquid and a weight, $\mathcal{W}$, placed on one side with the gage on the other side. The weight acting on the liquid through a 0.4-in.-diameter opening creates a pressure that is transmitted to the gage. This arrangement, with a series of weights, can be used to determine what a change in the dial movement, θ, in Fig. P2.50b. corresponds to in terms of a change in pressure. For a particular gage, some data are given below. Based on a plot of these data, determine the relationship between θ and the pressure, p, where p is measured in psi?

$\mathcal{W}$ (lb)	0	1.04	2.00	3.23	4.05	5.24	6.31
θ (deg.)	0	20	40	60	80	100	120



■ FIGURE P2.50

$$p = \frac{\mathcal{W}}{Area} = \frac{\mathcal{W}\,(lb)}{\frac{\pi}{4}(0.4\ in.)^2} = 7.96\,\mathcal{W}(lb) \qquad (1)$$

(where p is in psi)

From graph

$$\mathcal{W} = 0.0522\,\theta$$

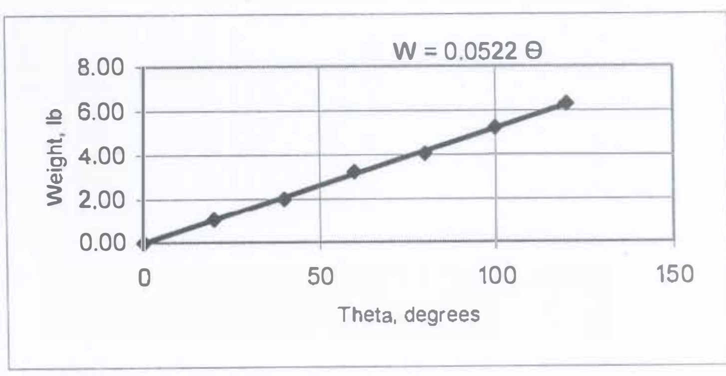
so that from Eq. (1)

$$\frac{p\ (psi)}{7.96} = 0.0522\,\theta$$

and

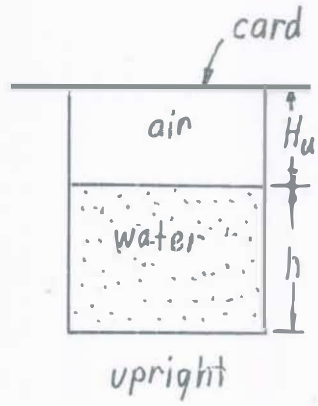
$$\underline{\underline{p(psi) = 0.416\,\theta}}$$

Theta, deg.	W, lb
0	0.00
20	1.04
40	2.00
60	3.23
80	4.05
100	5.24
120	6.31



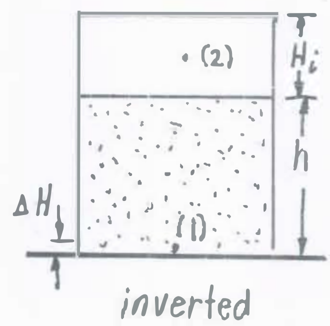
2.51

2.51 You partially fill a glass with water, place an index card on top of the glass, and then turn the glass upside down while holding the card in place. You can then remove your hand from the card and the card remains in place, holding the water in the glass. Explain how this works.



In order to hold the index card in place when the glass is inverted, the pressure at the card-water interface, p_1, must be $p_1 A = -\mathcal{W}$, where A is the area of the glass opening and $\mathcal{W}$ is the card weight. Thus, $p_1 = -\mathcal{W}/A$. Hence, $p_2 = p_1 - \gamma h$, or

$p_2 = -\mathcal{W}/A - \gamma h$ (gage).



Since the amount of air in the glass remains the same when it is inverted,

$\rho_u A H_u = \rho_i A H_i$, where u and i subscripts refer to the upright and inverted conditions. Thus,

(1) $H_i = \frac{\rho_u}{\rho_i} H_u$ But $p = \rho R T$ so that

(2) $\frac{\rho_u}{\rho_i} = \frac{(p_u/RT_u)}{(p_i/RT_i)} = \frac{p_u}{p_i}$ provided the temperature remains constant: $T_i = T_u$. Note: Since we are using the perfect gas law the pressures must be absolute — $p_u = p_{atm}$, $p_i = p_2 = -\mathcal{W}/A - \gamma h + p_{atm}$

Hence, from Eqs. (1) and (2):

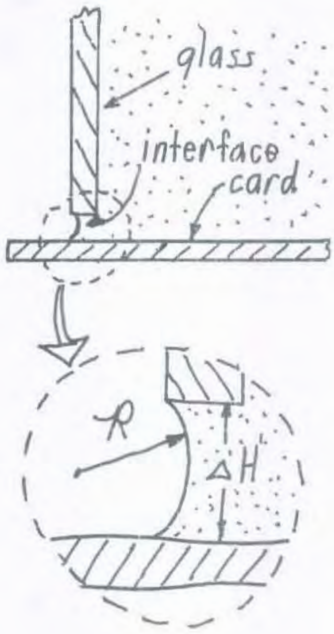
(3) $H_i = \left(\frac{p_{atm}}{p_{atm} - \mathcal{W}/A - \gamma h}\right) H_u$ That is, when the glass is inverted the column of air inside expans slightly, causing a small gap of size ΔH between the lip is the glass and the index card. From Eq. (3) this ΔH is

(4) $\Delta H = H_i - H_u = \left(\frac{p_{atm}}{p_{atm} - \mathcal{W}/A - \gamma h}\right) H_u - H_u = \left(\frac{\mathcal{W}/A + \gamma h}{p_{atm} - \mathcal{W}/A - \gamma h}\right) H_u$

If this gap is "large enough" the water would flow out of the glass and air into it. If it is "small enough," surface tension will allow the slight pressure difference across the air-water interface (i.e., $p_1 = -\mathcal{W}/A$) needed to prevent flow and thus keep the index card in place. Recall from Equation (1.21) in Section 1.9

(con't)

2.51 (con't)



that the pressure diffenence across an interface is proportional to the surface tension of the liquid, σ, and the radius of curvature, $\mathcal{R}$, of the interface.
That is, $p_1 \sim \sigma / \mathcal{R}$
Thus, for small enough gap, ΔH, which gives a small enough interface radius of curvature, $\mathcal{R}$, surface tension is large enough to keep the water from flowing and the index card remains in place.

Consider some typical numbers to obtain an approximation of the gap produced.

Assume $h = 3\,in. = 0.25\,ft$, $H_u = 2\,in. = 0.167\,ft$, $p_{atm} = 14.7\,psia$, and $\mathcal{W}/A \ll \gamma h$. That is, the weight of the card is much less than the weight of the water in the glass (i.e., $\mathcal{W} \ll \gamma A h$).
Hence, from Eq. (4):

$$\Delta H = \left(\frac{\gamma h}{p_{atm} - \gamma h} \right) H_u = \left[\frac{62.4 \frac{lb}{ft^3} (0.25\,ft)}{(14.7 \frac{lb}{in.^2})(144 \frac{in.^2}{ft^2}) - 62.4 \frac{lb}{ft^3} (0.25\,ft)} \right] (0.167\,ft)$$

or

$$\Delta H = 0.00124\,ft = 0.0149\,in.$$

This is apparently a small enough gap to allow surface tension to keep the water in the glass, air out of it, and the pressure at the water-card interface low enough to keep the card in place.

2.52

2.52 A piston having a cross-sectional area of 0.07 m² is located in a cylinder containing water as shown in Fig. P2.52. An open U-tube manometer is connected to the cylinder as shown. For $h_1 = 60$ mm and $h = 100$ mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.

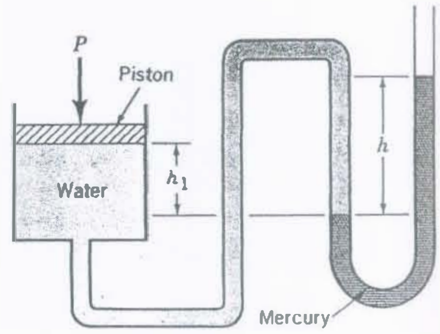


FIGURE P2.52

For equilibrium, $P = p_p A_p$ where p_p is the pressure acting on piston and A_p is the area of the piston. Also,

$$p_p + \gamma_{H_2O} h_1 - \gamma_{Hg} h = 0$$

or

$$p_p = \gamma_{Hg} h - \gamma_{H_2O} h_1$$

$$= \left(133 \frac{kN}{m^3}\right)(0.100\,m) - \left(9.80 \frac{kN}{m^3}\right)(0.060\,m)$$

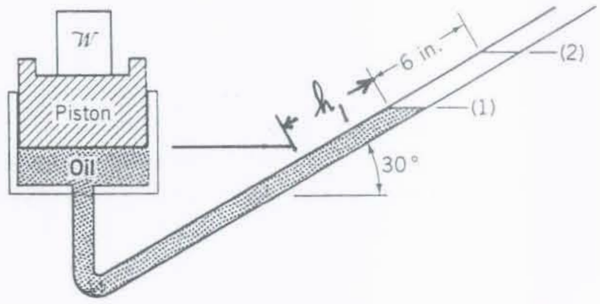
$$= 12.7 \frac{kN}{m^2}$$

Thus,

$$P = \left(12.7 \times 10^3 \frac{N}{m^2}\right)\left(0.07\,m^2\right) = \underline{\underline{889\,N}}$$

2.53

2.53 A 6-in.-diameter piston is located within a cylinder which is connected to a $\frac{1}{2}$-in.-diameter inclined-tube manometer as shown in Fig. P2.53. The fluid in the cylinder and the manometer is oil (specific weight = 59 lb/ft³). When a weight $\mathcal{W}$ is placed on the top of the cylinder the fluid level in the manometer tube rises from point (1) to (2). How heavy is the weight? Assume that the change in position of the piston is negligible.



■ FIGURE P2.53

With piston alone let pressure on face of piston = p_p, and manometer equation becomes

$$p_p - \gamma_{oil}\, h_1 \sin 30° = 0 \qquad (1)$$

With weight added pressure p_p increases to p_p' where

$$p_p' = p_p + \frac{\mathcal{W}}{A_p} \qquad (A_p \sim \text{area of piston})$$

and manometer equation becomes

$$p_p' - \gamma_{oil}\left(h_1 + \tfrac{6}{12}\text{ ft}\right) \sin 30° = 0 \qquad (2)$$

Subtract Eq.(1) from Eq.(2) to obtain

$$p_p' - p_p - \gamma_{oil}\left(\tfrac{6}{12}\text{ ft}\right)\sin 30° = 0$$

or

$$\frac{\mathcal{W}}{A_p} = \gamma_{oil}\left(\tfrac{6}{12}\text{ ft}\right)\sin 30°$$

so that

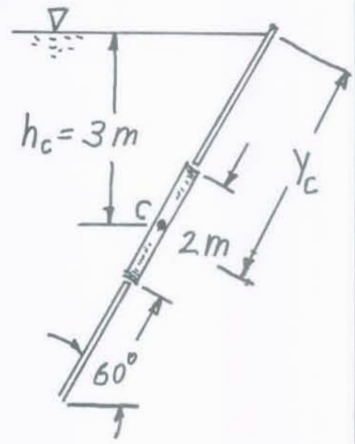
$$\frac{\mathcal{W}}{\frac{\pi}{4}\left(\frac{6}{12}\text{ ft}\right)^2} = \left(59\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\tfrac{6}{12}\text{ ft}\right)(0.5)$$

and

$$\mathcal{W} = \underline{\underline{2.90 \text{ lb}}}$$

2.54

2.54 A circular 2-m-diameter gate is located on the sloping side of a swimming pool. The side of the pool is oriented 60° relative to the horizontal bottom, and the center of the gate is located 3 m below the water surface. Determine the magnitude of the water force acting on the gate and the point through which it acts.



$F_R = p_c A = \gamma h_c A$, where $h_c = 3m$

Thus,

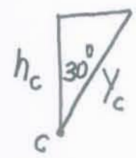
$$F_R = (9.8\ \frac{kN}{m^3})(3m)(\frac{\pi}{4}(2m)^2) = \underline{\underline{94.2\ kN}}$$

Also,

$$y_R - y_c = \frac{I_{xc}}{y_c A}$$, where for a circle $I_{xc} = \frac{\pi R^4}{4} = \frac{\pi (1m)^4}{4} = \frac{\pi}{4} m^4$

and $\cos 30° = \frac{h_c}{y_c}$ so that

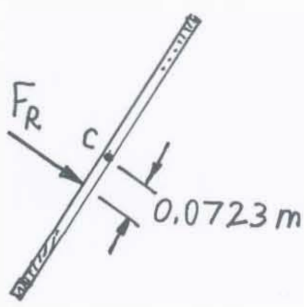
$$y_c = \frac{h_c}{\cos 30°} = \frac{3\ m}{\cos 30°} = 3.46\ m$$



Hence,

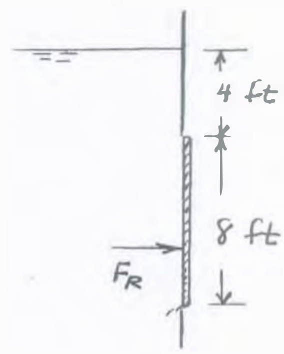
$$y_R - y_c = \frac{I_{xc}}{y_c A} = \frac{\frac{\pi}{4} m^4}{(3.46m)\frac{\pi}{4}(2m)^2} = \underline{\underline{0.0723\ m}}$$

Thus, the resultant force acts normal to the gate and 0.0723 m from the centroid, along the gate.



2.55

2.55 A vertical rectangular gate is 8 ft wide and 10 ft long and weighs 6000 lb. The gate slides in vertical slots in the side of a reservoir containing water. The coefficient of friction between the slots and the gate is 0.03. Determine the minimum vertical force required to lift the gate when the water level is 4 ft above the top edge of the gate.



$$F_R = \gamma h_c A$$

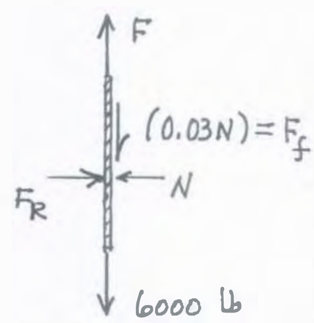
$$= \left(62.4 \frac{lb}{ft^3}\right)(8\ ft)(8\ ft \times 10\ ft)$$

$$= 39,900\ lb$$

$$\sum F_{horizontal} = 0$$

Thus,

$$N = F_R = 39,900\ lb$$



$$\sum F_{vertical} = 0$$

Thus,

$$F = 6000\ lb + (0.03)(39,900\ lb)$$

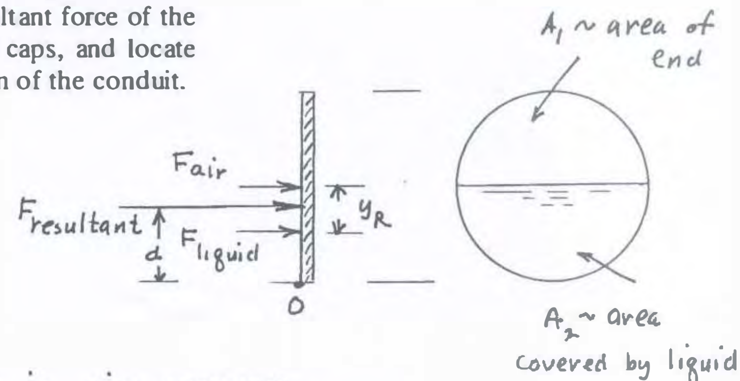
$$= \underline{\underline{7200\ lb}}$$

$F_f \sim$ maximum frictional force

$F \sim$ force to lift gate

2.56

2.56 A horizontal 2-m-diameter conduit is half filled with a liquid ($SG = 1.6$) and is capped at both ends with plane vertical surfaces. The air pressure in the conduit above the liquid surface is 200 kPa. Determine the resultant force of the fluid acting on one of the end caps, and locate this force relative to the bottom of the conduit.



$F_{air} = p A_1$ where p is air pressure

Thus,

$$F_{air} = \left(200 \times 10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(2\,m)^2 = 200\pi \times 10^3\ N$$

$F_{liquid} = \gamma h_c A_2$ where $h_c = \frac{4R}{3\pi}$ (see Fig. 2.18c)

Thus,

$$F_{liquid} = (1.6)\left(9.81 \times 10^3 \frac{N}{m^3}\right)\left[\frac{4(1\,m)}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2\,m)^2 = 10.5 \times 10^3\ N$$

For F_{liquid},

$$y_R = \frac{I_{xc}}{y_c A_2} + y_c \qquad \text{where } I_{xc} = 0.1098\ R^4 \quad \text{(see Fig. 2.18c)}$$

$$\text{and } y_c = h_c = \frac{4R}{3\pi}$$

Thus,

$$y_R = \frac{0.1098\,(1\,m)^4}{\left[\frac{4(1\,m)}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2\,m)^2} + \frac{4(1\,m)}{3\pi} = 0.5891\,m$$

Since $F_{resultant} = F_{air} + F_{liquid} = (200\pi + 10.5) \times 10^3\ N = \underline{\underline{639\ kN}}$,

we can sum moments about O to locate resultant to obtain

$$F_{resultant}\,(d) = F_{air}\,(1\,m) + F_{liquid}\,(1\,m - 0.5891\,m)$$

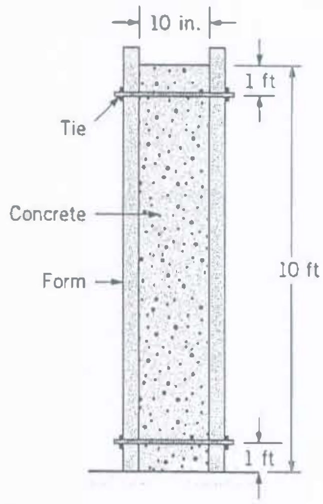
so that

$$d = \frac{(200\pi \times 10^3\ N)(1\,m) + (10.5 \times 10^3\ N)(0.4109\,m)}{639 \times 10^3\ N}$$

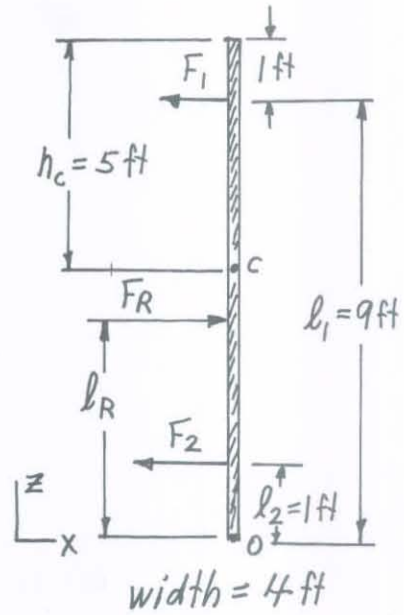
$$= \underline{\underline{0.990\ m \text{ above bottom of conduit}}}$$

2.57

2.57 Forms used to make a concrete basement wall are shown in Fig. P2.57. Each 4-ft-long form is held together by four ties—two at the top and two at the bottom as indicated. Determine the tension in the upper and lower ties. Assume concrete acts as a fluid with a weight of 150 lb/ft^3.



■ FIGURE P2.57



(1) $\Sigma F_x = 0$, or $F_1 + F_2 = F_R$

and

(2) $\Sigma M_0 = 0$, or $\ell_1 F_1 + \ell_2 F_2 = \ell_R F_R$, where $F_R = p_c A = \gamma h_c A$

Thus,

$$F_R = 150 \frac{lb}{ft^3}(5\,ft)(10\,ft)(4\,ft) = 30{,}000\,lb$$

Also,

$$\ell_R = 10\,ft - y_R = 10\,ft - y_c - (y_R - y_c) = 10\,ft - h_c - \frac{I_{xc}}{y_c A}$$

$$= 10\,ft - 5\,ft - \frac{\frac{1}{12}(4\,ft)(10\,ft)^3}{5\,ft\,(10\,ft)(4\,ft)}$$

$$= 5\,ft - 1.67\,ft$$

or

$$\ell_R = 3.33\,ft$$

Thus, from Eq. (2):

$$(9\,ft)F_1 + (1\,ft)F_2 = (3.33\,ft)(30{,}000\,lb) = 99{,}900\,ft \cdot lb$$

or

(3) $9F_1 + F_2 = 99{,}900$

From Eq. (1), $F_1 + F_2 = 30{,}000\,lb$, or $F_2 = 30{,}000 - F_1$

Thus, from Eq. (3),

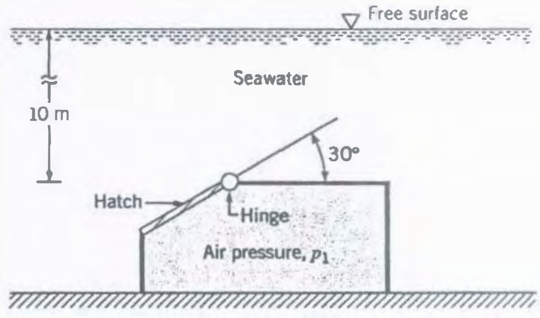
$$9F_1 + 30{,}000 - F_1 = 99{,}900$$

or

$F_1 = \underline{\underline{8{,}740\,lb}}$ so that $F_2 = 30{,}000\,lb - 8{,}740\,lb = \underline{\underline{21{,}260\,lb}}$

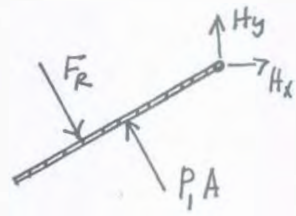
2.58

2.58 A structure is attached to the ocean floor as shown in Fig. P2.58. A 2-m-diameter hatch is located in an inclined wall and hinged on one edge. Determine the minimum air pressure, p_1, within the container to open the hatch. Neglect the weight of the hatch and friction in the hinge.



■ FIGURE P2.58

$F_R = \gamma h_c A$ where $h_c = 10\,m + \frac{1}{2}(2m)\sin 30°$
$= 10.5\,m$



Thus,

$$F_R = \left(10.1 \times 10^3 \frac{N}{m^3}\right)(10.5m)\left(\frac{\pi}{4}\right)(2m)^2$$
$$= 3.33 \times 10^5\,N$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad y_c = \frac{10\,m}{\sin 30°} + 1m = 21\,m$$

so that

$$y_R = \frac{\left(\frac{\pi}{4}\right)(1m)^4}{(21m)(\pi)(1m)^2} + 21\,m = 21.012\,m$$

For equilibrium,

$$\sum M_H = 0$$

so that

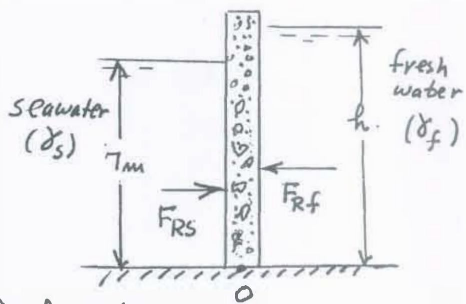
$$F_R\,(21.012m - 20\,m) = p_1(\pi)(1m)^2(1m)$$

and

$$p_1 = \frac{(3.33 \times 10^5\,N)(1.012\,m)}{\pi\,(1m)^2(1m)} = \underline{\underline{107\,kPa}}$$

2.59

2.59 A long, vertical wall separates seawater from freshwater. If the seawater stands at a depth of 7 m, what depth of freshwater is required to give a zero resultant force on the wall? When the resultant force is zero will the moment due to the fluid forces be zero? Explain.



For a zero resultant force

$$F_{Rs} = F_{Rf}$$

or

$$\gamma_s h_{cs} A_s = \gamma_f h_{cf} A_f$$

Thus, for a unit length of wall

$$\left(10.1 \frac{kN}{m^3}\right)\left(\frac{7m}{2}\right)(7m \times 1m) = \left(9.80 \frac{kN}{m^3}\right)\left(\frac{h}{2} m\right)(h \times 1m)$$

so that

$$\underline{\underline{h = 7.11\ m}}$$

In order for moment to be zero, F_{Rs} and F_{Rf} must be collinear.

For F_{RS}:

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(1m)(7m)^3}{\left(\frac{7}{2}m\right)(7m \times 1m)} + \frac{7}{2}m = 4.67m$$

Similarly for F_{Rf}:

$$y_R = \frac{\frac{1}{12}(1m)(7.11m)^3}{\left(\frac{7.11}{2}m\right)(7.11m \times 1m)} + \frac{7.11}{2}m = 4.74m$$

Thus, the distance to F_{Rs} from the bottom (point O) is $7m - 4.67m = 2.33m$. For F_{Rf} this distance is $7.11m - 4.74m = 2.37m$. The forces are not collinear. $\underline{\underline{\text{No.}}}$

2.60

2.60 A pump supplies water under pressure to a large tank as shown in Fig. P2.60. The circular-plate valve fitted in the short discharge pipe on the tank pivots about its diameter A–A and is held shut against the water pressure by a latch at B. Show that the force on the latch is independent of the supply pressure, p, and the height of the tank, h.

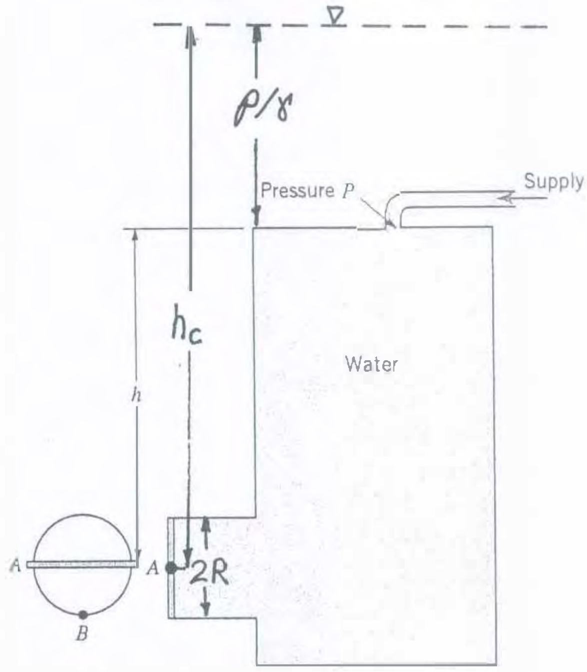
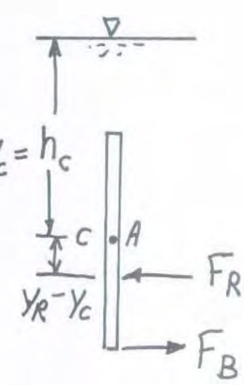


FIGURE P2.60

The pressure on the gate is the same as it would be for an open tank with a depth of

$$h_c = \frac{p + \gamma h}{\gamma}$$

as shown in the figure.



$\Sigma M_A = 0$, or

(1) $(y_R - y_c)\, F_R = R\, F_B$

where

$$F_R = p_c A = \gamma h_c (\pi R^2) = (p + \gamma h)(\pi R^2)$$

and

(2) $$y_R - y_c = \frac{I_{xc}}{y_c A} = \frac{\frac{\pi R^4}{4}}{\left(\frac{p+\gamma h}{\gamma}\right)\pi R^2} = \frac{R^2}{4\left(\frac{p}{\gamma} + h\right)}$$

Thus, from Eqs. (1) and (2)

$$F_B = \frac{(y_R - y_c)}{R} F_R = \frac{R}{4\left(\frac{p}{\gamma} + h\right)} (p + \gamma h)(\pi R^2)$$

or

$$F_B = \underline{\underline{\gamma \frac{\pi}{4} R^3}}$$, which is independent of both p and h.

2.61

2.61 A homogeneous, 4-ft-wide, 8-ft-long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in Fig. P2.61 Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.

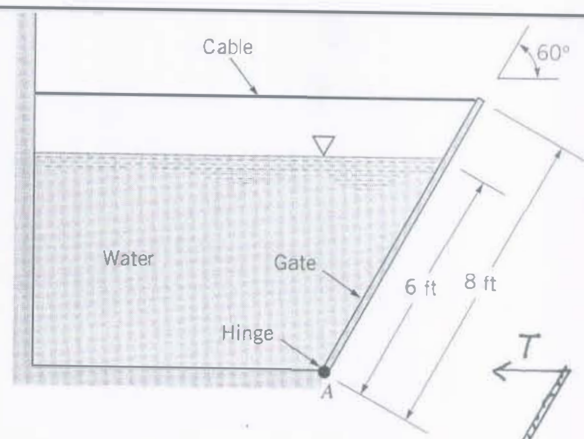
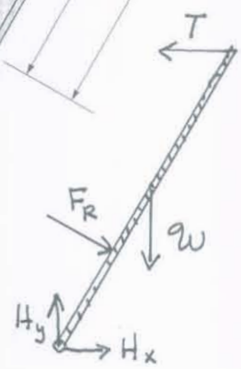


FIGURE P2.61



$$F_R = \gamma h_c A \quad \text{where } h_c = \left(\frac{6\,ft}{2}\right)\sin 60^\circ$$

Thus,

$$F_R = \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{6\,ft}{2}\right)(\sin 60^\circ)(6\,ft \times 4\,ft)$$

$$= 3890\ lb$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where } y_c = 3\,ft$$

so that

$$y_R = \frac{\frac{1}{12}(4\,ft)(6\,ft)^3}{(3\,ft)(6\,ft \times 4\,ft)} + 3\,ft = 4.0\ ft$$

For equilibrium,

$$\sum M_H = 0$$

and

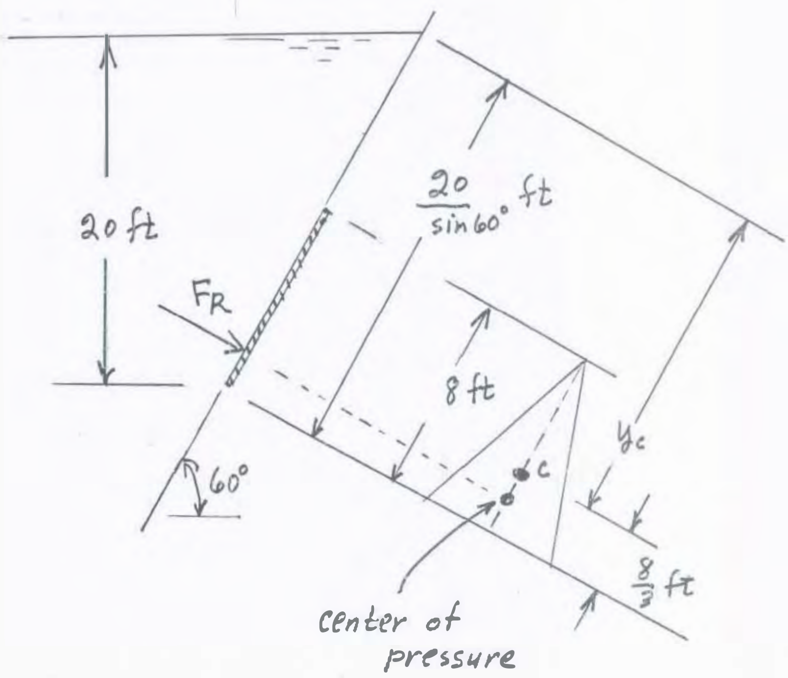
$$T\,(8\,ft)(\sin 60^\circ) = w\,(4\,ft)(\cos 60^\circ) + F_R\,(2\,ft)$$

$$T = \frac{(800\,lb)(4\,ft)(\cos 60^\circ) + (3890\,lb)(2\,ft)}{(8\,ft)(\sin 60^\circ)}$$

$$= \underline{\underline{1350\ lb}}$$

2.63

2.63 An area in the form of an isosceles triangle with a base width of 6 ft and an altitude of 8 ft lies in the plane forming one wall of a tank which contains a liquid having a specific weight of 79.8 lb/ft³. The side slopes upward making an angle of 60° with the horizontal. The base of the triangle is horizontal and the vertex is above the base. Determine the resultant force the fluid exerts on the area when the fluid depth is 20 ft above the base of the triangular area. Show, with the aid of a sketch, where the center of pressure is located.



$$y_c = \left(\frac{20}{\sin 60^\circ}\right)\text{ft} - \left(\frac{8}{3}\right)\text{ft}$$

$$= 20.43 \text{ ft}$$

$$h_c = y_c \sin 60^\circ$$

$$F_R = \gamma h_c A = \left(79.8 \frac{\text{lb}}{\text{ft}^3}\right)\left[(20.43 \text{ ft})\sin 60^\circ\right]\left(\tfrac{1}{2}\right)(6 \text{ ft} \times 8 \text{ ft})$$

$$= \underline{33,900 \text{ lb}}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad I_{xc} = \frac{1}{36}(6\text{ft})(8\text{ft})^3$$

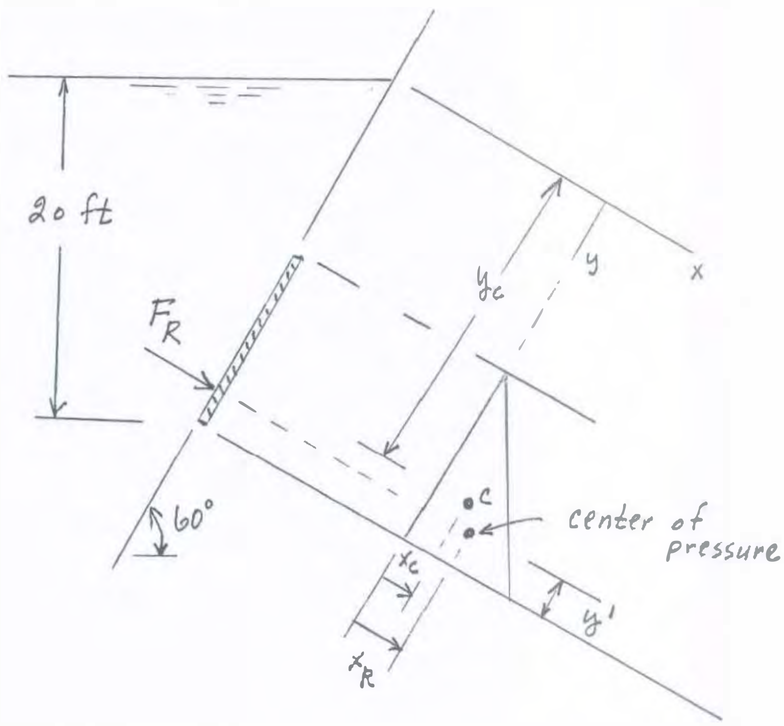
Thus,

$$y_R = \frac{\frac{1}{36}(6\text{ft})(8\text{ft})^3}{(20.43\text{ft})(\frac{1}{2})(6\text{ft}\times 8\text{ft})} + 20.43 \text{ ft} = 20.6 \text{ ft}$$

The force, F_R, acts through the center of pressure which is located a distance of $\frac{20}{\sin 60^\circ}$ ft $-$ 20.6 ft $=$ $\underline{2.49 \text{ ft}}$ above the base of the triangle as shown in sketch.

2.64

2.64 Solve Problem 2.63 if the isosceles triangle is replaced with a right triangle having the same base width and altitude as the isosceles triangle.



$F_R = \underline{\underline{33,900\ lb}}$

$y' = \underline{\underline{2.49\ ft}}$

(see solution to Problem 2.63)

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad \text{(Eq. 2.20)}$$

where

$$I_{xyc} = \frac{(6\ ft)^2(8\ ft)^2}{72} = 32\ ft^4 \qquad \text{(see Fig. 2.18 d)}$$

and $y_c = 20.43\ ft$ (see solution to Problem 2.63)

Thus,

$$x_R = \frac{32\ ft^4}{(20.43\ ft)(\frac{1}{2})(6\ ft \times 8\ ft)} + \frac{6}{3}\ ft = \underline{\underline{2.07\ ft}}$$

The force, F_R, acts through the center of pressure with coordinates $x_R = 2.07\ ft$ and $y' = 2.49\ ft$ (see sketch).

2.65

2.65 A vertical plane area having the shape shown in Fig. P2.65 is immersed in an oil bath (specific weight = 8.75 kN/m³). Determine the magnitude of the resultant force acting on one side of the area as a result of the oil.

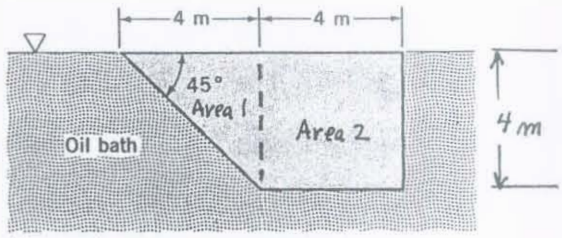


FIGURE P2.65

Break area into two parts as shown in figure.

For area 1:

$$F_{R_1} = \gamma h_{c_1} A_1$$

$$= \left(8.75 \frac{kN}{m^3}\right)\left(\frac{4\,m}{2}\right)(4\,m \times 4\,m) = 280\ kN$$

For area 2:

$$F_{R_2} = \gamma h_{c_2} A_2$$

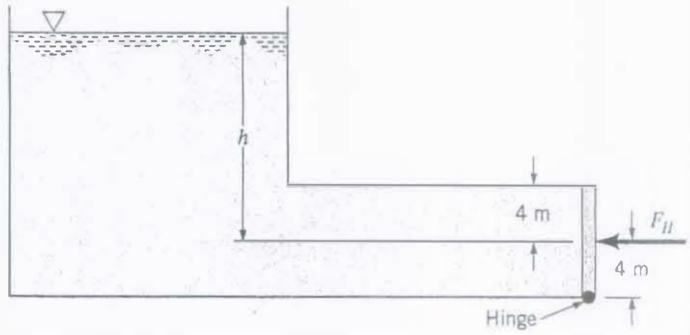
$$= \left(8.75 \frac{kN}{m^3}\right)\left(\frac{4\,m}{3}\right)\left(\frac{1}{2}\right)(4\,m \times 4\,m) = 93.3\ kN$$

Thus,

$$F_R = F_{R_1} + F_{R_2} = 280\ kN + 93.3\ kN = \underline{\underline{373\ kN}}$$

2.66

2.66 A 3-m-wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Fig. P2.66. The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum value for F_H is 3500 kN. (a) Determine the maximum water depth, h, above the center of the gate that can exist without the gate opening. (b) Is the answer the same if the gate is hinged at the top? Explain your answer.



■ FIGURE P2.66

For gate hinged at <u>bottom</u>

$\sum M_H = 0$

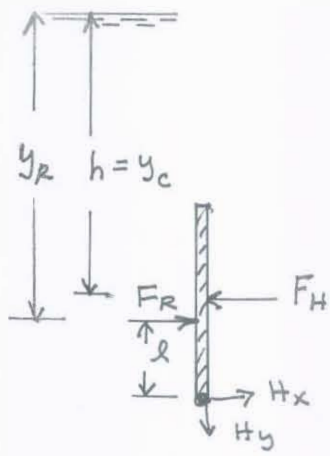
so that

$$(4m)\,F_H = \ell\, F_R \quad \text{(see figure)} \quad (1)$$

and

$$F_R = \gamma h_c A = \left(9.80\,\frac{kN}{m^3}\right)(h)(3m \times 8m)$$

$$= (9.80 \times 24\, h)\ kN$$



$\ell = h + 4 - y_R$

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(3m)(8m)^3}{h\,(3m \times 8m)} + h$$

$$= \frac{5.33}{h} + h$$

Thus,

$$\ell(m) = h + 4 - \left(\frac{5.33}{h} + h\right) = 4 - \frac{5.33}{h}$$

and from Eq. (1)

$$(4\,m)(3500\,kN) = \left(4 - \frac{5.33}{h}\right)(9.80 \times 24)(h)\ kN$$

so that

$$\underline{\underline{h = 16.2\ m}}$$

(con't)

2.66 (Con't)

For gate hinged at <u>top</u>

$\sum M_H = 0$

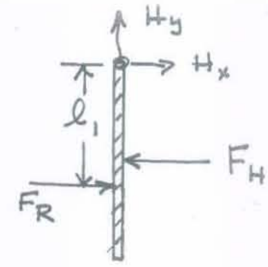
so that

$(4\,m) F_H = \ell_1 F_R$ (see figure) (1)

where

$\ell_1 = y_R - (h-4) = \left(\frac{5.33}{h} + h\right) - (h-4)$

$= \frac{5.33}{h} + 4$



$\ell_1 = y_R - (h-4)$

Thus, from Eq. (1)

$(4\,m)(3500\,kN) = \left(\frac{5.33}{h} + 4\right)(9.80 \times 24)(h)\ kN$

and

$$\underline{\underline{h = 13.5\,m}}$$

Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.

2.67

2.67 A gate having the cross section shown in Fig. P2.67 closes an opening 5 ft wide and 4 ft high in a water reservoir. The gate weighs 500 lb and its center of gravity is 1 ft to the left of AC and 2 ft above BC. Determine the horizontal reaction that is developed on the gate at C.

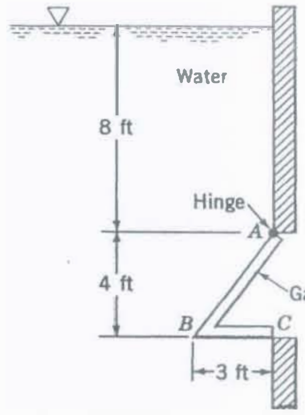
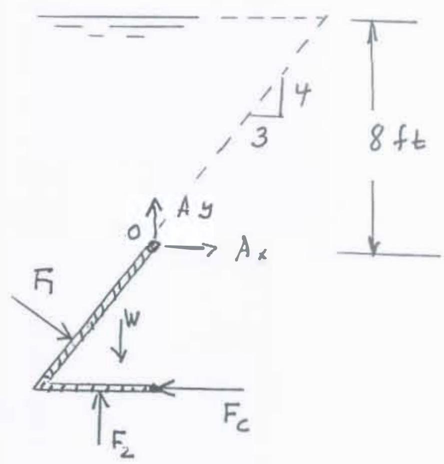


FIGURE P2.67



$F_1 = \gamma h_{c1} A_1$ where $h_{c1} = 8\,ft + 2\,ft$

Thus,

$F_1 = (62.4\,\frac{lb}{ft^3})(10\,ft)(5\,ft \times 5\,ft)$

$= 15,600\,lb$

To locate F_1,

$$y_1 = \frac{I_{xc}}{y_{c1}A_1} + y_{c1}$$

where $y_{c1} = \frac{8\,ft}{\frac{4}{5}} + 2.5\,ft = 12.5\,ft$

So that

$$y_1 = \frac{\frac{1}{12}(5\,ft)(5\,ft)^3}{(12.5\,ft)(5\,ft \times 5\,ft)} + 12.5\,ft = 12.67\,ft$$

Also,

$F_2 = p_2 A_2$ where $p_2 = \gamma_{H_2O}(8\,ft + 4\,ft)$

so that

$F_2 = \gamma_{H_2O}(12\,ft)(A_2) = (62.4\,\frac{lb}{ft^3})(12\,ft)(3\,ft \times 5\,ft) = 11,230\,lb$

For equilibrium,

$$\sum M_o = 0$$

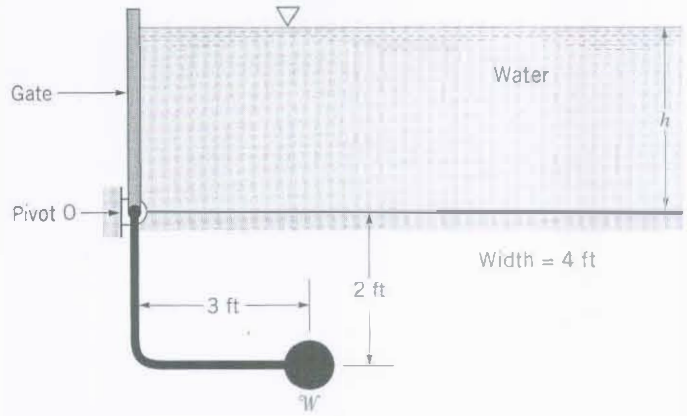
and $F_1\left(y_1 - \frac{8\,ft}{\frac{4}{5}}\right) + W(1\,ft) - F_2(\tfrac{1}{2})(3\,ft) - F_c(4\,ft)$

so that

$$F_c = \frac{(15,600\,lb)(12.67\,ft - 10\,ft) + (500\,lb)(1\,ft) - (11,230\,lb)(\frac{3}{2}\,ft)}{4\,ft} = \underline{\underline{6330\,lb}}$$

2.68

2.68 The massless, 4-ft-wide gate shown in Fig. P2.68 pivots about the frictionless hinge O. It is held in place by the 2000 lb counterweight, W. Determine the water depth, h.



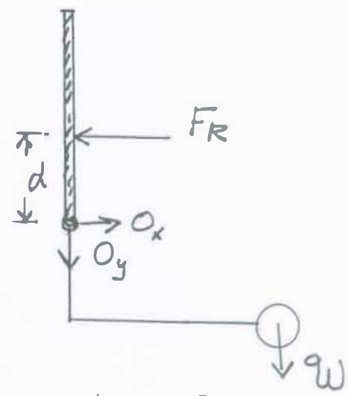
■ FIGURE P2.68

$$F_R = \gamma h_c A \quad \text{where} \quad h_c = \frac{h}{2}$$

Thus,

$$F_R = \gamma_{H_2O} \frac{h}{2} (h \times b)$$

$$= \gamma_{H_2O} \frac{h^2}{2} (4\,ft)$$



$b = 4\,ft$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4\,ft)(h^3)}{\frac{h}{2}(4\,ft \times h)} + \frac{h}{2}$$

$$= \frac{2}{3} h$$

For equilibrium,

$$\Sigma M_o = 0$$

$$F_R\, d = W (3\,ft) \quad \text{where} \quad d = h - y_R = \frac{h}{3}$$

so that

$$\frac{h}{3} = \frac{(2000\,lb)(3\,ft)}{(\gamma_{H_2O})(\frac{h^2}{2})(4\,ft)}$$

Thus,

$$h^3 = \frac{(3)(2000\,lb)(3\,ft)}{(62.4\,\frac{lb}{ft^3})(\frac{1}{2})(4\,ft)}$$

$$h = \underline{\underline{5.24\,ft}}$$

*2.69

*2.69 A 200-lb homogeneous gate of 10-ft width and 5-ft length is hinged at point A and held in place by a 12-ft-long brace as shown in Fig. P2.69. As the bottom of the brace is moved to the right, the water level remains at the top of the gate. The line of action of the force that the brace exerts on the gate is along the brace. **(a)** Plot the magnitude of the force exerted on the gate by the brace as a function of the angle of the gate, θ, for $0 \le \theta \le 90°$. **(b)** Repeat the calculations for the case in which the weight of the gate is negligible. Comment on the results as $\theta \to 0$.

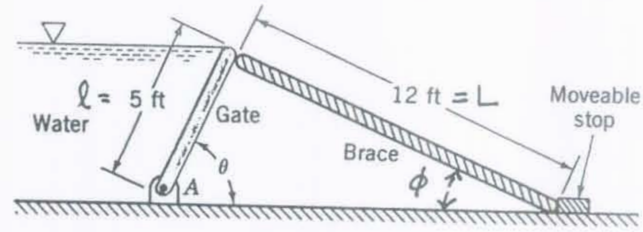
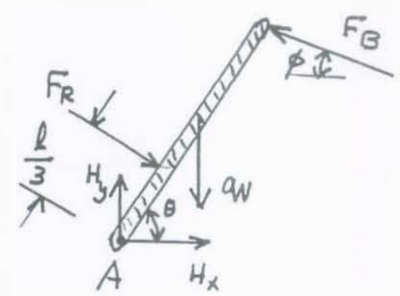


FIGURE P2.69



a) For the free-body-diagram of the gate (see figure),

$$\Sigma F_A = 0$$

so that

$$F_R\left(\frac{\ell}{3}\right) + W\left(\frac{\ell}{2}\cos\theta\right) = (F_B\cos\phi)(\ell\sin\theta) + (F_B\sin\phi)(\ell\cos\theta) \quad (1)$$

Also,

$$\ell\sin\theta = L\sin\phi \quad \text{(assuming hinge and end of brace at same elevation)}$$

or

$$\sin\phi = \frac{\ell}{L}\sin\theta$$

and

$$F_R = \gamma h_c A = \gamma\left(\frac{\ell\sin\theta}{2}\right)(\ell w)$$

where w is the gate width. Thus, Eq. (1) can be written as

$$\gamma\left(\frac{\ell^3}{6}\right)(\sin\theta)w + \frac{W\ell}{2}\cos\theta = F_B\,\ell\left(\cos\phi\sin\theta + \sin\phi\cos\theta\right)$$

so that

$$F_B = \frac{\left(\frac{\gamma\ell^2 w}{6}\right)\sin\theta + \frac{W}{2}\cos\theta}{\cos\phi\sin\theta + \sin\phi\cos\theta} = \frac{\left(\frac{\gamma\ell^2 w}{6}\right)\tan\theta + \frac{W}{2}}{\cos\phi\tan\theta + \sin\phi} \quad (2)$$

For $\gamma = 62.4\ lb/ft^3$, $\ell = 5\,ft$, $w = 10\,ft$, and $W = 200\,lb$,

$$F_B = \frac{\frac{(62.4\frac{lb}{ft^3})(5ft)^2(10ft)}{6}\tan\theta + \frac{200\,lb}{2}}{\cos\phi\tan\theta + \sin\phi} = \frac{2600\tan\theta + 100}{\cos\phi\tan\theta + \sin\phi} \quad (3)$$

(con't)

*2.69 (con't)

Since $\sin\phi = \frac{l}{L}\sin\theta$ and $l = 5\,ft$, $L = 12\,ft$

$$\sin\phi = \frac{5}{12}\sin\theta$$

and for a given θ, ϕ can be determined. Thus, Eq. (3) can be used to determine F_B for a given θ.

(b) For $W = 0$, Eq. (3) reduces to

$$F_B = \frac{2600\tan\theta}{\cos\phi\tan\theta + \sin\phi} \qquad (4)$$

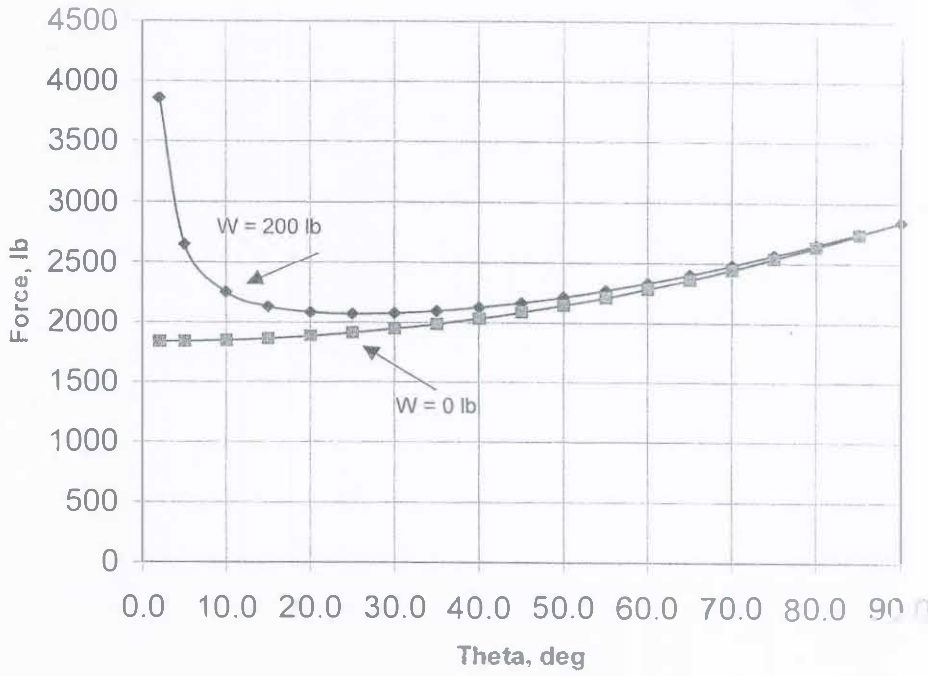
and Eq. (4) can be used to determine F_B for a given θ. Tabulated data of F_B vs. θ for both $W = 200\,lb$ and $W = 0\,lb$ are given below.

θ, deg	F_B, lb (W =200 lb)	F_B, lb (W = 0 lb)
90.0	2843	2843
85.0	2745	2736
80.0	2651	2633
75.0	2563	2536
70.0	2480	2445
65.0	2403	2360
60.0	2332	2282
55.0	2269	2210
50.0	2213	2144
45.0	2165	2085
40.0	2125	2032
35.0	2094	1985
30.0	2075	1945
25.0	2069	1911
20.0	2083	1884
15.0	2130	1863
10.0	2250	1847
5.0	2646	1838
2.0	3858	1836

A plot of the data is given on the following page.

(con't)

*2.69 (con't)



(b) (con't)

As $\theta \to 0$ the value of F_B can be determined from Eq.(+),

$$F_B = \frac{2600 \tan\theta}{\cos\phi \tan\theta + \sin\phi}$$

Since

$$\sin\phi = \frac{5}{12}\sin\theta$$

, it follows that

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}$$

and therefore

$$F_B = \frac{2600\tan\theta}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}\,\tan\theta + \frac{5}{12}\sin\theta} = \frac{2600}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta} + \frac{5}{12}\cos\theta}$$

Thus, as $\theta \to 0$

$$F_B \to \frac{2600}{1+\frac{5}{12}} = 1840 \text{ lb}$$

Physically this result means that for $\theta \equiv 0$, the value of F_B is indeterminate, but for any "very small" value of θ, F_B will approach 1840 lb.

2.70

2.70 An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700 \text{ kg/m}^3$ at a depth of 4 m, as shown in Fig. P2.70. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?

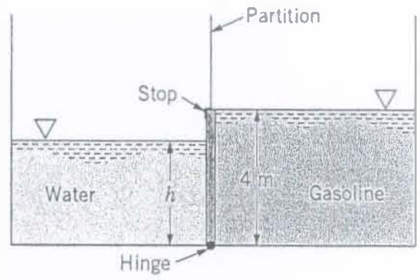
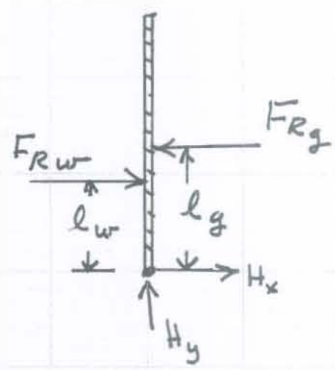


FIGURE P2.70



$F_{R_g} = \gamma_g \, h_{c_g} \, A_g$

where g refers to gasoline.

$F_{R_g} = \left(700 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2\text{m})(4\text{m} \times 2\text{m})$

$= 110 \times 10^3 \text{ N} = 110 \text{ kN}$

$F_{R_w} = \gamma_w \, h_{c_w} \, A_w$

where w refers to water.

$F_{R_w} = \left(9.80 \times 10^3 \frac{\text{N}}{\text{m}^3}\right)\left(\frac{h}{2}\right)(2\text{ m} \times h)$

where h is depth of water.

$F_{R_w} = (9.80 \times 10^3)\, h^2$

For equilibrium,

$\sum M_H = 0$

so that

$F_{R_w} \ell_w = F_{R_g} \ell_g$ with $\ell_w = \frac{h}{3}$ and $\ell_g = \frac{4}{3} \text{ m}$

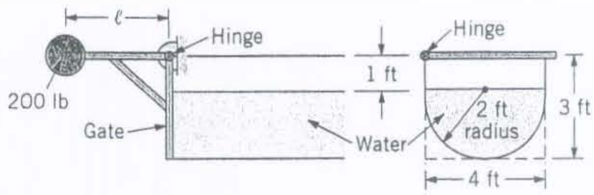
Thus, $(9.80 \times 10^3)(h^2)\left(\frac{h}{3}\right) = (110 \times 10^3 \text{N})\left(\frac{4}{3} \text{ m}\right)$

and $\underline{\underline{h = 3.55 \text{ m}}}$

which is the limiting value for h.

2.71

2.71 A 4-ft by 3-ft massless rectangular gate is used to close the end of the water tank shown in Fig. P2.71. A 200 lb weight attached to the arm of the gate at a distance ℓ from the frictionless hinge is just sufficient to keep the gate closed when the water depth is 2 ft, that is, when the water fills the semicircular lower portion of the tank. If the water were deeper the gate would open. Determine the distance ℓ.



■ FIGURE P2.71

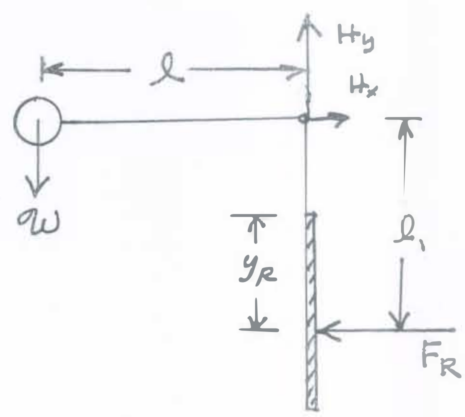
$F_R = \gamma h_c A$ where $h_c = \dfrac{4R}{3\pi}$ (see Fig. 2.18)

Thus,

$$F_R = \gamma_{H_2O}\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)$$

$$= \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{4\,(2ft)}{3\pi}\right)\left(\frac{\pi\,(2ft)^2}{2}\right)$$

$$= 333\ lb$$



To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$= \frac{0.1098\,R^4}{\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)} + \frac{4R}{3\pi} \quad \text{(see Fig. 2.18)}$$

$\ell_1 = 1\,ft + y_R$

$$= \frac{(0.1098)(2ft)^4}{\left(\frac{4\,(2ft)}{3\pi}\right)\frac{\pi(2ft)^2}{2}} + \frac{4\,(2ft)}{3\pi} = 1.178\ ft$$

For equilibrium,

$$\sum M_H = 0$$

so that

$$\mathcal{W}\ell = F_R\,(1\,ft + y_R)$$

and

$$\ell = \frac{(333\,lb)(1\,ft + 1.178\,ft)}{200\,lb} = \underline{\underline{3.63\ ft}}$$

2.72

2.72 A rectangular gate that is 2 m wide is located in the vertical wall of a tank containing water as shown in Fig. P2.72. It is desired to have the gate open automatically when the depth of water above the top of the gate reaches 10 m. **(a)** At what distance, d, should the frictionless horizontal shaft be located? **(b)** What is the magnitude of the force on the gate when it opens?

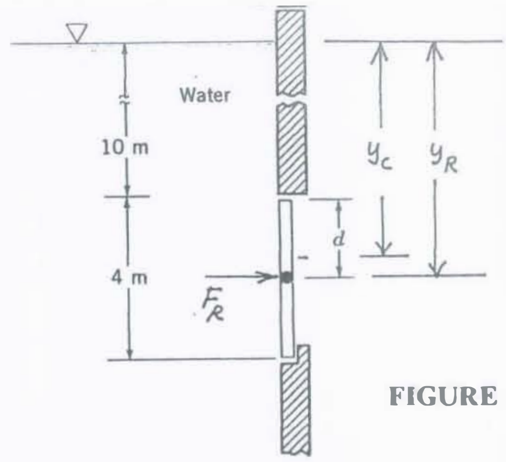


FIGURE P2.72

(a) As depth increases the center of pressure moves toward the centroid of the gate. If we locate hinge at y_R when depth $= 10\,m + d$, the gate will open automatically for any further increase in depth. Since,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(2m)(4m)^3}{(12\,m)(2m \times 4m)} + 12\,m = 12.11\,m$$

then

$$d = y_R - 10\,m = 12.11\,m - 10\,m = \underline{\underline{2.11\,m}}$$

(b) For the depth shown,

$$F_R = \gamma h_c A = \left(9.80\,\frac{kN}{m^3}\right)(12\,m)(2m \times 4m) = \underline{\underline{941\,kN}}$$

2.73

2.73 A thin 4-ft-wide, right-angle gate with negligible mass is free to pivot about a frictionless hinge at point O, as shown in Fig. P2.73. The horizontal portion of the gate covers a 1-ft-diameter drain pipe which contains air at atmospheric pressure. Determine the minimum water depth, h, at which the gate will pivot to allow water to flow into the pipe.

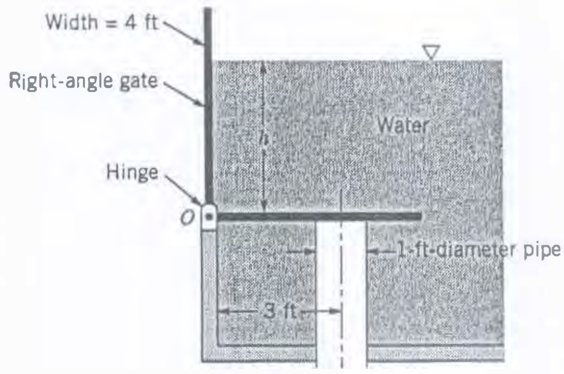
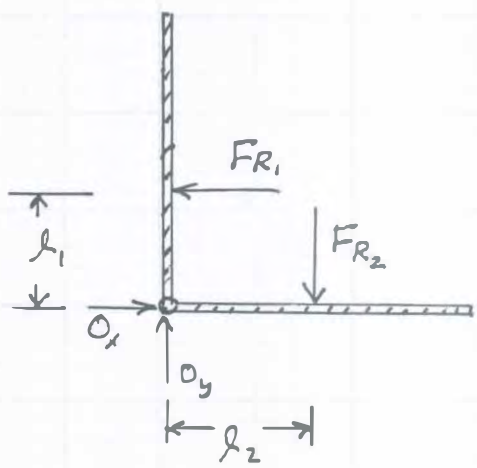


FIGURE P2.73



For equilibrium

$$\sum M_O = 0$$

$$F_{R_1} \times \ell_1 = F_{R_2} \times \ell_2 \qquad (1)$$

$$F_{R_1} = \gamma h_{c_1} A_1$$

$$= \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{h}{2}\right)(4 ft \times h)$$

$$= 125\, h^2$$

For the force on the horizontal portion of the gate (which is balanced by pressure on both sides except for the area of the pipe)

$$F_{R_2} = \gamma h \left(\frac{\pi}{4}\right)(1 ft)^2 = \left(62.4 \frac{lb}{ft^3}\right)(h)\left(\frac{\pi}{4}\right)(1 ft)^2$$

$$= 49.0\, h$$

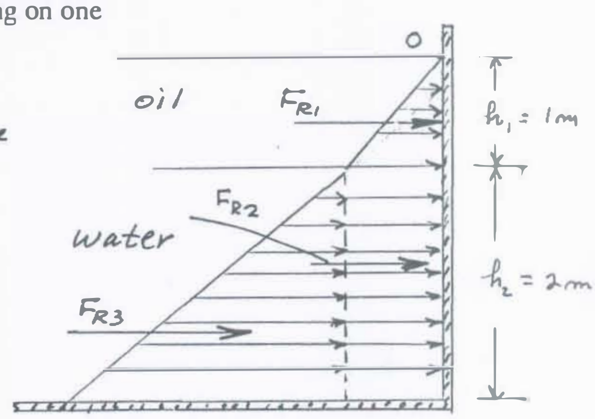
Thus, from Eq. (1) with $\ell_1 = \frac{h}{3}$ and $\ell_2 = 3 ft$

$$(125 h^2)\left(\frac{h}{3}\right) = (49.0 h)(3 ft)$$

$$h = \underline{\underline{1.88 ft}}$$

2.74

2.74 An open rectangular tank is 2 m wide and 4 m long. The tank contains water to a depth of 2 m and oil ($SG = 0.8$) on top of the water to a depth of 1 m. Determine the magnitude and location of the resultant fluid force acting on one end of the tank.



Let $w \sim$ width $= 2\,m$

Use the concept of the pressure prism (see figure).

$$F_{R1} = \gamma_{oil}\, h_{c1}\, A_1$$

so that

$$F_{R1} = (0.8)\left(9.81\,\frac{kN}{m^3}\right)\left(\frac{1\,m}{2}\right)(1\,m \times 2\,m)$$
$$= 7.85\ kN$$

$F_{R2} = p_2 A_2$ where p_2 is pressure at depth h_1. Thus,

$$F_{R2} = (\gamma_{oil}\, h_1)(h_2 \times w) = (0.8)\left(9.81\,\frac{kN}{m^3}\right)(1\,m)(2\,m \times 2\,m) = 31.4\ kN$$

Also,

$$F_{R3} = \gamma_{H_2O}\, h_{c3}\, A_3 \quad \text{so that}$$

$$F_{R3} = \gamma_{H_2O}\left(\frac{h_2}{2}\right)(h_2 \times w) = \left(9.80\,\frac{kN}{m^3}\right)\left(\frac{2\,m}{2}\right)(2\,m \times 2\,m) = 39.2\ kN$$

Thus,

$$F_R = F_{R1} + F_{R2} + F_{R3} = 7.85\,kN + 31.4\,kN + 39.2\,kN = \underline{\underline{78.5\,kN}}$$

To locate F_R sum moments around axis through O, so that

$$F_R\, d_R = F_{R1}\, d_1 + F_{R2}\, d_2 + F_{R3}\, d_3 \qquad (1)$$

where d_R is distance to F_R. Since F_{R1}, F_{R2}, and F_{R3} act through the centroids of their respective pressure prisms it follows that

$$d_1 = \tfrac{2}{3}(1\,m), \quad d_2 = 1\,m + 1\,m = 2\,m, \quad d_3 = 1\,m + \tfrac{2}{3}(2\,m)$$

and from Eq. (1)

$$d = \frac{(7.85\,kN)\left(\frac{2}{3}\right)(1\,m) + (31.4\,kN)(2\,m) + (39.2\,kN)\left(1\,m + \frac{4\,m}{3}\right)}{78.5\,kN}$$

$$= \underline{\underline{2.03\,m \quad (\text{below oil free surface})}}$$

*2.75

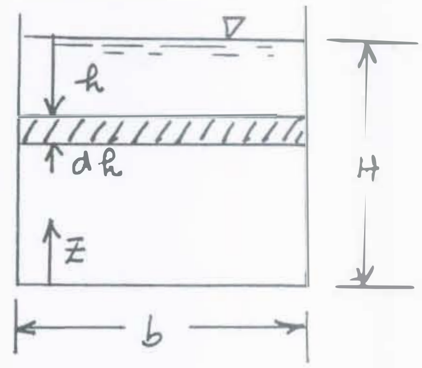
(con't)

*2.75 An open rectangular settling tank contains a liquid suspension that at a given time has a specific weight that varies approximately with depth according to the following data:

h (m)	γ (kN/m^3)
0	10.0
0.4	10.1
0.8	10.2
1.2	10.6
1.6	11.3
2.0	12.3
2.4	12.7
2.8	12.9
3.2	13.0
3.6	13.1

The depth $h = 0$ corresponds to the free surface. By means of numerical integration, determine the magnitude and location of the resultant force that the liquid suspension exerts on a vertical wall of the tank that is 6 m wide. The depth of fluid in the tank is 3.6 m.

The magnitude of the fluid force, F_R, can be found by summing the differential forces acting on the horizontal strip shown in the figure. Thus,



$$F_R = \int_0^H dF_R = b\int_0^H p\,dh \qquad (1)$$

where p is the pressure at depth h. To find p we use Eq. 2.4

$$\frac{dp}{dz} = -\gamma$$

and with $dz = -dh$

$$p(h) = \int_0^h \gamma\,dh \qquad (2)$$

Equation (2) can be integrated numerically using the trapezoidal rule, i.e., $I = \frac{1}{2}\sum_{i=1}^{n-1}(y_i + y_{i+1})(x_{i+1} - x_i)$ where $y \sim \gamma$, $x \sim h$ and n = number of data points. The pressure distribution is given below.

h, m	γ, kN/m^3	Pressure, kPa
0	10.0	0
0.4	10.1	4.02
0.8	10.2	8.08
1.2	10.6	12.24
1.6	11.3	16.62
2.0	12.3	21.34
2.4	12.7	26.34
2.8	12.9	31.46
3.2	13.0	36.64
3.6	13.1	41.86

(con't)

*2.75 (con't)

Equation (1) can now be integrated numerically using the trapezoidal rule with $y \sim p$ and $x \sim h$. The approximate value of the integral is $71.07 \frac{kN}{m}$.

Thus, with

$$\int_0^H p\,dh = 71.07 \frac{kN}{m}$$

$$F_R = (6\,m)\left(71.07 \frac{kN}{m}\right) = \underline{\underline{426\ kN}}$$

To locate F_R sum moments about axis formed by intersection of vertical wall and fluid surface. Thus,

$$F_R h_R = b\int_0^H h\,p\,dh \qquad (3)$$

The integrand, hp, can now be determined and is tabulated below.

h, m	Pressure, kPa	h *p, kN/m
0	0	0.00
0.4	4.02	1.61
0.8	8.08	6.46
1.2	12.24	14.69
1.6	16.62	26.59
2.0	21.34	42.68
2.4	26.34	63.22
2.8	31.46	88.09
3.2	36.64	117.25
3.6	41.86	150.70

Equation (3) can now be integrated numerically using the trapezoidal rule with $y \sim hp$ and $x \sim h$. The approximate value of the integral is $174.4\ kN$.

Thus, with

$$\int_0^H h\,p\,dh = 174.4\ kN$$

it follows from Eq. (3) that

$$h_R = \frac{b\int_0^H h\,p\,dh}{F_R} = \frac{(6\,m)(174.4\ kN)}{426\ kN} = 2.46\,m$$

The resultant force acts $\underline{\underline{2.46\ m \text{ below fluid surface}}}$.

2.76

2.76 The closed vessel of Fig. P2.76 contains water with an air pressure of 10 psi at the water surface. One side of the vessel contains a spout that is closed by a 6-in.-diameter circular gate that is hinged along one side as illustrated. The horizontal axis of the hinge is located 10 ft below the water surface. Determine the minimum torque that must be applied at the hinge to hold the gate shut. Neglect the weight of the gate and friction at the hinge.

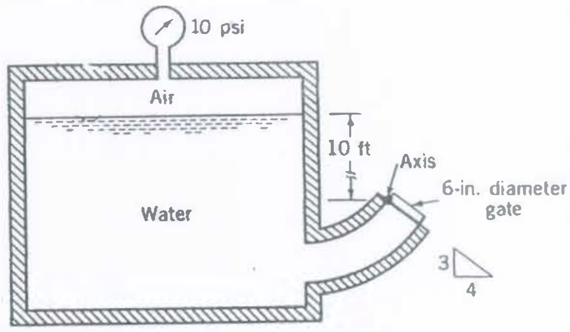
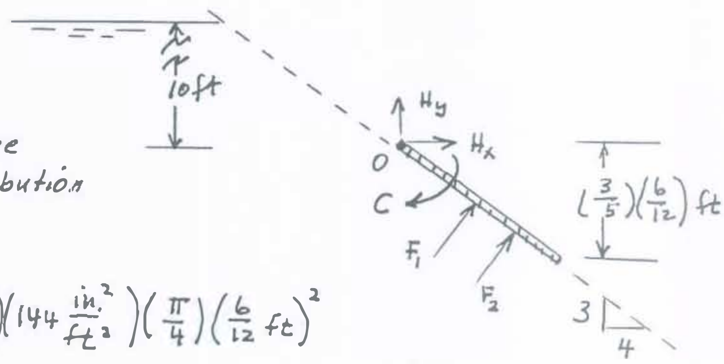


FIGURE P2.76

Let F_1 ~ force due to air pressure, and F_2 ~ force due to hydrostatic pressure distribution of water.



Thus,

$$F_1 = p_{air} A = \left(10 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2$$

$$= 283 \text{ lb}$$

and

$$F_2 = \gamma h_c A \quad \text{where} \quad h_c = 10 ft + \frac{1}{2}\left[\left(\frac{3}{5}\right)\left(\frac{6}{12}\right) ft\right] = 10.15 ft$$

so that

$$F_2 = \left(62.4 \frac{lb}{ft^3}\right)(10.15 ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2 = 124 \text{ lb}$$

Also,

$$y_{R2} = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad y_c = \frac{10 ft}{\frac{3}{5}} + \frac{1}{2}\left(\frac{6}{12} ft\right) = 16.92 ft$$

so that

$$y_{R2} = \frac{\left(\frac{\pi}{4}\right)\left(\frac{3}{12} ft\right)^4}{(16.92 ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2} + 16.92 ft = 16.92 ft$$

For equilibrium,

$$\sum M_o = 0$$

and

$$C = F_1 \left(\frac{3}{12} ft\right) + F_2 \left(y_{R2} - \frac{10 ft}{\frac{3}{5}}\right)$$

or

$$C = (283 lb)\left(\frac{3}{12} ft\right) + (124 lb)\left(16.92 ft - \frac{10 ft}{\frac{3}{5}}\right) = \underline{\underline{102 \text{ ft} \cdot \text{lb}}}$$

2.77

2.77 A 4-ft-tall, 8-in.-wide concrete (150 lb/ft^3) retaining wall is built as shown in Fig. P2.77. During a heavy rain, water fills the space between the wall and the earth behind it to a depth h. Determine the maximum depth of water possible withoutthe wall tipping over. The wall simply rests on the ground without being anchored to it.

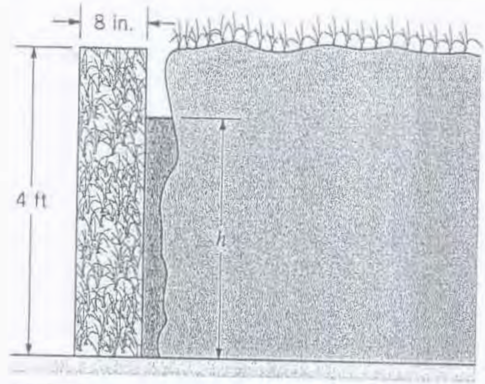


FIGURE P2.77

For equilibrium,

$\sum M_0 = 0$, or

(1) $\ell F_R = (4 \text{ in.}) W$, where with L = wall length,

$$W = \gamma_{concrete} \, \mathcal{V} = \left(150 \frac{lb}{ft^3}\right)\left(\frac{8}{12} ft\right)(4 ft) L = 400 L \text{ lb}$$

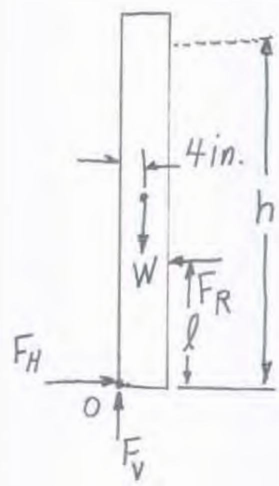
and

$$F_R = p_c A = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{h}{2}\right) L h = 31.2 L h^2$$

Also,

$$\ell = \frac{h}{2} - (y_R - y_c) = \frac{h}{2} - \frac{I_{xc}}{y_c A}$$

$$= \frac{h}{2} - \frac{\frac{1}{12} L h^3}{\left(\frac{h}{2}\right) L h} = \frac{h}{2} - \frac{h}{6} = \frac{h}{3}$$



Thus, Eq (1) becomes

$$\frac{h}{3}(31.2 L h^2) = \frac{4}{12}(400 L)$$

or

$$h = \underline{\underline{2.34 \text{ ft}}}$$

*2.78

***2.78** Water backs up behind a concrete dam as shown in Fig. P2.78. Leakage under the foundation gives a pressure distribution under the dam as indicated. If the water depth, h, is too great, the dam will topple over about its toe (point A). For the dimensions given, determine the maximum water depth for the following widths of the dam: ℓ = 20, 30, 40, 50, and 60 ft. Base your analysis on a unit length of the dam. The specific weight of the concrete is 150 lb/ft³.

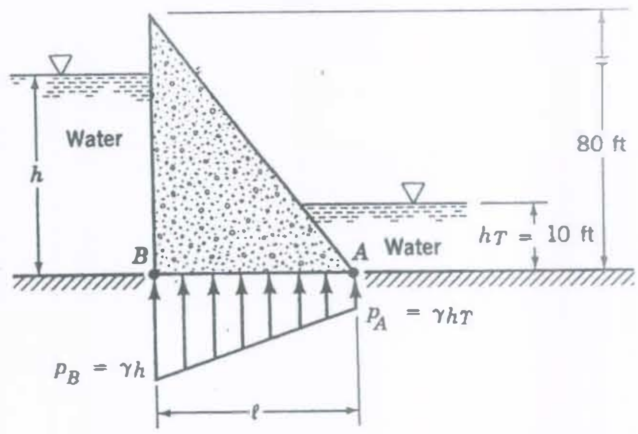
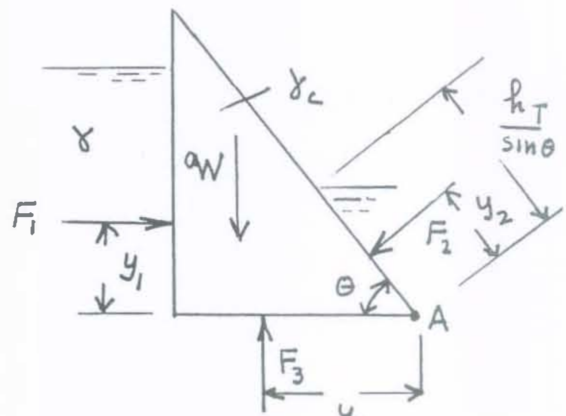


FIGURE P2.78

A free-body-diagram of the dam is shown in the figure at the right, where:



$$F_1 = \frac{\gamma h^2}{2} \quad (\text{for unit length})$$

$$\mathcal{W} = \gamma_c\left(\tfrac{1}{2}\right)(\ell)(80) = 40\gamma_c\,\ell$$

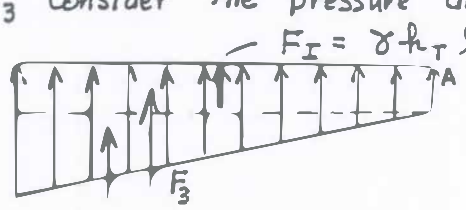
$$F_3 = \left(\frac{\gamma h + \gamma h_T}{2}\right)\ell$$

$$F_2 = \gamma\left(\frac{h_T}{2}\right)\left(\frac{h_T}{\sin\theta}\right) = \frac{\gamma h_T^2}{2\sin\theta}$$

$$y_1 = \frac{h}{3} \qquad y_2 = \frac{1}{3}\left(\frac{h_T}{\sin\theta}\right)$$

$$\tan\theta = \frac{80}{\ell}$$

To determine y_3 consider the pressure distribution on the bottom:



$$F_I = \gamma h_T\,\ell$$

$$F_{II} = \frac{\gamma}{2}(h - h_T)\,\ell$$

Summing moments about A,

$$F_3\, y_3 = F_I\left(\frac{\ell}{2}\right) + F_{II}\left(\frac{2}{3}\ell\right)$$

(con't)

*2.78 (con't)

so that

$$y_3 = \frac{F_I\left(\frac{\ell}{2}\right) + F_{II}\left(\frac{2}{3}\ell\right)}{F_3}$$

where $F_3 = F_I + F_{II}$. Substitution of expressions for F_I and F_{II} yields,

$$y_3 = \frac{\ell\left(\frac{h_T}{3} + \frac{2}{3}h\right)}{h + h_T}$$

For equilibrium of the dam, $\Sigma M_A = 0$, so that

$$F_1 y_1 - \mathcal{W}\left(\frac{2}{3}\ell\right) - F_2 y_2 + F_3 y_3 = 0 \qquad (1)$$

and with $\gamma = 62.4\ lb/ft^3$, $\gamma_c = 150\ lb/ft^3$, and $h_T = 10\ ft$, then:

$$F_1 = 31.2\,h^2 \qquad \mathcal{W} = 6000\,\ell \qquad F_2 = \frac{3120}{\sin\theta} \qquad y_2 = \frac{10/3}{\sin\theta}$$

$$F_3 = 31.2\,(h+10)\,\ell \qquad y_3 = \frac{\ell\left(\frac{10}{3} + \frac{2}{3}h\right)}{h + h_T} = \frac{(2h+10)\,\ell}{3(h+10)}$$

Substitution of these expressions into Eq. (1) yields,

$$(31.2\,h^2)\left(\frac{h}{3}\right) - (6000\ell)\left(\frac{2}{3}\ell\right) - \left(\frac{3120}{\sin\theta}\right)\left(\frac{10/3}{\sin\theta}\right) + \left[31.2\,(h+10)\ell\right]\left[\frac{(2h+10)\,\ell}{3(h+10)}\right] = 0$$

which can be simplified to

$$\frac{31.2}{3}h^3 + 20.8\,\ell^2 h - 3896\,\ell^2 - \frac{10{,}400}{\sin^2\theta} = 0 \qquad (2)$$

Thus, for a given ℓ, θ can be determined from the condition $\tan\theta = 80/\ell$, and Eq. (2) solved for h.

For the dam widths specified, the maximum water depths are given below. Note that for the two largest dam widths the water would overflow the dam before it would topple.

Dam width, ℓ, ft	Maximum depth, h, ft
20	48.2
30	61.1
40	71.8
50	81.1
60	89.1

2.79

2.79 (See Fluids in the News article titled "The Three Gorges Dam," Section 2.8.) (a) Determine the horizontal hydrostatic force on the 2309-m-long Three Gorges Dam when the average depth of the water against it is 175 m. (b) If all of the 6.4 billion people on Earth were to push horizontally against the Three Gorges Dam, could they generate enough force to hold it in place? Support your answer with appropriate calculations.

(a) $$F_R = \gamma h_c A = \left(9.80 \times 10^3 \frac{N}{m^3}\right)\left(\frac{175 m}{2}\right)(175 m \times 2,309 m)$$

$$= \underline{\underline{3.46 \times 10^{11} N}}$$

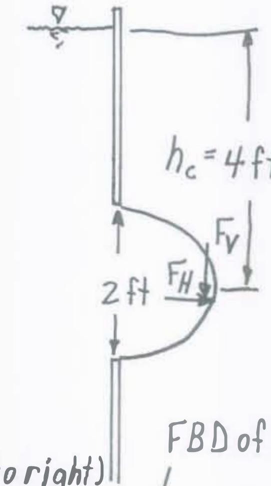
(b) Required average force per person $= \dfrac{3.46 \times 10^{11} N}{6.4 \times 10^9}$

$$= \underline{\underline{54.1 \frac{N}{person}}} \left(12.2 \frac{lb}{person}\right)$$

Yes. It is likely that enough force could be generated since required average force per person is relatively small.

2.81

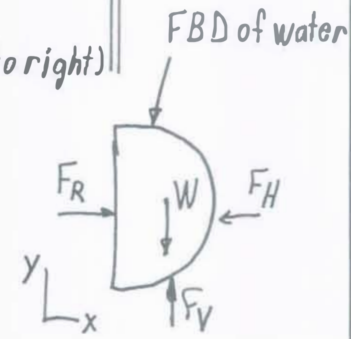
2.81 A 2-ft-diameter hemispherical plexiglass "bubble" is to be used as a special window on the side of an above-ground swimming pool. The window is to be bolted onto the vertical wall of the pool and faces outward, covering a 2-ft-diameter opening in the wall. The center of the opening is 4 ft below the surface. Determine the horizontal and vertical components of the force of the water on the hemisphere.



$\sum F_x = 0$, or $F_H = F_R = p_c A$

Thus,

$$F_H = \gamma h_c A = 62.4\,\frac{lb}{ft^3}(4\,ft)\,\frac{\pi}{4}(2\,ft)^2 = \underline{\underline{784\,lb}}\ \text{(to right)}$$



and

$\sum F_y = 0$, or $F_V = W = \gamma \mathcal{V} = \gamma\, \frac{4}{3}\pi R^3/2$,

where $R = 1\,ft$

Thus,

$$F_V = 62.4\,\frac{lb}{ft^3}\left(4\pi\,(1\,ft)^3/6\right) = \underline{\underline{131\,lb}}\ \text{(down on bubble)}$$

2.82

2.82 Two round, open tanks containing the same type of fluid rest on a table top as shown in Fig. P2.82. They have the same bottom area, A, but different shapes. When the depth, h, of the liquid in the two tanks is the same, the pressure force of the liquids on the bottom of the two tanks is the same. However, the force that the table exerts on the two tanks is different because the weight in each of the tanks is different. How do you account for this apparent paradox?

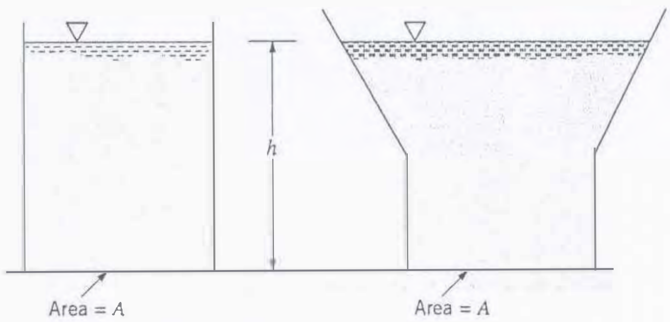
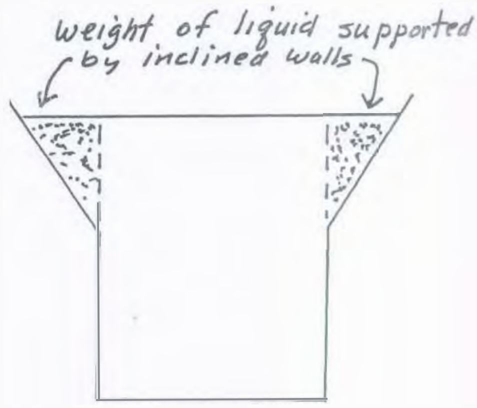


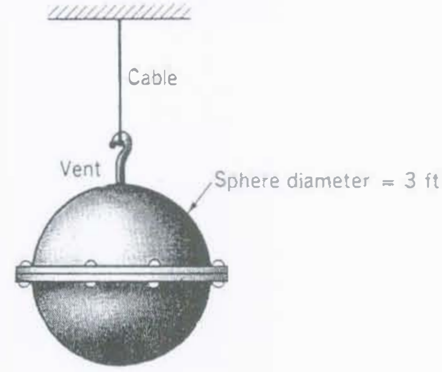
FIGURE P2.82



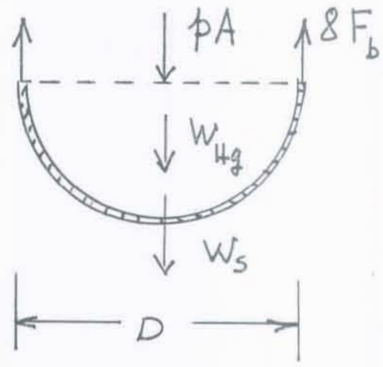
For the tank with the inclined walls, the pressure on the bottom is due to the weight of the liquid in the column directly above the bottom as shown by the dashed lines in the figure. This is the same weight as that for the tank with the straight sides. Thus, the pressure on the bottom of the two tanks is the same. The additional weight in the tank with the inclined walls is supported by the inclined walls, as illustrated in the figure.

2.83

2.83 Two hemispherical shells are bolted together as shown in Fig. P2.83. The resulting spherical container, which weighs 300 lb, is filled with mercury and supported by a cable as shown. The container is vented at the top. If eight bolts are symmetrically located around the circumference, what is the vertical force that each bolt must carry?



■ FIGURE P2.83



F_b ~ force in one bolt

p ~ pressure at mid-plane

A ~ area at mid-plane

W_{Hg} ~ weight of mercury in bottom half of shell

W_s ~ weight of bottom half of shell

For equilibrium,

$$\sum F_{vertical} = 0$$

Thus,

$$8F_b = pA + W_{Hg} + W_s$$

$$= \gamma_{Hg}\left(\frac{D}{2}\right)\left(\frac{\pi}{4}D^2\right) + \gamma_{Hg}\left(\frac{1}{2}\right)\left(\frac{\pi}{6}D^3\right) + \frac{1}{2}\,(300\text{ lb})$$

$$= \left(847\ \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{3\text{ ft}}{2}\right)\left(\frac{\pi}{4}\right)(3\text{ ft})^2 + \left(847\ \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{6}\right)(3\text{ ft})^3 + 150\text{ lb}$$

and

$$F_b = \underline{\underline{1890\text{ lb}}}$$

2.84

2.84 The 18-ft-long gate of Fig. P2.84 is a quarter circle and is hinged at H. Determine the horizontal force, P, required to hold the gate in place. Neglect friction at the hinge and the weight of the gate.

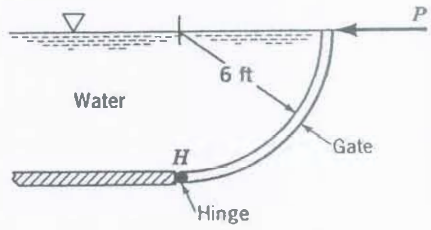
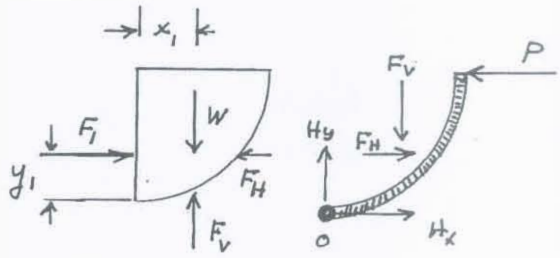


FIGURE P2.84



For equilibrium (from free-body-diagram of fluid mass),

$$\Sigma F_x = 0$$

so that

$$F_H = F_1 = \gamma h_{c1} A_1$$

$$= \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{6\,ft}{2}\right)(6\,ft \times 18\,ft) = 20{,}200\,lb$$

Similarly,

$$\Sigma F_y = 0$$

so that

$$F_V = W = \gamma_{H_2O} \times (\text{volume of fluid}) = \left(62.4 \frac{lb}{ft^3}\right)\left[\frac{\pi}{4}(6\,ft)^2 \times 18\,ft\right] = 31{,}800\,lb$$

Also,

$$x_1 = \frac{4(6\,ft)}{3\pi} = \frac{8}{\pi}\,ft \qquad (\text{see Fig. } 2.18e)$$

and

$$y_1 = \frac{6\,ft}{3} = 2\,ft$$

For equilibrium (from free-body-diagram of gate)

$$\Sigma M_0 = 0$$

so that

$$P(6\,ft) = F_H(y_1) + F_V(x_1)$$

or

$$P = \frac{(20{,}200\,lb)(2\,ft) + (31{,}800\,lb)\left(\frac{8}{\pi}\,ft\right)}{6\,ft} = \underline{\underline{20{,}200\,lb}}$$

2.85

2.85 The air pressure in the top of the two liter pop bottle shown in Video V2.5 and Fig. P2.85 is 40 psi, and the pop depth is 10 in. The bottom of the bottle has an irregular shape with a diameter of 4.3 in. **(a)** If the bottle cap has a diameter of 1 in. what is magnitude of the axial force required to hold the cap in place? **(b)** Determine the force needed to secure the bottom 2 inches of the bottle to its cylindrical sides. For this calculation assume the effect of the weight of the pop is negligible. **(c)** By how much does the weight of the pop increase the pressure 2 inches above the bottom? Assume the pop has the same specific weight as that of water.

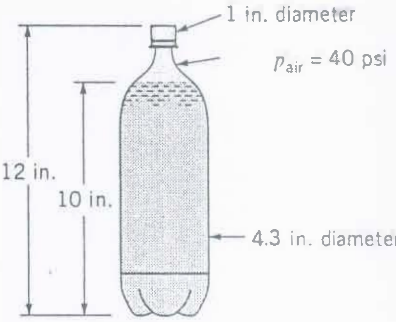


FIGURE P2.85

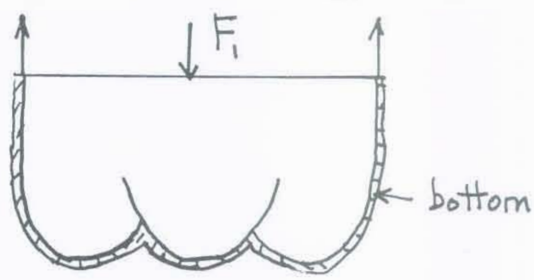
(a) $F_{cap} = p_{air} \times Area_{cap} = \left(40 \frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(1\,in.)^2 = \underline{\underline{31.4\ lb}}$

(b) $\Sigma F_{vertical} = 0$

$F_{sides} = F_1 = $ (pressure @ 2 in. above bottom) $\times$ (Area)

$= \left(40 \frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(4.3\,in.)^2$

$= \underline{\underline{581\ lb}}$



(c) $p = p_{air} + \gamma h$

$= 40 \frac{lb}{in.^2} + \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{8}{12}\,ft\right)\left(\frac{1}{144\ in.^2/ft^2}\right)$

$= 40 \frac{lb}{in.^2} + 0.289 \frac{lb}{in.^2}$

Thus, the increase in pressure due to weight = $\underline{\underline{0.289\ psi}}$ (which is less than 1% of air pressure).

2.86

2.86 Hoover Dam (see **Video 2.4**) is the highest arch-gravity type of dam in the United States. A cross section of the dam is shown in Fig. P2.86(*a*). The walls of the canyon in which the dam is located are sloped, and just upstream of the dam the vertical plane shown in Figure P2.86(*b*) approximately represents the cross section of the water acting on the dam. Use this vertical cross section to estimate the resultant horizontal force of the water on the dam, and show where this force acts.

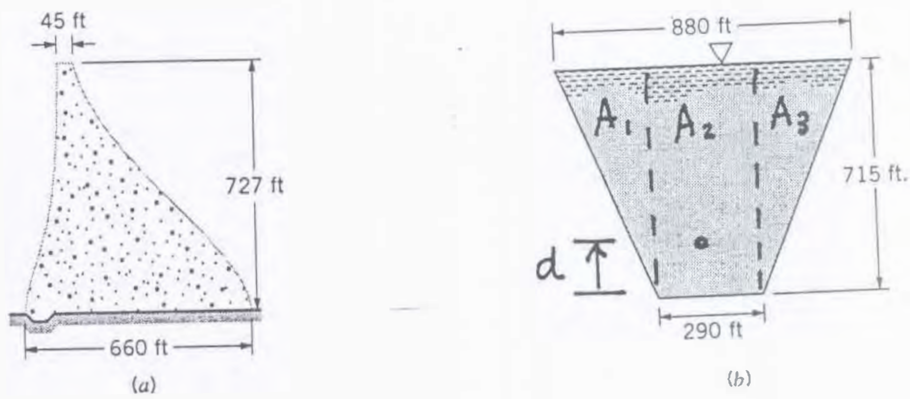


FIGURE P2.86

Break area into 3 parts as shown.

For area 1:

$$F_{R_1} = \gamma h_c A_1 = \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{1}{3}\right)(715\ ft)\left(\frac{1}{2}\right)(295\ ft)(715\ ft)$$

$$= 1.57 \times 10^9\ lb$$

For area 3: $F_{R_3} = F_{R_1} = 1.57 \times 10^9\ lb$

For area 2:

$$F_{R_2} = \gamma h_c A_2 = \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{1}{2}\right)(715\ ft)(290\ ft)(715\ ft)$$

$$= 4.63 \times 10^9\ lb$$

Thus,

$$F_R = F_{R_1} + F_{R_2} + F_{R_3} = 1.57 \times 10^9\ lb + 4.63 \times 10^9\ lb + 1.57 \times 10^9\ lb$$

$$= 7.77 \times 10^9\ lb$$

Since the moment of the resultant force about the base of the dam must be equal to the moments due to F_{R_1}, F_{R_2}, and F_{R_3}, it follows that

(con't)

2.86 (con't)

$$F_R \times d = F_{R_1}\left(\frac{2}{3}\right)(715\text{ ft}) + F_{R_2}\left(\frac{1}{2}\right)(715\text{ ft}) + F_{R_3}\left(\frac{2}{3}\right)(715\text{ ft})$$

and

$$d = \frac{(1.57\times10^9\text{ lb})\left(\frac{2}{3}\right)(715\text{ ft}) + (4.63\times10^9\text{ lb})\left(\frac{1}{2}\right)(715\text{ ft}) + (1.57\times10^9\text{ lb})\left(\frac{2}{3}\right)(715\text{ ft})}{7.77\times10^9\text{ lb}}$$

$$= 406\text{ ft}$$

Thus, the resultant horizontal force on the dam is 7.77×10^9 lb acting 406 ft up from the base of the dam along the axis of symmetry of the area.

2.87

2.87 A plug in the bottom of a pressurized tank is conical in shape as shown in Fig. P2.87. The air pressure is 40 kPa and the liquid in the tank has a specific weight of 27 kN/m³. Determine the magnitude, direction, and line of action of the force exerted on the curved surface of the cone within the tank due to the 40 kPa pressure and the liquid.

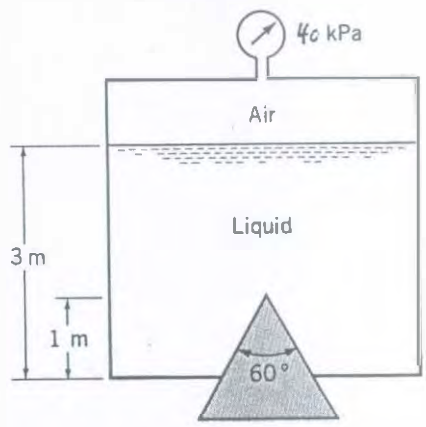
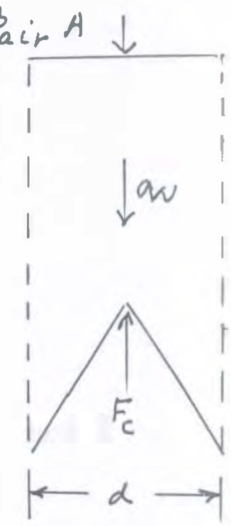


FIGURE P2.87



$\tan 30° = \dfrac{d/2}{1}$

$d = 2 \tan 30° = 1.155\,m$

volume of cone $= \frac{\pi}{3}\left(\frac{d}{2}\right)^2(1)$

For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F_c = p_{air} A + \mathcal{W}$$

where F_c is the force the cone exerts of the fluid.

Also,

$$p_{air} A = (40\,kPa)\left(\frac{\pi}{4}\right)(d^2)$$
$$= (40\,kPa)\left(\frac{\pi}{4}\right)(1.155\,m)^2 = 41.9\,kN$$

and

$$\mathcal{W} = \gamma\left[\frac{\pi}{4} d^2 (3\,m) - \frac{\pi}{3}\left(\frac{d}{2}\right)^2 (1\,m)\right]$$
$$= \gamma \pi d^2 \left[\frac{3\,m}{4} - \frac{1\,m}{12}\right]$$
$$= \left(27\,\frac{kN}{m^3}\right)(\pi)(1.155\,m)^2\left(\frac{2}{3}\,m\right) = 75.4\,kN$$

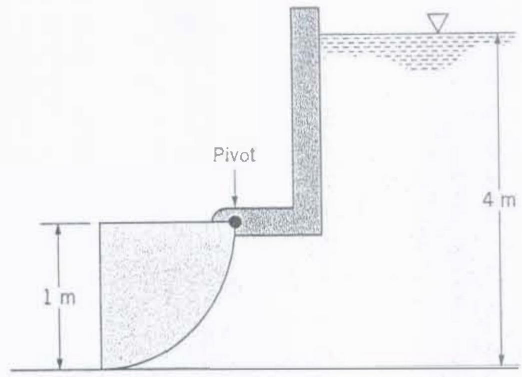
Thus,

$$F_c = 41.9\,kN + 75.4\,kN = 117\,kN$$

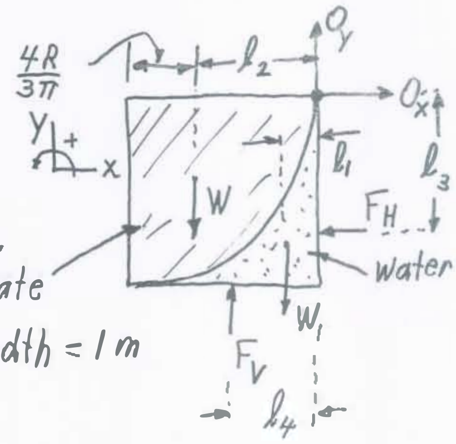
and the force on the cone has a magnitude of 117 kN and is directed vertically downward along the cone axis.

2.88

2.88 The homogeneous gate shown in Fig. P2.88 consists of one quarter of a circular cylinder and is used to maintain a water depth of 4 m. That is, when the water depth exceeds 4 m, the gate opens slightly and lets the water flow under it. Determine the weight of the gate per meter of length.



■ FIGURE P2.88



Consider the free body diagram of the gate and a portion of the water as shown.

gate width = 1 m

$\sum M_O = 0$, or

(1) $\ell_2 W + \ell_1 W_1 - F_H \ell_3 - F_V \ell_4 = 0$, where

(2) $F_H = \gamma h_c A = 9.8 \times 10^3 \frac{N}{m^3}(3.5\,m)(1\,m)(1\,m) = 34.3\,kN$

since for the vertical side, $h_c = 4m - 0.5m = 3.5m$

Also,

(3) $F_V = \gamma h_c A = 9.8 \times 10^3 \frac{N}{m^3}(4m)(1m)(1m) = 39.2\,kN$

Also,

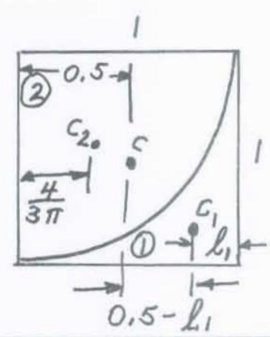
(4) $W_1 = \gamma(1m)^3 - \gamma(\frac{\pi}{4}(1m)^2)(1m) = 9.8\times10^3\frac{N}{m^3}[1 - \frac{\pi}{4}]m^3 = 2.10\,kN$

(5) Now, $\ell_4 = 0.5\,m$ and

(6) $\ell_3 = 0.5m + (y_R - y_c) = 0.5m + \frac{I_{xc}}{y_c A} = 0.5m + \frac{\frac{1}{12}(1m)(1m)^3}{3.5m(1m)(1m)} = 0.524\,m$

(7) and $\ell_2 = 1m - \frac{4R}{3\pi} = 1 - \frac{4(1m)}{3\pi} = 0.576\,m$

To determine ℓ_1, consider a unit square that consits of a quarter circle and the remainder as shown in the figure. The centroids of areas ① and ② are as indicated.



Thus,

$(0.5 - \frac{4}{3\pi})A_2 = (0.5 - \ell_1)A_1$

(con't)

2.88 (con't)

so that with $A_2 = \frac{\pi}{4}(1)^2 = \frac{\pi}{4}$ and $A_1 = 1 - \frac{\pi}{4}$ this gives

$$\left(0.5 - \frac{4}{3\pi}\right)\frac{\pi}{4} = (0.5 - \ell_1)\left(1 - \frac{\pi}{4}\right)$$

or

(8) $\ell_1 = 0.223\ m$

Hence, by combining Eqs (1) through (8):

$$(0.576m)W + (0.223m)(2.10\ kN) - (34.3\ kN)(0.524m) - (39.2\ kN)(0.5m) = 0$$

or

$W = \underline{\underline{64.4\ kN}}$

2.89

2.89 The concrete (specific weight = 150 lb/ft³) seawall of Fig. P2.89 has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).

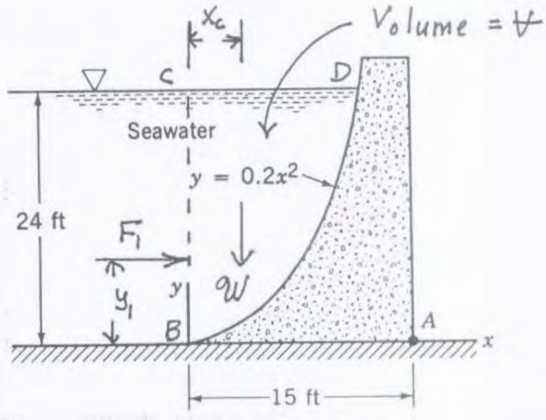


FIGURE P2.89

The components of the fluid force acting on the wall are F_1 and W as shown on the figure where

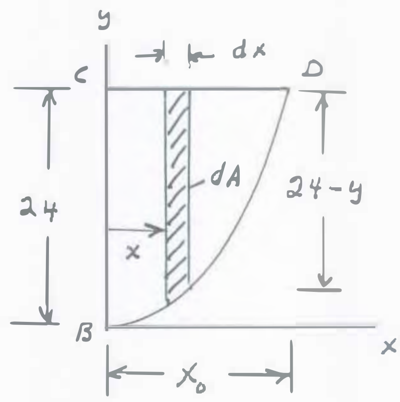
$$F_1 = \gamma h_c A = \left(64.0 \tfrac{lb}{ft^3}\right)\left(\tfrac{24\,ft}{2}\right)(24\,ft \times 1\,ft)$$

$$= 18{,}400\ lb \qquad \text{and} \qquad y_1 = \frac{24\,ft}{3} = 8\,ft$$

Also,

$$W = \gamma \forall$$

To determine $\forall$ find area BCD. Thus, (see figure to right)



$x_0 = \sqrt{120}$

(Note: All lengths in ft)

$$A = \int_0^{x_0} (24 - y)\, dx = \int_0^{x_0} (24 - 0.2x^2)\, dx$$

$$= \left[24x - \frac{0.2x^3}{3}\right]_0^{x_0}$$

and with $x_0 = \sqrt{120}$, $A = 175\ ft^2$ so that

$$\forall = A \times 1\,ft = 175\ ft^3$$

Thus,

$$W = \left(64.0 \tfrac{lb}{ft^3}\right)(175\ ft^3) = 11{,}200\ lb$$

To locate centroid of A:

$$x_c A = \int_0^{x_0} x\, dA = \int_0^{x_0} (24 - y)\, x\, dx = \int_0^{x_0} (24x - 0.2x^3)\, dx = 12x_0^2 - \frac{0.2x_0^4}{4}$$

and

$$x_c = \frac{12(\sqrt{120})^2 - \frac{0.2(\sqrt{120})^4}{4}}{175} = 4.11\ ft$$

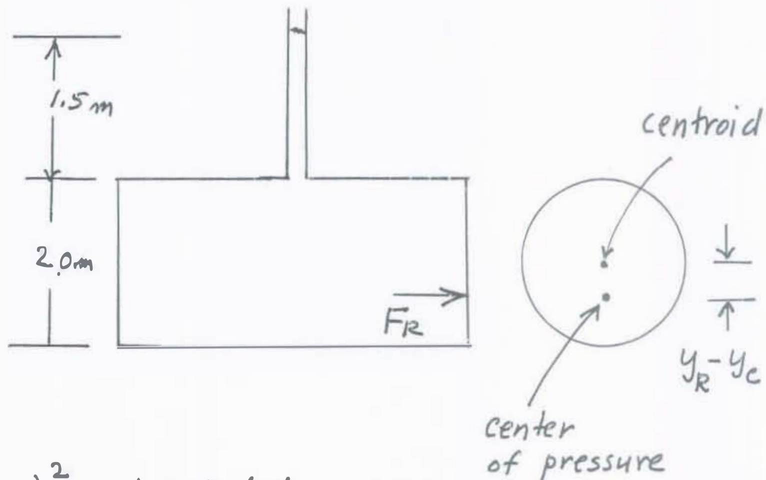
Thus,

$$M_A = F_1 y_1 - W(15 - x_c)$$

$$= (18{,}400\ lb)(8\,ft) - (11{,}200\ lb)(15\,ft - 4.11\,ft) = \underline{\underline{25{,}200\ ft \cdot lb}}$$

2.90

2.90 A cylindrical tank with its axis horizontal has a diameter of 2.0 m and a length of 4.0 m. The ends of the tank are vertical planes. A vertical, 0.1-m-diameter pipe is connected to the top of the tank. The tank and the pipe are filled with ethyl alcohol to a level of 1.5 m above the top of the tank. Determine the resultant force of the alcohol on one end of the tank and show where it acts.



$F_R = \gamma h_c A$

where $h_c = 1.5\,m + 1.0\,m = 2.5\,m$

so that

$$F_R = \left(7.74\,\frac{kN}{m^3}\right)(2.5\,m)\left(\frac{\pi}{4}\right)(2.0\,m)^2 = 60.8\,kN$$

Also,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

where $y_c = h_c$ so that

$$y_R = \frac{\frac{\pi (1\,m)^4}{4}}{(2.5\,m)\left(\frac{\pi}{4}\right)(2\,m)^2} + 2.5\,m = 2.60\,m$$

Thus, the resultant force has a magnitude of $\underline{60.8\,kN}$ and acts at a distance of $y_R - y_c = 2.60\,m - 2.50\,m = \underline{0.100\,m}$ below center of tank end wall.

2.91

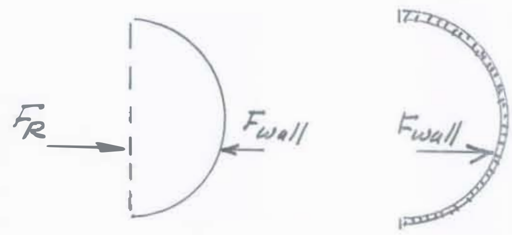
2.91 If the tank ends in Problem 2.90 are hemispherical, what is the magnitude of the resultant horizontal force of the alcohol on one of the curved ends?

For equilibrium,

$F_R = F_{wall}$ (see figure)

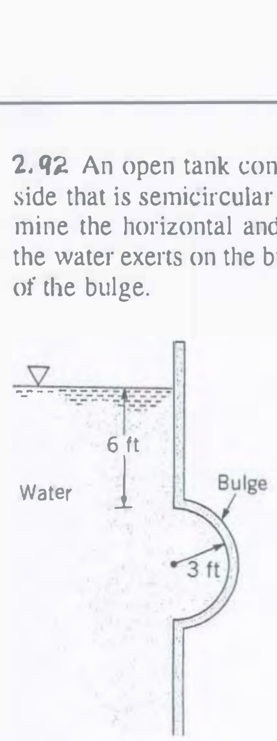
$= \underline{\underline{60.8 \text{ kN}}}$

(since solution for horizontal force the same as for Problem 2.90).



2.92

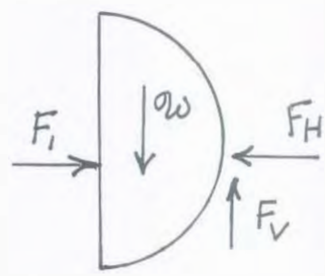
2.92 An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in Fig. P2.92. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge.



■ FIGURE P2.92

$F_H \sim$ horizontal force of wall on fluid

$F_V \sim$ vertical force of wall on fluid



$$\mathcal{W} = \gamma_{H_2O} \, \forall_{vol}$$
$$= \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{\pi (3 ft)^2}{2}\right)(1 \; ft)$$
$$= 882 \; lb$$

$$F_1 = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)(6 \; ft + 3 \; ft)(6 \; ft \times 1 \; ft)$$
$$= 3370 \; lb$$

For equilibrium, $F_V = \mathcal{W} = 882 \; lb \uparrow$

and $F_H = F_1 = 3370 \; lb \leftarrow$

The force the water exerts <u>on</u> the bulge is equal to, but opposite in direction to F_V and F_H above. Thus,

$$\underline{\underline{(F_H)_{wall} = 3370 \; lb \longrightarrow}}$$

$$\underline{\underline{(F_V)_{wall} = 882 \; lb \downarrow}}$$

2.93

2.93 A closed tank is filled with water and has a 4-ft-diameter hemispherical dome as shown in Fig. P2.93 A U-tube manometer is connected to the tank. Determine the vertical force of the water on the dome if the differential manometer reading is 7 ft and the air pressure at the upper end of the manometer is 12.6 psi.

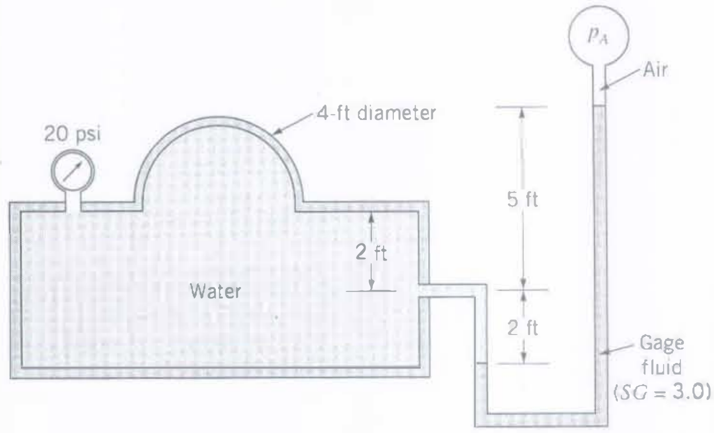
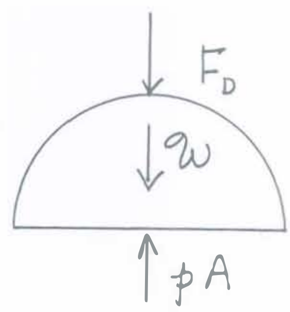


FIGURE P2.93



For equilibrium,

$$\Sigma F_{vertical} = 0$$

so that

$$F_D = pA - \mathcal{W} \qquad (1)$$

where F_D is the force the dome exerts on the fluid and p is the water pressure at the base of the dome. From the manometer,

$$p_A + \gamma_{gf}(7\,ft) - \gamma_{H_2O}(4\,ft) = p$$

so that

$$p = \left(12.6\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) + (3.0)\left(62.4\,\frac{lb}{ft^3}\right)(7\,ft) - \left(62.4\,\frac{lb}{ft^3}\right)(4\,ft)$$

$$= 2880\,\frac{lb}{ft^2}$$

Thus, from Eq. (1) with volume of sphere $= \frac{\pi}{6}(\text{diameter})^3$

$$F_D = \left(2880\,\frac{lb}{ft^2}\right)\left(\frac{\pi}{4}\right)(4\,ft)^2 - \frac{1}{2}\left[\frac{\pi}{6}(4\,ft)^3\right]\left(62.4\,\frac{lb}{ft^3}\right)$$

$$= 35{,}100\,lb$$

The force that the vertical force that the water exerts on the dome is $35{,}100\,lb \uparrow$.

2.94

2.94 A 3-m-diameter open cylindrical tank contains water and has a hemispherical bottom as shown in Fig. P2.94 Determine the magnitude, line of action, and direction of the force of the water on the curved bottom.

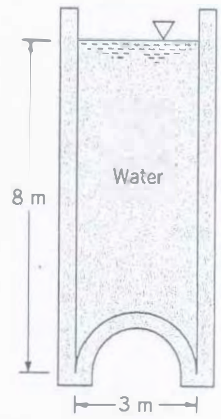


FIGURE P2.94

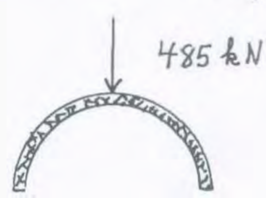
Force = weight of water supported by hemispherical bottom

$$= \gamma_{H_2O}\left[(\text{volume of cylinder}) - (\text{volume of hemisphere})\right]$$

$$= 9.80\,\frac{kN}{m^3}\left[\frac{\pi}{4}(3m)^2(8m) - \frac{\pi}{12}(3m)^3\right]$$

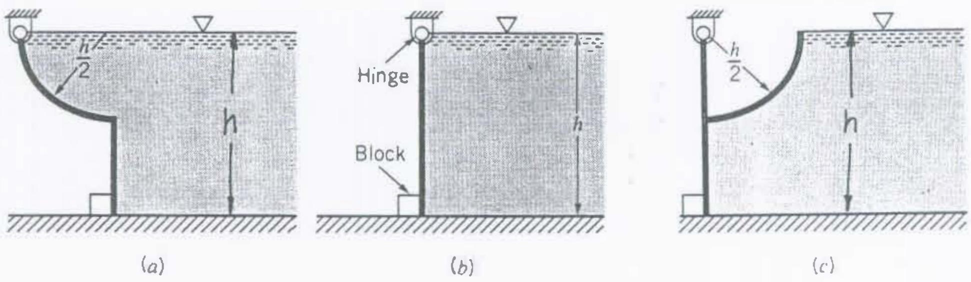
$$= \underline{\underline{485\ kN}}$$

The force is directed vertically downward, and due to symmetry it acts on the hemisphere along the vertical axis of the cylinder.



2.95

2.95 Three gates of negligible weight are used to hold back water in a channel of width b as shown in Fig. P2.95. The force of the gate against the block for gate (b) is R. Determine (in terms of R) the force against the blocks for the other two gates.

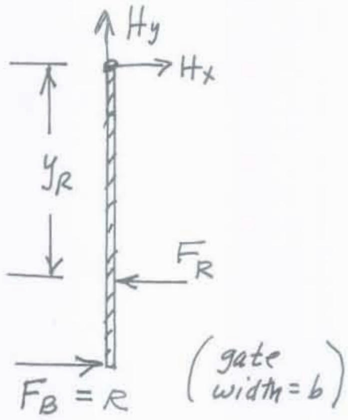


■ FIGURE P2.95

For case (b)

$$F_R = \gamma h_c A = \gamma \left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and $y_R = \frac{2}{3}h$



Thus,

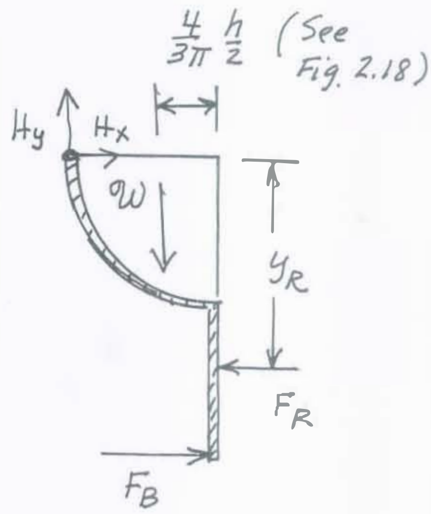
$$\Sigma M_H = 0$$

so that

$$hR = \left(\frac{2}{3}h\right)F_R$$

$$hR = \left(\frac{2}{3}h\right)\left(\frac{\gamma h^2 b}{2}\right)$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$



For case (a) on free-body-diagram shown

$F_R = \frac{\gamma h^2 b}{2}$ (from above) and

$y_R = \frac{2}{3}h$

and

$$\mathcal{W} = \gamma \times \text{Vol}$$

$$= \gamma \left[\frac{\pi \left(\frac{h}{2}\right)^2}{4}(b)\right]$$

$$= \frac{\pi \gamma h^2 b}{16}$$

Thus, $\Sigma M_H = 0$

so that

$$\mathcal{W}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3}h\right) = F_B\, h$$

and

$$\frac{\pi \gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right) = F_B\, h$$

(con't)

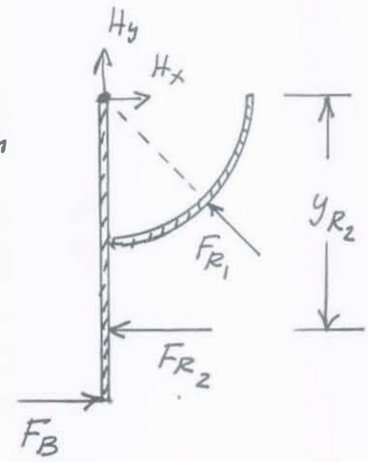
2.95 (Con't)

It follows that

$$F_B = \gamma h^2 b\,(0.390)$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \underline{\underline{1.17R}}$$



For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate

$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right) = \frac{3}{8}\gamma h^2 b$$

and

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus,

$$\Sigma M_H = 0$$

so that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B\, h$$

or

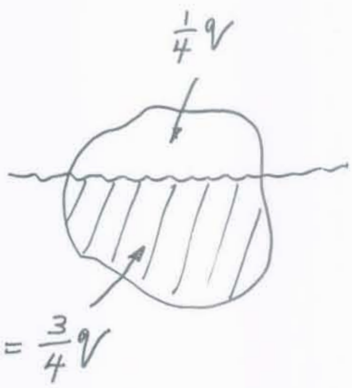
$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

2.97

2.97 A freshly cut log floats with one fourth of its volume protruding above the water surface. Determine the specific weight of the log.



$\mathcal{V}$ = log volume

$F_B = \mathcal{W}$ or

$\gamma_{H_2O}\, \mathcal{V}_{H_2O} = \gamma_{log}\, \mathcal{V}$

Thus,

$$\gamma_{log} = \gamma_{H_2O} \frac{\mathcal{V}_{H_2O}}{\mathcal{V}} = \gamma_{H_2O} \frac{\frac{3}{4}\mathcal{V}}{\mathcal{V}}$$

or

$$\gamma_{log} = \frac{3}{4}\gamma_{H_2O} = \frac{3}{4}\left(62.4\,\frac{lb}{ft^3}\right) = \underline{\underline{46.8\ \frac{lb}{ft^3}}}$$

2.98

2.98 A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 28 ft wide and 90 ft long. When unloaded its draft (depth of submergence) is 5 ft, and with the load of grain the draft is 7 ft. Determine: **(a)** the unloaded weight of the barge, and **(b)** the weight of the grain.

(a) For equilibrium,

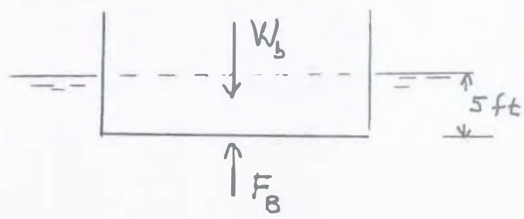
$$\sum F_{vertical} = 0$$

so that

$$W_b = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$= \left(62.4 \frac{lb}{ft^3}\right)(5\ ft \times 28\ ft \times 90\ ft)$$

$$= \underline{\underline{786{,}000\ lb}}$$



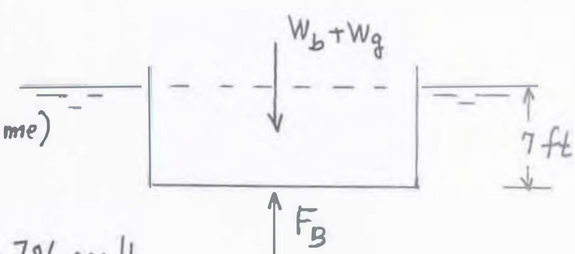
W_b ~ weight of barge (unloaded)

(b) $$\sum F_{vertical} = 0$$

$$W_b + W_g = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$W_g = \left(62.4 \frac{lb}{ft^3}\right)(7\ ft \times 28\ ft \times 90\ ft) - 786{,}000\ lb$$

$$= \underline{\underline{315{,}000\ lb}}$$



W_g ~ weight of grain

2.99

2.99 A tank of cross-sectional area A is filled with a liquid of specific weight γ_1 as shown in Fig. P2.99a. Show that when a cylinder of specific weight γ_2 and volume $\forall$ is floated in the liquid (see Fig. P2.99b), the liquid level rises by an amount $\Delta h = (\gamma_2 / \gamma_1) \forall / A$.

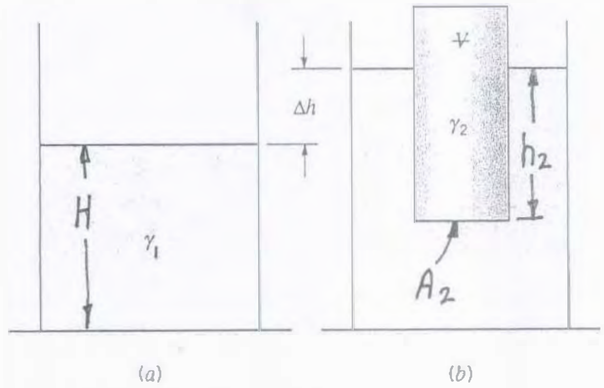


FIGURE P2.99

$\mathcal{W}$ = weight of cylinder $= \gamma_2 \forall$

For equilibrium,

$\mathcal{W}$ = weight of liquid displaced $= \gamma_1 h_2 A_2 = \gamma_1 \forall_2$ where $\forall_2 = h_2 A_2$

Thus,

$\gamma_2 \forall = \gamma_1 \forall_2$, or

$$\forall_2 = \frac{\gamma_2}{\gamma_1} \forall$$

However, the final volume within the tank is equal to the initial volume plus the volume, $\forall_2$, of the cylinder that is submerged. That is,

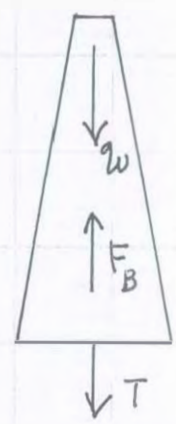
$$(H + \Delta h)A = HA + \forall_2$$

or

$$\Delta h = \frac{\forall_2}{A} = \underline{\underline{\frac{\gamma_2}{\gamma_1} \frac{\forall}{A}}}$$

2.100

2.100 When the Tucurui dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.



$\mathcal{W}$ ~ weight
F_B ~ buoyant force
T ~ tension in ropes

For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$T = F_B - \mathcal{W} \qquad (1)$$

For a truncated cone,

$$Volume = \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

where: r_1 = base radius
r_2 = top radius
h = height

Thus,

$$\forall_{tree} = \frac{(\pi)(100\,ft)}{3}\left[(4\,ft)^2 + (4\,ft \times 1\,ft) + (1\,ft)^2\right]$$

$$= 2200\ ft^3$$

For buoyant force,

$$F_B = \gamma_{H_2O} \times \forall_{tree} = \left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 137{,}000\ lb$$

For weight,

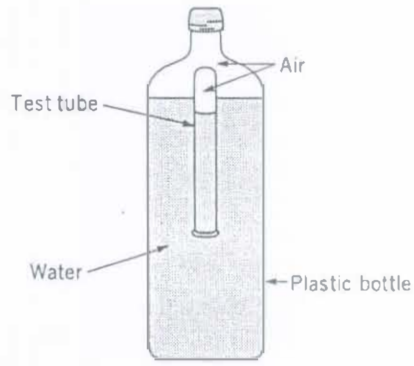
$$\mathcal{W} = \gamma_{tree} \times \forall_{tree} = (0.6)\left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 82{,}400\,lb$$

From Eq. (1)

$$T = 137{,}000\ lb - 82{,}400\ lb = \underline{\underline{54{,}600\ lb}}$$

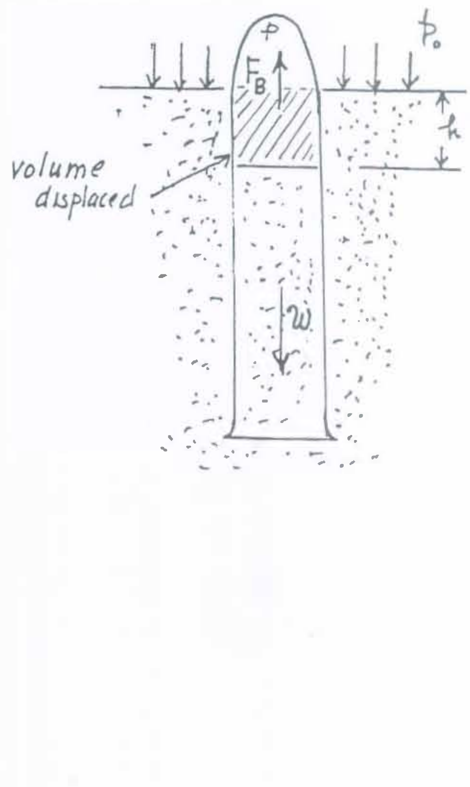
2.102

2.102 An inverted test tube partially filled with air floats in a plastic water-filled soft drink bottle as shown in Video V2.7 and Fig. P2.102. The amount of air in the tube has been adjusted so that it just floats. The bottle cap is securely fastened. A slight squeezing of the plastic bottle will cause the test tube to sink to the bottom of the bottle. Explain this phenomenon.



■ FIGURE P2.102

When the test tube is floating the weight of the tube, $\mathcal{W}$, is balanced by the buoyant force, F_B, as shown in the figure. The buoyant force is due to the displaced volume of water as shown. This displaced volume is due to the air pressure, p, trapped in the tube where $p = p_0 + \gamma_{H_2O} h$. When the bottle is squeezed, the air pressure in the bottle, p_0, is increased slightly and this in turn increases p, the pressure compressing the air in the test tube. Thus, the displaced volume is decreased with a subsequent decrease in F_B. Since $\mathcal{W}$ is constant, a decrease in F_B will cause the test tube to sink.



2.103

2.103 An irregularly shaped piece of a solid material weighs 8.05 lb in air and 5.26 lb when completely submerged in water. Determine the density of the material.

$$W(\text{in air}) = \rho g \times (\text{volume}) \qquad \text{where } \rho \sim \text{density of material}$$

$$W(\text{in water}) = \rho g \times (\text{volume}) - \text{buoyant force}$$

$$= \rho g \times (\text{volume}) - \rho_{H_2O}\, g \times (\text{volume})$$

Thus,

$$\frac{W(\text{in air})}{W(\text{in water})} = \frac{\rho}{\rho - \rho_{H_2O}} = \frac{1}{1 - \frac{\rho_{H_2O}}{\rho}}$$

or

$$\rho = \frac{\rho_{H_2O}}{1 - \frac{W(\text{in water})}{W(\text{in air})}} = \frac{1.94\ \frac{\text{slugs}}{\text{ft}^3}}{1 - \frac{5.26\ \text{lb}}{8.05\ \text{lb}}} = \underline{\underline{5.60\ \frac{\text{slugs}}{\text{ft}^3}}}$$

2.104

2.104 A 1-m-diameter cylindrical mass, M, is connected to a 2-m-wide rectangular gate as shown in Fig. P2.104. The gate is to open when the water level, h, drops below 2.5 m. Determine the required value for M. Neglect friction at the gate hinge and the pulley.

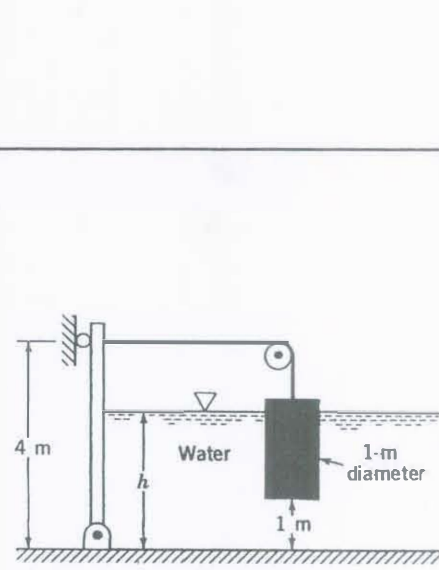
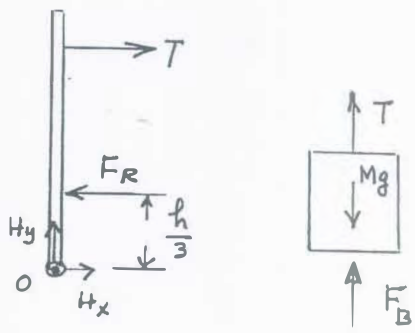


FIGURE P2.104

$$F_R = \gamma h_c A$$
$$= \gamma\left(\frac{h}{2}\right) h\,(2)$$
$$= \gamma h^2$$

where all lengths are in m.



For equilibrium,

$$\sum M_O = 0$$

so that

$$4\,T = \left(\frac{h}{3}\right) F_R = \gamma \frac{h^3}{3}$$

and

$$T = \frac{\gamma h^3}{12}$$

For the cylindrical mass $\sum F_{vertical} = 0$ and

$$T = Mg - F_B = Mg - \gamma \forall_{mass}$$

Thus,

$$M = \frac{T + \gamma \forall_{mass}}{g} = \frac{\frac{\gamma h^3}{12} + \gamma\left(\frac{\pi}{4}\right)(1)^2(h-1)}{g}$$

and for $h = 2.5\,m$

$$M = \frac{\left(9.80\times10^3\,\frac{N}{m^3}\right)\left[\frac{(2.5\,m)^3}{12} + \frac{\pi}{4}(1\,m)^2(2.5\,m - 1.0\,m)\right]}{9.81\,\frac{m}{s^2}}$$

$$= \underline{\underline{2480\,kg}}$$

2.105

2.105 When a hydrometer (see Fig. P2.105 and Video V2.8) having a stem diameter of 0.30 in. is placed in water, the stem protrudes 3.15 in. above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs 0.042 lb.

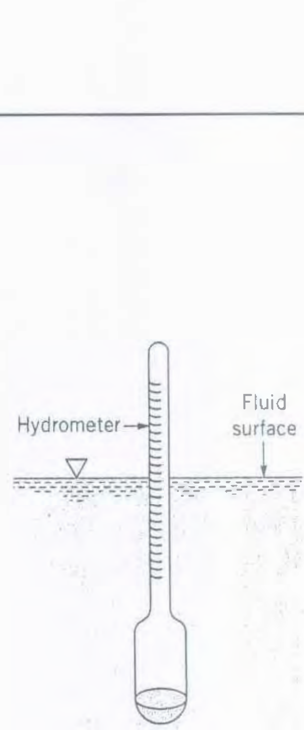
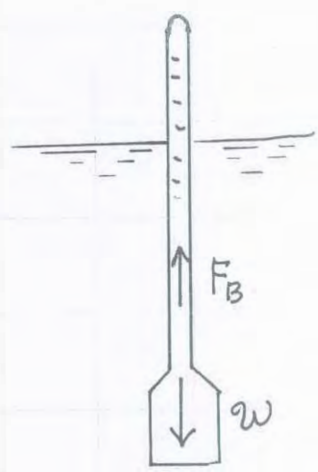


FIGURE P2.105

When the hydrometer is floating its weight, $\mathcal{W}$, is balanced by the buoyant force, F_B. For equilibrium,



$$\sum F_{vertical} = 0$$

Thus, for water

$$F_B = \mathcal{W}$$

$$(\gamma_{H_2O}) \forall_1 = \mathcal{W} \qquad (1)$$

where $\forall_1$ is the submerged volume. With the new liquid

$$(SG)(\gamma_{H_2O}) \forall_2 = \mathcal{W} \qquad (2)$$

Combining Eqs. (1) and (2) with $\mathcal{W}$ constant

$$(\gamma_{H_2O}) \forall_1 = (SG)(\gamma_{H_2O}) \forall_2$$

and

$$\forall_2 = \frac{\forall_1}{SG} \qquad (3)$$

(con't)

2.105 (Con't)

From Eq. (1)

$$\forall_1 = \frac{\mathcal{W}}{\gamma_{H_2O}} = \frac{0.042\ lb}{62.4\ \frac{lb}{ft^3}} = 6.73 \times 10^{-4}\ ft^3$$

so that from Eq. (3)

$$\forall_2 = \frac{6.73 \times 10^{-4}\ ft^3}{1.10} = 6.12 \times 10^{-4}\ ft^3$$

Thus, $\forall_1 - \forall_2 = (6.73 - 6.12) \times 10^{-4}\ ft^3 = 0.61 \times 10^{-4}\ ft^3$

To obtain this difference the change in length, $\Delta \ell$, is

$$\left(\frac{\pi}{4}\right)(0.30\ in.)^2 \Delta \ell = (0.61 \times 10^{-4}\ ft^3)\left(1728\ \frac{in.^3}{ft^3}\right)$$

$$\Delta \ell = 1.49\ in.$$

With the new liquid the stem would protrude

$$3.15\ in. + 1.49\ in. = \underline{\underline{4.64\ in.}}\ \text{above the surface}$$

2.106

2.106 A 2-ft-thick block constructed of wood (SG = 0.6) is submerged in oil (SG = 0.8). and has a 2-ft-thick aluminum (specific weight = 168 lb/ft^3) plate attached to the bottom as indicated in Fig. P2.106. Determine completely the force required to hold the block in the position shown. Locate the force with respect to point A.

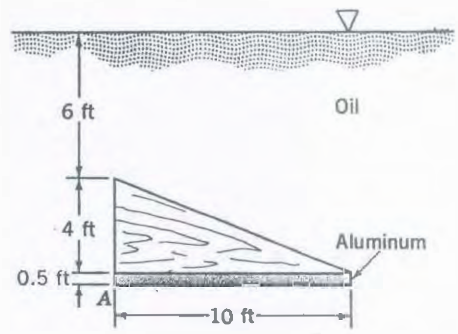
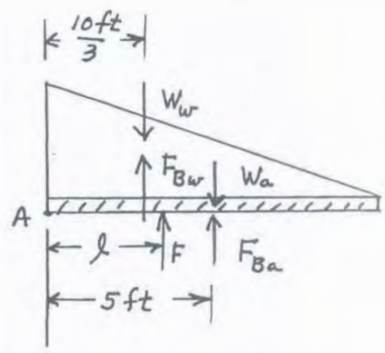


FIGURE P2.106



For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F = W_w - F_{Bw} + W_a - F_{Ba}$$

where:

w ~ wood
a ~ aluminum
F ~ force to hold block

$$W_w = (SG_w)(\gamma_{H_2O})\,\forall_w$$
$$= (0.6)\left(62.4\,\tfrac{lb}{ft^3}\right)\left(\tfrac{1}{2}\right)(10ft \times 4ft \times 2ft) = 1500\ lb$$

$$W_a = \left(168\,\tfrac{lb}{ft^3}\right)(0.5\,ft \times 10ft \times 2ft) = 1680\ lb$$

$$F_{Bw} = (SG_{oil})(\gamma_{H_2O})\,\forall_w = (0.8)\left(62.4\,\tfrac{lb}{ft^3}\right)\left(\tfrac{1}{2}\right)(10ft \times 4ft \times 2\,ft) = 2000\ lb$$

$$F_{Ba} = (SG_{oil})(\gamma_{H_2O})\,\forall_a = (0.8)\left(62.4\,\tfrac{lb}{ft^3}\right)(0.5ft \times 10ft \times 2ft) = 499\ lb$$

Thus,

$$F = 1500\ lb - 2000\ lb + 1680\ lb - 499\ lb = \underline{\underline{681\ lb\ \text{upward}}}$$

Also,

$$\sum M_A = 0$$

So that

$$\ell F = \left(\tfrac{10}{3}\,ft\right)(W_w - F_{Bw}) + (5\,ft)(W_a - F_{Ba})$$

or

$$\ell\,(681\ lb) = \left(\tfrac{10}{3}\,ft\right)(1500\ lb - 2000\ lb) + (5\,ft)(1680\ lb - 499\ lb)$$

and

$$\underline{\underline{\ell = 6.22\ ft\ \text{to right of point } A}}$$

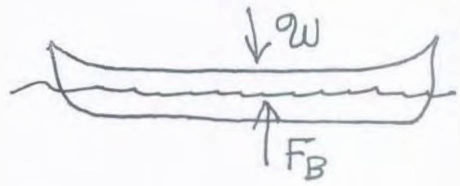
2.107

2.107 (See Fluids in the News article titled "Concrete canoe," Section 2.11.1.) How much extra water does a 147-lb concrete canoe displace compared to an ultralightweight 38-lb Kevlar canoe of the same size carrying the same load?

For equilibrium,

$$\sum F_{vertical} = 0$$

and $\mathcal{W} = F_B = \gamma_{H_2O} \forall$ and $\forall$ is displaced volume.



For concrete canoe,

$$147 \text{ lb} = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right) \forall_c$$

$$\forall_c = 2.36 \text{ ft}^3$$

For Kevlar canoe,

$$38 \text{ lb} = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right) \forall_K$$

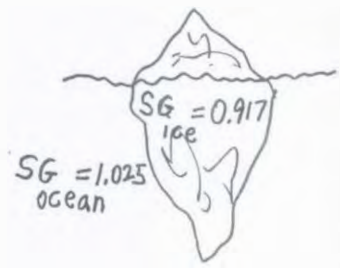
$$\forall_K = 0.609 \text{ ft}^3$$

Extra water displacement $= 2.36 \text{ ft}^3 - 0.609 \text{ ft}^3$

$= \underline{\underline{1.75 \text{ ft}^3}}$

2.108

2.108 An ice berg (specific gravity 0.917) floats in the ocean (specific gravity 1.025). What percent of the volume of the iceberg is under water?



For equilibrium,

$\mathcal{W}$ = weight of iceberg = F_B = buoyant force

or

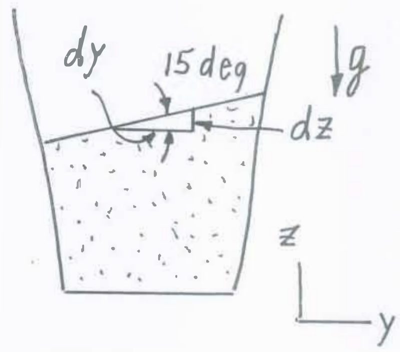
$\mathcal{V}_{ice}\,\gamma_{ice} = \mathcal{V}_{sub}\,\gamma_{ocean}$, where $\mathcal{V}_{sub}$ = volume of ice submerged.

Thus,

$$\frac{\mathcal{V}_{sub}}{\mathcal{V}_{ice}} = \frac{\gamma_{ice}}{\gamma_{ocean}} = \frac{SG_{ice}}{SG_{ocean}} = \frac{0.917}{1.025} = 0.895 = \underline{\underline{89.5\,\%}}$$

2.110

2.110 It is noted that while stopping, the water surface in a glass of water sitting in the cup holder of a car is slanted at an angle of 15° relative to the horizontal street. Determine the rate at which the car is decelerating.



$$\frac{dz}{dy} = -\frac{a_y}{g + a_z}$$

where $a_z = 0$ and $\frac{dz}{dy} = \tan 15° = 0.268$

Thus,

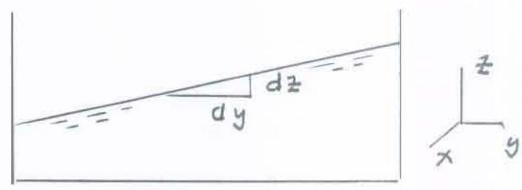
$$0.268 = -\frac{a_y}{g} = -\frac{a_y}{32.2\,ft/s^2}$$

or

$$a_y = -(0.268)(32.2\,\frac{ft}{s^2}) = \underline{\underline{-8.63\,\frac{ft}{s^2}}}$$

2.111

2.111 An open container of oil rests on the flatbed of a truck that is traveling along a horizontal road at 55 mi/hr. As the truck slows uniformly to a complete stop in 5 s, what will be the slope of the oil surface during the period of constant deceleration?



$$\text{slope} = \frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad (\text{Eq. } 2.28)$$

$$a_y = \frac{\text{final velocity} - \text{initial velocity}}{\text{time interval}}$$

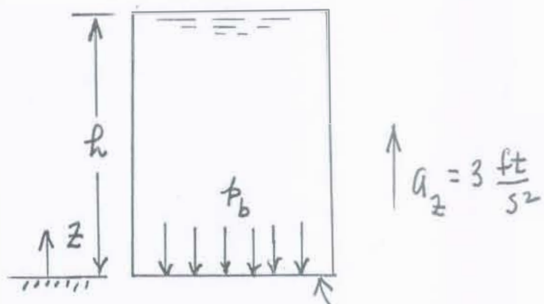
$$= \frac{0 - (55\ mph)\left(0.4470\ \frac{\frac{m}{s}}{mph}\right)}{5\ s} = -4.92\ \frac{m}{s^2}$$

Thus,

$$\frac{dz}{dy} = -\frac{\left(-4.92\ \frac{m}{s^2}\right)}{9.81\ \frac{m}{s^2} + 0} = \underline{\underline{0.502}}$$

2.112

2.112 A 5-gal, cylindrical open container with a bottom area of 120 in.^{2} is filled with glycerin and rests on the floor of an elevator. **(a)** Determine the fluid pressure at the bottom of the container when the elevator has an upward acceleration of 3 ft/s^2. **(b)** What resultant force does the container exert on the floor of the elevator during this acceleration? The weight of the container is negligible. (Note: 1 gal = 231 in.^{3})



(a) $\frac{dp}{dz} = -\rho(g + a_z)$ (Eq. 2.26)

hA = volume

$h\ (120\ in.^2) = (5\ gal)\left(\frac{231\ in.^3}{gal}\right)$

$h = 9.63\ in.$

Thus,

$$\int_0^{p_b} dp = -\rho(g + a_z)\int_h^0 dz$$

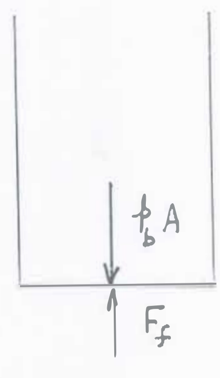
and

$$p_b = \rho(g + a_z)h$$

$$= \left(2.44\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2} + 3\ \frac{ft}{s^2}\right)\left(\frac{9.63}{12}\ ft\right)$$

$$= \underline{\underline{68.9\ \frac{lb}{ft^2}}}$$

(b) From free-body-diagram of container,



$$F_f = p_b A$$

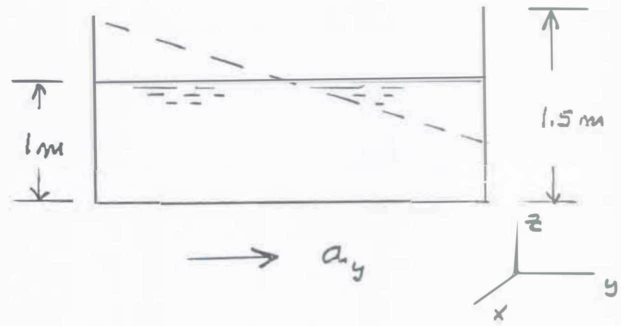
$$= \left(68.9\ \frac{lb}{ft^2}\right)(120\ in.^2)\left(\frac{1\ ft^2}{144\ in.^2}\right)$$

$$= 57.4\ lb$$

Thus, force of container on floor is $\underline{\underline{57.4\ lb\ downward.}}$

2.113

2.113 An open rectangular tank 1 m wide and 2 m long contains gasoline to a depth of 1 m. If the height of the tank sides is 1.5 m, what is the maximum horizontal acceleration (along the long axis of the tank) that can develop before the gasoline would begin to spill?



To prevent spilling,

$$\frac{dz}{dy} \geq -\frac{1.5\,m - 1.0\,m}{1\,m} = -0.50$$

(see figure).

Since,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad (\text{Eq. } 2.28)$$

or, with $a_z = 0$,

$$a_y = -\left(\frac{dz}{dy}\right) g$$

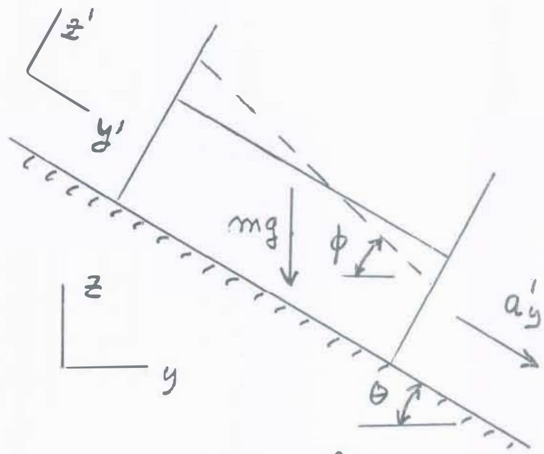
so that

$$(a_y)_{max} = -(-0.50)\left(9.81\,\frac{m}{s^2}\right) = \underline{\underline{4.91\,\frac{m}{s^2}}}$$

(Note: Acceleration could be either to the right or the left.)

2.114

2.114 If the tank of Problem 2.113 slides down a frictionless plane that is inclined at 30° with the horizontal, determine the angle the free surface makes with the horizontal.



$m \sim$ mass of tank and gasoline

From Newton's 2nd law,

$$\sum F_{y'} = m a_y'$$

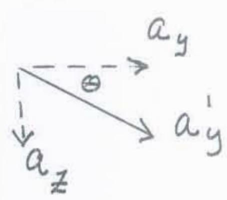
Since the only force in the y'-direction is the component of weight $(mg)\sin\theta$,

$$(mg)\sin\theta = m a_y'$$

so that

$$a_y' = g \sin\theta$$

and therefore



$$a_y = a_y' \cos\theta \qquad a_z = -a_y' \sin\theta$$

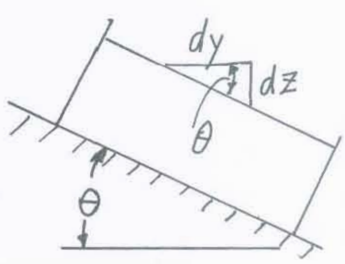
Also,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

$$= -\frac{a_y' \cos\theta}{g - a_y' \sin\theta} = -\frac{g\sin\theta\cos\theta}{g - g\sin\theta\sin\theta}$$

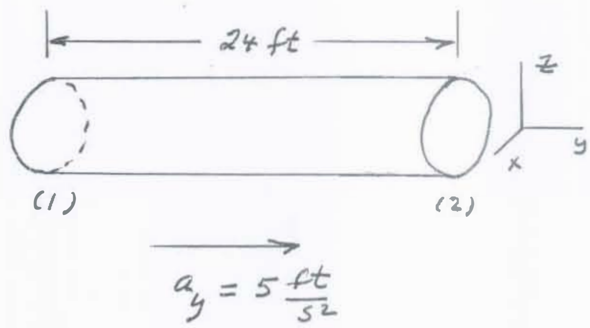
$$= -\frac{\sin\theta\cos\theta}{1-\sin^2\theta} = -\frac{\sin\theta\cos\theta}{\cos^2\theta} = -\tan\theta$$

Hence, $\frac{dz}{dy} = -\tan\theta$, so that the free surface is at <u>the same angle as the plane.</u>



2.115

2.115 A closed cylindrical tank that is 8 ft in diameter and 24 ft long is completely filled with gasoline. The tank, with its long axis horizontal, is pulled by a truck along a horizontal surface. Determine the pressure difference between the ends (along the long axis of the tank) when the truck undergoes an acceleration of 5 ft/s².



$$\frac{\partial p}{\partial y} = -\rho a_y \qquad \text{(Eq. 2.25)}$$

Thus,

$$\int_{p_1}^{p_2} dp = -\rho a_y \int_0^{24} dy$$

where $p = p_1$ at $y = 0$ and $p = p_2$ at $y = 24\ ft$,

and

$$p_2 - p_1 = -\rho a_y (24\ ft)$$

$$= -\left(1.32\ \frac{slugs}{ft^3}\right)\left(5\ \frac{ft}{s^2}\right)(24\ ft)$$

$$= -158\ \frac{lb}{ft^2}$$

or

$$p_1 - p_2 = \underline{\underline{158\ \frac{lb}{ft^2}}}$$

2.116

2.116 The open U-tube of Fig. P2.116 is partially filled with a liquid. When this device is accelerated with a horizontal acceleration, a, a differential reading, h, develops between the manometer legs which are spaced a distance ℓ apart. Determine the relationship between a, ℓ, and h.

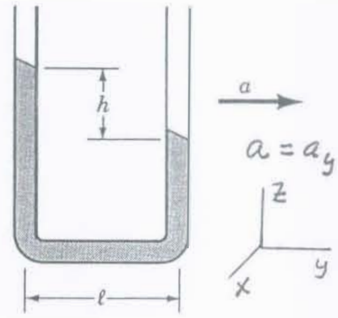


FIGURE P2.116

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

Since, $\frac{dz}{dy} = -\frac{h}{\ell}$ and $a_z = 0$

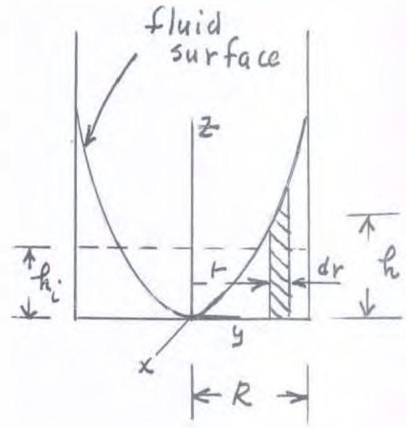
then $-\frac{h}{\ell} = -\frac{a}{g+0}$

or

$$\underline{\underline{h = \frac{a\ell}{g}}}$$

2.117

2.117 An open 1-m-diameter tank contains water at a depth of 0.7 m when at rest. As the tank is rotated about its vertical axis the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.



h_i ~ initial depth

Equation for surfaces of constant pressure (Eq. 2.32):

$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

For free surface with $h=0$ at $r=0$,

$$h = \frac{\omega^2 r^2}{2g}$$

The volume of fluid in rotating tank is given by

$$\forall_f = \int_0^R 2\pi r h \, dr = \frac{2\pi\, \omega^2}{2g}\int_0^R r^3 dr = \frac{\pi \omega^2 R^4}{4g}$$

Since the initial volume, $\forall_i = \pi R^2 h_i$, must equal the final volume,

$$\forall_f = \forall_i$$

so that

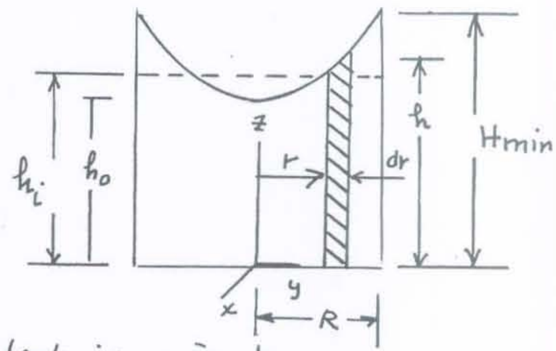
$$\frac{\pi \omega^2 R^4}{4g} = \pi R^2 h_i$$

or

$$\omega = \sqrt{\frac{4 g h_i}{R^2}} = \sqrt{\frac{4(9.81 \frac{m}{s^2})(0.7m)}{(0.5m)^2}} = \underline{\underline{10.5 \frac{rad}{s}}}$$

2.118

2.118 An open, 2-ft-diameter tank contains water to a depth of 3 ft when at rest. If the tank is rotated about its vertical axis with an angular velocity of 180 rev/min, what is the minimum height of the tank walls to prevent water from spilling over the sides?



For free surface,

$$h = \frac{\omega^2 r^2}{2g} + h_o \qquad \text{(Eq. 2.32)}$$

The volume of fluid in the rotating tank is given by

$$\forall_f = \int_0^R 2\pi r h \, dr = 2\pi \int_0^R \left(\frac{\omega^2 r^3}{2g} + h_o r \right) dr$$

$$= \frac{\pi \omega^2 R^4}{4g} + \pi h_o R^2$$

$$= \frac{\pi \left(180 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{1\,min}{60\,s}\right)^2 (1\,ft)^4}{4\left(32.2 \frac{ft}{s^2}\right)} + \pi h_o (1\,ft)^2$$

$$= \pi \, (2.76 + h_o) \; ft^3 \qquad (\text{with } h_o \text{ in ft})$$

Since the initial volume,

$$\forall_i = \pi R^2 h_i = \pi (1\,ft)^2 (3\,ft) = 3\pi \; ft^3$$

and the final volume must be equal,

$$\forall_f = \forall_i$$

or

$$\pi \, (2.76 + h_o) \; ft^3 = 3\pi \; ft^3$$

and

$$h_o = 0.240 \; ft$$

Thus, from the first equation (Eq. 2.32)

$$h = \frac{\omega^2 r^2}{2g} + 0.240 \; ft$$

and

$$H_{min} = \frac{\left(180 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{1\,min}{60\,s}\right)^2 (1\,ft)^2}{2\left(32.2 \frac{ft}{s^2}\right)} + 0.240\,ft = \underline{\underline{5.76\,ft}}$$

2.119

2.119 A child riding in a car holds a string attached to a floating, helium-filled balloon. As the car decelerates to a stop, the balloon tilts backwards. As the car makes a right-hand turn, the balloon tilts to the right. On the other hand, the child tends to be forced forward as the car decelerates and to the left as the car makes a right-hand turn. Explain these observed effects on the balloon and child.

A floating balloon attached to a string will align itself so that the string it normal to lines of constant pressure. Thus, if the car is not accelerating, the lines of p = constant pressure are horizontal (gravity acts vertically down), and the balloon floats "straight up" (i.e. $\theta = 0$). If forced to the side ($\theta \neq 0$), the balloon will return to the vertical ($\theta = 0$) equilibrium position in which the two forces T and $F_B - W$ line up

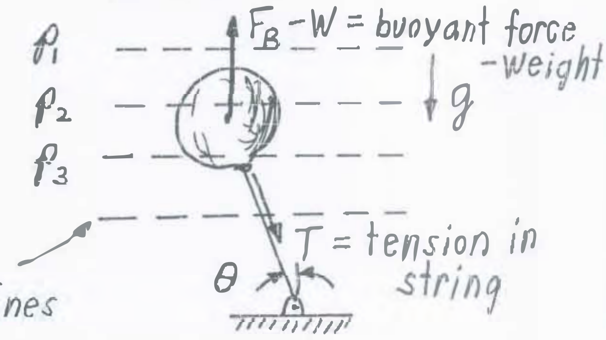


Fig. (1) No acceleration, $\theta = 0$ for equilibrium.

Consider what happens when the car decelerates with an amount $a_y < 0$. As show by Eq. (2.28), the lines of constant pressure are not horizontal, but have a slope of

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} = -\frac{a_y}{g} > 0 \text{ since } a_z = 0$$

and $a_y < 0$, Again, the balloon's equilibrium position is with the string normal to p = const. lines. That is, the balloon tilts back as the car stops.

When the car turns, $a_y = \frac{V^2}{R}$ (the centrifugal acceleration), the lines of p = const. are as shown, and the balloon tilts to the outside of the curve

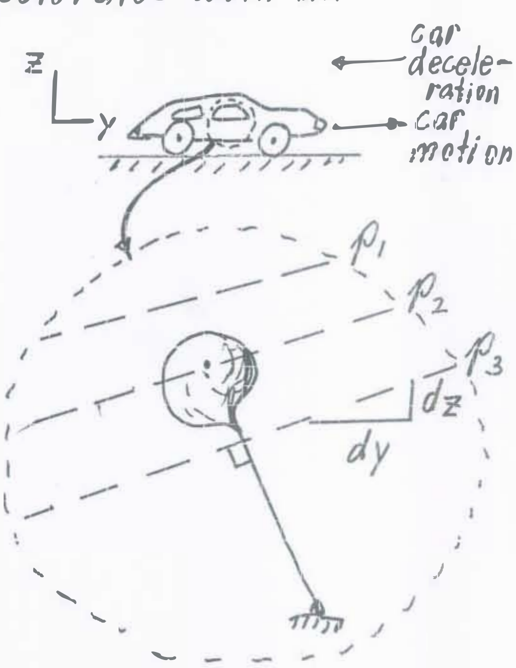
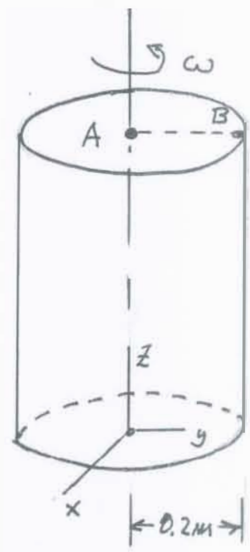


Fig. (2) Balloon aligned so that string is normal to p = constant lines

Fig. (3) Left turn; balloon tilts to right

2.120

2.120 A closed, 0.4-m-diameter cylindrical tank is completely filled with oil ($SG = 0.9$) and rotates about its vertical longitudinal axis with an angular velocity of 40 rad/s. Determine the difference in pressure just under the vessel cover between a point on the circumference and a point on the axis.



Pressure in a rotating fluid varies in accordance with the equation,

$$p = \frac{\rho \omega^2 r^2}{2} - \gamma z + \text{constant} \qquad (\text{Eq. } 2.33)$$

Since $z_A = z_B$,

$$p_B - p_A = \frac{\rho \omega^2}{2}\left(r_B^2 - r_A^2\right)$$

$$= \frac{(0.9)(10^3 \frac{kg}{m^3})(40 \frac{rad}{s})^2}{2}\left[(0.2\,m)^2 - 0\right]$$

$$= \underline{\underline{28.8\ kPa}}$$

2.121

2.121 (See Fluids in the News article titled "**Rotating mercury mirror telescope,**" Section 2.12.2.) The largest liquid mirror telescope uses a 6-ft-diameter tank of mercury rotating at 7 rpm to produce its parabolic-shaped mirror as shown in Fig. P2.121. Determine the difference in elevation of the mercury, Δh, between the edge and the center of the mirror.

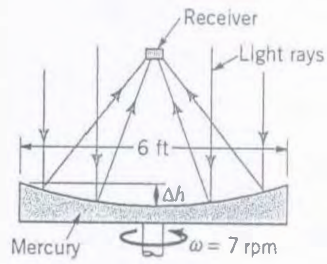


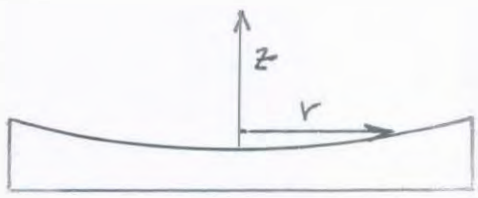
FIGURE P2.121

For free surface of rotating liquid,

$$z = \frac{\omega^2 r^2}{2g} + \text{constant} \qquad \text{(Eq. 2.32)}$$

Let $z = 0$ at $r = 0$ and therefore constant $= 0$. Thus,

$\Delta h = \Delta z$ for $r = 3\text{ ft}$ and with



$$\omega = (7\text{ rpm})\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 0.733\,\frac{\text{rad}}{\text{s}}$$

it follows that

$$\Delta h = \frac{\left(0.733\,\frac{\text{rad}}{\text{s}}\right)^2 (3\text{ ft})^2}{2\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)} = \underline{\underline{0.0751\text{ ft}}}$$

2.122

2.122 Force Needed to Open a Submerged Gate

Objective: A gate, hinged at the top, covers a hole in the side of a water filled tank as shown in Fig. P2.122 and is held against the tank by the water pressure. The purpose of this experiment is to compare the theoretical force needed to open the gate to the experimentally measured force.

Equipment: Rectangular tank with a rectangular hole in its side; gate that covers the hole and is hinged at the top; force transducer to measure the force needed to open the gate; ruler to measure the water depth.

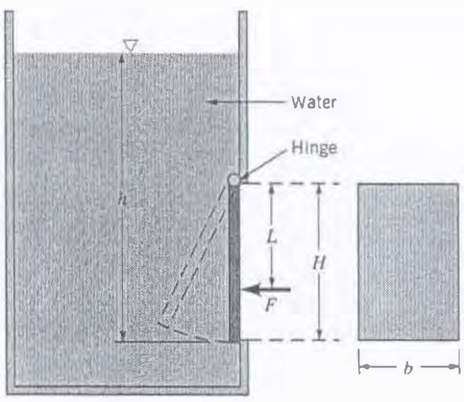
Experimental Procedure: Measure the height, H, and width, b, of the hole in the tank and the distance, L, from the hinge to the point of application of the force, F, that opens the gate. Fill the tank with water to a depth h above the bottom of the gate. Use the force transducer to determine the force, F, needed to slowly open the gate. Repeat the force measurements for various water depths.

Calculations: For arbitrary water depths, h, determine the theoretical force, F, needed to open the gate by equating the moment about the hinge from the water force on the gate to the moment produced by the applied force, F.

Graph: Plot the experimentally determined force, F, needed to open the gate as ordinates and the water depth, h, as abscissas.

Results: On the same graph, plot the theoretical force as a function of water depth.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P2.122

(con't)

2.122 (Con't)

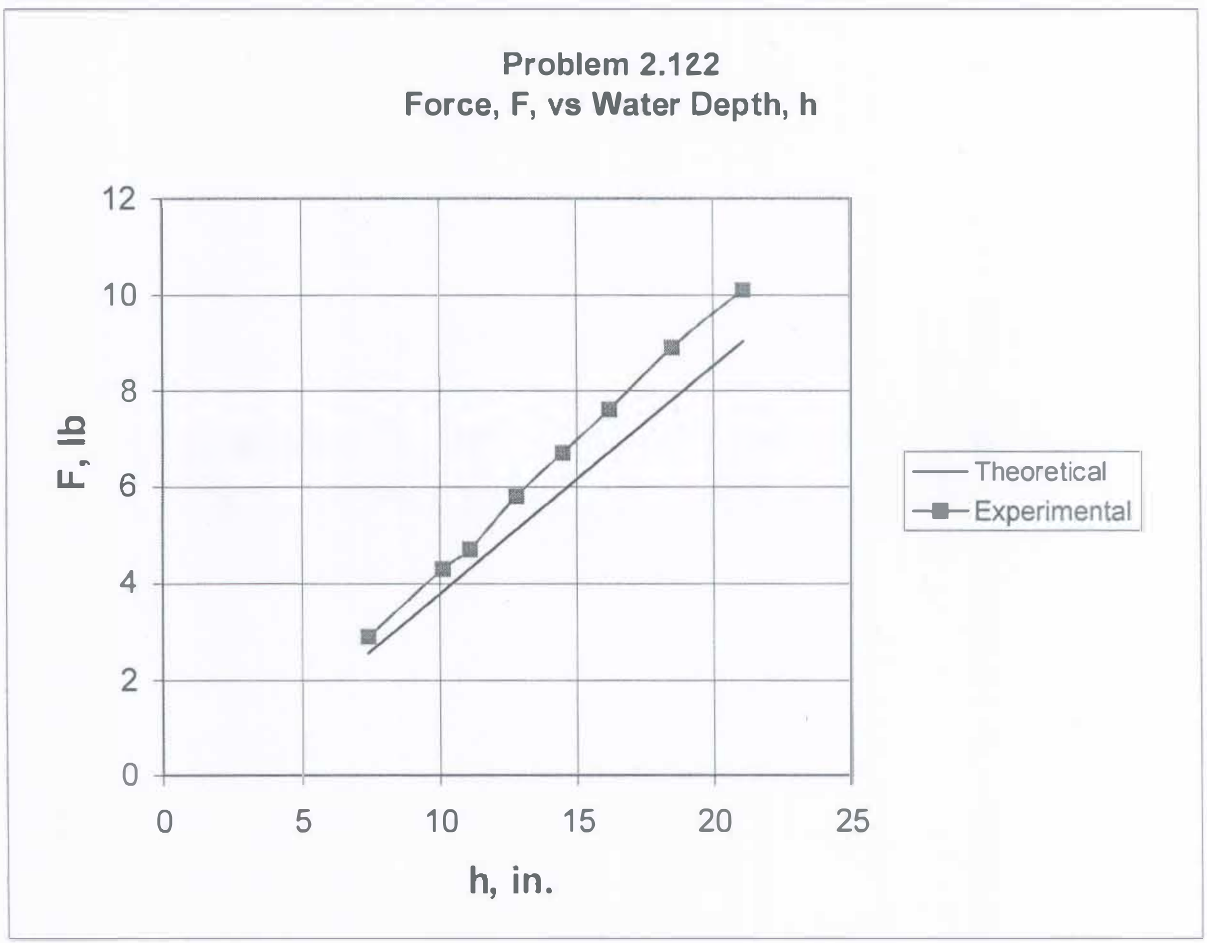
Solution for Problem 2.122: Force Needed to Open a Submerged Gate

L, in.	H, in.	b, in.	γ, lb/ft^3	I_{xc}, ft^4
5.5	6.0	4.0	62.4	0.003472

h, in.	F, lb	F_1, lb	$y_r - y_c$, ft	d, ft	F, lb
21.1	10.1	15.69	0.0138	0.264	9.03
18.5	8.9	13.43	0.0161	0.266	7.80
16.2	7.6	11.44	0.0189	0.269	6.71
14.5	6.7	9.97	0.0217	0.272	5.91
12.8	5.8	8.49	0.0255	0.276	5.11
11.1	4.7	7.02	0.0309	0.281	4.30
10.1	4.3	6.15	0.0352	0.285	3.83
7.4	2.9	3.81	0.0568	0.307	2.55

Since h > H, A = H*b = constant and I_{xc} = b*H^3/12 = constant.

F = F_1*d/L, where F_1 = γ*(h - H/2)*A, d = H/2 + (y_r - y_c), and y_r - y_c = I_{xc}/(h - H/2)*A



2.123

2.123 Hydrostatic Force on a Submerged Rectangle

Objective: A quarter-circle block with a vertical rectangular end is attached to a balance beam as shown in Fig. P2.123. Water in the tank puts a hydrostatic pressure force on the block which causes a clockwise moment about the pivot point. This moment is balanced by the counterclockwise moment produced by the weight placed at the end of the balance beam. The purpose of this experiment is to determine the weight, W, needed to balance the beam as a function of the water depth, h.

Equipment: Balance beam with an attached quarter-circle, rectangular cross-section block; pivot point directly above the vertical end of the beam to support the beam; tank; weights; ruler.

Experimental Procedure: Measure the inner radius, R_1, outer radius, R_2, and width, b, of the block. Measure the length, L, of the moment arm between the pivot point and the weight. Adjust the counter weight on the beam so that the beam is level when there is no weight on the beam and no water in the tank. Hang a known mass, m, on the beam and adjust the water level, h, in the tank so that the beam again becomes level. Repeat with different masses and water depths.

Calculations: For a given water depth, h, determine the hydrostatic pressure force, $F_R = \gamma h_c A$, on the vertical end of the block. Also determine the point of action of this force, a distance $y_R - y_c$ below the centroid of the area. Note that the equations for F_R and $y_R - y_c$ are different when the water level is below the end of the block ($h < R_2 - R_1$) than when it is above the end of the block ($h > R_2 - R_1$).

For a given water depth, determine the theoretical weight needed to balance the beam by summing moments about the pivot point. Note that both F_R and W produce a moment. However, because the curved sides of the block are circular arcs centered about the pivot point, the pressure forces on the curved sides of the block (which act normal to the sides) do not produce any moment about the pivot point. Thus the forces on the curved sides do not enter into the moment equation.

Graph: Plot the experimentally determined weight, W, as ordinates and the water depth, h, as abscissas.

Result: On the same graph plot the theoretical weight as a function of water depth.

Data: To proceed, print this page for reference when you work the problem and click here to bring up an EXCEL page with the data for this problem.

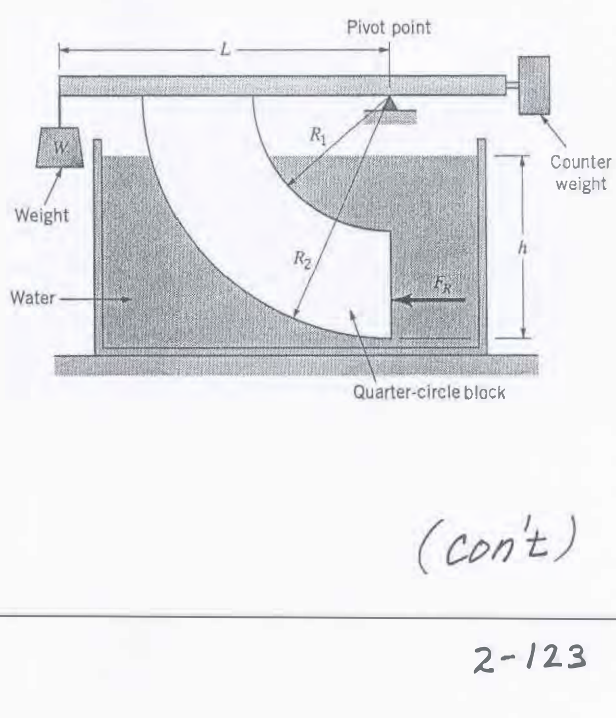


FIGURE P2.123

(con't)

2.123 (Con't)

Solution for Problem 2.123: Hydrostatic Force on a Submerged Rectangle

R_1, in.	R_2, in.	L, in.	b, in.		g, ft/s^2	γ, lb/ft^3	
5.0	9.0	12.0	3.0		32.2	62.4	

		Experimental				Theoretical
m, kg	h, in.	W, lb	F_R, lb	$y_r - y_c$, ft	**d**, ft	W, lb
0.00	0.00	0.00	0.00		0.750	0.000
0.02	1.11	0.04	0.07		0.719	0.048
0.04	1.58	0.09	0.14		0.706	0.095
0.06	1.92	0.13	0.20		0.697	0.139
0.10	2.51	0.22	0.34		0.680	0.232
0.12	2.76	0.26	0.41		0.673	0.278
0.14	2.99	0.31	0.48		0.667	0.323
0.16	3.20	0.35	0.55		0.661	0.367
0.18	3.41	0.40	0.63		0.655	0.413
0.20	3.60	0.44	0.70		0.650	0.456
0.22	3.80	0.48	0.78		0.644	0.504
0.24	3.99	0.53	0.86		0.639	0.551
0.26	4.17	0.57	0.94	0.0512	0.634	0.597
0.28	4.33	0.62	1.01	0.0476	0.631	0.637
0.30	4.50	0.66	1.08	0.0444	0.628	0.680
0.35	4.95	0.77	1.28	0.0376	0.621	0.794
0.40	5.39	0.88	1.47	0.0328	0.616	0.905
0.45	5.83	0.99	1.66	0.0290	0.612	1.016
0.50	6.27	1.10	1.85	0.0260	0.609	1.127
0.55	6.70	1.21	2.04	0.0236	0.607	1.236

W = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Sum moments about pivot to give $W*L = F_R*d$

For $h < R_2 - R_1$:

$F_R = \gamma*(h/2)*h*b$

$\mathbf{d} = R_2 - (h/3)$

For $h > R_2 - R_1$:

$F_R = \gamma*(h - (R_2 - R_1)/2)*(R_2 - R_1)*b$

$d = R_2 - (R_2 - R_1)/2 + (y_r - y_c)$

$y_r - y_c = I_{xc}/h_c*A$

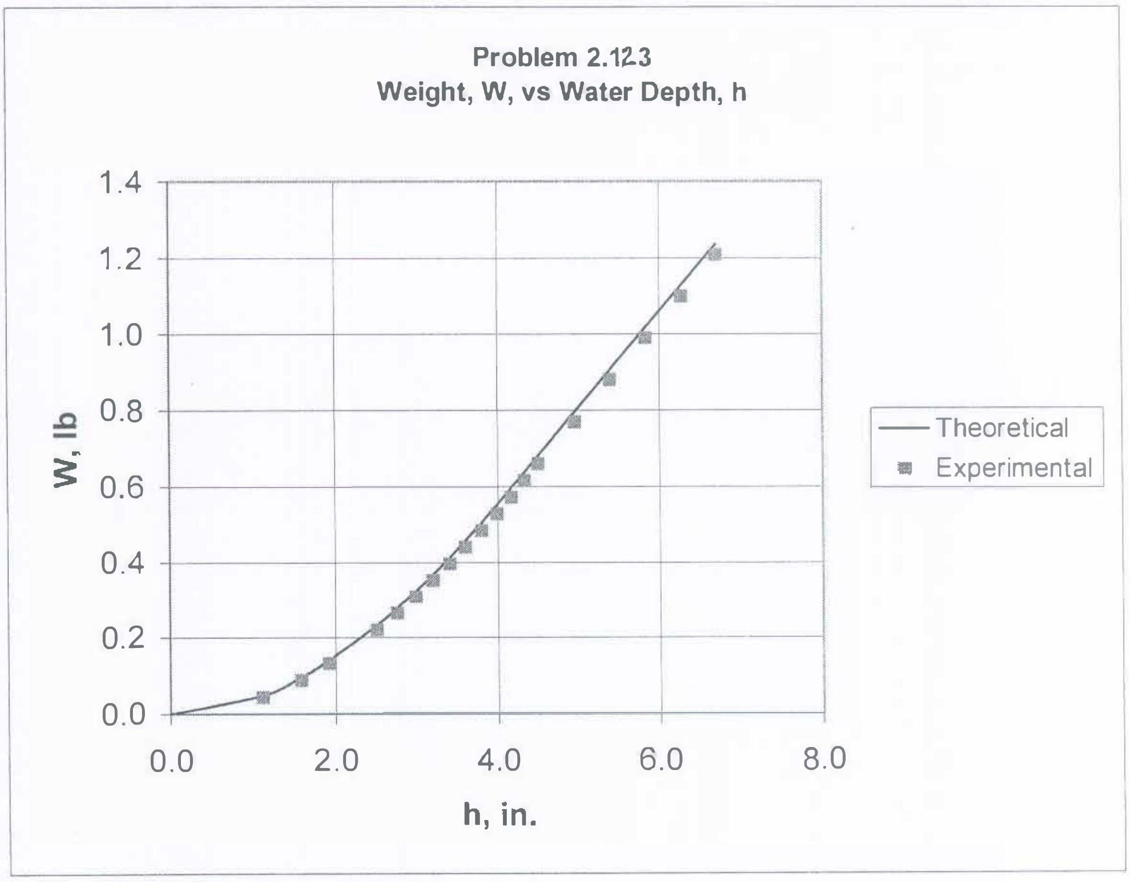
$I_{xc} = b*(R_2 - R_1)\text{^}3/12 = 0.000771\ \text{ft^4}$

$h_c = h - (R_2 - R_1)/2$

$A = b*(R_2 - R_1)$

(con't)

2.123 (Con't)


Problem 2.123
Weight, W, vs Water Depth, h
W, lb
h, in.
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.0
2.0
4.0
6.0
8.0
Theoretical
Experimental

2.124

2.124 Vertical Uplift Force on an Open-Bottom Box with Slanted Sides

Objective: When a box or form as shown in Fig. P2.124 is filled with a liquid, the vertical force of the liquid on the box tends to lift it off the surface upon which it sits, thus allowing the liquid to drain from the box. The purpose of this experiment is to determine the minimum weight, W, needed to keep the box from lifting off the surface.

Equipment: An open-bottom box that has vertical side walls and slanted end walls; weights; ruler; scale.

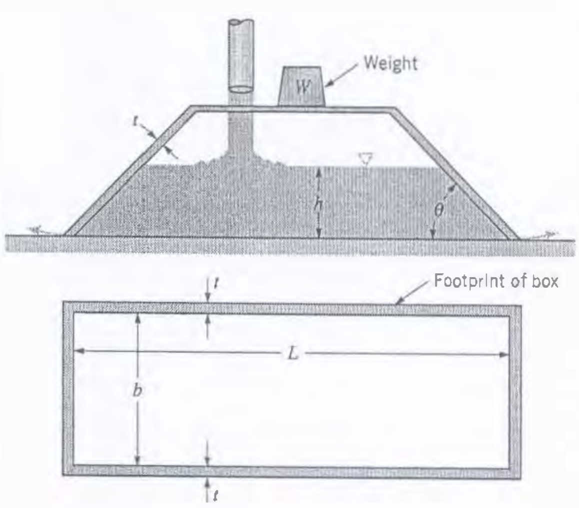
Experimental Procedure: Determine the weight, W_{box}, of the empty box and measure its length, L, width, b, wall thickness, t, and the angle of the ends, θ. Set the box on a smooth surface and place a known mass, m, on it. Slowly fill the box with water and note the depth, h, at which the net upward water force is equal to the total weight, $W + W_{box}$, where $W = mg$. This condition will be obvious because the friction force between the box and the surface on which it sits will be zero and the box will "float" effortlessly along the surface. Repeat for various masses and water levels.

Calculations: For an arbitrary water depth, h, determine the theoretical weight, W, needed to maintain equilibrium with no contact force between the box and the surface below it. This can be done by equating the total weight, $W + W_{box}$, to the net vertical hydrostatic pressure force on the box. Calculate this vertical pressure force for two different situations. (1) Assume the vertical pressure force is the vertical component of the pressure forces acting on the slanted ends of the box. (2) Assume the vertical upward force is that from part (1) plus the pressure force acting under the sides and ends of the box because of the finite thickness, t, of the box walls. This additional pressure force is assumed to be due to an average pressure of $p_{avg} = \gamma h/2$ acting on the "foot print" area of the box walls.

Graph: Plot the experimentally determined total weight, $W + W_{box}$, as ordinates and the water depth, h, as abscissas.

Results: On the same graph plot two theoretical total weight verses water depth curves—one involving only the slanted-end pressure force, and the other including the slanted end and the finite-thickness wall pressure forces.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P2.124

(con't)

2.124 (Con't)

Solution for Problem 2.124: Vertical Uplift Force on an Open-Bottom Box with Slanted Sides

θ, deg	L, in.	b, in.	t, in.	W_{box}, lb	γ, lb/ft^3
45	10.3	4.0	0.25	0.942	62.4

		Experimental		Theory 1		Theory 2
m, kg	h, in.	W + W_{box}, lb	h, in.	W + W_{box}, lb	p_{avg}, lb/ft^2	W + W_{box}, lb
0.00	2.06	0.942	0.00	0.000	0.00	0.000
0.05	2.23	1.052	0.25	0.009	0.65	0.047
0.10	2.42	1.162	0.50	0.036	1.30	0.111
0.15	2.53	1.272	0.75	0.081	1.95	0.194
0.20	2.67	1.382	1.00	0.144	2.60	0.295
0.25	2.81	1.491	1.25	0.226	3.25	0.414
0.30	2.94	1.601	1.50	0.325	3.90	0.551
0.35	3.06	1.711	1.75	0.442	4.55	0.706
0.40	3.16	1.821	2.00	0.578	5.20	0.879
			2.25	0.731	5.85	1.070
			2.50	0.903	6.50	1.279
			2.75	1.092	7.15	1.506
			3.00	1.300	7.80	1.752
			3.25	1.526	8.45	2.015

W = g*m = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Theory 1. Including only the slanted-end pressure force:

$W + W_{box} = \gamma$*Vol

Vol = b*h*h

Theory 2. Including the slanted-end pressure force and the finite-thickness wall pressure force:

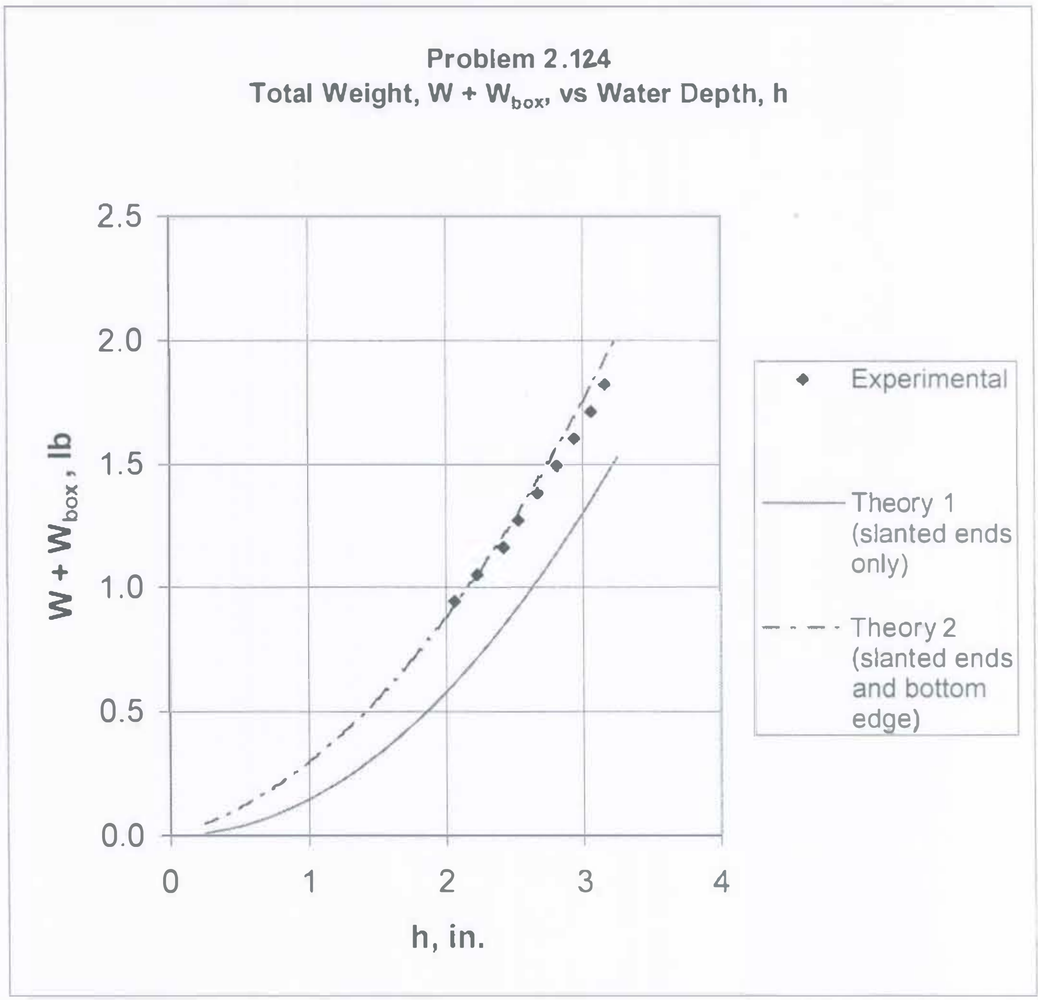
$W + W_{box} = \gamma$*Vol + p_{avg}*A

p_{avg} = 0.5*γ*h

A = (b + 2*t)*(L + 2*t/sinθ) - b*L = 8.33 in.^2 = 0.0579 ft^2

(con't)

2.124


Problem 2.124
Total Weight, W + W_{box}, vs Water Depth, h
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
W + W_{box}, lb
h, in.
Experimental
Theory 1 (slanted ends only)
Theory 2 (slanted ends and bottom edge)

2.125

2.125 Air Pad Lift Force

Objective: As shown in Fig. P2.125, it is possible to lift objects by use of an air pad consisting of an inverted box that is pressurized by an air supply. If the pressure within the box is large enough, the box will lift slightly off the surface, air will flow under its edges, and there will be very little frictional force between the box and the surface. The purpose of this experiment is to determine the lifting force, W, as a function of pressure, p, within the box.

Equipment: Inverted rectangular box; air supply; weights; manometer.

Experimental Procedure: Connect the air source and the manometer to the inverted square box. Determine the weight, W_{box}, of the square box and measure its length and width, L, and the wall thickness, t. Set the inverted box on a smooth surface and place a known mass, m, on it. Increase the air flowrate until the box lifts off the surface slightly and "floats" with negligible frictional force. Record the manometer reading, h, under these conditions. Repeat the measurements with various masses.

Calculations: Determine the theoretical weight that can be lifted by the air pad by equating the total weight, $W + W_{box}$, to the net vertical pressure force on the box. Here $W = mg$. Calculate this pressure force for two different situations. (1) Assume the pressure force is equal to the area of the box, $A = L^2$, times the pressure, $p = \gamma_m h$, within the box, where γ_m is the specific weight of the manometer fluid. (2) Assume that the net pressure force is that from part (1) plus the pressure force acting under the edges of the box because of the finite thickness, t, of the box walls. This additional pressure force is assumed to be due to an average pressure of $p_{avg} = \gamma_m h/2$ acting on the "foot print" area of the box walls, $4t(L + t)$.

Graph: Plot the experimentally determined total weight, $W + W_{box}$, as ordinates and the pressure within the box, p, as abscissas.

Results: On the same graph, plot two theoretical total weight verses pressure curves— one involving only the pressure times box area pressure force, and the other including the pressure times box area and the finite-thickness wall pressure forces.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

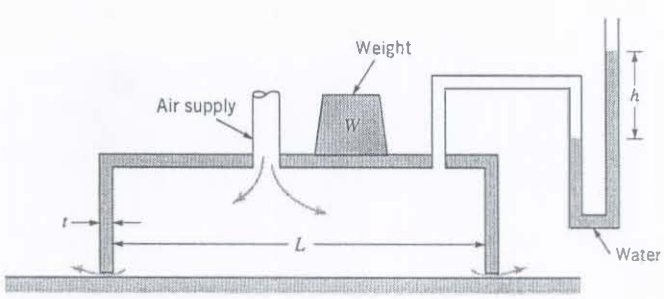


FIGURE P2.125

(Con't)

2.125 (con't)

Solution for Problem 2.125: Air Pad Lift Force

L, in.	t, in.	W_{box}, lb	γ_{H2O}, lb/ft^3
7.5	0.25	1.25	62.4

		Experiment		Theory 1	Theory 2
m, kg	h, in.	W + W_{box}, lb	p, lb/ft^2	W + W_{box}, lb	W + W_{box}, lb
0.0	0.54	1.25	2.81	1.10	1.17
0.1	0.64	1.47	3.33	1.30	1.39
0.2	0.74	1.69	3.85	1.50	1.61
0.3	0.82	1.91	4.26	1.67	1.78
0.4	0.94	2.13	4.89	1.91	2.04
0.5	1.04	2.35	5.41	2.11	2.26
0.6	1.12	2.57	5.82	2.28	2.43
0.7	1.23	2.79	6.40	2.50	2.67
0.8	1.32	3.01	6.86	2.68	2.87
0.9	1.42	3.23	7.38	2.88	3.08
1.0	1.52	3.45	7.90	3.09	3.30
1.1	1.63	3.67	8.48	3.31	3.54
1.2	1.72	3.89	8.94	3.49	3.73
1.3	1.83	4.11	9.52	3.72	3.97
1.4	1.96	4.33	10.19	3.98	4.26
1.5	2.06	4.55	10.71	4.18	4.47
1.6	2.12	4.77	11.02	4.31	4.60
1.7	2.23	4.99	11.60	4.53	4.84
1.8	2.32	5.21	12.06	4.71	5.04

W = g*m = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Theory 1. Involving only the pressure times the box area:

$W + W_{box} = p*L^2$

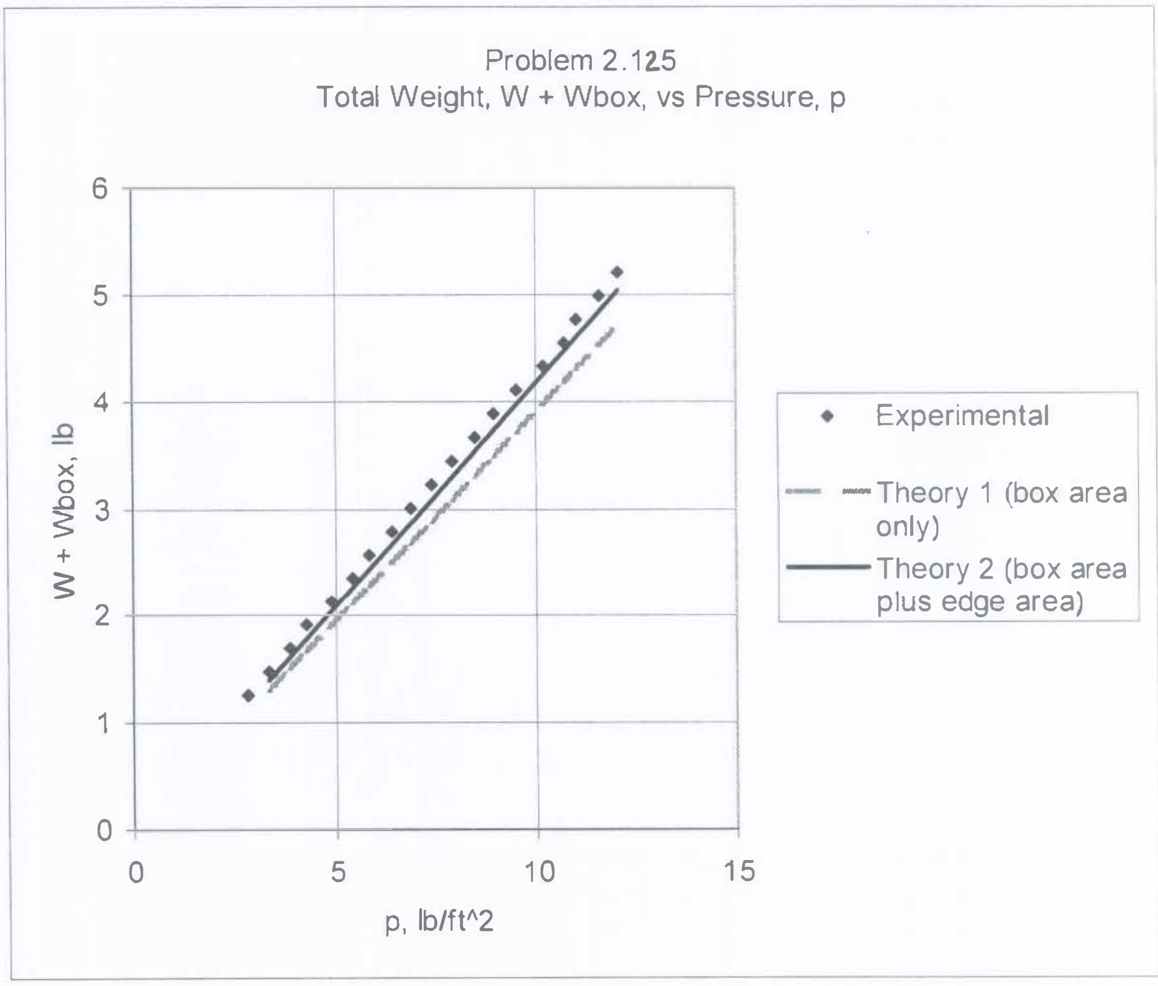
$p = \gamma_{H2O}*h$

Theory 2. Involving the pressure times the box area plus the average pressure times the edge area:

$W + W_{box} = p*L^2 + (p/2)*((L + 2t)^2 - L^2)$

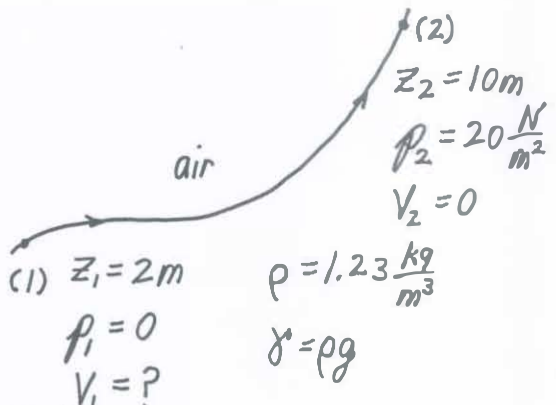
(con't)

2.125 (con't)


Problem 2.125
Total Weight, W + Wbox, vs Pressure, p
W + Wbox, lb
p, lb/ft^2
0
1
2
3
4
5
6
0
5
10
15
Experimental
Theory 1 (box area only)
Theory 2 (box area plus edge area)

3.2

3.2 Air flows steadily along a streamline from point (1) to point (2) with negligible viscous effects. The following conditions are measured: At point (1) $z_1 = 2$ m and $p_1 = 0$ kPa; at point (2) $z_2 = 10$ m, $p_2 = 20$ N/m^2, and $V_2 = 0$. Determine the velocity at point (1).



$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

Thus, with $p_1 = 0$ and $V_2 = 0$,

$$\tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \gamma z_2$$

or

$$\tfrac{1}{2}\left(1.23\frac{kg}{m^3}\right)V_1^2 = 20\frac{N}{m^2} + \left(1.23\frac{kg}{m^3}\right)9.81\frac{m}{s^2}(10m - 2m)$$

or

$$V_1^2 = \frac{2(20)}{1.23}\ \frac{N\cdot m}{kg} + 2\left(9.81\frac{m}{s^2}\right)(8m) = 189\frac{m^2}{s^2} \quad \left(\text{Note: } \frac{N\cdot m}{kg} = \frac{\left(\frac{kg\cdot m}{s^2}\right)m}{kg} = \frac{m^2}{s^2}\right)$$

Thus,

$\underline{\underline{V_1 = 13.7\ m/s}}$

3.3

3.3 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.3. The velocity is given by $\mathbf{V} = 10(1 + x)\hat{\mathbf{i}}$ ft/s, where x is in feet. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 50 psi, determine the pressure at (2) by: (i) integration of the pressure gradient obtained in **(a)**; (ii) application of the Bernoulli equation.

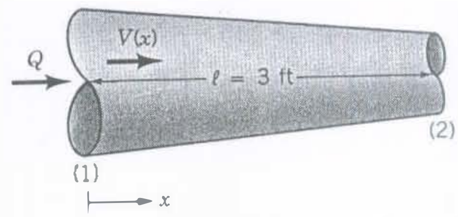


FIGURE P3.3

(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $V = 10(1+x)$ ft/s

$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s}$ or $\frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} = -\rho\,(10(1+x))(10)$

Thus, $\frac{\partial p}{\partial x} = -1.94 \frac{slugs}{ft^3}(10 \frac{ft}{s})^2(1+x)$, with x in feet

$= \underline{\underline{-194(1+x)\frac{lb}{ft^3}}}$

(b)(i) $\frac{dp}{dx} = -194(1+x)$ so that $\int_{p_1 = 50\,psi}^{p_2} dp = -194\int_{x_1=0}^{x_2=3}(1+x)\,dx$

or $p_2 = 50\,psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) = 50 - 10.1 = \underline{\underline{39.9\,psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = z_2$

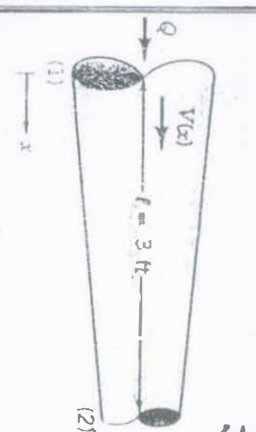
$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$ where $V_1 = 10(1+0) = 10\frac{ft}{s}$

$V_2 = 10(1+3) = 40\frac{ft}{s}$

Thus,

$p_2 = 50\,psi + \frac{1}{2}(1.94\frac{slugs}{ft^3})(10^2 - 40^2)\frac{ft^2}{s^2}\left(\frac{1\,ft^2}{144\,in^2}\right) = \underline{\underline{39.9\,psi}}$

3.4



3.4 Repeat Problem 3.3 if the pipe is vertical with the flow down.

(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ with $\theta = -90°$ and $V = 10(1+x)\,\frac{ft}{s}$

$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} + \gamma$ or $\frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} + \gamma = -\rho(10(1+x))(10) + \gamma$

Thus, $\frac{\partial p}{\partial x} = -1.94 \frac{slugs}{ft^3}\left(10\frac{ft}{s}\right)^2(1+x) + 62.4\,\frac{lb}{ft^3}$, with x in feet

$$= \underline{\underline{-194\,(1+x) + 62.4\ \frac{lb}{ft^3}}}$$

(b)(i) $\frac{dp}{dx} = -194\,(1+x) + 62.4$ so that $\int_{p_1 = 50psi}^{p_2} dp = \int_{x_1=0}^{x_2=3} [-194(1+x) + 62.4]\,dx$

or $p_2 = 50psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) + 62.4\,(3)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right)$

$= 50 - 10.1 + 1.3 = \underline{\underline{41.2\,psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = 0$, $z_2 = -3\,ft$

and $V_1 = 10(1+0) = 10\,\frac{ft}{s}$, $V_2 = 10(1+3) = 40\,\frac{ft}{s}$

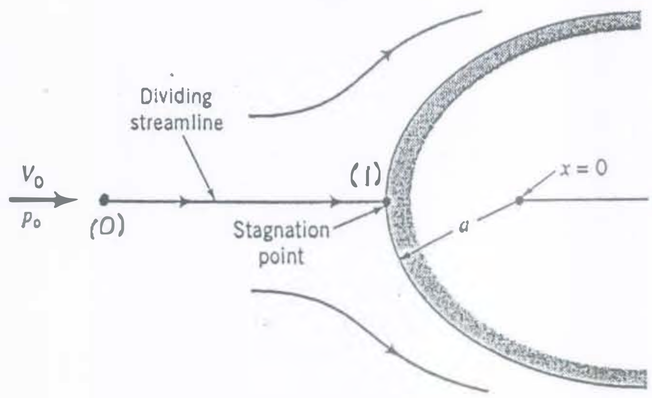
$p_2 = p_1 + \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) - \gamma z_2$

$= 50psi + \frac{1}{2}\left(1.94\,\frac{slug}{ft^3}\right)\left(10^2 - 40^2\right) - 62.4\,\frac{lb}{ft^3}(-3\,ft)$

$= \underline{\underline{41.2\,psi}}$

3.5

3.5 An incompressible fluid with density ρ flows steadily past the object shown in Video V3.7 and Fig. P3.5. The fluid velocity along the horizontal dividing streamline $(-\infty \leq x \leq -a)$ is found to be $V = V_0(1 + a/x)$, where a is the radius of curvature of the front of the object and V_0 is the upstream velocity. (a) Determine the pressure gradient along this streamline. (b) If the upstream pressure is p_0, integrate the pressure gradient to obtain the pressure $p(x)$ for $-\infty \leq x \leq -a$. (c) Show from the result of part (b) that the pressure at the stagnation point $(x = -a)$ is $p_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.



■ FIGURE P3.5

(a) $\frac{dp}{ds} = -\rho V \frac{dV}{ds}$ where $V = V_0\left(1 + \frac{a}{x}\right)$

Thus, $\frac{dV}{ds} = \frac{dV}{dx} = -\frac{V_0 a}{x^2}$

or

$$\frac{dp}{ds} = \frac{dp}{dx} = -\rho V_0\left(1 + \frac{a}{x}\right)\left(-\frac{V_0 a}{x^2}\right) = \underline{\underline{\rho a V_0^2\left(\frac{1}{x^2} + \frac{a}{x^3}\right)}}$$

(b) $$\int_{p_0}^{p} dp = \int_{x=-\infty}^{x} \frac{dp}{dx}\,dx = \rho a V_0^2 \int_{-\infty}^{x}\left(\frac{1}{x^2} + \frac{a}{x^3}\right)dx$$ Note: $p = p_0$ at $x = -\infty$

or

$$p - p_0 = \rho a V_0^2\left[-\frac{1}{x} - \frac{1}{2}\frac{a}{x^2}\right]_{-\infty}^{x}$$

Thus,

$$\underline{\underline{p = p_0 - \rho a V_0^2\left[\frac{1}{x} + \frac{a}{2x^2}\right]}}$$

(c) From part (b), when $x = -a$

$$p\Big|_{x=-a} = p_0 - \rho a V_0^2\left[-\frac{1}{a} + \frac{a}{2a^2}\right] = \underline{\underline{p_0 + \frac{1}{2}\rho V_0^2}}$$

From the Bernoulli equation $p_0 + \frac{1}{2}\rho V_0^2 = p_1 + \frac{1}{2}\rho V_1^2$

where

$$V_1 = V\Big|_{x=-a} = V_0\left(1 + \frac{a}{(-a)}\right) = 0$$

Thus, $p_1 = p_0 + \frac{1}{2}\rho V_0^2$ as expected.

3.6

3.6 What pressure gradient along the streamline, dp/ds, is required to accelerate water in a horizontal pipe at a rate of 30 m/s²?

$$\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s} \quad \text{where} \quad \theta = 0 \quad \text{and}$$

$$V\frac{\partial V}{\partial s} = a_s = 0 \frac{m}{s^2}$$

Thus,

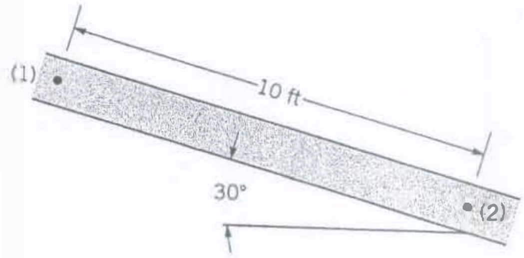
$$\frac{\partial p}{\partial s} = -\rho a_s = -999 \frac{kg}{m^3}\left(30 \frac{m}{s^2}\right) = -30{,}000\left(\frac{N}{m^2}\right)/m$$

or

$$\frac{\partial p}{\partial s} = \underline{\underline{-30.0\ kPa/m}}$$

3.7

3.7 A fluid with a specific weight of 100 lb/ft^3 and negligible viscous effects flows in the pipe shown in Fig. P3.7. The pressures at points (1) and (2) are 400 lb/ft^2 and 900 lb/ft^2, respectively. The velocities at points (1) and (2) are equal. Is the fluid accelerating uphill, downhill, or not accelerating? Explain.



■ FIGURE P3.7

If the flow is steady (i.e., not accelerating), then

(1) $$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

But $V_1 = V_2$. Thus, for steady flow

$p_1 + \gamma z_1 = p_2 + \gamma z_2$, where, if we set $z_2 = 0$, then $z_1 = (10\,ft)\sin 30° = 5\,ft$

For the given data, Eq. (1) becomes

$$\left(400 \frac{lb}{ft^2}\right) + \left(100 \frac{lb}{ft^3}\right)(5\,ft) = \left(900 \frac{lb}{ft^2}\right)$$

or

$$900 \frac{lb}{ft^2} = 900 \frac{lb}{ft^2}$$

That is, Eq. (1) (the steady flow equation) is valid.

The flow is <u>not accelerating.</u>

Note: If the flow were accelerating the pressure difference between points (1) and (2) would be different than the given $(900-400)\frac{lb}{ft^2} = 500\frac{lb}{ft^2}$

3.8

3.8 What pressure gradient along the streamline, dp/ds, is required to accelerate water upward in a vertical pipe at a rate of 30 ft/s^2? What is the answer if the flow is downward?

$$\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s}$$

where $\theta = 90°$ for up flow,
$\theta = -90°$ for down flow,
and $V\frac{\partial V}{\partial s} = a_s = 30\ \frac{ft}{s^2}$

Thus, for up flow

$$\frac{\partial p}{\partial s} = -62.4(1)\frac{lb}{ft^3} - 1.94\ \frac{slugs}{ft^3}\left(30\ \frac{ft}{s^2}\right) = -120.6\left(\frac{lb}{ft^2}\right)/ft = \underline{\underline{-0.838\ \frac{psi}{ft}}}$$

and for down flow

$$\frac{\partial p}{\partial s} = -62.4(-1)\frac{lb}{ft^3} - 1.94\ \frac{slugs}{ft^3}\left(30\frac{ft}{s}\right) = 4.20\ \left(\frac{lb}{ft^3}\right)/ft = \underline{\underline{0.0292\ \frac{psi}{ft}}}$$

3.9

3.9 Consider a compressible fluid for which the pressure and density are related by $p/\rho^n = C_0$, where n and C_0 are constants. Integrate the equation of motion along the streamline, Eq. 3.6, to obtain the "Bernoulli equation" for this compressible flow as $[n/(n-1)]p/\rho + V^2/2 + gz = \text{constant}$.

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant along a streamline}$$

and

$$\rho^n = \frac{p}{C_0} \quad \text{or} \quad \rho = \frac{p^{1/n}}{C_0^{1/n}} \quad \text{so that}$$

$$\int \frac{dp}{\rho} = C_0^{\frac{1}{n}} \int \frac{dp}{p^{\frac{1}{n}}} = C_0^{\frac{1}{n}} \int p^{-\frac{1}{n}} dp = C_0^{\frac{1}{n}} \frac{1}{\left(1-\frac{1}{n}\right)} p^{1-\frac{1}{n}} + \text{const.}$$

Thus,

$$\int \frac{dp}{\rho} = \frac{n}{n-1} p \left(\frac{C_0}{p}\right)^{\frac{1}{n}} = \frac{n}{n-1} \frac{p}{\rho}$$

Hence: $\underline{\underline{\frac{n}{n-1} \frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{constant along a streamline}}}$

3.10

3.10 An incompressible fluid flows steadily past a circular cylinder as shown in Fig. P3.10. The fluid velocity along the dividing streamline ($-\infty \le x \le -a$) is found to be $V = V_0(1 - a^2/x^2)$, where a is the radius of the cylinder and V_0 is the upstream velocity. **(a)** Determine the pressure gradient along this streamline. **(b)** If the upstream pressure is p_0, integrate the pressure gradient to obtain the pressure $p(x)$ for $-\infty \le x \le -a$. **(c)** Show from the result of part **(b)** that the pressure at the stagnation point ($x = -a$) is $p_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.

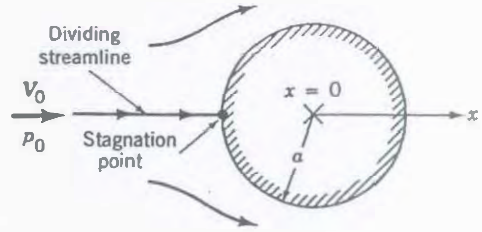


FIGURE P3.10

(a) $\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $\frac{\partial V}{\partial s} = \frac{\partial V}{\partial x}\frac{\partial x}{\partial s} = \frac{\partial V}{\partial x}$

$= V_0[-a^2]\left(\frac{-2}{x^3}\right) = \frac{2a^2 V_0}{x^3}$

Thus,

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial x} = -2\rho a^2 V_0^2 \left[1 - \left(\frac{a}{x}\right)^2\right] / x^3$$

(b) $\int_{p_0}^{p} dp = \int_{x=-\infty}^{x} \frac{dp}{dx} dx$ or $p - p_0 = -2\rho a^2 V_0^2 \int_{-\infty}^{x} \left[1 - \left(\frac{a}{x}\right)^2\right] \frac{dx}{x^3}$

$$= -2\rho a^2 V_0^2 \int_{-\infty}^{x} \left[x^{-3} - a^2 x^{-5}\right] dx$$

Thus,

$$p = p_0 + \rho V_0^2 \left[\left(\frac{a}{x}\right)^2 - \frac{1}{2}\left(\frac{a}{x}\right)^4\right] \quad \text{for} \quad -\infty \le x \le -a$$

(c) For $x = -a$, from part (b):

$$p\Big|_{x=-a} = p_0 + \rho V_0^2 \left[(-1)^2 - \frac{1}{2}(-1)^4\right] = p_0 + \frac{1}{2}\rho V_0^2$$

Note: Bernoulli equation from point (1) where $V_1 = V_0$, $p_1 = p_0$ and $z_1 = z_0$ to point (2) where $V_2 = 0$, $z_2 = z_0$ gives

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

or

$$p_2 = p_0 + \frac{1}{2}\rho V_0^2$$

3.11

3.11 Consider a compressible liquid that has a constant bulk modulus. Integrate "F = *m*a" along a streamline to obtain the equivalent of the Bernoulli equation for this flow. Assume steady, inviscid flow.

From Eq. 3.6

$dp + \frac{1}{2}\rho\, d(V^2) + \gamma\, dz = 0$ where $\gamma = \rho g$

and $dp = E_v \frac{d\rho}{\rho}$ where

E_v = bulk modulus = constant

(see Eq. 1.13)

Thus, along a streamline:

$E_v \cdot \frac{d\rho}{\rho} + \frac{1}{2}\rho\, d(V^2) + \rho g\, dz = 0$ or

$E_v \frac{d\rho}{\rho^2} + d\left(\frac{1}{2}V^2\right) + g\, dz = 0$ which can be integrated between between points (1) and (2) to give

$$E_v \int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho^2} + \int_{V_1}^{V_2} d\left(\tfrac{1}{2}V^2\right) + \int_{z_1}^{z_2} g\, dz = 0$$

or

$$-E_v\left[\frac{1}{\rho_2} - \frac{1}{\rho_1}\right] + \frac{1}{2}\left[V_2^2 - V_1^2\right] + g\left[z_2 - z_1\right] = 0$$

Hence:

$$\underline{\underline{gz - \frac{E_v}{\rho} + \frac{V^2}{2} = \text{constant along a streamline}}}$$

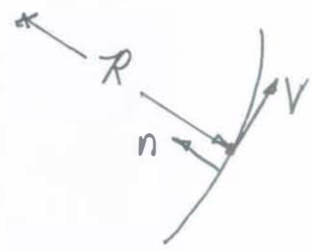
3.13

3.13 Air flows along a horizontal, curved streamline with a 20 ft radius with a speed of 100 ft/s. Determine the pressure gradient normal to the streamline.

$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{R}$, where $\frac{dz}{dn} = 0$ since the streamine is horizontal.

Thus,

$$\frac{\partial p}{\partial n} = -\frac{\rho V^2}{R} = \frac{-\left(0.00238 \frac{slug}{ft^3}\right)\left(100 \frac{ft}{s}\right)^2}{20\,ft}$$



$$= -1.19 \frac{slug}{ft^2 \cdot s^2}\left(1 \frac{lb}{\left(\frac{slug \cdot ft}{s^2}\right)}\right) = \underline{\underline{-1.19 \frac{lb}{ft^3}}}$$

3.14

3.14 Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in Fig. P3.14. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

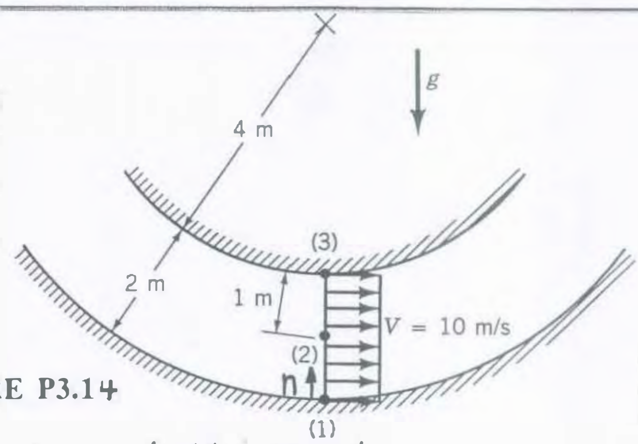


FIGURE P3.14

$$-\gamma\frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}} \quad \text{with } \frac{dz}{dn} = 1 \text{ and } V = 10\,m/s$$

Thus, with $\mathcal{R} = 6 - n$

$$\frac{dp}{dn} = -\gamma - \frac{\rho V^2}{6-n} \quad \text{or}$$

$$\int_{n=0}^{n} \frac{dp}{dn}\,dn = -\int_{n=0}^{n} \gamma\,dn - \int_{n=0}^{n} \frac{\rho V^2 dn}{6-n}$$

so that since γ and V are constants

$$p - p_1 = -\gamma n - \rho V^2 \int_{n=0}^{n} \frac{dn}{6-n}$$

Thus,

$$p = p_1 - \gamma n - \rho V^2 \ln\left(\frac{6}{6-n}\right)$$

With $p_1 = 40\,kPa$ and $n_2 = 1\,m$: $p_2 = 40\,kPa - 9.8 \times 10^3 \frac{N}{m^3}(1\,m) - 999\,\frac{kg}{m^3}\left(10\frac{m}{s}\right)^2 \ln\left(\frac{6}{5}\right)$

or

$$p_2 = \underline{\underline{12.0\ kPa}}$$

and

with $p_1 = 40\,kPa$ and $n_3 = 2\,m$: $p_3 = 40\,kPa - 9.80 \times 10^3 \frac{N}{m^3}(2\,m) - 999\,\frac{kg}{m^3}\left(10\frac{m}{s}\right)^2 \ln\left(\frac{6}{4}\right)$

or

$$p_3 = \underline{\underline{-20.1\ kPa}}$$

*3.15

*3.15 Water flows around a vertical two-dimensional bend with circular streamlines as is shown in Fig. P3.15 The pressure at point (1) is measured to be $p_1 = 25$ psi and the velocity across section a–a is as indicated in the table. Calculate and plot the pressure across section a–a of the channel [$p = p(z)$ for $0 \le z \le 2$ ft].

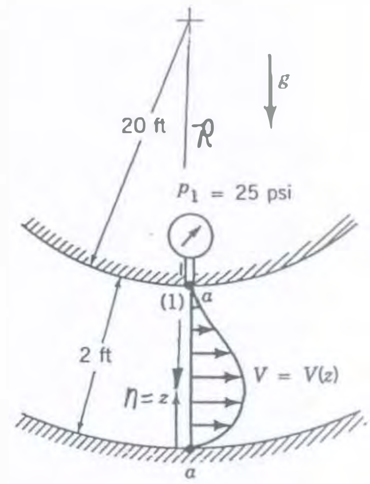


FIGURE P3.15

$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}},$$ with $\frac{dz}{dn} = 1$, $\mathcal{R} = 22-n$, and $V = V(n)$ as given in the table with $z = n$.

Thus,

$$\frac{dp}{dn} = -\gamma - \frac{\rho V^2}{(22-n)}$$

or

$$\int_{p}^{p_1} dp = -\int_{n}^{n=2} \gamma\, dn - \int_{n}^{n=2} \frac{\rho V^2}{(22-n)}\, dn$$

or

$$p_1 - p = -\gamma(2-n) - \rho \int_{n}^{n=2} \frac{V^2}{(22-n)}\, dn$$

Hence with $\gamma = 62.4\ \frac{lb}{ft^3}$, $\rho = 1.94\ \frac{slugs}{ft^3}$, and $p_1 = 25\ \frac{lb}{in.^2}\left(144\ \frac{in.^2}{ft^2}\right) = 3600\ \frac{lb}{ft^2}$ this gives

$$p = 3600 + 62.4(2-n) + 1.94 \int_{n}^{2} \frac{V^2}{(22-n)}\, dn\ , \quad \text{where } p \sim \frac{lb}{ft^2},\ n \sim ft \qquad (1)$$

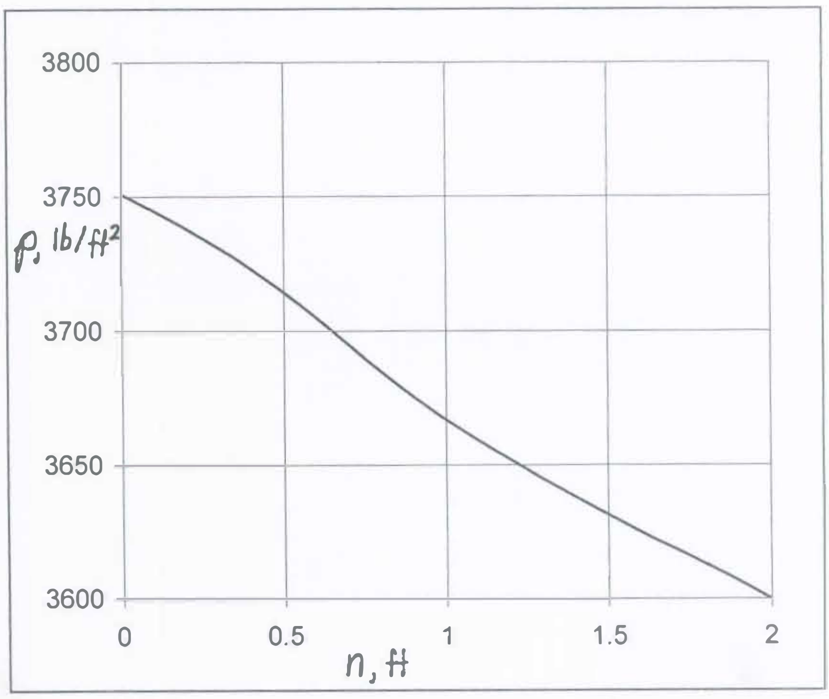
For $0 \le n \le 2$ use the data in the table ($V = V(n)$, where $n = z$) and integrate numerically to determine $p = p(n)$.

z (ft)	V (ft/s)
0	0
0.2	8.0
0.4	14.3
0.6	20.0
0.8	19.5
1.0	15.6
1.2	8.3
1.4	6.2
1.6	3.7
1.8	2.0
2.0	0

(con't)

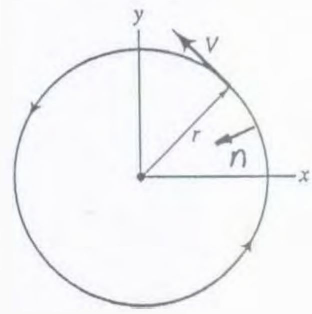
*3.15 (con't)

n, ft	value of integral	p, lb/ft^2
0	13.33	3751
0.2	13.04	3738
0.4	11.8	3723
0.6	8.98	3705
0.8	5.32	3685
1	2.37	3667
1.2	0.879	3652
1.4	0.361	3638
1.6	0.107	3625
1.8	0.02	3613
2	0	3600



3.16

3.16 Water in a container and air in a tornado flow in horizontal circular streamlines of radius r and speed V as shown in Video V3.6 and Fig. P3.16 Determine the radial pressure gradient, $\partial p/\partial r$, needed for the following situations: (a) The fluid is water with r = 3 in. and V = 0.8 ft/s. (b) The fluid is air with r = 300 ft and V = 200 mph.



■ FIGURE P3.16

For curved streamlines,

$$-\frac{dp}{dn} = \frac{\rho V^2}{\mathcal{R}} + \gamma \frac{dz}{dn}, \text{ or with } \frac{dz}{dn} = 0 \text{ (horizontal streamlines)}, \mathcal{R} = r,$$

and $\frac{d}{dn} = -\frac{d}{dr}$ this becomes

$$\frac{dp}{dr} = \frac{\rho V^2}{r}$$

a) With $r = \frac{3}{12}$ ft and $V = 0.8 \frac{ft}{s}$ and water ($\rho = 1.94 \frac{slugs}{ft^3}$),

$$\frac{dp}{dr} = \frac{1.94 \frac{slugs}{ft^3} (0.8 \frac{ft}{s})^2}{(\frac{3}{12} ft)} = 4.97 \frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{4.97 \frac{lb}{ft^3}}}$$

(b) With $r = 300 ft$ and $V = 200 \, mph \left(\frac{88 \frac{ft}{s}}{60 \, mph}\right) = 293 \frac{ft}{s}$

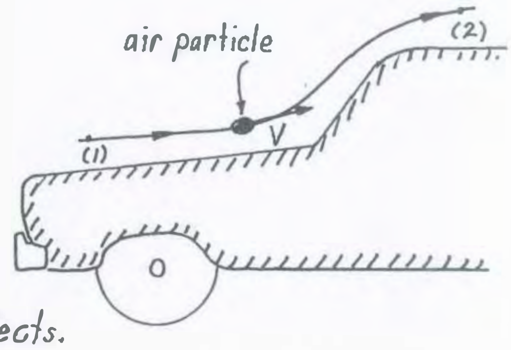
and air ($\rho = 0.00238 \frac{slugs}{ft^3}$),

$$\frac{dp}{dr} = \frac{0.00238 \frac{slugs}{ft^3} (293 \frac{ft}{s})^2}{300 \, ft} = 0.681 \frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{0.681 \frac{lb}{ft^3}}}$$

3.17

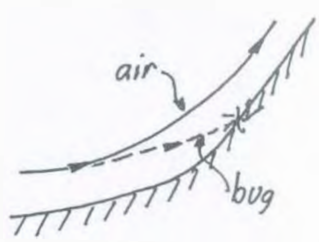
3.17 Air flows smoothly over the hood of your car and up past the windshield. However, a bug in the air does not follow the same path; it becomes splattered against the windshield. Explain why this is so.

An air particle flowing along streamline (1)-(2) is immersed in a pressure field produced by all of the surrounding air particles. Gravity and pressure effects precisely balance centrifugal acceleration effects. That is,



$$-\gamma \frac{\partial z}{\partial n} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}}$$, where γ and ρ are the specific weight and density of the air

A bug is more dense than air, $\rho_{bug} > \rho$, but it "feels" the same pressure field, which is not sufficient to make it turn as sharply as the air does. Hence, $\mathcal{R}_{bug} > \mathcal{R}$ and the bug hits the windshield.



3.19

3.19 At a given point on a horizontal streamline in flowing air, the static pressure is −2.0 psi (i.e., a vacuum) and the velocity is 150 ft/s. Determine the pressure at a stagnation point on that streamline.

$V_1 = 150 \frac{ft}{s}$

$p_1 = -2.0 psi$

$z_1 = z_2$

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

where $z_1 = z_2$ and $V_2 = 0$

Thus,

$$p_2 = p_1 + \tfrac{1}{2}\rho V_1^2 = \left(-2.0 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right) + \tfrac{1}{2}\left(0.00238 \frac{slugs}{ft^3}\right)\left(150 \frac{ft}{s}\right)^2$$

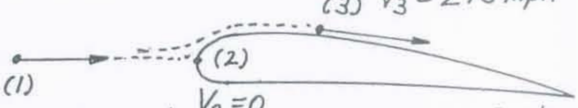
$$= -288 \frac{lb}{ft^2} + 26.8 \frac{slug \cdot ft}{ft^2 \cdot s^2}\left(\frac{1\ lb}{\frac{slug \cdot ft}{s^2}}\right)$$

$$= -261 \frac{lb}{ft^2} = \underline{\underline{-1.81\ psi}}$$

3.21

3.21 When an airplane is flying 200 mph at 5000-ft altitude in a standard atmosphere, the air velocity at a certain point on the wing is 273 mph relative to the airplane. What suction pressure is developed on the wing at that point? What is the pressure at the leading edge (a stagnation point) of the wing?

(a) $p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant}$



Thus, with $z_1 \approx z_2 \approx z_3$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^2 \text{ , but } p_1 = 0 \text{ so that}$$

$$p_3 = \frac{1}{2}\rho\left[V_1^2 - V_3^2\right] \quad \text{where} \quad V_1 = 200\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 293\,\frac{ft}{s}$$

and

$$V_3 = 273\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 400\,\frac{ft}{s}$$

or

$$p_3 = \frac{1}{2}\left(2.05 \times 10^{-3}\,\frac{slugs}{ft^3}\right)\left[293^2 - 400^2\right]\frac{ft^2}{s^2}$$

$$= \underline{\underline{-76.0\,\frac{lb}{ft^2}}} \quad (gage)$$

(b) Also,

$$p_2 = \frac{1}{2}\rho V_1^2 = \frac{1}{2}\left(2.05 \times 10^{-3}\,\frac{slugs}{ft^3}\right)\left(293\,\frac{ft}{s}\right)^2 = \underline{\underline{88.0\,\frac{lb}{ft^2}}} \ (gage)$$

3.22

3.22 Some animals have learned to take advantage of the Bernoulli effect without having read a fluid mechanics book. For example, a typical prairie dog burrow contains two entrances—a flat front door, and a mounded back door as shown in Fig. P3.22. When the wind blows with velocity V_0 across the front door, the average velocity across the back door is greater than V_0 because of the mound. Assume the air velocity across the back door is $1.07V_0$. For a wind velocity of 6 m/s. what pressure differences, $p_1 - p_2$, is generated to provide a fresh air flow within the burrow?

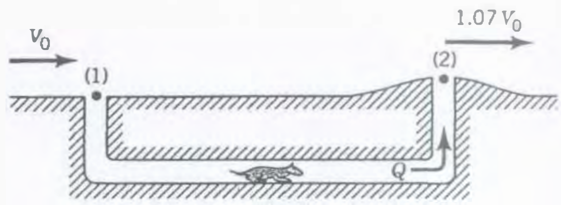


FIGURE P3.22

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Thus, with negligible gravitational effects (i.e. $z_1 \approx z_2$)

$$p_1 - p_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$$= \frac{1}{2}\left(1.23\,\frac{kg}{m^3}\right)\left(\left(1.07\left(6\,\frac{m}{s}\right)\right)^2 - \left(6\,\frac{m}{s}\right)^2\right)$$

or

$$p_1 - p_2 = \underline{\underline{3.21\,\frac{N}{m^2}}}$$

3.23

3.23 A loon is a diving bird equally at home "flying" in the air or water. What swimming velocity under water will produce a dynamic pressure equal to that when it flies in the air at 40 mph?

$$\frac{1}{2}\rho_{air}V_{air}^2 = \frac{1}{2}\rho_{H_2O}V_{H_2O}^2 \quad \text{or} \quad V_{H_2O} = \left[\frac{\rho_{air}}{\rho_{H_2O}}\right]^{\frac{1}{2}} V_{air}$$

Thus,

$$V_{H_2O} = \left[\frac{2.38\times10^{-3}\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}}\right](40\ mph) = \underline{\underline{1.40\ mph}}$$

3.24

3.24 A person thrusts his hand into the water while traveling 3 m/s in a motor boat. What is the maximum pressure on his hand?

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with } z_1 = z_2$$

$V_1 = 3\,\frac{m}{s}$

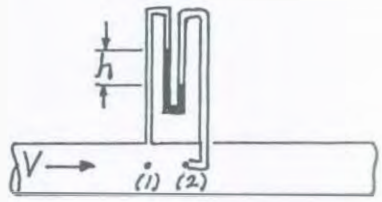
$p_1 = 0\ ,\ V_2 = 0$

Thus,

$$p_2 = \frac{\gamma}{2g} V_1^2 = \frac{1}{2}\rho V_1^2 \text{ or } p_2 = \frac{1}{2}\left(999\,\frac{kg}{m^3}\right)\left(3\,\frac{m}{s}\right)^2 = 4\,500\,\frac{N}{m^2} = \underline{\underline{4.50\ kPa}}$$

3.25

3.25 A Pitot-static tube is used to measure the velocity of helium in a pipe. The temperature and pressure are 40 °F and 25 psia. A water manometer connected to the Pitot-static tube indicates a reading of 2.3 in. Determine the helium velocity. Is it reasonable to consider the flow as incompressible? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with $z_1 = z_2$, $V_1 = V$, and $V_2 = 0$

Thus,

$$V_1 = \sqrt{2g\frac{(p_2 - p_1)}{\gamma}} = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

where

$$\rho = \frac{p}{RT} = \frac{25\,\frac{lb}{in.^2}\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.242\times10^4\,\frac{ft\cdot lb}{slug\cdot°R}\right)(460+40)°R} = 5.80\times10^{-4}\,\frac{slugs}{ft^3}$$

and since $\gamma_{H_2O} \gg \gamma_{He}$

$$p_2 - p_1 = \gamma_{H_2O}\, h = 62.4\,\frac{lb}{ft^3}\left(\frac{2.3}{12}\,ft\right) = 11.96\,\frac{lb}{ft^2}$$

Thus,

$$V_1 = \sqrt{\frac{2\left(11.96\,\frac{lb}{ft^2}\right)}{5.80\times10^{-4}\,\frac{slugs}{ft^3}}} = \underline{\underline{203\,\frac{ft}{s}}}$$

Note: $M = \frac{V}{c}$ where $c = \sqrt{kRT}$

Thus,

$$c = \left[1.66\left(1.242\times10^4\right)\frac{ft\cdot lb}{slug\cdot°R}(460+40)°R\right]^{1/2} = 3210\,\frac{ft}{s}$$

or

$$M = \frac{203\,\frac{ft}{s}}{3210\,\frac{ft}{s}} = 0.063 \ll 0.3$$

Thus, the flow can be considered incompressible.

3.26

3.26 An inviscid fluid flows steadily along the stagnation streamline shown in Fig. P3.26 and Video V3.7 starting with speed V_0 far upstream of the object. Upon leaving the stagnation point, point (1), the fluid speed along the surface of the object is assumed to be given by $V = 2\,V_0 \sin\theta$, where θ is the angle indicated. At what angular position, θ_2, should a hole be drilled to give a pressure difference of $p_1 - p_2 = \rho V_0^2/2$? Gravity is negligible.

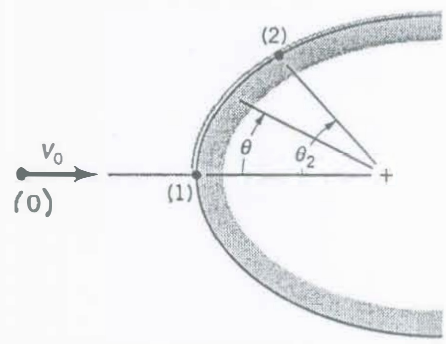


FIGURE P3.26

$$p_0 + \tfrac{1}{2}\rho V_0^2 = p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

where $V_1 = 0$

Thus,

$$p_1 - p_2 = \tfrac{1}{2}\rho\,(V_2^2 - V_1^2) = \tfrac{1}{2}\rho V_2^2$$

so that if

$$p_1 - p_2 = \tfrac{1}{2}\rho V_0^2 \quad \text{then } V_2 = V_0$$

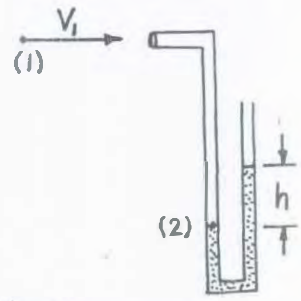
That is:

$$V_2 = 2\,V_0 \sin\theta_2 = V_0 \quad \text{or} \quad \sin\theta_2 = \tfrac{1}{2}$$

Hence, $\theta_2 = \underline{\underline{30^\circ}}$

3.27

3.27 A water-filled manometer is connected to a Pitot-static tube to measure a nominal airspeed of 50 ft/s. It is assumed that a change in the manometer reading of 0.002 in. can be detected. What is the minimum deviation from the 50 ft/s airspeed that can be detected by this system? Repeat the problem if the nominal airspeed is 5 ft/s.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ V_2 = 0$$

$z_1 \approx z_2$, and $p_2 = \gamma_{H_2O} h$

Thus,

$$\frac{V_1^2}{2g} = \frac{\gamma_{H_2O} h}{\gamma} \quad \text{or} \quad h = \frac{\rho V_1^2}{2\gamma_{H_2O}} = \frac{(0.00238 \frac{slugs}{ft^3})(V_1^2 \frac{ft^2}{s^2})(12 \frac{in.}{ft})}{2\,(62.4 \frac{lb}{ft^3})}$$

Hence, $h = 2.29 \times 10^{-4} V_1^2$, where $V_1 \sim ft/s$ and $h \sim in.$

For $V_1 = 50 \frac{ft}{s}$ this gives

$h = 2.29 \times 10^{-4} (50)^2 = 0.573$ in.

while for $V_1 = 5$ ft/s it gives

$h = 2.29 \times 10^{-4} (5)^2 = 0.00573$ in.

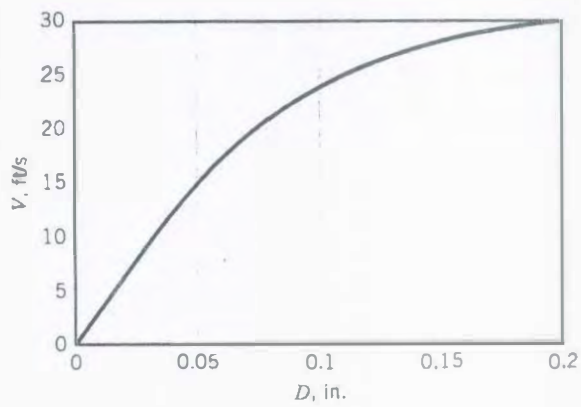
With $h \pm 0.002$ in. from these nominal values we obtain

h, in.	V_1, ft/s
0.571	49.9
0.573	50.0
0.575	50.1
0.00373	4.04
0.00573	5.00
0.00773	5.81

Thus, with $V_1 = 50$ ft/s the minimum air speed deviation that can be detected is $\underline{\pm 0.1 \text{ ft/s}}$; for $V_1 = 5$ ft/s it is $\underline{\pm 0.81 \text{ ft/s}}$.

3.28

3.28 (See Fluids in the News article titled **"Incorrect raindrop shape,"** Section 3.2.) The speed, V, at which a raindrop falls is a function of its diameter, D, as shown in Fig. P3.28. For what sized raindrop will the stagnation pressure be equal to half the internal pressure caused by surface tension. Recall from **Section 1.9** that the pressure inside a drop is $\Delta p = 4\sigma/D$ greater than the surrounding pressure, where σ is the surface tension.



■ FIGURE P3.28

Determine diameter D for which

$$\frac{1}{2}\rho V^2 = \frac{1}{2}[4\sigma/D], \text{ or}$$

$$\frac{1}{2}\left(0.00238\frac{\text{slugs}}{\text{ft}^3}\right)V^2 = \frac{1}{2}\left[4\left(5.03\times10^{-3}\frac{\text{lb}}{\text{ft}}\right)/D\right]$$

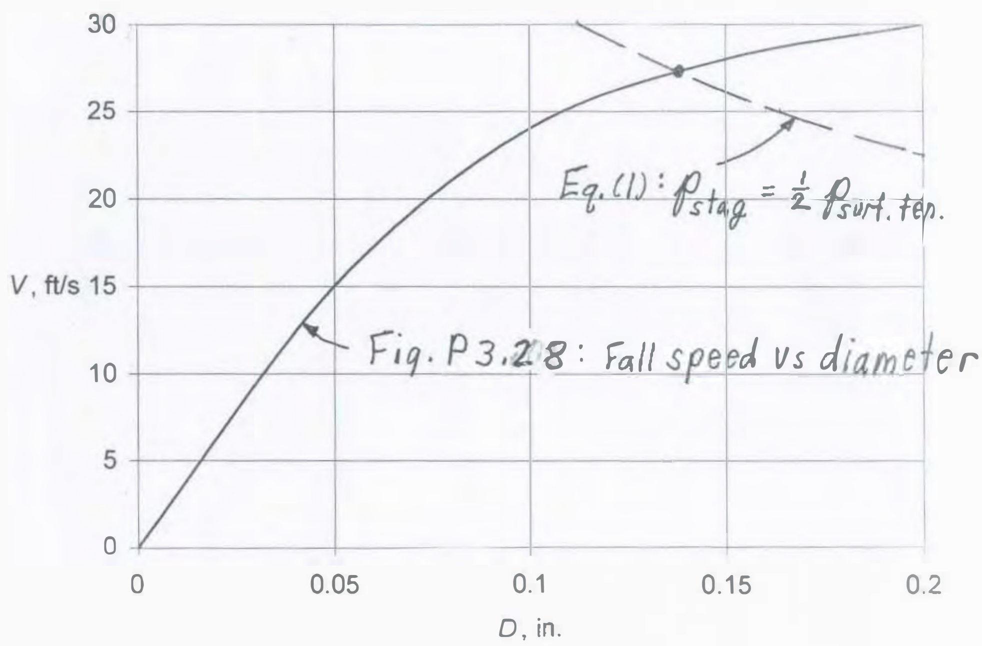
or

$$D = 8.45/V^2, \text{ where } D \sim \text{ft and } V \sim \text{ft/s}$$

or

$$D = 101/V^2, \text{ where } D \sim \text{in. and } V \sim \text{ft/s} \qquad (1)$$

Thus, there are 2 unknowns, D and V, and 2 equations, Eq. (1) and Fig. P3.28. The solution is given by the intersection of these two D-V graphs as shown below.



Thus, $D \approx \underline{\underline{0.14 \text{ in.} = 3.6 \text{ mm}}}$

3.29

3.29 (See Fluids in the News article titled "**Pressurized eyes,**" Section 3.5.) Determine the air velocity needed to produce a stagnation pressure equal to 10 mm of mercury.

$\frac{1}{2}\rho V^2 = p_{stag} = 10$ mm of mercury $= \gamma_{Hg} h$, where $\gamma_{Hg} = 133 \times 10^3 \frac{N}{m^3}$

Thus,

$$\frac{1}{2}(1.23 \frac{kg}{m^3}) V^2 = 10mm \left(\frac{1m}{1000mm}\right)(133 \times 10^3 \frac{N}{m^3})$$

or

$V = \underline{\underline{46.5\ m/s}}$

3.30

3.30 (See Fluids in the News article titled "**Bugged and plugged Pitot tubes,**" Section 3.5.) A airplane's Pitot tube used to indicated airspeed is partially plugged by an insect nest so that it measures 60% of the stagnation pressure rather than the actual stagnation pressure. If the airspeed indicator indicates that the plane is flying 150 mph, what is the actual airspeed?

When unplugged the air speed indicator would register a pressure difference of

$$\Delta p = \frac{1}{2}\rho V^2 = \frac{1}{2}\rho (150\ mph)^2$$

at 150 mph.

However, when plugged and the reading indicates 150 mph, the actual speed would be

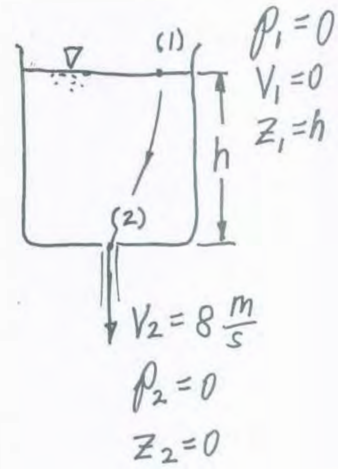
$$\Delta p = \frac{1}{2}\rho (150\ mph)^2 = 0.60\left[\frac{1}{2}\rho V^2\right]$$

or

$V = \underline{\underline{194\ mph}}$

3.32

3.32 Water flows through a hole in the bottom of a large, open tank with a speed of 8 m/s. Determine the depth of water in the tank. Viscous effects are negligible.



$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Thus, with $p_1 = p_2 = z_2 = V_1 = 0$,

$\gamma z_1 = \frac{1}{2}\rho V_2^2$, where $\gamma = \rho g$ and $z_1 = h$

so that

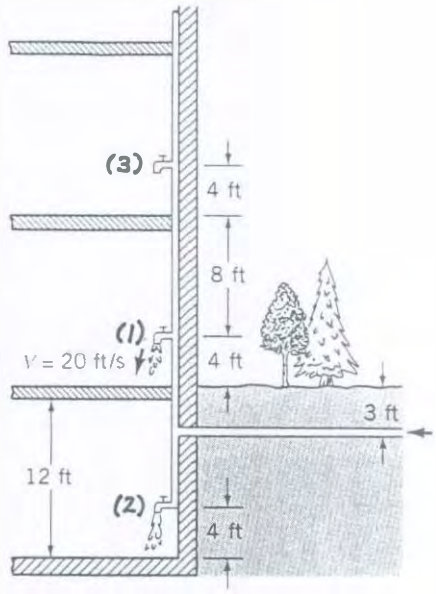
$$\rho g h = \frac{1}{2}\rho V_2^2$$

or

$$h = \frac{V_2^2}{2g} = \frac{(8\ \frac{m}{s})^2}{2(9.81\ \frac{m}{s^2})} = \underline{\underline{3.26\ m}}$$

3.33

3.33 Water flows from the faucet on the first floor of the building shown in Fig. P3.33 with a maximum velocity of 20 ft/s. For steady inviscid flow, determine the maximum water velocity from the basement faucet and from the faucet on the second floor (assume each floor is 12 ft tall).



■ FIGURE P3.33

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$

Thus, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $p_2 = p_1 = 0$ (free jet) and $V_1 = 20\ ft/s$, $z_1 = 4\ ft$, $z_2 = -8\ ft$

or

$$\frac{(20\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 4\ ft = \frac{V_2^2}{2(32.2\frac{ft}{s^2})} + (-8\ ft)$$

or $V_2 = \underline{\underline{34.2\ \frac{ft}{s}}}$

and $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ with $p_3 = p_1 = 0$ (free jet) and $V_1 = 20\ \frac{ft}{s}$, $z_1 = 4\ ft$, $z_3 = 16\ ft$

or

$$\frac{(20\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 4\ ft = \frac{V_3^2}{2(32.2\frac{ft}{s^2})} + 16\ ft$$

or

$$V_3 = \sqrt{20^2 - 2(32.2)(12)} = \sqrt{-373}$$

Impossible! No flow from second floor faucet.

*3.35

*3.35 An inviscid liquid drains from a large tank through a square duct of width b as shown in Fig. P3.35. The velocity of the fluid at the outlet is not precisely uniform because of the difference in elevation across the outlet. If $b \ll h$, this difference in velocity is negligible. For given b and h, determine v as a function of x and integrate the results to determine the average velocity, $V = Q/b^2$. Plot the velocity distribution, $v = v(x)$, across the outlet if $h = 1$ and $b = 0.1, 0.2, 0.4, 0.6, 0.8,$ and 1.0 m. How small must b be if the centerline velocity, v at $x = b/2$, is to be within 3% of the average velocity?

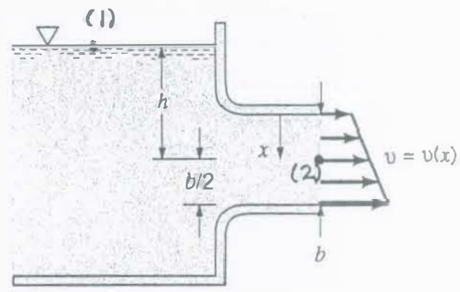


FIGURE P3.35

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{ , where } p_1 = p_2 = 0,\ V_1 = 0,\ V_2 = v,$$
$$z_1 = 0, \text{ and } z_2 = -h + \frac{b}{2} - x$$

Thus,

$$0 = \frac{v^2}{2g} + (-h + \frac{b}{2} - x) \quad \text{or} \quad v = \sqrt{2g\left(x + h - \frac{b}{2}\right)} \qquad (1)$$

Also,

$$Q = \int v\,dA = \int_{x=0}^{x=b} v\,b\,dx = \sqrt{2g}\int_0^b b\left(x + h - \frac{b}{2}\right)^{1/2} dx = b\sqrt{2g}\left(\frac{2}{3}\right)\left(x + h - \frac{b}{2}\right)^{3/2}\Big|_{x=0}^{x=b}$$

or

$$Q = \frac{2b}{3}\sqrt{2g}\left[\left(h + \frac{b}{2}\right)^{3/2} - \left(h - \frac{b}{2}\right)^{3/2}\right]$$

Hence, with $Q = AV = b^2 V$ this gives

$$V = \frac{2}{3b}\sqrt{2g}\left[\left(h + \frac{b}{2}\right)^{3/2} - \left(h - \frac{b}{2}\right)^{3/2}\right] \qquad (2)$$

Plot $v = v(x)$ from Eq. (1) from $x = 0$ to $x = b$ with $h = 1$ m and $b = 0.1, 0.2, 0.4, 0.6, 0.8,$ and 1.0 m. See the graph at the end of this problem solution.

Let V_c = centerline velocity $= v\big|_{x=\frac{b}{2}}$, where from Eq. (1):

$$V_c = \sqrt{2gh} \qquad (3)$$

Note that in the limiting case of $\frac{b}{2} = h$ the average velocity (see Eq. (2)) is

$$V\Big|_{\frac{b}{2}=h} = \frac{2}{3(2h)}\sqrt{2g}\left[(2h)^{3/2}\right] = \frac{2^{3/2}}{3}\sqrt{2gh} = 0.943\sqrt{2gh}$$
$$= 0.943\, V_c$$

(con't)

*3.35 (con't)

Thus, for $b=2h$ $\quad \frac{V_c}{V} = \frac{1}{0.943} = 1.060$

In the other limit as $b \to 0$ we can use the expansions (valid for small b) that $(h+\frac{b}{2})^{3/2} = h^{3/2}(1+\frac{b}{2h})^{3/2} \approx h^{3/2}(1+\frac{3}{2}(\frac{b}{2h})+\cdots)$ and

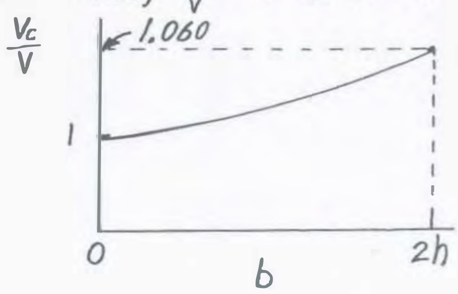
$$(h-\tfrac{b}{2})^{3/2} = h^{3/2}(1-\tfrac{b}{2h})^{3/2} \approx h^{3/2}(1-\tfrac{3}{2}(\tfrac{b}{2h})+\cdots)$$

Hence, Eq. (2) in the limit $\frac{b}{2h} \to 0$ gives

$$V\Big]_{b\to 0} = \frac{2}{3b}\sqrt{2g}\left[(h^{3/2})\left[1+\frac{3}{2}\left(\frac{b}{2h}\right)-1+\frac{3}{2}\left(\frac{b}{2h}\right)\right]\right] = \frac{2}{3b}\sqrt{2g}\,h^{3/2}\left(\frac{3b}{2h}\right)$$

or

$V\Big]_{b\to 0} = \sqrt{2gh}$, as is to be expected. Thus, $\frac{V_c}{V} \to 1$ as $b \to 0$



We are to determine the value of b that gives $V_c - V = 0.03V$, or

$$\frac{V_c}{V} = 1.03$$

That is, from Eqs. (2) and (3):

$$\sqrt{2gh} = 1.03\left(\frac{2}{3b}\right)\sqrt{2g}\left[\left(h+\frac{b}{2}\right)^{3/2}-\left(h-\frac{b}{2}\right)^{3/2}\right], \text{ or with } \eta \equiv \frac{b}{2h}$$

$$3\eta = 1.03\left[(1+\eta)^{3/2}-(1-\eta)^{3/2}\right]$$

Hence, find the root of the function $F(\eta) = 1.03\left[(1+\eta)^{3/2}-(1-\eta)^{3/2}\right]-3\eta$ i.e., η such that $F(\eta)=0$. By using a standard root-finding computer program we obtain

$\eta = 0.779$

Thus, $\eta = 0.779 = \frac{b}{2h}$

or

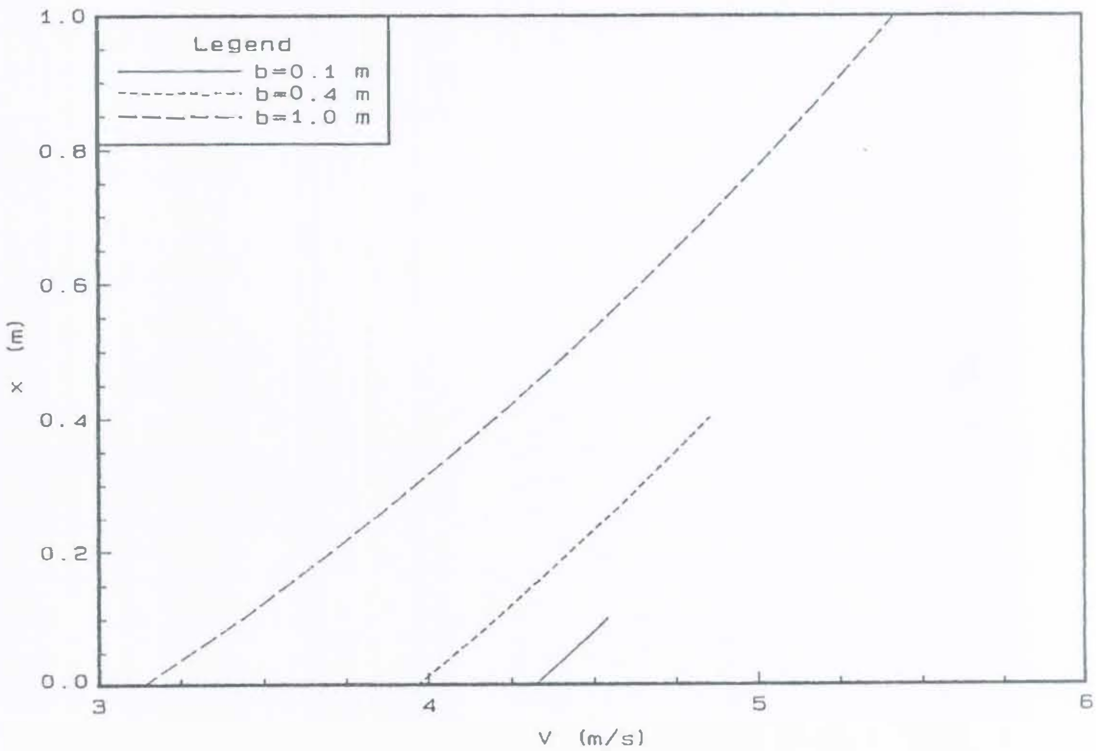
$b = 2(0.779)h = \underline{\underline{1.56\,h}}$

For $b \le 1.56h$ it follows that the centerline velocity is within 3% of the average velocity.

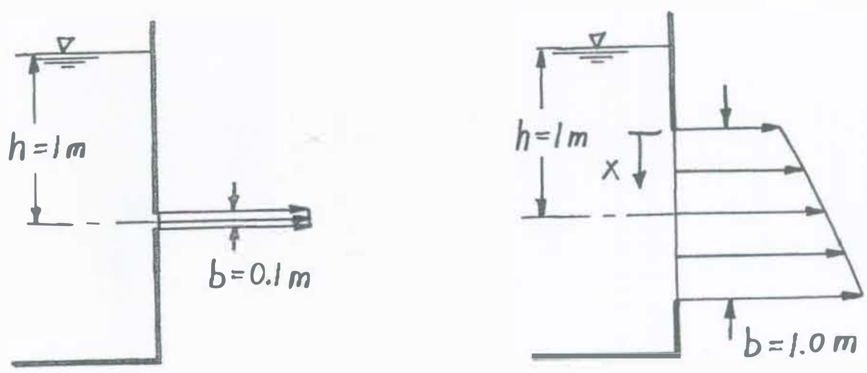
(con't)

*3.35 (con't)

Typical velocity profiles are shown below.



The velocity profiles for $b=0.1m$ and $b=1.0m$ are drawn to scale below.



3.36

3.36 Several holes are punched into a tin can as shown in Fig. P3.36. Which of the figures represents the variation of the water velocity as it leaves the holes? Justify your choice.

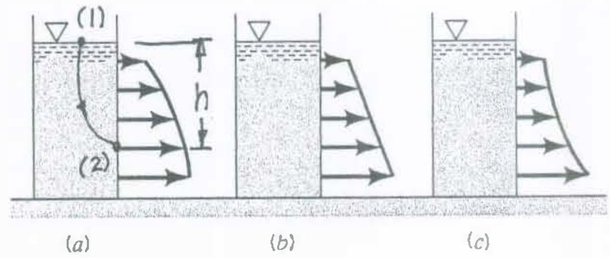


FIGURE P3.36

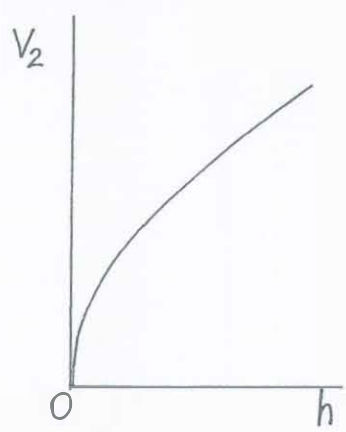
$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$ so that with $V_1 = 0$, $p_1 = 0$ and $z_1 = h_1$ at the free surface, then

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ or with $p_2 = 0$ (free jet) and $z_2 = h_2$

or

$h_1 = \frac{V_2^2}{2g} + h_2$ so that $V_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2gh}$

Thus,



or

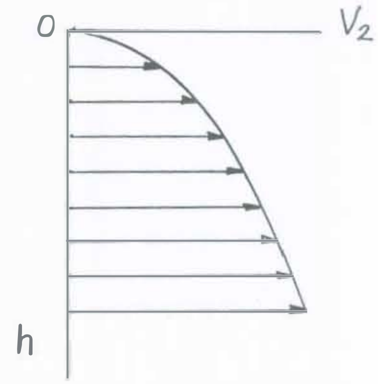
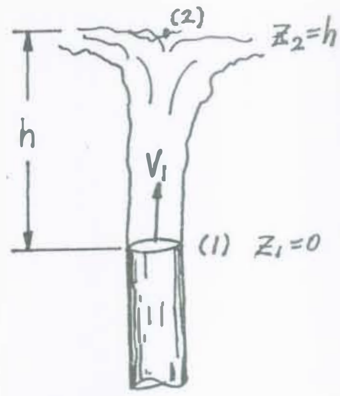


Fig. (a) is correct distribution

3.37

3.37 Water flows from a garden hose nozzle with a velocity of 15 m/s. What is the maximum height that it can reach above the nozzle?



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{but } p_1=0$$

$p_2=0$

$V_2=0$

$V_1=15\ m/s$

Thus,

$$h=\frac{V_1^2}{2g}=\frac{(15\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}=\underline{\underline{11.5\ m}}$$

3.38

3.38 Water flows from a pressurized tank, through a 6-in.-diameter pipe, exits from a 2-in.-diameter nozzle, and rises 20 ft above the nozzle as shown in Fig. P3.38. Determine the pressure in the tank if the flow is steady, frictionless, and incompressible.

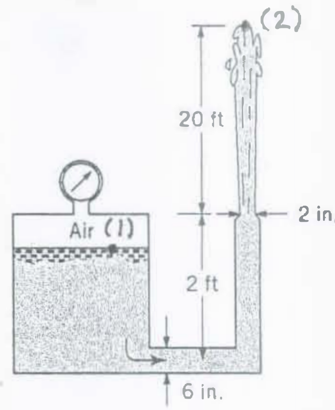


FIGURE P3.38

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2,$$

where $V_1 = 0$, $V_2 = 0$, $z_1 = 2\,ft$, $z_2 = 22\,ft$, and $p_2 = 0$

Thus,

$$\frac{p_1}{\gamma} = z_2 - z_1$$

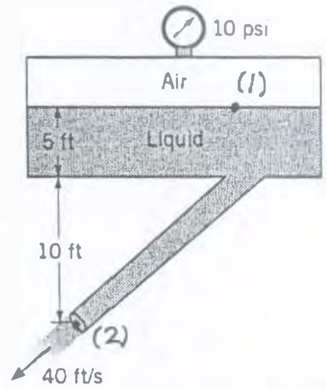
or

$$p_1 = \gamma(z_2 - z_1) = \left(62.4\,\frac{lb}{ft^3}\right)(22\,ft - 2\,ft) = \underline{\underline{1248\,\frac{lb}{ft^2}}}$$

Note: The diameter of the pipe or nozzle are not needed.

3.39

3.39 An inviscid, incompressible liquid flows steadily from the large pressurized tank shown in Fig. P.3.39. The velocity at the exit is 40 ft/s. Determine the specific gravity of the liquid in the tank.



■ FIGURE P3.39

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$$p_1 = 10\,\frac{lb}{in.^2}\left(144\,\frac{in.^2}{ft^2}\right) = 1440\,\frac{lb}{ft^2}, \quad p_2 = 0,$$

$$z_1 = 15\,ft, \quad z_2 = 0, \quad V_1 = 0, \text{ and } V_2 = 40\,\frac{ft}{s}$$

Thus,

$$\frac{1440\ lb/ft^2}{\gamma} + 15\,ft = \frac{(40\ ft/s)^2}{2\,(32.2\ ft/s^2)}$$

or

$$\gamma = 146.3\,\frac{lb}{ft^3}$$

Hence,

$$SG = \frac{\gamma}{\gamma_{H_2O}} = \frac{146\ lb/ft^3}{62.4\ lb/ft^3} = \underline{\underline{2.34}}$$

3.40

3.40 Water flows from the tank shown in Fig. P3.40. If viscous effects are negligible determine the value of h in terms of H and the specific gravity, SG, of the manometer fluid.

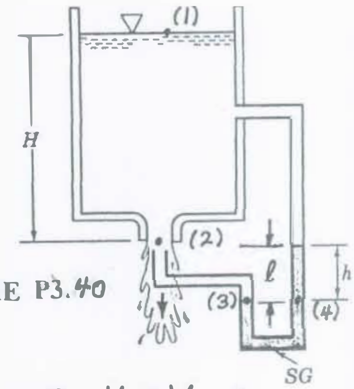


FIGURE P3.40

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = 0$, $V_1 = V_2 = 0$ and $z_1 - z_2 = H$

Thus,

$$\frac{p_2}{\gamma} = H \qquad (1)$$

But, $p_3 = p_2 + \gamma\ell = p_4 = p_1 + \gamma(H + \ell - h) + SG\,\gamma h$

or

$$p_2 = \gamma(H - h + SGh) \qquad (2)$$

Combine Eqns. (1) and (2) to give:

$H = (H + (SG-1)h)$

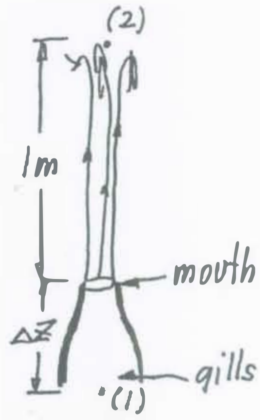
or

$(SG-1)h = 0$

Thus, if $SG \neq 1$, then $\underline{\underline{h = 0 \text{ for any } SG}}$

3.41

3.41 (See Fluids in the News article titled "**Armed with a water jet for hunting,**" Section 3.4.) Determine the pressure needed in the gills of an archerfish if it can shoot a jet of water 1 m vertically upward. Assume steady, inviscid flow.



From the Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

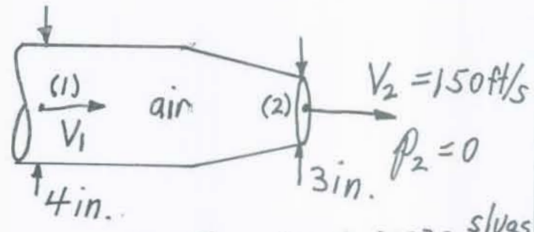
Assume $V_1 \approx 0$ (large gills), $\Delta z \ll 1\,m$ (small fish), $p_2 = 0$ (free jet), and $V_2 = 0$ (top of vertical water jet).

Thus,

$$\frac{p_1}{\gamma} = z_2 - z_1 \quad \text{or} \quad p_1 = \gamma(z_2 - z_1) = 9.80 \times 10^3 \frac{N}{m^3}(1\,m) = 9.80 \times 10^3 \frac{N}{m^2} = \underline{\underline{9.80\,kPa}}$$

3.43

3.43 Air flows steadily through a horizontal 4-in.-diameter pipe and exits into the atmosphere through a 3-in.-diameter nozzle. The velocity at the nozzle exit is 150 ft/s. Determine the pressure in the pipe if viscous effects are negligible.



$z_1 = z_2 \quad \rho = 0.00238 \frac{slugs}{ft^3}$

From Bernoulli's equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

Thus, with $z_1 = z_2$, $p_2 = 0$, and $V_2 = 150 \frac{ft}{s}$,

$$p_1 = \tfrac{1}{2}\rho(V_2^2 - V_1^2)$$

But, $A_1V_1 = A_2V_2$, or $V_1 = \left(\dfrac{\frac{\pi}{4} D_2^2}{\frac{\pi}{4} D_1^2}\right) V_2 = \left(\dfrac{D_2}{D_1}\right)^2 V_2 = \left(\dfrac{3\text{ in.}}{4\text{ in.}}\right)^2 (150 \frac{ft}{s}) = 84.4 \frac{ft}{s}$

Thus,

$$p_1 = \tfrac{1}{2}\left(0.00238 \frac{slugs}{ft^3}\right)\left[\left(150 \frac{ft}{s}\right)^2 - \left(84.4 \frac{ft}{s}\right)^2\right] = 18.3 \frac{slug}{ft \cdot s^2}\left(\frac{1\, lb}{\frac{slug \cdot ft}{s^2}}\right)$$

or

$$p_1 = 18.3 \frac{lb}{ft^2} = \underline{\underline{0.127\, psi}}$$

3.44

3.44 A fire hose nozzle has a diameter of $1\frac{1}{8}$ in. According to some fire codes, the nozzle must be capable of delivering at least 250 gal/min. If the nozzle is attached to a 3-in.-diameter hose, what pressure must be maintained just upstream of the nozzle to deliver this flowrate?

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with $z_1 = z_2$, $p_2 = 0$

and $Q = \left(250\ \frac{gal}{min}\right)\left(231\ \frac{in^3}{gal}\right)\left(\frac{1\ ft^3}{1728\ in^3}\right)\left(\frac{1\ min}{60\ s}\right) = 0.557\ \frac{ft^3}{s}$

Thus,

$$p_1 = \frac{\gamma}{2g}\left[V_2^2 - V_1^2\right] \quad \text{where} \quad V_2 = \frac{Q}{A_2} = \frac{0.557\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{1.125}{12}\right)^2 ft^2} = 80.7\ \frac{ft}{s}$$

and

$$V_1 = \frac{Q}{A_1} = \frac{0.557\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2 ft^2} = 11.34\ \frac{ft}{s}$$

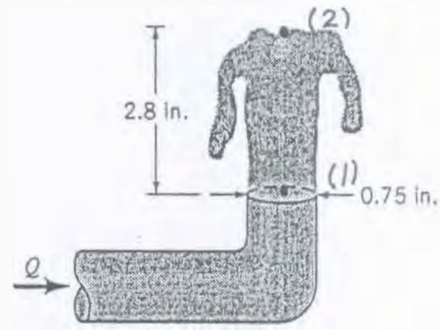
so that with $\frac{\gamma}{g} = \rho$

$$p_1 = \frac{1}{2}\left(1.94\ \frac{slugs}{ft^3}\right)\left[80.7^2 - 11.34^2\right]\frac{ft^2}{s^2}$$

$$= 6190\ \frac{lb}{ft^2} = \underline{\underline{43.0\ psi}}$$

3.45

3.45 Water flowing from the 0.75-in.-diameter outlet shown in Video V8.14 and Fig. P3.45 rises 2.8 inches above the outlet. Determine the flowrate.



■ FIGURE P3.45

The flowrate is $Q = A_1 V_1$, where from the Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Thus, with $p_1 = p_2 = z_1 = V_2 = 0$ we obtain

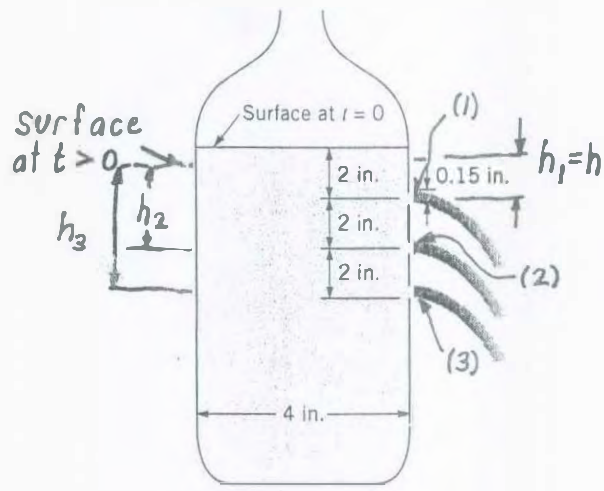
$$V_1 = \sqrt{2 g z_2} = \sqrt{2\,(32.2\ \mathrm{ft/s^2})\,(2.8/12)\,\mathrm{ft}} = 3.88\ \mathrm{ft/s}$$

so that

$$Q = A_1 V_1 = \frac{\pi}{4}\left(\frac{0.75}{12}\,\mathrm{ft}\right)^2 \left(3.88\ \frac{\mathrm{ft}}{\mathrm{s}}\right) = \underline{\underline{0.0119\ \frac{\mathrm{ft}^3}{\mathrm{s}}}}$$

3.46

3.46 Pop (with the same properties as water) flows from a 4-in. diameter pop container that contains three holes as shown in Fig. P3.46 (see Video 3.9). The diameter of each fluid stream is 0.15 in., and the distance between holes is 2 in. If viscous effects are negligible and quasi-steady conditions are assumed, determine the time at which the pop stops draining from the top hole. Assume the pop surface is 2 in. above the top hole when $t = 0$. Compare your results with the time you measure from the video.



■ FIGURE P3.46

$$Q = Q_1 + Q_2 + Q_3 = -A_T \frac{dh}{dt}$$

where $Q_i = V_i A_i = \sqrt{2gh_i}\, A_i$ $(i = 1,2,3)$ and $A_1 = A_2 = A_3 = \frac{\pi}{4}\left(\frac{0.15}{12}\text{ ft}\right)^2 = 1.227 \times 10^{-4}\text{ ft}^2$

$$A_T = \frac{\pi}{4}\left(\frac{4}{12}\text{ ft}\right)^2 = 0.0873\text{ ft}^2$$

Thus,

$$\sqrt{2g}\, A_1\left[\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3}\right] = -A_T \frac{dh}{dt}, \text{ where } h_1 = h,\ h_2 = h + L,\ h_3 = h + 2L \text{ and } L = 2\text{ in.}$$

Hence,

$$-\left(\sqrt{2g}\, A_1 / A_T\right)\int_0^t dt = \int_L^0 \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

where t is the time it take for the free surface to reach the upper hole ($h=0$).

or

$$t = \frac{A_T}{A_1\sqrt{2g}}\int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

$$= \frac{0.0873\text{ ft}^2}{\left(1.227\times10^{-4}\text{ ft}^2\right)\left[(2)(32.2\text{ ft/s}^2)\right]^{1/2}}\int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

Thus,

$$t = 88.7\int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)} \quad \text{where } L = \frac{2}{12}\text{ ft} = 0.1667\text{ ft}$$

Note: With L in feet, this equation gives t in seconds.

(con't)

3.46 (con't)

The numerical value of the integral is obtained by using the trapezoidal rule since the closed form analytical solution is not given in integral tables. The EXCEL spreadsheet used for this is given below.

$$t = 88.7 \int_0^L f(h)\, dh \quad \text{where } f(h) = \frac{1}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$

$$\approx 88.7 \left[\frac{1}{2} \sum_{i=1}^{20} (f_i + f_{i+1})(h_{i+1} - h_i) \right] = \left(88.7 \frac{s}{\sqrt{ft}}\right)\left[0.120\sqrt{ft}\right] = \underline{\underline{10.7\,s}}$$

h, in.	h, ft	f(h), 1/ft$^{1/2}$	(1/2)*(f_i + f_{i+1})*(h_{i+1} - h_i), ft$^{1/2}$	i
0.0	0.0000	1.015	0.00804	1
0.1	0.0083	0.914	0.00743	2
0.2	0.0167	0.870	0.00711	3
0.3	0.0250	0.837	0.00686	4
0.4	0.0333	0.810	0.00665	5
0.5	0.0417	0.786	0.00646	6
0.6	0.0500	0.764	0.00629	7
0.7	0.0583	0.745	0.00614	8
0.8	0.0667	0.728	0.00600	9
0.9	0.0750	0.712	0.00587	10
1.0	0.0833	0.697	0.00575	11
1.1	0.0917	0.684	0.00564	12
1.2	0.1000	0.671	0.00554	13
1.3	0.1083	0.659	0.00544	14
1.4	0.1167	0.647	0.00535	15
1.5	0.1250	0.637	0.00526	16
1.6	0.1333	0.627	0.00518	17
1.7	0.1417	0.617	0.00510	18
1.8	0.1500	0.608	0.00503	19
1.9	0.1583	0.599	0.00496	20
2.0	0.1667	0.591		21
	Sum of column = integral =		0.12011	

Thus, t = 88.7*0.12011 = 10.7 s

3.47

3.47 Water (assumed inviscid and incompressible) flows steadily in the vertical variable-area pipe shown in Fig. P3.47. Determine the flowrate if the pressure in each of the gages reads 50 kPa..

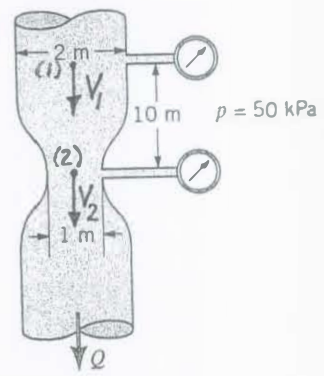


FIGURE P3.47

From the Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2,$$

where $p_1 = p_2 = 50\ kPa$

Thus,

(1) $$\frac{1}{2}\rho(V_2^2 - V_1^2) = \gamma(z_1 - z_2)$$

Also, $A_1 V_1 = A_2 V_2$, or

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{\frac{\pi}{4}D_2^2}{\frac{\pi}{4}D_1^2}\right)V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{1m}{2m}\right)^2 V_2 = \frac{1}{4}V_2$$

Hence, Eq. (1) becomes

$$\frac{1}{2}\rho\left[V_2^2 - \frac{1}{16}V_2^2\right] = \rho g(z_1 - z_2)$$

or

$$\frac{15}{16}V_2^2 = 2g(z_1 - z_2) = 2\left(9.81\frac{m}{s}\right)(10m)$$

or

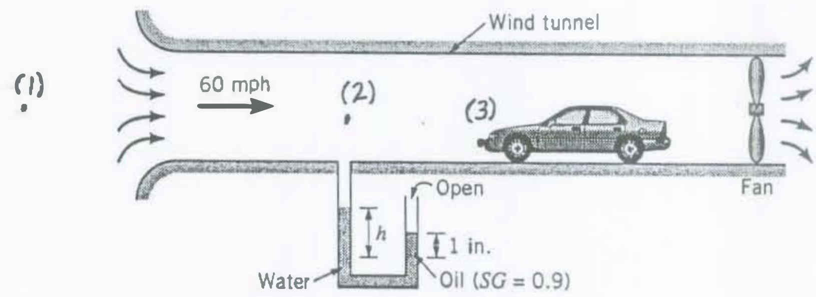
$$V_2 = 14.5\frac{m}{s}$$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4}(1m)^2\left(14.5\frac{m}{s}\right) = \underline{\underline{11.4\frac{m^3}{s}}}$$

3.48

3.48 Air is drawn into a wind tunnel used for testing automobiles as shown in Fig. P3.48. (a) Determine the manometer reading, h, when the velocity in the test section is 60 mph. Note that there is a 1-in. column of oil on the water in the manometer. (b) Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section.



■ FIGURE P3.48

(a) $$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where

$z_1 = z_2$, $p_1 = 0$, and $V_1 \approx 0$

Thus, with $V_2 = 60\ mph = 88\ \frac{ft}{s}$,

$$\frac{p_2}{\gamma} = -\frac{V_2^2}{2g} \quad or$$

$$p_2 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}\left(0.00238\ \frac{slugs}{ft^3}\right)\left(88\ \frac{ft}{s}\right)^2 = -9.22\ \frac{lb}{ft^2}$$

But $p_2 + \gamma_{H_2O}h - \gamma_{oil}\left(\frac{1}{12}ft\right) = 0$ where $\gamma_{oil} = 0.9\gamma_{H_2O} = 0.9\left(62.4\ \frac{lb}{ft^3}\right) = 56.2\ \frac{lb}{ft^3}$

Thus,

$$-9.22\ \frac{lb}{ft^2} + 62.4\ \frac{lb}{ft^3}(h\ ft) - 56.2\ \frac{lb}{ft^3}\left(\frac{1}{12}ft\right) = 0, \quad or\ h = \underline{\underline{0.223\ ft}}$$

(b) $$\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$$

where

$z_2 = z_3$ and $V_3 = 0$

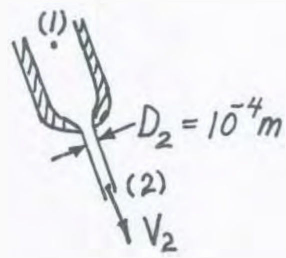
Thus,

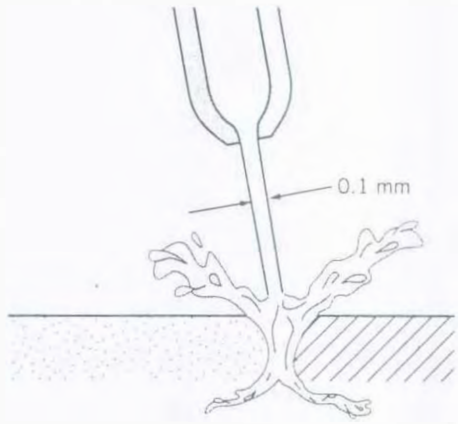
$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_3}{\gamma} \quad or$$

$$p_3 - p_2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}\left(0.00238\ \frac{slugs}{ft^3}\right)\left(88\ \frac{ft}{s}\right)^2 = \underline{\underline{9.22\ \frac{lb}{ft^2}}}$$

3.49

3.49 Small-diameter, high-pressure liquid jets can be used to cut various materials as shown in Fig. P3.49. If viscous effects are negligible, estimate the pressure needed to produce a 0.10-mm-diameter water jet with a speed of 700 m/s. Determine the flowrate.





■ FIGURE P3.49

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } V_1 \approx 0,\ z_1 \approx z_2, \text{ and } p_2 = 0$$

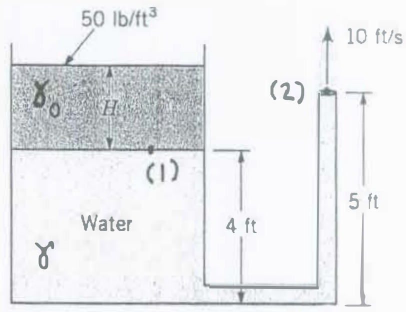
Thus $p_1 = \frac{1}{2}\frac{\gamma}{g}V_2^2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}(999\,\frac{kg}{m^3})(700\,\frac{m}{s})^2 = \underline{\underline{2.45 \times 10^5\,\frac{kN}{m^2}}}$

Also,

$Q = V_2 A_2 = 700\,\frac{m}{s}\left[\frac{\pi}{4}(10^{-4}m)^2\right] = \underline{\underline{5.50 \times 10^{-6}\,\frac{m^3}{s}}}$

3.50

3.50 Water (assumed inviscid and incompressible) flows steadily with a speed of 10 ft/s from the large tank shown in Fig. P3.50. Determine the depth, H, of the layer of light liquid (specific weight = 50 lb/ft^3) that covers the water in the tank.



■ FIGURE P3.50

From the Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = \gamma_0 H$, $V_1 = 0$, $p_2 = 0$, $z_1 = 4\,ft$, and $z_2 = 5\,ft$

Thus,

$$\frac{\gamma_0}{\gamma} H + z_1 = \frac{V_2^2}{2g} + z_2 \quad \text{so that with } V_2 = 10\,ft/s,$$

$$\left(\frac{50\,lb/ft^3}{62.4\,lb/ft^3}\right) H + 4\,ft = \frac{(10\,ft/s)^2}{2(32.2\,ft/s^2)} + 5\,ft$$

Therefore,

$$H = \underline{\underline{3.19\,ft}}$$

3.51

3.51 Water flows through the pipe contraction shown in Fig. P3.51. For the given 0.2-m difference in manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

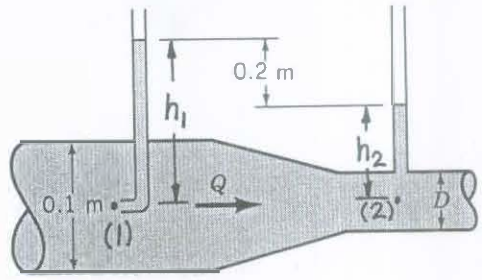


FIGURE P3.51

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{or with } z_1 = z_2 \text{ and } V_1 = 0$$

$$V_2 = \sqrt{2g\,\frac{(p_1 - p_2)}{\gamma}}$$

but $p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

$$V_2 = \sqrt{2g\,\frac{0.2\gamma}{\gamma}} = \sqrt{2g(0.2)}$$

or

$$Q = A_2 V_2 = \frac{\pi}{4} D^2 V_2 = \frac{\pi}{4} D^2 \sqrt{2(9.81)(0.2)} = \underline{\underline{1.56\, D^2 \frac{m^3}{s}}} \text{ when } D \sim m$$

3.52

3.52 Water flows through the pipe contraction shown in Fig. P3.52. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

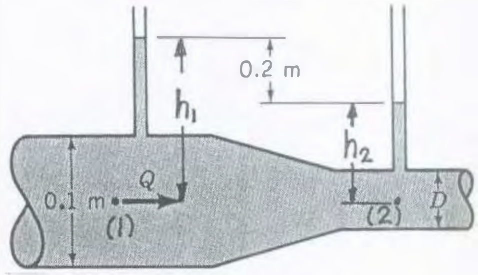


FIGURE P3.52

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with} \quad A_1V_1 = A_2V_2$$

Thus, with $z_1 = z_2$ or $V_2 = \frac{\left(\frac{\pi}{4}D_1^2\right)}{\left(\frac{\pi}{4}D_2^2\right)}V_1 = \left(\frac{0.1}{D}\right)^2 V_1$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g}$$

but

$p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

$$\frac{0.2\gamma}{\gamma} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{\frac{0.2\,(2g)}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

and

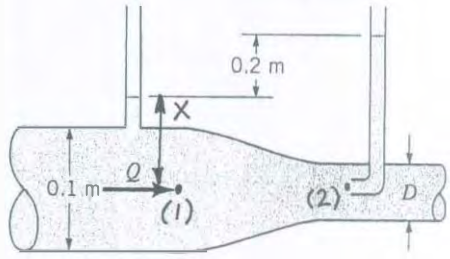
$$Q = A_1V_1 = \frac{\pi}{4}(0.1)^2\sqrt{\frac{0.2\,(2\,(9.81))}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

or

$$Q = \frac{0.0156\,D^2}{\sqrt{(0.1)^4 - D^4}} \ \frac{m^3}{s} \quad \text{when } D \sim m$$

3.53

3.53 Water flows through the pipe contraction shown in Fig. P3.53. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.



■ FIGURE P3.53

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$ and $V_2 = 0$.

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma}$$

But

$\frac{p_1}{\gamma} = x$ and $\frac{p_2}{\gamma} = 0.2m + x$ so that

$$x + \frac{V_1^2}{2g} = 0.2m + x \quad \text{or}$$

$$V_1 = \sqrt{2g(0.2m)} = \left(2\left(9.81\frac{m}{s^2}\right)(0.2m)\right)^{1/2} = 1.98\frac{m}{s}$$

Thus,

$$Q = A_1V_1 = \frac{\pi}{4}(0.1m)^2\left(1.98\frac{m}{s}\right) = \underline{\underline{0.0156\frac{m^3}{s} \text{ for any } D}}$$

3.54

3.54 A 0.15-m-diameter pipe discharges into a 0.10-m-diameter pipe. Determine the velocity head in each pipe if they are carrying 0.12 m^3/s of kerosene.

$$V_1 = \frac{Q}{A_1} = \frac{0.12 \frac{m^3}{s}}{\frac{\pi}{4}(0.15 m)^2} = 6.79 \frac{m}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{0.12 \frac{m^3}{s}}{\frac{\pi}{4}(0.10 m)^2} = 15.27 \frac{m}{s}$$

Thus,

$$\frac{V_1^2}{2g} = \frac{(6.79 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \underline{\underline{2.35 \, m}}$$

and

$$\frac{V_2^2}{2g} = \frac{(15.27 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \underline{\underline{11.9 \, m}}$$

3.55

3.55 Carbon tetrachloride flows in a pipe of variable diameter with negligible viscous effects. At point A in the pipe the pressure and velocity are 20 psi and 30 ft/s, respectively. At location B the pressure and velocity are 23 psi and 14 ft/s. Which point is at the higher elevation and by how much?

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \qquad \text{with } \gamma = 99.5\,\frac{lb}{ft^3}$$

or

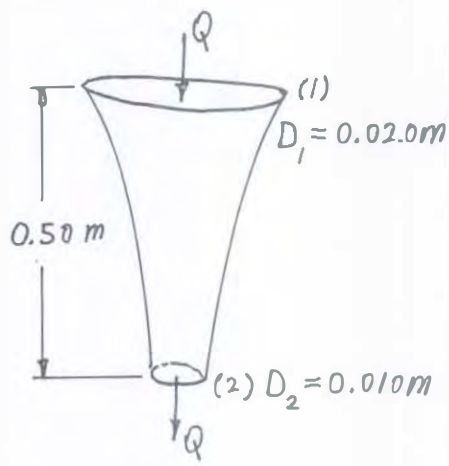
$$z_B - z_A = \frac{p_A - p_B}{\gamma} + \frac{V_A^2 - V_B^2}{2g} = \frac{(20-23)\frac{lb}{in.^2}\left(144\,\frac{in.^2}{ft^2}\right)}{99.5\,\frac{lb}{ft^3}} + \frac{(30^2 - 14^2)\frac{ft^2}{s^2}}{2\left(32.2\,\frac{ft}{s^2}\right)}$$

or

$$z_B - z_A = \underline{\underline{6.59\text{ ft},\quad B \text{ is above } A}}$$

3.56

3.56 The circular stream of water from a faucet is observed to taper from a diameter of 20 mm to 10 mm in a distance of 50 cm. Determine the flowrate.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = p_2 = 0$, $z_2 = 0$, $z_1 = 0.50m$

and

$$V_1 = \frac{Q}{A_1}, \quad V_2 = \frac{Q}{A_2}$$

Thus,

$$\left(\frac{Q}{A_1}\right)^2 + 2gz_1 = \left(\frac{Q}{A_2}\right)^2 \text{ or } Q = \left[\frac{2gz_1}{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)}\right]^{\frac{1}{2}} = \frac{A_2\sqrt{2gz_1}}{\sqrt{1-(A_2/A_1)^2}}$$

or since

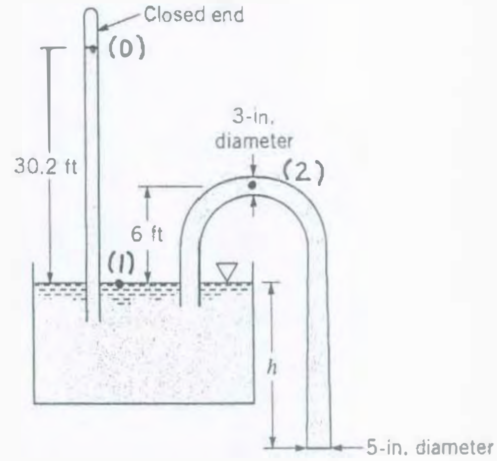
$$\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 \text{ we obtain}$$

$$Q = A_2\frac{\sqrt{2gz_1}}{\sqrt{1-(D_2/D_1)^4}} = \frac{\pi}{4}(0.010\,m)^2\left[\frac{2(9.81\frac{m}{s^2})(0.50m)}{1-\left(\frac{0.010}{0.020}\right)^4}\right]^{\frac{1}{2}}$$

$$= \underline{\underline{2.54 \times 10^{-4}\frac{m^3}{s}}}$$

3.57

3.57 Water is siphoned from the tank shown in Fig. P3.57. The water barometer indicates a reading of 30.2 ft. Determine the maximum value of h allowed without cavitation occurring. Note that the pressure of the vapor in the closed end of the barometer equals the vapor pressure.



■ FIGURE P3.57

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } p_1=0,\ V_1=0,\ p_2=p_{vapor}$$

Thus, $\qquad z_1=0,\ z_2=6\text{ ft}$

$$0=\frac{p_{vapor}}{\gamma}+\frac{V_2^2}{2g}+6\text{ ft}$$

but $p_0+30.2\text{ ft}\,\gamma=p_1$ or since $p_0=p_{vapor}$, $\dfrac{p_{vapor}}{\gamma}=-30.2\text{ ft}$

Hence,

$$0=-30.2\text{ ft}+\frac{V_2^2}{2g}+6\text{ ft} \quad \text{or} \quad \frac{V_2^2}{2g}=24.2\text{ ft} \quad \text{or} \quad V_2^2=\left[2\left(32.2\tfrac{\text{ft}}{\text{s}^2}\right)(24.2\text{ ft})\right]$$

Thus,

$$V_2=39.5\ \frac{\text{ft}}{\text{s}}$$

Since $V_3A_3=V_2A_2$, $\quad V_3=\dfrac{A_2}{A_3}V_2=\dfrac{D_2^2}{D_3^2}V_2=\left(\dfrac{3\text{ in.}}{5\text{ in.}}\right)^2\left(39.5\ \dfrac{\text{ft}}{\text{s}}\right)$

or

$$V_3=14.2\ \frac{\text{ft}}{\text{s}}$$

However,

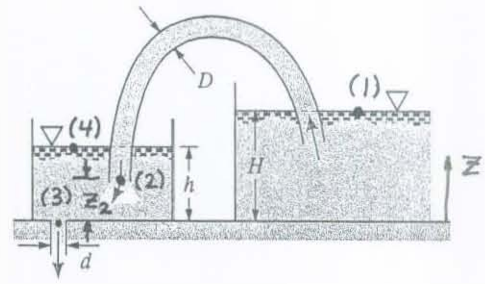
$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3 \quad \text{or} \quad V_3=\sqrt{2gh}$$

Thus,

$$14.2\ \frac{\text{ft}}{\text{s}}=\sqrt{2\left(32.2\tfrac{\text{ft}}{\text{s}^2}\right)h\text{ ft}} \quad \text{or} \quad \underline{\underline{h=3.13\text{ ft}}}$$

*3.58

*3.58 As shown in Fig. P3.58, water from a large reservoir flows without viscous effects through a siphon of diameter D and into a tank. It exits from a hole in the bottom of the tank as a stream of diameter d. The surface of the reservoir remains H above the bottom of the tank. For steady-state conditions, the water depth in the tank, h, is constant. Plot a graph of the depth ratio h/H as a function of the diameter ratio d/D.



■ FIGURE P3.58

From the Bernoulli equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

where $p_1=V_1=0$, $z_1=H$, and at the "free jet" end of the siphon,

$p_2=\gamma(h-z_2)$.

Thus, Eq. (1) becomes

$$H=(h-z_2)+\frac{V_2^2}{2g}+z_2=h+\frac{V_2^2}{2g}$$

or

(1) $$V_2=\sqrt{2g(H-h)}$$

Also,

$$\frac{p_4}{\gamma}+\frac{V_4^2}{2g}+z_4=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3,$$ where $p_4=V_4=p_3=z_3=0$ and $z_4=h$

Thus,

$$h=\frac{V_3^2}{2g}$$ or

(2) $$V_3=\sqrt{2gh}$$

Also, for constant liquid levels in the tanks, $Q_2=Q_3$

or

$$A_2V_2=A_3V_3$$

so that

(3) $$\frac{\pi}{4}D^2V_2=\frac{\pi}{4}d^2V_3$$

From Eqs. (1), (2), and (3):

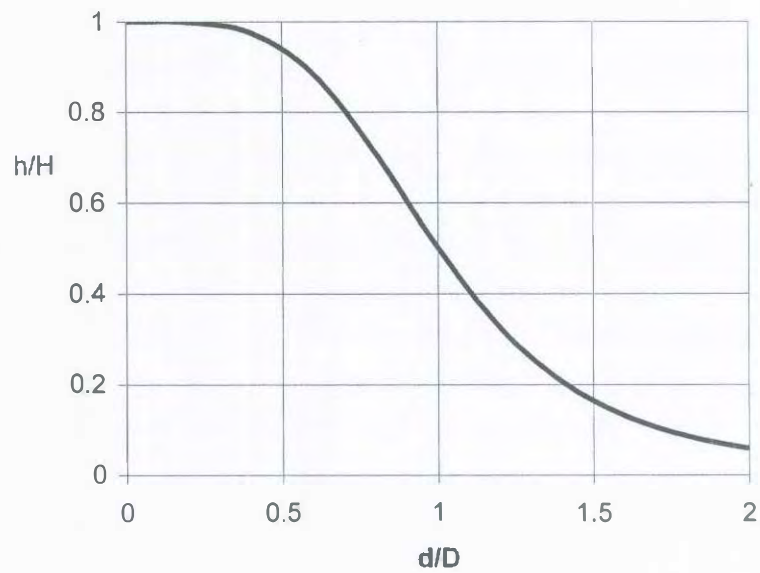
$$D^2\sqrt{2g(H-h)}=d^2\sqrt{2gh}$$ or $$H-h=\left(\frac{d}{D}\right)^4h$$

Thus,

$$\underline{\underline{\frac{h}{H}=\frac{1}{1+(d/D)^4}}}$$ This result is plotted on the next page.

(con't)

3.58 (con't)



3.59

3.59 A smooth plastic, 10-m-long garden hose with an inside diameter of 20 mm is used to drain a wading pool as is shown in Fig. P3.59. If viscous effects are neglected, what is the flowrate from the pool?

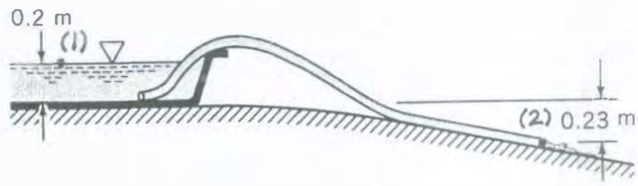


FIGURE P3.59

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = p_2 = 0$, $z_1 = 0.2\,m$, $z_2 = -0.23\,m$, and $V_1 = 0$

Thus,

$$V_2 = \sqrt{2g(z_1 - z_2)} = \left(2\left(9.81\,\frac{m}{s^2}\right)\left(0.2\,m - (-0.23\,m)\right)\right)^{\frac{1}{2}}$$

$$= 2.90\,\frac{m}{s}$$

or

$$Q = A_2 V_2 = \frac{\pi}{4}(0.020\,m)^2\left(2.90\,\frac{m}{s}\right) = \underline{\underline{9.11 \times 10^{-4}\,\frac{m^3}{s}}}$$

3.60

3.60 Water exits a pipe as a free jet and flows to a height h above the exit plane as shown in Fig. P3.60. The flow is steady, incompressible, and frictionless. (a) Determine the height h. (b) Determine the velocity and pressure at section (1).

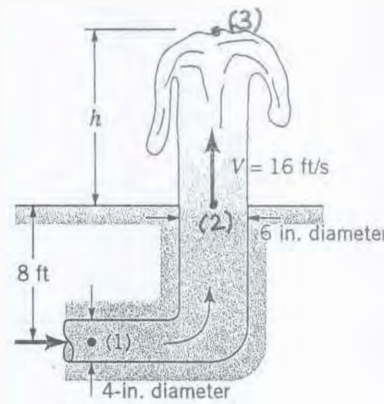


FIGURE P3.60

(a) From the Bernoulli eqn.,

$$\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3, \text{ where } p_2=p_3=0, \text{ and } V_3=0.$$

Thus,

$$\frac{V_2^2}{2g}=z_3-z_2=h$$

or

$$h=\frac{V_2^2}{2g}=\frac{(16\ \text{ft/s})^2}{2(32.2\ \text{ft/s})}=\underline{\underline{3.98\ \text{ft}}}$$

(b) Also, $A_1V_1=A_2V_2$

or

$$V_1=\frac{A_2}{A_1}V_2=\frac{\frac{\pi}{4}(6\text{in.})^2}{\frac{\pi}{4}(4\text{in.})^2}\left(16\frac{\text{ft}}{\text{s}}\right)=\underline{\underline{36.0\frac{\text{ft}}{\text{s}}}}$$

From the Bernoulli equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2,$$

or since $\gamma=\rho g$,

$$p_1=p_2+\frac{1}{2}\rho(V_2^2-V_1^2)+\gamma(z_2-z_1) \text{ where } p_2=0$$

Thus,

$$p_1=\frac{1}{2}\left(1.94\frac{\text{slugs}}{\text{ft}^3}\right)\left[\left(16\frac{\text{ft}}{\text{s}}\right)^2-\left(36.0\frac{\text{ft}}{\text{s}}\right)^2\right]+62.4\frac{\text{lb}}{\text{ft}^3}(8\text{ft})$$

$$=-1009\left(\frac{\text{slugs}\cdot\text{ft}}{\text{s}^2}\right)/\text{ft}^2+499\frac{\text{lb}}{\text{ft}^2}$$

$$=\underline{\underline{-510\frac{\text{lb}}{\text{ft}^2}}}$$

3.61

3.61 Water flows steadily from a large, closed tank as shown in Fig. P3.61. The deflection in the mercury manometer is 1 in. and viscous effects are negligible. (a) Determine the volume flowrate. (b) Determine the air pressure in the space above the surface of the water in the tank.

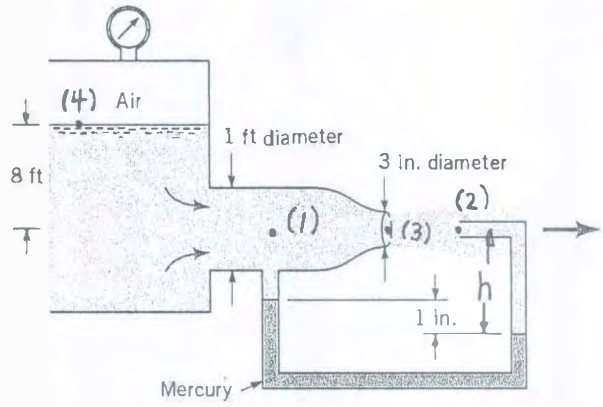


FIGURE P3.61

(a) From the Bernoulli equation,

(1) $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\text{, where } V_2=0 \text{ and } z_1=z_2$$

Also, for the manometer,

$$p_2+\gamma_{H_2O}h=p_1+\gamma_{H_2O}(h-1\text{in.})+\gamma_{Hg}(1\text{in.})$$

or

$$p_2-p_1=(\gamma_{Hg}-\gamma_{H_2O})(1\text{in.})=\gamma_{H_2O}(SG_{Hg}-1)(1\text{in.})$$

$$=\left(62.4\frac{lb}{ft^3}\right)(13.56-1)\left(\frac{1}{12}ft\right)=65.3\frac{lb}{ft^2}$$

Thus, from Eq(1),

$$\frac{V_1^2}{2g}=\frac{p_2-p_1}{\gamma}=\frac{65.3\ lb/ft^2}{62.4\ lb/ft^3}=1.046\ ft$$

so that

$$V_1=\sqrt{(2)(32.2\tfrac{ft}{s^2})(1.046ft)}=8.21\frac{ft}{s}$$

Hence,

$$Q=A_1V_1=\frac{\pi}{4}(1ft)^2(8.21\tfrac{ft}{s})=\underline{\underline{6.45\ \frac{ft^3}{s}}}$$

(b) From the Bernoulli equation,

(2) $$\frac{p_4}{\gamma}+\frac{V_4^2}{2g}+z_4=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3\text{, where } V_4=0,\ p_3=0\text{, and } V_3=\frac{Q}{A_3}$$

Thus,

$$V_3=\frac{6.45\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{3}{12}ft)^2}=131\frac{ft}{s}$$

Hence, from Eq.(2),

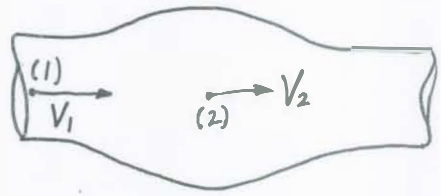
$$\frac{p_4}{\gamma}+z_4=\frac{V_3^2}{2g}+z_3\text{, or } p_4=\frac{1}{2}\rho V_3^2+\gamma(z_3-z_4)$$

Hence,

$$p_4=\frac{1}{2}(1.94\tfrac{slug}{ft^3})(131\tfrac{ft}{s})^2+62.4\tfrac{lb}{ft^3}(-8ft)=16,150\frac{lb}{ft^2}=\underline{\underline{112\ psi}}$$

3.62

3.62 Blood ($SG = 1$) flows with a velocity of 0.5 m/s in an artery. It then enters an aneurysm in the artery (i.e., an area of weakened and stretched artery walls that cause a ballooning of the vessel) whose cross-sectional area is 1.8 times that of the artery. Determine the pressure difference between the blood in the aneurysm and that in the artery. Assume the flow is steady and inviscid.



$A_2 = 1.8 A_1$

From the Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

where $z_1 = z_2$ and $V_1 = 0.5 \frac{m}{s}$

Thus,

(1) $$p_2 - p_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

However,

$$\rho = \rho_{H_2O} SG_{blood} = \rho_{H_2O}(1) = 999 \frac{kg}{m^3}$$

and

$$V_1 A_1 = V_2 A_2 \text{ or}$$

$$V_2 = \frac{A_1}{A_2} V_1 = \left(\frac{1}{1.8}\right) V_1$$

Thus, Eq (1) becomes

$$p_2 - p_1 = \frac{1}{2}\left(999 \frac{kg}{m^3}\right)\left[\left(0.5 \frac{m}{s}\right)^2 - \left(\frac{1}{1.8}\right)^2 \left(0.5 \frac{m}{s}\right)^2\right]$$

$$= 86.3 \left(\frac{kg \cdot m}{s^2}\right)/m^2 = 86.3 \frac{N}{m^2} = \underline{\underline{86.3 \text{ Pa}}}$$

3.63

3.63 Water flows steadily through the variable area pipe shown in Fig. P3.63 with negligible viscous effects. Determine the manometer reading, H, if the flowrate is 0.5 m^3/s and the density of the manometer fluid is 600 kg/m^3.

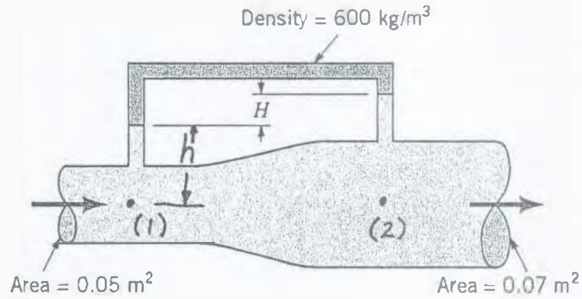


FIGURE P3.63

From the Bernoulli equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2,\quad \text{where } z_1=z_2$$

Thus,

(1) $$p_2-p_1=\frac{\gamma}{2g}(V_1^2-V_2^2)=\frac{1}{2}\rho(V_1^2-V_2^2)$$

But, $Q=A_1V_1=A_2V_2$ so that

$$V_1=\frac{Q}{A_1}=\frac{0.5\frac{m^3}{s}}{0.05\,m^2}=10\frac{m}{s}\quad\text{and}\quad V_2=\frac{Q}{A_2}=\frac{0.5\frac{m^3}{s}}{0.07\,m^2}=7.14\frac{m}{s}$$

Hence, from Eq. (1):

(2) $$p_2-p_1=\frac{1}{2}\left(999\frac{kg}{m^3}\right)\left[\left(10\frac{m}{s}\right)^2-\left(7.14\frac{m}{s}\right)^2\right]=24.5\times10^3\left(\frac{kg\cdot m}{s^2}\right)/m^2$$
$$=24.5\times10^3\frac{N}{m^2}$$

For the manometer,

$$p_1-\gamma_{H_2O}h-\gamma_{man}H=p_2-\gamma_{H_2O}(h+H)$$

so that

(3) $$p_2-p_1=\gamma_{H_2O}(h+H)-\gamma_{H_2O}h-\gamma_{man}H=(\gamma_{H_2O}-\gamma_{man})H=g(\rho_{H_2O}-\rho_{man})H$$

Hence, from Eqs (2) and (3):

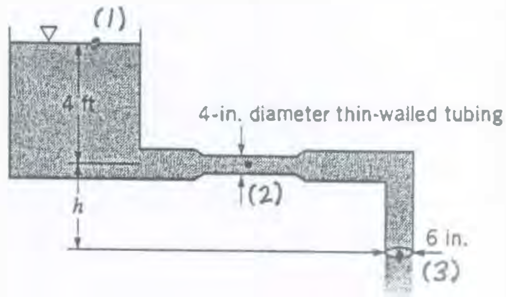
$$24.5\times10^3\frac{N}{m^2}=9.81\frac{m}{s^2}\left(999\frac{kg}{m^3}-600\frac{kg}{m^3}\right)H$$

or

$$H=\underline{\underline{6.26\,m}}$$

3.64

3.64 Water flows steadily with negligible viscous effects through the pipe shown in Fig. P3.64. It is known that the 4-in. diameter section of thin-walled tubing will collapse if the pressure within it becomes less than 10 psi below atmospheric pressure. Determine the maximum value that h can have without causing collapse of the tubing.



■ FIGURE P3.64

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = 0$, $V_1 = 0$, $z_2 = 0$, and $p_2 = -10\,\frac{lb}{in.^2}\left(144\,\frac{in.^2}{ft^2}\right) = -1440\,\frac{lb}{ft^2}$

Thus, with $z_1 = 4\,ft$,

$$4\,ft = \frac{-1440\,lb/ft^2}{62.4\,lb/ft^3} + \frac{V_2^2}{2(32.2\,ft/s^2)}$$

or $V_2 = 41.7\,\frac{ft}{s}$

Also,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$$

where

$p_3 = 0$, $z_3 = -h$, and $V_3 = \frac{A_2 V_2}{A_3} = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{4\,in.}{6\,in.}\right)^2 \left(41.7\,\frac{ft}{s}\right)$

$= 18.5\,\frac{ft}{s}$

Thus,

$$4\,ft = -h + \frac{(18.5\,ft/s)^2}{2(32.2\,ft/s^2)}$$

or

$h = \underline{\underline{1.31\,ft}}$

3.65

3.65 Helium flows through a 0.30-m-diameter horizontal pipe with a temperature of 20 °C and a pressure of 200 kPa (abs) at a rate of 0.30 kg/s. If the pipe reduces to 0.25-m-diameter determine the pressure difference between these two sections. Assume incompressible, inviscid flow.

$D_1 = 0.3\,m$ $\qquad D_2 = 0.25\,m$

$p_1 = 200\,kPa\ abs$

$T_1 = 20°C$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$

Thus,

$$(1) \quad p_1 - p_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) \qquad \text{where } \rho = \frac{p_1}{RT_1} = \frac{200 \times 10^3 \frac{N}{m^2}}{\left(2077 \frac{N \cdot m}{kg \cdot K}\right)(273+20)\,K}$$

or

$$\rho = 0.329 \frac{kg}{m^3}$$

Also,

$$\dot{m} = \rho A_1 V_1 = 0.30 \frac{kg}{s}$$

so that

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{0.30 \frac{kg}{s}}{\left(0.329 \frac{kg}{m^3}\right)\frac{\pi}{4}(0.3m)^2} = 12.9 \frac{m}{s}$$

and

$$A_1 V_1 = A_2 V_2 \quad \text{or}$$

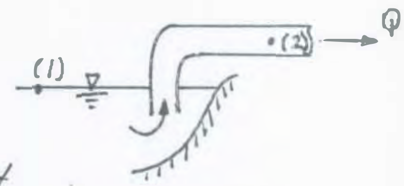
$$V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{0.3m}{0.25m}\right)^2 \left(12.9 \frac{m}{s}\right) = 18.6 \frac{m}{s}$$

Thus, from Eq.(1):

$$p_1 - p_2 = \frac{1}{2}\left(0.329 \frac{kg}{m^3}\right)\left(18.6^2 - 12.9^2\right)\frac{m^2}{s^2} = \underline{\underline{29.5\ Pa}}$$

3.66

3.66 Water is pumped from a lake through an 8-in. pipe at a rate of 10 ft^3/s. If viscous effects are negligible, what is the pressure in the suction pipe (the pipe between the lake and the pump) at an elevation 6 ft above the lake?



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

where $p_1=0$, $V_1=0$, $z_1=0$, $z_2=6.0 \text{ ft}$
and

$$V_2=\frac{Q}{A_2}=\frac{4Q}{\pi D_2^2}=\frac{4\left(10\frac{ft^3}{s}\right)}{\pi\left(\frac{8}{12}ft\right)^2}=28.6\frac{ft}{s}$$

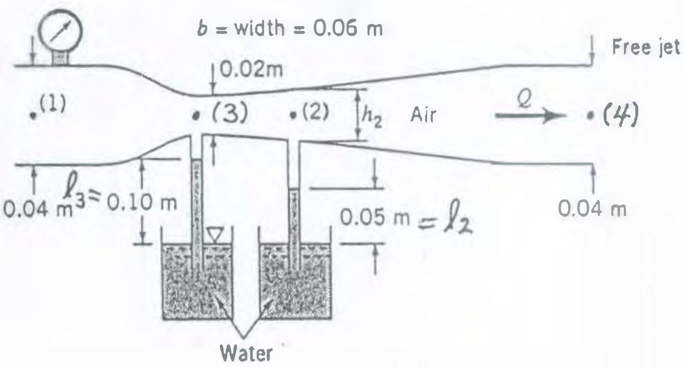
Thus,

$$p_2=-\gamma z_2-\frac{1}{2}\rho V_2^2=-62.4\frac{lb}{ft^3}(6.0\,ft)-\frac{1}{2}\left(1.94\frac{slugs}{ft^3}\right)\left(28.6\frac{ft}{s}\right)^2$$

$$=-1168\frac{lb}{ft^2}=\underline{\underline{-8.11\text{ psi}}}$$

3.67

3.67 Air flows through a Venturi channel of rectangular cross section as shown in **Video V3.10** and Fig. P3.67. The constant width of the channel is 0.06 m and the height at the exit is 0.04 m. Compressibility and viscous effects are negligible. **(a)** Determine the flowrate when water is drawn up 0.10 m in a small tube attached to the static pressure tap at the throat where the channel height is 0.02 m. **(b)** Determine the channel height, h_2, at section (2) where, for the same flowrate as in part **(a)**, the water is drawn up 0.05 m. **(c)** Determine the pressure needed at section (1) to produce this flow.



■ FIGURE P3.67

(a) For steady, inviscid, incompressible flow: $(\gamma = 12.0 \frac{N}{m^3})$

(1) $\frac{p_3}{\gamma} + \frac{V_3^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_3 = -\gamma_{H_2O}\ell_3 = 9.80\times10^3\frac{N}{m^3}(0.1m)$

$= -980 \frac{N}{m^2}$

Also, $A_3V_3 = A_4V_4$ so that $V_3 = \frac{(0.04m \times 0.06m)}{(0.02m \times 0.06m)} V_4 = 2V_4$

Thus, Eqn. (1) becomes

$$\frac{-980\frac{N}{m^2}}{12.0\frac{N}{m^3}} + \frac{4V_4^2}{2(9.81\frac{m}{s^2})} = \frac{V_4^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_4 = 23.1\frac{m}{s}$$

Hence,

$$Q = A_4V_4 = (0.04m \times 0.06m)(23.1\frac{m}{s}) = \underline{\underline{0.0554\frac{m^3}{s}}}$$

(2) (b) $\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_2 = -\gamma_{H_2O}\ell_2 = 9.80\times10^3\frac{N}{m^3}(0.05m)$

$= -490\frac{N}{m^2}$

From part (a), $V_4 = 23.1\frac{m}{s}$

Thus, Eqn. (2) becomes

$$\frac{-490\frac{N}{m^2}}{12.0\frac{N}{m^3}} + \frac{V_2^2}{2(9.81\frac{m}{s^2})} = \frac{(23.1\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_2 = 36.5\frac{m}{s}$$

But $V_2A_2 = V_4A_4$ so that

$$(36.5\frac{m}{s})(0.06m)h_2 = (23.1\frac{m}{s})(0.06m)(0.04m) \quad \text{or} \quad h_2 = \underline{\underline{0.0253\,m}}$$

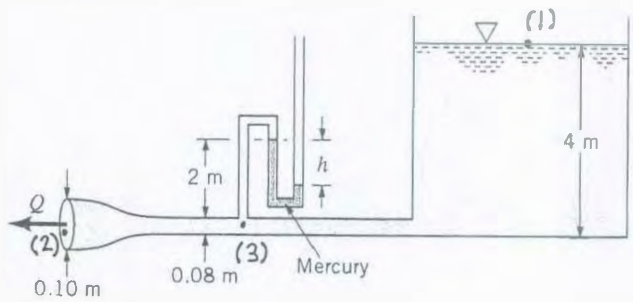
(3) (c) Also, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$ and $A_1V_1 = A_4V_4$

But since $A_1 = (0.04m \times 0.06m) = A_4$ then $V_1 = V_4$ and Eqn. (3) gives

$p_1 = p_4 = \underline{\underline{0}}$

3.68

3.68 Water flows steadily from the large open tank shown in Fig. P3.68. If viscous effects are negligible, determine (a) the flowrate, Q, and (b) the manometer reading, h.



■ FIGURE P3.68

(a) From the Bernoulli equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2, \text{ where } p_1 = p_2 = 0,\ V_1 = 0,\ z_1 = 4\,m, \text{ and } z_2 = 0$$

Thus,

$$\gamma z_1 = \tfrac{1}{2}\rho V_2^2, \text{ or } \rho g z_1 = \tfrac{1}{2}\rho V_2^2 \text{ so that } V_2 = \sqrt{2 g z_1}$$

or

$$V_2 = \sqrt{2(9.81\,m/s^2)(4\,m)} = 8.86\,m/s$$

Hence,

$$Q = A_2 V_2 = \tfrac{\pi}{4}(0.10\,m)^2(8.86\,m/s) = \underline{\underline{0.0696\,m^3/s}}$$

(b) From the Bernoulli equation,

$$p_3 + \tfrac{1}{2}\rho V_3^2 + \gamma z_3 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_3, \text{ where } z_2 = z_3 \text{ and } p_2 = 0$$

so that

$$p_3 = \tfrac{1}{2}\rho(V_2^2 - V_3^2)$$

Also, $A_2 V_2 = A_3 V_3$ so that $V_3 = \frac{A_2}{A_3} V_2 = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{0.1\,m}{0.08\,m}\right)^2 8.86\,m/s = 13.84\,m/s$

Hence,

$$p_3 = \tfrac{1}{2}(999\,kg/m^3)\left[(8.86\,m/s)^2 - (13.84\,m/s)^2\right] = -56{,}500\,N/m^2 \qquad (1)$$

Also, from the manometer,

$$p_3 = -\gamma_{Hg} h + \gamma_{H_2O}(2\,m + (0.08/2)\,m)$$

$$= -(133 \times 10^3\,N/m^3)\,h + (9.80 \times 10^3\,N/m^3)(2.04\,m)$$

$$= -133 \times 10^3 h + 1.99 \times 10^4\ N/m^2, \text{ where } h \sim m \qquad (2)$$

Thus, from Eqs. (1) and (2):

$$-5.65 \times 10^4\,N/m^2 = -133 \times 10^3 h + 1.99 \times 10^4\,N/m^2$$

or

$$h = \underline{\underline{0.574\,m}}$$

3.69

3.69 Water from a faucet fills a 16-oz glass (volume = 28.9 in.³) in 20 s. If the diameter of the jet leaving the faucet is 0.60 in., what is the diameter of the jet when it strikes the water surface in the glass which is positioned 14 in. below the faucet?

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with $p_1 = p_2 = 0$, $z_1 = 14 \text{ in.}$, $z_2 = 0$

Thus,

$$V_2 = \sqrt{2g\left(z_1 + \frac{V_1^2}{2g}\right)} \quad \text{where } V_1 = \frac{Q}{A_1} = \frac{\forall}{A_1 t}$$

or

$$V_1 = \frac{(28.9 \text{ in}^3)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^3}{\frac{\pi}{4}\left(\frac{0.60}{12}\right)^2 \text{ft}^2 (20 \text{ s})} = 0.426 \frac{\text{ft}}{\text{s}}$$

Hence,

$$V_2 = \sqrt{2\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{14}{12} \text{ ft} + \frac{\left(0.426 \frac{\text{ft}}{\text{s}}\right)^2}{2\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)}\right)} = 8.67 \frac{\text{ft}}{\text{s}}$$

But,

$$A_1 V_1 = A_2 V_2 \quad \text{so that} \quad D_1^2 V_1 = D_2^2 V_2$$

or

$$D_2 = \left(\frac{V_1}{V_2}\right)^{\frac{1}{2}} D_1 = \left(\frac{0.426 \frac{\text{ft}}{\text{s}}}{8.76 \frac{\text{ft}}{\text{s}}}\right)^{\frac{1}{2}} (0.60 \text{ in.}) = \underline{\underline{0.132 \text{ in.}}}$$

3.70

3.70 Air flows steadily through a converging–diverging rectangular channel of constant width as shown in Fig. P3.70 and Video V3.10. The height of the channel at the exit and the exit velocity are H_0 and V_0, respectively. The channel is to be shaped so that the distance, d, that water is drawn up into tubes attached to static pressure taps along the channel wall is linear with distance along the channel. That is, $d = (d_{max}/L)\,x$, where L is the channel length and d_{max} is the maximum water depth (at the minimum channel height; $x = L$). Determine the height, $H(x)$, as a function of x and the other important parameters.

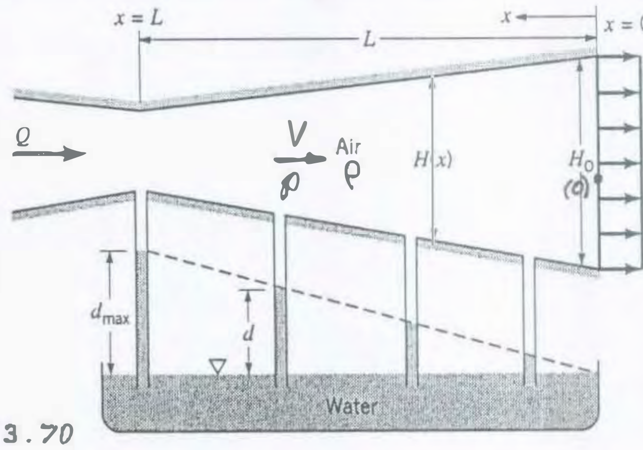


FIGURE P3.70

$$p + z\gamma + \tfrac{1}{2}\rho V^2 = p_0 + z_0\gamma + \tfrac{1}{2}\rho V_0^2 \quad \text{where } \rho = \text{air density}$$

where

$$z = z_0, \quad p_0 = 0, \quad p = -\gamma_{H_2O}\, d = -\gamma_{H_2O}\frac{d_{max}}{L}x$$

Thus,

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho V_0^2$$

But

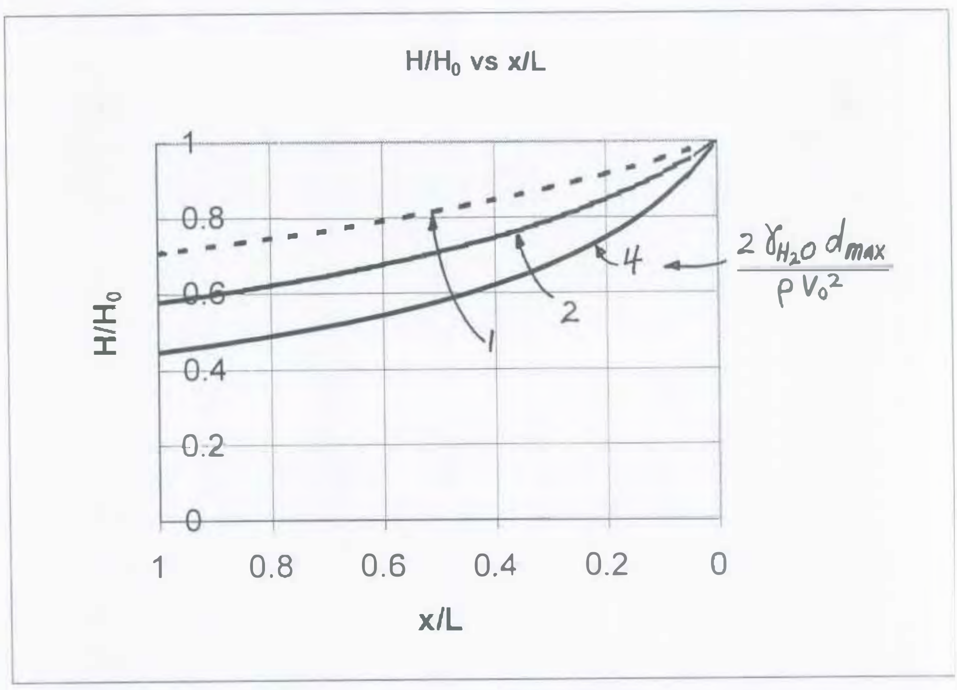
$$AV = A_0V_0, \quad \text{or} \quad V = \frac{A_0}{A}V_0 = \frac{H_0}{H}V_0 \quad \text{so that}$$

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \tfrac{1}{2}\rho\left(\frac{H_0}{H}V_0\right)^2 = \tfrac{1}{2}\rho V_0^2$$

or

$$\frac{H}{H_0} = \frac{1}{\sqrt{1 + \left(\dfrac{2\gamma_{H_2O}\, d_{max}}{\rho V_0^2}\right)\dfrac{x}{L}}}$$

Typical shapes are shown below.



3.71

3.71 The device shown in Fig. P3.71 is used to spray an appropriate mixture of water and insecticide. The flowrate from tank A is to be $Q_A = 0.02$ gal/min when the water flowrate through the hose is $Q = 1$ gal/min. Determine the pressure needed at point (1) and the diameter, D, of the device. For the diameter determined above, plot the ratio of insecticide flowrate to water flowrate as a function of water flowrate, Q, for $0.1 \le Q \le 1$ gal/min. Can this device be used to provide a reasonably constant ratio of insecticide to water regardless of the water flowrate? Explain.

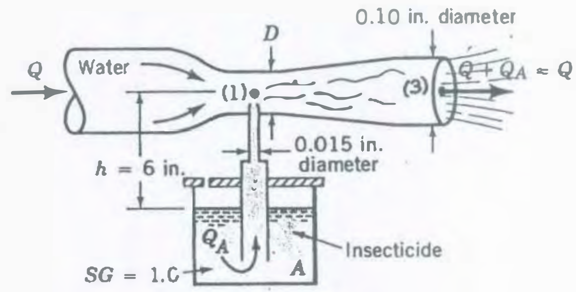
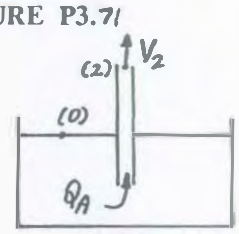


FIGURE P3.71



$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{, where } p_0 = 0,\ V_0 = 0 \qquad (0)$$

$z_0 = 0$, $z_1 = 0.5\,ft$, and $V_2 = \frac{Q_A}{A_2}$ with

$$Q_A = 0.02\,\frac{gal}{min}\left(\frac{231}{1728}\,\frac{ft^3}{gal}\right)\left(\frac{1\,min}{60\,sec}\right) = 4.46 \times 10^{-5}\,\frac{ft^3}{s}$$

Thus,

$$V_2 = \frac{4.46 \times 10^{-5}\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.015}{12}\,ft\right)^2} = 36.3\,\frac{ft}{s}$$

Hence,

$$p_2 = -\tfrac{1}{2}\rho V_2^2 - \gamma z_2 = -\tfrac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left(36.3\,\frac{ft}{s}\right)^2 - \left(62.4\,\frac{lb}{ft^3}\right)(0.5\,ft) = -1310\,\frac{lb}{ft^2}$$

Now assume $p_1 = p_2$ and neglect the kinetic energy of the insecticide compared to that of the water at (1). That is,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \text{, where } z_1 = z_3,\ V_1 = \frac{Q}{A_1}\text{, and } V_3 = \frac{Q}{A_3} \qquad (1)$$

Thus, with

$$Q = 1\,\frac{gal}{min}\left(\frac{231}{1728}\,\frac{ft^3}{gal}\right)\left(\frac{1\,min}{60\,sec}\right) = 2.23 \times 10^{-3}\,\frac{ft^3}{s} \text{ we have}$$

$$V_3 = \frac{2.23 \times 10^{-3}\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.10}{12}\,ft\right)^2} = 40.8\,\frac{ft}{s} \text{ so that Eq. (1) gives}$$

$$\frac{-1310\,\frac{lb}{ft^2}}{62.4\,\frac{lb}{ft^3}} + \frac{V_1^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = \frac{\left(40.8\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} \text{, or } V_1 = 54.9\,\frac{ft}{s}$$

Thus, $\frac{\pi}{4}D^2 V_1 = Q$ or

$$D = \left[\frac{4Q}{\pi V_1}\right]^{\frac{1}{2}} = \left[\frac{4\left(2.23 \times 10^{-3}\,\frac{ft^3}{s}\right)}{\pi\left(54.9\,\frac{ft}{s}\right)}\right]^{\frac{1}{2}} = 7.19 \times 10^{-3}\,ft = \underline{\underline{0.0863\ in.}}$$

With this diameter determine $\frac{Q_A}{Q}$ with $0.1 \le Q \le 1\,\frac{gal}{min}$

(con't)

3.71	(con't)

From Eq. (1):

$$p_1 + \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V_3^2 \quad \text{or with } V_1 = \frac{Q}{A_1} \text{ and } V_3 = \frac{Q}{A_3}$$

$$p_1 = \frac{1}{2}\rho Q^2\left[\frac{1}{A_3^2} - \frac{1}{A_1^2}\right] = \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)Q^2\left[\frac{1}{\left[\frac{\pi}{4}\left(\frac{0.1}{12}ft\right)^2\right]^2} - \frac{1}{\left[\frac{\pi}{4}\left(7.19\times10^{-3}ft\right)^2\right]^2}\right]$$

or

$$p_1 = -2.62\times10^{8}\,Q^2\ \frac{lb}{ft^2}, \text{ where } Q \sim \frac{ft^3}{s} \qquad (2)$$

Also, from Eq. (0) with $p_2 = p_1$

$$0 = \frac{p_1}{\gamma} + \frac{V_2^2}{2g} + z_2, \quad \text{or} \quad p_1 = -\frac{1}{2}\rho V_2^2 - \gamma z_2$$

where

$$V_2 = \frac{Q_A}{A_2} = \frac{Q_A}{\frac{\pi}{4}\left(\frac{0.015}{12}ft\right)^2} = 8.15\times10^{5}\,Q_A\ \frac{ft}{s}, \text{ with } Q_A \sim \frac{ft^3}{s}$$

Thus,

$$p_1 = -\frac{1}{2}\left(1.94\,\frac{slug}{ft^3}\right)\left(8.15\times10^5\,Q_A\,\frac{ft}{s}\right)^2 - \left(62.4\,\frac{lb}{ft^3}\right)(0.5\,ft)$$

or

$$p_1 = -6.44\times10^{11}\,Q_A^2 - 31.2\ \frac{lb}{ft^2}, \text{ where } Q_A \sim \frac{ft^3}{s} \qquad (3)$$

Combine Eqs. (2) and (3) to give

$$2.62\times10^8\,Q^2 = 6.44\times10^{11}\,Q_A^2 + 31.2$$

or

$$\left(\frac{Q_A}{Q}\right)^2 = 4.07\times10^{-4} - \frac{4.84\times10^{-11}}{Q^2}, \text{ where } Q \sim \frac{ft^3}{s}$$

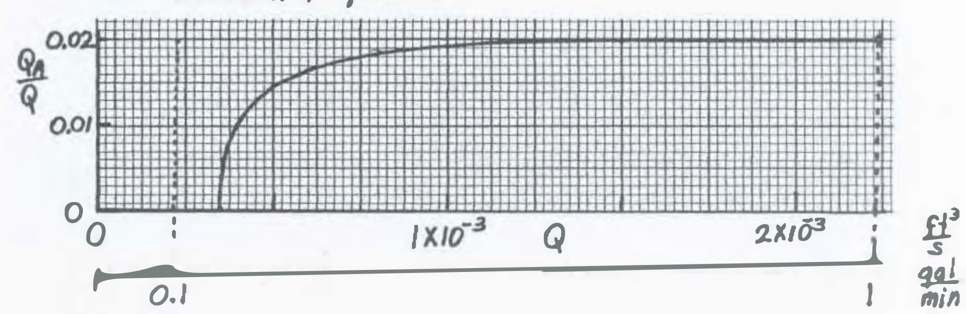
Thus,

$$\frac{Q_A}{Q} = 0.0202\sqrt{1 - \frac{1.19\times10^{-7}}{Q^2}}, \text{ where } Q \sim \frac{ft^3}{s} \qquad (4)$$

Plot Eq. (4) from $Q = 0.1\,\frac{gal}{min} = 2.23\times10^{-4}\,\frac{ft^3}{s}$ to $Q = 1\,\frac{gal}{min} = 2.23\times10^{-3}\,\frac{ft^3}{s}$

Note: $\frac{Q_A}{Q} = 0$ when $Q = (1.19\times10^{-7})^{\frac{1}{2}} = 3.45\times10^{-4}\,\frac{ft^3}{s}$

With $\frac{Q_A}{Q} < 3.45\times10^{-4}\,\frac{ft^3}{s}$, Eq. (4) gives the square root of a negative number — not physically possible. With $Q = 3.45\times10^{-4}$ Eq. (2) gives $p_1 = -31.2\,\frac{lb}{ft^2}$, the minimum needed to draw the insecticide up the 0.5 foot elevation to point (2)



3.72

3.72 If viscous effects are neglected and the tank is large, determine the flowrate from the tank shown in Fig. P3.72

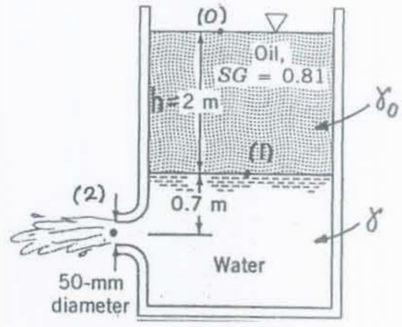


FIGURE P3.72

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = p_0 + \gamma_0 h = \gamma_0 h$$

$z_1 = 0.7\,m$, $z_2 = 0$, and $V_1 = 0$

Thus,

$$\frac{\gamma_0 h}{\gamma} + z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad V_2 = \sqrt{2g\left(\frac{\gamma_0 h}{\gamma} + z_1\right)} \quad \text{where } \frac{\gamma_0}{\gamma} = 0.81$$

and

$$Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2$$

Thus,

$$Q = \frac{\pi}{4}(0.050\,m)^2 \sqrt{2\left(9.81\,\frac{m}{s}\right)\left(0.81(2\,m) + 0.7\,m\right)} = \underline{\underline{0.0132\,\frac{m^3}{s}}}$$

3.73

3.73 Water flows steadily downward in the pipe shown in Fig. 3.73 with negligible losses. Determine the flowrate.

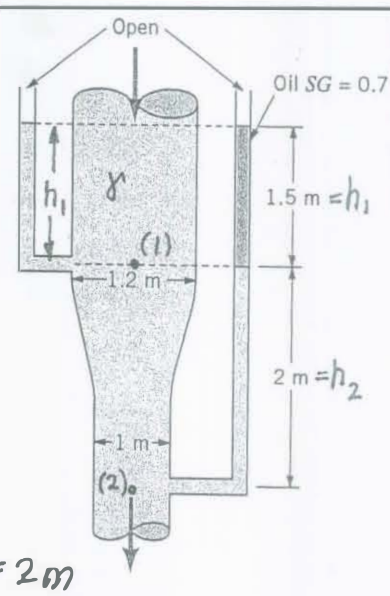


FIGURE P3.73

From the Bernoulli equation,

(1) $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$, where $z_1 - z_2 = 2m$

and

$A_1 V_1 = A_2 V_2$, or $\frac{\pi}{4}(1.2m)^2 V_1 = \frac{\pi}{4}(1m)^2 V_2$

or

(2) $V_1 = 0.694 V_2$

Also, from the manometers,

$p_1 = \gamma h_1$ and $p_2 = \gamma_{oil} h_1 + \gamma h_2$, where $\gamma_{oil} = 0.7\gamma$

Thus,

$p_2 - p_1 = \gamma(0.7h_1 + h_2) - \gamma h_1$

or

(3) $\frac{p_2 - p_1}{\gamma} = h_2 - 0.3h_1 = 2m - 0.3(1.5m) = 1.55m$

Now, from Eq. (1),

$z_1 - z_2 = \frac{p_2 - p_1}{\gamma} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

which, when combined with Eqs. (2) and (3), gives:

$2m = 1.55m + \frac{V_2^2}{2(9.81 m/s^2)}(1 - (0.694)^2)$

or

$V_2 = 4.13 \frac{m}{s}$

Hence,

$Q = A_2 V_2 = \frac{\pi}{4}(1m)^2(4.13 \frac{m}{s}) = \underline{\underline{3.24 \frac{m^3}{s}}}$

3.74

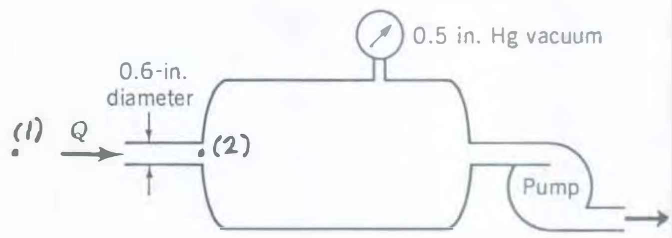


FIGURE P3.74

3.74 Air at 80 °F and 14.7 psia flows into the tank shown in Fig. P3.74. Determine the flowrate in ft³/s, lb/s, and slugs/s. Assume incompressible flow.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad \text{where } z_1 = z_2,\ p_1 = 0,\ V_1 = 0$$

Thus,

$$V_2 = \sqrt{-2g\frac{p_2}{\gamma}} = \sqrt{-2\frac{p_2}{\rho}}$$

where $\rho = \frac{p}{RT} = \frac{\left(14.7\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{\left(1716\frac{ft\cdot lb}{slugs\cdot °R}\right)(460+80)°R} = 2.28\times10^{-3}\frac{slugs}{ft^3}$

Hence, with $p_2 = -\gamma_{Hg}h = -\left(847\frac{lb}{ft^3}\right)\left(\frac{0.5}{12}ft\right) = -35.3\frac{lb}{ft^2}$

$$V_2 = \left[-2\frac{\left(-35.3\frac{lb}{ft^2}\right)}{2.28\times10^{-3}\frac{slugs}{ft^3}}\right]^{1/2} = 176\frac{ft}{s}$$

Thus,

$$Q = A_2V_2 = \frac{\pi}{4}\left(\frac{0.6}{12}ft\right)^2\left(176\frac{ft}{s}\right) = \underline{\underline{0.346\frac{ft^3}{s}}}$$

$$\dot{m} = \rho Q = \left(2.28\times10^{-3}\frac{slugs}{ft^3}\right)\left(0.346\frac{ft^3}{s}\right) = \underline{\underline{7.89\times10^{-4}\frac{slugs}{s}}}$$

and

$$g\dot{m} = \left(32.2\frac{ft}{s^2}\right)\left(7.89\times10^{-4}\frac{slugs}{ft^3}\right) = \underline{\underline{0.0254\frac{lb}{s}}}$$

3.75

3.75 Water flows from a large tank as shown in Fig. P3.75. Atmospheric pressure is 14.5 psia and the vapor pressure is 1.60 psia. If viscous effects are neglected, at what height, h, will cavitation begin? To avoid cavitation, should the value of D_1 be increased or decreased? To avoid cavitation, should the value of D_2 be increased or decreased? Explain.

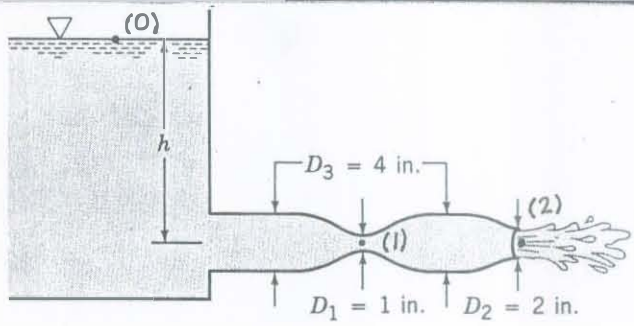


FIGURE P3.75

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1$$ where $p_0=14.5\text{ psia}$, $p_1=1.60\text{ psia}$, $z_0=h$, $z_1=0$, and $V_0=0$

Thus,

$$h=\frac{p_1-p_0}{\gamma}+\frac{V_1^2}{2g} \qquad (1)$$

However,

$$A_1V_1=A_2V_2 \quad \text{or} \quad V_1=\left(\frac{D_2}{D_1}\right)^2 V_2$$

where

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ with $p_0=p_2$ and $z_2=0$

Thus,

$$\frac{V_2^2}{2g}=h$$

so that

$$\frac{V_1^2}{2g}=\frac{\left(\frac{D_2}{D_1}\right)^4 V_2^2}{2g}=\left(\frac{D_2}{D_1}\right)^4 h \qquad (2)$$

Combine Eqs. (1) and (2) to obtain

$$h=\frac{p_1-p_0}{\gamma}+\left(\frac{D_2}{D_1}\right)^4 h$$

or

$$h=\frac{p_0-p_1}{\gamma\left[\left(\frac{D_2}{D_1}\right)^4-1\right]}=\frac{(14.5-1.60)\frac{\text{lb}}{\text{in.}^2}\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)}{62.4\frac{\text{lb}}{\text{ft}^3}\left[\left(\frac{2\text{ in.}}{1\text{ in.}}\right)^4-1\right]}=\underline{\underline{1.98\text{ ft}}} \qquad (3)$$

From Eq. (3) it is seen that h increases in increasing D_1 and decreasing D_2. Thus, to avoid cavitation (i.e. to have h small enough) $\underline{\underline{D_1 \text{ should be increased and } D_2 \text{ decreased.}}}$

3.76

3.76 Water flows into the sink shown in Fig. P3.76 and **Video V5.1** at a rate of 2 gal/min. If the drain is closed, the water will eventually flow through the overflow drain holes rather than over the edge of the sink. How many 0.4-in.-diameter drain holes are needed to ensure that the water does not overflow the sink? Neglect viscous effects.

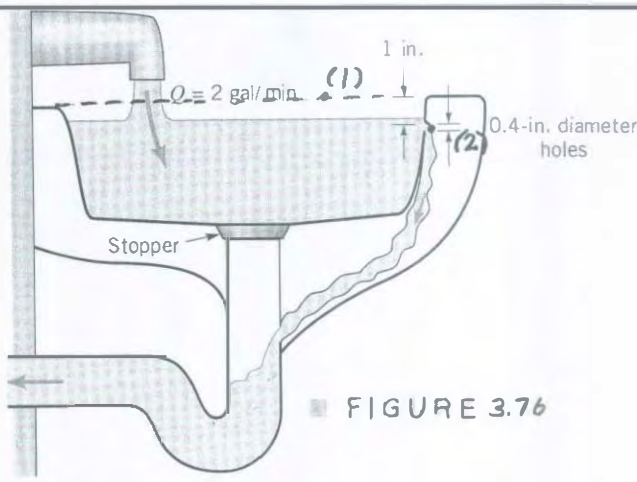


FIGURE 3.76

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{ , where } p_1 = 0,\ V_1 = 0, \text{ and } z_2 = 0,\ p_2 = 0$$

Thus,

$$z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad V_2 = \sqrt{2gz_1} = \left[2\left(32.2\,\frac{ft}{s}\right)\left(\frac{1+0.2}{12}\,ft\right)\right]^{1/2} = 2.54\,\frac{ft}{s}$$

Also,

$$Q = n A_2 V_2 = n\, C_c \frac{\pi}{4} d_2^2 V_2 \text{ , where } n = \text{number of holes required,}$$

$d_2 = 0.4$ in., and C_c = contraction coef. $= 0.61$ (see Fig. 3.14)

Thus, with

$$Q = 2\,\frac{gal}{min}\left(\frac{1\ min}{60\,s}\right)\left(\frac{231\ in.^3}{1\ gal}\right)\left(\frac{1\ ft^3}{1728\ in.^3}\right) = 4.46\times10^{-3}\,\frac{ft^3}{s},$$

$$n = \frac{4Q}{\pi C_c d_2^2 V_2} = \frac{4\,(4.46\times10^{-3}\,ft^3/s)}{\pi(0.61)\left(\frac{0.4}{12}\right)^2 ft^2\,(2.54\,ft/s)} = 3.30$$

Thus, $\underline{\underline{4}}$ holes are needed.

3.77

3.77 What pressure, p_1, is needed to produce a flowrate of $0.09\ ft^3/s$ from the tank shown in Fig. P3.77?

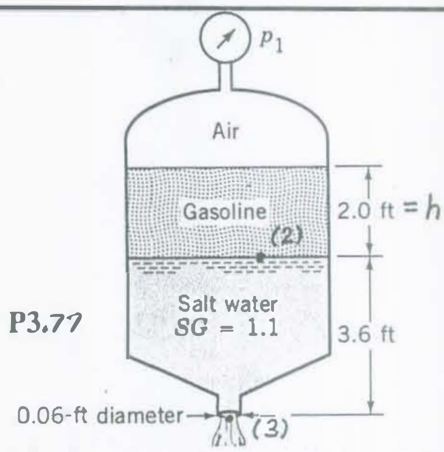


FIGURE P3.77

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$ where $p_2 = p_1 + \gamma_0 h$, $p_3 = 0$, $z_2 = 3.6\ ft$, $z_3 = 0$ and $V_2 = 0$

Thus,

$$\frac{p_1 + \gamma_0 h}{\gamma} + z_2 = \frac{V_3^2}{2g}$$

where $Q = A_3 V_3 = \frac{\pi}{4} D_3^2 V_3$

or

$$V_3 = \frac{4Q}{\pi D_3^2} = \frac{4\left(0.09\ \frac{ft^3}{s}\right)}{\pi (0.06\ ft)^2} = 31.8\ \frac{ft}{s}$$

Thus,

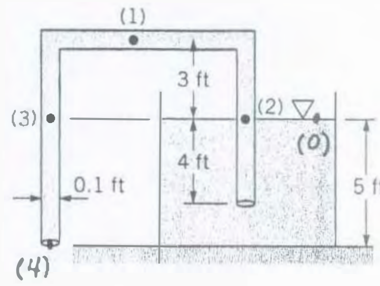
$$p_1 = \gamma\left(\frac{V_3^2}{2g} - z_2\right) - \gamma_0 h = \left(1.1\left(62.4\ \frac{lb}{ft^3}\right)\right)\left[\frac{\left(31.8\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} - 3.6\ ft\right] - 42.5\ \frac{lb}{ft^3}(2.0\ ft)$$

or

$$p_1 = 746\ \frac{lb}{ft^2} = \underline{\underline{5.18\ psi}}$$

3.78

3.78 Water is siphoned from the tank shown in Fig. P3.78. Determine the flowrate from the tank and the pressures at points (1), (2), and (3) if viscous effects are negligible.



■ FIGURE P3.78

From the Bernoulli equation,

$$p_0 + \tfrac{1}{2}\rho V_0^2 + \gamma z_0 = p_4 + \tfrac{1}{2}\rho V_4^2 + \gamma z_4, \text{ where } p_0 = p_4 = 0,\ V_0 = 0,\ z_0 = 5\,ft,$$
and $z_4 = 0$

Thus,

$$\gamma z_0 = \tfrac{1}{2}\rho V_4^2, \text{ or } V_4 = \sqrt{2\gamma z_0/\rho} = \sqrt{2 g z_0} = \sqrt{2(32.2\tfrac{ft}{s^2})(5\,ft)} = 17.94\tfrac{ft}{s}$$

Hence,

$$Q = A_4 V_4 = \tfrac{\pi}{4}(0.1\,ft)^2(17.94\tfrac{ft}{s}) = \underline{\underline{0.141\tfrac{ft^3}{s}}}$$

For p_1: $p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_4 + \tfrac{1}{2}\rho V_4^2 + \gamma z_4$, which with $p_4 = 0$, $z_4 = 0$, $z_1 = 8\,ft$, and $V_1 = V_4$ (since $A_1 = A_4$) becomes

$$p_1 = -\gamma z_1 = -(62.4\,lb/ft^3)(8\,ft) = \underline{\underline{-499\tfrac{lb}{ft^2}}}$$

For p_3: $p_3 + \tfrac{1}{2}\rho V_3^2 + \gamma z_3 = p_4 + \tfrac{1}{2}\rho V_4^2 + \gamma z_4$, which with $p_4 = 0$, $z_4 = 0$, $z_3 = 5\,ft$, and $V_3 = V_4$ (since $A_3 = A_4$) becomes

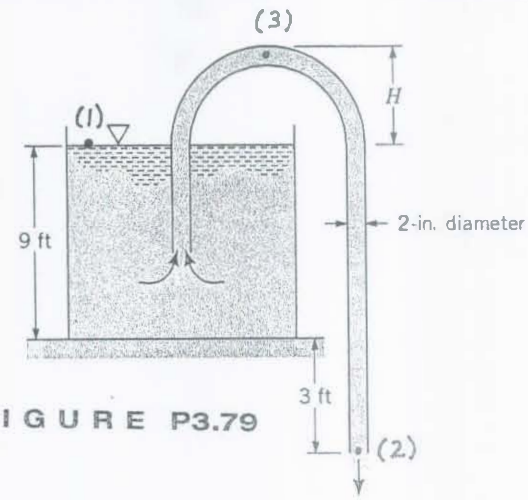
$$p_3 = -\gamma z_3 = -(62.4\,lb/ft^3)(5\,ft) = \underline{\underline{-312\ lb/ft^2}}$$

For p_2: Since $z_2 = z_3$ and $V_2 = V_3$ it follows that

$$p_2 = p_3 = \underline{\underline{-312\ lb/ft^2}}$$

3.79

3.79 Water is siphoned from a large tank and discharges into the atmosphere through a 2-in.-diameter tube as shown in Fig. P3.79. The end of the tube is 3 ft below the tank bottom, and viscous effects are negligible. (a) Determine the volume flowrate from the tank. (b) Determine the maximum height, H, over which the water can be siphoned without cavitation occurring. Atmospheric pressure is 14.7 psia, and the water vapor pressure is 0.26 psia.



■ FIGURE P3.79

(a) From the Bernoulli equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2, \text{ where } p_1=p_2=0 \text{ and } V_1=0.$$

Thus,

$$z_1=\frac{V_2^2}{2g}+z_2$$

or

$$V_2=\sqrt{2g(z_1-z_2)}=\sqrt{(2)(32.2\tfrac{ft}{s^2})(9\,ft+3\,ft)}=27.8\,\frac{ft}{s}$$

Hence,

$$Q=A_2V_2=\frac{\pi}{4}\left(\frac{2}{12}ft\right)^2\left(27.8\frac{ft}{s}\right)=\underline{\underline{0.607\,\frac{ft^3}{s}}}$$

(b) From the Bernoulli equation,

$$\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2, \text{ where } V_2=V_3 \text{ since } Q=A_2V_2=A_3V_3 \text{ and } A_2=A_3$$

Thus, with $z_3-z_2=H+9\,ft+3\,ft=H+12\,ft$,

$$p_3+\gamma(z_3-z_2)=p_2$$

where $p_2=14.7\,psia$ and $p_3=0.26\,psia$

Hence,

$$\left(62.4\frac{lb}{ft^3}\right)(H+12\,ft)=(14.7-0.26)\frac{lb}{in.^2}\ \frac{144\,in.^2}{ft^2}$$

or

$$H=\underline{\underline{21.3\,ft}}$$

3.80

3.80 Determine the manometer reading, h, for the flow shown in Fig. P3.80

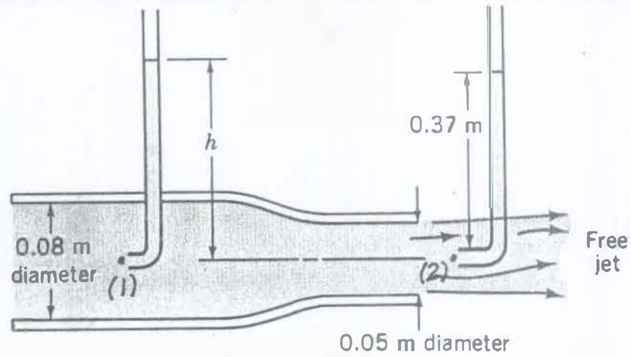


FIGURE P3.80

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $z_1 = z_2$, $V_1 = 0$, and $V_2 = 0$

Thus,

$$p_1 = p_2$$

However, $p_1 = \gamma h$ and $p_2 = \gamma (0.37 m)$
so that

$$h = \underline{\underline{0.37\ m}}$$

3.81

3.81 Air flows steadily through the variable area pipe shown in Fig. P3.81. Determine the flowrate if viscous and compressibility effects are negligible.

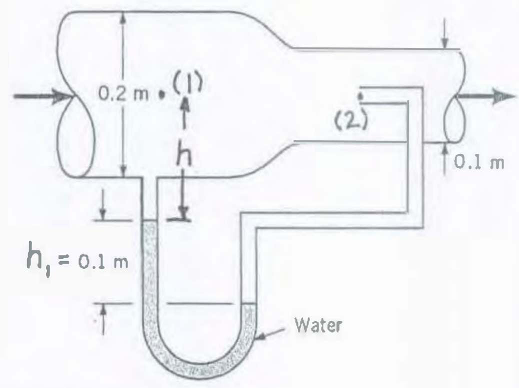


FIGURE P3.81

From the Bernoulli equation,

(1) $\frac{p_1}{\gamma_{air}} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_{air}} + \frac{V_2^2}{2g} + z_2$, where $z_1 = z_2$ and $V_2 = 0$

and

(2) $Q = A_1 V_1$

Also, from the manometer

(3) $p_1 + \gamma_{air} h + \gamma_{H_2O} h_1 = p_2 + \gamma_{air}(h + h_1)$

But $\gamma_{H_2O} >> \gamma_{air}$ so that Eq.(3) becomes

$p_2 = p_1 + \gamma_{H_2O} h_1$ or $\frac{p_2}{\gamma_{air}} = \frac{p_1}{\gamma_{air}} + \frac{\gamma_{H_2O}}{\gamma_{air}} h_1$

Hence, from Eq. (1):

$$\frac{p_1}{\gamma_{air}} + \frac{V_1^2}{2g} = \frac{p_1}{\gamma_{air}} + \left(\frac{\gamma_{H_2O}}{\gamma_{air}}\right) h_1$$

or

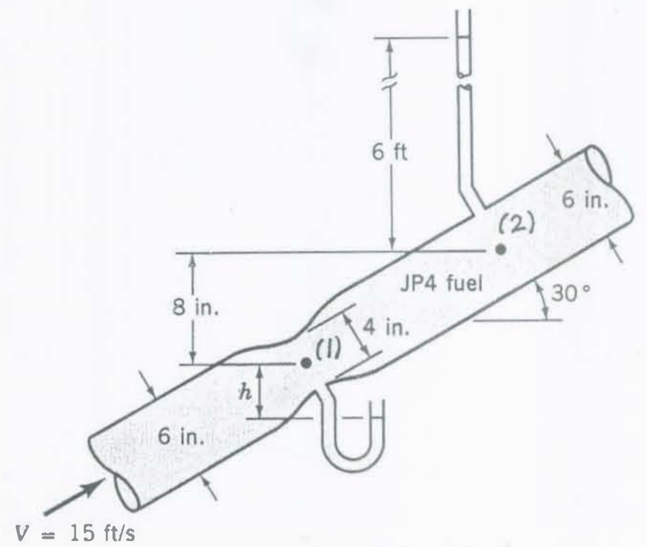
$$V_1 = \sqrt{2g\left(\frac{\gamma_{H_2O}}{\gamma_{air}}\right)h} = \sqrt{2\left(9.81\,\frac{m}{s^2}\right)\left(\frac{9.80\times10^3\,\frac{N}{m^3}}{12.0\,\frac{N}{m^3}}\right)(0.1m)} = 40.0\,\frac{m}{s}$$

Thus, from Eq. (2),

$$Q = \frac{\pi}{4}(0.2m)^2\left(40.0\,\frac{m}{s}\right) = \underline{\underline{1.26\,\frac{m^3}{s}}}$$

3.82

3.82 JP-4 fuel ($SG = 0.77$) flows through the Venturi meter shown in Fig. P3.82 with a velocity of 15 ft/s in the 6-in. pipe. If viscous effects are negligible, determine the elevation, h, of the fuel in the open tube connected to the throat of the Venturi meter.



■ FIGURE P3.82

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = 0,\ z_2 = \frac{8}{12}\text{ ft}, \quad (1)$$

and $V_2 = 15$ ft/s

Also, $A_1V_1 = A_2V_2$

or

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{6\text{ in.}}{4\text{ in.}}\right)^2\left(15\,\frac{\text{ft}}{\text{s}}\right) = 33.75\,\frac{\text{ft}}{\text{s}}$$

Thus, with $\frac{p_2}{\gamma} = 6$ ft Eq. (1) becomes

$$\frac{p_1}{\gamma} + \frac{\left(33.75\,\frac{\text{ft}}{\text{s}}\right)^2}{2\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)} = 6\text{ ft} + \frac{\left(15\,\frac{\text{ft}}{\text{s}}\right)^2}{2\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)} + \frac{8}{12}\text{ ft}$$

or

$$\frac{p_1}{\gamma} = -7.53\text{ ft}$$

But $\frac{p_1}{\gamma} = -h$ so that $h = \underline{\underline{7.53\text{ ft}}}$

3.83

3.83 Repeat Problem 3.82 if the flowing fluid is water rather than JP-4 fuel.

Note from the solution to Problem 3.82 that the value of γ is not needed. Thus, $\underline{\underline{h = 7.53 \text{ ft for either water or JP-4 fuel.}}}$

3.84

3.84 Oil flows through the system shown in Fig. P3.84 with negligible losses. Determine the flowrate.

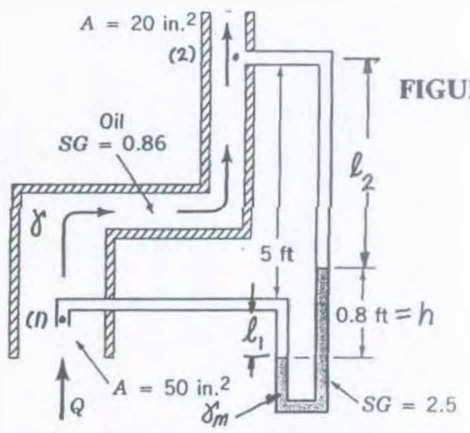


FIGURE P3.84

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ where $z_1=0$, $z_2=5\text{ ft}$, and $V_1=0$

Also, $V_2=\frac{Q}{A_2}$

Thus,

$$\frac{p_1-p_2}{\gamma}=z_2+\frac{V_2^2}{2g} \qquad (1)$$

where $p_1+\gamma\ell_1=p_2+\gamma\ell_2+\gamma_m h$

or

$$\frac{p_1-p_2}{\gamma}=\ell_2-\ell_1+\frac{\gamma_m}{\gamma}h$$

with $\ell_2-\ell_1=5\text{ ft}-h$

Thus, the manometer equation gives

$$\frac{p_1-p_2}{\gamma}=5\text{ ft}+\left(\frac{\gamma_m}{\gamma}-1\right)h \qquad (2)$$

Combine Eqs. (1) and (2), using $z_2=5\text{ ft}$, to obtain

$$\frac{V_2^2}{2g}=\left(\frac{\gamma_m}{\gamma}-1\right)h=\left(\frac{SG_m}{SG}-1\right)h$$

or

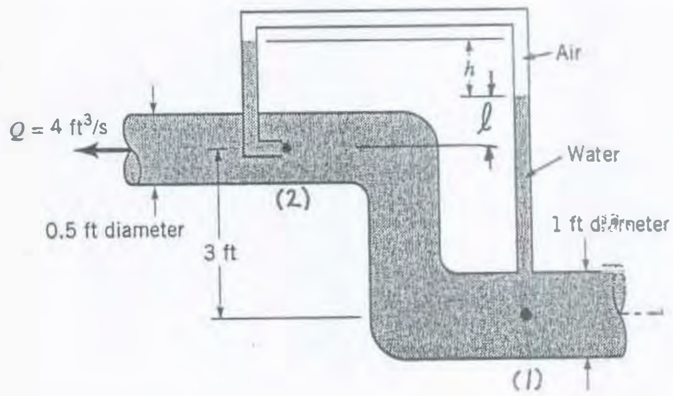
$$V_2=\sqrt{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{2.5}{0.86}-1\right)(0.8\text{ ft})}=9.91\frac{\text{ft}}{\text{s}}$$

Thus,

$$Q=A_2V_2=\left(20\text{ in.}^2\times\frac{1\text{ ft}^2}{144\text{ in.}^2}\right)\left(9.91\frac{\text{ft}}{\text{s}}\right)=\underline{\underline{1.38\frac{\text{ft}^3}{\text{s}}}}$$

3.85

3.85 Water, considered an inviscid, incompressible fluid, flows steadily as shown in Fig. P3.85 Determine h.



■ FIGURE P3.85

$$p_1 + \gamma z_1 + \frac{1}{2}\rho V_1^2 = p_2 + \gamma z_2 + \frac{1}{2}\rho V_2^2$$

where $z_1 = 0$, $z_2 = 3\ ft$, $V_2 = 0$, and $V_1 = \frac{Q}{A_1} = \dfrac{4\ \frac{ft^3}{s}}{\frac{\pi}{4}(1\ ft)^2} = 5.09\ \frac{ft}{s}$

Thus,

$$p_1 + \frac{1}{2}\left(1.94\ \frac{slugs}{ft^3}\right)\left(5.09\ \frac{ft}{s}\right)^2 = p_2 + 62.4\ \frac{lb}{ft^3}(3\ ft)$$

or

$$p_1 - p_2 = 162\ \frac{lb}{ft^2} \qquad (1)$$

But from the manometer,

$$p_1 - \gamma(\ell + 3\ ft) + \gamma(h + \ell) = p_2$$

or

$$p_1 - 62.4\ \frac{lb}{ft^3}(3\ ft) + 62.4\ \frac{lb}{ft^3}\, h = p_2$$

Hence,

$p_1 = p_2 + 187 - 62.4h$ which when combined with Eq. (1) gives

$$p_2 + 187 - 62.4h - p_2 = 162$$

or

$$h = \underline{\underline{0.400\ ft}}$$

3.86

3.86 Determine the flowrate through the submerged orifice shown in Fig. P3.86 if the contraction coefficient is $C_c = 0.63$.

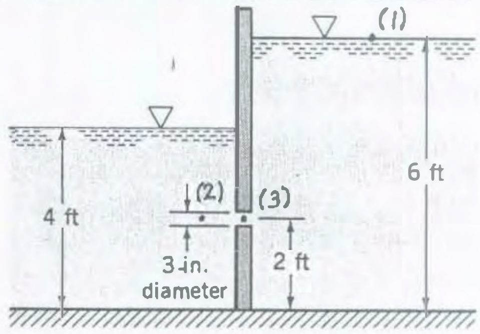


FIGURE P3.86

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = 0$, $V_1 = 0$, $z_1 = 4\,ft$, $z_2 = 0$, and $\frac{p_2}{\gamma} = 2\,ft$

Thus,

$$4\,ft = 2\,ft + \frac{V_2^2}{2(32.2\,\frac{ft}{s})}$$

or

$$V_2 = 11.34\,\frac{ft}{s}$$

so that

$$Q = A_2 V_2 = C_c A_3 V_2 = (0.63)\frac{\pi}{4}\left(\frac{3}{12}\,ft\right)^2\left(11.34\,\frac{ft}{s}\right) = \underline{\underline{0.351\,\frac{ft^3}{s}}}$$

3.87

3.87 An inexpensive timer is to be made from a funnel as indicated in Fig. P3.87. The funnel is filled to the top with water and the plug is removed at time $t = 0$ to allow the water to run out. Marks are to be placed on the wall of the funnel indicating the time in 15-s intervals, from 0 to 3 min (at which time the funnel becomes empty). If the funnel outlet has a diameter of $d = 0.1$ in., draw to scale the funnel with the timing marks for funnels with angles of $\theta = 30$, 45, and 60°. Repeat the problem if the diameter is changed to 0.05 in.

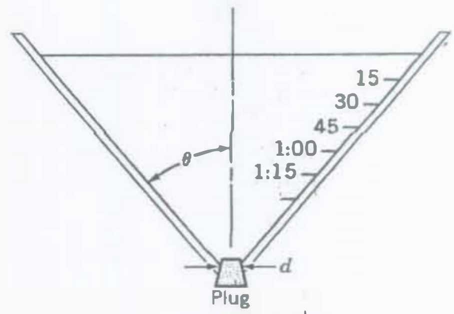
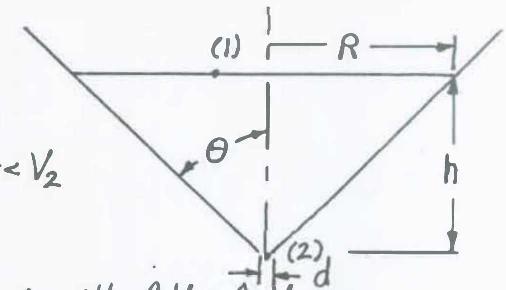


FIGURE P3.87



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 0$,
$z_2 = 0$, and $V_1 = -\frac{dh}{dt} << V_2$
if $R >> \frac{d}{2}$

Thus,

$V_2 = \sqrt{2gh}$ which when combined with $A_1 V_1 = A_2 V_2$ gives

$$-A_1 \frac{dh}{dt} = A_2 \sqrt{2gh} \quad \text{or} \quad -\pi R^2 \frac{dh}{dt} = \frac{\pi}{4} d^2 \sqrt{2gh} \qquad (1)$$

where $R = h \tan\theta$

Thus, Eq. (1) becomes $\quad -h^2 \tan^2\theta \frac{dh}{dt} = \frac{d^2}{4}\sqrt{2gh}$

or

$$h^{3/2} dh = \frac{-d^2\sqrt{2g}}{4\tan^2\theta}\, dt$$ which can be integrated from $h = h_0$ at $t = 0$ as

$$\int_{h_0}^{h} h^{3/2} dh = -\frac{d^2\sqrt{2g}}{4\tan^2\theta} \int_0^t dt \quad \text{or} \quad \frac{2}{5}\left[h^{5/2} - h_0^{5/2}\right] = -\frac{d^2\sqrt{2g}}{4\tan^2\theta}\, t$$

Thus,

$$h = \left[h_0^{5/2} - \frac{5 d^2 \sqrt{2g}\, t}{8\tan^2\theta}\right]^{2/5} \qquad (2)$$

Since $h = 0$ when $t = 3\text{min} = 180\text{s}$ it follows that,

$$h_0^{5/2} = \frac{5 d^2 \sqrt{2(32.2 \frac{ft}{s^2})}\,(180 s)}{8 \tan^2\theta}$$

which when combined with Eq. (2) gives

$$h = \left[\frac{5 d^2 \sqrt{2(32.2 \frac{ft}{s^2})}\,(180 s)}{8\tan^2\theta}\right]^{2/5} \left(1 - \frac{t}{180}\right)^{2/5}$$

or

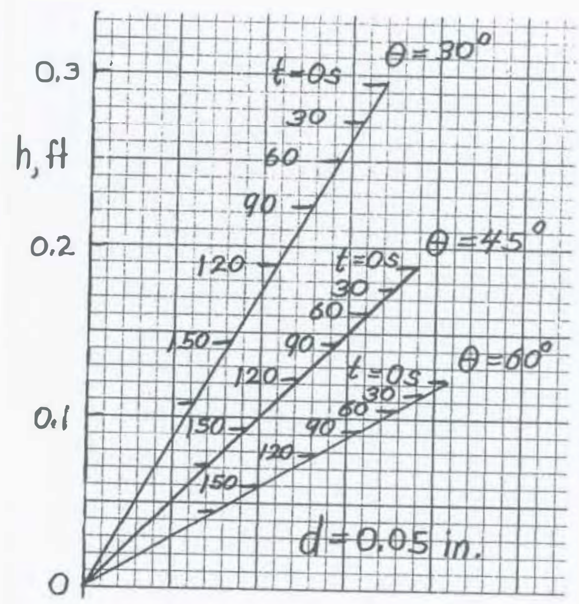
$$h = 15.2\left(\frac{d}{\tan\theta}\right)^{4/5}\left(1 - \frac{t}{180}\right)^{2/5}$$ where $h \sim$ ft, $d \sim$ ft, and $t \sim$ s

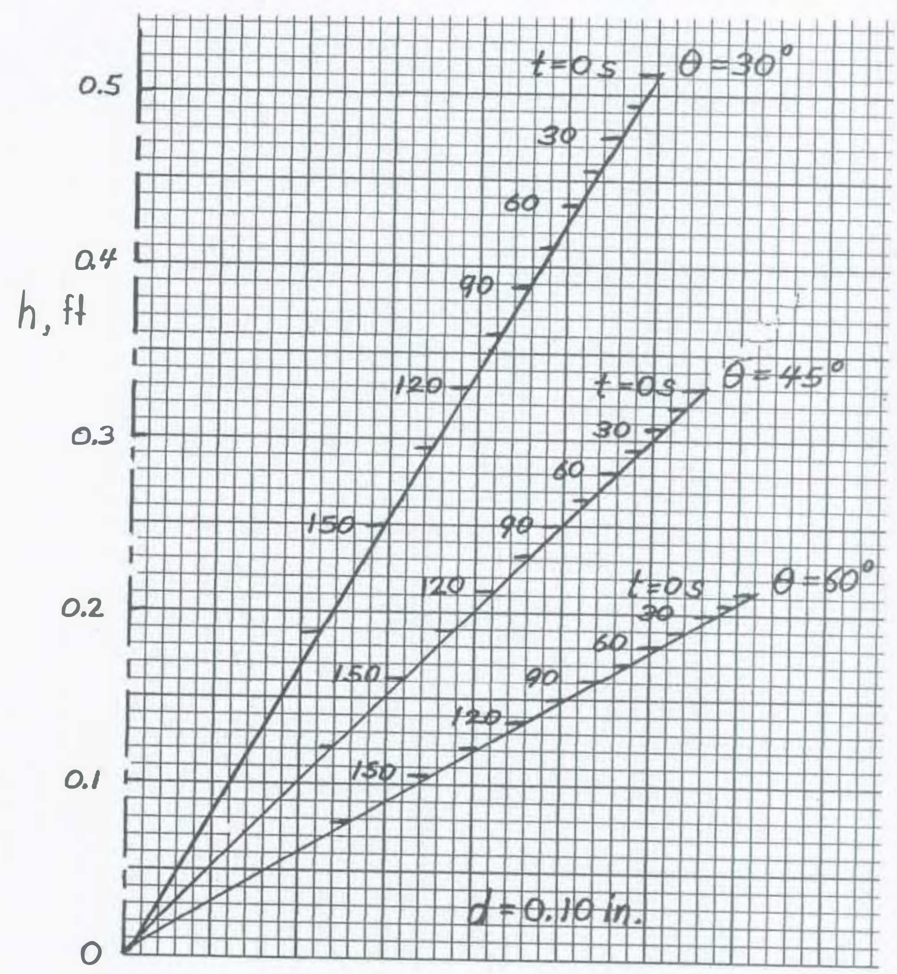
For $t = 0, 15, 30, \ldots, 180$ s calculate h from Eq. (3) with $\theta = 30, 45$, and $60°$ and $d = 0.1$ and 0.05 in. The calculated data for $d = 0.05$ in. and $\theta = 30$ deg. are shown in the table below. Other data are graphed. (con't)

3.87 (con't)

For d = 0.0500 in and theta = 30.00 deg

t, s	h, ft
0.00	+2.941E-01
15.00	+2.841E-01
30.00	+2.734E-01
45.00	+2.621E-01
60.00	+2.501E-01
75.00	+2.371E-01
90.00	+2.229E-01
105.00	+2.072E-01
120.00	+1.895E-01
135.00	+1.689E-01
150.00	+1.436E-01
165.00	+1.089E-01
180.00	+0.000E+00





3.88

3.88 A long water trough of triangular cross section is formed from two planks as is shown in Fig. P3.88 A gap of 0.1 in. remains at the junction of the two planks. If the water depth initially was 2 ft, how long a time does it take for the water depth to reduce to 1 ft.?

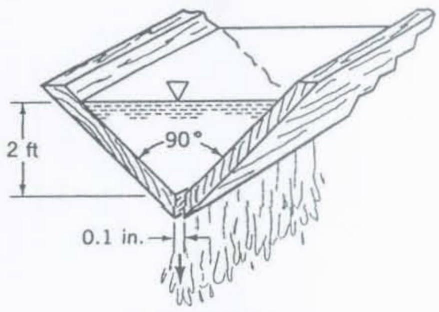
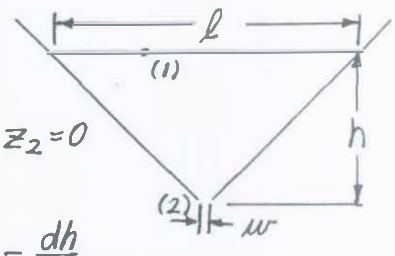


FIGURE P3.88



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad (1)$$

where $p_1=0$, $p_2=0$, $z_1=h$, and $z_2=0$

Also $V_1A_1=V_2A_2$ or since $\ell \gg w$ it follows that $V_1 \ll V_2$, where $V_1=-\frac{dh}{dt}$

Thus, Eq.(1) gives

$V_2=\sqrt{2gh}$ so that

$-A_1\frac{dh}{dt}=A_2\sqrt{2gh}$ with $A_1=b\ell=2bh$ and $A_2=bw$ where b is the tank length.

Thus,

$$-2bh\frac{dh}{dt}=bw\sqrt{2gh}$$

or

$\sqrt{h}\,dh=-w\sqrt{\frac{g}{2}}\,dt$ which can be integrated to give

$$\int_{h_i=2}^{h_f=1} h^{\frac{1}{2}}\,dh=-w\sqrt{\frac{g}{2}}\int_{t_i=0}^{t_f} dt$$

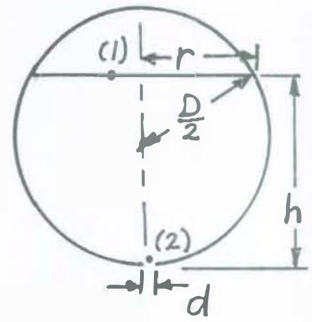
or

$$t_f=\frac{2}{3w}\sqrt{\frac{2}{g}}\left[h_i^{3/2}-h_f^{3/2}\right]=\frac{2}{3\left(\frac{0.1}{12}\right)ft}\sqrt{\frac{2}{32.2\,\frac{ft}{s^2}}}\left[2^{3/2}-1^{3/2}\right]ft^{3/2}$$

$$=\underline{\underline{36.5\ s}}$$

*3.89

*3.89 A spherical tank of diameter D has a drain hole of diameter d at its bottom. A vent at the top of the tank maintains atmospheric pressure within the tank. The flow is quasisteady and inviscid and the tank is full of water initially. Determine the water depth as a function of time, $h = h(t)$, and plot graphs of $h(t)$ for tank diameters of 1, 5, 10, and 20 ft if $d = 1$ in.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = h$, $z_2 = 0$ and $V_1 = -\frac{dh}{dt} << V_2$ if $r >> d$

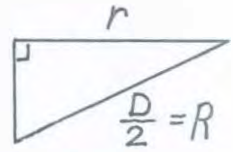
Thus,

$V_2 = \sqrt{2gh}$ which when combined with $A_1V_1 = A_2V_2$ gives

$$-A_1\frac{dh}{dt} = A_2\sqrt{2gh} \quad \text{or} \quad -\pi r^2\frac{dh}{dt} = \frac{\pi}{4}d^2\sqrt{2gh} \qquad (1)$$

where $R^2 = r^2 + (h-R)^2$

with $R = \frac{D}{2}$ = radius of tank



Thus, $r = \sqrt{R^2-(h-R)^2}$ so that Eq. (1) becomes

$$-[R^2-(h-R)^2]\frac{dh}{dt} = \frac{d^2}{4}\sqrt{2gh}$$

or

$$(h^{3/2} - 2Rh^{1/2})\,dh = \frac{d^2\sqrt{2g}}{4}\,dt$$

which can be integrated from the initial time and depth ($t=0$, $h=2R$) to an arbitrary time and depth (t,h) as

$$\int_{2R}^{h}(h^{3/2} - 2Rh^{1/2})\,dh = \frac{d^2\sqrt{2g}}{4}\int_0^t dt$$

or

$$\frac{2}{5}\left(h^{5/2} - (2R)^{5/2}\right) - \frac{4}{3}R\left(h^{3/2} - (2R)^{3/2}\right) = \frac{d^2\sqrt{2g}}{4}\,t \qquad (2)$$

Use $d = \frac{1}{12}$ ft and $g = 32.2\ \frac{ft}{s^2}$ and plot $h = h(t)$ for values of $R = 0.5$, 2.5, 5, and 10 ft

Note: It is easier to solve Eq. (2) as $t = t(h)$ rather than $h = h(t)$

Note: The time taken to empty the tank, t_e, is obtained from Eq. (2) with $h = 0$ as

$$t_e = \frac{64\, R^{5/2}}{15\, d^2\sqrt{g}}$$

(con't)

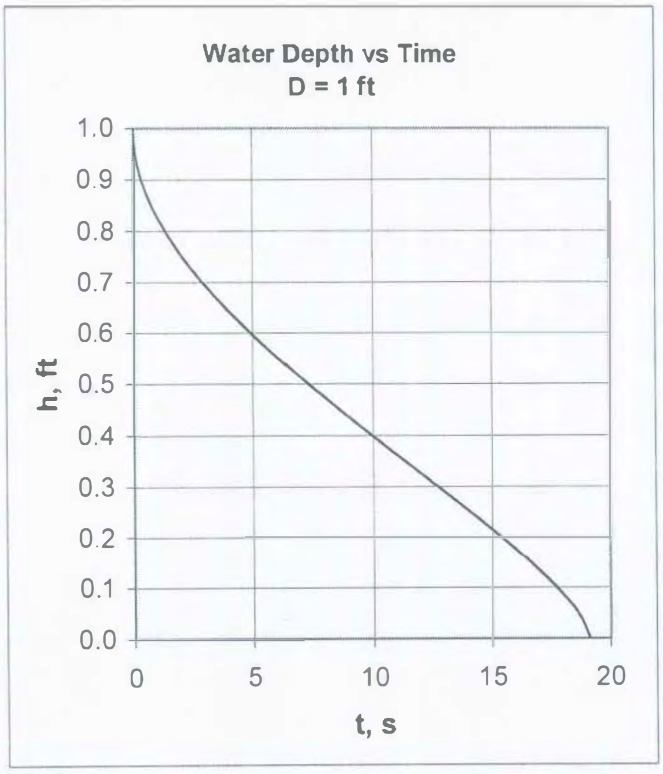
*3.89 (con't.)

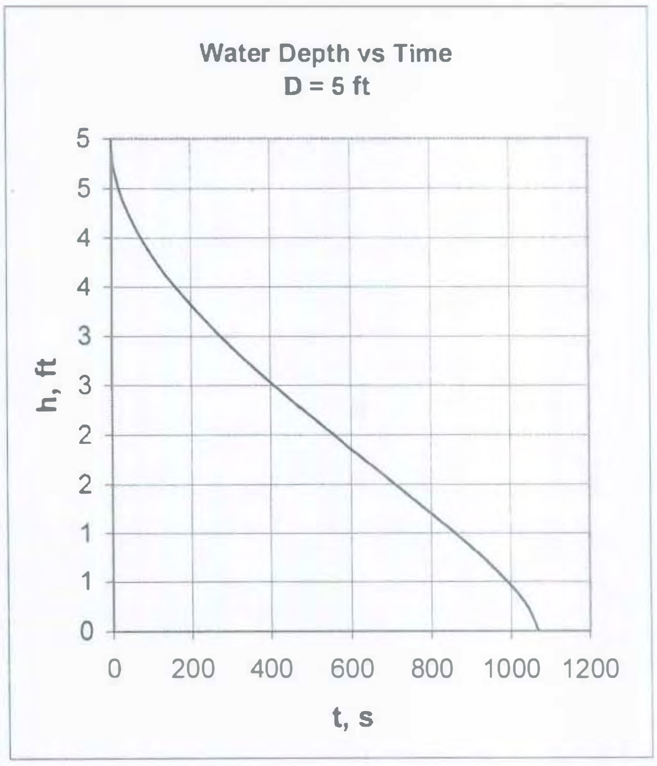
Results of an EXCEL Program to calculate h(t) from Eqn. (2):

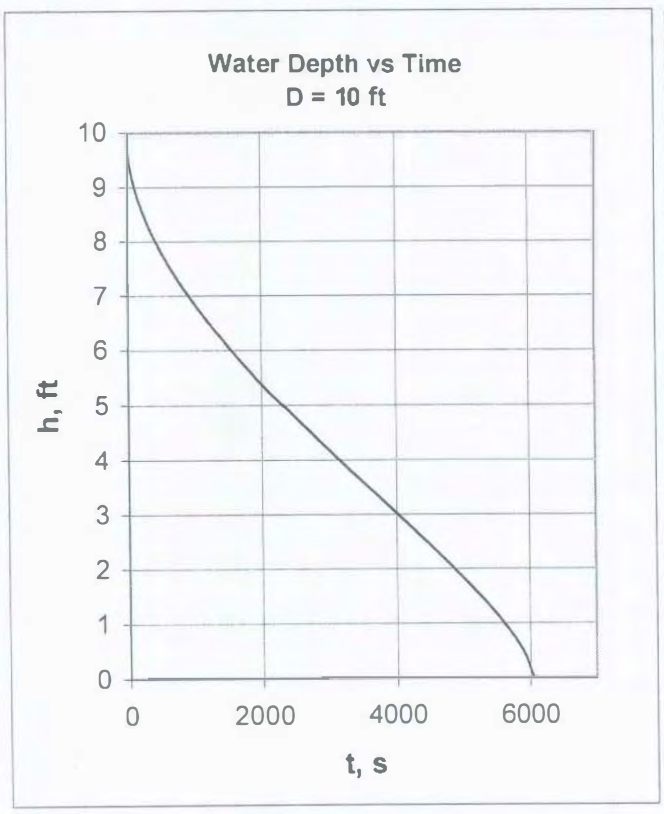
D = 1 ft		D = 5 ft		D = 10 ft		D = 20 ft	
t, s	h, ft	t, s	h, ft	t, s	h, ft	t, s	h, ft
0.00	1.000	0	5.000	0	10.00	0	20
0.09	0.950	5	4.750	28	9.50	158	19
0.35	0.900	19	4.500	110	9.00	620	18
0.77	0.850	43	4.250	242	8.50	1370	17
1.34	0.800	75	4.000	422	8.00	2390	16
2.05	0.750	114	3.750	647	7.50	3661	15
2.89	0.700	161	3.500	913	7.00	5163	14
3.84	0.650	215	3.250	1216	6.50	6876	13
4.91	0.600	274	3.000	1552	6.00	8778	12
6.06	0.550	339	2.750	1917	5.50	10846	11
7.30	0.500	408	2.500	2308	5.00	13055	10
8.60	0.450	481	2.250	2718	4.50	15376	9
9.94	0.400	556	2.000	3143	4.00	17782	8
11.31	0.350	632	1.750	3577	3.50	20237	7
12.69	0.300	710	1.500	4014	3.00	22706	6
14.06	0.250	786	1.250	4445	2.50	25144	5
15.37	0.200	859	1.000	4862	2.00	27502	4
16.61	0.150	929	0.750	5253	1.50	29714	3
17.72	0.100	990	0.500	5603	1.00	31695	2
18.62	0.050	1041	0.250	5889	0.50	33311	1
19.14	0.000	1070	0.000	6053	0.00	34239	0

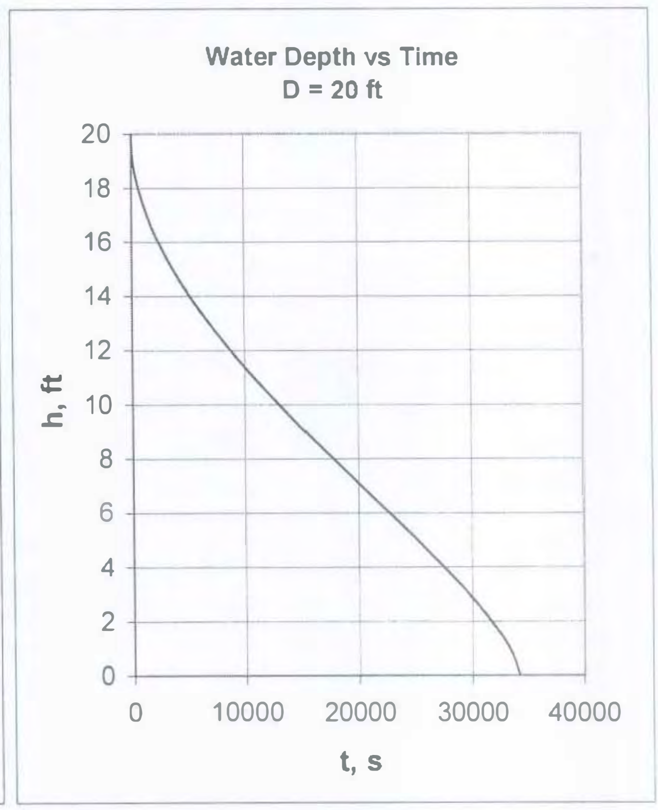
See next page for graphs of above results.

*3.89 (con't)


Water Depth vs Time
D = 1 ft
h, ft
t, s
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20

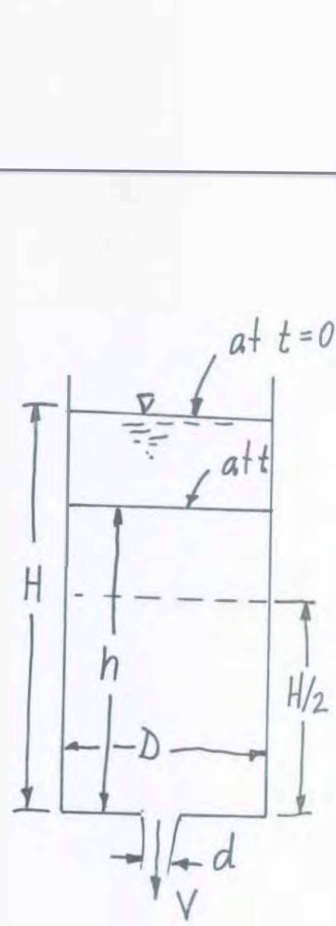

Water Depth vs Time
D = 5 ft
h, ft
t, s
5
5
4
4
3
3
2
2
1
1
0
0
200
400
600
800
1000
1200


Water Depth vs Time
D = 10 ft
h, ft
t, s
10
9
8
7
6
5
4
3
2
1
0
0
2000
4000
6000


Water Depth vs Time
D = 20 ft
h, ft
t, s
20
18
16
14
12
10
8
6
4
2
0
0
10000
20000
30000
40000

3.90

3.90 When the drain plug is pulled, water flows from a hole in the bottom of a large, open cylindrical tank. Show that if viscous effects are negligible and if the flow is assumed to be quasisteady, then it takes 3.41 times longer to empty the entire tank than it does to empty the first half of the tank. Explain why this is so.



$$Q = AV = \frac{\pi}{4} d^2 V = A_{tank}\left(-\frac{dh}{dt}\right)$$

where

$$V = \sqrt{2gh} \quad \text{and} \quad A_{tank} = \frac{\pi}{4} D^2$$

Thus,

$$d^2 \sqrt{2gh} = D^2\left(-\frac{dh}{dt}\right)$$

or

$$\frac{dh}{\sqrt{h}} = -\sqrt{2g}\left(\frac{d}{D}\right)^2 dt$$

Integrate from $h = H$ at $t = 0$ to h at t:

$$\int_H^h \frac{dh}{\sqrt{h}} = -\sqrt{2g}\left(\frac{d}{D}\right)^2 \int_0^t dt$$

or

$$2\sqrt{h}\Big|_H^h = -\sqrt{2g}\left(\frac{d}{D}\right)^2 t$$

or

$$t = \frac{2}{\sqrt{2g}}\left(\frac{D}{d}\right)^2\left[\sqrt{H} - \sqrt{h}\right]$$

Thus, to empty the tank,

$$t\Big|_{h=0} = \frac{2}{\sqrt{2g}}\left(\frac{D}{d}\right)^2 \sqrt{H}$$

and to half empty the tank,

$$t\Big|_{h=H/2} = \frac{2}{\sqrt{2g}}\left(\frac{D}{d}\right)^2\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right] = \frac{2}{\sqrt{2g}}\left(\frac{D}{d}\right)^2 \sqrt{H}\left[1 - \frac{1}{\sqrt{2}}\right]$$

Thus,

$$\frac{t|_0}{t|_{H/2}} = \frac{\frac{2}{\sqrt{2g}}\left(\frac{D}{d}\right)^2\sqrt{H}}{\frac{2}{\sqrt{2g}}\left(\frac{D}{d}\right)^2\sqrt{H}\left[1 - \frac{1}{\sqrt{2}}\right]} = \frac{1}{\left[1 - \frac{1}{\sqrt{2}}\right]} = \underline{\underline{3.41}}$$

*3.91

*3.91 The surface area, A, of the pond shown in Fig. P3.91 varies with the water depth, h, as shown in the table. At time $t = 0$ a valve is opened and the pond is allowed to drain through a pipe of diameter D. If viscous effects are negligible and quasisteady conditions are assumed, plot the water depth as a function of time from when the valve is opened ($t = 0$) until the pond is drained for pipe diameters of $D = 0.5, 1.0, 1.5, 2.0, 2.5$, and 3.0 ft. Assume $h = 18$ ft at $t = 0$.

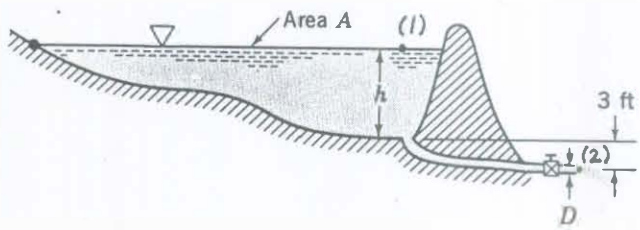


FIGURE P3.91

h (ft)	A [acres (1 acre = 43,560 ft^2)]
0	0
2	0.3
4	0.5
6	0.8
8	0.9
10	1.1
12	1.5
14	1.8
16	2.4
18	2.8

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $p_1 = 0$, $p_2 = 0$, $z_1 = h$, $z_2 = -3\,ft$
and $V_1 = -\frac{dh}{dt} \ll V_2$

Thus, $V_2 = \sqrt{2g(h+3)}$ which when combined with $A_1 V_1 = A_2 V_2$ gives

$-A_1 \frac{dh}{dt} = \frac{\pi}{4} D^2 \sqrt{2g(h+3)}$ where $A_1 = A_1(h)$ as given.

This can be rearranged and integrated to give

$$\int_{18\,ft}^{h} A_1 \frac{dh}{\sqrt{h+3}} = -\frac{\pi}{4}\sqrt{2g}\,D^2 \int_0^t dt = -\frac{\pi}{4} D^2 \sqrt{2g}\; t = -\frac{\pi}{4} D^2 \sqrt{2 \times 32.2}\; t$$

or

$$t = \frac{0.159}{D^2} \int_h^{18} A_1 \frac{dh}{\sqrt{h+3}} \text{, where } t \sim s,\ A_1 \sim ft^2, \text{ and } h \sim ft \qquad (1)$$

Note: It is easier to determine t as a function of h rather than h as a function of t

Note: $t \sim D^{-2}$

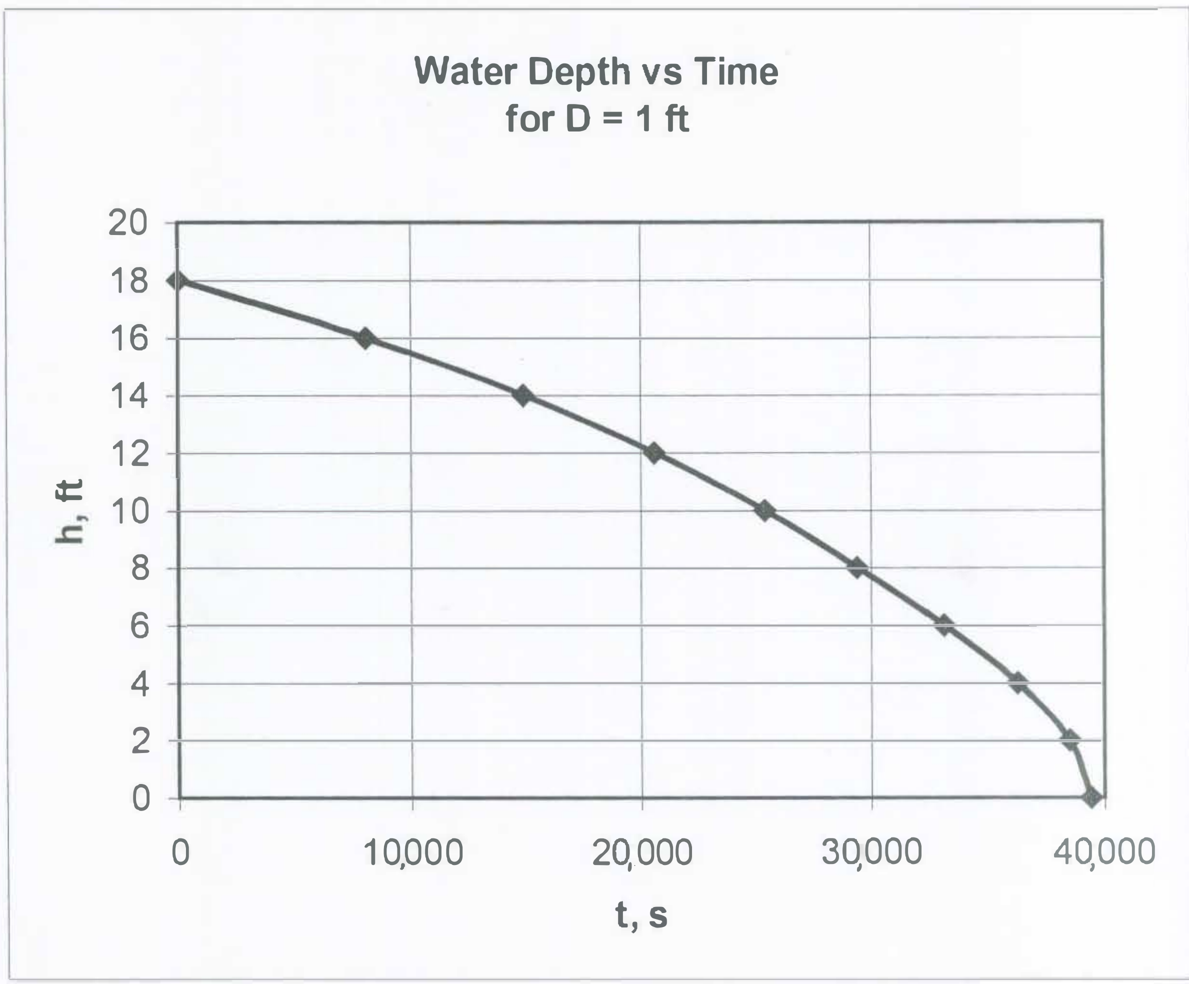
(con't)

*3.91 (con't)

An EXCEL Program using a trapezoidal integration approzimation was used to calculate the results shown below.

			D = 0.5 ft	D = 1.0 ft	D = 1.5 ft	D = 2.0 ft	D = 2.5 ft	D = 3.0 ft
h, ft	A, acres	A, ft^2	t, s	t, s	t, s	t, s	t, s	t, s
18	2.8	121968	0	0	0	0	0	0
16	2.4	104544	32181	8045	3576	2011	1287	894
14	1.8	78408	59530	14882	6614	3721	2381	1654
12	1.5	65340	82354	20589	9150	5147	3294	2288
10	1.1	47916	101536	25384	11282	6346	4061	2820
8	0.9	39204	117506	29377	13056	7344	4700	3264
6	0.8	34848	132412	33103	14712	8276	5296	3678
4	0.5	21780	145035	36259	16115	9065	5801	4029
2	0.3	13068	153988	38497	17110	9624	6160	4277
0	0	0	157704	39426	17523	9857	6308	4381

The graph for D = 1 ft is shown below. The shape of the curve is the same for any D.



3.92

3.92 Water flows through a horizontal branching pipe as shown in Fig. P3.92. Determine the pressure at section (3).

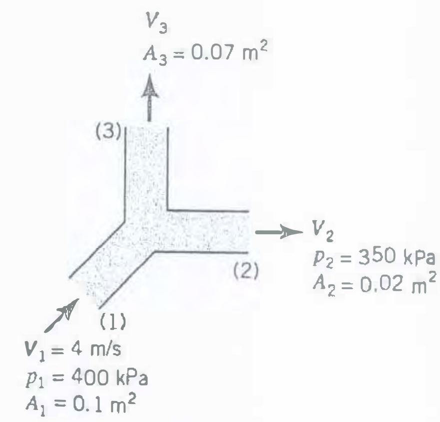


FIGURE P3.92

$Q_1 = Q_2 + Q_3$ or $V_3 = \dfrac{Q_1 - Q_2}{A_3}$ where $Q_1 = A_1 V_1 = 0.1 m^2 (4 \frac{m}{s}) = 0.4 \frac{m^3}{s}$

Also $Q_2 = A_2 V_2$ where $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$
with $z_1 = z_2$

Thus,

$$\frac{400 kPa}{9.80 \frac{kN}{m^3}} + \frac{(4 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \frac{350 kPa}{9.80 \frac{kN}{m^3}} + \frac{V_2^2}{2(9.81 \frac{m}{s^2})}$$

or

$V_2 = 10.78 \frac{m}{s}$

Thus,

$$V_3 = \frac{0.4 \frac{m^3}{s} - 0.02 m^2 (10.78 \frac{m}{s})}{0.07 m^2} = 2.63 \frac{m}{s}$$

Then from $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ with $z_1 = z_3$
we obtain

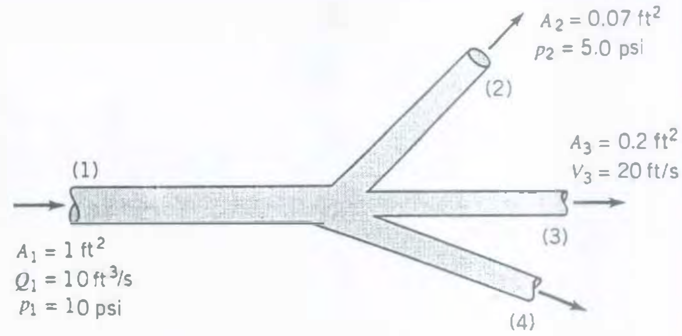
$$p_3 = p_1 + \frac{\gamma}{2g}(V_1^2 - V_3^2) = 400 kPa + \frac{9.80 \frac{kN}{m^3}}{2(9.81 \frac{m}{s^2})}(4^2 - 2.63^2)\frac{m^2}{s^2}$$

or

$p_3 = (400 + 4.54) \frac{kN}{m^2} = \underline{\underline{404.5 kPa}}$

3.93

3.93 Water flows through the horizontal branching pipe shown in Fig. P3.93 at a rate of 10 ft^3/s. If viscous effects are negligible, determine the water speed at section (2), the pressure at section (3), and the flowrate at section (4).



■ FIGURE P3.93

From (1) to (2): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $z_1 = z_2$, $p_1 = 10\,psi$, $p_2 = 5\,psi$, and $V_1 = \frac{Q_1}{A_1}$ or $V_1 = (10\,\frac{ft^3}{s})/(1\,ft^2) = 10\,\frac{ft}{s}$

Thus, with $\gamma = \rho g$

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{(10\,\frac{ft}{s})^2}{2} = \frac{(5\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{V_2^2}{2} \quad \text{or} \quad \underline{\underline{V_2 = 29.0\,\frac{ft}{s}}}$$

From (1) to (3): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ where $z_1 = z_3$, $p_1 = 10\,psi$, $V_1 = 10\,\frac{ft}{s}$ and $V_3 = 20\,\frac{ft}{s}$

Thus,

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + \frac{(10\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = \frac{p_3}{62.4\,\frac{lb}{ft^3}} + \frac{(20\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$p_3 = 1150\,\frac{lb}{ft^2} = \underline{\underline{7.98\,psi}}$$

Also,

$$Q_4 = Q_1 - Q_2 - Q_3 = Q_1 - A_2V_2 - A_3V_3$$

or

$$Q_4 = 10\,\frac{ft^3}{s} - 0.07\,ft^2(29.0\,\frac{ft}{s}) - 0.2\,ft^2(20\,\frac{ft}{s}) = \underline{\underline{3.97\,\frac{ft^3}{s}}}$$

3.94

3.94 Water flows from a large tank through a large pipe that splits into two smaller pipes as shown in Fig. P3.94 If viscous effects are negligible, determine the flowrate from the tank and the pressure at point (1).

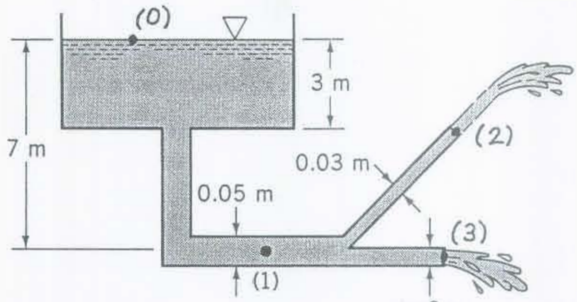


FIGURE P3.94

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_0 = 0,\ p_2 = 0,\ V_0 = 0,\ z_0 = 7m \text{ and } z_2 = 4m$$

Thus,

$$V_2 = \sqrt{2g(z_0 - z_2)} = \sqrt{2(9.81\frac{m}{s^2})(7-4)m} = 7.67\frac{m}{s}$$

Similarly

$$V_3 = \sqrt{2g(z_0 - z_3)} = \sqrt{2(9.81\frac{m}{s^2})(7m)} = 11.7\frac{m}{s}$$

Thus, $Q = Q_2 + Q_3 = \frac{\pi}{4}D_2^2 V_2 + \frac{\pi}{4}D_3^2 V_3^2$

or

$$Q = \frac{\pi}{4}\left[(0.03m)^2(7.67\frac{m}{s}) + (0.02m)^2(11.7\frac{m}{s})\right] = \underline{\underline{9.10 \times 10^{-3}\frac{m^3}{s}}}$$

Also,

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{where } z_1 = 0 \text{ and}$$

or

$$V_1 = \frac{Q}{A_1} = \frac{9.10\times10^{-3}\frac{m^3}{s}}{\frac{\pi}{4}(0.05m)^2} = 4.63\frac{m}{s}$$

$$p_1 = \gamma\left[z_0 - \frac{V_1^2}{2g}\right] = 9.80\times10^3\frac{N}{m^3}\left[7m - \frac{(4.63\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}\right] = 5.79\times10^4\frac{N}{m^2}$$

or

$$p_1 = \underline{\underline{57.9\ kPa}}$$

3.95

3.95 An air cushion vehicle is supported by forcing air into the chamber created by a skirt around the periphery of the vehicle as shown in Fig. P3.95 The air escapes through the 3-in. clearance between the lower end of the skirt and the ground (or water). Assume the vehicle weighs 10,000 lb and is essentially rectangular in shape, 30 by 65 ft. The volume of the chamber is large enough so that the kinetic energy of the air within the chamber is negligible. Determine the flowrate, Q, needed to support the vehicle. If the ground clearance were reduced to 2 in., what flowrate would be needed? If the vehicle weight were reduced to 5000 lb and the ground clearance maintained at 3 in., what flowrate would be needed?

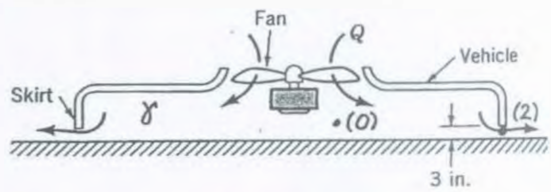


FIGURE P3.95

To support the load $p_0 = \frac{W}{A_0}$ where W = vehicle weight and $A_0 = (30\,ft)(65\,ft) = 1950\,ft^2$

Also,

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_2 = 0,\ V_0 = 0, \text{ and } z_0 = z_2$$

so that

$$V_2 = \sqrt{\frac{2p_0}{\rho}} \quad \text{or} \quad V_2 = \sqrt{\frac{2W}{A_0\rho}}$$

With h = ground clearance it follows that

$$Q = A_2V_2 = 2h(L+b)V_2 \quad \text{where } L = 65\,ft \text{ and } b = 30\,ft$$

Thus,

$$Q = 2h(65\,ft + 30\,ft)\sqrt{\frac{2W}{(1950\,ft^2)(2.38\times10^{-3}\,\frac{slug}{ft^3})}}$$

or

$$Q = 124.7\,h\sqrt{W}\ \frac{ft^3}{s} \quad \text{where } h \sim ft \text{ and } W \sim lb$$

Thus, if $h = \frac{3}{12}\,ft$ and $W = 10{,}000\,lb$, then $Q = \underline{\underline{3\,120\,\frac{ft^3}{s}}}$

if $h = \frac{2}{12}\,ft$ and $W = 10{,}000\,lb$, then $Q = \underline{\underline{2\,080\,\frac{ft^3}{s}}}$

and if $h = \frac{3}{12}\,ft$ and $W = 5\,000\,lb$, then $Q = \underline{\underline{2\,200\,\frac{ft^3}{s}}}$

3.96

3.96 Water flows from the pipe shown in Fig. P3.96 as a free jet and strikes a circular flat plate. The flow geometry shown is axisymmetrical. Determine the flowrate and the manometer reading, H.

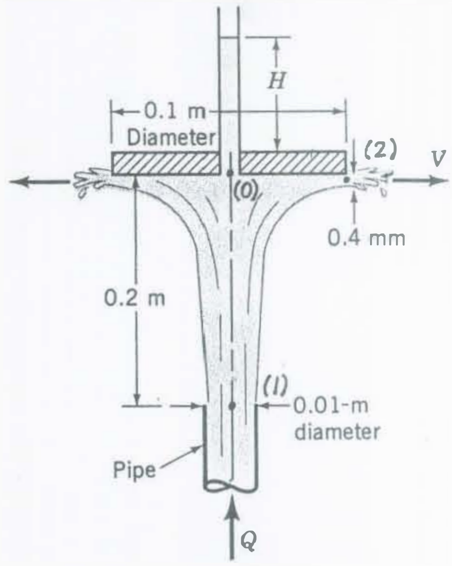


FIGURE P3.96

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\,, \text{ where } p_1 = 0,\ p_2 = 0,\ z_1 = 0, \text{ and } z_2 = 0.2\,m$$

Thus,

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} + z_2 \quad \text{where} \quad A_1 V_1 = A_2 V_2 = Q \qquad (1)$$

or $V_1 = \frac{A_2}{A_1} V_2 = \frac{\pi D_2 h}{\frac{\pi}{4} D_1^2} V_2 = \frac{4 D_2 h}{D_1^2} V = \frac{4\,(0.1\,m)(4\times10^{-4}\,m)}{(0.01\,m)^2} V_2 = 1.6\,V_2$

Hence, Eq. (1) gives

$(1.60 V_2)^2 = V_2^2 + 2(9.81\,\frac{m}{s^2})(0.2\,m)$ or $V_2 = 1.59\,\frac{m}{s}$

so that

$$Q = A_2 V_2 = \pi\,(0.1\,m)(4\times10^{-4}\,m)(1.59\,\tfrac{m}{s}) = \underline{\underline{2.00\times10^{-4}\,\tfrac{m^3}{s}}}$$

Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0\,, \text{ where } V_0 = 0,\ z_0 = 0.2\,m,\ V_1 = 1.60\,V_2$$

or $V_1 = 1.60\,(1.59\,\frac{m}{s}) = 2.54\,\frac{m}{s}$, and $p_1 = 0$

Thus,

$$H = \frac{p_0}{\gamma} = \frac{V_1^2}{2g} - z_0 = \frac{(2.54\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})} - 0.2\,m = \underline{\underline{0.129\,m}}$$

3.97

3.97 Air flows from a hole of diameter 0.03 m in a flat plate as shown in Fig. P3.97. A circular disk of diameter D is placed a distance h from the lower plate. The pressure in the tank is maintained at 1 kPa. Determine the flowrate as a function of h if viscous effects and elevation changes are assumed negligible and the flow exits radially from the circumference of the circular disk with uniform velocity.

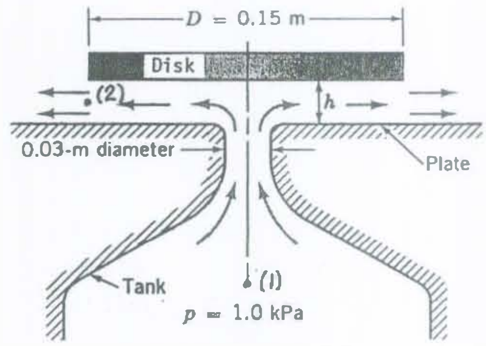


FIGURE P3.97

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_0 = 1\,\frac{kN}{m^2},\ p_2 = 0,\ z_0 \approx z_2,\ \text{and } V_0 = 0$$

Thus,

$$V_2 = \sqrt{\frac{2p_0}{\rho}} = \sqrt{\frac{2\left(1\times 10^3\,\frac{N}{m^2}\right)}{1.23\,\frac{kg}{m^3}}} = 40.3\,\frac{m}{s}$$

so that

$$Q = A_2 V_2 = \pi D_2 h V_2 = \pi (0.15\,m) h \left(40.3\,\frac{m}{s}\right)$$

or

$$Q = \underline{\underline{19.0\,h\ \frac{m^3}{s}}} \quad \text{where } h \sim m$$

3.98

3.98 A conical plug is used to regulate the air flow from the pipe shown in Fig. P3.98. The air leaves the edge of the cone with a uniform thickness of 0.02 m. If viscous effects are negligible and the flowrate is 0.50 m³/s, determine the pressure within the pipe.

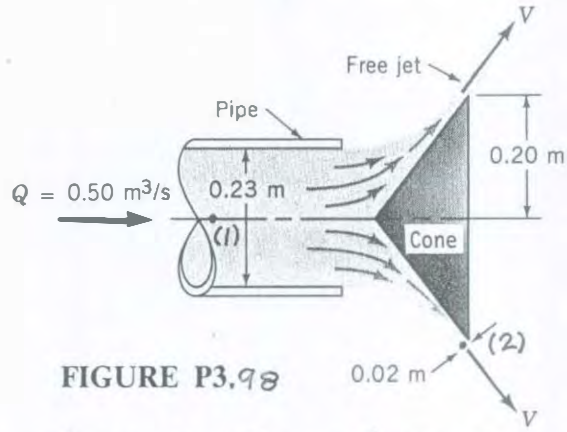


FIGURE P3.98

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad \text{where } z_1=z_2 \text{ and } p_2=0$$

Also,

$$V_1=\frac{Q}{A_1}=\frac{0.5\,\frac{m^3}{s}}{\frac{\pi}{4}(0.23m)^2}=12.0\,\frac{m}{s}$$

and

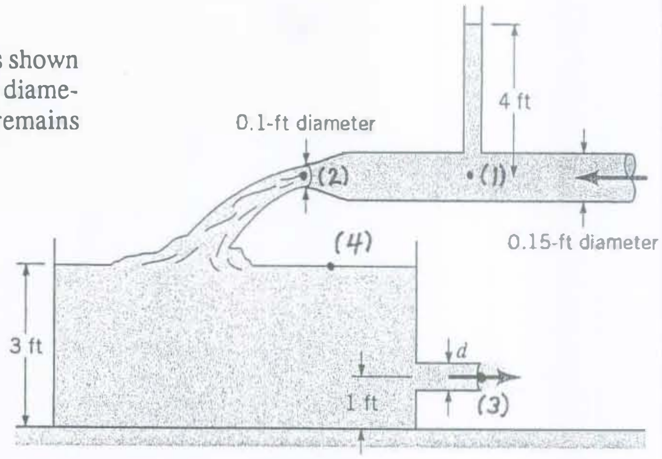
$$V_2=\frac{Q}{A_2}=\frac{Q}{2\pi Rh}=\frac{0.5\,\frac{m^3}{s}}{2\pi(0.2m)(0.02m)}=19.9\,\frac{m}{s}$$

Thus,

$$p_1=\frac{1}{2}\rho(V_2^2-V_1^2)=\frac{1}{2}\left(1.23\,\frac{kg}{m^3}\right)\left(19.9^2-12.0^2\right)\frac{m^2}{s^2}=\underline{\underline{155\,\frac{N}{m^2}}}$$

3.99

3.99 Water flows steadily from a nozzle into a large tank as shown in Fig. P3.99. The water then flows from the tank as a jet of diameter d. Determine the value of d if the water level in the tank remains constant. Viscous effects are negligible.



■ FIGURE P3.99

From the Bernoulli equation,

(1) $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$, where $z_1 = z_2$ and $p_2 = 0$

Also,

$\frac{p_1}{\gamma} = 4\text{ ft}$ and $A_1V_1 = A_2V_2$

or

$\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D_2^2V_2$ so that

(2) $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{0.10\text{ ft}}{0.15\text{ ft}}\right)^2 V_2 = 0.444\,V_2$

Thus, from Eqs. (1) and (2),

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$

or

$4\text{ ft} = \frac{(V_2^2 - V_1^2)}{2g} = \frac{(1-(0.444)^2)V_2^2}{2(32.2\text{ ft/s}^2)}$

Hence,

$V_2 = 17.9\,\frac{\text{ft}}{\text{s}}$

so that

$Q_2 = A_2V_2 = \frac{\pi}{4}(0.10\text{ft})^2(17.9\,\frac{\text{ft}}{\text{s}}) = 0.1407\,\frac{\text{ft}^3}{\text{s}}$

Also,

$Q_3 = Q_2$ where $Q_3 = A_3V_3$ and $V_3 = \sqrt{2g(z_4 - z_3)} = \sqrt{2(32.2\frac{\text{ft}}{\text{s}^2})(3\text{ft} - 1\text{ft})}$

$= 11.35\,\frac{\text{ft}}{\text{s}}$

Hence,

$\frac{\pi}{4}d^2(11.35\,\frac{\text{ft}}{\text{s}}) = 0.1407\,\frac{\text{ft}^3}{\text{s}}$

or

$d = \underline{\underline{0.126\text{ ft}}}$

3.100

3.100 A small card is placed on top of a spool as shown in Fig. P3.100. It is not possible to blow the card off the spool by blowing air through the hole in the center of the spool. The harder one blows, the harder the card "sticks" to the spool. In fact, by blowing hard enough it is possible to keep the card against the spool with the spool turned upside down. (*Note:* It may be necessary to use a thumb tack to prevent the card from sliding from the spool.) Explain this phenomenon.

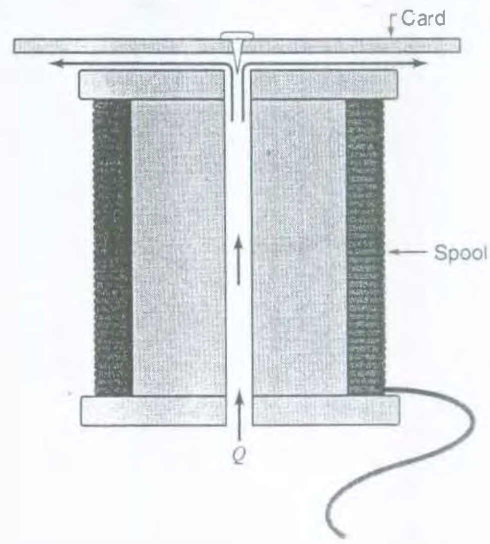
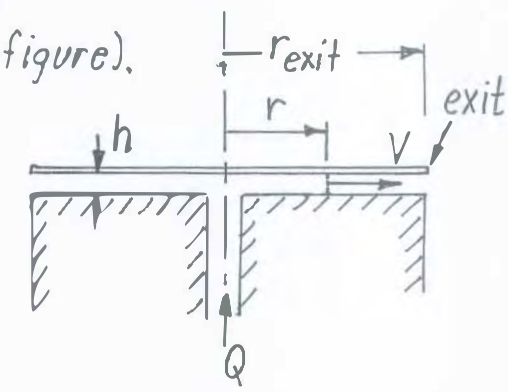


FIGURE P3.100

As the air flows radially outward in the gap between the card and the spool it slows down since the flow area increases with r, the radial distance from the center. That is,

$Q = 2\pi r h V$, or $V = \dfrac{Q}{2\pi h r}$ (see the figure). (1)



If viscous effects are not important, then

$$\frac{p}{\gamma} + \frac{V^2}{2g} = \text{constant} = \frac{p_{exit}}{\gamma} + \frac{V_{exit}^2}{2g}$$

or since $p_{exit} = 0$ (a free jet) it follows that

$$p = \tfrac{1}{2}\rho\left(V_{exit}^2 - V^2\right), \text{ where from Eq. (1) } V_{exit}^2 - V^2 = \left(\frac{Q}{2\pi h}\right)^2\left[\frac{1}{r_{exit}^2} - \frac{1}{r^2}\right]$$

But $r_{exit} > r$ so that $p < 0$. There is a vacuum within the gap. The card is sucked against the spool. The harder one blows through the spool (larger Q), the larger the vacuum, and the harder the card is held against the spool.

3.101

3.101 Water flows down the sloping ramp shown in Fig. P3.101 with negligible viscous effects. The flow is uniform at sections (1) and (2). For the conditions given show that three solutions for the downstream depth, h_2, are obtained by use of the Bernoulli and continuity equations. However, show that only two of these solutions are realistic. Determine these values.

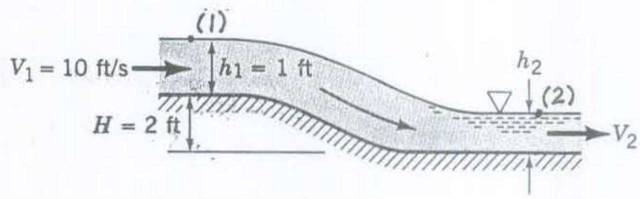


FIGURE P3.101

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 3\,ft$, and $z_2 = h_2$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{(1\,ft)(10\,\frac{ft}{s})}{h_2} = \frac{10}{h_2}$$

Thus, Eq. (1) becomes

$$\frac{(10\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} + 3\,ft = \frac{(\frac{10}{h_2})^2}{2(32.2\,\frac{ft}{s^2})} + h_2$$

or

$$64.4 h_2^3 - 293 h_2^2 + 100 = 0$$

By using a root finding program the three roots to this cubic equation are found to be:

$h_2 = 0.630\,ft$

$h_2 = 4.48\,ft$

or

$h_2 =$ a negative root Clearly it is not possible (physically) to have $h_2 < 0$. Thus, $\underline{\underline{h_2 = 0.630\,ft}}$ or $\underline{\underline{h_2 = 4.48\,ft}}$

3.102

3.102 Water flows in a rectangular channel that is 2.0 m wide as shown in Fig. P3.102. The upstream depth is 70 mm. The water surface rises 40 mm as it passes over a portion where the channel bottom rises 10 mm. If viscous effects are negligible, what is the flowrate?

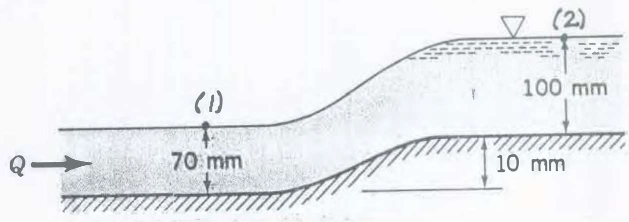


FIGURE P3.102

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ p_2 = 0,\ z_1 = 0.07\,m, \quad (1)$$

and $z_2 = (0.01 + 0.10)m = 0.11\,m$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{0.07\,m}{0.10\,m} V_1 = 0.7 V_1$$

Thus, Eq. (1) becomes

$$\left[1 - 0.7^2\right] V_1^2 = 2\left(9.81 \tfrac{m}{s^2}\right)(0.11 - 0.07)m \quad \text{or } V_1 = 1.24 \tfrac{m}{s}$$

Hence,

$$Q = A_1 V_1 = (0.07m)(2.0m)\left(1.24 \tfrac{m}{s}\right) = \underline{\underline{0.174 \ \tfrac{m^3}{s}}}$$

*3.103

*3.103 Water flows up the ramp shown in Fig. P3.103 with negligible viscous losses. The upstream depth and velocity are maintained at $h_1 = 0.3$ m and $V_1 = 6$ m/s. Plot a graph of the downstream depth, h_2, as a function of the ramp height, H, for $0 \leq H \leq 2$ m. Note that for each value of H there are three solutions, not all of which are realistic.

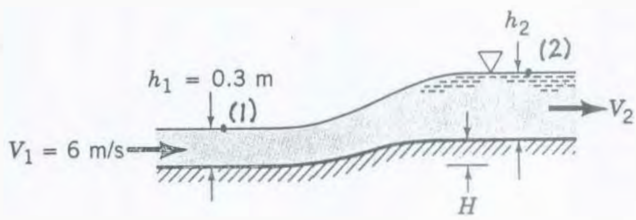


FIGURE P3.103

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ p_2 = 0,\ z_1 = 0.3m, \text{ and } z_2 = H + h_2 \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{(0.3m)(6\frac{m}{s})}{h_2} = \frac{1.8}{h_2} \quad \text{where } h_2 \sim m$$

Thus, Eq. (1) becomes

$$\frac{V_1^2}{2g} + 0.3\,m = \frac{\left(\frac{1.8}{h_2}\right)^2}{2g} + (H + h_2) \quad \text{or with } V_1 = 6\frac{m}{s},$$

$$(6\tfrac{m}{s})^2 + 2(9.81\tfrac{m}{s^2})(0.3 - H - h_2)m = \left(\frac{1.8}{h_2}\right)^2 \frac{m^2}{s^2}$$

which can be written as:

$$h_2^3 - (2.135 - H)h_2^2 + 0.1651 = 0 \qquad (2)$$

For $0 \leq H \leq 2m$ solve Eq. (2) for h_2

Rather than solving a cubic equation for h_2 (give H), one can directly solve for H (given h_2). From Eq. (2):

$$H = 2.135 - h_2 - \frac{0.1651}{h_2^2} \qquad (3)$$

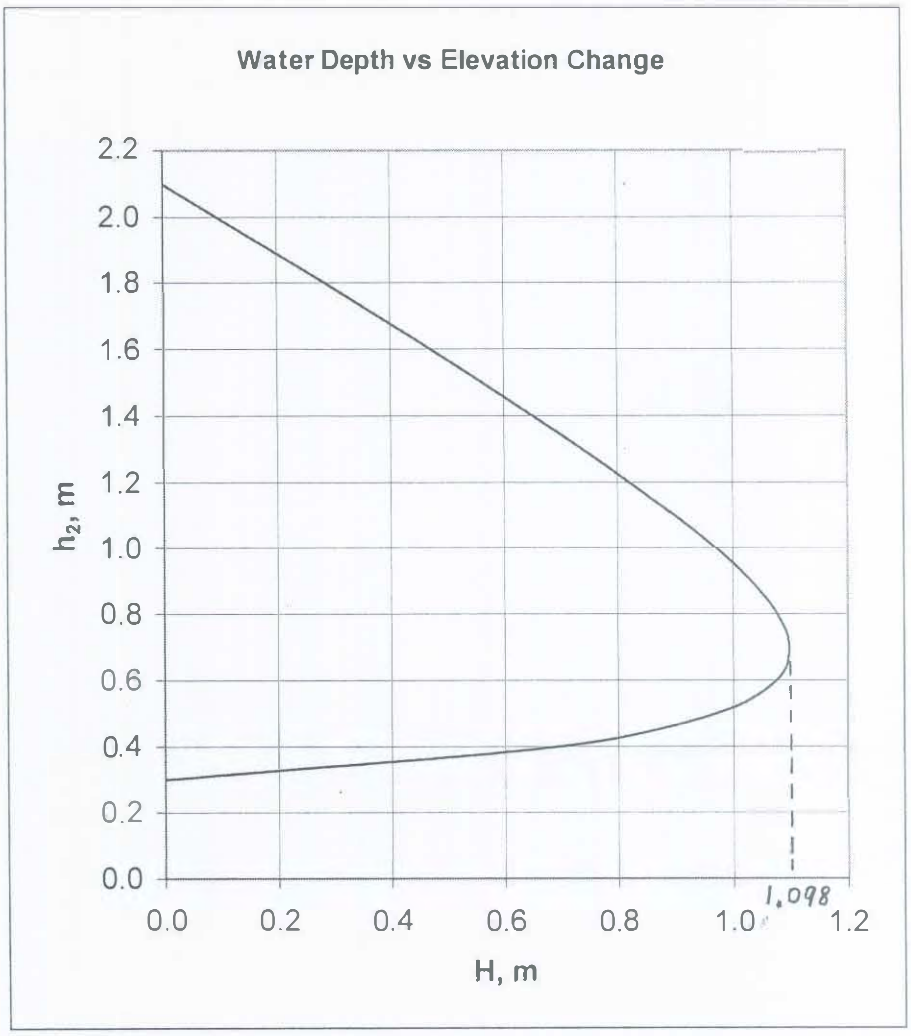
A graph of Eq. (2) or (3) is given on the following page.

(con't)

*3.103 (con't)

The results of an EXCEL Program to calculate H for given values of h_2 are shown below.

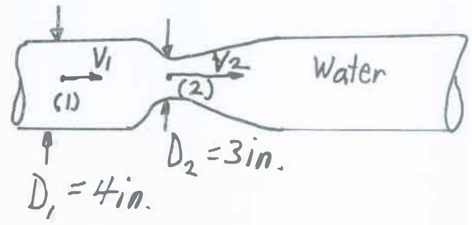
h_2, m	H, m
0.3	0.001
0.4	0.703
0.5	0.975
0.6	1.076
0.7	1.098
0.8	1.077
0.9	1.031
1.0	0.970
1.1	0.899
1.2	0.820
1.3	0.737
1.4	0.651
1.5	0.562
1.6	0.471
1.7	0.378
1.8	0.284
1.9	0.189
2.0	0.094
2.1	-0.002



For $H \geq 1.098$ m there are no real, positive roots of Eq. (2). That is, for the given upstream conditions ($V_1 = 6\ \frac{m}{s}$ and $h_1 = 0.3$ m) we must have $H < 1.098$ m. It would not be possible to have the flow go up a ramp of greater height than this without increasing either V_1 and/or h_1. The two possible water depths for a given H are plotted above.

3.105

3.105 A Venturi meter with a minimum diameter of 3 in. is to be used to measure the flowrate of water through a 4-in.-diameter pipe. Determine the pressure difference indicated by the pressure gage attached to the flow meter if the flowrate is 0.5 ft^3/s and viscous effects are negligible.



$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}} \quad \text{, where } Q = 0.5\,\frac{ft^3}{s} \text{ and } \rho = 1.94\,\frac{slugs}{ft^3}$$

Thus, since $A_2/A_1 = (D_2/D_1)^2$,

$$0.5\,\frac{ft^3}{s} = \frac{\pi}{4}\left(\frac{3}{12}\,ft\right)^2 \sqrt{\frac{2(p_1 - p_2)}{(1.94\,\frac{slugs}{ft^3})[1 - (3in./4in.)^4]}}$$

or

$$p_1 - p_2 = 68.8\,\frac{slug}{s^2\,ft} = 68.8\left(\frac{slug \cdot ft}{s^2}\right)/ft^2 = \underline{\underline{68.8\,\frac{lb}{ft^2}}}$$

3.106

3.106 Determine the flowrate through the Venturi meter shown in Fig. P3.106 if ideal conditions exist.

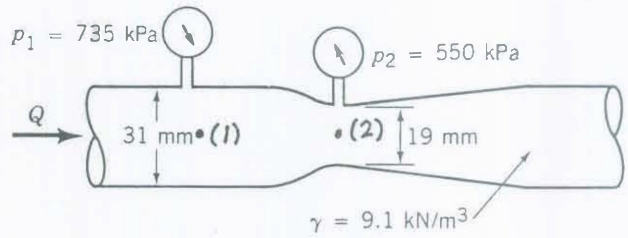


FIGURE P3.106

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$ and $A_1V_1 = A_2V_2$

or

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

Thus,

$$\frac{p_1}{\gamma} + \frac{\left(\frac{D_2}{D_1}\right)^4 V_2^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

or

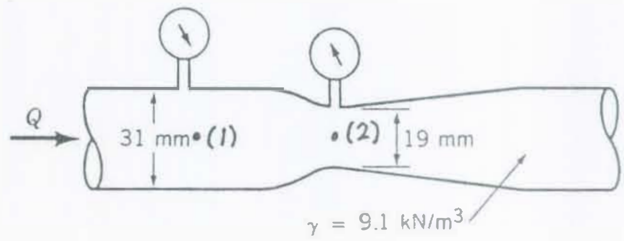
$$V_2 = \sqrt{\frac{2g\frac{(p_1 - p_2)}{\gamma}}{1 - \left(\frac{D_2}{D_1}\right)^4}} = \sqrt{\frac{2\left(9.81\frac{m}{s^2}\right)\frac{(735-550)kPa}{\left(9.1\frac{kN}{m^3}\right)}}{1-\left(\frac{19\ mm}{31\ mm}\right)^4}} = 21.5\frac{m}{s}$$

so that

$$Q = A_2V_2 = \frac{\pi}{4}D_2^2V_2 = \frac{\pi}{4}(0.019m)^2\left(21.5\frac{m}{s}\right) = \underline{\underline{6.10 \times 10^{-3}\frac{m^3}{s}}}$$

3.107

3.107 For what flowrate through the Venturi meter of Prob. 3.106 will cavitation begin if p_1 = 275 kPa gage, atmospheric pressure is 101 kPa (abs), and the vapor pressure is 3.6 kPa (abs)?



(1) $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $z_1 = z_2$, $p_2 = 3.6$ kPa(abs)

and $p_1 = (275+101)$ kPa(abs) $= 376$ kPa(abs)

Thus, with $A_1V_1 = A_2V_2$

or

$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$ Eq. (1) becomes

$$V_2 = \sqrt{\frac{2g\left(\frac{p_1 - p_2}{\gamma}\right)}{1-\left(\frac{D_2}{D_1}\right)^4}} = \left[\frac{2\left(9.81\frac{m}{s}\right)\frac{(376-3.6)\,kPa}{9.1\ kN/m^3}}{1-\left(\frac{19\ mm}{31\ mm}\right)^4}\right]^{1/2}$$

or

$V_2 = 30.6\ \frac{m}{s}$

Thus,

$$Q = A_2V_2 = \frac{\pi}{4}D_2^2V_2 = \frac{\pi}{4}(0.019\ m)^2\left(30.6\ \frac{m}{s}\right) = \underline{\underline{8.68\times10^{-3}\ \frac{m^3}{s}}}$$

3.108

3.108 What diameter orifice hole, d, is needed if under ideal conditions the flowrate through the orifice meter of Fig. P3.108 is to be 30 gal/min of seawater with $p_1 - p_2 = 2.37$ lb/in.^{2}? The contraction coefficient is assumed to be 0.63.

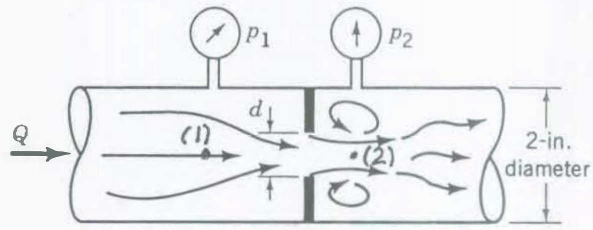


FIGURE P3.108

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{(1)}$$

where $z_1 = z_2$, $C_c = 0.63$, and $p_1 - p_2 = 2.37\,psi$

With

$$Q = \left(30\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{231\,in.^3}{1\,gal}\right)\left(\frac{1\,ft^3}{1728\,in^3}\right) = 0.0668\,\frac{ft^3}{s}$$ and $\gamma = 64.0\,\frac{lb}{ft^3}$

it follows that

$$V_1 = \frac{Q}{A_1} = \frac{0.0668\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2} = 3.06\,\frac{ft}{s}$$

Thus, Eq(1) gives

$$V_2 = \sqrt{V_1^2 + 2g\left(\frac{p_1 - p_2}{\gamma}\right)} = \sqrt{\left(3.06\,\frac{ft}{s}\right)^2 + 2\left(32.2\,\frac{ft}{s^2}\right)\left(\frac{2.37 \times 144\,\frac{lb}{ft^2}}{64.0\,\frac{lb}{ft^3}}\right)}$$

or

$$V_2 = 18.8\,\frac{ft}{s}$$

Thus, since

$$Q = A_2 V_2 = C_c \frac{\pi}{4} d^2 V_2$$ it follows that

$$d = \left[\frac{4Q}{\pi C_c V_2}\right]^{1/2} = \left[\frac{4 \times 0.0668\,\frac{ft^3}{s}}{\pi\,(0.63)\left(18.8\,\frac{ft}{s}\right)}\right]^{1/2} = 0.0847\,ft = \underline{\underline{1.016\,in.}}$$

3.109

3.109 Water flows over a weir plate (see Video V10.13) which has a parabolic opening as shown in Fig. P3.109. That is, the opening in the weir plate has a width $CH^{1/2}$, where C is a constant. Determine the functional dependence of the flowrate on the head, $Q = Q(H)$.

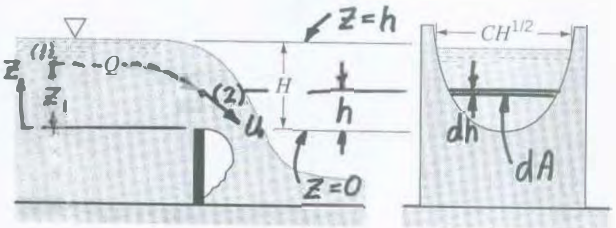


FIGURE P3.109

$Q = \int u \, dA$ where u is a function of h.

That is, from $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $\frac{p_1}{\gamma} = H - z_1$, $V_2 \equiv u$

$\frac{p_2}{\gamma} = 0$ ("free jet")

and $z_2 = H - h$

or

$$(H - z_1) + \frac{V_1^2}{2g} + z_1 = 0 + \frac{u^2}{2g} + (H - h)$$

Thus,

$$u = \sqrt{2gh + V_1^2} \approx \sqrt{2gh} \quad \text{if } V_1 \text{ is "small"}$$

Also,

$dA = C\sqrt{z}\, dz$ (i.e. $dA = 0\, dz$ for $z = 0$; $dA = C\sqrt{H}$ for $z = H$) so that

$$Q = \int_{z=0}^{z=H} \sqrt{2g}\sqrt{h}\, C\sqrt{z}\, dz \quad \text{where } h = H - z.$$

Thus, $Q = C\sqrt{2g}\int_0^H \sqrt{zH - z^2}\, dz$, where

$$\int_0^H \sqrt{zH - H^2}\, dz = \frac{1}{2}\left[\left(z - \frac{H}{2}\right)\sqrt{Hz - z^2} + \left(\frac{H}{2}\right)^2 \sin^{-1}\left[\left(z - \frac{H}{2}\right)/(H/2)\right]\right]_{z=0}^{z=H}$$

which reduces to:

$$Q = \frac{\pi C}{8}\sqrt{2g}\, H^2 \qquad \text{That is } \underline{\underline{Q \sim H^2}}$$

Alternatively, $Q = VA$ where the average velocity is proportional to $\sqrt{H}$ (i.e. $V \sim \sqrt{2gH}$) and the total flow area is proportional to $H^{3/2}$ (i.e. $A \sim H \times (CH^{1/2}) = CH^{3/2}$). Thus,

$$Q \sim \sqrt{2gH}\,(CH^{3/2}) = C\sqrt{2g}\, H^2$$

That is, $Q \sim H^2$ as obtained above.

3.110

3.110 A weir (see Video V10.13) of trapezoidal cross section is used to measure the flowrate in a channel as shown in Fig. P3.110 If the flowrate is Q_0 when $H = \ell/2$, what flowrate is expected when $H = \ell$?

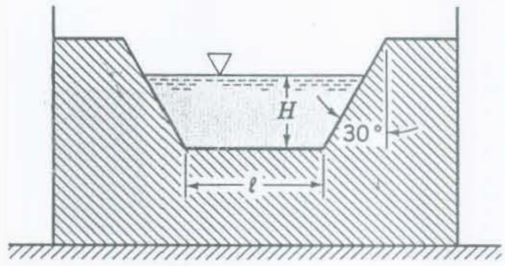


FIGURE P3.110

$Q = AV$ where it is expected that V is a function of the head, H.
That is, $V \sim \sqrt{2gH}$

Also, from the geometry $A = \frac{1}{2} H (\ell + \ell_T)$ where $\ell_T = \ell + 2H \tan 30°$

Thus, $A = H(\ell + H \tan 30°)$ so that

$Q = C_1 \sqrt{2g}\,(\ell + H \tan 30°)\, H^{3/2}$ where C_1 is a constant

Let Q_0 = flowrate when $H = \frac{\ell}{2}$

and Q_ℓ = flowrate when $H = \ell$

Thus,

$$\frac{Q_0}{Q_\ell} = \frac{C_1\sqrt{2g}\,(\ell + \frac{\ell}{2}\tan 30°)(\frac{\ell}{2})^{3/2}}{C_1\sqrt{2g}\,(\ell + \ell \tan 30°)(\ell)^{3/2}} = \frac{(1 + \frac{1}{2}\tan 30°)}{(1 + \tan 30°)(2^{3/2})} = 0.289$$

or

$$\underline{\underline{Q_\ell = 3.46\, Q_0}}$$

3.111

3.111 The flowrate in a water channel is sometimes determined by use of a device called a Venturi flume. As shown in Fig. P3.111, this device consists simply of a hump on the bottom of the channel. If the water surface dips a distance of 0.07 m for the conditions shown, what is the flowrate per width of the channel? Assume the velocity is uniform and viscous effects are negligible.

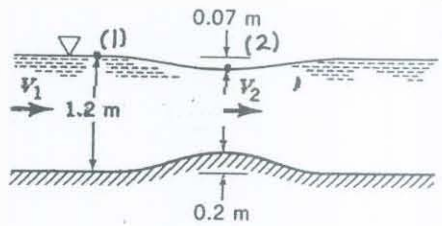


FIGURE P3.111

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with } p_1 = 0,\ p_2 = 0,\ z_1 = 1.2\,m, \tag{1}$$

and $z_2 = 1.2\,m - 0.07\,m = 1.13\,m$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{1.2\,m \quad V_1}{(1.2 - 0.07 - 0.2)\,m} = 1.29\,V_1$$

Thus, from Eq. (1):

$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 \quad \text{or } \left[(1.29)^2 - 1\right] V_1^2 = 2\left(9.81\,\frac{m}{s^2}\right)(1.2 - 1.13)\,m$$

or $V_1 = 1.438\,\frac{m}{s}$

Hence,

$$q = h_1 V_1 = \left(1.438\,\frac{m}{s}\right)(1.2\,m) = \underline{\underline{1.73\,\frac{m^2}{s}}}$$

3.112

3.112 Water flows under the inclined sluice gate shown in Fig. P3.112 Determine the flowrate if the gate is 8 ft wide.

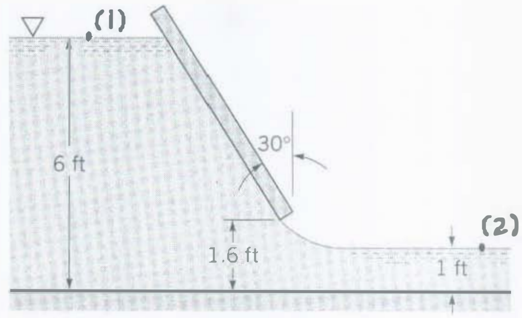


FIGURE P3.112

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ where $p_1=0$, $p_2=0$, $z_1=6\,ft$, and $z_2=1\,ft$

Thus,

$$\frac{V_1^2}{2g}+6\,ft=\frac{V_2^2}{2g}+1\,ft \qquad (1)$$

But $A_1V_1=A_2V_2$, or

$$V_2=\frac{A_1}{A_2}V_1=\frac{6\,ft}{1\,ft}V_1=6\,V_1$$

Hence, Eq. (1) becomes

$$\frac{V_1^2}{2g}+6\,ft=\frac{(6)^2V_1^2}{2g}+1\,ft$$

or

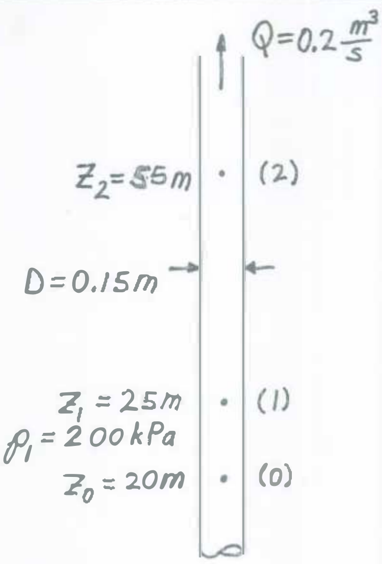
$$[6^2-1]V_1^2=2\left(32.2\frac{ft}{s^2}\right)(6-1)ft \quad \text{o} \quad V_1=3.03\frac{ft}{s}$$

Hence,

$$Q=A_1V_1=6\,ft\,(8\,ft)\left(3.03\frac{ft}{s}\right)=\underline{\underline{145\frac{ft^3}{s}}}$$

3.113

3.113 Water flows in a vertical pipe of 0.15-m diameter at a rate of 0.2 m³/s and a pressure of 200 kPa at an elevation of 25 m. Determine the velocity head and pressure head at elevations of 20 and 55 m.



$$V = \frac{Q}{A} = \frac{0.2 \frac{m^3}{s}}{\frac{\pi}{4}(0.15m)^2} = 11.3 \frac{m}{s} = V_0 = V_2$$

At point (0):

$$\frac{V_0^2}{2g} = \frac{(11.3 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \underline{\underline{6.51 m}}$$

and

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{or} \quad \frac{p_0}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_0$$

or

$$\frac{p_0}{\gamma} = \frac{200 \frac{kN}{m^2}}{9.80 \frac{kN}{m^3}} + (25-20)m = \underline{\underline{25.4 m}}$$

Similarly at point (2):

$$\frac{V_0^2}{2g} = \frac{V_2^2}{2g} = \underline{\underline{6.51 m}}$$

and

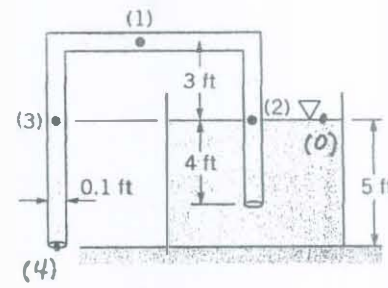
$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{or} \quad \frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_2$$

or

$$\frac{p_2}{\gamma} = \frac{200 \frac{kN}{m^2}}{9.80 \frac{kN}{m^3}} + (25-55)m = \underline{\underline{-9.59 m}}$$

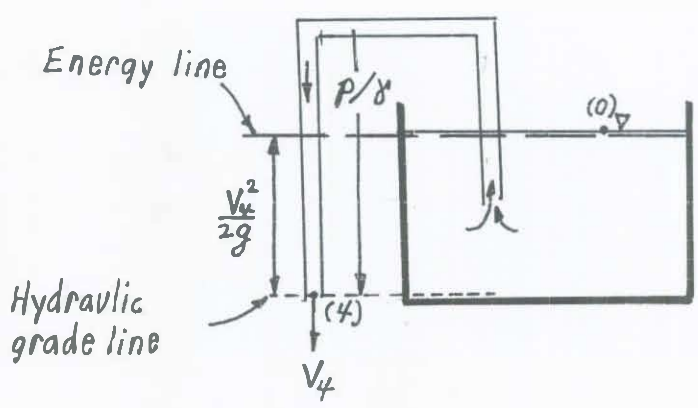
3.114

3.114 Draw the energy line and the hydraulic grade line for the flow shown in Problem 3.78



■ FIGURE P3.78

For inviscid flow with no pumps or turbines, the energy line is horizontal at the elevation of the free surface of the tank. The hydraulic grade line is one velocity head, $V^2/2g$, below the energy line. Since $V_4 = \sqrt{2g(z_0 - z_4)}$ it follows that the hydraulic grade line is $V_4^2/2g = (z_0 - z_4) = 5\ \text{ft}$ below the free surface at the exit of the pipe. Also, since the pipe is a constant diameter, the velocity is constant throught the pipe. Hence, the hydraulic grade line is horizontal, 5 ft below the free surface. Note that since the pipe is above the hydraulic grade line, the pressure throughout the pipe is less than atmospheric.



3.115

3.115 Draw the energy line and the hydraulic grade line for the flow of Problem 3.75

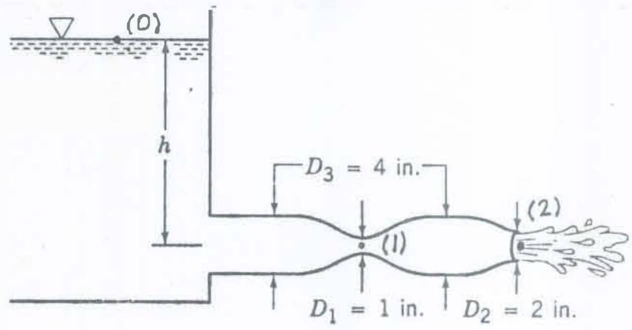


FIGURE P3.75

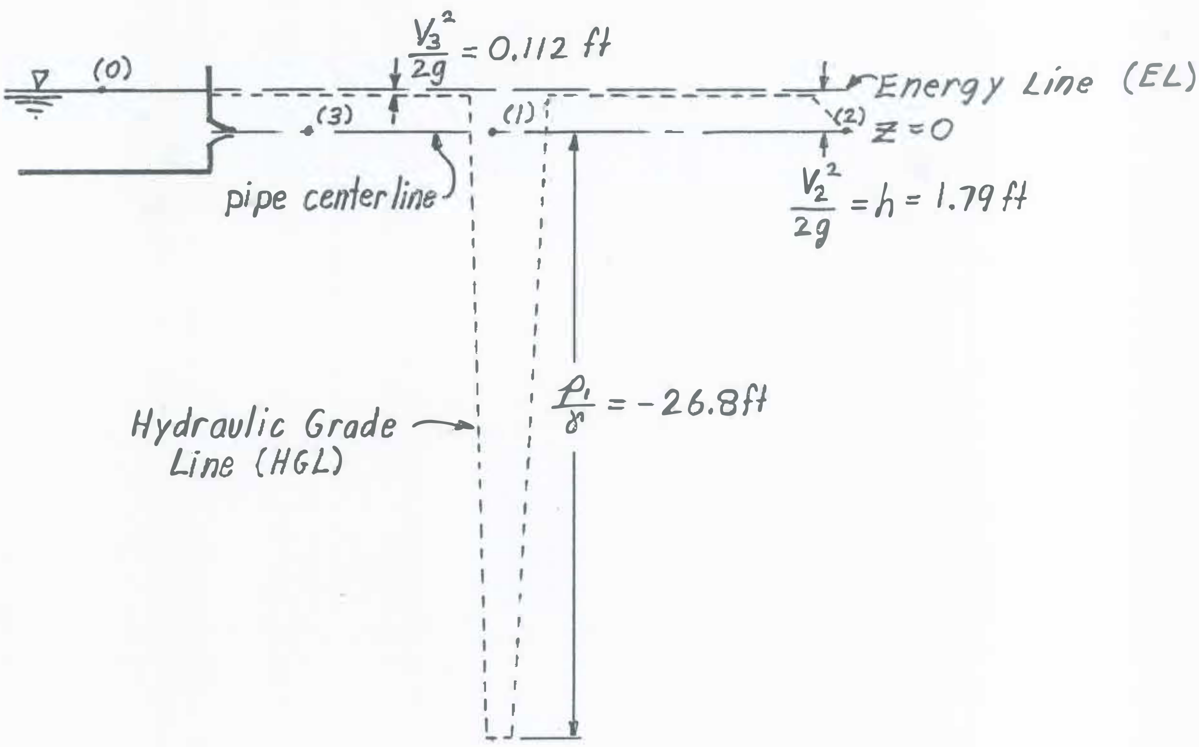
For inviscid flow with no pumps or turbines, the energy line is horizontal, a distance h above the outlet. From Problem 3.75 we obtain $h = 1.79$ ft.

The hydraulic grade line is $\frac{V^2}{2g}$ below the energy line, starting at the free surface where $V_0 = 0$ and ending at the pipe exit where $p_2 = 0$ and $\frac{V_2^2}{2g} = h$. At point (1) the pressure head is $p_1/\gamma = (2.88 - 14.5)\frac{\text{lb}}{\text{in.}^2}\left(\frac{144\,\text{in.}^2}{\text{ft}^2}\right)/62.4\frac{\text{lb}}{\text{ft}^3} = -26.8$ ft, and $z_1 = 0$.

In the 4 in. pipe $V_3 = A_2 V_2/A_3 = \left(\frac{D_2}{D_3}\right)^2 V_2$ so that

$$\frac{V_3^2}{2g} = \left(\frac{D_2}{D_3}\right)^4 \frac{V_2^2}{2g} = \left(\frac{D_2}{D_3}\right)^4 h = \left(\frac{2}{4}\right)^4 (1.79\text{ ft}) = 0.112\text{ ft}$$

The corresponding EL and HGL are drawn to scale below.



3.116

3.116 Draw the energy line and hydraulic grade line for the flow shown in Problem 3.64.

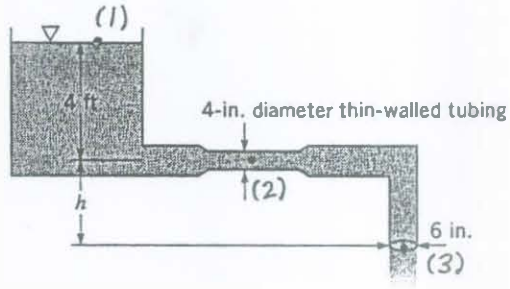


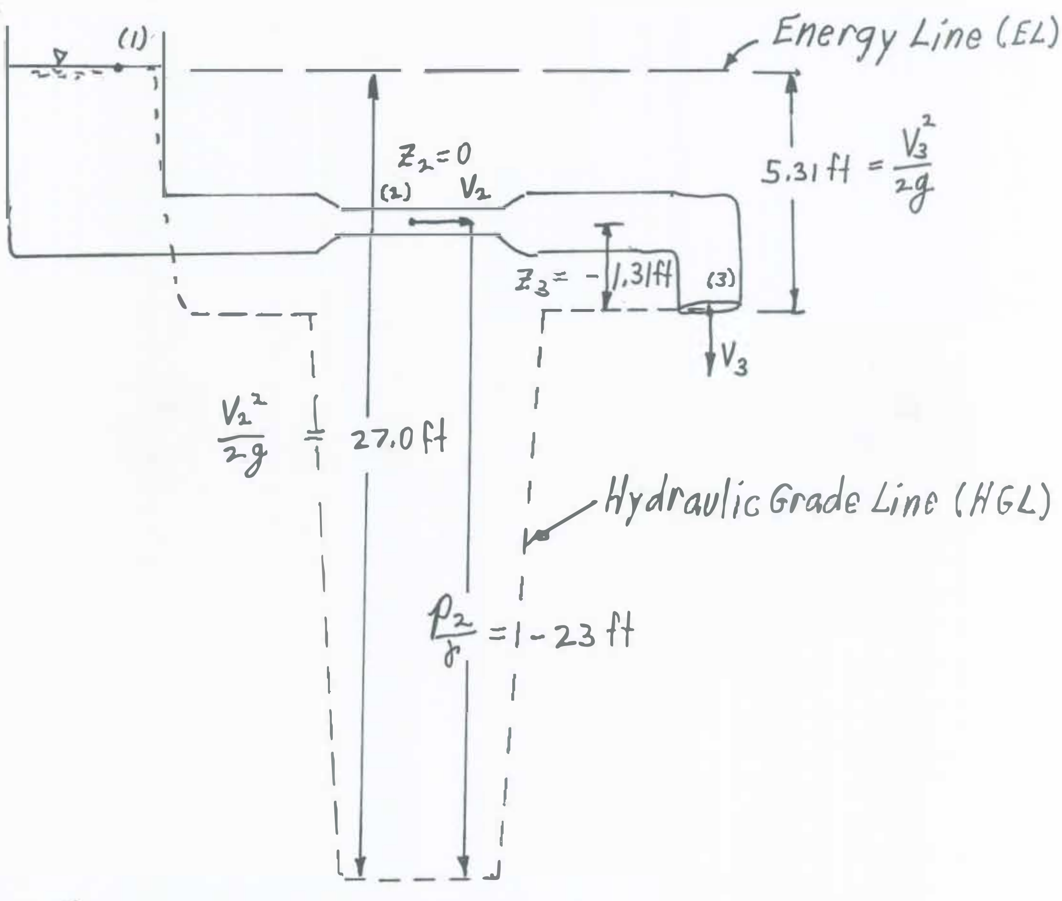
FIGURE P3.64

For steady, inviscid flow with no pumps or turbines the energy line is horizontal, a distance of $h + 4\,ft = 1.31\,ft + 4\,ft = 5.31\,ft$ above the outlet. (See solution to problem 3.64 for values of h, p_2, V_2, and p_3, V_3.) The hydraulic grade line is one velocity head, $V^2/2g$, below the energy line.

Thus, with $V_1^2/2g = 0$, $V_2^2/2g = (41.7\,\frac{ft}{s})^2/(2(32.2\,\frac{ft}{s^2})) = 27.0\,ft$,

and $V_3^2/2g = (18.5\,\frac{ft}{s})^2/(2(32.2\,\frac{ft}{s^2})) = 5.31\,ft$

the following EL and HGL are obtained:



Note: $\frac{p_2}{\gamma} = -144\,\frac{lb}{ft^2} / (62.4\,\frac{lb}{ft^3}) = 23.\,ft$

3.118

3.118 Pressure Distribution between Two Circular Plates

Objective: According to the Bernoulli equation, a change in velocity can cause a change in pressure. Also, for an incompressible flow, a change in flow area causes a change in velocity. The purpose of this experiment is to determine the pressure distribution caused by air flowing radially outward in the gap between two closely spaced flat plates as shown in Fig. P3.118.

Equipment: Air supply with a flow meter; two circular flat plates with static pressure taps at various radial locations from the center of the plates; spacers to maintain a gap of height b between the plates; manometer; barometer; thermometer.

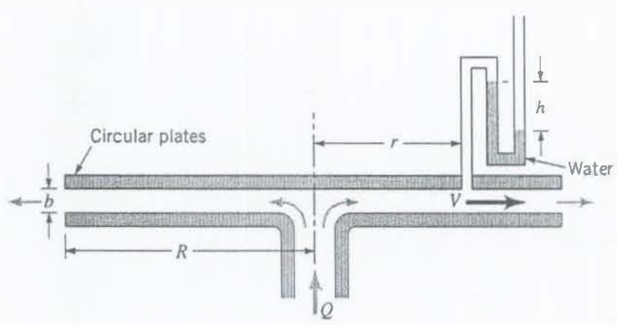
Experimental Procedure: Measure the radius, R, of the plates and the gap width, b, between them. Adjust the air supply to provide the desired, constant flowrate, Q, through the inlet pipe and the gap between the flat plates. Attach the manometer to the static pressure tap located a radial distance r from the center of the plates and record the manometer reading, h. Repeat the pressure measurements (for the same Q) at different radial locations. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings to obtain the experimentally determined pressure distribution, $p = p(r)$, within the gap. That is, $p = -\gamma_m h$, where γ_m is the specific weight of the manometer fluid. Also use the Bernoulli equation ($p/\gamma + V^2/2g$ = constant) and the continuity equation (AV = constant, where $A = 2\pi rb$) to determine the theoretical pressure distribution within the gap between the plates. Note that the flow at the edge of the plates ($r = R$) is a free jet ($p = 0$). Also note that an increase in r causes an increase in A, a decrease in V, and an increase in p.

Graph: Plot the experimentally measured pressure head, p/γ, in feet of air as ordinates and radial location, r, as abscissas.

Results: On the same graph, plot the theoretical pressure head distribution as a function of radial location.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.118

(con't)

3.118 (con't)

Solution for Problem 3.118 Pressure Distribuition between Two Circular Plates

Q, ft^3/s	R, in.	b, in.	H_{atm}, in. Hg	T, deg F	γ_{H2O}, lb/ft^3
0.879	5.0	0.125	29.09	83	62.4

		Experiment	Theory	
r, in.	h, in.	p/γ, ft	V, ft/s	p/γ, ft
0.7	-9.05	-663.75	220.8	-740.7
1.0	-6.02	-441.52	161.2	-387.2
1.5	-2.02	-148.15	107.4	-163.1
2.0	-0.96	-70.41	80.6	-84.7
2.5	-0.48	-35.20	64.5	-48.4
3.0	-0.24	-17.60	53.7	-28.7
3.5	-0.13	-9.53	46.0	-16.8
4.0	-0.03	-2.20	40.3	-9.1
4.5	-0.01	-0.73	35.8	-3.8
5.0	0.00	0.00	32.2	0.0

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.09/12ft) = 2053 lb/ft^2

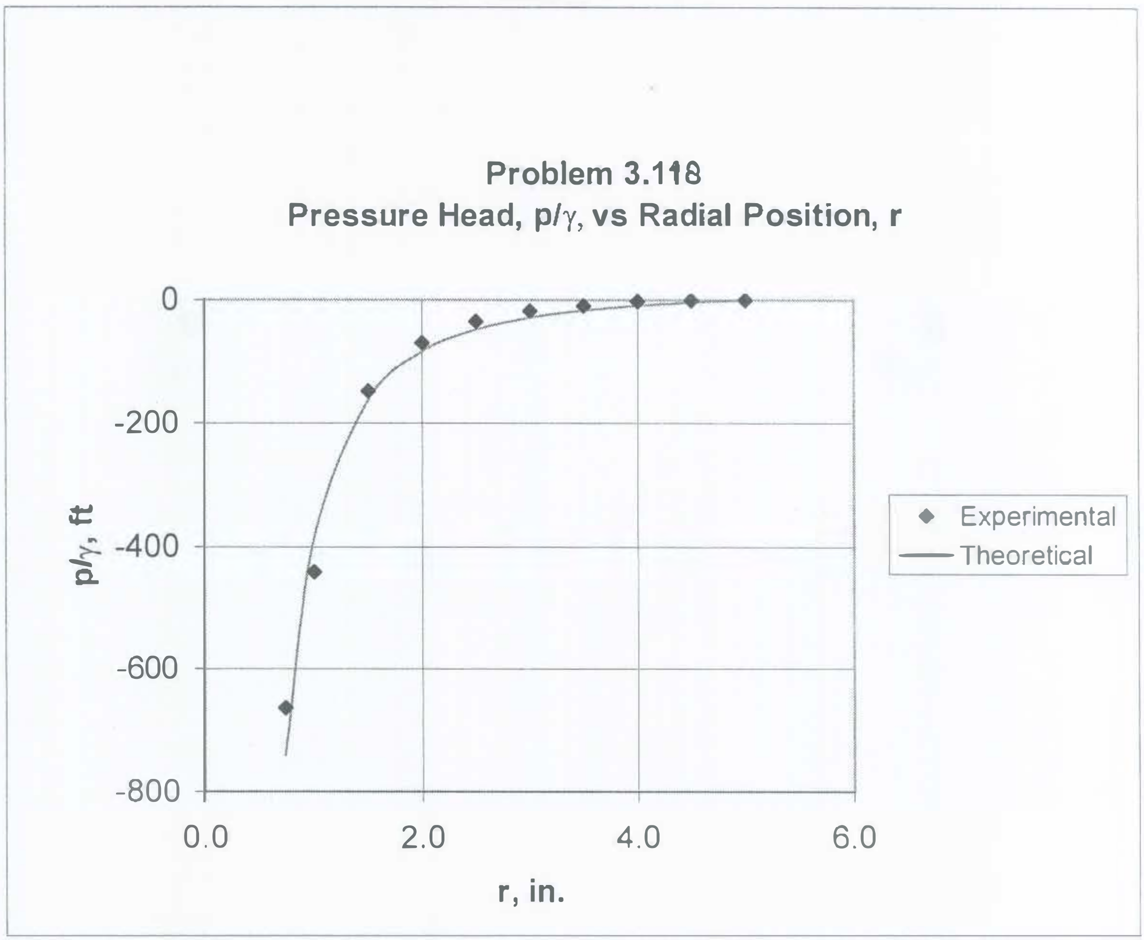
R = 1716 ft lb/slug deg R

T = 83 + 460 = 543 deg R

Thus, ρ = 0.00220 slug/ft^3 and $\gamma = \rho * g$ = 0.00220*32.2 = 0.0709 lb/ft^3

$p/\gamma = \gamma_{H20} * h/\gamma$

V = Q/(2πrb) = 0.879 ft^s/(2*3.1415*(0.125/12)ft*r)



3.119.

3.119 Calibration of a Nozzle Flow Meter

Objective: As shown in Section 3.6.3 of the text, the volumetric flowrate, Q, of a given fluid through a nozzle flow meter is proportional to the square root of the pressure drop across the meter. Thus, $Q = Kh^{1/2}$, where K is the meter calibration constant and h is the manometer reading that measures the pressure drop across the meter (see Fig. P3.119). The purpose of this experiment is to determine the value of K for a given nozzle flow meter.

Equipment: Pipe with a nozzle flow meter; variable speed fan; exit nozzle to produce a uniform jet of air; Pitot static tube; manometers; barometer; thermometer.

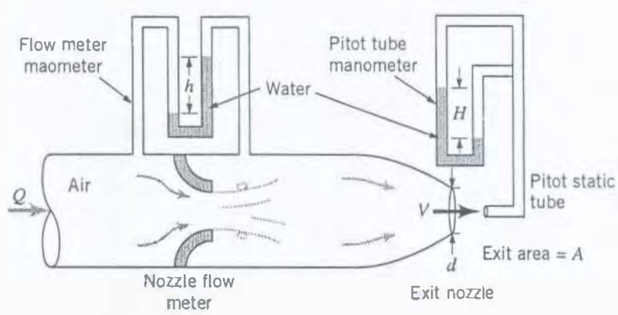
Experimental Procedure: Adjust the fan speed control to give the desired flowrate, Q. Record the flow meter manometer reading, h, and the Pitot tube manometer reading, H. Repeat the measurements for various fan settings (i.e., flowrates). Record the nozzle exit diameter, d. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated from the perfect gal law.

Calculations: For each fan setting determine the flowrate, $Q = VA$, where V and A are the air velocity at the exit and the nozzle exit area, respectively. The velocity, V, can be determined by using the Bernoulli equation and the Pitot tube manometer data, H (see Equation 3.16).

Graph: Plot flowrate, Q, as ordinates and flow meter manometer reading, h, as abscissas on a log-log graph. Draw the best-fit straight line with a slope of ½ through the data.

Results: Use your data to determine the calibration constant, K, in the flow meter equation $Q = Kh^{1/2}$.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.119

(con't)

Solution for Problem 3.119: Calibration of a Nozzle Flow Meter

d, in.	H_{atm}, in. Hg	T, deg F
1.169	29.01	75

h, in.	H, in.	Δp, lb/ft^2	V, ft/s	Q, ft^3/s
11.6	5.6	29.1	162	1.20
11.1	5.4	28.1	159	1.18
10.7	5.2	27.0	156	1.16
10.1	4.9	25.5	151	1.13
9.6	4.7	24.4	148	1.10
8.8	4.3	22.4	142	1.06
7.9	3.9	20.3	135	1.00
7.2	3.6	18.7	130	0.97
6.1	3.1	16.1	120	0.90
5.4	2.7	14.0	112	0.84
4.5	2.3	12.0	104	0.77
3.8	2.0	10.4	97	0.72
2.9	1.5	7.8	84	0.62
2.1	1.1	5.7	72	0.53
1.0	0.6	3.1	53	0.39

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.01/12 ft) = 2048 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

Thus, ρ = 0.00223 slug/ft^3

$V = (2*\Delta p/\rho)^{1/2}$

Q = AV where

$A = \pi d^2/4$ = π*(1.169/12 ft)^2/4 = 7.45E-3 ft^2

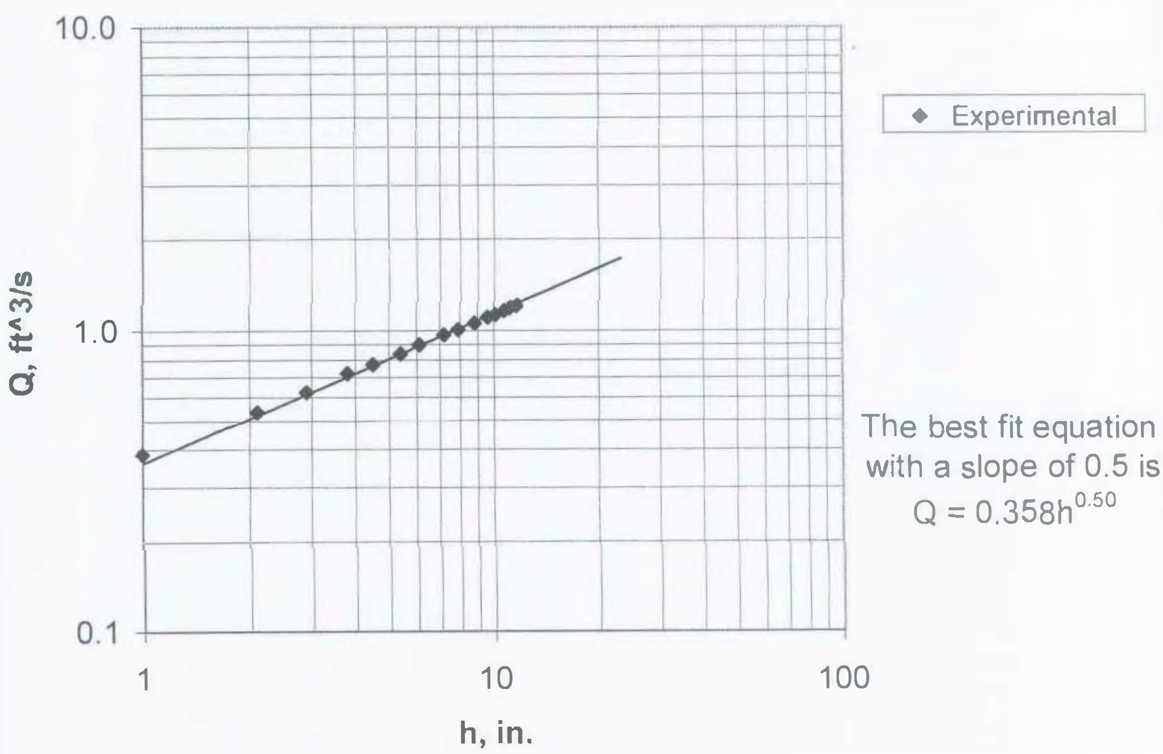
From the graph, $Q = K\,h^{1/2} = 0.358\,h^{1/2}$ where Q is in ft^3/s and h is in in.

Thus, K = $\underline{0.358\ ft^3/(s*in.^{1/2})}$

(con't)

3.119 (con't)

Problem 3.119
Flow Rate, Q, vs Manometer Reading, h



3.120

3.120 Pressure Distribution in a Two-Dimensional Channel

Objective: According to the Bernoulli equation, a change in velocity can cause a change in pressure. Also, for an incompressible flow, a change in flow area causes a change in velocity. The purpose of this experiment is to determine the pressure distribution caused by air flowing within a two-dimensional, variable area channel as shown in Fig. P3.120.

Equipment: Air supply with a flow meter; two-dimensional channel with one curved side and one flat side; static pressure taps at various locations along both walls of the channel; ruler; manometer; barometer; thermometer.

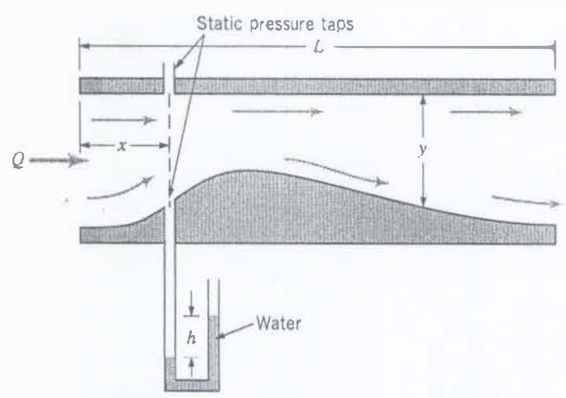
Experimental Procedure: Measure the constant width, b, of the channel and the channel height, y, as a function of distance, x, along the channel. Adjust the air supply to provide the desired, constant flowrate, Q, through the channel. Attach the manometer to the static pressure tap located a distance, x, from the origin and record the manometer reading, h. Repeat the pressure measurements (for the same Q) at various locations on both the flat and the curved sides of the channel. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings, h, to calculate the pressure within the channel, $p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. Convert this pressure into the pressure head, p/γ, where $\gamma = g\rho$ is the specific weight of air. Also use the Bernoulli equation ($p/\gamma + V^2/2g =$ constant) and the continuity equation ($AV = Q$, where $A = yb$) to determine the theoretical pressure distribution within the channel. Note that the air leaves the end of the channel ($x = L$) as a free jet ($p = 0$).

Graph: Plot the experimentally determined pressure head, p/γ, as ordinates and the distance along the channel, x, as abscissas. There will be two curves—one for the curved side of the channel and another for the flat side.

Results: On the same graph, plot the theoretical pressure distribution within the channel.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.120

(con't)

3.120 (con't)

Solution for Problem 3.120: Pressure Distribution in a Two-Dimensional Channel

b, in.	Q, ft^3/s	H_{atm}, in. Hg	T, deg F	L, in.
2.0	1.32	28.96	71	21.75

x, in.	y, in.	h, in. flat side	h, in. curved side	Experimental p/γ, ft flat side	Experimental p/γ, ft curved side	Theory p/γ, ft
0.75	2.00	0.28	0.31	20.2	22.3	0.0
2.50	2.00	0.21	0.37	15.1	26.6	0.0
4.00	1.28	-0.42	0.03	-30.2	2.3	-50.5
4.63	1.05	-0.77	-1.63	-55.5	-117.4	-92.2
5.38	1.05	-1.01	-1.05	-72.7	-75.6	-92.2
8.14	1.29	-0.63	-0.62	-45.4	-44.7	-49.2
10.75	1.54	-0.32	-0.31	-23.0	-22.3	-24.1
13.25	1.77	-0.15	-0.15	-10.8	-10.8	-9.7
15.78	2.00	-0.05	0.00	-3.6	0.0	0.0
21.75	2.00	0.00	0.00	0.0	0.0	0.0

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} \cdot H_{atm}$ = 847 lb/ft^3*(28.96/12 ft) = 2044 lb/ft^2

R = 1716 ft lb/slug deg R

T = 71 + 460 = 531 deg R

Thus, ρ = 0.00224 slug/ft^3 and γ = ρ*g = 0.00224 slug/ft^3*(32.2 ft/s^2) = 0.0722 lb/ft^3

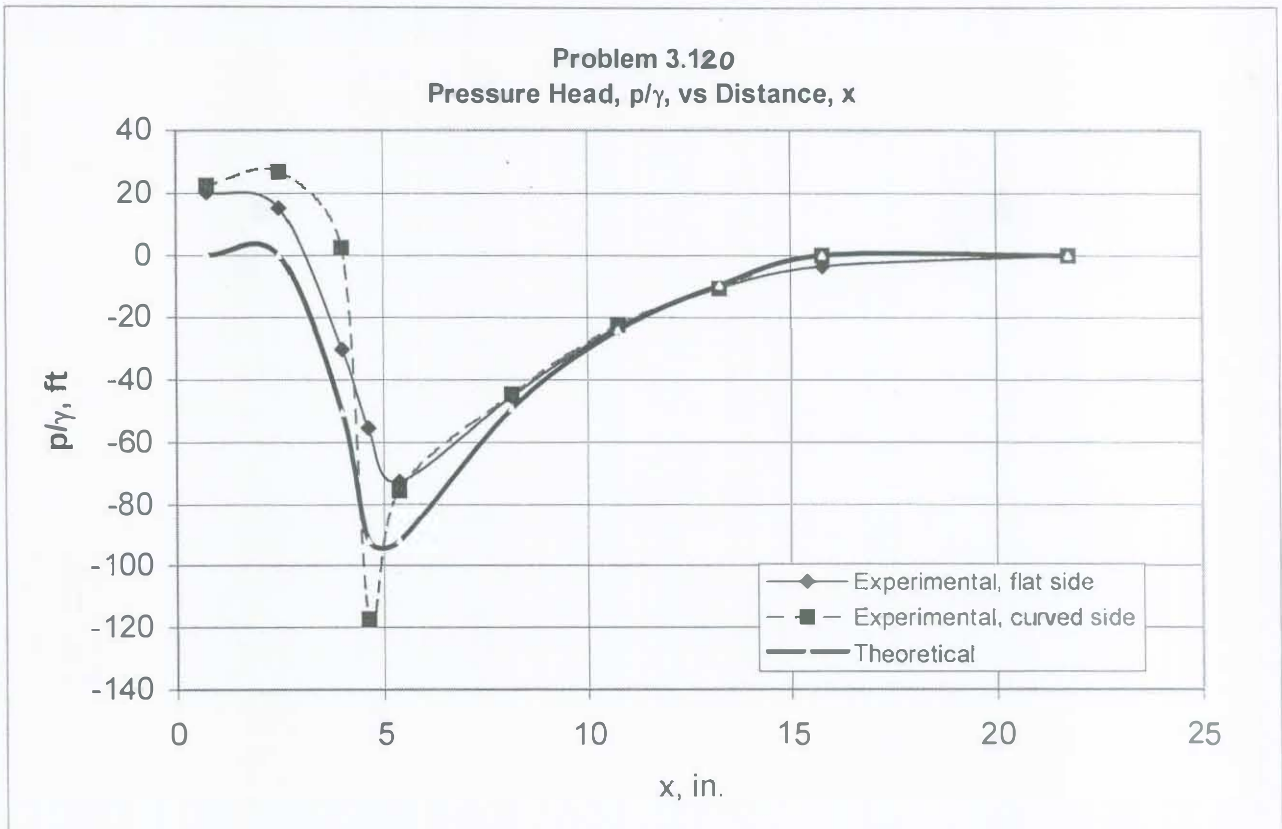
$p/\gamma = \gamma_{H2O} * h/\gamma$

Theoretical:

$p/\gamma = V_{exit}^2/2g - V^2/2g$ where

V = Q/A = Q/(b*y) and

$V_{exit} = Q/A_{exit}$ = (1.32 ft^3/s/)*(2 *2 /144 ft^2) = 47.5 ft/s



3.121

3.121 Sluice Gate Flowrate

Objective: The flowrate of water under a sluice gate as shown in Fig. P3.121 is a function of the water depths upstream and downstream of the gate. The purpose of this experiment is to compare the theoretical flowrate with the experimentally determined flowrate.

Equipment: Flow channel with pump and control valve to provide the desired flowrate in the channel; sluice gate; point gage to measure water depth; float; stop watch.

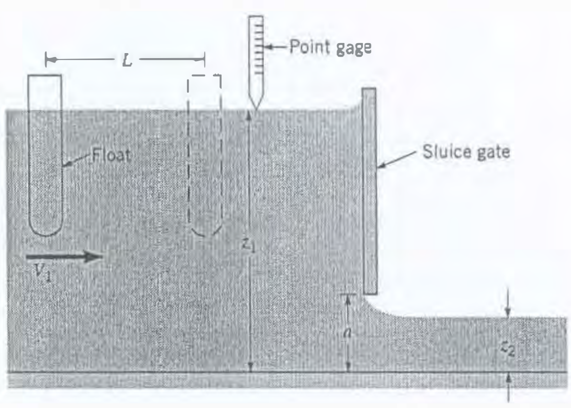
Experimental Procedure: Adjust the vertical position of the sluice gate so that the bottom of the gate is the desired distance, a, above the channel bottom. Measure the width, b, of the channel (which is equal to the width of the gate). Turn on the pump and adjust the control valve to produce the desired water depth upstream of the sluice gate. Insert a float into the water upstream of the gate and measure the water velocity, V_1, by recording the time, t, it takes the float to travel a distance L. That is, $V_1 = L/t$. Use a point gage to measure the water depth, z_1, upstream of the gate. Adjust the control valve to produce various water depths upstream of the gate and repeat the measurements.

Calculations: For each water depth used, determine the flowrate, Q, under the sluice gate by using the continuity equation $Q = A_1V_1 = b\,z_1V_1$. Use the Bernoulli and continuity equations to determine the theoretical flowrate under the sluice gate (see Equation 3.21). For these calculations assume that the water depth downstream of the gate, z_2, remains at 61% of the distance between the channel bottom and the bottom of the gate. That is, $z_2 = 0.61a$.

Graph: Plot the experimentally determined flowrate, Q, as ordinates and the water depth, z_1, upstream of the gate as abscissas.

Results: On the same graph, plot the theoretical flowrate as a function of water depth upstream of the gate.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.121

(con't)

3.121 (Con't)

Solution for problem 3.121: Sluice Gate Flowrate

a, in.	b, in.	L, ft	z₂, ft
1.2	6.0	4.0	0.061

		Experimental		Theoretical
z_1, ft	t, s	V_1, ft/s	Q, ft^3/s	Q, ft^3/s
0.183	4.2	0.952	0.087	0.091
0.267	5.0	0.800	0.107	0.114
0.343	5.2	0.769	0.132	0.132
0.453	6.2	0.645	0.146	0.155
0.569	6.4	0.625	0.178	0.175
0.725	7.0	0.571	0.207	0.200
0.877	8.6	0.465	0.204	0.222

Experimental:

$V_1 = L/t$

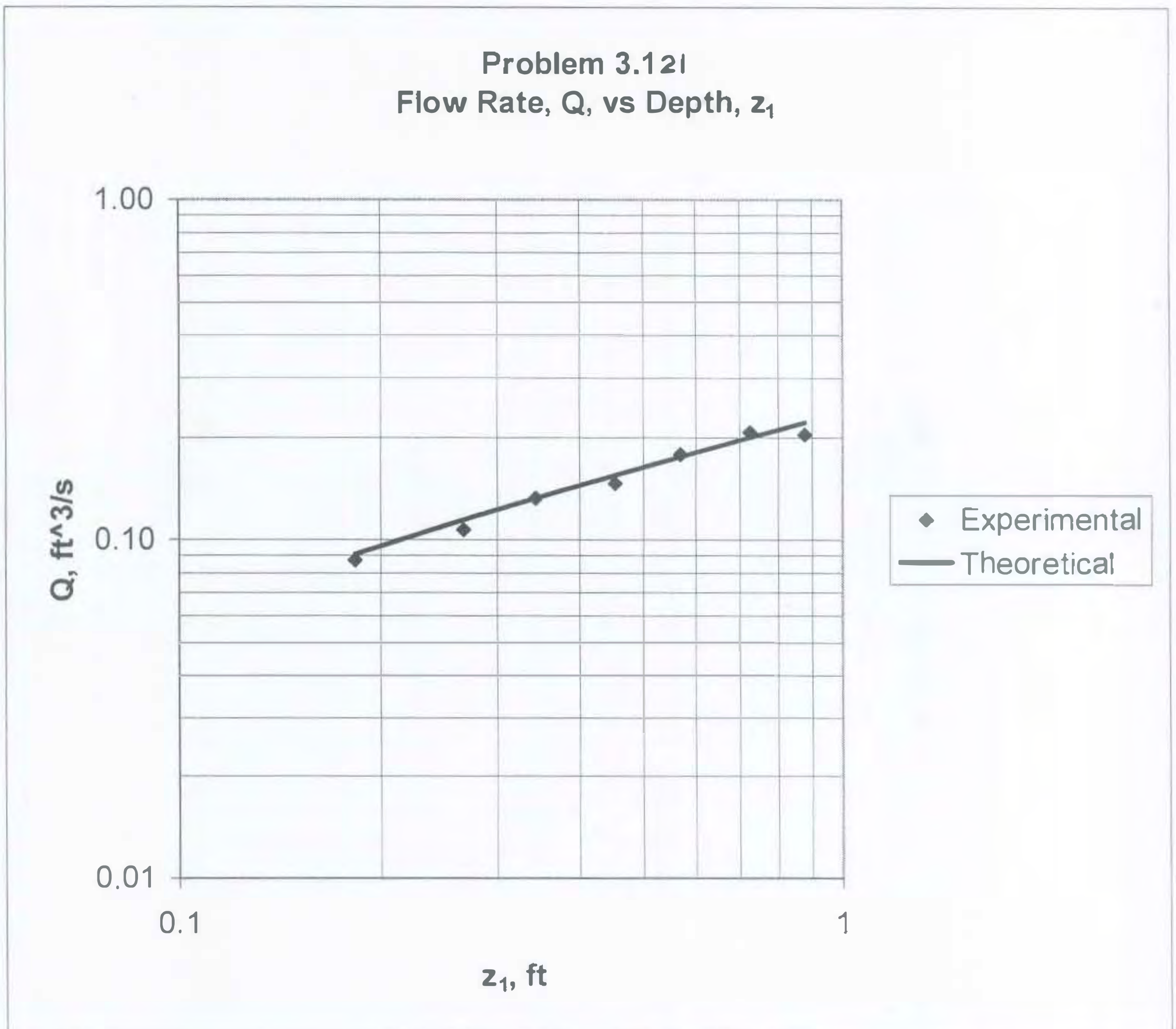
$Q = V_1 b z_1$

Theoretical:

$Q = b*z_2^{3/2}*(2*g)^{1/2}*[((z_1/z_2) - 1)/(1 - (z_2/z_1)^2)]^{1/2}$

where

$z_2 = 0.61*a$



4.4

4.4 The x- and y-components of a velocity field are given by $u = -(V_0/\ell)\,x$ and $v = -(V_0/\ell)\,y$, where V_0 and ℓ are constants. Make a sketch of the velocity field in the first quadrant ($x > 0, y > 0$) by drawing arrows representing the fluid velocity at representative locations.

$u = -(V_0/\ell)x$ and $v = -(V_0/\ell)y$ so that

$$V = \sqrt{u^2+v^2} = (V_0/\ell)\sqrt{(-x)^2+(-y^2)} = (V_0/\ell)\sqrt{x^2+y^2}$$

Thus, with $r = \sqrt{x^2+y^2}$ = radial distance from the origin,

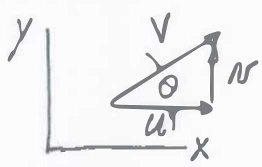
$$V = (V_0/\ell)\,r$$

Hence, $V = V_0$ on $r = \ell$; $V = 2V_0$ on $r = 2\ell$; $V = \frac{1}{2}V_0$ on $r = \frac{1}{2}\ell$; etc.

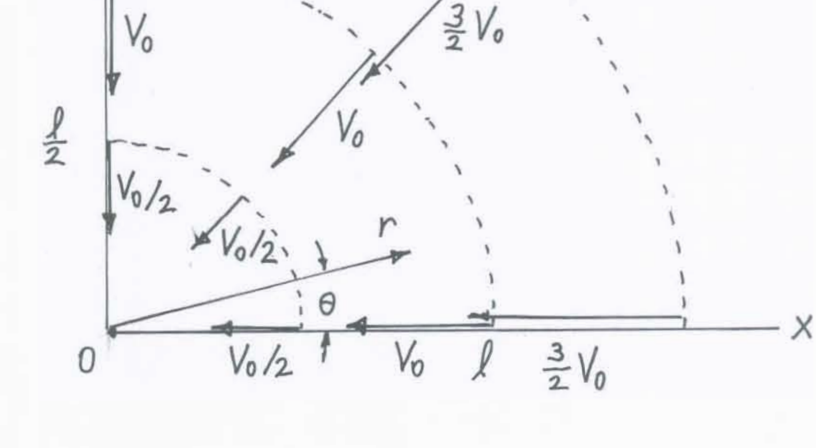
Also, the direction of the fluid motion relative to the x axis is

$\theta = \arctan(v/u)$ or

$$\tan\theta = \frac{v}{u} = \frac{-(V_0/\ell)y}{-(V_0/\ell)x} = \frac{y}{x}$$



Thus, on the x axis ($y=0$), $\tan\theta = 0$, or $\theta = 0°$ or $180°$ ($180°$ for $x>0$)
and on the y axis ($x=0$), $\tan\theta = \pm\infty$, or $\theta = 90°$ or $270°$ ($270°$ for $y>0$)
The velocity field looks as shown below. In the 1st quadrand, both $x>0$ and $y>0$ so that both $u<0$ and $v<0$.



4.5

4.5 A two-dimensional velocity field is given by $u = 1 + y$ and $v = 1$. Determine the equation of the streamline that passes through the origin. On a graph, plot this streamline.

$u = 1 + y$ and $v = 1$ so the streamlines are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{1}{1+y}$$

Thus,

$$\int (1+y)\,dy = \int dx \quad \text{or}$$

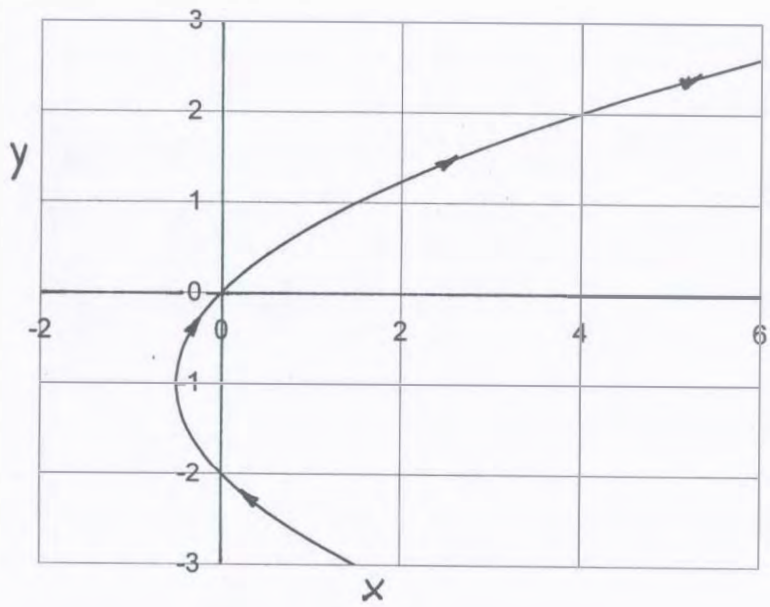
$y + \frac{1}{2}y^2 = x + C$, where C is a constant.

For the streamline that goes through $x = y = 0$, $C = 0$.

Hence,

$$\underline{\underline{x = y + \tfrac{1}{2}y^2}}$$

This streamline is plotted below. Note that since $v = 1 > 0$, the direction of flow is as shown.



4.6

4.6 The velocity field of a flow is given by $V = (5z - 3)\hat{\mathbf{i}} + (x + 4)\hat{\mathbf{j}} + 4y\hat{\mathbf{k}}$ ft/s, where x, y, and z are in feet. Determine the fluid speed at the origin ($x = y = z = 0$) and on the x axis ($y = z = 0$).

$u = 5z - 3$, $v = x + 4$, $w = 4y$

Thus, at the origin $u = -3$, $v = 4$, $w = 0$

so that

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(-3)^2 + 4^2} = \underline{\underline{5 \text{ ft/s}}}$$

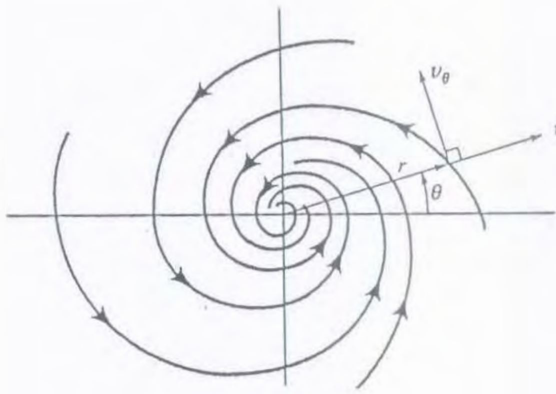
Similarly, on the x axis $u = -3$, $v = x + 4$, $w = 0$

so that

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(-3)^2 + (x+4)^2} = \underline{\underline{\sqrt{x^2 + 8x + 25}}} \text{ ft/s, where } x \sim \text{ft}$$

4.7

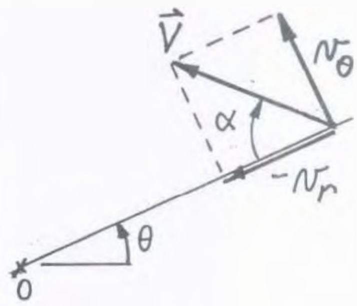
4.7 A flow can be visualized by plotting the velocity field as velocity vectors at representative locations in the flow as shown in Video V4.2 and Fig. E4.1. Consider the velocity field given in polar coordinates by $v_r = -10/r$ and $v_\theta = 10/r$. This flow approximates a fluid swirling into a sink as shown in Fig. P4.7 Plot the velocity field at locations given by $r = 1$, 2, and 3 with $\theta = 0$, 30, 60, and 90 deg.



■ FIGURE P4.7

With $v_r = -10/r$ and $v_\theta = 10/r$ then

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-10/r)^2 + (10/r)^2} = \frac{14.14}{r}$$



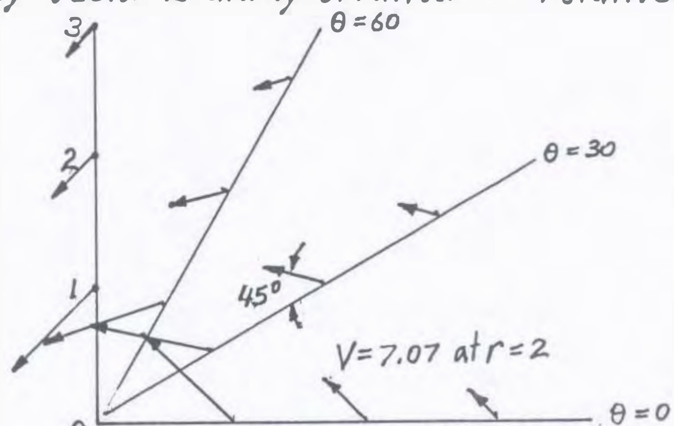
The angle α between the radial direction and the velocity vector is given by

$$\tan\alpha = \frac{v_\theta}{-v_r} = \frac{10/r}{-(-10/r)} = 1$$

Thus, $\alpha = 45°$ for any r, θ

(i.e. the velocity vector is alway oriented 45° relative to radial lines)

Note: V is independent of θ.



4.8

4.8 The velocity field of a flow is given by $\mathbf{V} = 20y/(x^2 + y^2)^{1/2}\hat{\mathbf{i}} - 20x/(x^2 + y^2)^{1/2}\hat{\mathbf{j}}$ ft/s, where x and y are in feet. Determine the fluid speed at points along the x axis; along the y axis. What is the angle between the velocity vector and the x axis at points $(x, y) = (5, 0)$, $(5, 5)$, and $(0, 5)$?

$$u = \frac{20y}{(x^2+y^2)^{1/2}} \quad , \quad v = -\frac{20x}{(x^2+y^2)^{1/2}}$$

Thus, $V = \sqrt{u^2+v^2}$ or

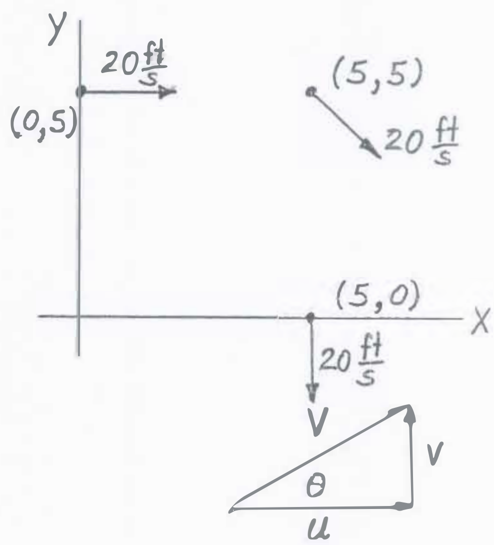
$$V = \left[\frac{400x^2 + 400y^2}{(x^2+y^2)}\right]^{1/2} = \underline{\underline{20\,\frac{ft}{s} \text{ for any } x, y}}$$

Also,

$$\tan\theta = \frac{v}{u} = \frac{\dfrac{-20x}{(x^2+y^2)^{1/2}}}{\dfrac{20y}{(x^2+y^2)^{1/2}}}$$

or

$$\tan\theta = -\frac{x}{y}$$



Thus, for $(x,y) = (5,0)$

$\tan\theta = -\infty$ or $\theta = \underline{\underline{-90^\circ}}$

for $(x,y) = (5,5)$

$\tan\theta = -1$ or $\theta = \underline{\underline{-45^\circ}}$

for $(x,y) = (0,5)$

$\tan\theta = 0$ or $\theta = \underline{\underline{0^\circ}}$

4.9

4.9 The components of a velocity field are given by $u = x + y$, $v = xy^3 + 16$, and $w = 0$. Determine the location of any stagnation points ($V = 0$) in the flow field.

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(x+y)^2 + (xy^3+16)^2} = 0$$

or

$u = x + y = 0$ so that $x = -y$

and

$v = xy^3 + 16 = 0$ so that $xy^3 = -16$

Hence, $(-y)y^3 = -16$, or $y = 2$

Therefore, $V = 0$ at $\underline{\underline{x = -2,\ y = 2}}$

4.10

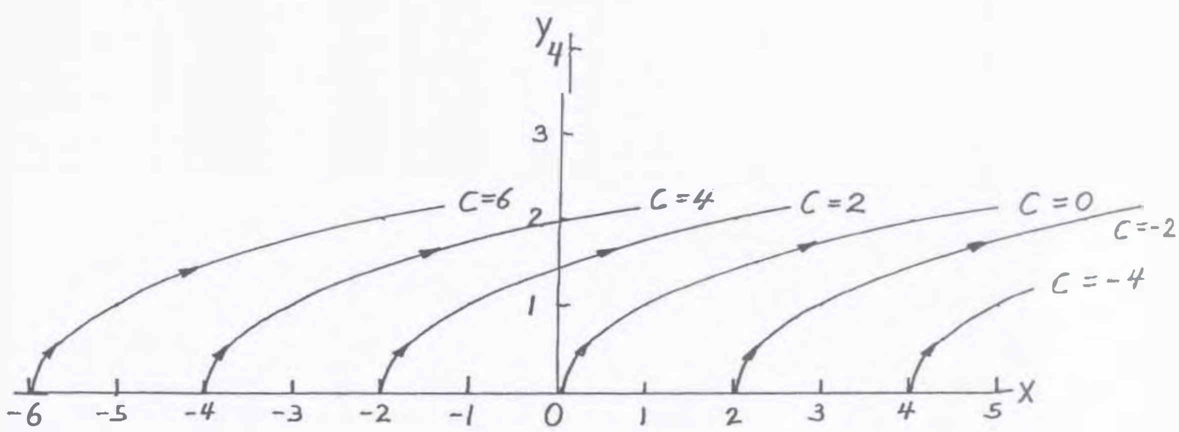
4.10 The x and y components of velocity for a two-dimensional flow are $u = 6y$ ft/s and $v = 3$ ft/s, where y is in feet. Determine the equation for the streamlines and sketch representative streamlines in the upper half plane.

$u = 6y$, $v = 3$ where streamlines are obtained from

$\frac{dy}{dx} = \frac{v}{u} = \frac{3}{6y}$ or $2y\,dy = dx$ which can be integrated to give

$\underline{\underline{y^2 = x + C}}$, where C is a constant.

Representative streamlines corresponding to different values of C are shown below.



Note that for $y>0$, $u>0$ (i.e., the flow is from left to right)

4.11

4.11 Show that the streamlines for a flow whose velocity components are $u = c(x^2 - y^2)$ and $v = -2cxy$, where c is a constant, are given by the equation $x^2y - y^3/3$ = constant. At which point (points) is the flow parallel to the y axis? At which point (points) is the fluid stationary?

$u = c(x^2 - y^2)$, $v = -2cxy$

Streamlines given by $y = f(x)$ are such that $\frac{dy}{dx} = \frac{v}{u}$

Consider the function $x^2y - \frac{y^3}{3} = \text{const.}$ (1)

Note: It is not easy to write this explicitly as $y = f(x)$

However, we can differentiate Eq. (1) to give

$$2xy\,dx + x^2 dy - y^2 dy = 0 \text{ , or}$$

$$(x^2 - y^2)dy + 2xy\,dx = 0$$

Thus, the lines in the x-y plane given by Eq. (1) have a slope

$$\frac{dy}{dx} = \frac{-2xy}{(x^2-y^2)} \quad \text{or for any constant } c, \quad \underline{\underline{\frac{dy}{dx} = \frac{-2cxy}{c(x^2-y^2)}}} \equiv \frac{v}{u}$$

i.e. the function $x^2y - \frac{y^3}{3} = \text{const.}$ represents the streamlines of the given flow.

The flow is parallel to the x-axis when $\frac{dy}{dx} = 0$, or $v = 0$.

This occurs when either $x = 0$ or $y = 0$, i.e., the x-axis or the y-axis

The flow is parallel to the y-axis when $\frac{dy}{dx} = \infty$, or $u = 0$.

This occurs when $\underline{\underline{x = \pm y}}$

The fluid has zero velocity at $\underline{\underline{x = y = 0}}$

4.12

4.12 A velocity field is given by $\mathbf{V} = x\hat{\mathbf{i}} + x(x-1)(y+1)\hat{\mathbf{j}}$, where u and v are in ft/s and x and y are in feet. Plot the streamline that passes through $x = 0$ and $y = 0$. Compare this streamline with the streakline through the origin.

$u = x$, $v = x(x-1)(y+1)$ where the streamlines are obtained from

$$\frac{dy}{dx} = \frac{v}{u} = \frac{x(x-1)(y+1)}{x} = (x-1)(y+1)$$

or

$$\int \frac{dy}{(y+1)} = \int (x-1)\,dx$$

which when integrated gives

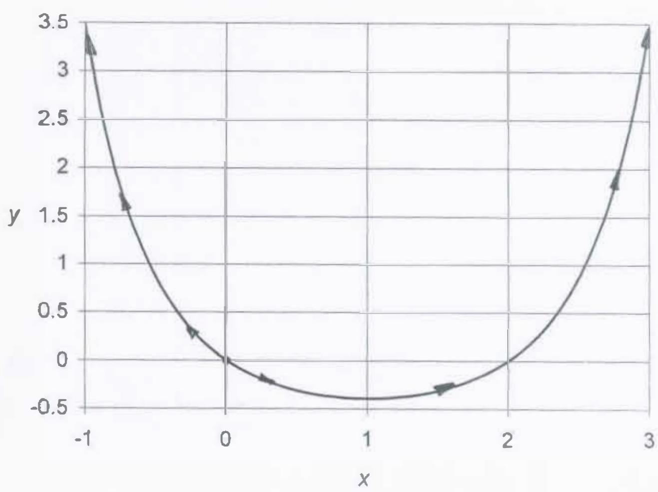
$$\ln(y+1) = \tfrac{1}{2}x^2 - x + C \text{, where } C \text{ is a constant} \qquad (1)$$

For the streamline that passes through the origin $x = y = 0$ the value of C is found from Eq. (1) as

$\ln(1) = C$, or $C = 0$

Thus, $\ln(y+1) = \frac{1}{2}x^2 - x$ or $\underline{\underline{y = e^{(\frac{1}{2}x^2 - x)} - 1}}$

This streamline is plotted below.



Note: The streamline is symmetrical about its low point of $x = 1$, $y = -0.393$. At $x = y = 0$ the velocity is 0.

For $x < 0$, $u < 0$ and for $x > 0$, $u > 0$. Thus, the fluid flows from the origin $(x = y = 0)$.

Since the flow is steady, <u>streaklines are the same as streamlines.</u>

4.13

4.13 From time $t = 0$ to $t = 5$ hr radioactive steam is released from a nuclear power plant accident located at $x = -1$ mile and $y = 3$ miles. The following wind conditions are expected: $\mathbf{V} = 10\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$ mph for $0 < t < 3$ hr, $\mathbf{V} = 15\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$ mph for $3 < t < 10$ hr, and $\mathbf{V} = 5\hat{\mathbf{i}}$ mph for $t > 10$ hr. Draw to scale the expected streakline of the steam for $t = 3$, 10, and 15 hr.

For $0<t<3hr$, $u = 10\,mph$ and $v = -5\,mph$

For $3<t<10hr$, $u = 15\,mph$ and $v = 8\,mph$

For $t>10hr$, $u = 5\,mph$ and $v = 0$

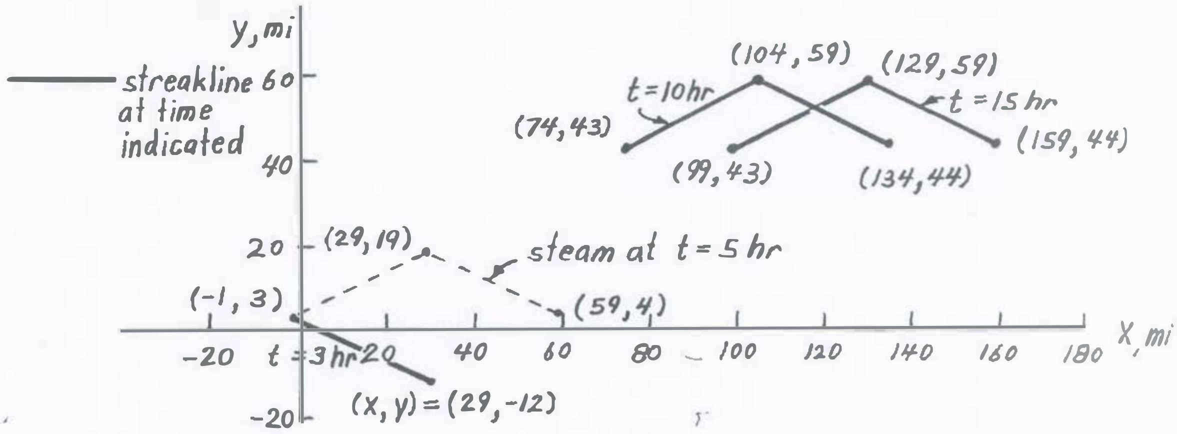
The streakline is the location (at time t) of steam released earlier.

a) At $t=3hr$ steam is still being released. From $t=0$ to $t=3hr$ it has traveled in the direction $\frac{dy}{dx} = \frac{v}{u} = -\frac{5}{10} = -0.5$ and the first of the steam is at $x = -1mi + (10mph)(3hr) = 29mi$ and $y = 3mi + (-5mph)(3hr) = -12mi$ at $t=3hr$. See figure below.

b) At $t=5hr$ steam release stops. From $t=3hr$ to $t=5hr$ the steams travels $\Delta x = u\,\Delta t = (15mph)(5-3)hr = 30mi$ "east" and $\Delta y = v\,\Delta t = (8mph)(5-3)hr = 16mi$ "north". See figure below. For $t>5hr$ the streakline does not "grow" (i.e., no more steam released), it merely maintains its shape it had at $t=5hr$ ($\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 0$) and translates. From $t=5hr$ to $t=10hr$ it moves $\Delta x = u\Delta t = (15mph)(10-5)hr = 75mi$ farther "east" and $\Delta y = v\Delta t = (8mph)(10-5)hr = 40mi$ farther "north". See figure below.

c) For $10<t<15hr$ the steam moves $\Delta x = (5mph)(15-10)hr = 25mi$ "east" and $\Delta y = v\Delta t = 0\,mi$ "north".

The above is shown in the figure below.



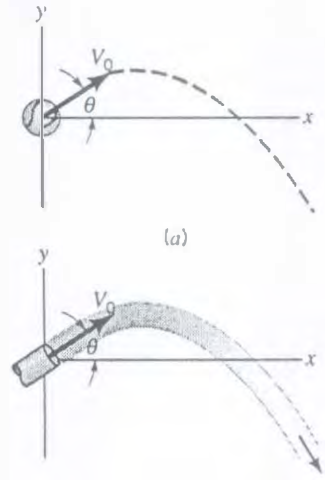
*4.14

4.14 Consider a ball thrown with initial speed V_0 at an angle of θ as shown in Fig. P4.14a. As discussed in beginning physics, if friction is negligible the path that the ball takes is given by

$$y = (\tan\theta)x - [g/(2\,V_0^2\cos^2\theta)]x^2$$

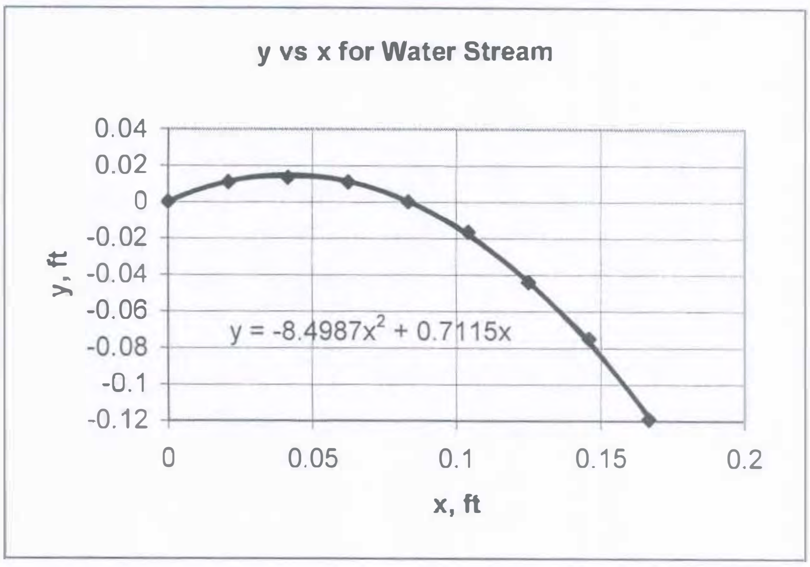
That is, $y = c_1x + c_2x^2$, where c_1 and c_2 are constants. The path is a parabola. The pathline for a stream of water leaving a small nozzle is shown in Fig. P4.14b and Video V4.12. The coordinates for this water stream are given in the following table. (a) Use the given data to determine appropriate values for c_1 and c_2 in the above equation and, thus, show that these water particles also follow a parabolic pathline. (b) Use your values of c_1 and c_2 to determine the speed of the water, V_0, leaving the nozzle.

x, in.	y, in.
0	0
0.25	0.13
0.50	0.16
0.75	0.13
1.0	0.00
1.25	−0.20
1.50	−0.53
1.75	−0.90
2.00	−1.43



■ FIGURE P4.14

An EXCEL Program was used to plot the x-y data and to fit a second order curve to the data. The results are shown below.



Thus, with $y = c_1x + c_2x^2$ it follows that

$c_1 = 0.7115 = \tan\theta$ or $\theta = 35.4°$

and

$$c_2 = -8.4987 = -\frac{g}{2V_0^2\cos^2\theta}$$

or

$$V_0^2 = \frac{32.2}{2(8.4987)\cos^2(35.4°)} = 2.85\,\frac{ft^2}{s^2}$$

Thus, $\underline{\underline{V_0 = 1.69\,\frac{ft}{s}}}$

4.15

4.15 The x and y components of a velocity field are given by $u = x^2y$ and $v = -xy^2$. Determine the equation for the streamlines of this flow and compare with those in Example 4.2. Is the flow in this problem the same as that in Example 4.2? Explain.

Streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = -\frac{xy^2}{x^2y} = -\frac{y}{x}$

or $\frac{dy}{y} = -\frac{dx}{x}$ which can be integrated as:

$\int \frac{dy}{y} = -\int \frac{dx}{x}$ Thus, $\ln y = -\ln y + \tilde{C}$, where $\tilde{C}$ is a constant.

Thus, $\underline{\underline{xy = C}}$

Note: These streamlines are the same shape (same "flow pattern") as in Example 4.2 — but the velocity fields are different. However, the ratios $\frac{v}{u}$ are the same:

$\frac{v}{u} = -\frac{xy^2}{x^2y} = -\frac{y}{x}$ for this problem

and

$\frac{v}{u} = \frac{(V_0/\ell)(+y)}{(V_0/\ell)(-x)} = -\frac{y}{x}$ for Example 4.2.

4.16

4.16 A flow in the x–y plane is given by the following velocity field: $u = 3$ and $v = 6$ m/s for $0 < t < 20$ s; $u = -4$ and $v = 0$ m/s for $20 < t < 40$ s. Dye is released at the origin ($x = y = 0$) for $t = 0$. (a) Draw the pathline at $t = 30$ s for two particles that were released from the origin—one released at $t = 0$ and the other released at $t = 20$ s. (b) On the same graph draw the streamlines at times $t = 10$ s and $t = 30$ s.

(a) For the particle released at $t=0$, $u = 3\,\frac{m}{s}$ and $v = 6\,\frac{m}{s}$ for $0<t<20s$. During this time the flow is steady and the pathline has a slope $\frac{dy}{dx} = \frac{v}{u} = \frac{6}{3} = 2$. At $t=0$, $x=y=0$ and at $t=20$, $x = (3\,\frac{m}{s})(20s) = 60m$ and $y = (6\,\frac{m}{s})(20s) = 120m$

For $20<t<30$, $u = -4\,\frac{m}{s}$ and $v=0$, so that the flow is steady and the pathline has a slope of $\frac{dy}{dx} = 0$. The particle moves from $x = 60m$ to $x = 60 + (-4\,\frac{m}{s})(30-20)s = +20m$, but keeps the $y = 120m$ location during $20<t<30s$. This pathline is shown in the figure below.

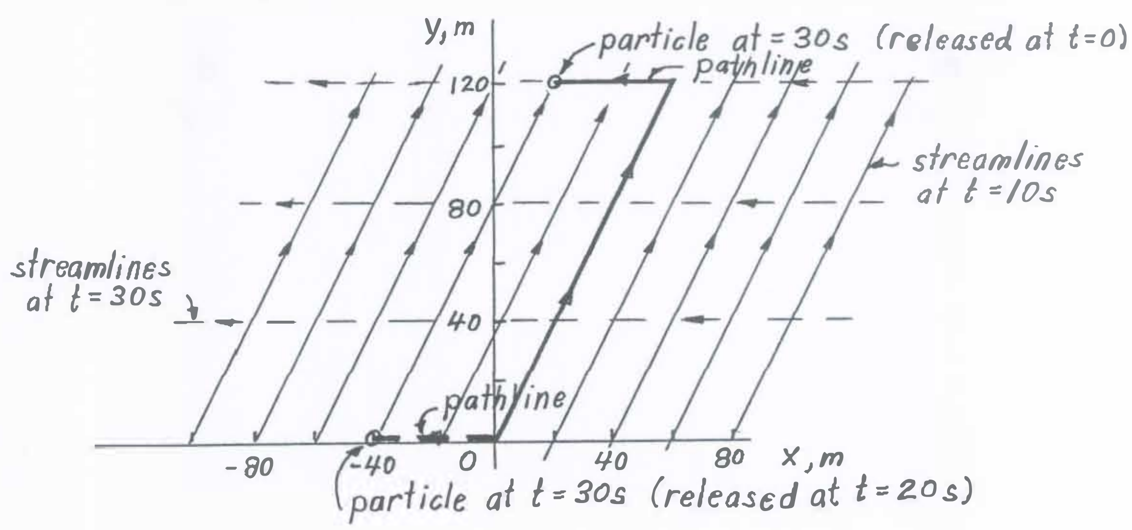
For the particle released at the origin at $t = 20s$ it follows that $u = -4\,\frac{m}{s}$ and $v=0$. Thus, the corresponding pathline extends from $x=0$ to $x = (-4\,\frac{m}{s})(30-20)s = -40m$ at $t = 30s$. This pathline is shown in the figure below.

(b) At $t = 10s$, streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = \frac{6}{3} = 2$

or $y = 2x + C_1$, where $C_1 = const.$

At $t = 30s$, streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = 0$

or $y = C_2$, where $C_2 = const.$ These lines are shown below.



4.17

4.17 In addition to the customary horizontal velocity components of the air in the atmosphere (the "wind"), there often are vertical air currents (thermals) caused by buoyant effects due to uneven heating of the air as indicated in Fig. P4.17. Assume that the velocity field in a certain region is approximated by $u = u_0$, $v = v_0(1 - y/h)$ for $0 < y < h$, and $u = u_0$, $v = 0$ for $y > h$. Plot the shape of the streamline that passes through the origin for values of $u_0/v_0 = 0.5$, 1, and 2.

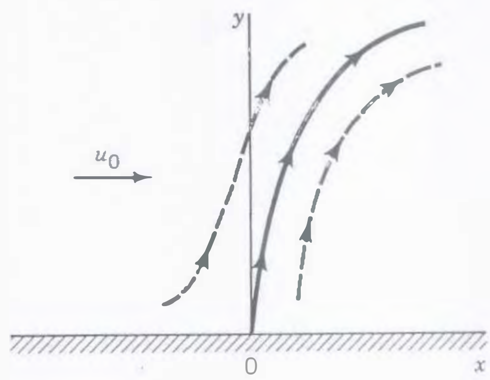


FIGURE P4.17

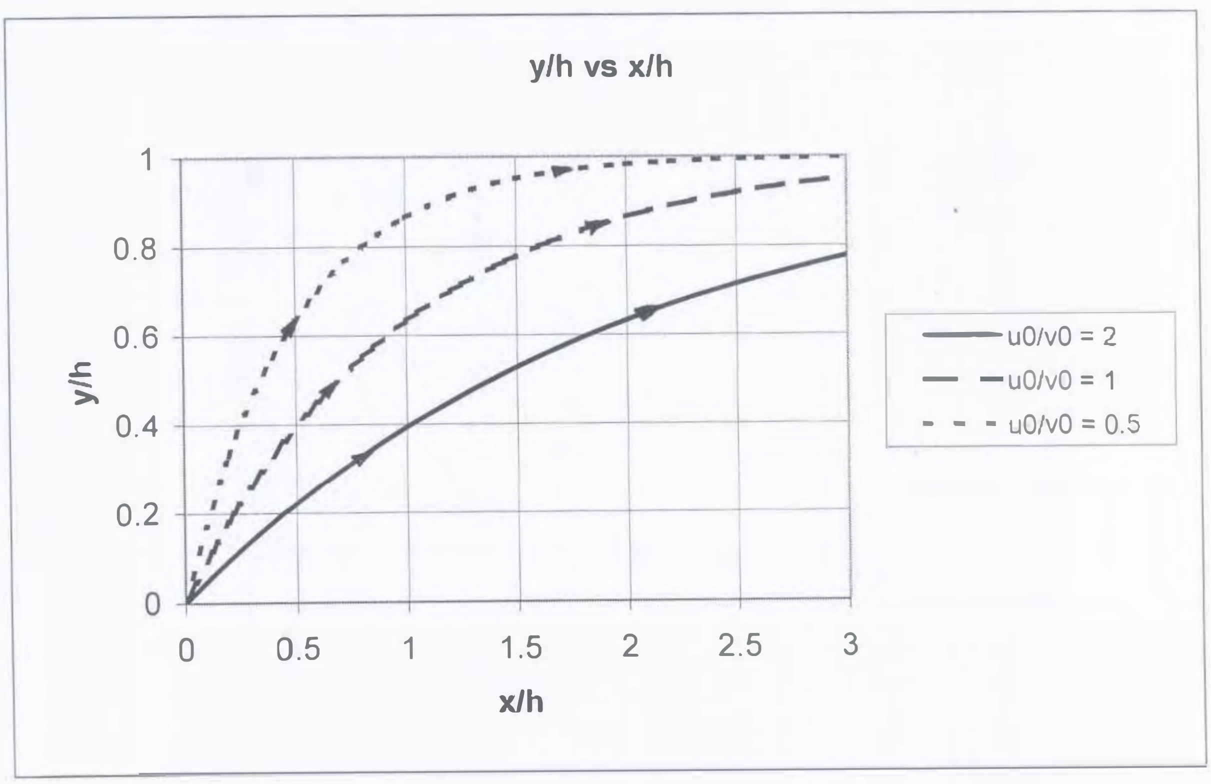
$u = u_0$, $v = v_0\left(1-\frac{y}{h}\right)$ for $0<y<h$ so that streamlines for $y<h$ are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{v_0\left(1-\frac{y}{h}\right)}{u_0} \quad \text{or} \quad \int_0^y \frac{dy}{\left(1-\frac{y}{h}\right)} = \frac{v_0}{u_0}\int_0^x dx$$

Thus, $-h\ln\left(1-\frac{y}{h}\right) = \frac{v_0}{u_0}x$ Note: The lower limits of integration $(x=0,\ y=0)$ insure that this equation is for the streamline through the origin.

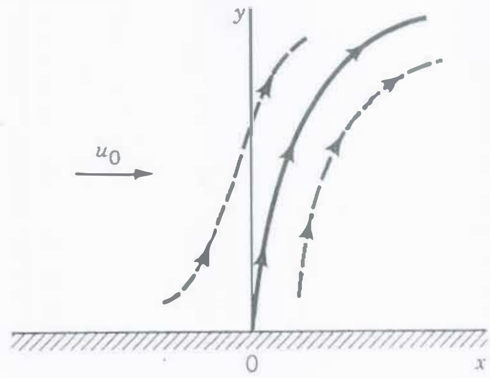
This streamline

$\underline{x = -h\left(\frac{u_0}{v_0}\right)\ln\left(1-\frac{y}{h}\right)}$ is plotted below.



*4.18

***4.18** Repeat Problem 4.17 using the same information except that $u = u_0 y/h$ for $0 \le y \le h$ rather than $u = u_0$. Use values of $u_0/v_0 = 0, 0.1, 0.2, 0.4, 0.6, 0.8$, and 1.0.



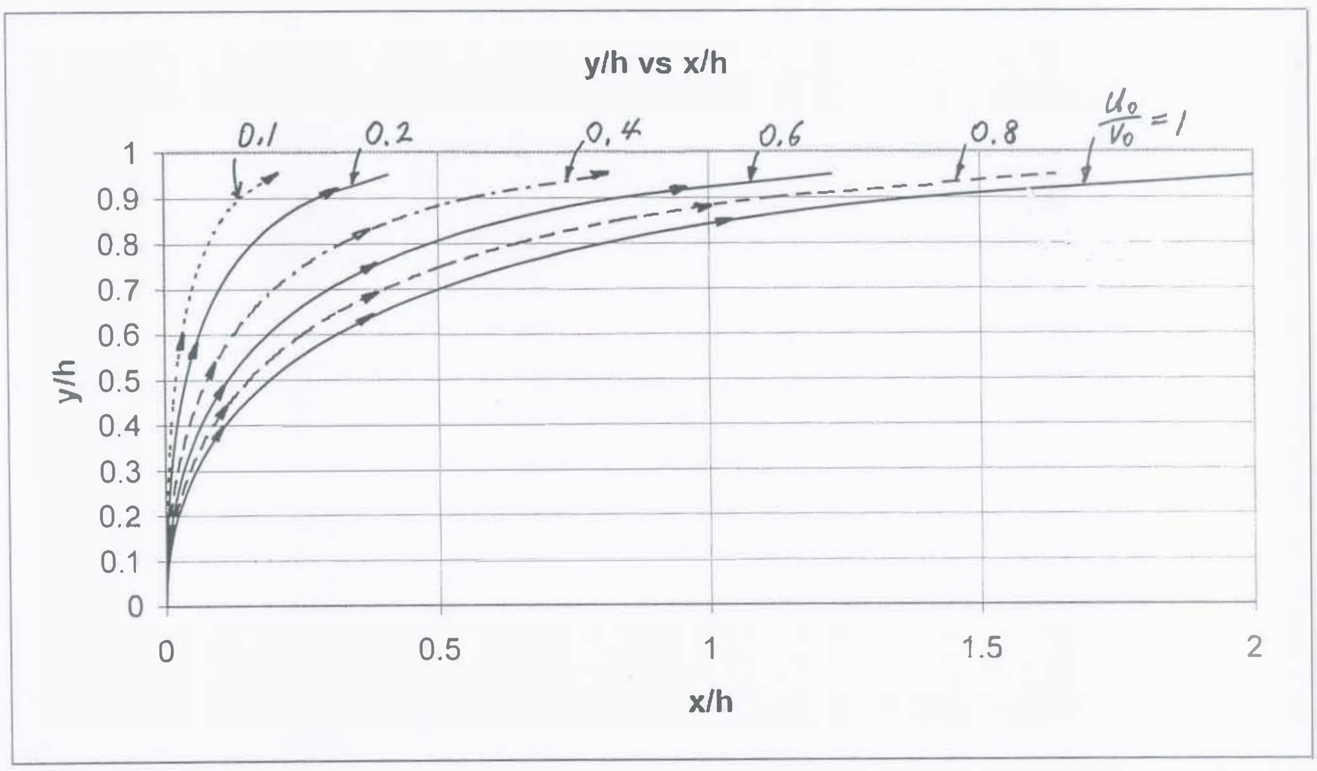
$u = \frac{u_0 y}{h}$, $v = v_0(1-\frac{y}{h})$ for $0<y<h$ so that streamlines for $y<h$ are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{v_0(1-\frac{y}{h})}{u_0\frac{y}{h}} = \frac{v_0}{u_0}\,\frac{(h-y)}{y}$$ or with $x=0$ when $y=0$

$$\int_0^y \frac{y}{(h-y)}\,dy = \int_0^x \frac{v_0}{u_0}\,dx$$ This integrates to give

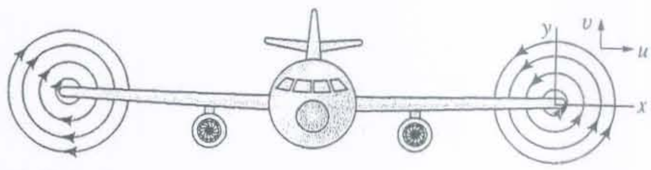
$$-y - h\ln(h-y) + h\ln(h) = \frac{v_0}{u_0}x \quad \text{or} \quad \frac{x}{h} = \left(\frac{u_0}{v_0}\right)\left[\ln\left(\frac{h}{h-y}\right) - \frac{y}{h}\right]$$

This streamline is plotted below for $0 \le \frac{y}{h} \le 1$, with $\frac{u_0}{v_0} = 0, 0.1, 0.2, 0.4, 0.6, 0.8$, and 1.0 The values were calculated and plotted using an EXCEL Program.



4.19

4.19 As shown in Video V4.6 and Fig. P4.19, a flying airplane produces swirling flow near the end of its wings. In certain circumstances this flow can be approximated by the velocity field $u = -Ky/(x^2 + y^2)$ and $v = Kx/(x^2 + y^2)$, where K is a constant depending on various parameter associated with the airplane (i.e., its weight, speed) and x and y are measured from the center of the swirl. (a) Show that for this flow the velocity is inversely proportional to the distance from the origin. That is, $V = K/(x^2 + y^2)^{1/2}$. (b) Show that the streamlines are circles.



■ FIGURE P4.19

(a) $V = \sqrt{u^2 + v^2} = \left[\dfrac{(-Ky)^2}{(x^2+y^2)^2} + \dfrac{(Kx)^2}{(x^2+y^2)^2}\right]^{1/2} = \dfrac{K}{\sqrt{x^2+y^2}}$

or

$\underline{\underline{V = \dfrac{K}{r}}}$, where $r = \sqrt{x^2+y^2}$

(b) Streamlines are given by $\dfrac{dy}{dx} = \dfrac{v}{u} = \dfrac{\dfrac{Kx}{(x^2+y^2)}}{\dfrac{-Ky}{(x^2+y^2)}} = -\dfrac{x}{y}$

Thus,

$y\,dy = -x\,dx$ which when integrated gives

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$, where C_1 is a constant.

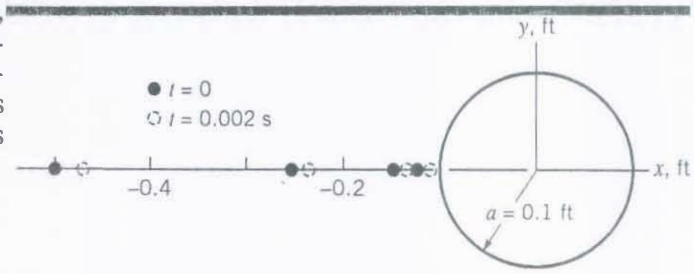
or

$\underline{\underline{x^2 + y^2 = \text{Constant}}}$

4.20

4.20 (See Fluids in the News article titled **"Follow those particles,"** Section 4.1.) Two photographs of four particles in a flow past a sphere are superposed as shown in Fig. P4.20. The time interval between the photos is $\Delta t = 0.002$ s. The locations of the particles, as determined from the photos, are shown in the table. **(a)** Determine the fluid velocity for these particles. **(b)** Plot a graph to compare the results of part **(a)** with the theoretical velocity which is given by $V = V_0(1 + a^3/x^3)$, where a is the sphere radius and V_0 is the fluid speed far from the sphere.

Particle	x at $t = 0$ s (ft)	x at $t = 0.002$ s (ft)
1	−0.500	−0.480
2	−0.250	−0.232
3	−0.140	−0.128
4	−0.120	−0.112



■ FIGURE P4.20

The fluid velocity (which is assumed to be the same as the particle velocity) can be estimated by

$V = \Delta x / \Delta t$

Thus, for particle (1): $V_1 = [-0.480\,ft - (-0.500\,ft)]/(0.002\,s) = 10\ ft/s$

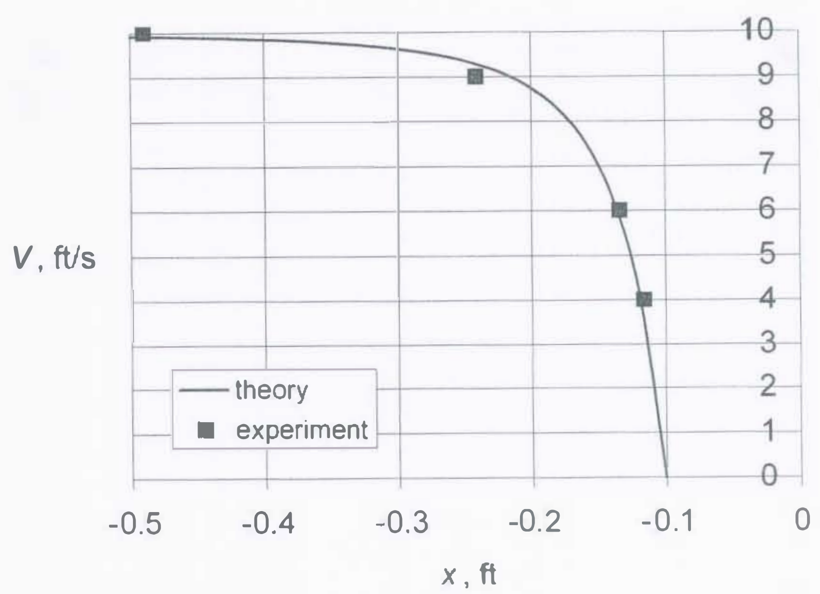
During to 0.002s time interval the average location of the particle was

$x = [(-0.480\,ft) + (-0.500\,ft)] = -0.0490\ ft$

By similar calculations the following experimental results were obtained:

Particle	x, ft	V, ft/s
1	−0.490	10
2	−0.241	9
3	−0.134	6
4	−0.116	4

These experimental results and the theoretical results ($V = V_0(1 + a^3/x^3)$, where $V_0 = 10$ ft/s and $a = 0.1$ ft) are plotted in the figure below.



4.21

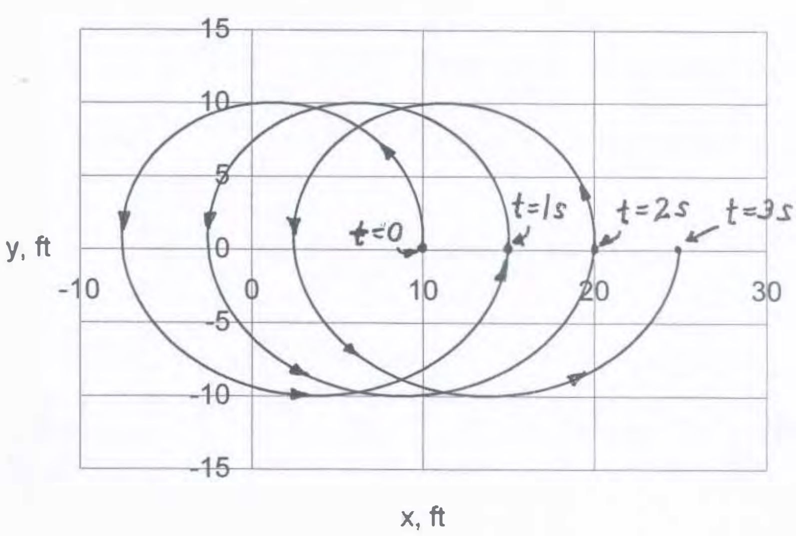
4.21 (See Fluids in the News article titled "Winds on Earth and Mars," Section 4.1.4.) A 10-ft-diameter dust devil that rotates one revolution per second travels across the Martian surface (in the x-direction) with a speed of 5 ft/s. Plot the pathline etched on the surface by a fluid particle 10 ft from the center of the dust devil for time $0 \le t \le 3$ s. The particle position is given by the sum of that for a stationary swirl $[x = 10\ \cos(2\pi t), y = 10\ \sin(2\pi t)]$ and that for a uniform velocity ($x = 5t$, y = constant), where x and y are in feet and t is in seconds.

The path line is given by

$x = 10\cos(2\pi t) + 5t$
and
$y = 10\sin(2\pi t)$, where $x \sim ft$, $y \sim ft$, and $t \sim s$

This path is plotted for $0 \le t \le 3s$ below.

Particle Path



4.22

4.22 The x- and y-components of a velocity field are given by $u = (V_0/\ell)x$ and $v = -(V_0/\ell)y$, where V_0 and ℓ are constants. Plot the streamlines for this flow and determine the acceleration field.

Since $u = (V_0/\ell)x$ and $v = -(V_0/\ell)y$, the streamlines are given by

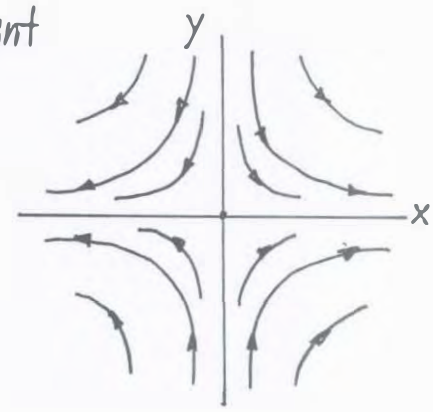
$$\frac{dy}{dx} = \frac{v}{u} = \frac{-(V_0/\ell)y}{(V_0/\ell)x} = -\frac{y}{x} \quad \text{or}$$

$\dfrac{dy}{y} = -\dfrac{dx}{x}$ which can be integrated to give

$\displaystyle\int \frac{dy}{y} = -\int \frac{dx}{x}$ or $\ln y = -\ln x + \text{constant}$

Hence, the streamlines are $xy = \text{constant}$

Typical streamlines (hyperbolas) are sketched in the figure to the right.



Also,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (V_0/\ell)x(V_0/\ell) + 0 = (V_0/\ell)^2 x$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 0 + (-(V_0/\ell)y)(-V_0/\ell) = (V_0/\ell)^2 y$$

Thus,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \underline{\underline{(V_0/\ell)^2[x\hat{i} + y\hat{j}]}}$$

4.23

4.23 A velocity field is given by $u = cx^2$ and $v = cy^2$, where c is a constant. Determine the x and y components of the acceleration. At what point (points) in the flow field is the acceleration zero?

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (cx^2)(2cx) = \underline{\underline{2c^2x^3}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (cy^2)(2cy) = \underline{\underline{2c^2y^3}}$$

Thus, $\vec{a} = a_x\hat{i} + a_y\hat{j} = 0$ at $\underline{\underline{(x,y) = (0,0)}}$

4.24

4.24 Determine the acceleration field for a three-dimensional flow with velocity components $u = -x$, $v = 4x^2y^2$, and $w = x - y$.

$u = -x$, $v = 4x^2y^2$, and $w = x - y$ so that

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$= 0 + (-x)(-1) + 4x^2y^2(0) + (x-y)(0) = x$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$= 0 + (-x)(8xy^2) + (4x^2y^2)(8x^2y) + (x-y)(0)$$
$$= -8x^2y^2 + 32x^4y^3 = 8x^2y^2(4x^2y - 1)$$

and

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$
$$= 0 + (-x)(1) + (4x^2y^2)(-1) + (x-y)(0)$$
$$= -x - 4x^2y^2$$

Thus,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$
$$\underline{\underline{= x\hat{i} + 8x^2y^2(4x^2y-1)\hat{j} - (x+4x^2y^2)\hat{k}}}$$

4.26

4.26 The velocity of air in the diverging pipe shown in Fig. P4.26 is given by $V_1 = 4t$ ft/s and $V_2 = 2t$ ft/s, where t is in seconds. **(a)** Determine the local acceleration at points (1) and (2). **(b)** Is the average convective acceleration between these two points negative, zero, or positive? Explain.

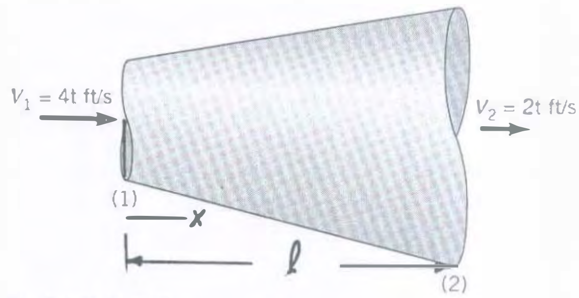


FIGURE P4.26

a) $\left.\frac{\partial u}{\partial t}\right|_{(1)} = 4\,\frac{ft}{s^2}$ and $\left.\frac{\partial u}{\partial t}\right|_{(2)} = 2\,\frac{ft}{s^2}$

b) convective acceleration along the pipe $= u\,\frac{\partial u}{\partial x}$ where $u > 0$. At any time, t, $V_2 < V_1$. Thus, between (1) and (2)

$$\frac{\partial u}{\partial x} \approx \frac{V_2 - V_1}{\ell} < 0$$

Hence, $u\,\frac{\partial u}{\partial x} < 0$ or the average convective acceleration is negative.

4.27

4.27 Water flows in a pipe so that its velocity triples every 20 s. At $t = 0$ it has $u = 5$ ft/s. That is, $\mathbf{V} = u(t)\hat{\mathbf{i}} = 5\,(3^{t/20})\hat{\mathbf{i}}$ ft/s. Determine the accleration when $t = 0$, 10, and 20 s.

$u = 5(3^{t/20})$, $v = 0$, $w = 0$

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} = \frac{\partial u}{\partial t}\hat{i}$ since $\vec{V}\cdot\nabla\vec{V} \equiv 0$ because $\vec{V}$ is not a a function of $x, y,$ or z.

Since $\frac{\partial u}{\partial t} = 5\left[3^{t/20}\ln(3)\frac{1}{20}\right] = 0.275\,(3^{t/20})\frac{ft}{s^2}$ with $t \sim s$

it follows that

$\vec{a} = \underline{\underline{0.275\hat{i}\,\frac{ft}{s^2}}}$ at $t = 0$

$\vec{a} = \underline{\underline{0.476\hat{i}\,\frac{ft}{s^2}}}$ at $t = 10\,s$

and

$\vec{a} = \underline{\underline{0.825\hat{i}\,\frac{ft}{s^2}}}$ at $t = 20\,s$

4.28

4.28 When a valve is opened, the velocity of water in a certain pipe is given by $u = 10(1 - e^{-t})$, $v = 0$, and $w = 0$, where u is in ft/s and t is in seconds. Determine the maximum velocity and maximum acceleration of the water.

$V = \sqrt{u^2 + v^2 + w^2} = 10(1 - e^{-t})$ so that $\frac{dV}{dt} = 10e^{-t} > 0$ for all t

Thus, $V_{max} = V\Big|_{t=\infty} = \underline{\underline{10\,\frac{ft}{s}}}$

Also, $\vec{a} = a_x\hat{i}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$ with $\frac{\partial u}{\partial x} = 0$

Thus, $a_x = \frac{\partial u}{\partial t} = 10e^{-t}$, so that $a_{x_{max}} = a_x\Big|_{t=\infty} = \underline{\underline{10\,\frac{ft}{s^2}}}$

4.29

4.29 The velocity of the water in the pipe shown in Fig. P4.29 is given by $V_1 = 0.50t$ m/s and $V_2 = 1.0t$ m/s, where t is in seconds. Determine the local acceleration at points (1) and (2). Is the average convective acceleration between these two points negative, zero, or positive? Explain.

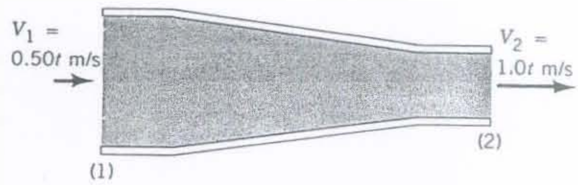


FIGURE P4.29

$$\frac{\partial V_1}{\partial t} = 0.5\ \frac{m}{s^2}$$

$$\frac{\partial V_2}{\partial t} = 1.0\ \frac{m}{s^2}$$

Since $V_2 > V_1$ it follows that $\frac{\partial V}{\partial x} > 0$. Also, $V > 0$ so that the convective acceleration, $V\frac{\partial V}{\partial x}$, is positive.

4.30

4.30 A shock wave is a very thin layer (thickness = ℓ) in a high-speed (supersonic) gas flow across which the flow properties (velocity, density, pressure, etc.) change from state (1) to state (2) as shown in Fig. P4.30. If $V_1 = 1800$ fps, $V_2 = 700$ fps, and $\ell = 10^{-4}$ in., estimate the average deceleration of the gas as it flows across the shock wave. How many g's deceleration does this represent?

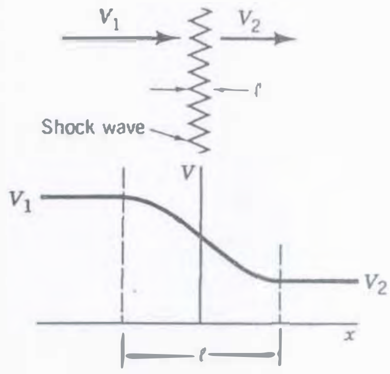


FIGURE P4.30

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ so with $\vec{V} = u(x)\hat{i}$, $\vec{a} = a_x\hat{i} = u\frac{\partial u}{\partial x}\hat{i}$

Without knowing the actual velocity distribution, $u = u(x)$, the acceleration can be approximated as

$$a_x = u\frac{\partial u}{\partial x} \approx \frac{(V_1+V_2)}{2}\,\frac{(V_2-V_1)}{\ell} = \frac{(1800+700)\,\text{fps}}{2}\,\frac{(700-1800)\,\text{fps}}{\left(\frac{10^{-4}}{12}\right)\text{ft}}$$

or

$$\underline{\underline{a_x = -1.65\times10^{11}\,\frac{\text{ft}}{\text{s}^2}}} \qquad \text{This is } \frac{a_x}{g} = \frac{-1.65\times10^{11}\,\frac{\text{ft}}{\text{s}^2}}{32.2\,\frac{\text{ft}}{\text{s}^2}} = \underline{\underline{-5.12\times10^{9}}}$$

4.32

4.32 As a valve is opened, water flows through the diffuser shown in Fig. P4.32 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 =$ 10 ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

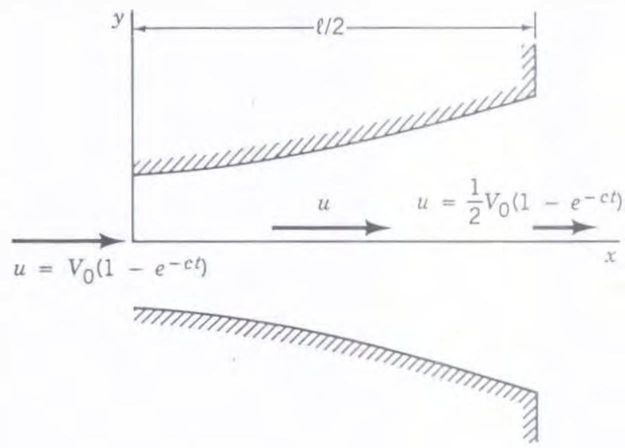


FIGURE P4.32

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x,t)$, $v = 0$, and $w = 0$ this becomes

$\vec{a} = (\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x})\hat{i} = a_x\hat{i}$, where $u = V_0(1-e^{-ct})(1-\frac{x}{\ell})$

Thus,

$$a_x = V_0(1-\tfrac{x}{\ell})c\,e^{-ct} + V_0^2(1-e^{-ct})^2(1-\tfrac{x}{\ell})(-\tfrac{1}{\ell})$$

or

$$a_x = V_0(1-\tfrac{x}{\ell})\left[c\,e^{-ct} - \tfrac{V_0}{\ell}(1-e^{-ct})^2\right]$$

If $a_x = 0$ for any x at $t = 1$ s we must have

$\left[c\,e^{-ct} - \frac{V_0}{\ell}(1-e^{-ct})^2\right] = 0$ With $V_0 = 10$ and $\ell = 5$

$c\,e^{-c} - \frac{10}{5}(1-e^{-c})^2 = 0$ The solution (root) of this equation is $\underline{\underline{C = 0.490\ \frac{1}{s}}}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$). The flowrate increases with time, but the fluid flows to an area of lower velocity.

4.33

4.33 A fluid flows along the x axis with a velocity given by $\mathbf{V} = (x/t)\hat{\mathbf{i}}$, where x is in feet and t in seconds. (a) Plot the speed for $0 \le x \le 10$ ft and $t = 3$ s. (b) Plot the speed for $x = 7$ ft and $2 \le t \le 4$ s. (c) Determine the local and convective acceleration. (d) Show that the acceleration of any fluid particle in the flow is zero. (e) Explain physically how the velocity of a particle in this unsteady flow remains constant throughout its motion.

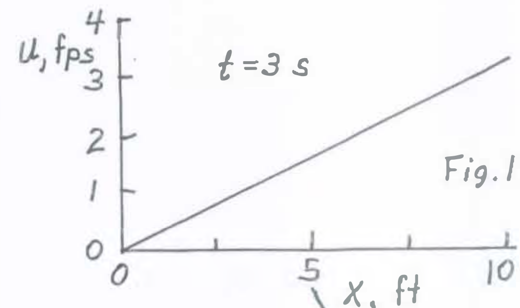


Fig.1

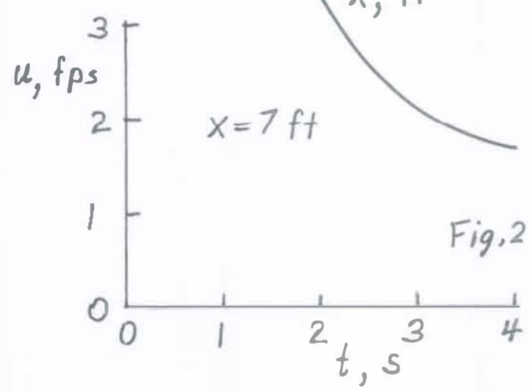


Fig.2

(a) $u = \frac{x}{t}\ \frac{ft}{s}$ so at $t = 3\,s$, $u = \frac{x}{3}\ \frac{ft}{s}$

(b) For $x = 7\,ft$, $u = \frac{7}{t}\ \frac{ft}{s}$

(c) $\frac{\partial u}{\partial t} = \underline{\underline{-\frac{x}{t^2}}}$ and $u\frac{\partial u}{\partial x} = \frac{x}{t}\left(\frac{1}{t}\right) = \underline{\underline{\frac{x}{t^2}}}$

(d) For any fluid particle $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ which with $v = 0$, $w = 0$ becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = \left(-\frac{x}{t^2} + \frac{x}{t^2}\right)\hat{\imath} \equiv 0$$

(e) The particles flow into areas of higher velocity (see Fig.1), but at any given location the velocity is decreasing in time (see Fig.2). For the given velocity field the local and convective accelerations are equal and opposite, giving zero acceleration throughout.

4.34

4.34 A hydraulic jump is a rather sudden change in depth of a liquid layer as it flows in an open channel as shown in Fig. P4.34 and **Video V10.12**. In a relatively short distance (thickness = ℓ) the liquid depth changes from z_1 to z_2, with a corresponding change in velocity from V_1 to V_2. If V_1 = 1.20 ft/s, V_2 = 0.30 ft/s, and ℓ = 0.02 ft, estimate the average deceleration of the liquid as it flows across the hydraulic jump. How many g's deceleration does this represent?

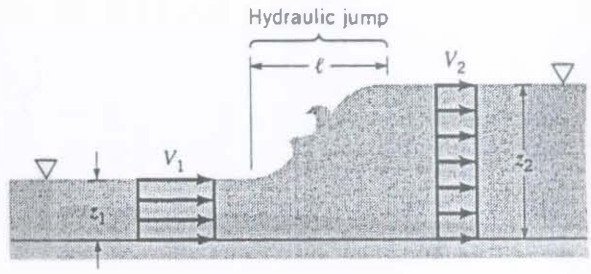


FIGURE P4.34

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} \quad \text{so with } \vec{V} = u(x)\hat{i}\,, \quad \vec{a} = a_x\hat{i} = u\frac{\partial u}{\partial x}\hat{i}$$

Without knowing the actual velocity distribution, $u = u(x)$, the acceleration can be approximated as

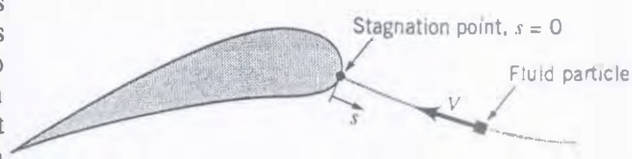
$$a_x = u\frac{\partial u}{\partial x} \approx \frac{1}{2}(V_1+V_2)\frac{(V_2-V_1)}{\ell} = \frac{1}{2}(1.20+0.30)\frac{ft}{s}\,\frac{(0.30-1.20)\frac{ft}{s}}{0.02\,ft}$$

$$= \underline{\underline{-33.8\,\frac{ft}{s^2}}}$$

Thus, $\frac{|a_x|}{g} = \frac{33.8\,\frac{ft}{s^2}}{32.2\,\frac{ft}{s^2}} = \underline{\underline{1.05}}$

4.35

4.35 A fluid particle flowing along a stagnation streamline, as shown in **Video V4.9** and Fig. P4.35, slows down as it approaches the stagnation point. Measurements of the dye flow in the video indicate that the location of a particle starting on the stagnation streamline a distance $s = 0.6$ ft upstream of the stagnation point at $t = 0$ is given approximately by $s = 0.6e^{-0.5t}$, where t is in seconds and s is in feet. **(a)** Determine the speed of a fluid particle as a function of time, $V_{particle}(t)$, as it flows along the streamline. **(b)** Determine the speed of the fluid as a function of position along the streamline, $V = V(s)$. **(c)** Determine the fluid acceleration along the streamline as a function of position, $a_s = a_s(s)$.



■ FIGURE P4.35

(a) With $s = 0.6\,e^{-0.5t}$ it follows that

$$V_{particle} = \frac{ds}{dt} = 0.6\,(-0.5)\,e^{-0.5t} = \underline{\underline{-0.3\,e^{-0.5t}}} \quad \text{ft/s}$$

(b) From part (a),

$$V = (-0.5)\left[0.6\,e^{-0.5t}\right] \quad \text{where } s = 0.6\,e^{-0.5t}$$

Thus,

$$V = (-0.5)[s], \quad \text{or} \quad V = \underline{\underline{-0.5s}} \ \text{ft/s where } s \sim \text{ft}$$

(c) For steady flow, $a_s = V\,\frac{dV}{ds}$

Thus, with $V = -0.5s$ and $\frac{dV}{ds} = -0.5$,

$$a_s = (-0.5\,s)(-0.5) = \underline{\underline{0.25\,s \ \text{ft/s}^2}} \ \text{where } s \sim \text{ft}$$

Note: For $s > 0$, a_s is positive — the particle's acceleration is to the right. Since the particle is moving to the left, a positive a_s for this case implies that the particle is decelerating (as it must be for this stagnation point flow).

4.36

4.36 A nozzle is designed to accelerate the fluid from V_1 to V_2 in a linear fashion. That is, $V = ax + b$, where a and b are constants. If the flow is constant with $V_1 = 10$ m/s at $x_1 = 0$ and $V_2 = 25$ m/s at $x_2 = 1$ m, determine the local acceleration, the convective acceleration, and the acceleration of the fluid at points (1) and (2).

With $u = ax + b$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$ can be written as

$\vec{a} = a_x \hat{i}$ where $a_x = u \frac{\partial u}{\partial x}$. (1)

Since $u = V_1 = 10 \frac{m}{s}$ at $x = 0$ and $u = V_2 = 25 \frac{m}{s}$ at $x = 1$ we obtain

$$10 = 0 + b$$

$$25 = a + b \quad \text{so that } a = 15 \text{ and } b = 10$$

That is, $u = (15x + 10) \frac{m}{s}$, where $x \sim m$, so that from Eq.(1)

$$a_x = (15x + 10)\frac{m}{s}\left(15 \frac{1}{s}\right) = \underline{\underline{(225x + 150)\frac{m}{s^2}}}$$

Note: The local acceleration is zero, $\underline{\underline{\frac{\partial \vec{V}}{\partial t} = 0}}$, and the convective acceleration is $u \frac{\partial u}{\partial x} \hat{i} = \underline{\underline{(225x + 150)\hat{i} \frac{m}{s^2}}}$

At $x = 0$, $\vec{a} = \underline{\underline{150 \hat{i} \frac{m}{s^2}}}$; at $x = 1\,m$, $\vec{a} = \underline{\underline{375 \hat{i} \frac{m}{s^2}}}$

4.37

4.37 Repeat Problem 4.36 with the assumption that the flow is not steady, but at the time when $V_1 = 10$ m/s and $V_2 = 25$ m/s, it is known that $\partial V_1/\partial t = 20$ m/s^2 and $\partial V_2/\partial t = 60$ m/s^2.

With $u=u(x,t)$, $v=0$, and $w=0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ can be written as

$\vec{a} = a_x\,\hat{i}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$, with $u = a(t)x + b(t)$. (1)

At the given time $(t=t_o)$ $u = V_1 = 10\frac{m}{s}$ at $x=0$ and $u = V_2 = 25\frac{m}{s}$ at $x=1m$

Thus, $10 = 0 + b(t_o)$

$25 = a(t_o) + b(t_o)$ so that $a(t_o) = 15$ and $b(t_o) = 10$

Also at $t=t_o$, $\frac{\partial u}{\partial t} = \frac{\partial V_1}{\partial t} = 20\,\frac{m}{s^2}$ at $x=0$

and $\frac{\partial u}{\partial t} = \frac{\partial V_2}{\partial t} = 60\,\frac{m}{s^2}$ at $x=1m$ Note: These are local accelerations at time $t=t_o$

The convective acceleration at $x=0$ (Eq. (1) is

$$u\frac{\partial u}{\partial x} = (ax+b)(a) = (15(0)+10)\frac{m}{s}(15\,\frac{1}{s}) = 150\,\frac{m}{s^2}$$

while at $x=1$ it is

$$u\frac{\partial u}{\partial x} = (15(1)+10)\frac{m}{s}(15\,\frac{1}{s}) = 375\frac{m}{s^2}$$

The fluid acceleration at $t=t_o$ is

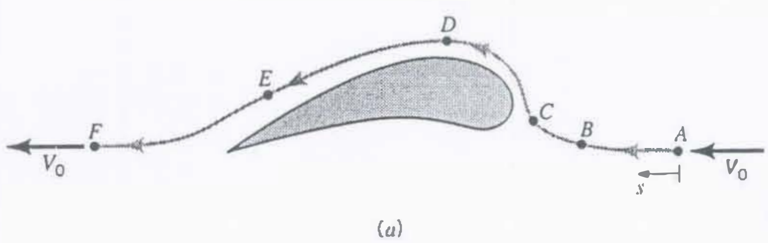
$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = (20+150)\,\hat{i}\,\frac{m}{s^2} = 170\,\hat{i}\,\frac{m}{s^2} \text{ at } x=0$$

and

$$\vec{a} = (60+375)\,\hat{i}\,\frac{m}{s^2} = 435\,\hat{i}\,\frac{m}{s^2} \text{ at } x=1m$$

4.38

4.38 An incompressible fluid flows past a turbine blade as shown in Fig. P4.38*a* and **Video V4.9**. Far upstream and downstream of the blade the velocity is V_0. Measurements show that the velocity of the fluid along streamline *A–F* near the blade is as indicated in Fig. P4.38*b*. Sketch the streamwise component of acceleration, a_s, as a function of distance, s, along the streamline. Discuss the important characteristics of your result.



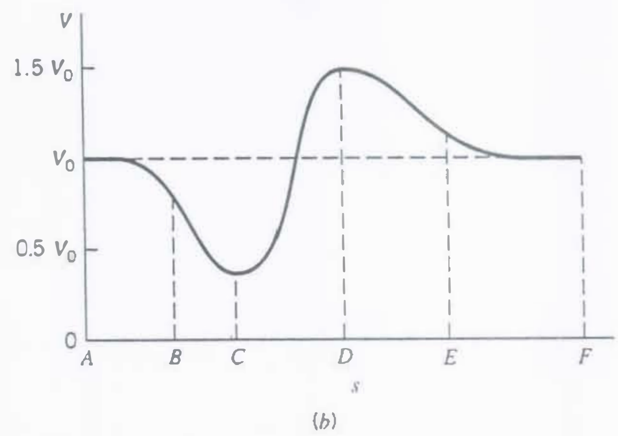
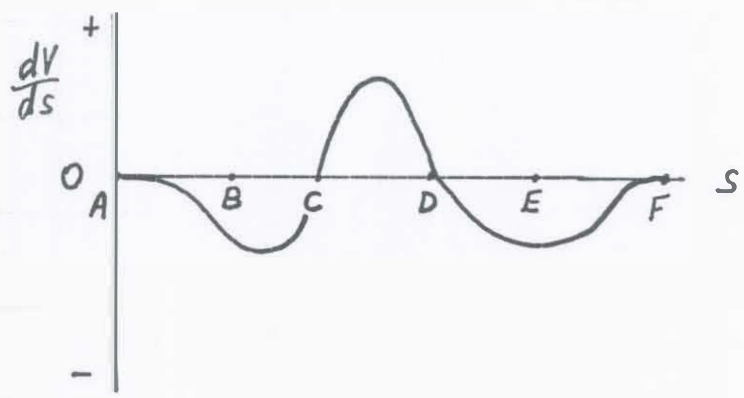
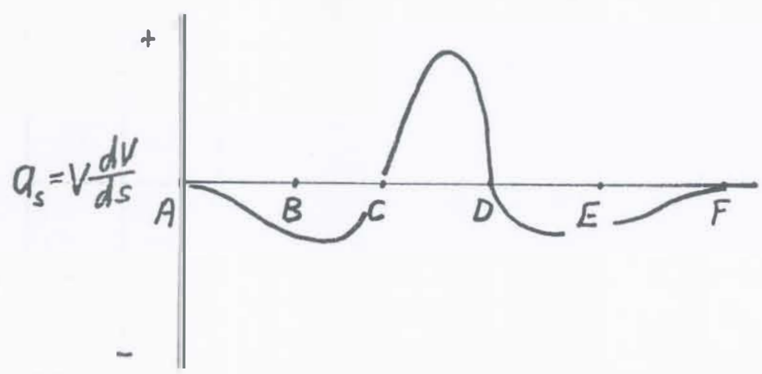


FIGURE P4.38

$a_s = V\frac{dV}{ds}$ where from the figure of $V = V(s)$ the function $\frac{dV}{ds}$ has the following shape.



Hence, the product $V\frac{dV}{ds}$ has the shape shown below.



The fluid decelerates from A to C, accelerates from C to D, and the decelerates again from D to F. The net acceleration from A to F is zero (i.e., $V_A = V_0 = V_F$).

4.39

4.39 Air flows steadily through a variable area pipe with a velocity of $\mathbf{V} = u(x)\hat{\mathbf{i}}$ ft/s, where the approximate measured values of $u(x)$ are given in the table. Plot the acceleration as a function of x for $0 \le x \le 12$ in. Plot the acceleration if the flowrate is increased by a factor of N (i.e., the values of u are increased by a factor of N), for $N = 2, 4, 10$.

x (in.)	u (ft/s)	x (in.)	u (ft/s)
0	10.0	7	20.1
1	10.2	8	17.4
2	13.0	9	13.5
3	20.1	10	11.9
4	28.3	11	10.3
5	28.4	12	10.0
6	25.8	13	10.0

Since $u = u(x)$, $v=0$, and $w=0$ it follows that $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ simplifies to $\vec{a} = a_x\hat{i}$ where $a_x = u\frac{\partial u}{\partial x}$ (1)

The values u are given in the table; the corresponding values of $\frac{\partial u}{\partial x}$ can be obtained by an approximate numerical differentiation. The results are shown below for the given date (i.e. with $N=1$).

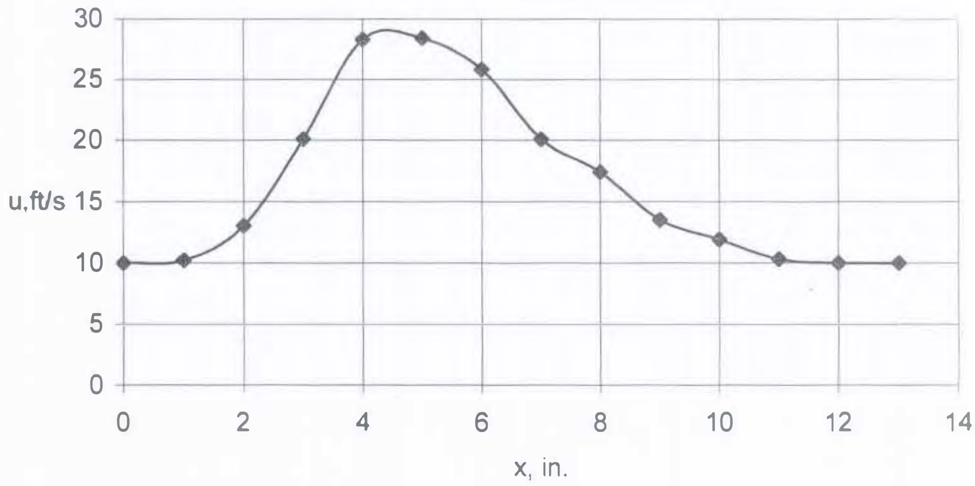
Note that since $a_x = u\frac{\partial u}{\partial x}$ it follows that and increase in velocity from u to Nu increases the acceleration from a_x to $N^2 a_x$

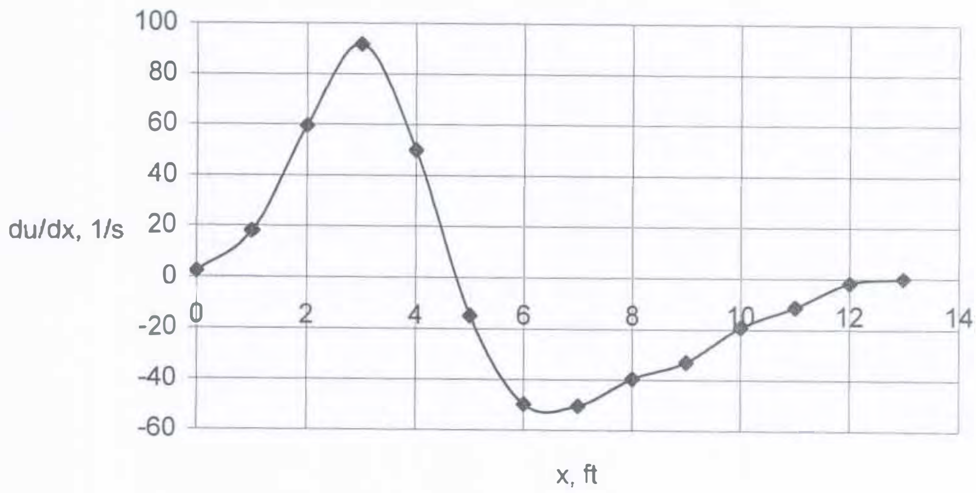
x, in.	u, ft/s	du/dx, 1/s	u du/dx
0	10	2.4	24
1	10.2	18	184
2	13	59.4	772
3	20.1	91.8	1845
4	28.3	49.8	1409
5	28.4	-15	-426
6	25.8	-49.8	-1285
7	20.1	-50.4	-1013
8	17.4	-39.6	-689
9	13.5	-33	-446
10	11.9	-19.2	-228
11	10.3	-11.4	-117
12	10	-1.8	-18
13	10	0	0

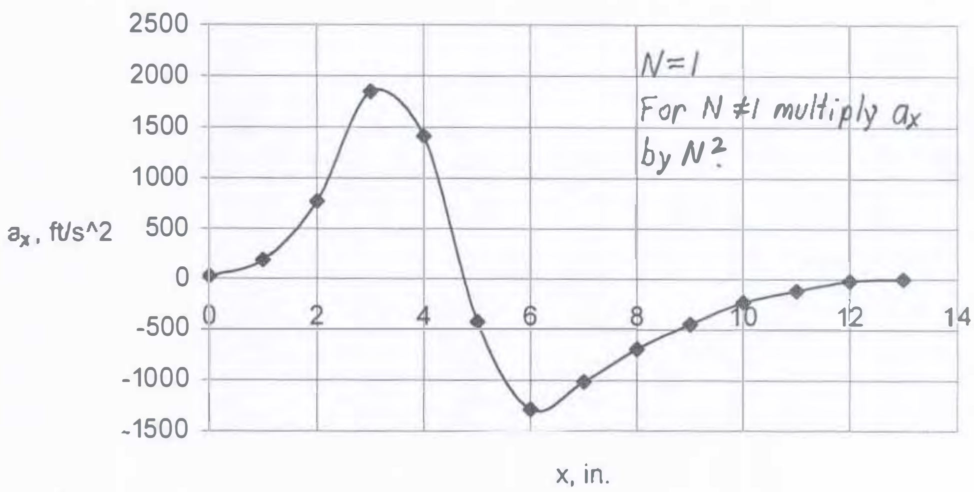
The results are plotted on the next page.

(con't)

*4.39 (con't)


30
25
20
u,ft/s 15
10
5
0
0
2
4
6
8
10
12
14
x, in.


100
80
60
40
du/dx, 1/s 20
0
-20
-40
-60
0
2
4
6
8
10
12
14
x, ft


2500
2000
1500
1000
a_x, ft/s^2 500
0
-500
-1000
-1500
0
2
4
6
8
10
12
14
x, in.
N=1
For N ≠ 1 multiply a_x by N².

*4.40

*4.40 As is indicated in Fig. P4.40, the speed of exhaust in a car's exhaust pipe varies in time and distance because of the periodic nature of the engine's operation and the damping effect with distance from the engine. Assume that the speed is given by $V = V_0[1 + ae^{-bx}\sin(\omega t)]$, where $V_0 = 8$ fps, $a = 0.05$, $b = 0.2\ \text{ft}^{-1}$, and $\omega = 50$ rad/s. Calculate and plot the fluid acceleration at $x = 0, 1, 2, 3, 4,$ and 5 ft for $0 \le t \le \pi/25$ s.

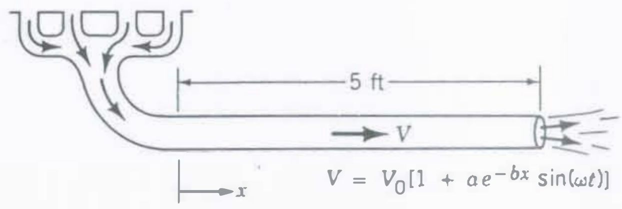


FIGURE P4.40

Since $u = u(x,t)$, $v = 0$, and $w = 0$ it follows that

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} = a_x\hat{i}\ , \quad \text{where} \quad a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \qquad (1)$$

Thus, with $u = V_0[1 + a e^{-bx}\sin(\omega t)]$ Eq. (1) gives

$$a_x = V_0 a\omega e^{-bx}\cos(\omega t) + V_0[1 + a e^{-bx}\sin(\omega t)]V_0 a(-b)e^{-bx}\sin(\omega t)$$

$$= V_0 a e^{-bx}[\omega\cos(\omega t) - V_0 b\sin(\omega t)(1 + a e^{-bx}\sin(\omega t))]$$

With $V_0 = 8\ \frac{ft}{s}$, $a = 0.05$, $b = 0.2\ \frac{1}{ft}$, and $\omega = 50\ \frac{rad}{s}$ this becomes

$$a_x = 0.4\ e^{-0.2x}[50\cos(50t) - 1.6\sin(50t)(1 + 0.05\ e^{-0.2x}\sin(50t))]\ \frac{ft}{s^2} \qquad (2)$$

where $t \sim s$ and $x \sim ft$

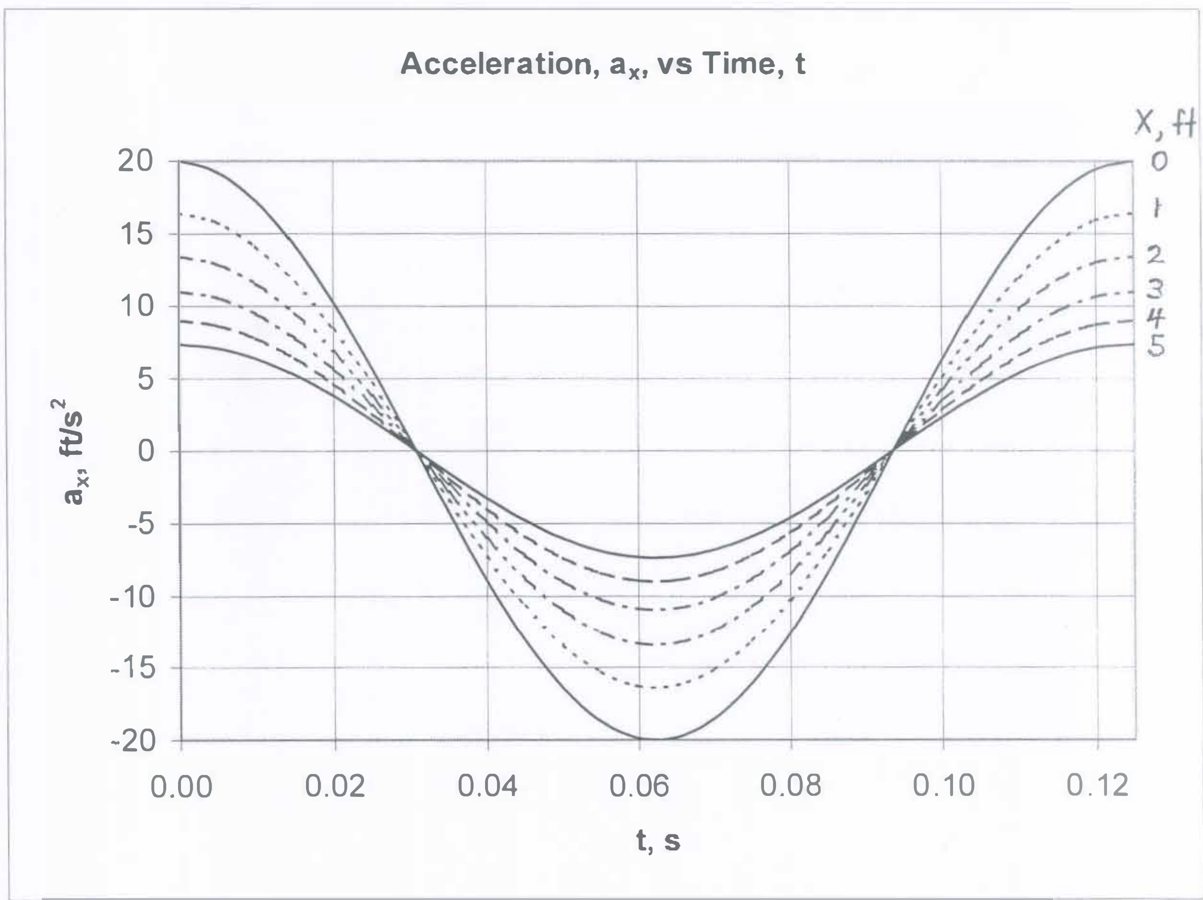
Plot a_x from Eq. (2) for $0 \le t \le \frac{\pi}{25}$ s with $x = 0, 1, 2, 3, 4,$ and 5 ft.

An Excel Program was used to calculate a_x from Eq. (2). The results are shown on the next page.

(con't)

4.40* (con't)

t, s	Acceleration at various x locations, ft/s^2					
	x = 0 ft	x = 1 ft	x = 2 ft	x= 3 ft	x = 4 ft	x = 5 ft
0.000	20.00	16.37	13.41	10.98	8.99	7.36
0.005	19.22	15.73	12.88	10.55	8.64	7.07
0.010	17.24	14.11	11.56	9.46	7.75	6.34
0.015	14.18	11.61	9.51	7.79	6.38	5.22
0.020	10.24	8.39	6.87	5.63	4.61	3.77
0.025	5.67	4.65	3.81	3.12	2.55	2.09
0.030	0.74	0.61	0.51	0.42	0.34	0.28
0.035	-4.23	-3.46	-2.83	-2.31	-1.89	-1.55
0.040	-8.93	-7.31	-5.98	-4.90	-4.01	-3.28
0.045	-13.08	-10.71	-8.76	-7.17	-5.87	-4.81
0.050	-16.42	-13.44	-11.00	-9.01	-7.37	-6.04
0.055	-18.73	-15.34	-12.56	-10.28	-8.42	-6.89
0.060	-19.89	-16.29	-13.33	-10.92	-8.94	-7.32
0.065	-19.81	-16.22	-13.28	-10.87	-8.90	-7.29
0.070	-18.51	-15.15	-12.41	-10.16	-8.32	-6.81
0.075	-16.06	-13.14	-10.76	-8.81	-7.21	-5.90
0.080	-12.61	-10.32	-8.45	-6.91	-5.66	-4.63
0.085	-8.37	-6.85	-5.61	-4.59	-3.76	-3.07
0.090	-3.62	-2.96	-2.42	-1.98	-1.62	-1.32
0.095	1.36	1.12	0.92	0.75	0.62	0.51
0.100	6.26	5.13	4.20	3.44	2.82	2.31
0.105	10.77	8.82	7.22	5.92	4.84	3.97
0.110	14.61	11.96	9.80	8.02	6.57	5.38
0.115	17.54	14.36	11.76	9.63	7.88	6.45
0.120	19.38	15.87	12.99	10.64	8.71	7.13
0.125	20.01	16.38	13.41	10.98	8.99	7.36



4.41

4.41 Water flows over the crest of a dam with speed V as shown in Fig. P4.41. Determine the speed if the magnitude of the normal acceleration at point (1) is to equal the acceleration of gravity, g.

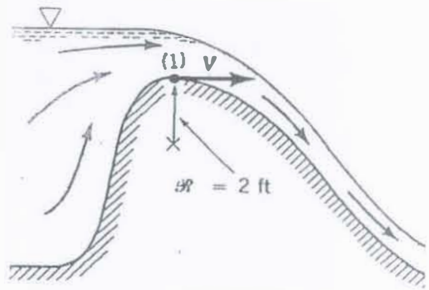


FIGURE P4.41

$$a_n = \frac{V^2}{\mathcal{R}} \quad \text{or with } a_n = 32.2\,\frac{ft}{s^2}, \quad V = \sqrt{a_n \mathcal{R}} = \sqrt{\left(32.2\,\frac{ft}{s^2}\right)(2\,ft)}$$

$$= \underline{\underline{8.02\,\frac{ft}{s}}}$$

4.42

4.42 Assume that the streamlines for the wingtip vortices from an airplane (see Fig. P4.19 and **Video V4.6**) can be approximated by circles of radius r and that the speed is $V = K/r$, where K is a constant. Determine the streamline acceleration, a_s, and the normal acceleration, a_n, for this flow.

■ FIGURE P4.19

$a_s = V\frac{dV}{ds}$, where, since $V = \frac{K}{r}$, $\frac{dV}{ds} = 0$

Thus,

$a_s = \underline{\underline{0}}$

Also,

$$a_n = \frac{V^2}{\mathcal{R}} = \frac{(K/r)^2}{r} = \underline{\underline{\frac{K}{r^3}}}$$

4.43

4.43 A fluid flows past a sphere with an upstream velocity of $V_0 = 40$ m/s as shown in Fig. P4.43 From a more advanced theory it is found that the speed of the fluid along the front part of the sphere is $V = \frac{3}{2}V_0 \sin\theta$. Determine the streamwise and normal components of acceleration at point A if the radius of the sphere is $a = 0.20$ m.

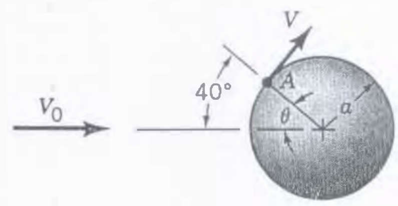


FIGURE P4.43

$$V = \frac{3}{2}V_0 \sin\theta = \frac{3}{2}\left(40\,\frac{m}{s}\right)\sin\theta = 60\sin\theta\ \frac{m}{s} \qquad (1)$$

$$a_n = \frac{V^2}{R} = \frac{(60\sin 40°)^2\ \frac{m^2}{s^2}}{0.2\,m} = \underline{\underline{7440\ \frac{m}{s^2}}}$$

and

$$a_s = V\frac{\partial V}{\partial s} = (60\sin\theta)\frac{\partial V}{\partial s}\ , \text{ where } \frac{\partial V}{\partial s} = \frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}$$

From Eq. (1), $\frac{\partial V}{\partial \theta} = 60\cos\theta$

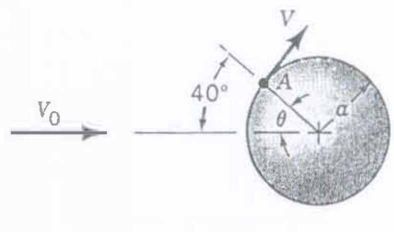
Also $s = a\theta = 0.2\,\theta$ m, where $\theta \sim$ rad, so that $\frac{\partial \theta}{\partial s} = \frac{1}{0.2\,m}$

Thus, for $\theta = 40°$

$$a_s = \left(60\sin 40°\,\frac{m}{s}\right)\left(60\cos 40°\,\frac{m}{s}\right)\left(\frac{1}{0.2\,m}\right) = \underline{\underline{8860\ \frac{m}{s^2}}}$$

*4.44

***4.44** For flow past a sphere as discussed in Problem 4.43, plot a graph of the streamwise acceleration, a_s, the normal acceleration, a_n, and the magnitude of the acceleration as a function of θ for $0 \le \theta \le 90°$ with $V_0 = 50$ ft/s and $a = 0.1$, 1.0, and 10 ft. Repeat for $V_0 = 5$ ft/s. At what point is the acceleration a maximum; a minimum?



$$a_n = \frac{V^2}{\mathcal{R}} = \frac{\left(\frac{3}{2}V_0 \sin\theta\right)^2}{a} = \underline{\underline{\frac{9V_0^2}{4a}\sin^2\theta}} \qquad (1)$$

and $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}$, where $\frac{\partial V}{\partial \theta} = \frac{3}{2}V_0\cos\theta$ and $s = a\theta$

or $\frac{\partial \theta}{\partial s} = \frac{1}{a}$

Thus,

$$a_s = \left(\tfrac{3}{2}V_0\sin\theta\right)\left(\tfrac{3}{2}V_0\cos\theta\right)\frac{1}{a} = \underline{\underline{\frac{9V_0^2}{4a}\sin\theta\cos\theta}} \qquad (2)$$

Hence the magnitude of the acceleration is

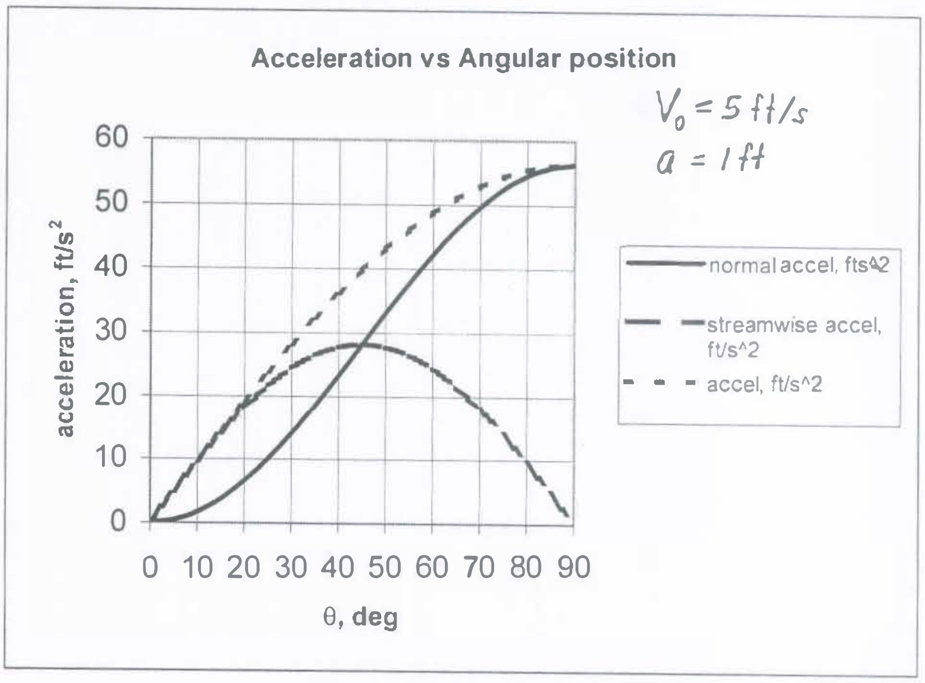
$$|\vec{a}| = \sqrt{a_n^2 + a_s^2} = \frac{9V_0^2}{4a}\sqrt{\sin^4\theta + \sin^2\theta\cos^2\theta} = \frac{9V_0^2}{4a}\sin\theta\sqrt{\sin^2\theta + \cos^2\theta}$$

or

(3) $|\vec{a}| = \underline{\underline{\frac{9V_0^2}{4a}\sin\theta}}$ Thus, $\underline{\underline{|\vec{a}|_{min} = 0 \text{ at } \theta = 0,\ |\vec{a}|_{max} = \frac{9V_0^2}{4a} \text{ at } \theta = 90°}}$

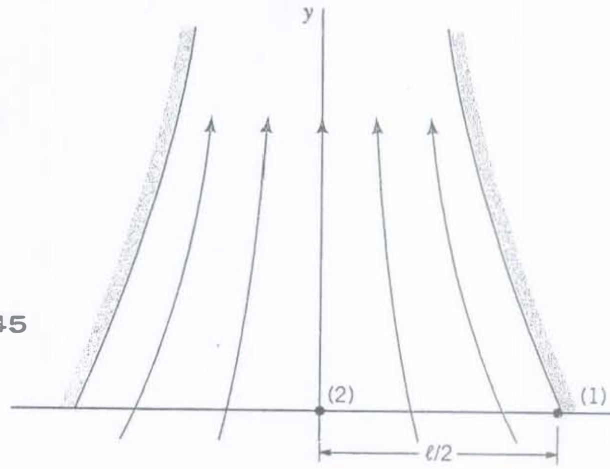
An Excel Program was used to calculate a_s, a_n, and a from Eqns. (1), (2), and (3). The results are shown below. The results for other values are similar if the factor V_0^2/a is accounted for. The following data is for $V_0 = 5$ ft/s, $a = 1$ ft

θ, deg	a_n, ft/s^2	a_s, ft/s^2	a, ft/s^2
0	0.0	0.0	0.0
5	0.4	4.9	4.9
10	1.7	9.6	9.8
15	3.8	14.1	14.6
20	6.6	18.1	19.2
25	10.0	21.5	23.8
30	14.1	24.4	28.1
35	18.5	26.4	32.3
40	23.2	27.7	36.2
45	28.1	28.1	39.8
50	33.0	27.7	43.1
55	37.7	26.4	46.1
60	42.2	24.4	48.7
65	46.2	21.5	51.0
70	49.7	18.1	52.9
75	52.5	14.1	54.3
80	54.6	9.6	55.4
85	55.8	4.9	56.0
90	56.3	0.0	56.3



4.45

*4.45 The velocity components for steady flow through the nozzle shown in Fig. P4.45 are $u = -V_0 x/\ell$ and $v = V_0[1 + (y/\ell)]$, where V_0 and ℓ are constants. Determine the ratio of the magnitude of the acceleration at point (1) to that at point (2).



■ FIGURE P4.45

$$a = \sqrt{a_x^2 + a_y^2}, \quad \text{where} \qquad (1)$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(-\frac{V_0}{\ell}x\right)\left(-\frac{V_0}{\ell}\right) + V_0\left[1+\frac{y}{\ell}\right](0) = \left(\frac{V_0}{\ell}\right)^2 x$$

and

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \left(-\frac{V_0}{\ell}x\right)(0) + V_0\left[1+\frac{y}{\ell}\right]\left(\frac{V_0}{\ell}\right) = \left(\frac{V_0}{\ell}\right)^2(\ell + y)$$

Thus, from Eq. (1),

$$a = \left(\frac{V_0}{\ell}\right)^2\sqrt{x^2 + (\ell+y)^2}$$

so that,

$$a_1 = \left(\frac{V_0}{\ell}\right)^2\sqrt{\frac{\ell^2}{4} + \ell^2} = \frac{V_0^2}{\ell}\sqrt{\frac{5}{4}}$$

and

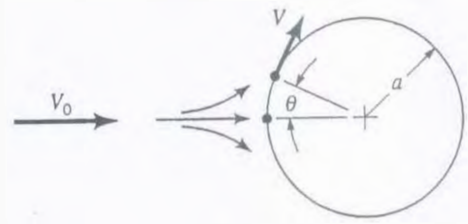
$$a_2 = \left(\frac{V_0}{\ell}\right)^2\sqrt{0 + \ell^2} = \frac{V_0^2}{\ell}$$

Hence,

$$\frac{a_1}{a_2} = \frac{\frac{V_0^2}{\ell}\sqrt{\frac{5}{4}}}{\frac{V_0^2}{\ell}} = \sqrt{\frac{5}{4}} = \underline{\underline{1.118}}$$

*4.46

*4.46 A fluid flows past a circular cylinder of radius a with an upstream speed of V_0 as shown in Fig. P4.46. A more advanced theory indicates that if viscous effects are negligible, the velocity of the fluid along the surface of the cylinder is given by $V = 2V_0 \sin\theta$. Determine the streamline and normal components of acceleration on the surface of the cylinder as a function of V_0, a, and θ and plot graphs of a_s and a_n for $0 \leq \theta \leq 90°$ with $V_0 = 10$ m/s and $a = 0.01, 0.10, 1.0,$ and 10.0 m.



■ FIGURE P4.46

$$a_n = \frac{V^2}{\mathcal{R}} = \frac{(2V_0 \sin\theta)^2}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin^2\theta}}$$

and

$$a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}\quad , \text{ where } \frac{\partial V}{\partial \theta} = 2V_0\cos\theta \text{ and } s = a\theta$$

$$\text{or } \frac{\partial \theta}{\partial s} = \frac{1}{a}$$

Thus,

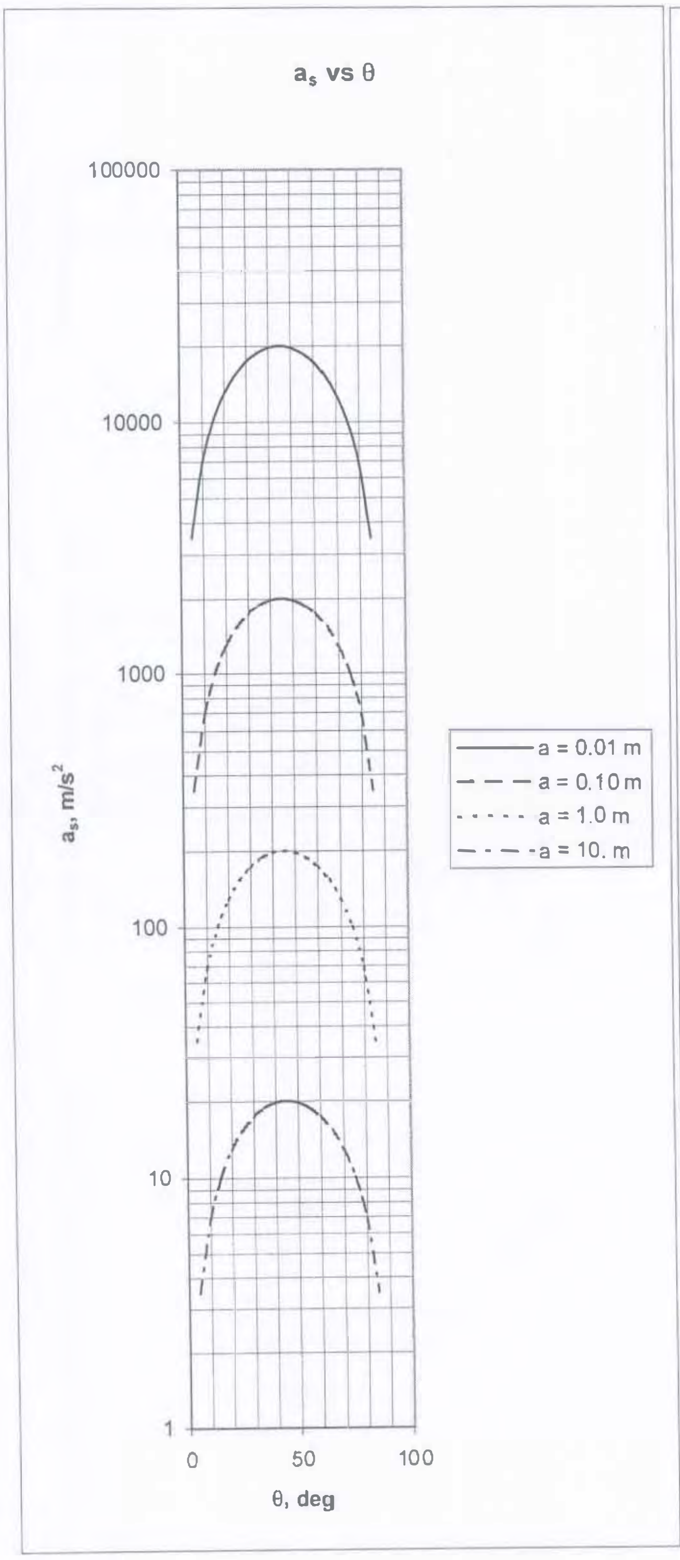
$$a_s = (2V_0 \sin\theta)(2V_0\cos\theta)\frac{1}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin\theta\cos\theta}}$$

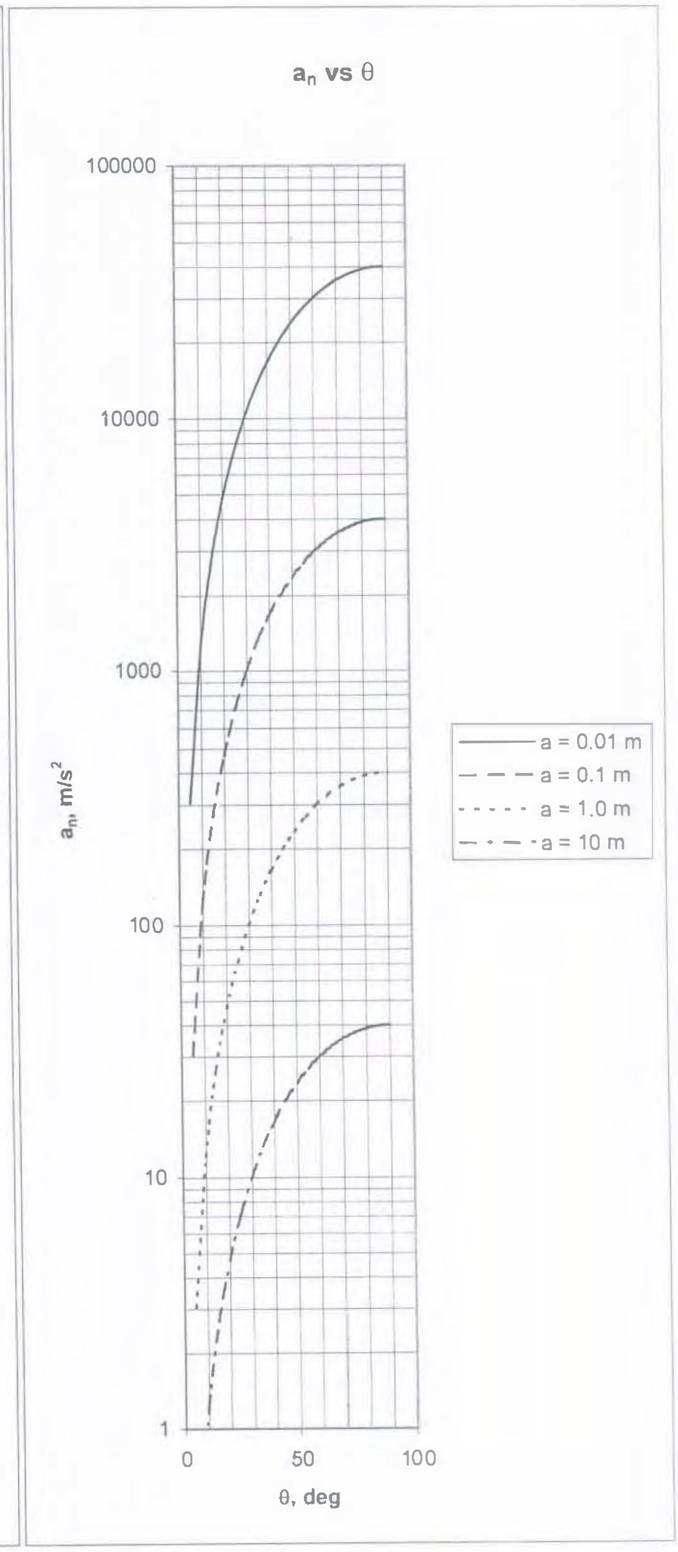
These results with $V_0 = 10\frac{m}{s}$ and $a = 0.01, 0.10, 1.0,$ and 10.0 m are plotted below.

	a = 0.01 m	a = 0.10 m	a = 1.0 m	a = 10 m	a = 0.10 m	a = 0.10 m	a = 1.0 m	a = 10 m
θ, deg	a_s, ft/s^2	a_s, ft/s^2	a_s, ft/s^2	a_s, ft/s^2	a_n, ft/s^2	a_n, ft/s^2	a_n, ft/s^2	a_n, ft/s^2
0	0	0	0	0.00	0	0	0	0.00
5	3473	347	35	3.47	304	30	3	0.30
10	6840	684	68	6.84	1206	121	12	1.21
15	10000	1000	100	10.00	2679	268	27	2.68
20	12856	1286	129	12.86	4679	468	47	4.68
25	15321	1532	153	15.32	7144	714	71	7.14
30	17321	1732	173	17.32	10000	1000	100	10.00
35	18794	1879	188	18.79	13160	1316	132	13.16
40	19696	1970	197	19.70	16527	1653	165	16.53
45	20000	2000	200	20.00	20000	2000	200	20.00
50	19696	1970	197	19.70	23473	2347	235	23.47
55	18794	1879	188	18.79	26840	2684	268	26.84
60	17321	1732	173	17.32	30000	3000	300	30.00
65	15321	1532	153	15.32	32856	3286	329	32.86
70	12856	1286	129	12.86	35321	3532	353	35.32
75	10000	1000	100	10.00	37321	3732	373	37.32
80	6840	684	68	6.84	38794	3879	388	38.79
85	3473	347	35	3.47	39696	3970	397	39.70
90	0	0	0	0.00	40000	4000	400	40.00

(con't)

*4.46 (con't)


a_s vs θ
100000
10000
1000
100
10
1
a_s, m/s^2
0
50
100
θ, deg
a = 0.01 m
a = 0.10 m
a = 1.0 m
a = 10. m


a_n vs θ
100000
10000
1000
100
10
1
a_n, m/s^2
0
50
100
θ, deg
a = 0.01 m
a = 0.1 m
a = 1.0 m
a = 10 m

4.47

4.47 Determine the x and y components of acceleration for the flow given in Problem 4.11. If $c > 0$, is the particle at point $x = x_0 > 0$ and $y = 0$ accelerating or decelerating? Explain. Repeat if $x_0 < 0$.

Since $u = c(x^2 - y^2)$ and $v = -2cxy$ it follows that

$\vec{a} = a_x \hat{i} + a_y \hat{j}$, where

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = c(x^2 - y^2)(2cx) + (-2cxy)(-2cy)$$

or

$$\underline{\underline{a_x = 2c^2 x(x^2 + y^2)}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = c(x^2 - y^2)(-2cy) + (-2cxy)(-2cx)$$

or

$$\underline{\underline{a_y = 2c^2 y(x^2 + y^2)}}$$

For $x = x_0$ and $y = 0$ we obtain:

$u = cx_0^2$, $v = 0$

and

$a_x = 2c^2 x_0^3$, $a_y = 0$

Thus, with $c>0$ and $x_0>0$ it follows that $u>0$, $a_x>0$; i.e., the fluid is <u>accelerating</u>.

With $c>0$ and $x_0<0$ it follows that $u>0$, $a_x<0$; i.e., the fluid is <u>decelerating</u>.

4.48

4.48 When flood gates in a channel are opened, water flows along the channel downstream of the gates with an increasing speed given by $V = 4(1 + 0.1t)$ ft/s, for $0 \le t \le 20$ s, where t is in seconds. For $t > 20$ s the speed is a constant $V = 12$ ft/s. Consider a location in the curved channel where the radius of curvature of the streamlines is 50 ft. For $t = 10$ s determine **(a)** the component of acceleration along the streamline, **(b)** the component of acceleration normal to the streamline, and **(c)** the net acceleration (magnitude and direction). Repeat for $t = 30$ s.

$V = 4(1+0.1t)$ ft/s for $0 \le t \le 20s$ and $V = 12$ ft/s for $t > 20s$

$a_s = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$ where $\frac{\partial V}{\partial s} = 0$

Thus,

$a_s = \frac{\partial V}{\partial t}$ and $a_n = \frac{V^2}{R}$, where $R = 50\,ft$

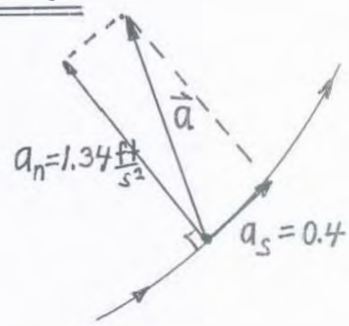
(1) For $t = 10s$:

(a) $a_s = \frac{\partial V}{\partial t} = 4(0.1) = \underline{\underline{0.4\,\frac{ft}{s^2}}}$

(b) $a_n = V^2/R = [4(1+0.1(10))]^2\,ft^2/s^2/(50\,ft) = \underline{\underline{1.28\,ft/s^2}}$

and

(c) $a = (a_n^2 + a_s^2)^{1/2} = \left[\left(0.4\frac{ft}{s^2}\right)^2 + \left(1.28\frac{ft}{s^2}\right)^2\right]^{1/2} = \underline{\underline{1.34\,\frac{ft}{s^2}}}$



(2) For $t = 30s$:

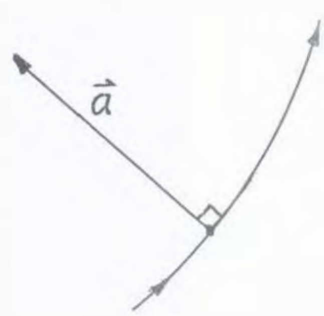
(a) Since $V = 12$ ft/s = constant, $\frac{\partial V}{\partial t} = 0$ and $\frac{\partial V}{\partial s} = 0$ so that

$a_s = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = \underline{\underline{0}}$

(b) $a_n = V^2/R = (12\,ft/s)^2/(50\,ft) = \underline{\underline{2.88\,\frac{ft}{s^2}}}$

and

(c) $a = (a_n^2 + a_s^2)^{1/2} = a_n = \underline{\underline{2.88\,\frac{ft}{s^2}}}$



4.49

4.49 Water flows steadily through the funnel shown in Fig. P4.49. Throughout most of the funnel the flow is approximately radial (along rays from O) with a velocity of $V = c/r^2$, where r is the radial coordinate and c is a constant. If the velocity is 0.4 m/s when $r = 0.1$ m, determine the acceleration at points A and B.

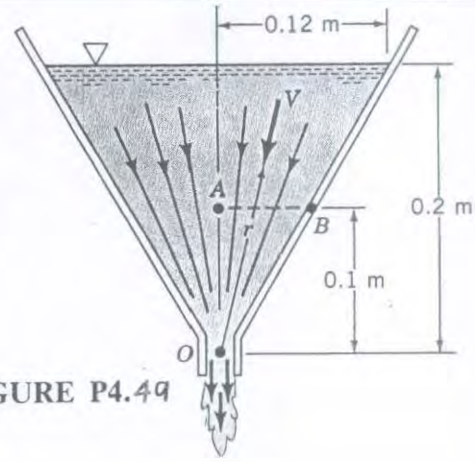


FIGURE P4.49

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$, where $V = \frac{c}{r^2}$

Since $V = 0.4\,\frac{m}{s}$ when $r = 0.1m$ it follows that

$c = Vr^2 = (0.4\,\frac{m}{s})(0.1\,m)^2 = 4 \times 10^{-3}\,\frac{m^3}{s}$, or $V = \frac{4 \times 10^{-3}}{r^2}\,\frac{m}{s}$, where $r \sim m$

Thus,

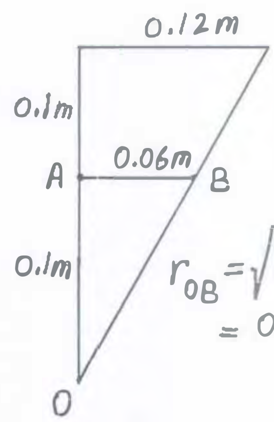
$$a_s = -\left(\frac{c}{r^2}\right)\left(-\frac{2c}{r^3}\right) = \frac{2c^2}{r^5}$$

At point A:

$$a_s = \frac{2(4 \times 10^{-3}\,\frac{m^3}{s})^2}{(0.1\,m)^5} = \underline{\underline{3.20\,\frac{m}{s^2}}}$$

At point B:

$$a_s = \frac{2(4 \times 10^{-3}\,\frac{m^3}{s})^2}{(0.1167\,m)^5} = \underline{\underline{1.48\,\frac{m}{s^2}}}$$



$r_{OB} = \sqrt{(0.1)^2 + (0.06)^2}$
$= 0.1167m$

4.50

4.50 Water flows through the slit at the bottom of a two-dimensional water trough as shown in Fig. P4.50. Throughout most of the trough the flow is approximately radial (along rays from O) with a velocity of $V = c/r$, where r is the radial coordinate and c is a constant. If the velocity is 0.04 m/s when $r = 0.1$ m, determine the acceleration at points A and B.

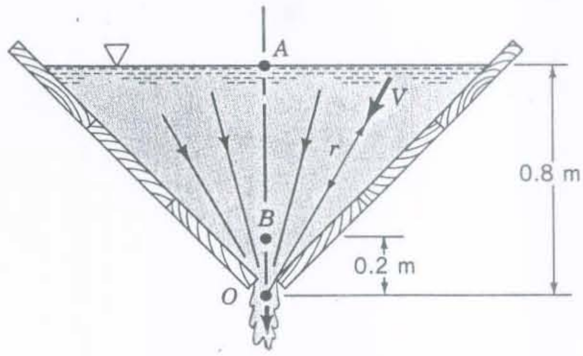


FIGURE P4.50

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V \frac{\partial V}{\partial s} = -V \frac{\partial V}{\partial r}$, where $V = \frac{c}{r}$

Since $V = 0.04 \frac{m}{s}$ when $r = 0.1 m$ it follows that

$c = Vr = (0.04 \frac{m}{s})(0.1 m) = 4 \times 10^{-3} \frac{m^2}{s}$, or $V = \frac{4 \times 10^{-3}}{r} \frac{m}{s}$, where $r \sim m$

Thus,

$$a_s = -\left(\frac{c}{r}\right)\left(-\frac{c}{r^2}\right) = \frac{c^2}{r^3}$$

At point A:

$$a_s = \frac{(4 \times 10^{-3} \frac{m^2}{s})^2}{(0.8 m)^3} = \underline{\underline{3.13 \times 10^{-5} \frac{m}{s^2}}}$$

At point B:

$$a_s = \frac{(4 \times 10^{-3} \frac{m^2}{s})^2}{(0.2 m)^3} = \underline{\underline{2.00 \times 10^{-3} \frac{m}{s^2}}}$$

4.51

4.51 Air flows from a pipe into the region between two parallel circular disks as shown in Fig. P4.51. The fluid velocity in the gap between the disks is closely approximated by $V = V_0 R/r$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for r = 1, 2, or 3 ft if V_0 = 5 ft/s and R = 3 ft.

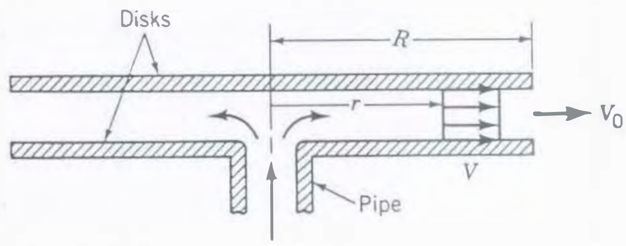


FIGURE P4.51

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial r}$, where $V = \frac{V_0 R}{r}$

Since $V_0 = 5\,\frac{ft}{s}$ and $R = 3\,ft$, $V = \frac{15}{r}\,\frac{ft}{s}$, where $r \sim ft$

Thus,

$$a_s = \left(\frac{V_0 R}{r}\right)\left(-\frac{V_0 R}{r^2}\right) = -\frac{V_0^2 R^2}{r^3} = -\frac{(5\,\frac{ft}{s})^2 (3\,ft)^2}{r^3\,ft^3} = -\frac{225}{r^3}\,\frac{ft}{s^2},\ r \sim ft$$

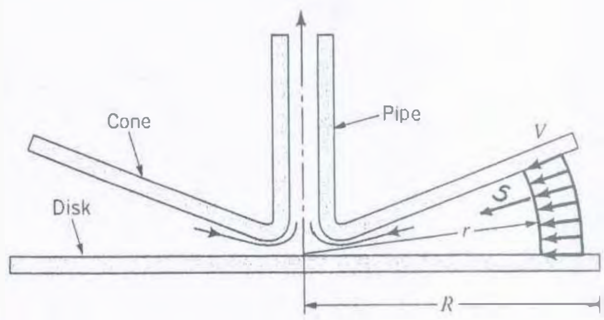
At $r = 1\,ft$, $a_s = \underline{\underline{-225\,\frac{ft}{s^2}}}$

At $r = 2\,ft$, $a_s = \underline{\underline{-28.1\,\frac{ft}{s^2}}}$

At $r = 3\,ft$, $a_s = \underline{\underline{-8.33\,\frac{ft}{s^2}}}$

4.52

4.52 Air flows into a pipe from the region between a circular disk and a cone as shown in Fig. P4.52. The fluid velocity in the gap between the disk and the cone is closely approximated by $V = V_0 R^2/r^2$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for $r = 0.5$ and 2 ft if $V_0 = 5$ ft/s and $R = 2$ ft.



■ FIGURE P4.52

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e. the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$ since r and s are pointed in opposite directions.

Thus, with $V = V_0 R^2/r^2$ it follows that

$$a_s = -(V_0 R^2/r^2)(-2V_0 R^2/r^3) = 2V_0^2 R^4/r^5$$

$$= 2(5\,\text{ft/s})^2(2\,\text{ft})^4/r^5 = 800/r^5 \ \frac{\text{ft}}{\text{s}^2}, \text{ where } r \sim \text{ft}$$

At $r = 0.5$ ft, $a_s = 800/(0.5)^5 \frac{\text{ft}}{\text{s}^2} = \underline{\underline{25{,}600 \frac{\text{ft}}{\text{s}^2}}}$

At $r = 2$ ft, $a_s = 800/(2.0)^5 \frac{\text{ft}}{\text{s}^2} = \underline{\underline{25 \frac{\text{ft}}{\text{s}^2}}}$

4.53

4.53 Air flows steadily through a long pipe with a speed of $u = 50 + 0.5x$, where x is the distance along the pipe in feet, and u is in ft/s. Due to heat transfer into the pipe, the air temperature, T, within the pipe is $T = 300 + 10x$ °F. Determine the rate of change of the temperature of air particles as they flow past the section at $x =$ ft.

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}, \text{ where}$$

$u = 50 + 0.5x$, $v = 0$, $w = 0$, and

$T = 300 + 10x$

Thus,

$$\frac{DT}{Dt} = 0 + u\frac{\partial T}{\partial x} + 0 + 0$$

$$= (50 + 0.5x)(10) = 500 + 5x \ \frac{°F}{s}, \text{ where } x \sim ft$$

Hence, at $x = 5\ ft$,

$$\frac{DT}{Dt} = 500 + 5(5) = \underline{\underline{525 \ \frac{°F}{s}}}$$

4.54

4.54 A company produces a perishable product in a factory located at $x = 0$ and sells the product along the distribution route $x > 0$. The selling price of the product, P, is a function of the lenght of time after it was produced, t, and the location at which it is sold, x. That is, $P = P(x, t)$. At a given location the price of the product decreases in time (it is perishable) according to $\partial P/\partial t = -8$ dollars/hr. In addition, because of shipping costs the price increases with distance from the factory according to $\partial P/\partial x = 0.2$ dollars/mi. If the manufacturer wishes to sell the product for the same 100-dollar price anywhere along the distribution route, determine how fast he must travel along the rout.

$$\frac{DP}{Dt} = \frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x}, \text{ where}$$

$$\frac{\partial P}{\partial t} = -8\,\frac{\text{dollars}}{\text{hr}} \text{ and } \frac{\partial P}{\partial x} = 0.2\,\frac{\text{dollars}}{\text{mi}}$$

But, $P = 100$ dollars anywhere, so that $\frac{DP}{Dt} = 0$

Hence,

$$0 = \frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} \quad \text{or}$$

$$u = -\frac{\partial P/\partial t}{\partial P/\partial x} = -\frac{(-8\text{ dollars/hr})}{(0.2\text{ dollars/mi})} = \underline{\underline{40\,\frac{\text{mi}}{\text{hr}}}}$$

4.55

4.55 Assume the temperature of the exhaust in an exhaust pipe can be approximated by $T = T_0(1 + ae^{-bx})[1 + c\cos(\omega t)]$, where $T_0 = 100\ °C$, $a = 3$, $b = 0.03\ m^{-1}$, $c = 0.05$, and $\omega = 100$ rad/s. If the exhaust speed is a constant 3 m/s, determine the time rate of change of temperature of the fluid particles at $x = 0$ and $x = 4$ m when $t = 0$.

Since $u = 3\ \frac{m}{s}$, $v = 0$, and $w = 0$ it follows that

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}$$

Thus,

$$\frac{DT}{Dt} = T_0(1 + ae^{-bx})(-c\,\omega\sin(\omega t)) + uT_0(1 + c\cos(\omega t))(-ab\,e^{-bx})$$

When $t = 0$:

$\frac{DT}{Dt} = -abuT_0(1+c)\,e^{-bx}$, or with the given data,

$$\frac{DT}{Dt} = -(3)(0.03\tfrac{1}{m})(3\tfrac{m}{s})(100\,°C)(1+0.05)\,e^{-0.03x}$$

$$= 28.4\,e^{-0.03x}\ \frac{°C}{s},\ \text{where } x \sim m$$

Thus, $\frac{DT}{Dt} = \underline{\underline{-28.4\ \frac{°C}{s}}}$ at $x = 0$, $t = 0$

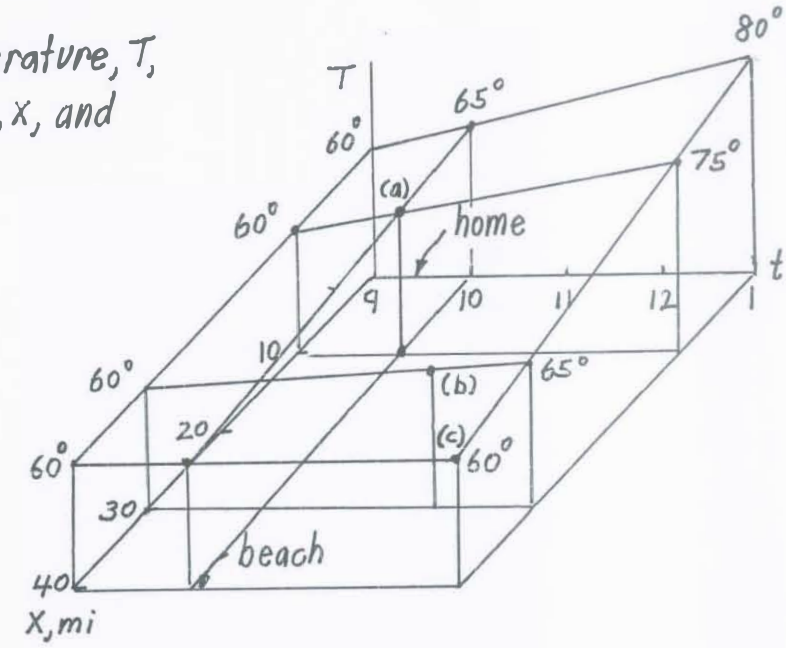
and

$\frac{DT}{Dt} = \underline{\underline{-25.1\ \frac{°C}{s}}}$ at $x = 4\,m$, $t = 0$

4.56

4.56 A bicyclist leaves from her home at 9 A.M. and rides to a beach 40 mi away. Because of a breeze off the ocean, the temperature at the beach remains 60 °F throughout the day. At the cyclist's home the temperature increases linearly with time, going from 60 °F at 9 A.M. to 80 °F by 1 P.M. The temperature is assumed to vary linearly as a function of position between the cyclist's home and the beach. Determine the rate of change of temperature observed by the cyclist for the following conditions: **(a)** as she pedals 10 mph through a town 10 mi from her home at 10 A.M.; **(b)** as she eats lunch at a rest stop 30 mi from her home at noon; **(c)** as she arrives enthusiastically at the beach at 1 P.M., pedaling 20 mph.

From the given data the temperature, T, varies as a function of location, x, and time, t, as shown in the figure.



Thus, $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}$

(a) At $x = 10\,mi$ and $t = 10\,am$,

$$\frac{\partial T}{\partial t} = \frac{(75^\circ - 60^\circ)}{4\,hr} = \frac{15}{4}\,{}^\circ/hr$$

and

$$\frac{\partial T}{\partial x} = \frac{(60^\circ - 65^\circ)}{40\,mi} = -\frac{1}{8}\,{}^\circ/mi$$

Thus, with $u = 10\,mi/hr$,

$$\frac{DT}{Dt} = \frac{15}{4}\,{}^\circ/hr + 10\frac{mi}{hr}\left(-\frac{1}{8}\,{}^\circ/mi\right)$$

$$= \underline{\underline{2.5\,{}^\circ/hr}}$$

(b) At noon with $u = 0$ (resting) and $\frac{\partial T}{\partial t} = \frac{(65^\circ - 60^\circ)}{4\,hr} = \frac{5}{4}\,{}^\circ/hr$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\partial T}{\partial t} = \frac{5}{4}\,{}^\circ/hr = \underline{\underline{1.25\,{}^\circ/hr}}$$

(c) Upon arival at the beach with $u = 20\,mph$, $\frac{\partial T}{\partial t} = 0$, and $\frac{\partial T}{\partial x} = \frac{(60^\circ - 80^\circ)}{40\,mi}$
$= -0.5\,{}^\circ/mi$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = u\frac{\partial T}{\partial x} = 20\frac{mi}{hr}(-0.5\,{}^\circ/mi) = \underline{\underline{-10^\circ/hr}}$$

4.57

4.57 The temperature distribution in a fluid is given by $T = 10x + 5y$, where x and y are the horizontal and vertical coordinates in meters and T is in degrees centigrade. Determine the time rate of change of temperature of a fluid particle traveling (a) horizontally with $u = 20$ m/s, $v = 0$ or (b) vertically with $u = 0$, $v = 20$ m/s.

$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$, where $\frac{\partial T}{\partial t} = 0$

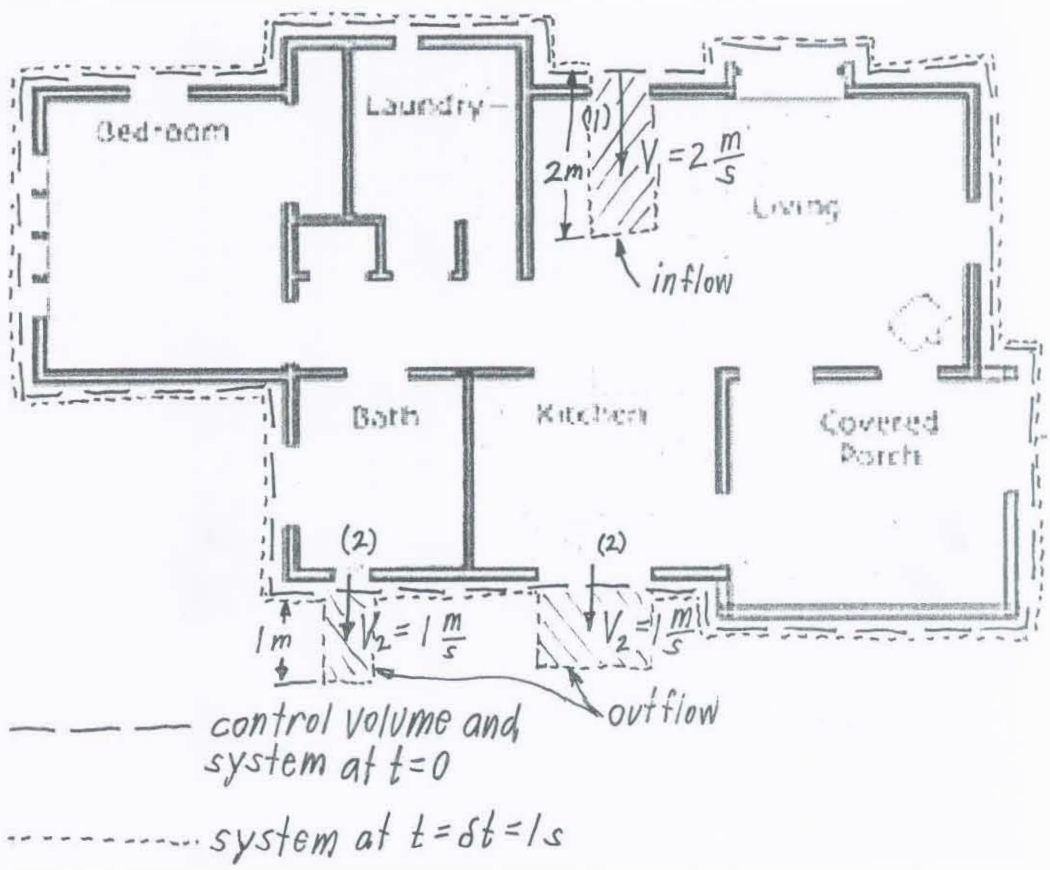
Thus, if $u = 20\frac{m}{s}$ and $v = 0$, then $\frac{DT}{Dt} = u\frac{\partial T}{\partial x} = (20\frac{m}{s})(10\frac{°C}{m}) = \underline{\underline{200\frac{°C}{s}}}$

and if $u = 0$ and $v = 20\frac{m}{s}$, then $\frac{DT}{Dt} = v\frac{\partial T}{\partial y} = (20\frac{m}{s})(5\frac{°C}{m}) = \underline{\underline{100\frac{°C}{s}}}$

4.59

4.59 The wind blows through the front door of a house with a speed of 2 m/s and exits with a speed of 1 m/s through two windows on the back of the house. Consider the system of interest for this flow to be the air within the house at time $t = 0$. Draw a simple sketch of the house and show an appropriate control volume for this flow. On the sketch, show the position of the system at time $t = 1$ s.

Since the air enters at 2 m/s and leaves at 1 m/s, the air at the entrance and exit has moved $\ell_1 = V_1 \delta t = 2\,m/s\,(1s) = 2m$ and $\ell_2 = V_2\,\delta t = 1\,m/s\,(1s) = 1\,m$, respectively. The control volume, which coincides with the system at $t=0$, and the system at $t = 1s$ are shown below.



4.60

4.60 Water flows through a duct of square cross section as shown in Fig. P4.60 with a constant, uniform velocity of $V = 20$ m/s. Consider fluid particles that lie along line A–B at time $t = 0$. Determine the position of these particles, denoted by line A'–B', when $t = 0.20$ s. Use the volume of fluid in the region between lines A–B and A'–B' to determine the flowrate in the duct. Repeat the problem for fluid particles originally along line C–D; along line E–F. Compare your three answers.

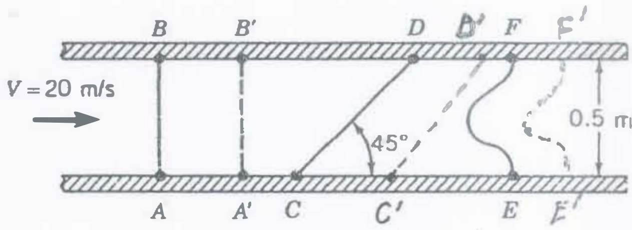


FIGURE P4.60

Since V is constant in time and space, all particles on line AB move a distance $\ell = V\,\Delta t = (20\,\frac{m}{s})(0.2s) = 4m$ from $t=0$ to $t=0.2s$
Thus, the volume of $ABA'B'$ is $\forall_{ABA'B'} = (0.5m)^2(4m) = 1.00\ m^3$
so that

$$Q = \frac{\forall_{ABA'B'}}{\Delta t} = \frac{1.00\ m^3}{0.2\,s} = \underline{\underline{5.0\,\frac{m^3}{s}}}$$

Similarly from $t=0$ to $t=0.2s$ the fluid along lines CD and EF move to $C'D'$ and $E'F'$, respectively. Also, $\forall_{CDC'D'} = \forall_{EFE'F'} = \forall_{ABA'B'}$ so that we obtain $Q = \frac{\forall}{\Delta t} = 5.0\frac{m^3}{s}$ regardless which line we consider.

4.61

4.61 Repeat Problem 4.60 if the velocity profile is linear from 0 to 20 m/s across the duct as shown in Fig. P4.61.

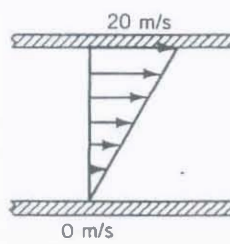


FIGURE P4.61

From $t=0$ to $t=0.1\,s$ the particle initially at B travels a distance $\ell_B = V_B\,\Delta t = (20\,\frac{m}{s})(0.1s) = 2\,m$ as show. Particle A remain fixed since $V_A = 0$. Since the velocity profile is linear, line AB remains straight, but "tilts" as indicated. Thus, the volume of fluid crossing the initial line AB is $\forall_{ABB'} = \frac{1}{2}\,\ell_B\,A = \frac{1}{2}(2m)(0.5m)^2 = 0.25\,m^3$ so that

$$Q = \frac{\forall_{ABB'}}{\Delta t} = \frac{0.25\,m^3}{0.1s} = \underline{\underline{2.5\,\frac{m^3}{s}}}$$

Since $\forall_{CDD'} = \forall_{EFF'} = V_{ABB'}$ it follows that the same value of Q is obtained regardless which volume is used.

4.62

4.62 In the region just downstream of a sluice gate, the water may develop a reverse flow region as is indicated in Fig. P4.62 and Video V10.9 The velocity profile is assumed to consist of two uniform regions, one with velocity $V_a = 10$ fps and the other with $V_b = 3$ fps. Determine the net flowrate of water across the portion of the control surface at section (2) if the channel is 20 ft wide.

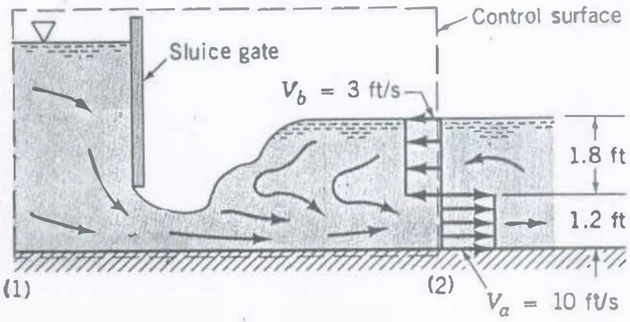


FIGURE P4.62

$$Q = V_a A_a - V_b A_b = (10 \tfrac{ft}{s})(1.2ft)(20ft) - (3\tfrac{ft}{s})(1.8ft)(20ft)$$
$$= \underline{\underline{132 \tfrac{ft^3}{s}}}$$

4.63

4.63 At time $t = 0$ the valve on an initially empty (perfect vacuum, $\rho = 0$) tank is opened and air rushes in. If the tank has a volume of $\forall_0$ and the density of air within the tank increases as $\rho = \rho_\infty(1 - e^{-bt})$, where b is a constant, determine the time rate of change of mass within the tank.

For $t \geq 0$, $\rho = \rho_0[1 - e^{-bt}]$ so that M = mass of air in tank $= \rho\forall_0 = \rho_0\forall_0[1 - e^{-bt}]$

Thus, $\dfrac{dM}{dt} = \underline{\underline{\rho_0 \forall_0 b e^{-bt}}}$

4.65

4.65 Water enters the bend of a river with the uniform velocity profile shown in Fig. P4.65. At the end of the bend there is a region of separation or reverse flow. The fixed control volume $ABCD$ coincides with the system at time $t = 0$. Make a sketch to indicate (a) the system at time $t = 5$ s and (b) the fluid that has entered and exited the control volume in that time period.

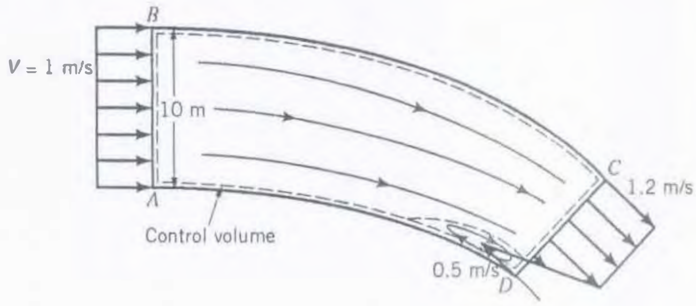
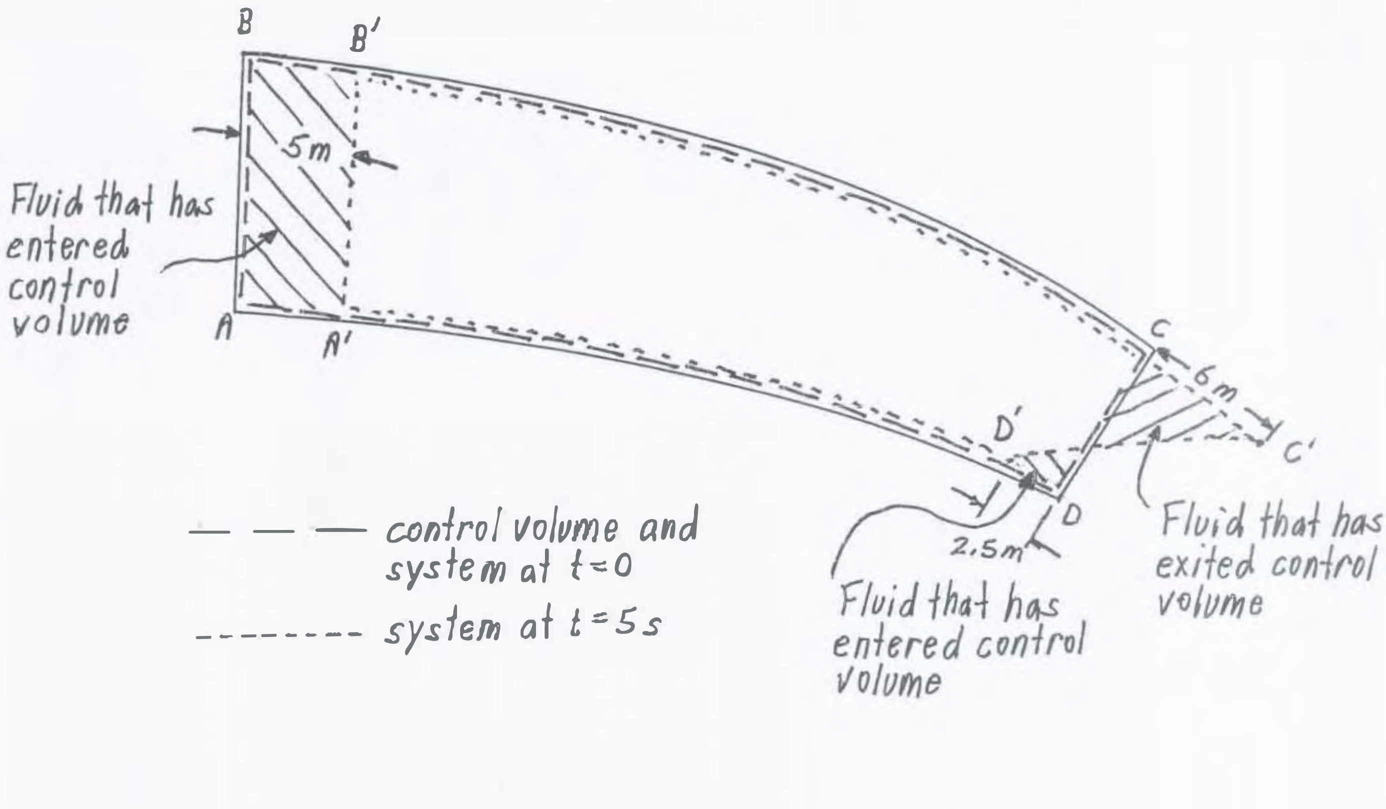


FIGURE P4.65

Since the distance the fluid travels in time $\delta t = 5s$ is $\ell = V\delta t$, the fluid at A-B when $t=0$ has traveled $\ell = (1 m/s)(5s) = 5m$ when $t = \delta t = 5s$. This is shown in the figure below. Similarly, the fluid across C-D at $t=0$ has moved as indicated when $t=\delta t = 5s$. Thus, the boundary of the system at $t=5s$ are as show in the figure below. The fluid that entered and exited the control volume in that time period is also shown.



4.66

4.66 A layer of oil flows down a vertical plate as shown in Fig. P4.66 with a velocity of $\mathbf{V} = (V_0/h^2)(2hx - x^2)\,\hat{\mathbf{j}}$ where V_0 and h are constants. (a) Show that the fluid sticks to the plate and that the shear stress at the edge of the layer ($x = h$) is zero. (b) Determine the flowrate across surface AB. Assume the width of the plate is b. (Note: The velocity profile for laminar flow in a pipe has a similar shape. See Video V6.13)

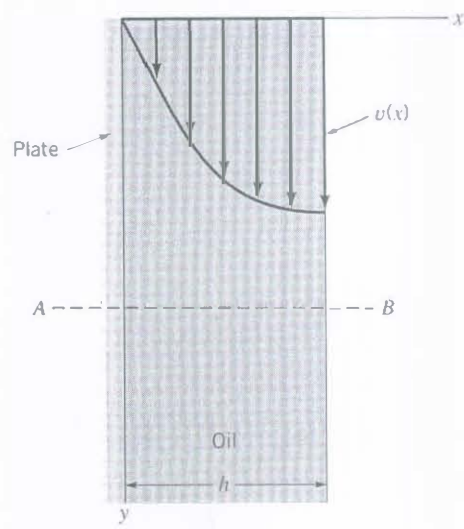


FIGURE P4.66

a) $v = \frac{V_0}{h^2}(2hx - x^2)$

Thus,

$$v\Big|_{x=0} = \frac{V_0}{h^2}(0-0) = 0 \quad \text{and}$$

$$\tau\Big|_{x=h} = \mu \frac{dv}{dx}\Big|_{x=h} = \mu \frac{V_0}{h^2}\left[2h - 2x\right|_{x=h} = 0$$

Hence, the fluid sticks to the plate and there is no shear stress at the free surface.

b) $$Q_{AB} = \int v\,dA = \int_{x=0}^{x=h} v\,b\,dx = \int_0^h \frac{V_0}{h^2}(2hx - x^2)\,b\,dx$$

or

$$Q_{AB} = \frac{V_0 b}{h^2}\left[hx^2 - \frac{1}{3}x^3\right|_0^h = \underline{\underline{\frac{2}{3}V_0 h b}}$$

4.67

4.67 Water flows in the branching pipe shown in Fig. P4.67 with uniform velocity at each inlet and outlet. The fixed control volume indicated coincides with the system at time $t = 20$ s. Make a sketch to indicate **(a)** the boundary of the system at time $t = 20.1$ s, **(b)** the fluid that left the control volume during that 0.1-s interval, and **(c)** the fluid that entered the control volume during that time interval.

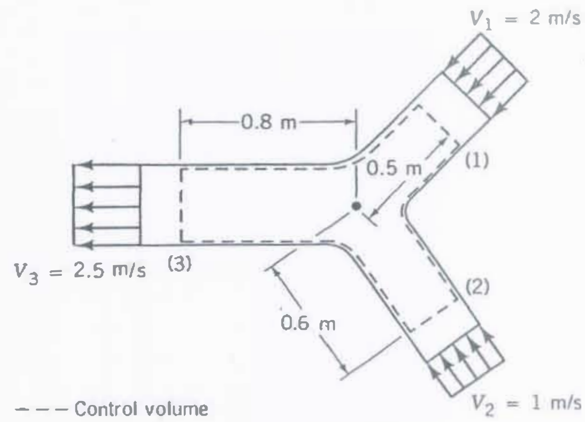


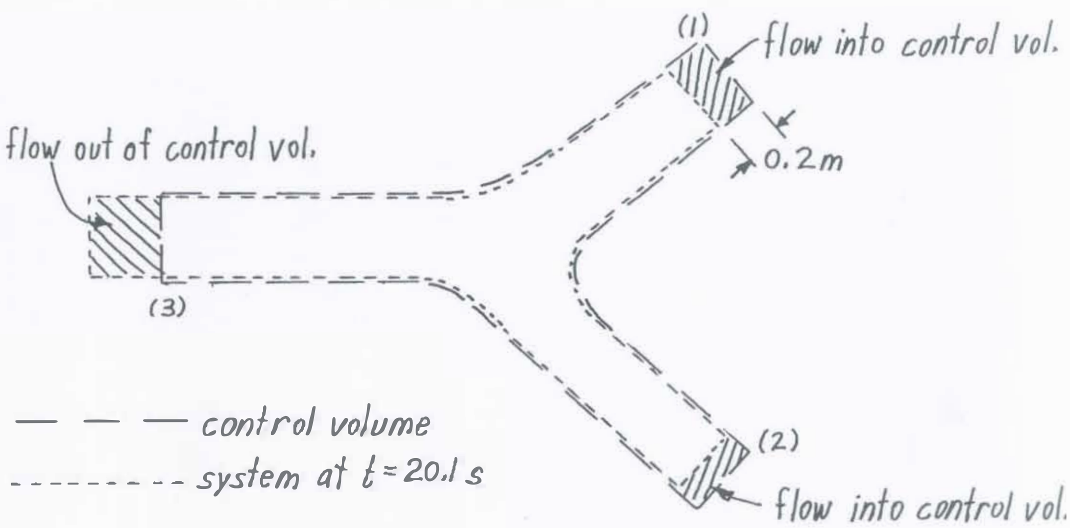
FIGURE P4.67

Since V is constant, the fluid travels a distance $\ell = V\Delta t$ in time Δt. Thus, $\ell_1 = V_1 \Delta t = (2\frac{m}{s})(20.1-20)s = 0.2\,m$

$$\ell_2 = V_2 \Delta t = (1\,\tfrac{m}{s})(20.1-20)s = 0.1\,m$$

and $\ell_3 = V_3 \Delta t = (2.5\frac{m}{s})(20.1-20)s = 0.25\,m$

The system at $t = 20.1\,s$ and the fluid that has entered or exited the control volume are indicated in the figure below.



4.68

4.68 Two plates are pulled in opposite directions with speeds of 1.0 ft/s as shown in Fig. P4.68. The oil between the plates moves with a velocity given by $\mathbf{V} = 10\, y\hat{\mathbf{i}}$ ft/s, where y is in feet. The fixed control volume $ABCD$ coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 0.2$ s and **(b)** the fluid that has entered and exited the control volume in that time period.

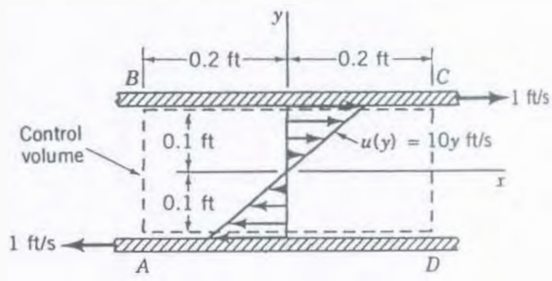
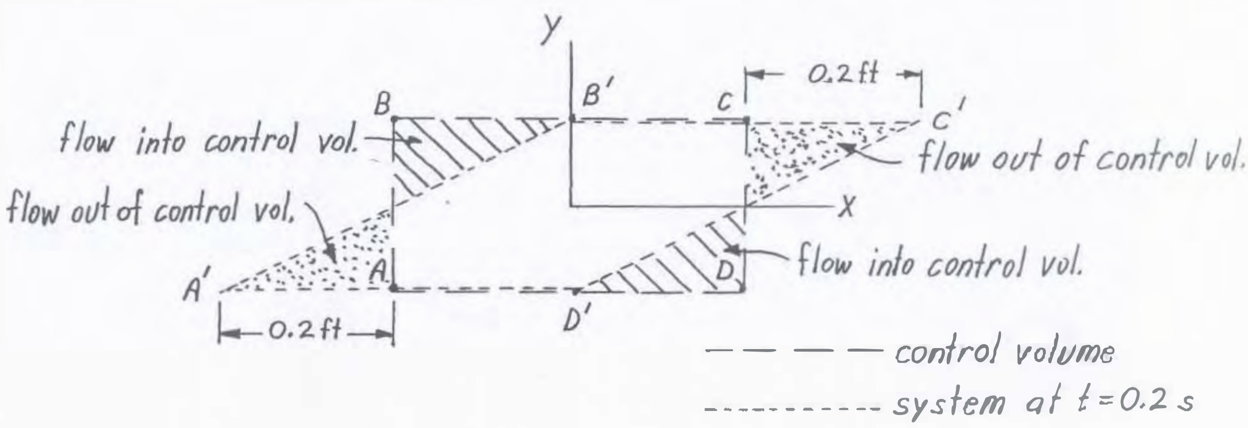


FIGURE P4.68

Since $\vec{V} = u(y)\hat{i} = 10y\,\hat{i}$ it follows that the fluid flows in the x-direction a distance of $\Delta x = u\,\Delta t = 10y\,(0.2)\text{ ft} = 2y\text{ ft}$ from $t=0$ to $t=0.2$ s. The lines A-B and C-D (the ends of the original system location) deform into lines A′-B′ and C′-D′ as shown in the figure below. The portions of the system that have entered and exited the control volume during this time are indicated.



4.69

4.69 Water is squirted from a syringe with a speed of $V = 5$ m/s by pushing in the plunger with a speed of $V_p = 0.03$ m/s as shown in Fig. P4.69. The surface of the deforming control volume consists of the sides and end of the cylinder and the end of the plunger. The system consists of the water in the syringe at $t = 0$ when the plunger is at section (1) as shown. Make a sketch to indicate the control surface and the system when $t = 0.5$ s.

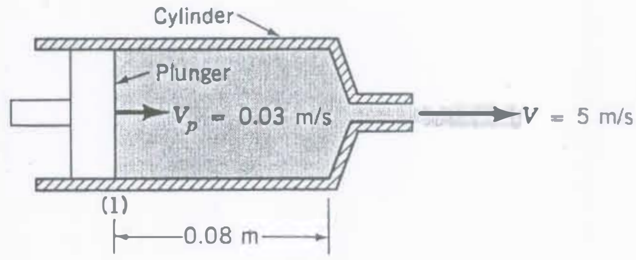
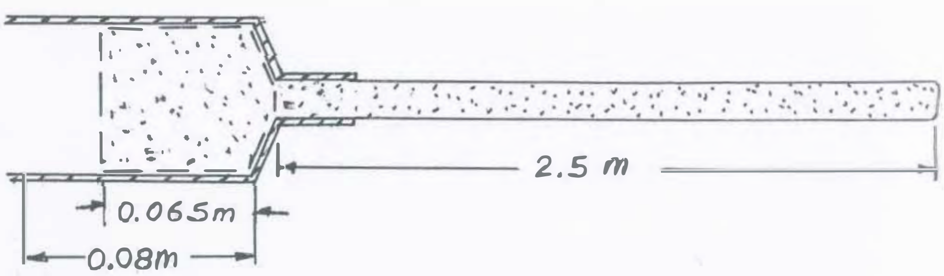


FIGURE P4.69

During the $t = 0.5$s time interval the plunger moves $\ell_1 = V_p\,\delta t = 0.015$m and the water initially at the exit moves $\ell_2 = V\,\delta t = 2.5$m. The corresponding control surfaces and systems at $t=0$ and $t=0.5$s shown in the figure below.



— — — control volume at $t=0.5$s

system at $t=0.5$s

4.70

4.70 Water enters a 5-ft-wide, 1-ft-deep channel as shown in Fig. P4.70. Across the inlet the water velocity is 6 ft/s in the center portion of the channel and 1 ft/s in the remainder of it. Farther downstream the water flows at a uniform 2 ft/s velocity across the entire channel. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate (a) the system at time $t = 0.5$ s and (b) the fluid that has entered and exited the control volume in that time period.

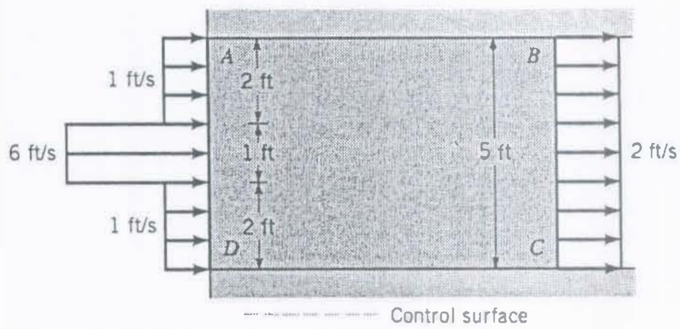
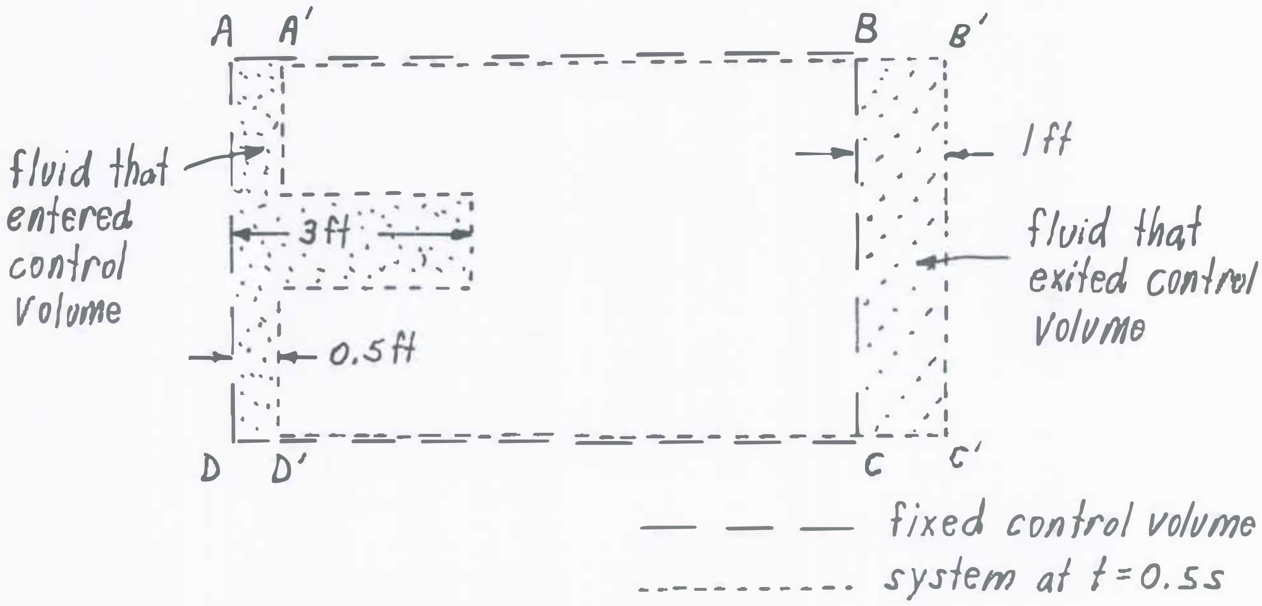


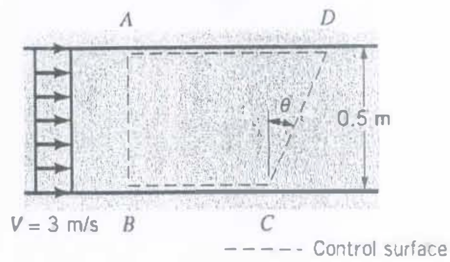
FIGURE P4.70

During the $t = 0.5$ s time interval the fluid that was along line BC at time $t=0$ has moved to the right a distance $\ell = V\,t = 2\frac{ft}{s}(0.5s) = 1\,ft$. Similarly, portions of the fluid along line AD have moved $\ell = 1\frac{ft}{s}(0.5s) = 0.5\,ft$ and $\ell = 6\frac{ft}{s}(0.5s) = 3\,ft$. This assumes the $1\frac{ft}{s}$ and $6\frac{ft}{s}$ fluid streams do not mix or intermingle during the 0.5 s time interval. See figure below.



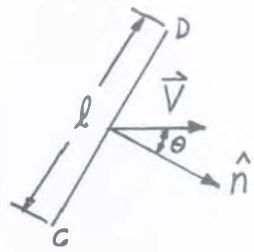
4.71

4.71 Water flows through the 2-m-wide rectangular channel shown in Fig. P4.71 with a uniform velocity of 3 m/s. **(a)** Directly integrate Eq. 4.16 with $b = 1$ to determine the mass flowrate (kg/s) across section CD of the control volume. **(b)** Repeat part (a) with $b = 1/\rho$, where ρ is the density. Explain the physical interpretation of the answer to part **(b)**.



■ FIGURE P4.71

a) $$\dot{B}_{out} = \int_{CS_{out}} \rho b \, \vec{V}\cdot\hat{n} \, dA \qquad (1)$$



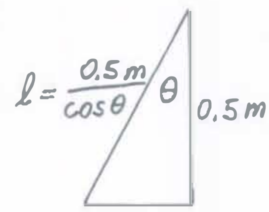
With $b = 1$ and $\vec{V}\cdot\hat{n} = V\cos\theta$ this becomes

$$\dot{B}_{out} = \int_{CD} \rho V\cos\theta \, dA = \rho V\cos\theta \int_{CD} dA$$

$$= \rho V\cos\theta \, A_{CD}, \quad \text{where } A_{CD} = \ell\,(2\,m)$$

$$= \left(\frac{0.5\,m}{\cos\theta}\right)(2\,m)$$

$$= \left(\frac{1}{\cos\theta}\right) m^2$$



Thus, with $V = 3\,m/s$,

$$\dot{B}_{out} = \left(3\,\frac{m}{s}\right)\cos\theta\left(\frac{1}{\cos\theta}\right)m^2\left(999\,\frac{kg}{m^3}\right) = \underline{\underline{3000\,\frac{kg}{s}}}$$

b) With $b = 1/\rho$ Eq. (1) becomes

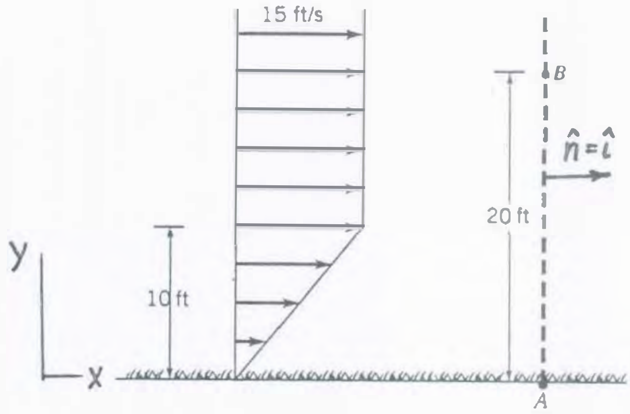
$$\dot{B}_{out} = \int_{CD} \vec{V}\cdot\hat{n}\, dA = \int_{CD} V\cos\theta \, dA = V\cos\theta \, A_{CD}$$

$$= \left(3\,\frac{m}{s}\right)\cos\theta\left(\frac{1}{\cos\theta}\right)m^2 = \underline{\underline{3.00\,\frac{m^3}{s}}}$$

With $b = 1/\rho = \dfrac{1}{\left(\frac{mass}{Vol}\right)} = \dfrac{vol}{mass}$ it follows that "B = volume" (i.e., $b = \frac{B}{mass}$) so that $\int \vec{V}\cdot\hat{n}\,dA = \dot{B}_{out}$ represents the volume flowrate (m^3/s) from the control volume.

4.72

4.72 The wind blows across a field with an approximate velocity profile as shown in Fig. P4.72. Use Eq. 4.16 with the parameter b equal to the velocity to determine the momentum flowrate across the vertical surface $A-B$, which is of unit depth into the paper.



■ FIGURE P4.72

$$\dot{\vec{B}}_{AB} = \int_{AB} \rho \vec{b}\, \vec{V}\cdot\hat{n}\, dA = \int_{AB} \rho \vec{V}\, \vec{V}\cdot\hat{n}\, dA = \rho \int_{y=0}^{y=20\,ft} (V\hat{i})[(V\hat{i})\cdot\hat{i}](1\,ft)\,dy$$

$$= \rho\hat{i}\int_0^{20} V^2\, dy$$

But, $V = \frac{15}{10}y\ \frac{ft}{s}$ for $0 \le y \le 10\,ft$ (i.e., $V=0$ at $y=0$; $V=15\frac{ft}{s}$ at $y=10$)
and $V = 15\frac{ft}{s}$ for $y \ge 10\,ft$

Thus,

$$\dot{\vec{B}}_{AB} = \rho\hat{i}\left[\int_0^{10}\left(\frac{15}{10}y\right)^2 dy + \int_{10}^{20}(15)^2 dy\right] = \rho\hat{i}\left[2.25\,\frac{y^3}{3}\Big|_0^{10} + 225\,y\Big|_0^{10}\right]$$

$$= 0.00238\,\frac{slugs}{ft^3}\left[750\,\frac{ft^4}{s^2} + 2250\,\frac{ft^4}{s^2}\right]\hat{i}$$

$$= \underline{\underline{7.14\hat{i}\,\frac{slug\ ft}{s^2}}} = 7.14\,\hat{i}\ lb$$

5.5

5.5 Water enters a cylindrical tank through two pipes at rates of 250 and 100 gal/min (see Fig. P5.5). If the level of the water in the tank remains constant, calculate the average velocity of the flow leaving the tank through an 8-in. inside-diameter pipe.

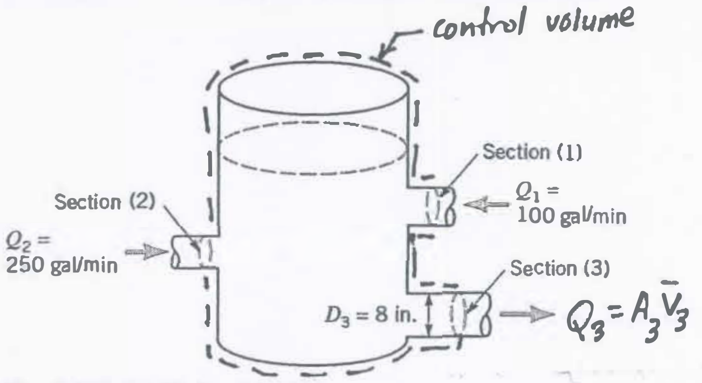


FIGURE P6.6

For steady and incompressible flow through the control volume shown

$$Q_3 = Q_1 + Q_2$$

or

$$\bar{V}_3 = \frac{1}{A_3}\left(Q_1 + Q_2\right) = \frac{1}{\frac{\pi d_3^2}{4}}\left(Q_1 + Q_2\right)$$

$$\bar{V}_3 = \frac{1}{\frac{\pi (8 \text{ in.})^2}{4}}\left(100\,gpm + 250\,gpm\right)\left(231\,\frac{in.^3}{gal}\right)\left(\frac{1}{60\,\frac{s}{min}}\right)\left(\frac{1}{12\,\frac{in.}{ft}}\right)$$

$$\bar{V}_3 = \underline{\underline{2.23}}\ \frac{ft}{s}$$

5.6

5.6 Water flows out through a set of thin, closely spaced blades as shown in Fig. 5.6 with a speed of $V = 10$ ft/s around the entire circumference of the outlet. Determine the mass flowrate through the inlet pipe.

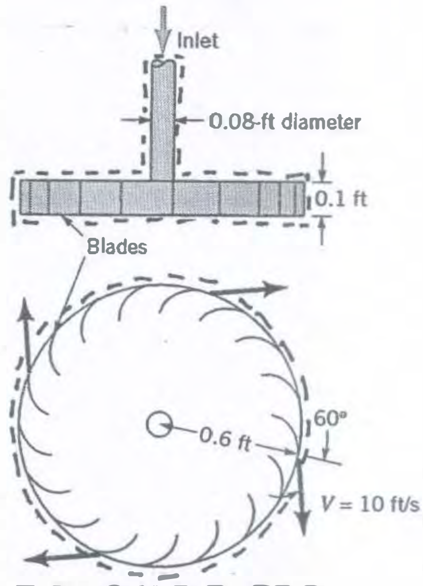


FIGURE P5.6

Use the control volume contained within the broken lines shown in the sketch above.

From the conservation of mass principle

$$\dot{m}_{inlet} = \dot{m}_{outlet}$$

Also

$$\dot{m}_{outlet} = \rho A_{outlet} V_{outlet} \cos 60^\circ$$

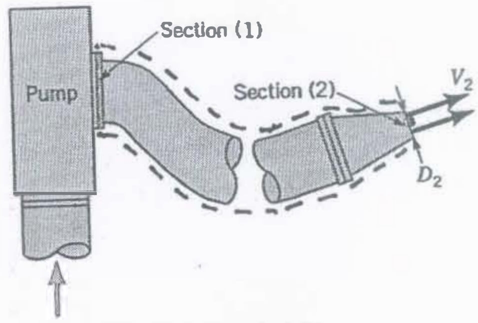
$$= \rho\, 2\pi r_{outlet} h V_{outlet} \cos 60^\circ$$

$$= \left(1.94 \frac{slugs}{ft^3}\right) 2\pi (0.6ft)(0.1ft)\left(10\frac{ft}{s}\right)\cos 60^\circ$$

$$= \underline{\underline{3.66}} \frac{slugs}{s}$$

5.7

5.7 The pump shown in Fig. P5.7 produces a steady flow of 10 gal/s through the nozzle. Determine the nozzle exit diameter, D_2, if the exit velocity is to be $V_2 = 100$ ft/s.



■ FIGURE P5.7

For steady flow $Q_1 = Q_2$, where $Q_1 = 10\,\frac{gal}{s}\left(231\,\frac{in.^3}{gal}\right)\left(\frac{1\,ft^3}{1728\,in.^3}\right) = 1.337\,\frac{ft^3}{s}$

Thus, with $V_2 = 100\,\frac{ft}{s}$,

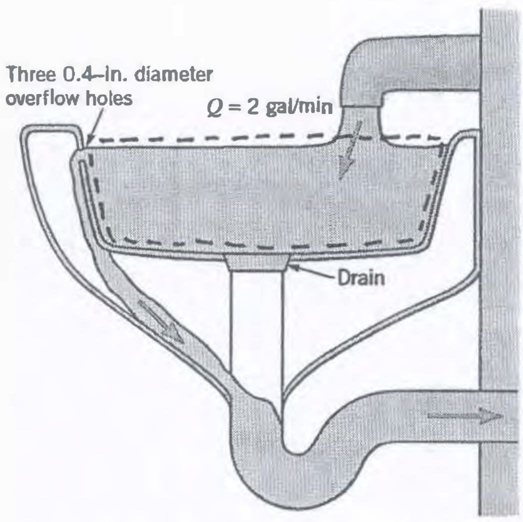
$$1.337\,\frac{ft^3}{s} = A_2 V_2 = \frac{\pi}{4} D_2^2 \left(100\,\frac{ft}{s}\right)$$

or

$$D_2 = 0.130\,ft = \underline{\underline{1.57\ in.}}$$

5.8

5.8 Water flows into a sink as shown in Video V5.1 and Fig. P5.8 at a rate of 2 gallons per minute. Determine the average velocity through each of the three 0.4-in.-diameter overflow holes if the drain is closed and the water level in the sink remains constant.



■ FIGURE P5.8

$Q_1 = Q_2$ for the control volume indicated,

where

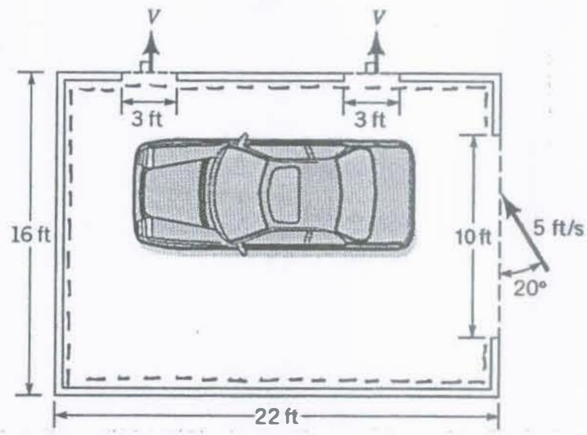
$$Q_1 = 2\,\frac{gal}{min}\left(\frac{1\,min}{60\,s}\right)\frac{1}{7.48\,\frac{gal}{ft^3}} = 0.00446\,\frac{ft^3}{s}$$

Thus,

$$Q_1 = A_2 V_2 \quad or \quad V_2 = \frac{Q_1}{A_2} = \frac{0.00446\,\frac{ft^3}{s}}{3\left[\frac{\pi}{4}\left(\frac{0.4}{12}ft\right)^2\right]} = \underline{\underline{1.70\,\frac{ft}{s}}}$$

5.9

5.9 The wind blows through a 7 ft × 10 ft garage door opening with a speed of 5 ft/s as shown in Fig. P5.9. Determine the average speed, V, of the air through the two 3 ft × 4 ft openings in the windows.



■ FIGURE P5.9

For steady incompressible flow

$$Q_{\substack{garage\\door}} = Q_{window} + Q_{window}$$

or

$$A_{\substack{garage\\door}} V_{\substack{normal\ to\\garage\ door}} = A_{window} V + A_{window} V$$

so the average speed, V, of the air through the two windows is

$$V = \frac{A_{\substack{garage\\door}} V_{normal\ to\ garage\ door}}{2A_{window}} = \frac{(7ft)(10ft)(5\frac{ft}{s})\sin 20^\circ}{2(3ft)(4ft)} = \underline{\underline{4.99\ \frac{ft}{s}}}$$

5.10

5.10 The human circulatory system consists of a complex branching pipe network ranging in diameter from the aorta (largest) to the capillaries (smallest). The average radii and the number of these vessels is shown in the table below. Does the average blood velocity increase, decrease, or remain constant as it travels from the aorta to the capillaries?

Vessel	Average Radius, mm	Number	r^2N, mm^2
Aorta	12.5	1	156
Arteries	2.0	159	636
Arterioles	0.03	1.4×10^7	12,600
Capillaries	0.006	3.9×10^9	140,400

The average blood velocity, V, is related to the blood mass flow, $\dot{m}$, by

$$V = \frac{\dot{m}}{\rho A N}$$

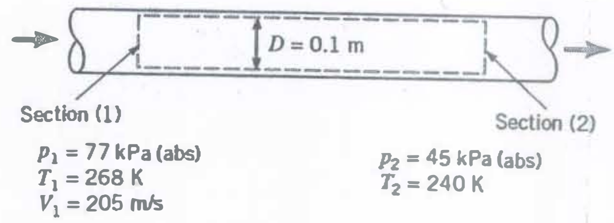
where ρ is blood density, A is vessel cross section area (πr^2) and N is number of vessels. So for constant $\dot{m}$ and ρ,

$$V = \dot{m}/(\rho \pi r^2 N)$$

and since the r^2N product becomes larger, the average velocity <u>becomes smaller</u>.

5.11

5.11 Air flows steadily between two cross sections in a long, straight section of 0.1-m inside diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.11. If the average air velocity at section (1) is 205 m/s, determine the average air velocity at section (2).



■ FIGURE P5.11

This analysis is similar to the one of Example 5.2. For steady flow between sections (1) and (2)

$$\dot{m}_2 = \dot{m}_1$$

or

$$\rho_2 A_2 \overline{V}_2 = \rho_1 A_1 \overline{V}_1$$

Thus

$$\overline{V}_2 = \frac{\rho_1}{\rho_2}\frac{A_1}{A_2}\overline{V}_1 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.8) to get

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}\frac{T_2}{T_1} \qquad (2)$$

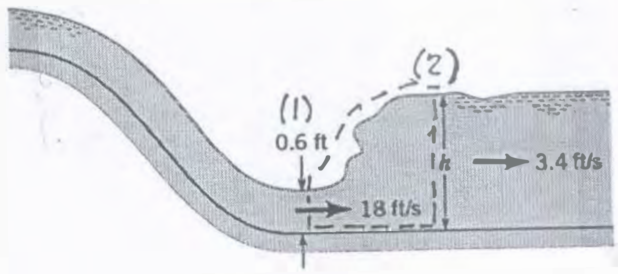
Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\overline{V}_2 = \frac{p_1 T_2}{p_2 T_1}\overline{V}_1 = \frac{[77\ kPa(abs)](240K)}{[45\ kPa(abs)](268K)}\left(205\ \frac{m}{s}\right)$$

$$\overline{V}_2 = 314\ \frac{m}{s}$$

5.12

5.12 A hydraulic jump (see Video V10.10) is in place downstream from a spillway as indicated in Fig. P5.12. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.



■ FIGURE P5.12

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

Thus

$$\bar{V}_1 h_1 = \bar{V}_2 h_2$$

and

$$h_2 = \frac{\bar{V}_1 h_1}{\bar{V}_2} = \frac{\left(18\ \frac{ft}{s}\right)(0.6\ ft)}{\left(3.4\ \frac{ft}{s}\right)} = \underline{\underline{3.18\ ft}}$$

5.13

5.13 An evaporative cooling tower (see Fig. P5.13) is used to cool water from 110 to 80°F. Water enters the tower at a rate of 250,000 lbm/hr. Dry air (no water vapor) flows into the tower at a rate of 151,000 lbm/hr. If the rate of wet air flow out of the tower is 156,900 lbm/hr, determine the rate of water evaporation in lbm/hr and the rate of cooled water flow in lbm/hr.

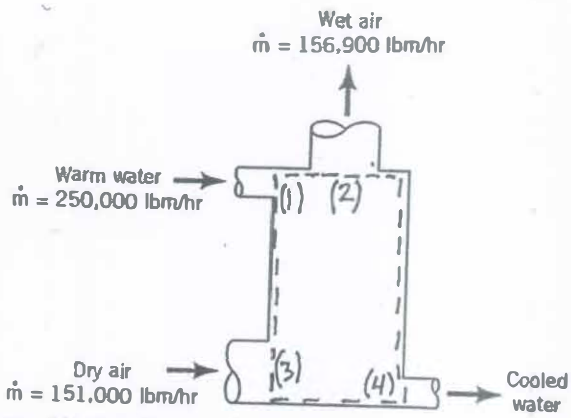


FIGURE P5.13

For steady flow of dry air

$$\dot{m}_3 = \dot{m}_{2,\text{dry air}} \qquad (1)$$

For steady flow of water

$$\dot{m}_1 = \dot{m}_{2,\text{water}} + \dot{m}_4 \qquad (2)$$

Also

$$\dot{m}_2 = \dot{m}_{2,\text{dry air}} + \dot{m}_{2,\text{water}} \qquad (3)$$

Combining Eqs. 1 and 3 we get

$$\dot{m}_{2,\text{water}} = \dot{m}_2 - \dot{m}_3 = \text{rate of water evaporation}$$

So

$$\dot{m}_{2,\text{water}} = 156,900\ \frac{\text{lbm}}{\text{hr}} - 151,000\ \frac{\text{lbm}}{\text{hr}} = \underline{\underline{5900\ \frac{\text{lbm}}{\text{hr}}}}$$

From Eq. 2 we get

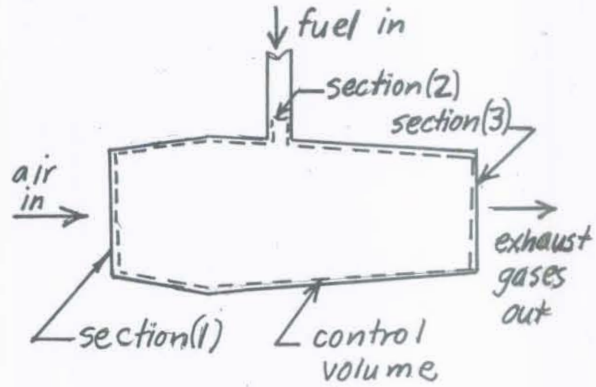
$$\dot{m}_4 = \dot{m}_1 - \dot{m}_{2,\text{water}} = \text{rate of cooled water flow}$$

or

$$\dot{m}_4 = 250,000\ \frac{\text{lbm}}{\text{hr}} - 5900\ \frac{\text{lbm}}{\text{hr}} = \underline{\underline{244,000\ \frac{\text{lbm}}{\text{hr}}}}$$

5.14

5.14 At cruise conditions, air flows into a jet engine at a steady rate of 65 lbm/s. Fuel enters the engine at a steady rate of 0.60 lbm/s. The average velocity of the exhaust gases is 1500 ft/s relative to the engine. If the engine exhaust effective cross-sectional area is 3.5 ft^2, estimate the density of the exhaust gases in lbm/ft^3.



For steady flow

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

or

$$\rho_3 A_3 \overline{V}_3 = \dot{m}_1 + \dot{m}_2$$

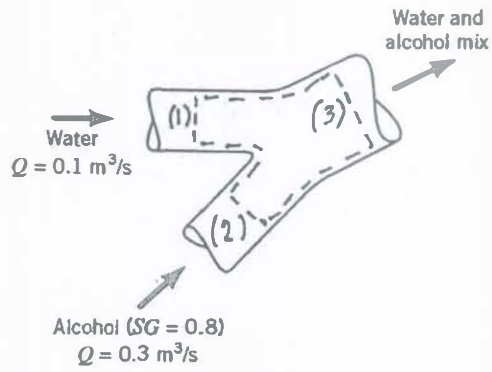
Thus

$$\rho_3 = \frac{\dot{m}_1 + \dot{m}_2}{A_3 \overline{V}_3} = \frac{65 \frac{lbm}{s} + 0.60 \frac{lbm}{s}}{(3.5\ ft^2)\left(1500 \frac{ft}{s}\right)}$$

$$\rho_3 = \underline{\underline{0.0125}} \ \frac{lbm}{ft^3}$$

5.15

5.15 Water at 0.1 m^3/s and alcohol ($SG=0.8$) at 0.3 m^3/s are mixed in a *y*-duct as shown in Fig. 5.15. What is the average density of the mixture of alcohol and water?



■ FIGURE P5.15

For steady flow

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

or

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 Q_3 \qquad (1)$$

Also, since the water and alcohol may be considered incompressible

$$Q_1 + Q_2 = Q_3 \qquad (2)$$

Combining Eqs. 1 and 2 we get

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 (Q_1 + Q_2)$$

or

$$\rho_3 = \frac{\rho_1 Q_1 + \rho_2 Q_2}{Q_1 + Q_2}$$

and

$$\rho_3 = \rho_1 \frac{(Q_1 + SG_2 Q_2)}{Q_1 + Q_2}$$

Thus

$$\rho_3 = \frac{\left(999 \frac{kg}{m^3}\right)\left[0.1 \frac{m^3}{s} + (0.8)(0.3 \frac{m^3}{s})\right]}{0.1 \frac{m^3}{s} + 0.3 \frac{m^3}{s}} = \underline{\underline{849 \frac{kg}{m^3}}}$$

5.16

5.16 Freshwater flows steadily into an open 55-gal drum initially filled with seawater. The freshwater mixes thoroughly with the seawater and the mixture overflows out of the drum. If the freshwater flowrate is 10 gal/min, estimate the time in seconds required to decrease the difference between the density of the mixture and the density of fresh water by 50%.

A fixed, non-deforming control volume that contains the water mixture in the 55-gal drum is used. Fresh water enters the control volume with density, ρ_1, and volume flowrate, Q_1. The mixture is assumed to be homogeneous throughout the control volume and leaves the control volume with density, ρ_2, and volume flowrate, Q_2. Application of the conservation of mass equation (Eq. 5.5) to the flow through this control volume yields

$$\frac{\partial}{\partial t}\int_{cv} \rho_2 \, d\forall + \rho_2 Q_2 - \rho_1 Q_1 = 0 \qquad (1)$$

Since the fluids involved are incompressible, $Q_1 = Q_2 = Q$. Also, the volume of the control volume is constant. Thus Eq. 1 leads to

$$\forall_{cv} \frac{d(\rho_2/\rho_1)}{dt} + \frac{\rho_2}{\rho_1} Q = Q$$

or

$$\frac{d(\rho_2/\rho_1)}{dt} + \left(\frac{\rho_2}{\rho_1}\right)\frac{Q}{\forall_{cv}} = \frac{Q}{\forall_{cv}} \qquad (2)$$

The solution of Eq. 2 is

$$\frac{\rho_2}{\rho_1} = C\, e^{-\frac{Q}{\forall_{cv}} t} + 1.0 \qquad (3)$$

At $t = 0$, $\rho_2/\rho_1 = \dfrac{\rho_{seawater}}{\rho_{fresh\ water}} = \dfrac{(1.99 \ \mathrm{slugs/ft^3})}{(1.94 \ \mathrm{slugs/ft^3})} = 1.026$

thus

$$C = 0.026$$

(con't)

5.16 (Con't)

Then for

$$\rho_{2,f} - \rho_1 = 0.5(\rho_{2,i} - \rho_1)$$

where

$\rho_{2,f}$ = final mixture density

$\rho_{2,i}$ = initial mixture density

we have

$$\frac{\rho_{2,f}}{\rho_1} = 0.5\left(\frac{\rho_{2,i}}{\rho_1} + 1\right) = 0.5(1.026 + 1) = 1.013$$

Substituting this value and other givens into Eq. 3 we get

$$1.013 = 0.026\, e^{-\frac{(10\ gal/min)}{(55\ gal)(60\ \frac{min}{s})} t} + 1.0$$

and

$$t = \underline{\underline{229}}\ s$$

5.17

5.17 A water jet pump (see Fig. P5.17) involves a jet cross-sectional area of 0.01 m^2, and a jet velocity of 30 m/s. The jet is surrounded by entrained water. The total cross-sectional area associated with the jet and entrained streams is 0.075 m^2. These two fluid streams leave the pump thoroughly mixed with an average velocity of 6 m/s through a cross-sectional area of 0.075 m^2. Determine the pumping rate (i.e., the entrained fluid flowrate) involved in liters/s.

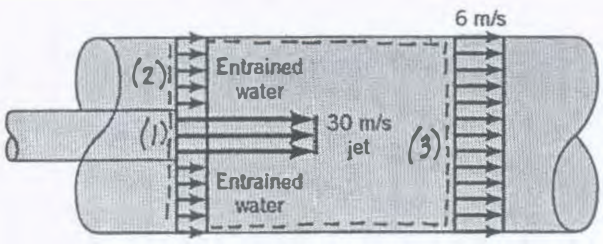


FIGURE P5.17

For steady incompressible flow through the control volume

$$Q_1 + Q_2 = Q_3$$

or

$$\bar{V}_1 A_1 + Q_2 = \bar{V}_3 A_3$$

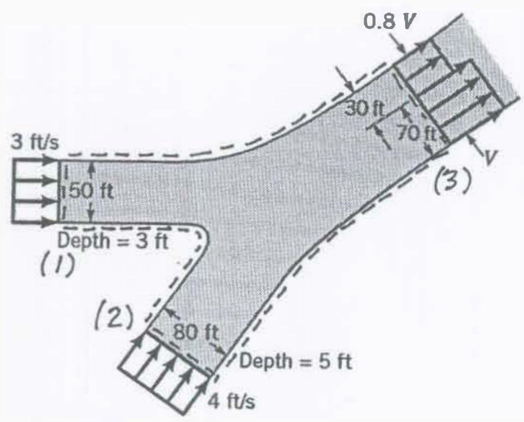
Thus

$$Q_2 = \bar{V}_3 A_3 - \bar{V}_1 A_1 = \left[\left(6\ \frac{m}{s}\right)\left(0.075\ m^2\right) - \left(30\ \frac{m}{s}\right)\left(0.01\ m^2\right)\right]\left(1000\ \frac{\text{liters}}{m^3}\right)$$

$$Q_2 = 150\ \frac{\text{liters}}{s}$$

5.18

5.18 Two rivers merge to form a larger river as shown in Fig. P5.18. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown and the depth is 6 ft. Determine the value of V.



■ FIGURE P5.18

Use the control volume shown within broken lines in the sketch above. We note that $\dot{m} = \rho A V$ and from the conservation of mass principle we get

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0.8V} + \dot{m}_V$$

Thus

$$\rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0.8V}\, 0.8\, V + \rho A_V V$$

and

$$V = \frac{A_1 V_1 + A_2 V_2}{A_{0.8V}(0.8) + A_V} = \frac{(50\text{ft})(3\text{ft})(3\frac{\text{ft}}{\text{s}}) + (80\text{ft})(5\text{ft})(4\frac{\text{ft}}{\text{s}})}{(30\text{ft})(6\text{ft})(0.8) + (70\text{ft})(6\text{ft})}$$

$$V = \underline{\underline{3.63}}\ \frac{\text{ft}}{\text{s}}$$

5.19

5.19 Various types of attachments can be used with the shop vac shown in Video V5.2. Two such attachments are shown in Fig. P5.19—a nozzle and a brush. The flowrate is 1 ft^3/s. **(a)** Determine the average velocity through the nozzle entrance, V_n. **(b)** Assume the air enters the brush attachment in a radial direction all around the brush with a velocity profile that varies linearly from 0 to V_b along the length of the bristles as shown in the figure. Determine the value of V_b.

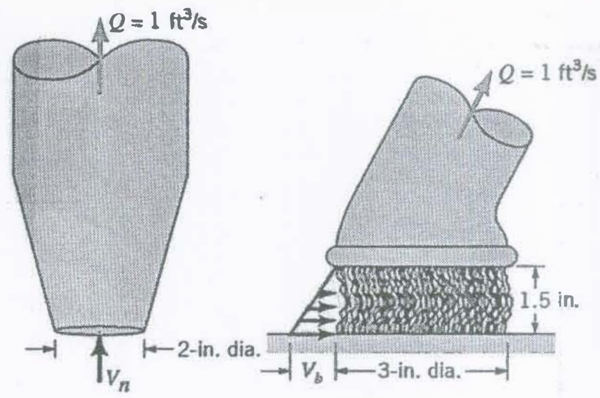


FIGURE P5.19

(a) $Q_1 = Q_2$ where $Q_2 = 1\,\frac{ft^3}{s}$

Thus,

$$A_1 V_1 = Q_2 \quad \text{or} \quad V_1 \equiv V_n = \frac{1\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2}$$

so

$$V_n = \underline{\underline{45.8\,\frac{ft}{s}}}$$

(b) $Q_3 = Q_4$ where $Q_4 = 1\,\frac{ft^3}{s}$ and $Q_3 = \overline{V}_3 A_3$ where

$\overline{V}_3$ = average velocity at (3) $= \frac{1}{2} V_b$ and

$A_3 = \pi D_3 h_3$

Thus,

$$\frac{1}{2} V_b \left[\pi\left(\frac{3}{12}\,ft\right)\left(\frac{1.5}{12}\,ft\right)\right] = 1\,\frac{ft^3}{s}, \text{ or}$$

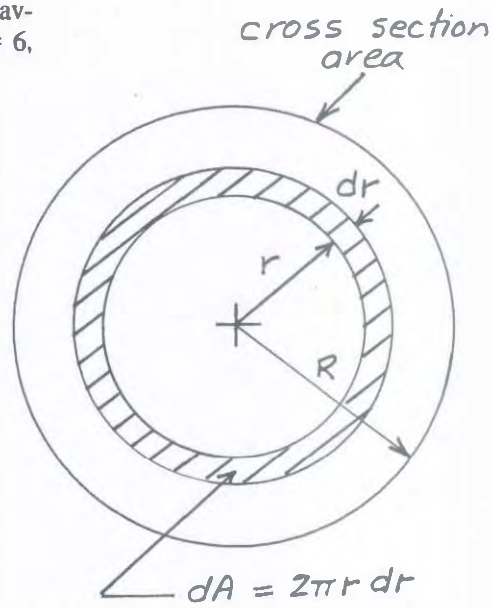
$$V_b = \underline{\underline{20.4\,\frac{ft}{s}}}$$

5.20

5.20 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c\left(\frac{R-r}{R}\right)^{1/n}\hat{\mathbf{i}}$$

where u_c = centerline velocity, r = local radius, R = pipe radius, and $\hat{\mathbf{i}}$ = unit vector along pipe centerline. Determine the ratio of average velocity, $\bar{u}$, to centerline velocity, u_c, for (a) $n = 4$, (b) $n = 6$, (c) $n = 8$, (d) $n = 10$. Compare the different velocity profiles.



For any cross section area

$$\dot{m} = \rho A \bar{u} = \int_A \rho\, \vec{V}\cdot\hat{n}\, dA$$

Also

$$\vec{V}\cdot\hat{n} = \vec{V}\cdot\hat{i} = u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}}$$

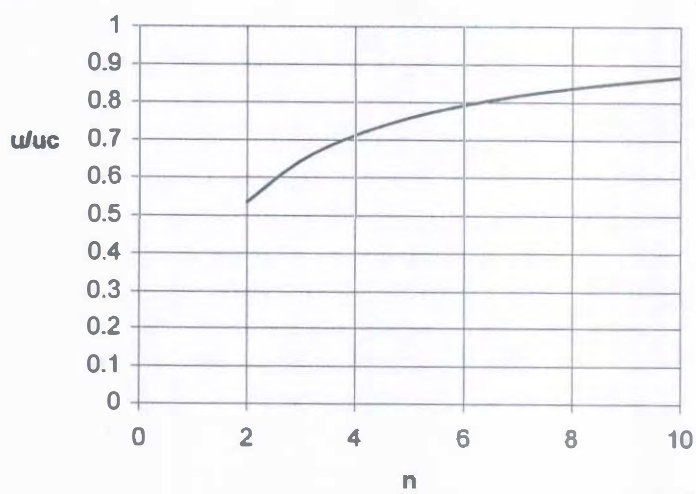
Thus for a uniformly distributed density, ρ, over area A

$$\bar{u} = \frac{\int_0^R u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}} 2\pi r\, dr}{\pi R^2}$$

and

$$\frac{\bar{u}}{u_c} = 2\int_0^R \left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = \frac{2n^2}{2n^2+3n+1}$$

n	$\frac{\bar{u}}{u_c}$
4	0.711
6	0.791
8	0.837
10	0.866



The different velocity profiles (including for laminar flow) are compared in Fig. 8.18. Since the profile for $n = 4$ is not practically significant, it is not shown.

5.21

5.21 As shown in Fig. P5.21, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity V. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where u is in ft/s and y is in ft. Determine the value of V.

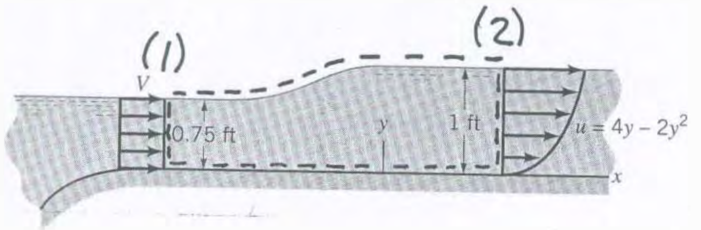


FIGURE P5.21

Use the control volume indicated by the broken lines in the sketch above.

From the conservation of mass principle

$$Q_1 = Q_2$$

$$V_1 A_1 = \int_{A_2} u\,dA \quad \int_0^{1\,ft} (4y - 2y^2)\, b\, dy$$

$$V(0.75\,ft)\,b = 3\left[\frac{4y^2}{2} - \frac{2y^3}{3}\right]_0^{1\,ft} b = \frac{4b}{3}\,\frac{ft^3}{s}$$

$$V = \frac{4}{3(0.75)} = \underline{\underline{1.78}}\ \frac{ft}{s}$$

5.22

5.22 A water flow situation is described by the velocity field equation

$$\mathbf{V} = (3x + 2)\hat{\mathbf{i}} + (2y - 4)\hat{\mathbf{j}} - 5z\hat{\mathbf{k}} \text{ ft/s}$$

where x, y, and z are in feet. **(a)** Determine the mass flow rate through the rectangular area in the plane corresponding to $z = 2$ feet having corners at $(x, y, z) = (0, 0, 2), (5, 0, 2), (5, 5, 2)$, and $(0, 5, 2)$ as shown in Fig. P5.22a. **(b)** Show that mass is conserved in the control volume having corners at $(x, y, z) = (0, 0, 2), (5, 0, 2), (5, 5, 2), (0, 5, 2), (0, 0, 0), (5, 0, 0), (5, 5, 0)$, and $(0, 5, 0)$ as shown in Fig. P5.22b.

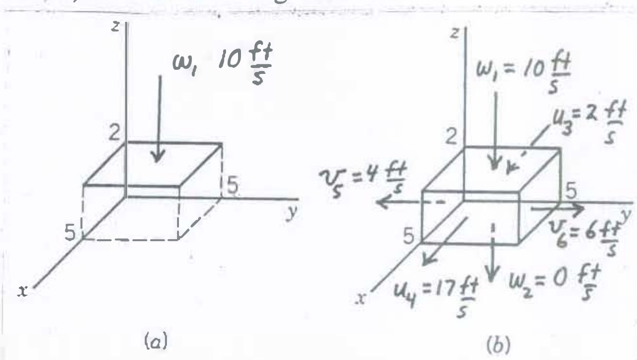


FIGURE P5.22

(a) The general expression for mass flowrate across area A_1 is

$$\dot{m}_1 = \int_{A_1} \rho \vec{V} \cdot \hat{n}\, dA$$

Since the z-direction component of velocity, w_1, is uniformly distributed over A_1, we can use

$$\dot{m}_1 = \rho(w_1 A_1) = \left(1.94\ \frac{slugs}{ft^3}\right)\left(10\ \frac{ft}{s}\right)\left(25\ ft^2\right)$$

or

$$\underline{\underline{\dot{m}_1 = 485\ \frac{slugs}{s}}}$$

(b) If $\int_{CS} \rho \vec{V} \cdot \hat{n}\, dA = 0$, then mass is conserved.

However $\int_{CS} \rho \vec{V} \cdot \hat{n}\, dA = \sum \dot{m}$ and since the component of velocity normal to each plane area of the control volume is uniformly distributed over that area we have

$$\sum \dot{m} = \rho\left(-w_1 A_1 + w_2 A_2 - u_3 A_3 + u_4 A_4 + v_5 A_5 + v_6 A_6\right)$$

$$\sum \dot{m} = \rho\left(-250\ \frac{ft^3}{s} + 0\ \frac{ft^3}{s} - 20\ \frac{ft^3}{s} + 170\ \frac{ft^3}{s} + 40\ \frac{ft^3}{s} + 60\ \frac{ft^3}{s}\right)$$

$\sum \dot{m} = 0$ and mass is conserved.

5.23

5.23 An incompressible flow velocity field (water) is given as

$$\mathbf{V} = -\frac{1}{r}\,\hat{\mathbf{e}}_r + \frac{1}{r}\,\hat{\mathbf{e}}_\theta \text{ m/s}$$

where r is in meters. **(a)** Calculate the mass flowrate through the cylindrical surface at $r = 1$ m from $z = 0$ to $z = 1$ m as shown in Fig. P5.23a. **(b)** Show that mass is conserved in the annular control volume from $r = 1$ m to $r = 2$ m and $z = 0$ to $z = 1$ m as shown in Fig. P5.23b.

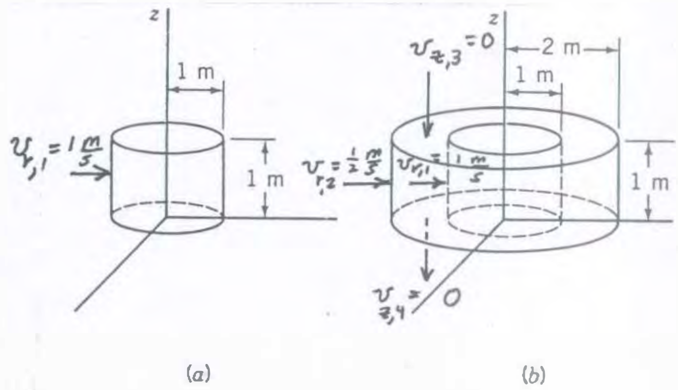


FIGURE P5.23

(a) The general expression for mass flowrate across cylindrical area A_1 is

$$\dot{m}_1 = \int_{A_1} \rho\, \vec{V}\cdot\hat{n}\, dA$$

Since the radial direction component of velocity, v_r, is uniformly distributed over A_1 we can use

$$\dot{m}_1 = \rho\,(v_{r,1} A_1) = \left(999\,\frac{kg}{m^3}\right)\left(1\,\frac{m}{s}\right)\left(2\pi\, m^2\right)$$

$$\dot{m}_1 = \underline{\underline{6280\,\frac{kg}{s}}}$$

(b) If $\int_{cs} \rho \vec{V}\cdot\hat{n}\, dA = 0$, then mass is conserved.

However $\int_{cs} \rho \vec{V}\cdot\hat{n}\, dA = \Sigma \dot{m}$ and since the component of velocity normal to each cylindrical and plane area of the control volume is uniformly distributed over that area we have

$$\Sigma \dot{m} = \rho\left(v_{r,1} A_1 - v_{r,2} A_2 - v_{z,3} A_3 + v_{z,4} A_4\right)$$

$$\Sigma \dot{m} = \rho\left(2\pi\,\frac{m^3}{s} - 2\pi\,\frac{m^3}{s} - 0\,\frac{m^3}{s} + 0\,\frac{m^3}{s}\right)$$

$\Sigma \dot{m} = 0$ and mass is conserved.

5.24

5.24 Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.24. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also U. If the x direction velocity profile at section (2) is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at x where the boundary layer thickness is δ.

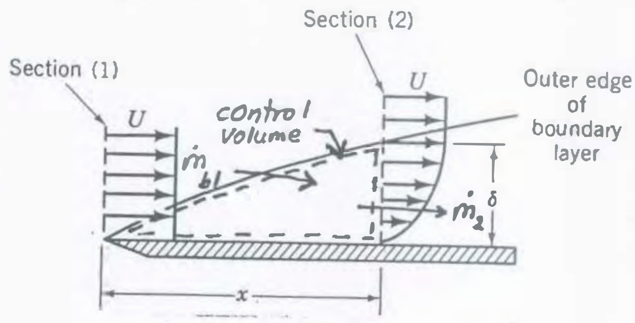


FIGURE P5.24

From the conservation flow through the control volume shown in the figure we have

$$\dot{m}_{bl} = \dot{m}_2 = \int_{A_2} \rho \vec{V} \cdot \hat{n}\, dA$$

For incompressible flow

$$\rho Q_{bl} = \rho U \ell \delta \int_0^1 \left(\frac{y}{\delta}\right)^{\frac{1}{7}} d\left(\frac{y}{\delta}\right)$$

where

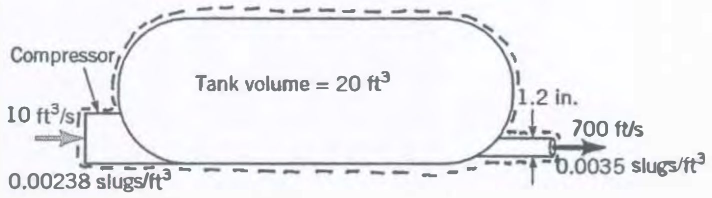
ℓ = width of the plate

and thus

$$Q_{bl} = \frac{7}{8} U \ell \delta$$

5.25

5.25 Air at standard conditions enters the compressor shown in Fig. P5.25 at a rate of 10 ft^3/s. It leaves the tank through a 1.2-in.-diameter pipe with a density of 0.0035 $slugs/ft^3$ and a uniform speed of 700 ft/s. (a) Determine the rate (slugs/s) at which the mass of air in the tank is increasing or decreasing. (b) Determine the average time rate of change of air density within the tank.



■ FIGURE P5.25

Use the control volume within the broken lines.

(a) From the conservation of mass principle we get

$$\frac{DM_{sys}}{Dt} = \dot{m}_{in} - \dot{m}_{out} = \rho_{in} Q_{in} - \rho_{out} A_{out} V_{out}$$

$$\frac{DM_{sys}}{Dt} = \left(0.00238 \frac{slug}{ft^3}\right)\left(10 \frac{ft^3}{s}\right) - \left(0.0035 \frac{slug}{ft^3}\right) \frac{\pi (1.2\ in.)^2}{\left(144 \frac{in^2}{ft^2}\right)} \left(700 \frac{ft}{s}\right)$$

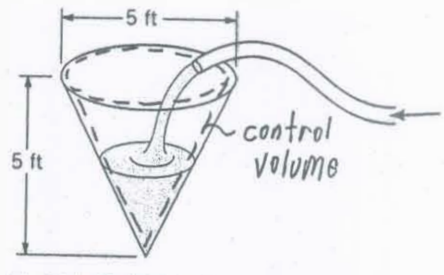
$$\frac{DM_{sys}}{Dt} = \underline{\underline{0.00456}} \frac{slug}{s} \quad \text{increasing}$$

(b) $$\frac{DM_{sys}}{Dt} = \frac{D(\rho V_{sys})}{Dt} = V_{sys} \frac{D\rho}{Dt} = 0.00456 \frac{slug}{s}$$

so $$\frac{D\rho}{Dt} = \frac{0.00456 \frac{slug}{s}}{V_{sys}} = \frac{0.00456 \frac{slug}{s}}{20\ ft^3} = \underline{\underline{2.28 \times 10^{-4}}} \frac{slug}{ft^3\, s}$$

5.26

5.26 Estimate the time required to fill with water a cone-shaped container (see Fig. P5.26) 5 ft high and 5 ft across at the top if the filling rate is 20 gal/min.



■ FIGURE P5.26

From application of the conservation of mass principle to the control volume shown in the figure we have

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}\cdot\hat{n}\, dA = 0$$

For incompressible flow

$$\frac{\partial \forall}{\partial t} - Q = 0$$

or

$$\int_0^{\forall} d\forall = Q\int_0^{t} dt$$

Thus

$$t = \frac{\forall}{Q} = \frac{\pi D^2 h}{12\, Q} = \frac{\pi\,(5\text{ ft})^2(5\text{ ft})\left(1728\ \frac{\text{in.}^3}{\text{ft}^3}\right)}{(12)\left(20\ \frac{\text{gal}}{\text{min}}\right)\left(231\ \frac{\text{in.}^3}{\text{gal}}\right)}$$

and

$$t = \underline{\underline{12.2\text{ min}}}$$

5.29

5.29 A hypodermic syringe (see Fig. P5.29) is used to apply a vaccine. If the plunger is moved forward at the steady rate of 20 mm/s and if vaccine leaks pass the plunger at 0.1 of the volume flowrate out the needle opening, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are 20 mm and 0.7 mm.

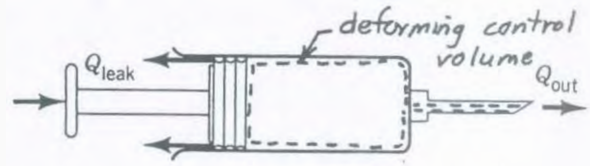


FIGURE P5.29

Using a deforming control volume and the conservation of mass principle (Eq. 5.17) as outlined in Example 5.8, we obtain (see Eq. 8 of Example 5.8)

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = 0 \qquad (1)$$

Since $\rho =$ constant, $Q_{leak} = 0.1\, Q_2$ and $Q_2 = A_2 V_2$ we obtain from Eq. 1

$$1.1\, A_2 V_2 = A_1 V_p$$

or

$$V_2 = \left(\frac{A_1}{A_2}\right)\frac{V_p}{1.1} = \left(\frac{d_1^2}{d_2^2}\right)\frac{V_p}{1.1} = \frac{(20\,mm)^2(20\,mm/s)}{(0.7\,mm)^2(1.1)\left(1000\,\frac{mm}{m}\right)}$$

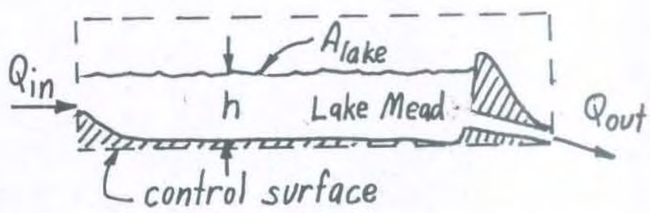
and

$$V_2 = \underline{\underline{14.8}}\ \frac{m}{s}$$

5.30

5.30 The Hoover Dam (see **Video V2.4**) backs up the Colorado River and creates Lake Mead, which is approximately 115 miles long and has a surface area of approximately 225 square miles. If during flood conditions the Colorado River flows into the lake at a rate of 45,000 cfs and the outflow from the dam is 8000 cfs, how many feet per 24-hour day will the lake level rise?

For the control volume shown:



$$\dot{m}_{in} - \dot{m}_{out} = \frac{\partial}{\partial t}\int_{\substack{cv\\ water}} \rho \, d\forall$$

or since $\dot{m} = \rho Q$,

$$Q_{in} - Q_{out} = \frac{\partial}{\partial t}(A_{lake} h) = A_{lake}\frac{dh}{dt}$$

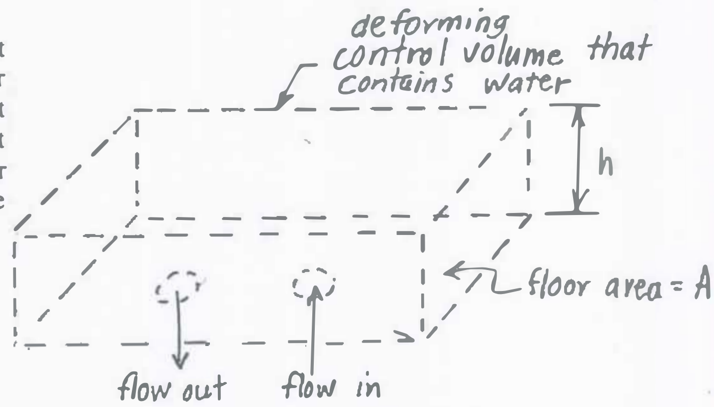
Thus,

$$\frac{dh}{dt} = \frac{Q_{out} - Q_{in}}{A_{lake}} = \frac{(45{,}000 - 8{,}000)\frac{ft^3}{s}}{225\ mi^2\left(5280\frac{ft}{mi}\right)^2} = 5.90 \times 10^{-6}\ \frac{in.}{s}$$

$$= 5.90 \times 10^{-6}\ \frac{in.}{s}\left(3{,}600\frac{s}{hr}\right)\left(24\frac{hr}{day}\right) = \underline{\underline{0.510\ \frac{ft}{day}}}$$

5.31

5.31 Storm sewer backup causes your basement to flood at the steady rate of 1 in. of depth per hour. The basement floor area is 1500 ft^2. What capacity (gal/min) pump would you rent to **(a)** keep the water accumulated in your basement at a constant level until the storm sewer is blocked off, **(b)** reduce the water accumulation in your basement at a rate of 3 in./hr even while the backup problem exists?



For a deforming control volume that contains the water over the basement floor (see sketch above), the conservation of mass principle (Eq. 5.17) leads to

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}_r \cdot \hat{n} \, dA = 0$$

or for constant fluid density and area (A)

$$A \frac{dh}{dt} - Q_{in} + Q_{out} = 0 \qquad (1)$$

(a) For part a, Eq. 1 leads to

$$Q_{out} = Q_{in}$$

To evaluate Q_{in}, we use Eq. 1 with $Q_{out} = 0$. Thus,

$$Q_{in} = A \frac{dh}{dt} = (1500 \text{ ft}^2)\left(1 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right) = 125 \frac{\text{ft}^3}{\text{hr}}$$

and

$$Q_{out} = \left(125 \frac{\text{ft}^3}{\text{hr}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right) = \underline{\underline{15.6}} \frac{\text{gal}}{\text{min}}$$

(b) For part b, Eq. 1 yields

$$Q_{out} = Q_{in} - A \frac{dh}{dt}$$

$$Q_{out} = 15.6 \frac{\text{gal}}{\text{min}} - (1500 \text{ ft}^2)\left(-3 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right)$$

$$Q_{out} = \underline{\underline{62.4}} \frac{\text{gal}}{\text{min}}$$

5.32

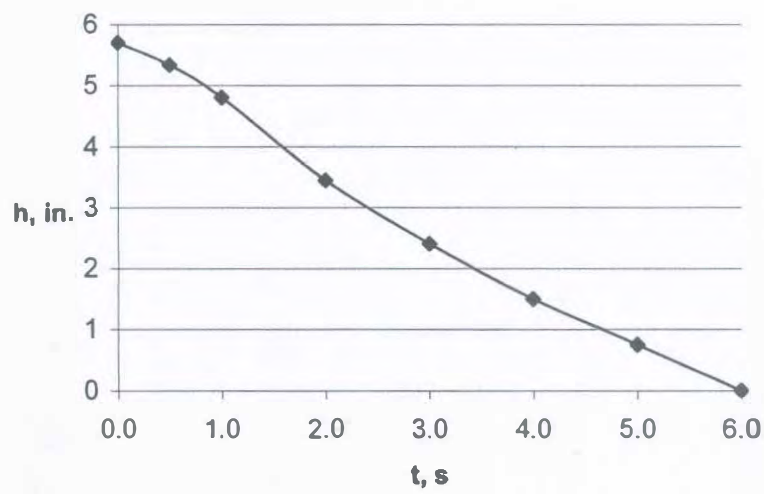
5.32 (See Fluids in the News article "New 1.6 gpf standards," Section 5.1.2.) When a toilet is flushed, the water depth, h, in the tank as a function of time, t, is as given in the table. The size of the rectangular tank is 19 in. by 7.5 in. (a) Determine the volume of water used per flush, gpf. (b) Plot the flowrate for $0 \le t \le 6$ s.

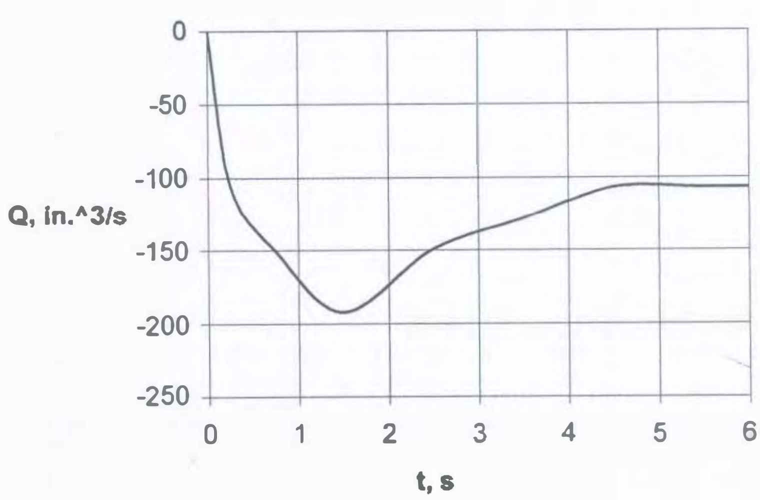
t (s)	h (in.)
0	5.70
0.5	5.33
1.0	4.80
2.0	3.45
3.0	2.40
4.0	1.50
5.0	0.75
6.0	0

(a) Volume of water per flush $= 5.70\,in.\,(19\,in. \times 7.5\,in.) = 812\,in.^3$

$$= 812\,in.^3\left(\frac{1\,gal}{231\,in.^3}\right) = \underline{\underline{3.52\,gal.}}$$

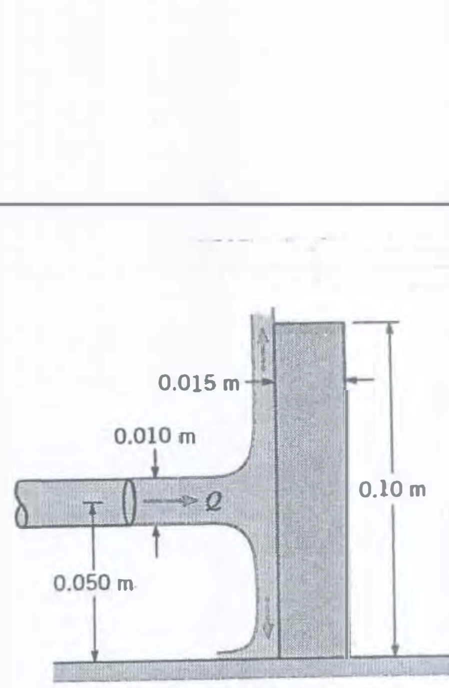
(b) $Q = \dfrac{d(\text{volume in tank})}{dt} = A_{tank}\dfrac{dh}{dt}$, where $\dfrac{dh}{dt}$ is obtained by numerical differentiation of the h vs t data shown below. The resulting Q vs t results are also shown below.





5.38

5.38 A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in **Video V5.96** and Fig. P5.38. Determine the minimum volume flowrate needed to tip the block.



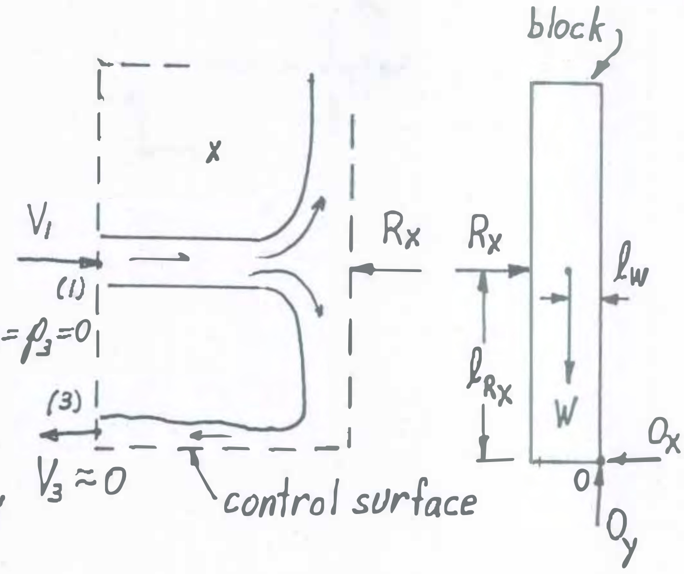
■ FIGURE P5.38

From the free body diagram of the block when it is ready to tip $\Sigma M_0 = 0$, or

$R_x \ell_{R_x} = W \ell_w$ where R_x is the fore that the water puts on the block.

Thus,

$$R_x = \frac{W \ell_w}{\ell_{R_x}} = \frac{6\,N\left(\frac{0.015}{2}\,m\right)}{0.050\,m} = 0.90\,N$$



For the control volume shown the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_x$$

becomes

$$V_1 \rho (-V_1) A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

$$V_1 = \sqrt{\frac{0.9\,N}{\left(999\,\frac{kg}{m^3}\right)\frac{\pi}{4}(0.01\,m)^2}} = 3.39\,\frac{m}{s}$$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.01\,m)^2\left(3.39\,\frac{m}{s}\right) = \underline{\underline{2.66 \times 10^{-4}\,\frac{m^3}{s}}}$$

5.39

5.39 Determine the anchoring force required to hold in place the conical nozzle attached to the end of the laboratory sink faucet shown in Fig. P5.39 when the water flowrate is 10 gal/min. The nozzle weight is 0.2 lb. The nozzle inlet and exit inside diameters are 0.6 and 0.2 in., respectively. The nozzle axis is vertical and the axial distance between sections (1) and (2) is 1.2 in. The pressure at section (1) is 68 psi.

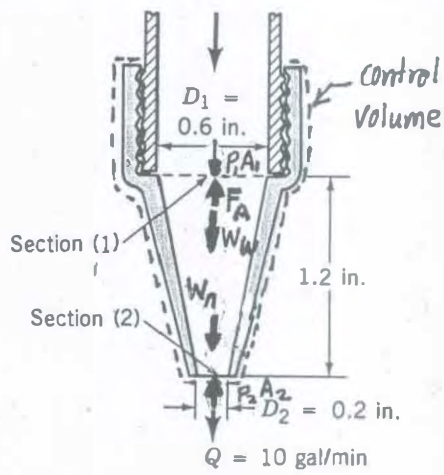


FIGURE P5.39

The analysis leading to the solution of this problem is similar to the one outlined in Example 5.10. Included in the control volume are the nozzle and the water in the nozzle at an instant. Application of the vertical or z-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume leads to

$$F_A = \rho w_1^2 A_1 - \rho w_2^2 A_2 + W_n + P_1 A_1 + W_w - P_2 A_2 \qquad (1)$$

which is Eq. 4 of Example 5.10.

The conservation of mass equation yields

$$\dot{m} = \rho w_1 A_1 = \rho w_2 A_2$$

thus Eq. 1 becomes

$$F_A = \dot{m}(w_1 - w_2) + W_n + P_1 A_1 + W_w - P_2 A_2 \qquad (2)$$

The different terms in Eq. 2 are calculated below.

$$\dot{m} = \rho Q = \left(1.94\ \frac{slugs}{ft^3}\right)\left(10\ \frac{gal}{min}\right)\frac{1}{\left(7.48\ \frac{gal}{ft^3}\right)}\frac{1}{\left(60\ \frac{s}{min}\right)} = 0.0432\ \frac{slug}{s}$$

$$w_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{\left(10\ \frac{gal}{min}\right)\left(12\ \frac{in.}{ft}\right)^2}{\frac{\pi (0.6\ in.)^2}{4}\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)} = 11.4\ \frac{ft}{s}$$

$$w_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{\left(10\ \frac{gal}{min}\right)\left(12\ \frac{in.}{ft}\right)^2}{\frac{\pi (0.2\ in.)^2}{4}\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)} = 102\ \frac{ft}{s}$$

$$P_1 A_1 = P_1 \frac{\pi D_1^2}{4} = \left(68\ \frac{lb}{in.^2}\right)\frac{\pi (0.6\ in.)^2}{4} = 19.2\ lb$$

(con't)

5.39 (Con't)

$$W_w = \rho g \forall_w = \rho g \frac{\pi}{12}\left(D_1^2 + D_2^2 + D_1 D_2\right) h$$

$$W_w = \left(1.94\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right)\left(1\ \frac{lb}{slug\ \frac{ft}{s^2}}\right)\frac{\pi}{12}\left[(0.6\ in.)^2 + (0.2\ in.)^2 + (0.6\ in.)(0.2\ in.)\right]\frac{(1.2\ in.)}{\left(1728\ \frac{in.^3}{ft^3}\right)}$$

$$W_w = 0.00591\ lb$$

$$P_2 A_2 = P_2 \pi \frac{D_2^2}{4} = \left(0\ \frac{lb}{in.^2}\right)\pi \frac{(0.2\ in.)^2}{4} = 0\ lb$$

Thus with Eq. 2

$$F_A = \left(0.0432\ \frac{slug}{s}\right)\left(11.4\ \frac{ft}{s} - 102\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\ \frac{ft}{s^2}}\right) + 0.2\ lb + 19.2\ lb + 0.00591\ lb - 0\ lb$$

$$F_A = \underline{\underline{15.5\ lb}}$$

5.40

5.40 Water flows through a horizontal, 180° pipe bend as is illustrated in Fig. P5.40. The flow cross section area is constant at a value of 9000 mm^2. The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.

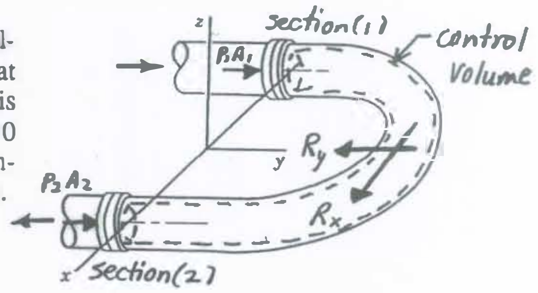


FIGURE P5.40

This analysis is similar to the one of Example 5.11. A fixed, non-deforming control volume that contains the water within the elbow between sections (1) and (2) at an instant is used. The horizontal forces acting on the contents of the control volume in the x and y directions are shown. Application of the x-direction component of the linear momentum equation (Eq. 5.22) leads to

$$R_x = \underline{\underline{0}}$$

Application of the y-direction component of the linear momentum equation yields

$$-v_1 \rho v_1 A_1 - v_2 \rho v_2 A_2 = p_1 A_1 - R_y + p_2 A_2$$

or

$$R_y = \rho A_1 v_1 (v_1 + v_2) + p_1 A_1 + p_2 A_2$$

Thus

$$R_y = \left(999 \frac{kg}{m^3}\right) \frac{(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2} \left(15 \frac{m}{s}\right)\left(15 \frac{m}{s} + 15 \frac{m}{s}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right) + \frac{(210\ kPA)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPA}}\right)}$$

$$+ \frac{(165\ kPA)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPA}}\right)}$$

$$R_y = \underline{\underline{7420}}\ N$$

5.41

5.41 Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.41 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.

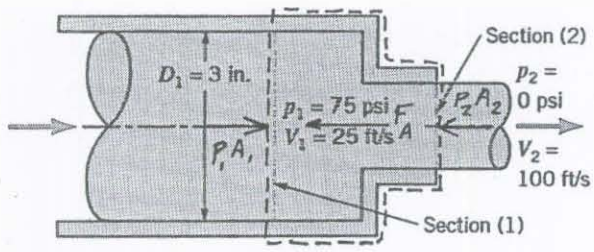


FIGURE P5.41

For this problem we include in the control volume the nozzle as well as the water at an instant between sections (1) and (2) as indicated in the sketch above. The horizontal forces acting on the contents of the control volume are shown in the sketch. Note that the atmospheric forces cancel out and are not shown. Application of the horizontal or x-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = p_1 A_1 - F_A - p_2 A_2 \qquad (1)$$

From the conservation of mass equation (Eq. 5.12) we obtain

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus Eq. (1) may be expessed as

$$\dot{m}(u_2 - u_1) = p_1 A_1 - F_A - p_2 A_2$$

or

$$F_A = p_1 A_1 - p_2 A_2 + \dot{m}(u_2 - u_1) = p_1 \frac{\pi}{4} D_1^2 - p_2 \frac{\pi D_2^2}{4} - \rho u_1 \frac{\pi}{4} D_1^2 (u_2 - u_1)$$

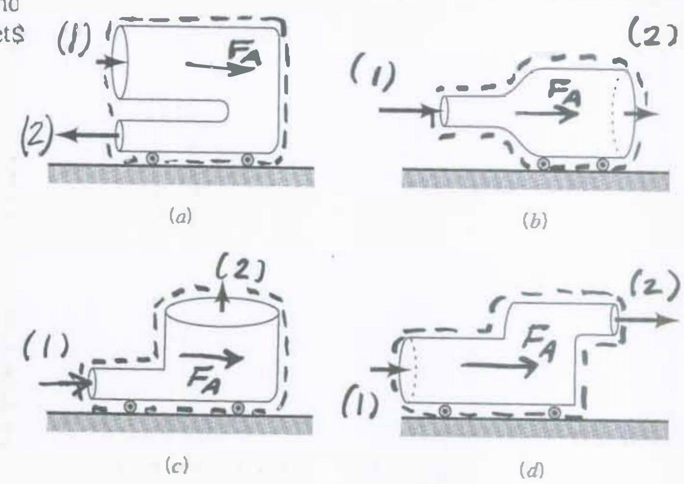
and

$$F_A = \left(75 \frac{lb}{in^2}\right) \frac{\pi (3 \text{ in.})^2}{4} - 0 \text{ lb} - \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right) \frac{\pi (3 \text{ in.})^2}{4 \left(144 \frac{in.^2}{ft^2}\right)} \left(100 \frac{ft}{s} - 25 \frac{ft}{s}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)$$

$$F_A = \underline{\underline{352}} \text{ lb}$$

5.42

5.42 The four devices shown in Fig. P5.42 rest on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.



■ FIGURE P5.42

We apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force F_A. If F_A is in the direction shown in the sketches, motion will be to the left. If F_A is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$, there is no horizontal motion.

For sketch (a)

$$-V_1\rho V_1 A_1 - V_2\rho V_2 A_2 = F_A$$

Since F_A is to the left, motion is to the right.

For sketch (b)

$$-V_1\rho V_1 A_1 + V_2\rho V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$, then F_A is to the left and motion is to the right.

For sketch (c) (note: flow is into CV at (1))

$$-V_1\rho V_1 A_1 = F_A$$

and F_A is to the left so motion is to the right.

For sketch (d)

$$-V_1\rho V_1 A + V_2\rho V_2 A_2 = F_A$$

and from conservation of mass

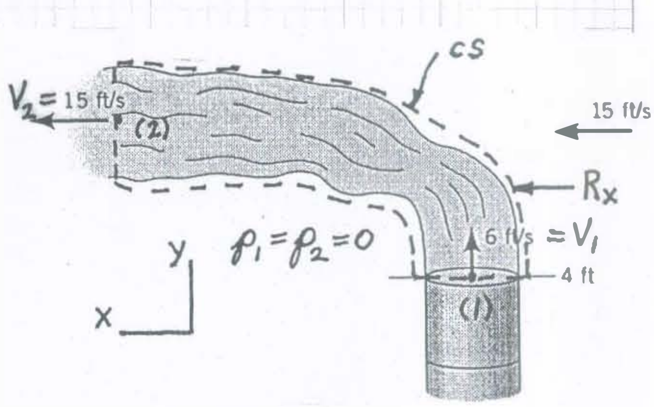
$$\rho V_1 A_1 = \rho V_2 A_2$$

and $V_1 < V_2$

so F_A is to the right and motion is to the left.

5.43

5.43 Exhaust (assumed to have the properties of standard air) leaves the 4-ft-diameter chimney shown in **Video V5.4** and Fig. P5.43 with a speed of 6 ft/s. Because of the wind, after a few diameters downstream the exhaust flows in a horizontal direction with the speed of the wind, 15 ft/s. Determine the horizontal component of the force that the blowing wind puts on the exhaust gases.



■ FIGURE P5.43

For the control volume indicated the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \Sigma F_x \quad \text{becomes}$$

$V_2 \rho V_2 A_2 = R_x$, where R_x is the net horizontal force that the wind puts on the exhaust gases.

Thus,

$$R_x = \dot{m}_2 V_2 \quad \text{where } \dot{m}_2 = \rho A_2 V_2 = \rho A_1 V_1 \quad (\text{i.e. } \dot{m}_1 = \dot{m}_2)$$

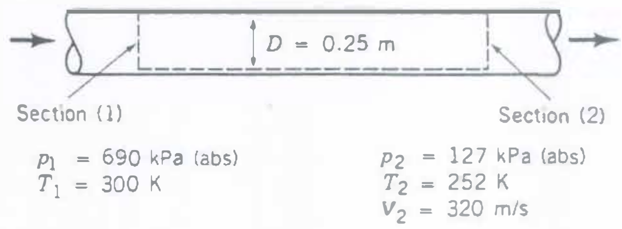
or $\dot{m}_2 = \left(0.00238 \frac{slugs}{s}\right)\left[\frac{\pi}{4}(4ft)^2\right]\left(6 \frac{ft}{s}\right) = 0.179 \frac{slugs}{s}$

Hence,

$$R_x = 0.179 \frac{slugs}{s}\left(15 \frac{ft}{s}\right) = 2.69 \frac{slug \cdot ft}{s^2} = \underline{\underline{2.69 \, lb}}$$

5.44

5.44 Air flows steadily between two cross section in a long, straight section of 12-in.-inside diameterpipe. The static temperature and pressure at each section are indicated in Fig P5.44. If the average air velocity at section (2) is 320 m/s, determine the average air velocity at section (1). Determine the frictional force exerted by the pipe wall on the air flowing between sections (1) and (2). Assume uniform velocity distributions at each section.



■ FIGURE P5.44

This analysis is similar to the one of Example 5.2.
For steady flow between sections (1) and (2)

$$\dot{m}_1 = \dot{m}_2$$

or

$$\rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2$$

Thus

$$\overline{V}_1 = \frac{\rho_2}{\rho_1}\frac{A_2}{A_1}\overline{V}_2 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.8) to get

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\frac{T_1}{T_2} \qquad (2)$$

Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\overline{V}_1 = \frac{p_2}{p_1}\frac{T_1}{T_2}\overline{V}_2 = \frac{[127\ kPa(abs)](300\ K)}{[690\ kPa(abs)](252\ K)}\left(320\ \frac{m}{s}\right)$$

$$\overline{V}_1 = 70.1\ \frac{m}{s}$$

(con't)

5.44 (con't)

The analysis for this problem is similar to the one of Example 5.12. For the control volume shown in the sketch above application of the axial component of the linear momentum equation leads to

$$-\bar{V}_1 \rho_1 \bar{V}_1 A_1 + \bar{V}_2 \rho_2 \bar{V}_2 A_2 = p_1 A_1 - R_x - p_2 A_2$$

From the conservation of mass principle

$$\dot{m} = \rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$$

Also the ideal equation of state is

$$\rho_2 = \frac{p_2}{R T_2}$$

Thus

$$R_x = \frac{p_2}{R T_2} A_2 \bar{V}_2 \left(\bar{V}_1 - \bar{V}_2\right) + A\left(p_1 - p_2\right) = \frac{\pi D^2}{4}\left[\frac{p_2}{R T_2} \bar{V}_2\left(\bar{V}_1 - \bar{V}_2\right) + \left(p_1 - p_2\right)\right]$$

$$R_x = \frac{\pi (12\text{ in.})^2}{4}\left(0.0254\,\frac{\text{m}}{\text{in.}}\right)^2\left[\frac{(127\text{ kPa})\left(320\,\frac{\text{m}}{\text{s}}\right)\left(70.1\,\frac{\text{m}}{\text{s}} - 320\,\frac{\text{m}}{\text{s}}\right)\left(1000\,\frac{\text{N}}{\text{kPa}\cdot\text{m}^2}\right)}{\left(286.9\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(252\text{ K})\left(1\,\frac{\text{N}\cdot\text{m}}{\text{J}}\right)\left(1\,\frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)} + (690\text{ kPa} - 127\text{ kPa})\left(1000\,\frac{\text{N}}{\text{kPa}\cdot\text{m}^2}\right)\right]$$

$$R_x = \underline{\underline{30{,}800\text{ N}}}$$

5.45

5.45 Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown in Fig. P5.45 in place. Atmospheric pressure is 100 kPa. The gage pressure at section (1) is 100 kPa. At section (2), the water exits to the atmosphere.

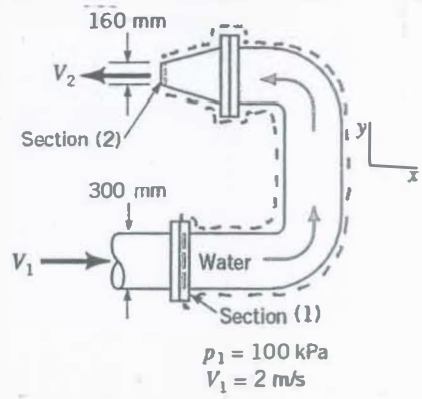


FIGURE P5.45

The control volume shown in the sketch above is used. Application of the y direction component of the linear momentum equation yields

$$R_y = \underline{\underline{0}}$$

Application of the x direction linear momentum equation leads to

$$-u_1 \rho u_1 A_1 - u_2 \rho u_2 A_2 = p_1 A_1 - R_x + p_2 A_2$$

From the conservation of mass equation

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus

$$R_x = \rho u_1 A_1 (u_1 + u_2) + p_1 A_1 + p_2 A_2 = \rho u_1 \frac{\pi D_1^2}{4}\left(u_1 + \frac{D_1^2}{D_2^2} u_1\right) + p_1 \frac{\pi D_1^2}{4} + (0) A_2$$

or

$$R_x = \left(999 \frac{kg}{m^3}\right)\left(2 \frac{m}{s}\right)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left[\left(2 \frac{m}{s}\right) + \frac{(300\ mm)^2}{(160\ mm)^2}\left(2 \frac{m}{s}\right)\right]$$

$$+ (100\ kPa)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left(1000 \frac{N}{m^2 \cdot kPa}\right)$$

and

$$R_x = \underline{\underline{8340\ N}}$$

5.46

5.46 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.46. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.

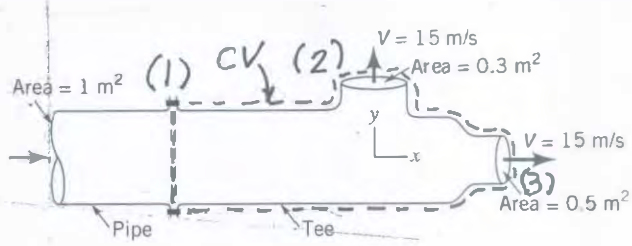


FIGURE P5.46

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1 \rho V_1 A_1 + V_3 \rho V_3 A_3 = P_1 A_1 - P_3 A_3 - P_{atm}(A_1 - A_3) + F_x$$

$$= (P_{1_{gage}} + P_{atm})A_1 - (P_{3_{gage}} + P_{atm})A_3 - P_{atm}(A_1 - A_3) + F_x$$

$$= P_{1_{gage}} A_1 + F_x \qquad (1)$$

To get V_1 we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1 V_1 = A_2 V_2 + A_3 V_3$

so $V_1 = \dfrac{A_2 V_2 + A_3 V_3}{A_1} = \dfrac{(0.3\,m^2)(15\,m/s) + (0.5\,m^2)(15\,m/s)}{1\,m^2} = 12\,m/s$

To estimate $P_{1_{gage}}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{P_{1gage}}{\rho} + \frac{V_1^2}{2} = \frac{P_{2gage}}{\rho} + \frac{V_2^2}{2}$$

$$P_{1_{gage}} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[\left(15\,\frac{m}{s}\right)^2 - \left(12\,\frac{m}{s}\right)^2\right]}{2}\left(\frac{1\,N \cdot s^2}{kg \cdot m}\right)$$

$$P_{1_{gage}} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq. (1) we get:

$$\left[-\left(12\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(12\,\frac{m}{s}\right)(1\,m^2) + \left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.5\,m^2)\right]\left(\frac{1\,N \cdot s^2}{kg \cdot m}\right) =$$

$$\left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $F_x = \underline{\underline{72{,}000\,N}} \leftarrow$

For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2 \rho V_2 A_2 = F_y$$

$$\left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.3\,m^2) = \underline{\underline{67{,}400\,N}} \uparrow = F_y$$

5.47

5.47 A converging elbow (see Fig. P5.47) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m³ between sections (1) and (2). The water volume flowrate is 0.4 m³/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place.

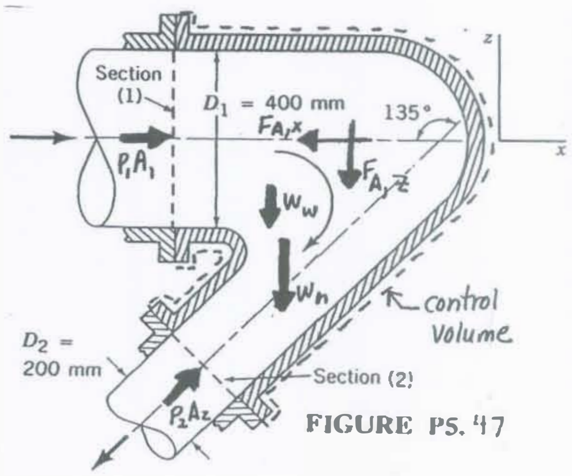


FIGURE P5.47

A control volume that contains the elbow and the water within the elbow between sections (1) and (2) as shown in the sketch above is used. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 - V_2 \cos 45^\circ \rho V_2 A_2 = P_1 A_1 - F_{A,x} + P_2 A_2 \cos 45^\circ$$

From conservation of mass

$$\dot{m} = \rho u_1 A_1 = \rho V_2 A_2 = \rho Q \qquad (1)$$

Thus

$$F_{A,x} = \frac{\rho Q^2}{A_1} + \frac{\rho Q^2 \cos 45^\circ}{A_2} + P_1 A_1 + P_2 A_2 \cos 45^\circ = \frac{\rho Q^2}{\frac{\pi D_1^2}{4}} + \frac{\rho Q^2 \cos 45^\circ}{\frac{\pi D_2^2}{4}} + P_1 \frac{\pi D_1^2}{4} + P_2 \frac{\pi D_2^2}{4} \cos 45^\circ$$

$$F_{A,x} = \left(999 \frac{kg}{m^3}\right)\left(0.4 \frac{m^3}{s}\right)^2 \frac{1}{\left(\frac{\pi}{4}\right)} \left[\frac{\left(1000 \frac{mm}{m}\right)^2}{(400\ mm)^2} + \frac{\cos 45^\circ \left(1000 \frac{mm}{m}\right)^2}{(200\ mm)^2}\right]\left(\frac{1\ N}{kg \frac{m}{s^2}}\right)$$

$$+ \frac{\pi}{4} \frac{\left(1000 \frac{N}{kPa\, m^2}\right)}{\left(1000 \frac{mm}{m}\right)^2}\left[(150\ kPa)(400\ mm)^2 + (90\ kPa)(200\ mm)^2 \cos 45^\circ\right]$$

$$\underline{\underline{F_{A,x} = 25{,}700\ N}}$$

Application of the vertical or z direction component of the linear momentum equation leads to

$$-V_2 \sin 45^\circ \rho V_2 A_2 = P_2 A_2 \sin 45^\circ - F_{A,z} - W_w - W_e$$

which when combined with Eq. 1 gives

$$F_{A,z} = \frac{\rho Q^2 \sin 45^\circ}{A_2} + P_2 A_2 \sin 45^\circ - W_w - W_e = \frac{\rho Q^2 \sin 45^\circ}{\frac{\pi D_2^2}{4}} + P_2 \frac{\pi D_2^2}{4} \sin 45^\circ - \rho g \forall_w - m_e g$$

(con't)

5.47 (con't)

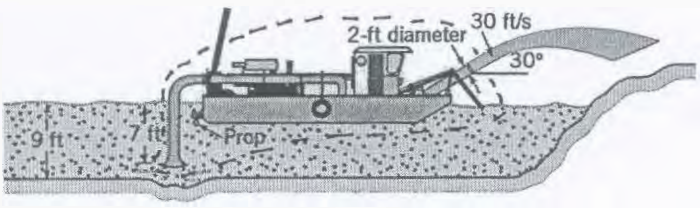
$$F_{A,z} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(0.4\,\frac{m^3}{s}\right)^2}{\frac{\pi}{4}\frac{(200\,mm)^2}{\left(1000\,\frac{mm}{m}\right)^2}}\sin 45^\circ\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) + \frac{(90\,kPa)\,\pi\,(200\,mm)^2\sin 45^\circ}{4\left(1000\,\frac{mm}{m}\right)^2}$$

$$-\left(999\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(0.2\,m^3\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) - (12\,kg)\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)$$

$$F_{A,z} = \underline{\underline{8920\ N}}$$

5.48

5.48 The hydraulic dredge shown in Fig. P5.48 is used to dredge sand from a river bottom. Estimate the thrust needed from the propeller to hold the boat stationary. Assume the specific gravity of the sand/water mixture is $SG = 1.2$.



■ FIGURE P5.48

Using the control volume shown by the broken line in the sketch above we use the horizontal or x component of the linear momentum equation to get

$$F_x = \rho_2 A_2 V_2 V_{2x} = \rho_{H_2O}(sg)_2 \frac{\pi d_2^2}{4} V_2 V_2 \cos 30^\circ$$

where section 1 is where flow enters the control volume vertically and section 2 is where flow leaves the control volume at an angle of 30° from the horizontal direction. Note that there is no horizontal direction linear momentum flow at section 1.

$$F_x = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)(1.4)\frac{\pi (2\text{ ft})^2}{4}\left(30 \frac{\text{ft}}{\text{s}}\right)\left(30 \frac{\text{ft}}{\text{s}}\right)\cos 30^\circ \left(\frac{1 \text{ lb}}{\frac{\text{ft slug}}{\text{s}^2}}\right)$$

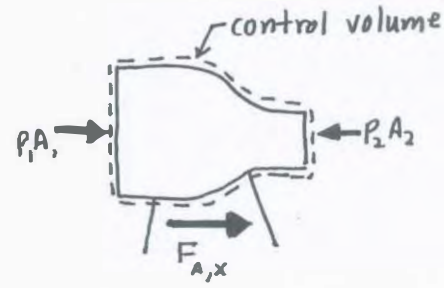
$$F_x = \underline{\underline{6650}} \text{ lb}$$

5.49

5.49 A static thrust stand is to be designed for testing a specific jet engine. Knowing the following conditions for a typical test,

intake air velocity = 700 ft/s
exhaust gas velocity = 1640 ft/s
intake cross section area = 10 ft²
intake static pressure = 11.4 psia
intake static temperature = 480 °R
exhaust gas pressure = 0 psi

estimate a nominal thrust to design for.



The analysis for this problem is similar to the one of Example 5.14. A control volume that contains the entire engine and the fluid in the engine as indicated in the sketch is used. Application of the horizontal or x direction component of the linear momentum equation leads to

$$-u_1\rho_1 u_1 A_1 + u_2\rho_2 u_2 A_2 = p_1 A_1 + F_{A,x}$$

or

$$F_{A,x} = -u_1\rho_1 u_1 A_1 + u_2\rho_2 u_2 A_2 - p_1 A_1$$

The conservation of mass principle yields

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

Thus

$$F_{A,x} = \rho_1 u_1 A_1 (u_2 - u_1) - p_1 A_1$$

or since

$$\rho_1 = \frac{p_1}{RT_1}$$

then

$$F_{A,x} = \frac{p_1}{RT_1} u_1 A_1 (u_1 - u_2) \quad p_1 A_1$$

$$F_{A,x} = \frac{\left(11.4\ \frac{lb}{in.^2}\right)\left(700\ \frac{ft}{s}\right)\left(10\ ft^2\right)\left(1640\frac{ft}{s} - 700\frac{ft}{s}\right)\left(144\ \frac{in.^2}{ft^2}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(480\,{}^\circ R)} - \left(11.4\ \frac{lb}{in.^2} - 14.7\frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)\left(10\ ft^2\right)$$

and

$$F_{A,x} = \underline{\underline{17{,}900\ lb}}$$

5.50

5.50 A horizontal circular cross section jet of air having a diameter of 6 in. strikes a conical deflector as shown in Fig. P5.50. A horizontal anchoring force of 5 lb is required to hold the cone in place. Estimate the nozzle flow rate in ft³/s. The magnitude of the velocity of the air remains constant.

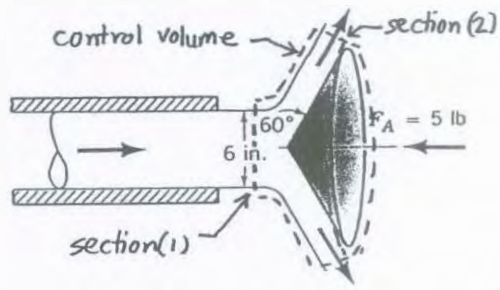


FIGURE P5.50

The control volume shown in the sketch is used. Application of the axial or x-direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

With the conservation of mass principle we can conclude for this incompressible flow that

$$u_1 A_1 = u_2 A_2 = Q$$

Also

$$u_2 = V \cos 60^\circ$$

and

$$u_1 = V = \frac{Q}{A_1}$$

Thus

$$-V\rho Q + V\cos 60^\circ \rho Q = -F_{A,x} = -\frac{Q^2}{A_1}\rho + \frac{Q^2}{A_1}\cos 60^\circ \rho$$

or

$$Q = \left[\frac{F_{A,x} A_1}{\rho(1-\cos 60^\circ)}\right]^{\frac{1}{2}} = \left[\frac{F_{A,x}\left(\frac{\pi D_1^2}{4}\right)}{\rho(1-\cos 60^\circ)}\right]^{\frac{1}{2}}$$

Thus

$$Q = \left[\frac{(5\ \text{lb})(\pi)(6\ \text{in.})^2}{\left(0.00238\frac{\text{slugs}}{\text{ft}^3}\right)(1-\cos 60^\circ)(4)\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right)\left(\frac{1\ \text{lb}}{\text{slug}\ \frac{\text{ft}}{\text{s}^2}}\right)}\right]^{\frac{1}{2}}$$

and

$$Q = \underline{\underline{28.7}}\ \frac{\text{ft}^3}{\text{s}}$$

5.51

5.51 A vertical, circular cross-sectional jet of air strikes a conical deflector as indicated in Fig. P5.51. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass (kg) of the deflector. The magnitude of velocity of the air remains constant.

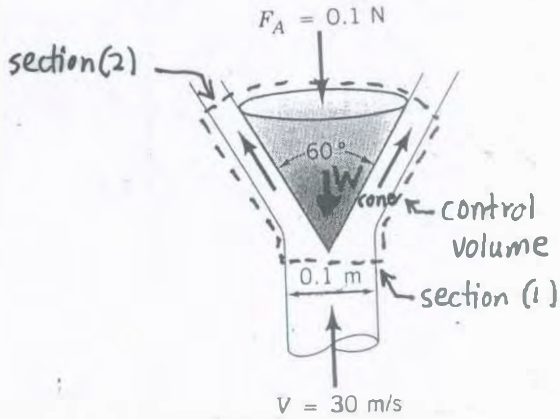


FIGURE P5.51

To determine the mass of the conical deflector we use the stationary, non-deforming control volume shown in the sketch above. Application of the vertical direction component of the linear momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\dot{m}\,(-V_1 + V_2 \cos 30°) = -F_A - W_{cone}$$

or

$$W_{cone} = m_{cone}\, g = \dot{m}\,(V_1 - V_2 \cos 30°) - F_A = \rho A_1 V_1 (V_1 - V_2 \cos 30°) - F_A \quad (1)$$

However

$$V_1 = V_2$$

and

$$A_1 = \frac{\pi D_1^2}{4}$$

Thus Eq. 1 can be expressed as

$$m_{cone} = \rho \frac{\pi D_1^2}{4g} V_1 (V_1 - V_1 \cos 30°) - \frac{F_A}{g}$$

or

$$m_{cone} = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.1\,m)^2 \left(30 \frac{m}{s}\right)\left[30 \frac{m}{s} - \left(30 \frac{m}{s}\right)\cos 30°\right]}{(4)\left(9.81 \frac{m}{s^2}\right)} - \frac{0.1\,N}{\left(9.81 \frac{m}{s^2}\right)\left(\frac{1\,N}{kg \cdot \frac{m}{s^2}}\right)}$$

and

$$m_{cone} = \underline{\underline{0.108\ kg}}$$

5.52

5.52 Water flows from a large tank into a dish as shown in Fig. P5.5 **(a)** If at the instant shown the tank and the water in it weigh W_1 lb, what is the tension, T_1, in the cable supporting the tank? **(b)** If at the instant shown the dish and the water in it weigh W_2 lb, what is the force, F_2, needed to support the dish?

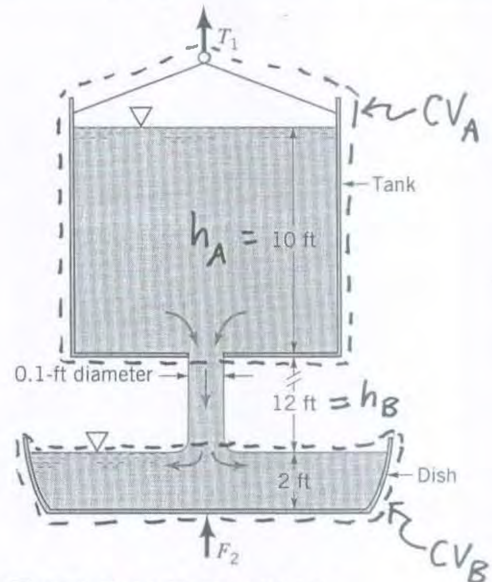


FIGURE P5.52

For part (a) we apply the vertical component of the linear momentum equation to the contents of control volume A, CV_A, to get

$$-V_{out}\,\rho V_{out} A_{out} = T_1 - W_1 \qquad (1)$$

To get value of V_{out} we apply Bernoulli's equation to the flow from the free surface of the water in the tank to the tank outlet to get

$$V_{out} = \sqrt{2gh_A} = \sqrt{(2)\left(32.2\ \frac{ft}{s^2}\right)(10\ ft)} = 25.4\ \frac{ft}{s}$$

Then from Eq.(1) we get

$$\frac{-\left(25.4\ \frac{ft}{s}\right)\left(1.94\ \frac{slugs}{ft^3}\right)\left(25.4\ \frac{ft}{s}\right)\frac{\pi\,(0.1\ ft)^2}{4}}{1\ \frac{slug\cdot ft}{lb\cdot s^2}} = T_1 - W_1$$

and

$$\underline{\underline{T_1 = W_1 - 9.8\ lb}}$$

For part (b) we apply the vertical component of the linear momentum equation to the contents of CV_B to get

$$V_{\substack{into\\CV_B}}\ \dot{m}_{\substack{into\\CV_B}} = F_2 - W_2 \qquad (2)$$

To get $V_{\substack{into\\CV_B}}$ we use Bernoulli's equation between free surface of water in tank to free surface of water in dish to get

$$V_{\substack{into\\CV_B}} = \sqrt{2g(h_A + h_B)} = \sqrt{2\left(32.2\ \frac{ft}{s^2}\right)(10\ ft + 12\ ft)} = 37.6\ \frac{ft}{s}$$

For $\dot{m}_{\substack{into\\CV_B}}$ we use from conservation of mass, $\dot{m}_{\substack{into\\CV_B}} = \dot{m}_{\substack{out\ of\\tank}} = \rho V_{out} A_{out}$

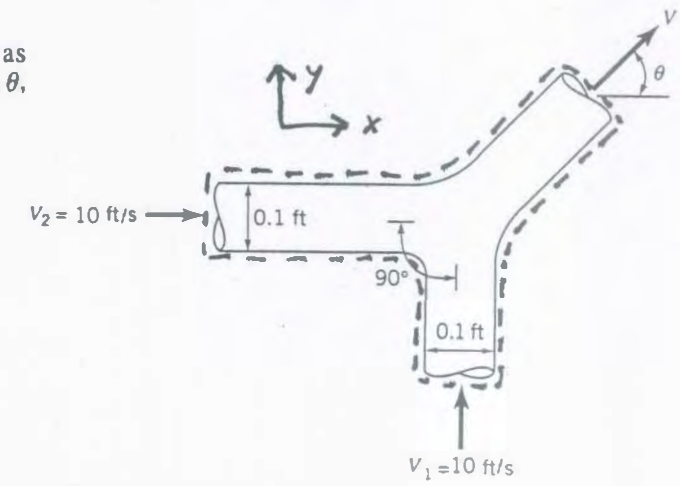
So from Eq. (2) we get

$$\left(37.6\ \frac{ft}{s}\right)\left(1.94\ \frac{slug}{ft^3}\right)\left(25.4\ \frac{ft}{s}\right)\frac{\pi\,(0.1\ ft)^2}{4}\left(\frac{1\ lb\cdot s^2}{slug\cdot ft}\right) = F_2 - W_2$$

and $\underline{\underline{F_2 = W_2 + 14.7\ lb}}$

5.53

5.53 Two water jets of equal size and speed strike each other as shown in Fig. P5.53. Determine the speed, V, and direction, θ, of the resulting combined jet. Gravity is negligible.



■ FIGURE P5.53

For the control volume shown in the sketch above the linear momentum equation for the x and y directions are, for the x direction

$$-V_2 \rho V_2 A_2 + (V\cos\theta)\rho V A = 0 \qquad (1)$$

and for the y direction

$$-V_1 \rho V_1 A_1 + (V\sin\theta)\rho V A = 0 \qquad (2)$$

Also for conservation of mass we have

$$\rho_1 V_1 A_1 + \rho V_2 A_2 - \rho V A = 0 \qquad (3)$$

From Eqs. 1 and 2 we get

$$\frac{V_2^2 A_2}{V_1^2 A_1} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

so

$$\theta = \cot^{-1}\frac{V_2^2 A_2}{V_1^2 A_1} = \cot^{-1}\left[\frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}\right] = 45^\circ$$

Now, combining Eqs. 2 and 3 we get

$$-V_1^2 A_1 + V\sin\theta\,(V_1 A_1 + V_2 A_2) = 0$$

or

$$V = \frac{V_1^2 A_1}{\sin\theta\,(V_1 A_1 + V_2 A_2)}$$

$$V = \frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}{(\sin 45^\circ)\left[\left(10\,\frac{ft}{s}\right)\frac{\pi(0.1\,ft)^2}{4} + \left(10\,\frac{ft}{s}\right)\frac{\pi(0.1\,ft)^2}{4}\right]}$$

and

$$V = \underline{\underline{7.07}}\ \frac{ft}{s}$$

5.54

5.54 Assuming frictionless, incompressible, one-dimensional flow of water through the horizontal tee connection sketched in Fig. P5.54, estimate values of the x and y components of the force exerted by the tee on the water. Each pipe has an inside diameter of 1 m.

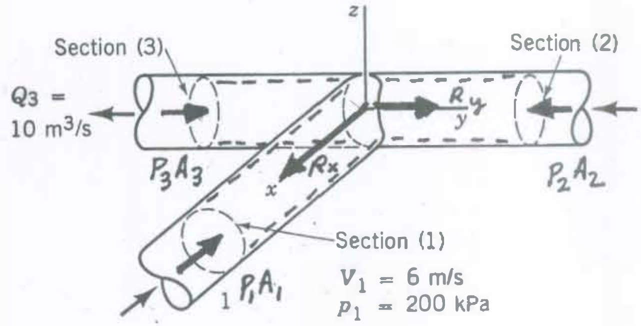


FIGURE P5.54

We can use the x and y components of the linear momentum equation (Eq. 5.22) to determine the x and y components of the reaction force exerted by the water on the tee. For the control volume containing water in the tee, Eq. 5.22 leads to

$$R_x = P_1A_1 + V_1\rho Q_1 = P_1\frac{\pi D_1^2}{4} + V_1\rho Q_1 \qquad (1)$$

and

$$R_y = P_2\frac{\pi D_2^2}{4} - P_3\frac{\pi D_3^2}{4} + V_2\rho Q_2 - V_3\rho Q_3 \qquad (2)$$

The reaction forces in Eqs. 1 and 2 are actually exerted by the tee on the water in the control volume. The reaction of the water on the tee is equal in magnitude but opposite in direction.

Conservation of mass (Eq. 5.4) leads to

$$Q_2 = Q_3 - Q_1 = Q_3 - V_1\frac{\pi D_1^2}{4} = 10\frac{m^3}{s} - \left(6\frac{m}{s}\right)\frac{\pi(1m)^2}{4} = 5.288\frac{m^3}{s}$$

Also

$$Q_1 = V_1\frac{\pi D_1^2}{4} = \left(6\frac{m}{s}\right)\frac{\pi(1m)^2}{4} = 4.712\frac{m^3}{s}$$

Further

$$V_2 = \frac{Q_2}{\frac{\pi D_2^2}{4}} = \frac{\left(5.288\frac{m^3}{s}\right)}{\frac{\pi(1m)^2}{4}} = 6.733\frac{m}{s}$$

and

$$V_3 = \frac{Q_3}{\frac{\pi D_3^2}{4}} = \frac{\left(10\frac{m^3}{s}\right)}{\frac{\pi(1m)^2}{4}} = 12.73\frac{m}{s}$$

(con't)

5.54 (con't)

Because the flow is incompressible and frictionless we assume that Bernoulli's equation (Eq. 5.74) is valid throughout the control volume. Thus

$$p_3 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_3^2\right) = 200\text{ kPa} + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(12.73\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_3 = 137\text{ kPa}$$

Also

$$p_2 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) = 200\text{ kPa} + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(6.733\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_2 = 195.3\text{ kPa}$$

With Eq. 1

$$R_x = \left(200{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1\,m)^2 + \left(6\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(4.712\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) = 185{,}000\,N = 185\,kN$$

and the x-direction component of force exerted by the water on the tee is $\underline{\underline{-185\ kN}}$.

With Eq. 2

$$R_y = \left(195{,}300\ \frac{N}{m^2}\right)\frac{\pi}{4}(1\,m)^2 - \left(137{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1\,m)^2 + \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)5.2$$

$$+ \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(5.288\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

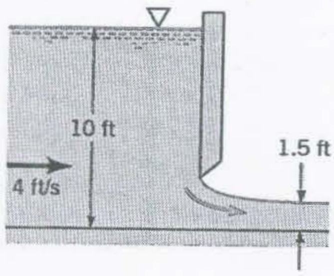
or

$$R_y = -45{,}800\,N = -45.8\,kN$$

and the y-direction component of force exerted by the water on the tee is $\underline{\underline{+45.8}}$ kN.

5.55

5.55 Determine the magnitude of the horizontal component of the anchoring force required to hold in place the sluice gate shown in Fig. 5.55. Compare this result with the size of the horizontal component of the anchoring force required to hold in place the sluice gate when it is closed and the depth of water upstream is 10 ft.



■ FIGURE P5.55

This analysis is similar to the one of Example 5.15. The control volumes of Fig. E 5.15 are appropriate for use in solving this problem. When the sluice gate is closed (see Figs. E5.15a and E5.15c) application of the x direction component of the linear momentum equation leads to

$$R_x = \frac{1}{2}\gamma H^2 = \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(10\,ft)^2 = \underline{\underline{3120}}\ \frac{lb}{ft}$$

When the sluice gate is open (see Figs. E5.15b and E5.15d) application of the x direction component of the linear momentum equation leads to

$$R_x = \frac{1}{2}\gamma H^2 - \frac{1}{2}\gamma h^2 - F_f + \rho u_1^2 H - \rho u_2^2 h$$

The exit velocity u_2 may be expressed in terms of the inlet velocity u_1 with the conservation of mass equation as follows

$$u_2 = u_1 \frac{H}{h}$$

Thus

$$R_x = \frac{1}{2}\gamma H^2 - \frac{1}{2}\gamma h^2 - F_f + \rho u_1^2 H - \rho u_1^2 \frac{H^2}{h}$$

Assuming F_f is negligibly small, we obtain

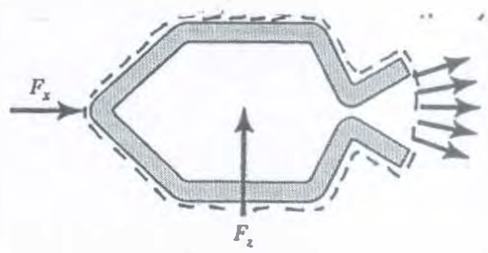
$$R_x = \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(10\,ft)^2 - \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(1.5\,ft)^2$$
$$+\left(1.94\,\frac{slugs}{ft^3}\right)\left(4\,\frac{ft}{s}\right)^2(10\,ft)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right) - \left(1.94\,\frac{slugs}{ft^3}\right)\left(4\,\frac{ft}{s}\right)^2\frac{(10\,ft)^2}{(1.5\,ft)}\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$R_x = \underline{\underline{1290}}\ \frac{lb}{ft}$$

Thus it takes considerably less force to hold in place the sluice gate when it is opened as compared to when it is closed.

5.56

5.56 The rocket shown in Fig. P5.56. is held stationary by the horizontal force, F_x, and the vertical force, F_z. The velocity and pressure of the exhaust gas are 5000 ft/s and 20 psia at the nozzle exit, which has a cross section area of 60 in.^{2}. The exhaust mass flowrate is constant at 21 lbm/s Determine the value of the restraining force F_x. Assume the exhaust flow is essentially horizontal.



■ FIGURE P5.56.

The control volume contains the rocket and the fluid within the rocket as indicated in the sketch. Application of the x direction component of the linear momentum equation yields

$$\underbrace{\frac{\partial}{\partial t}\int_{cv} u\,\rho\, d\forall}_{0 \text{ because the rocket is stationary}} + V_1\rho_1 V_1 A_1 = F_x - p_1 A_1$$

or

$$F_x = p_1 A_1 + V_1 \rho_1 V_1 A_1$$

But

$$\dot{m} = \rho_1 A_1 V_1$$

thus

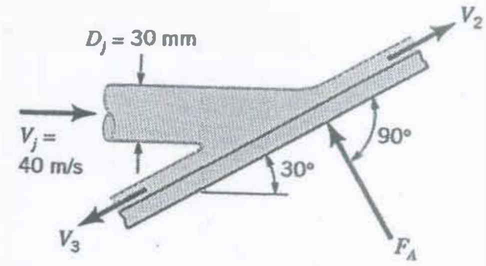
$$F_x = p_1 A_1 + V_1 \dot{m}$$

$$F_x = \left(20\,\frac{\text{lb}_f}{\text{in.}^2} - 14.7\,\frac{\text{lb}_f}{\text{in.}^2}\right)(60\ \text{in.}^2) + \left(5000\,\frac{\text{ft}}{\text{s}}\right)\left(21\,\frac{\text{lb}_m}{\text{s}}\right)\frac{1}{\left(32.2\,\frac{\text{lb}_m \cdot \text{ft}}{\text{lb}_f\, \text{s}^2}\right)}$$

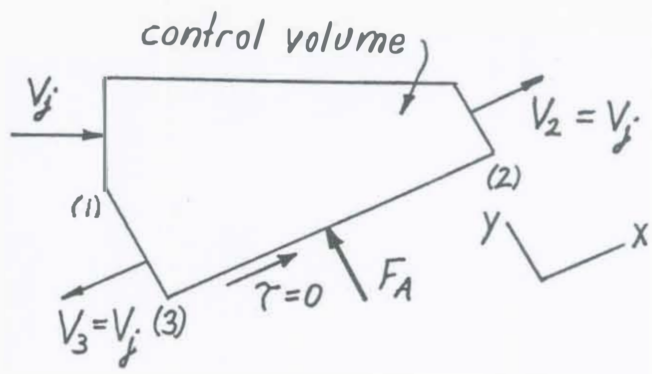
$$F_x = \underline{\underline{3580\ \text{lb}_f}}$$

5.57

5.57 A horizontal circular jet of air strikes a stationary flat plate as indicated in Fig. 5.57. The jet velocity is 40 m/s and the jet diameter is 30 mm. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine: **(a)** the magnitude of F_A, the anchoring force required to hold the plate stationary; **(b)** the fraction of mass flow along the plate surface in each of the two directions shown; **(c)** the magnitude of F_A, the anchoring force required to allow the plate to move to the right at a constant speed of 10 m/s.



■ FIGURE P5.57



The non-deforming control volume shown in the sketch above is used.

(a) To determine the magnitude of F_A we apply the component of the linear momentum equation (Eq. 5.22) along the direction of F_A. Thus, $\int_{cs} v \rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_y$, or

$$F_A = \dot{m} V_j \sin 30^\circ = \rho A_j V_j V_j \sin 30^\circ = \frac{\rho \pi D_j^2}{4} V_j^2 \sin 30^\circ$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030\,m)^2}{(4)} \left(40 \frac{m}{s}\right)^2 (\sin 30^\circ)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) = \underline{\underline{0.696\,N}}$$

(b) To determine the fraction of mass flow along the plate surface in each of the 2 directions shown in the sketch above, we apply the component of the linear momentum equation parallel to the surface of the plate, $\int_{cs} u \rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_x$, to obtain

$$R_{\text{along plate surface}} = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_j V_j \cos 30^\circ \qquad (1)$$

(con't)

5.57 (con't)

Since the air velocity magnitude remains constant, the value of $R_{\text{along plate surface}}$ is zero.* Thus from Eq. 1 we obtain

$$\dot{m}_3 V_3 = \dot{m}_2 V_2 - \dot{m}_j V_j \cos 30^\circ \qquad (2)$$

Since $V_3 = V_2 = V_j$, Eq. 2 becomes

$$\dot{m}_3 = \dot{m}_2 - \dot{m}_j \cos 30^\circ \qquad (3)$$

From conservation of mass we conclude that

$$\dot{m}_j = \dot{m}_2 + \dot{m}_3 \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$\dot{m}_3 = \dot{m}_j - \dot{m}_3 - \dot{m}_j \cos 30^\circ$$

or

$$\dot{m}_3 = \dot{m}_j \frac{(1 - \cos 30^\circ)}{2} = \dot{m}_j (0.0670)$$

and

$$\dot{m}_2 = \dot{m}_j (1 - 0.067) = \dot{m}_j (0.933)$$

Thus, $\underline{\underline{\dot{m}_2 \text{ involves } 93.3\% \text{ of } \dot{m}_j}}$ and $\underline{\underline{\dot{m}_3 \text{ involves } 6.7\% \text{ of } \dot{m}_j}}$.

(c) To determine the magnitude of F_A required to allow the plate to move to the right at a constant speed of $10 \frac{m}{s}$, we use a non-deforming control volume like the one in the sketch above that moves to the right with a speed of $10 \frac{m}{s}$. The translating control volume linear momentum equation (Eq. 5.29) leads to

$$F_A = \frac{\rho \pi D_j^2}{4} \left(V_j - 10 \frac{m}{s}\right)^2 \sin 30^\circ$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030\,m)^2}{4} \left(40 \frac{m}{s} - 10 \frac{m}{s}\right)^2 (\sin 30^\circ) \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$F_A = \underline{\underline{0.391\ N}}$$

*Since $V_1 = V_2 = V_3$ and $\rho_1 = \rho_2 = \rho_3$ and $z_1 = z_2 = z_3$ it follows that the Bernoulli equation is valid from $1 \to 2$ and $1 \to 3$.
Thus, there are no viscous effects (Bernoulli equation is valid only for inviscid flow) so that $\tau = 0$. Hence, $R_{\text{along plate}} = 0$.

5.58

5.58 Water is sprayed radially outward over 180° as indicated in Fig. P5.58. The jet sheet is in the horizontal plane. If the jet velocity at the nozzle exit is 20 ft/s, determine the direction and magnitude of the resultant horizontal anchoring force required to hold the nozzle in place.

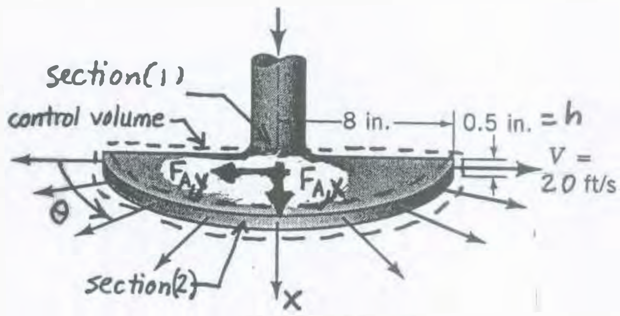


FIGURE P5.58

The control volume includes the nozzle and water between sections (1) and (2) as indicated in the sketch above. Application of the y direction component of the linear momentum equation yields

$$\int_{cs} v \rho \vec{V} \cdot \hat{n}\, dA = -F_{A,y}$$

or

$$F_{A,y} = -\rho \int_0^{\pi} (-V_2 \cos\theta)(V_2)\, h R\, d\theta = \rho h R V_2^2 (\sin\pi - \sin 0)$$

and

$$F_{A,y} = \underline{\underline{0}}$$

Application of the x direction component of the linear momentum equation leads to

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = F_{A,x}$$

or

$$F_{A,x} = \rho \int_0^{\pi} (V_2 \sin\theta)(V_2)\, h R\, d\theta = \rho h R V_2^2 (\cos 0 - \cos\pi)$$

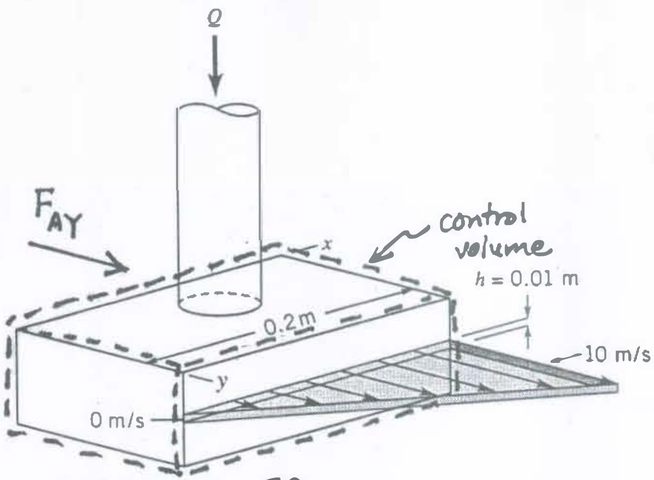
and

$$F_{A,x} = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{(0.5\ \text{in.})\,(8\ \text{in.})\,\left(20\ \frac{\text{ft}}{\text{s}}\right)^2 (2)}{\left(12\ \frac{\text{in.}}{\text{ft}}\right)\left(12\ \frac{\text{in.}}{\text{ft}}\right)} \left(\frac{1\ \text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)$$

$$F_{A,x} = \underline{\underline{43\ \text{lb}}}$$

5.59

5.59 A sheet of water of uniform thickness ($h = 0.01$ m) flows from the device shown in Fig. P5.59. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the y component of anchoring force necessary to hold this device stationary.



■ FIGURE P5.59

A control volume that contains the box portion of the device and the water in the box as shown in the sketch above is used. Application of the y-direction component of the linear momentum equation yields

$$F_{AY} = \int_{A_{slit}} v \rho \vec{V} \cdot \hat{n}\, dA = \rho \int_0^{0.2} v^2 h\, dx$$

The variation of v with x is linear or

$$v = 50x \ \frac{m}{s}$$

Thus

$$F_{AY} = \rho \int_0^{0.2} (50x)^2 h\, dx = \rho (50)^2 h \left. \frac{x^3}{3} \right|_0^{0.2}$$

or

$$F_{AY} = \left(999 \frac{kg}{m^3}\right)\left(50 \frac{1}{s}\right)^2 (0.01 m) \frac{(0.2 m)^3}{3} \left(1 \frac{N \cdot s^2}{kg \cdot m}\right)$$

and

$$F_{AY} = \underline{\underline{66.6 N}}$$

5.60

5.60 A variable mesh screen produces a linear and axisymmetric velocity profile as indicated in Fig. P5.60 in the air flow through a 2-ft-diameter circular cross section duct. The static pressures upstream and downstream of the screen are 0.2 and 0.15 psi and are uniformly distributed over the flow cross section area. Neglecting the force exerted by the duct wall on the flowing air, calculate the screen drag force.

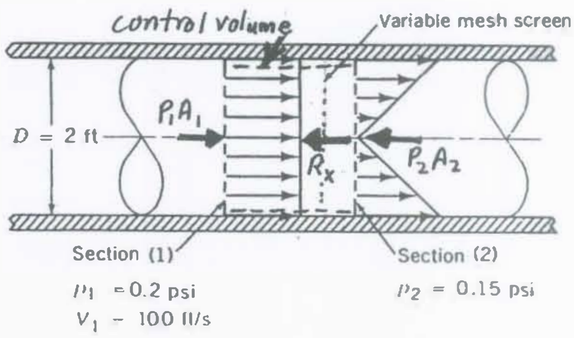


FIGURE P5.60

Application of the axial component of the linear momentum equation to the flow through the control volume shown in the sketch leads to

$$-V_1 \rho V_1 A_1 + \int_0^R u_2 \rho u_2 \, 2\pi r \, dr = P_1 A_1 - R_x - P_2 A_2$$

or

$$R_x = \rho V_1^2 \frac{\pi D_1^2}{4} - 2\pi\rho \int_0^R \left(u_{max} \frac{r}{R}\right)^2 r \, dr + P_1 \frac{\pi D_1^4}{4} - P_2 \frac{\pi D_2^2}{4} \qquad (1)$$

The value of u_{max} may be obtained from conservation of mass as follows

$$\rho V_1 \frac{\pi D_1^2}{4} = \rho \int_0^R \left(u_{max} \frac{r}{R}\right) 2\pi r \, dr$$

Thus $$u_{max} = \frac{V_1 D_1^2 R}{(2)(4)\int_0^R r^2 dr} = \frac{3}{2} V_1 = \frac{3}{2}\left(100 \frac{ft}{s}\right) = 150 \frac{ft}{s}$$

From Eq. 1

$$R_x = \left(0.00238 \frac{slug}{ft^3}\right)\left(100 \frac{ft}{s}\right)^2 \frac{\pi (2ft)^2}{4}\left(\frac{1 \ lb}{slug \cdot \frac{ft}{s^2}}\right) - 2\pi\left(0.00238 \frac{slug}{ft^3}\right)\left(150 \frac{ft}{s}\right)^2 \frac{(2ft)^2}{16}\left(\frac{1 \ lb}{slug \cdot \frac{ft}{s^2}}\right)$$
$$+\left(0.2 \frac{lb}{in.^2}\right)\frac{\pi (2ft)^2}{4}\left(144 \frac{in.^2}{ft^2}\right) - \left(0.15 \frac{lb}{in.^2}\right)\frac{\pi (2ft)^2}{4}\left(144 \frac{in.^2}{ft^2}\right)$$

$$R_x = \underline{\underline{13.3 \ lb}}$$

5.61

5.61 Water flows vertically upward in a circular cross section pipe as shown in Fig. P5.61. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c\left(\frac{R-r}{R}\right)^{1/7}\hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe radius, and r = radius from pipe axis. Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

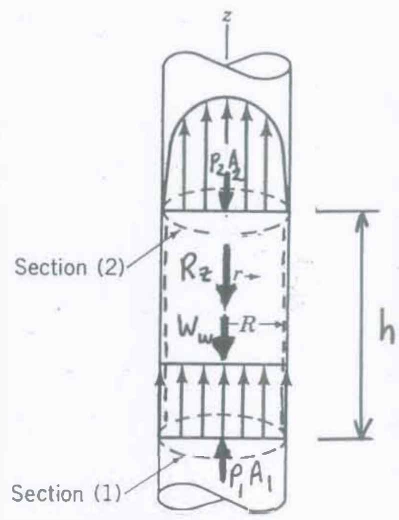


FIGURE P5.61

The analysis for this problem is similar to the one of Example 5.13. The control volume contains the fluid only between sections (1) and (2) as indicated in the sketch. Application of the vertical or z component of the linear momentum equation leads to

$$-w_1\rho w_1 A_1 + \int_0^R w_2\rho w_2\, 2\pi r\,dr = p_1A_1 - R_z + p_2A_2 - W_w$$

Thus

$$p_1 - p_2 = \frac{R_z}{A} - \rho w_1^2 + \frac{\rho 2\pi}{A}\int_0^R\left[w_c\left(\frac{R-r}{R}\right)^{\frac{1}{7}}\right]^2 r\,dr + \frac{W_w}{A} \qquad (1)$$

The weight of the water in the control volume may be expressed as

$$W_w = g\rho A h$$

The value of w_c may be obtained from the conservation of mass equation as follows

$$\rho w_1 A_1 = \int_0^R \rho w_c\left(\frac{R-r}{R}\right)^{\frac{1}{7}} 2\pi r\,dr$$

or

$$w_c = \frac{w_1 A_1}{2\pi\int_0^R\left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr} \qquad (2)$$

To evaluate the integral $\int_0^R\left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr$ we substitute

$$\alpha = \frac{R-r}{R} \qquad (3)$$

then

$$d\alpha = -\frac{dr}{R} \qquad (4)$$

(con't)

5.61 (con't)

and
$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr = -\int_1^0 \alpha^{\frac{1}{7}}(1-\alpha)R^2 d\alpha = \frac{49}{120}R^2 \qquad (5)$$

Combining Eqs. 2 and 5 we obtain

$$\omega_c = \frac{60}{49}\,\omega_1$$

Thus from Eq. 1

$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \frac{\rho(2)(60)^2 w_1^2}{R^2 (49)^2}\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr + g\rho h \qquad (6)$$

To evaluate the integral $\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr$ we use Eqs. 3 and 4.

Thus

$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr = -\int_1^0 \alpha^{2/7}(1-\alpha)R^2 d\alpha = \frac{49}{144}R^2$$

and Eq. 6 becomes

$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \rho(1.02)w_1^2 + g\rho h$$

or

$$\underline{\underline{p_1 - p_2 = \frac{R_z}{\pi R^2} + 0.02\rho w_1^2 + g\rho h}}$$

Note that in contrast to the result of Example 5.13, only a very small portion of the pressure drop is due to a change in the momentum flow between sections 1 and 2 in this case.

5.62

5.62 In a laminar pipe flow that is fully developed, the axial velocity profile is parabolic, that is,

$$u = u_c\left[1 - \left(\frac{r}{R}\right)^2\right]$$

as is illustrated in Fig. P5. . Compare the axial direction momentum flowrate calculated with the average velocity, $\bar{u}$, with the axial direction momentum flowrate calculated with the nonuniform velocity distribution taken into account.

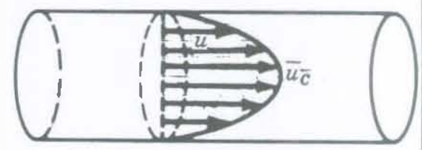


FIGURE P5.62

The axial direction momentum flowrate based on a uniform velocity profile with $u = \bar{u}$ is

$$MF_{x,\,uniform} = \bar{u}\rho\bar{u}A = \rho\bar{u}^2\pi R^2$$

The axial direction momentum flowrate based on the non-uniform parabolic velocity profile is

$$MF_{x,\,non\text{-}uniform} = \int_0^R u\rho u 2\pi r\,dr = \rho u_c^2 2\pi R^2 \int_0^1 \left[1-\left(\frac{r}{R}\right)^2\right]^2\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)$$

$$MF_{x,\,non\text{-}uniform} = \frac{\rho u_c^2 \pi R^2}{3}$$

To obtain a relationship between $\bar{u}$ and $u_{\text{¢}}$ we use the conservation of mass equation as follows

$$\rho\bar{u}\pi R^2 = \rho 2\pi R^2 u_c \int_0^R \left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)$$

Thus

$$\bar{u} = \frac{u_c}{2}$$

and

$$\underline{\underline{MF_{x,\,non\text{-}uniform}}} = \frac{4}{3}\rho\bar{u}^2\pi R^2 = \underline{\underline{\frac{4}{3}MF_{x,\,uniform}}}$$

5.64

5.64 A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.64 and Video V5.6. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the x direction component of anchoring force required to (a) hold the vane stationary, (b) confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.

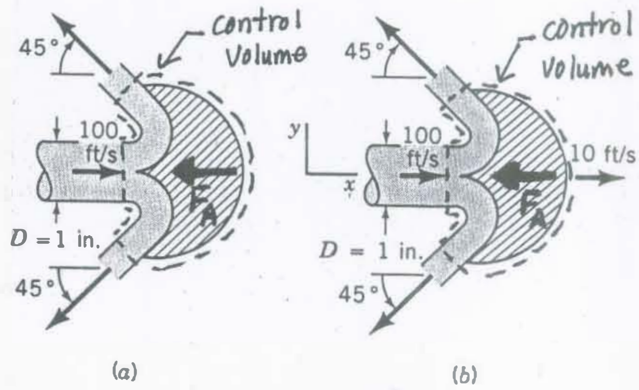


FIGURE P5.64

(a) To determine the x-direction component of anchoring force required to hold the vane stationary we use the stationary control volume shown above and the x-direction component of the linear momentum equation (Eq. 5.22). Thus,

$$F_A = \dot{m}(V_1 + V_2 \cos 45°) = \rho A_1 V_1 (V_1 + V_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} V_1 (V_1 + V_2 \cos 45°)$$

or

$$F_A = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{\pi (1 \text{ in.})^2 \left(100 \frac{\text{ft}}{\text{s}}\right)}{(4)\left(12 \frac{\text{in.}}{\text{ft}}\right)^2} \left[\left(100 \frac{\text{ft}}{\text{s}}\right) + \left(100 \frac{\text{ft}}{\text{s}}\right)\cos 45°\right] \left(\frac{1 \text{ lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}}\right)$$

and

$$\underline{\underline{F_A = 181 \text{ lb}}}$$

(b) To determine the x-direction component of anchoring force required to confine the vane to a constant speed of $10 \frac{\text{ft}}{\text{s}}$ to the right we use a control volume moving to the right with a speed of $10 \frac{\text{ft}}{\text{s}}$ and the x-direction component of the linear momentum equation for a translating control volume (Eq. 5.29). Thus,

$$F_A = \rho A_1 W_1 (W_1 + W_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} W_1 (W_1 + W_2 \cos 45°) \qquad (1)$$

We note that

$$W_1 = V_1 - 10 \frac{\text{ft}}{\text{s}} = 100 \frac{\text{ft}}{\text{s}} - 10 \frac{\text{ft}}{\text{s}} = 90 \frac{\text{ft}}{\text{s}}$$

Thus, Eq. 1 leads to

$$F_A = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{\pi (1 \text{ in.})^2}{4 \left(12 \frac{\text{in.}}{\text{ft}}\right)^2} \left(90 \frac{\text{ft}}{\text{s}}\right) \left[90 \frac{\text{ft}}{\text{s}} + \left(90 \frac{\text{ft}}{\text{s}}\right)\cos 45°\right] \left(\frac{1 \text{ lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}}\right)$$

or

$$\underline{\underline{F_A = 146 \text{ lb}}}$$

5.65

5.65 How much power is transferred to the moving vane of Problem 5.65?

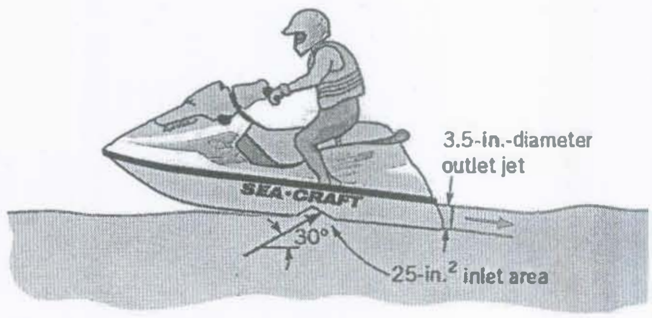
Power $= F_A V$, where from Problem 5.66 $F_A = 146\,lb$

Thus,

$$\text{Power} = \frac{(146\,lb)(10\,\frac{ft}{s})}{\left(550\,\frac{ft.lb}{s.hp}\right)} = \underline{\underline{2.65}}\ hp$$

5.66

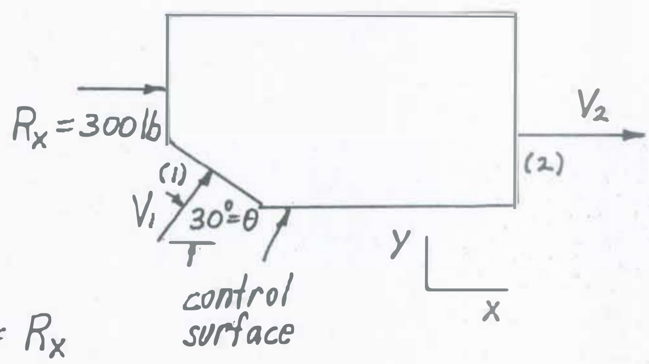
5.66 The thrust developed to propel the jet ski shown in **Video V9.11** and Fig. P5.66 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300-lb thrust? Assume the inlet and outlet jets of water are free jets.



■ FIGURE P5.66

For the control volume indicated the x-component of the momentum equation

$$\int_{cs} u\,\rho\,\vec{V}\cdot\hat{n}\,dA = \sum F_x \quad \text{becomes}$$



(1) $(V_1 \cos 30°)\rho(-V_1)A_1 + V_2\,\rho(+V_2)A_2 = R_x$

where we have assumed that $p = 0$ on the entire control surface and that the exiting water jet is horizontal.

With $\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ Eq. (1) becomes

$$R_x = \dot{m}(V_2 - V_1 \cos\theta) = \rho V_1 A_1 (V_2 - V_1 \cos 30°) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{25\,in.^2}{\frac{\pi}{4}(3.5\,in.)^2} V_1 = 2.60\,V_1 \qquad (2)$$

By combining Eqs. (1) and (2):

$$R_x = \rho V_1^2 A_1 (2.60 - \cos 30°)$$

or

$$V_1 = \left[\frac{300\,lb}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(\frac{25}{144}\,ft^2\right)(2.60 - \cos 30°)}\right]^{1/2} = 22.7\,\frac{ft}{s}$$

Thus,

$$Q = A_1 V_1 = \left(\frac{25}{144}\,ft^2\right)\left(22.7\,\frac{ft}{s}\right) = \underline{\underline{3.94\,\frac{ft^3}{s}}}$$

5.67

5.67 (See Fluids in the News article titled **"Where the plume goes,"** Section 5.2.2.) Air flows into the jet engine shown in Fig. P5.67 at a rate of 9 slugs/s and a speed of 300 ft/s. Upon landing, the engine exhaust exits through the reverse thrust mechanism with a speed of 900 ft/s in the direction indicated. Determine the reverse thrust applied by the engine to the airplane. Assume the inlet and exit pressures are atmospheric and that the mass flowrate of fuel is negligible compared to the air flowrate through the engine.

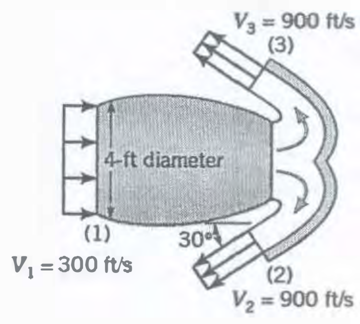
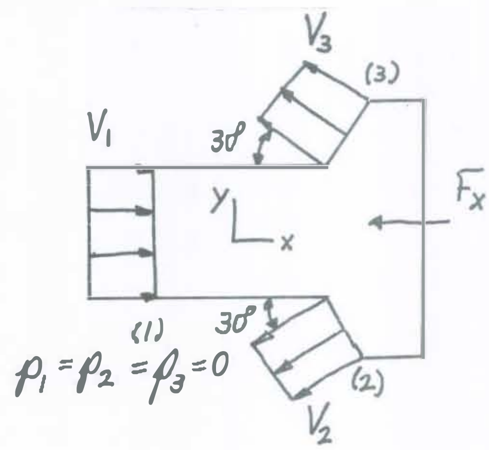


FIGURE P5.67



The momentum equation (x-component),

$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \Sigma F_x$, for the control volume shown can be written as

$$V_1 \rho (-V_1) A_1 + (-V_2 \cos 30^\circ) \rho V_2 A_2 + (-V_3 \cos 30^\circ) \rho V_3 A_3 = -F_x$$

or

$$F_x = (\rho V_1 A_1) V_1 + (\rho V_2 A_2) V_2 \cos 30^\circ + (\rho V_3 A_3) V_3 \cos 30^\circ \qquad (1)$$

But from conservation of mass,

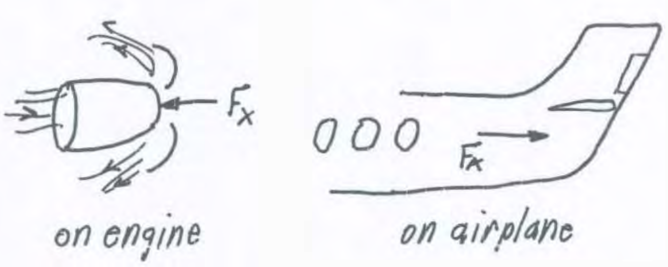
$$\rho V_1 A_1 = \rho V_2 A_2 + \rho V_3 A_3 = \dot{m} = 9 \, slugs/s$$

Also, $V_2 = V_3$, so that Eq. (1) becomes

$$F_x = \dot{m}(V_1 + V_2 \cos 30^\circ) = 9 \frac{slugs}{s}\left(300 \frac{ft}{s} + 900 \cos 30^\circ \frac{ft}{s}\right)$$

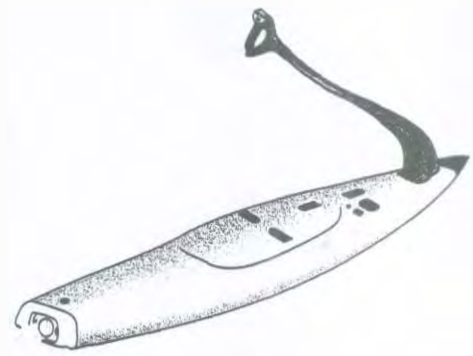
$$= 9710 \frac{slugs \cdot ft}{s^2} = \underline{\underline{9170 \ lb}}$$

Note direction of F_x on engine and engine on airplane.

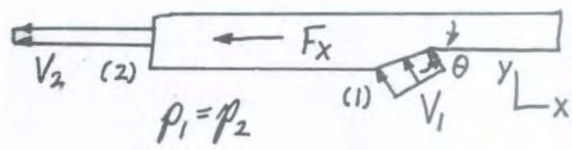


5.68

5.68 (See Fluids in the News article titled **"Motorized surfboard,"** Section 5.2.2.) The thrust to propel the powered surfboard shown in Fig. P5.68 is a result of water pumped through the board that exits as a high-speed 2.75-in.-diameter jet. Determine the flowrate and the velocity of the exiting jet if the thrust is to be 300 lb. Neglect the momentum of the water entering the pump.



■ FIGURE P5.



The x-component of the momentum equation, $\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \sum F_x$, for the control volume shown is

$$(-V_1 \cos\theta)\rho(-V_1)A_1 + (-V_2)\rho V_2 A_2 = -F_x$$

or

$$F_x = \rho V_2^2 A_2 - \rho V_1^2 A_1 \cos\theta \approx \rho V_2^2 A_2$$ if the momentum of the entering water is neglected.

Thus,

$$300 \text{ lb} = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) V_2^2 \left(\frac{\pi}{4}\left(\frac{2.75}{12} \text{ ft}^2\right)\right)$$

or

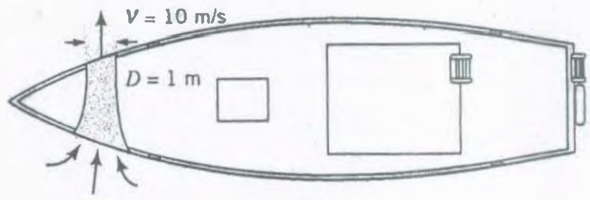
$$V_2 = \underline{\underline{61.2 \frac{\text{ft}}{\text{s}}}}$$

and

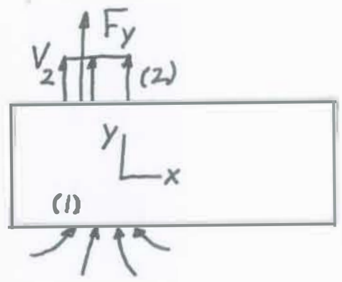
$$Q = A_2 V_2 = \frac{\pi}{4}\left(\frac{2.75}{12} \text{ ft}\right)^2 \left(61.2 \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{2.52 \frac{\text{ft}^3}{\text{s}}}}$$

5.69

5.69 (See Fluids in the News article titled **"Bow thrusters,"** Section 5.2.2) The bow thruster on the boat shown in Fig. P5.69 is used to turn the boat. The thruster produces a 1-m-diameter jet of water with a velocity of 10 m/s. Determine the force produced by the thruster. Assume that the inlet and outlet pressures are zero and that the momentum of the water entering the thruster is negligible.



■ FIGURE P5.69



The y-component of the momentum equation,

$\int_{cs} v \rho \vec{V} \cdot \hat{n} \, dA = \sum F_y$, for the control volume shown is,

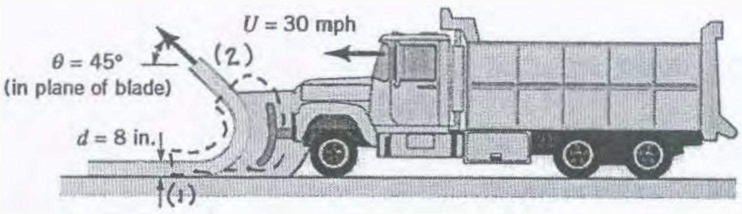
$$\int_{(1)} v \rho \vec{V} \cdot \hat{n} \, dA + V_2 \rho V_2 A_2 = F_y$$

If the momentum of the entering water is negligible the equation becomes

$$F_y = \rho V_2^2 A_2 = 999 \frac{kg}{m^3} \left(10 \frac{m}{s}\right)^2 \left(\frac{\pi}{4} (1m)^2\right) = 78{,}500 \frac{kg \cdot m}{s^2} = \underline{\underline{78.5 \text{ kN}}}$$

5.70

5.70 A snowplow mounted on a truck clears a path 12 *ft* through heavy wet snow, as shown in Figure P5.70. The snow is 8 *in.* deep and its density is 10 *lbm/ft*^{3}. The truck travels at 30 *mph*. The snow is discharged from the plow at an angle of 45° from the direction of travel and 45° above the horizontal, as shown in Figure P5.70. Estimate the force required to push the plow.



■ FIGURE P5.70

To estimate the force required to push the snowplow we use the control volume shown in the sketch above and Eq. 5.29. We neglect the friction force between the plow and the road surface. We also neglect any force associated with the flow deflecting air. We only consider how much force is required to turn wet snow 135°.

For the wet snow "flow" we get from Eq. 5.29

$$F_x = \dot{m}\,(W_1 + W_2 \cos 45°)$$

Since

$$\dot{m} = \rho A W_1$$

we assume $W_2 = W_1$ and get

$$F_x = \rho A W_1^2 (1 + \cos 45°)$$

Then

$$F_x = \frac{\left(10\ \frac{lbm}{ft^3}\right)\left(\frac{8\ in.}{12\ in./ft}\right)(12\ ft)\left[\left(30\ \frac{mi}{hr}\right)\frac{\left(5280\ \frac{ft}{mi}\right)}{(3600\ s/hr)}\right]^2 (1+0.707)}{\left(32.174\ \frac{lbm \cdot ft}{lb \cdot s^2}\right)}$$

$$F_x = \underline{\underline{8220}}\ lb$$

5.75

5.75 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.75. The exit cross section area of each of the two nozzles is 0.04 in.^{2} and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.

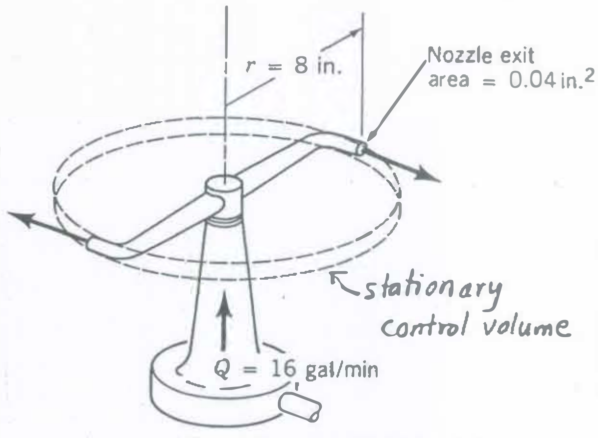


FIGURE P5.75

This is similar to Example 5.17.

(a) To determine the resisting torque required to hold the sprinkler head stationary we use the moment-of-momentum torque equation (Eq. 5.50). Thus,

$$T_{shaft} = \dot{m} r_2 V_{\theta,2} = \rho Q r_2 V_{\theta,2} \qquad (1)$$

For $V_{\theta,2}$ we use

$$V_{\theta,2} = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\ \frac{gal}{min}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{2\left(0.04\ in.^2\right)\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)}$$

or

$$V_{\theta,2} = 64.17\ \frac{ft}{s}$$

With Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\ \frac{slugs}{ft^3}\right)\left(16\ \frac{gal}{min}\right)\left(8\ in.\right)\left(64.17\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)\left(12\ \frac{in.}{ft}\right)}$$

and

$$T_{shaft} = \underline{\underline{2.96\ ft\cdot lb}}$$

(b) To determine the resisting torque associated with a sprinkler speed of 500 $\frac{rev}{min}$ we use Eq. 1 again. However, with rotation we have

$$V_{\theta,2} = W_2 - U_2 \qquad (2)$$

For W_2 we use

$$W_2 = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\ \frac{gal}{min}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{(2)\left(0.04\ in.^2\right)\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)} = 64.17\ \frac{ft}{s}$$

(con't)

5.75 (con't)

For U_2 we use

$$U_2 = r_2\omega = \frac{(8\ in.)\left(500\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)}{\left(12\ \frac{in.}{ft}\right)\left(60\ \frac{s}{min}\right)} = 34.91\ \frac{ft}{s}$$

Thus with Eq. 2 we have

$$V_{\theta,2} = 64.17\ \frac{ft}{s} - 34.91\ \frac{ft}{s} = 29.26\ \frac{ft}{s}$$

and with Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\ \frac{slugs}{ft^3}\right)\left(16\ \frac{gal}{min}\right)(8\ in.)\left(29.26\ \frac{ft}{s}\right)}{\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)\left(12\ \frac{in.}{ft}\right)}\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)$$

and

$$T_{shaft} = \underline{\underline{1.35\ ft\cdot lb}}$$

(c) To determine the angular velocity of the sprinkler if no resisting torque is applied we use the combination of Eqs. 1 and 2 to obtain

$$U_2 = W_2$$

or

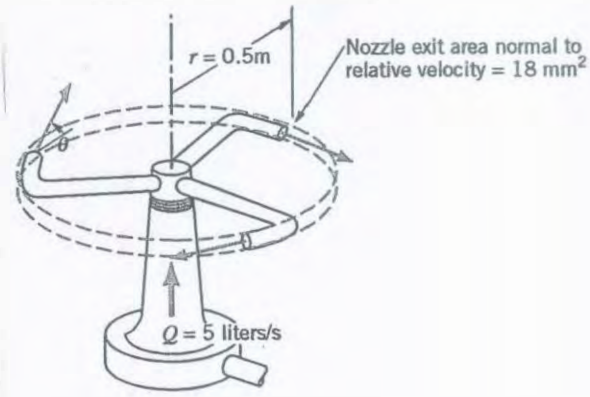
$$\omega = \frac{W_2}{r_2} = \frac{\left(64.17\ \frac{ft}{s}\right)\left(12\ \frac{in.}{ft}\right)}{(8\ in.)} = 96.3\ \frac{rad}{s}$$

The rotor speed, N, is thus

$$N = \left(96.3\ \frac{rad}{s}\right)\frac{\left(60\ \frac{s}{min}\right)}{\left(2\pi\ \frac{rad}{rev}\right)} = \underline{\underline{920\ \frac{rev}{min}}}$$

5.76

5.76 Five liters/s of water enter the rotor shown in **Video V5.10** and Fig. P5.76 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm². How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = 0°$, (b) $\theta = 30°$, (c) $\theta = 60°$?



■ FIGURE P5.76

To determine the torque required to hold the rotor stationary we use the moment-of-momentum torque equation (Eq. 5.50) to obtain

$$T_{shaft} = \dot{m}\, r_{out} V_{out} \cos\theta \qquad (1)$$

We note that

$$\dot{m} = \rho Q \qquad (2)$$

and

$$V_{out} = \frac{Q}{3A_{\substack{nozzle\\exit}}} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$T_{shaft} = \frac{\rho Q^2 r_{out} \cos\theta}{3\, A_{\substack{nozzle\\exit}}} \qquad (4)$$

To determine the rotor angular velocity associated with zero shaft torque we again use the moment-of-momentum torque equation (Eq. 5.50) to obtain, this time with rotation,

$$T_{shaft} = \dot{m}\, r_{out} \left(W_{out} \cos\theta - U_{out} \right) \qquad (5)$$

We note that

$$U_{out} = r_{out}\,\omega \qquad (6)$$

and

$$W_{out} = \frac{Q}{3A_{\substack{nozzle\\exit}}} \qquad (7)$$

(con't)

5.76 (con't)

Combining Eqs. 2,5,6 and 7 we get

$$T_{shaft} = \rho Q r_{out} \left(\frac{Q}{3 A_{\substack{nozzle\\exit}}} \cos\theta - r_{out}\,\omega \right) \qquad (8)$$

(a) For $\theta = 0°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999 \frac{kg}{m^3}\right)\left(5 \frac{liters}{s}\right)^2 (0.5 m)(\cos 0°)\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1 N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1000 \frac{liters}{m^3}\right)^2 (3)(18 mm^2)}$$

or

$$T_{shaft} = \underline{\underline{231 \ N \cdot m}}$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{Q \cos\theta}{3 A_{\substack{nozzle\\exit}} r_{out}} = \frac{\left(5 \frac{liters}{s}\right)(\cos 0°)\left(1000 \frac{mm}{m}\right)^2}{3(18 mm^2)\left(1000 \frac{liters}{m^3}\right)(0.5 m)} = \underline{\underline{185 \frac{rad}{s}}}$$

(b) For $\theta = 30°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999 \frac{kg}{m^3}\right)\left(5 \frac{liters}{s}\right)^2 (0.5 m)(\cos 30°)\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1 N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1000 \frac{liters}{m^3}\right)^2 (3)(18 mm^2)}$$

or

$$T_{shaft} = \underline{\underline{200 \ N \cdot m}}$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5 \frac{liters}{s}\right)(\cos 30°)\left(1000 \frac{mm}{m}\right)^2}{3(18 mm^2)\left(1000 \frac{liters}{m^3}\right)(0.5 m)} = \underline{\underline{160 \frac{rad}{s}}}$$

(c) For $\theta = 60°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999 \frac{kg}{m^3}\right)\left(5 \frac{liters}{s}\right)^2 (0.5 m)(\cos 60°)\left(1000 \frac{mm}{m}\right)^2 \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1000 \frac{liters}{m^3}\right)^2 (3)(18 mm^2)}$$

or

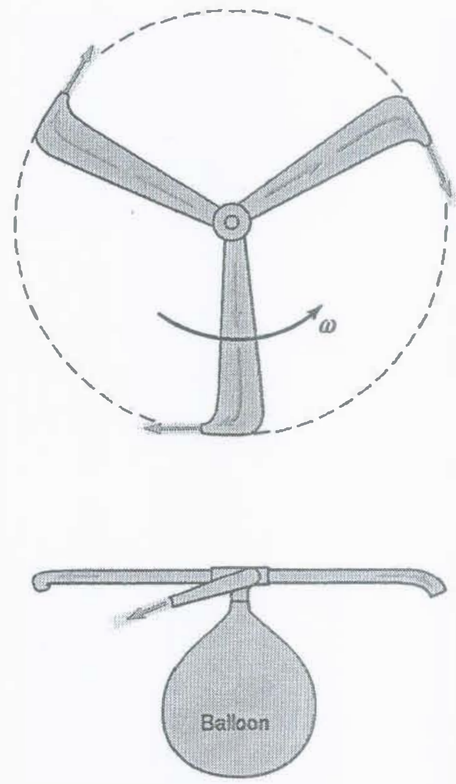
$$T_{shaft} = \underline{\underline{116 \ N \cdot m}}$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5 \frac{liters}{s}\right)(\cos 60°)\left(1000 \frac{mm}{m}\right)^2}{(3)(18 mm^2)\left(1000 \frac{liters}{m^3}\right)(0.5 m)} = \underline{\underline{92.5 \frac{rad}{s}}}$$

5.77

5.77 Shown in Fig. P5.77 is a toy "helicopter" powered by air escaping from a balloon. The air from the balloon flows radially through each of the three propeller blades and out through small nozzles at the tips of the blades. Explain physically how this flow can cause the rotation necessary to rotate the blades to produce the needed lifting force.



■ FIGURE P5.77

As the air flowing radially out through each propeller blade turns to flow out through the nozzle at the blade tip, it exerts a tangential force to the inside surface of the blade. Further, the velocity increase of the air flowing out of each nozzle results in additional force in the opposite direction. These two forces move the blades counter clockwise as shown. The rotating blades experience a lifting force from the air flowing over the blades because of the downward turning of the air.

5.78

5.78 A simplified sketch of a hydraulic turbine runner is shown in Fig. P5.78. Relative to the rotating runner, water enters at section (1) (cylindrical cross section area A_1 at $r_1 = 1.5$ m) at an angle of 100° from the tangential direction and leaves at section (2) (cylindrical cross section area A_2 at $r_2 = 0.85$ m) at an angle of 50° from the tangential direction. The blade height at sections (1) and (2) is 0.45 m and the volume flowrate through the turbine is 30 m³/s. The runner speed is 130 rpm in the direction shown. Determine the shaft power developed.

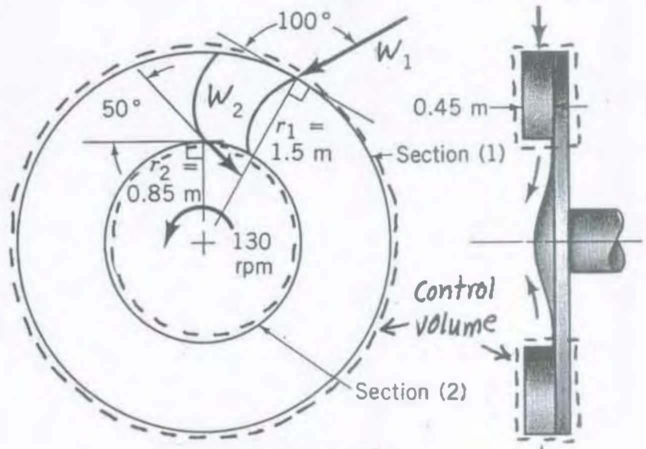


FIGURE P5.78

The stationary and non-deforming control volume shown in the sketch is used. Equation 5.53 can be used to determine to determine the shaft power. Thus

$$\dot{W}_{shaft} = -\dot{m}_1 \left(U_1 V_{\theta 1} \right) + \dot{m}_2 \left(U_2 V_{\theta 2} \right) \qquad (1)$$

With the conservation of mass equation we can conclude that

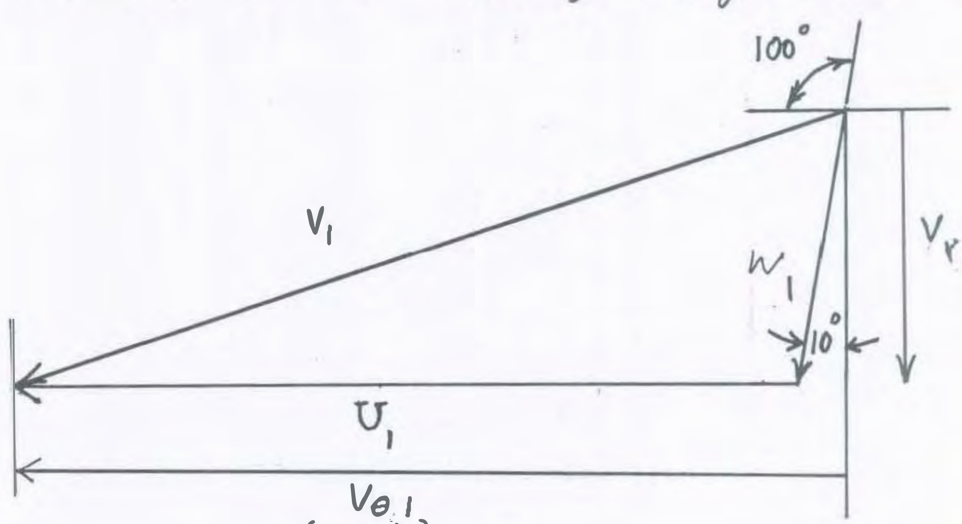
$$\dot{m}_1 = \dot{m}_2 = \rho Q = \left(999 \ \frac{kg}{m^3}\right)\left(30 \ \frac{m^3}{s}\right) = 30{,}000 \ \frac{kg}{s}$$

The blade velocities are easily obtained as follows.

$$U_1 = r_1 \omega = \frac{(1.5\, m)\left(130 \ \frac{rev}{min}\right)\left(2\pi \ \frac{rad}{rev}\right)}{\left(60 \ \frac{s}{min}\right)} = 20.4 \ \frac{m}{s}$$

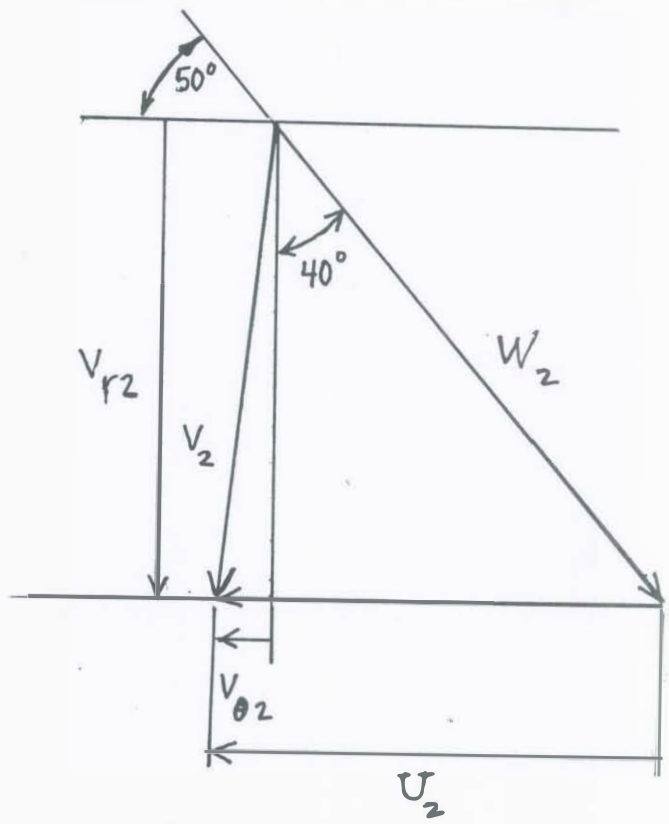
$$U_2 = r_2 \omega = \frac{(0.85\, m)\left(130 \ \frac{rev}{min}\right)\left(2\pi \ \frac{rad}{rev}\right)}{\left(60 \ \frac{s}{min}\right)} = 11.6 \ \frac{m}{s}$$

The tangential velocities, $V_{\theta,1}$ and $V_{\theta,2}$ may be obtained with the help of the velocity triangles sketched below.



(con't)

5.78 (con't)



With the velocity triangle for section (1) we see that

$$V_{\theta 1} = U_1 + W_1 \sin 10^\circ \qquad (2)$$

Also

$$W_1 \cos 10^\circ = V_{r1}$$

and

$$V_{r1} = \frac{Q}{A_1} = \frac{Q}{2\pi r_1 h_1} = \frac{\left(30 \frac{m^3}{s}\right)}{2\pi(1.5m)(0.45m)} = 7.07 \frac{m}{s}$$

Thus

$$W_1 = \frac{V_{r1}}{\cos 10^\circ} = \frac{\left(7.07 \frac{m}{s}\right)}{\cos 10^\circ} = 7.18 \frac{m}{s}$$

and with Eq. 2

$$V_{\theta 1} = 20.4 \frac{m}{s} + \left(7.18 \frac{m}{s}\right) \sin 10^\circ = 21.6 \frac{m}{s}$$

with the velocity triangle for section (2) we conclude that

$$V_{\theta 2} = U_2 - W_2 \sin 40^\circ \qquad (3)$$

(con't)

5.78 (con't)

Also

$$W_2 \cos 40^\circ = V_{r2} = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 h_2} = \frac{\left(30\ \frac{m^3}{s}\right)}{2\pi(0.85m)(0.45m)} = 12.5\ \frac{m}{s}$$

and

$$W_2 = \frac{V_{r2}}{\cos 40^\circ} = \frac{\left(12.5\ \frac{m}{s}\right)}{\cos 40^\circ} = 16.3\ \frac{m}{s}$$

Thus from Eq. 3

$$V_{\theta 2} = 11.6\ \frac{m}{s} - \left(16.3\ \frac{m}{s}\right)\sin 40^\circ = 1.12\ \frac{m}{s}$$

Finally, with Eq. 1 we obtain

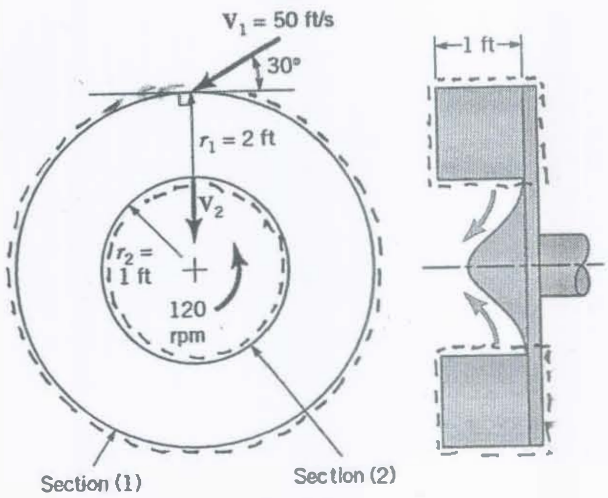
$$\dot{W}_{shaft} = \left[-\left(30{,}000\ \frac{kg}{s}\right)\left(20.4\ \frac{m}{s}\right)\left(21.6\ \frac{m}{s}\right) + \left(30{,}000\ \frac{kg}{s}\right)\left(11.6\ \frac{m}{s}\right)\left(1.12\ \frac{m}{s}\right)\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)$$

$$\dot{W}_{shaft} = \underline{\underline{-12.8 \times 10^6}}\ \frac{N\cdot m}{s} = \underline{\underline{-12.8 \times 10^6\ W}} = \underline{\underline{-12.8\ MW}}$$

The minus sign means power out of the control volume.

5.79

5.79 A water turbine with radial flow has the dimensions shown in Fig.P5.79. The absolute entering velocity is 50 ft/s, and it makes an angle of 30° with the tangent to the rotor. The absolute exit velocity is directed radially inward. The angular speed of the rotor is 120 rpm. Find the power delivered to the shaft of the turbine.



■ FIGURE P5.79

The stationary and non-deforming control volume shown in the sketch above is used. We use Eq. 5.53 to determine the shaft power involved. Thus

$$\dot{W}_{shaft} = -\dot{m}_1 U_1 V_{\theta 1} \qquad (1)$$

The mass flowrate may be obtained from (2)

$$\dot{m}_1 = \rho V_{r1} A_1 = \rho V_{r1} 2\pi r_1 h_1$$

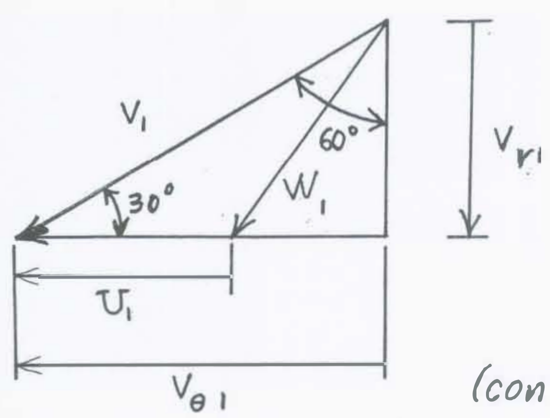
where

V_{r1} = radial component of velocity at section(1)

The blade velocity at section (1) is

$$U_1 = r_1 \omega = (2\ ft)\left(120\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)\frac{1}{\left(60\ \frac{s}{min}\right)} = 25.1\ \frac{ft}{s}$$

The values of $V_{\theta 1}$ and V_{r1} may be obtained with the help of a velocity triangle for the flow at section (1) as sketched below.



(con't)

5.79 (con't)

With the velocity triangle we conclude that

$$V_{r1} = V_1 \sin 30° = V_1 \cos 60° = \left(50 \frac{ft}{s}\right)(\sin 30°) = 25 \frac{ft}{s}$$

Then from Eq. 2

$$\dot{m}_1 = \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right) 2\pi (2\,ft)(1\,ft) = 610 \frac{slugs}{s}$$

Also with the triangle we see that

$$V_{\theta 1} = V_1 \cos 30° = V_1 \sin 60° = \left(50 \frac{ft}{s}\right) \cos 30° = 43.3 \frac{ft}{s}$$

Then, with Eq. 1 we obtain

$$\dot{W}_{shaft} = -\left(610 \frac{slugs}{s}\right)\left(25.1 \frac{ft}{s}\right)\left(43.3 \frac{ft}{s}\right)\left(\frac{1\,lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$\dot{W}_{shaft} = -\underline{\underline{6.63 \times 10^5}} \frac{ft \cdot lb}{s}$$

In horsepower we have

$$\dot{W}_{shaft} = -\left(6.63 \times 10^5 \frac{ft \cdot lb}{s}\right) \frac{1}{\left(550 \frac{ft \cdot lb}{s \cdot hp}\right)} = \underline{\underline{-1200}}\ hp$$

5.80

5.80 Shown in Fig. P5.80 are front and side views of a centrifugal pump rotor or impeller. If the pump delivers 200 liters/s of water and the blade exit angle is 35° from the tangential direction, determine the power requirement associated with flow leaving at the blade angle. The flow entering the rotor blade row is essentially radial as viewed from a stationary frame.

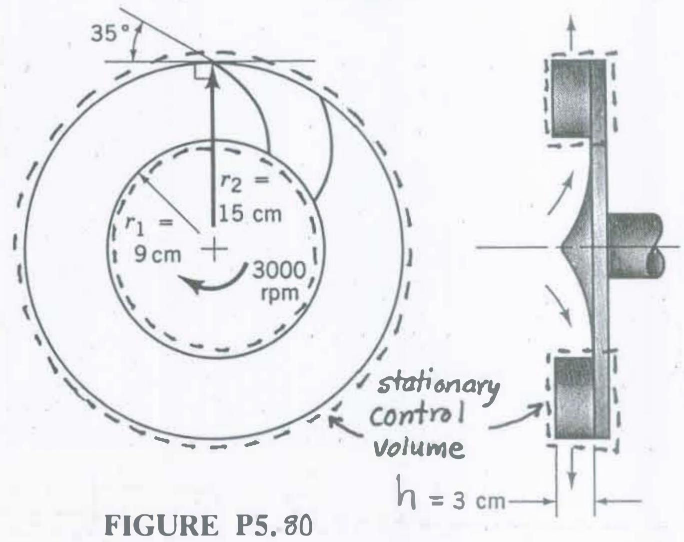


FIGURE P5.80

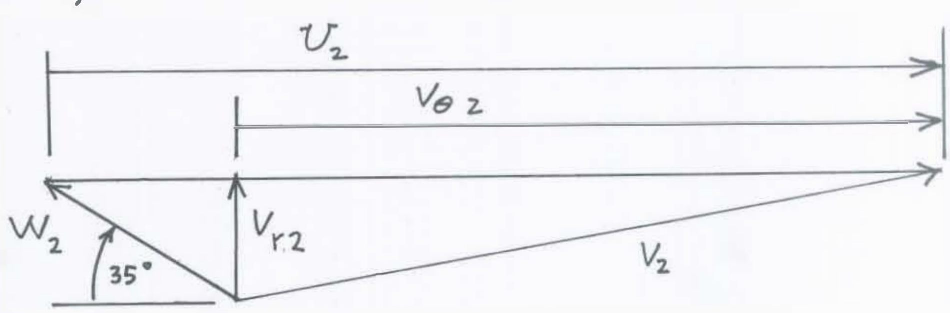
To determine the power, $\dot{W}_{\text{shaft net in}}$, we use the moment-of-momentum power equation (Eq. 5.53) to obtain

$$\dot{W}_{\text{shaft net in}} = \dot{m}\, U_2 V_{\theta 2} = \rho Q U_2 V_{\theta 2} \qquad (1)$$

We obtain U_2 from

$$U_2 = r_2 \omega = \frac{(15\ cm)(3000\ rpm)\left(2\pi \frac{rad}{rev}\right)}{\left(100\ \frac{cm}{m}\right)\left(60\ \frac{s}{min}\right)} = 47.12\ \frac{m}{s}$$

To determine $V_{\theta,2}$ we use the velocity triangle sketched below.



to get

$$V_{\theta 2} = U_2 - \frac{V_{r2}}{\tan 35^\circ} \qquad (3)$$

For V_{r2} we use

$$V_{r2} = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 h} = \frac{\left(200\ \frac{liters}{s}\right)\left(100\ \frac{cm}{m}\right)\left(100\ \frac{cm}{m}\right)}{\left(1000\ \frac{liters}{m^3}\right) 2\pi (15\ cm)(3\ cm)} = 7.074\ \frac{m}{s}$$

(con't)

5.80 (con't)

From Eq. 2 we obtain

$$V_{\theta 2} = 47.12 \frac{m}{s} - \frac{7.074 \frac{m}{s}}{\tan 35°} = 37.02 \frac{m}{s}$$

Thus with Eq. 1 we get

$$\dot{W}_{\substack{shaft \\ net\ in}} = \frac{\left(999 \frac{kg}{m^3}\right)\left(200 \frac{liters}{s}\right)\left(47.12 \frac{m}{s}\right)\left(37.02 \frac{m}{s}\right)}{\left(1000 \frac{liters}{m^3}\right)}\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

or

$$\dot{W}_{\substack{shaft \\ net\ in}} = 3.48 \times 10^5 \frac{N \cdot m}{s}$$

and

$$\dot{W}_{\substack{shaft \\ net\ in}} = \underline{\underline{348}}\ kW$$

5.81

5.81 The velocity triangles for water flow through a radial pump rotor are as indicated in Fig. P5. 1. **(a)** Determine the energy added to each unit mass (kg) of water as it flows through the rotor. **(b)** Sketch an appropriate blade section.

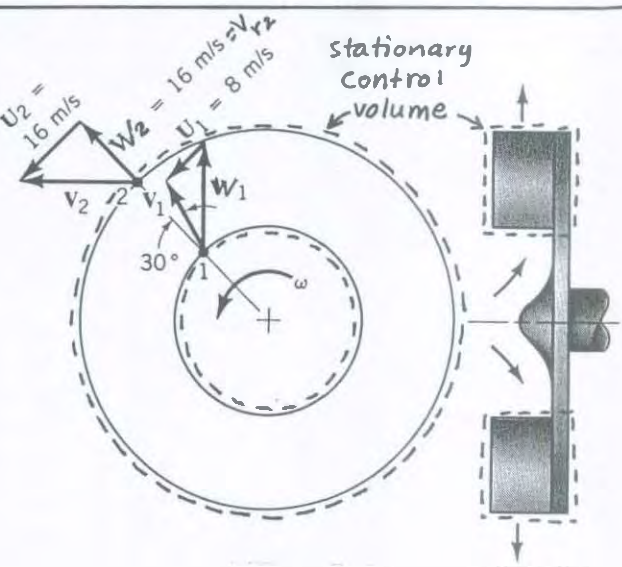


FIGURE P5. 1

(a) To determine the energy per unit mass added to the water flowing through the rotor we use the moment-of-momentum work equation (Eq. 5.54) to get

$$w_{\substack{shaft\\ net\ in}} = U_1 V_{\theta 1} + U_2 V_{\theta 2} \qquad (1)$$

We note from the section (2) velocity triangle that

$$V_{\theta 2} = U_2$$

To ascertain $V_{\theta 1}$ we note from the section (1) velocity triangle that

$$V_{\theta 1} = V_{r1} \tan 30^\circ \qquad (2)$$

From conservation of mass between sections (1) and (2) we conclude that

$$V_{r1} A_1 = V_{r2} A_2 = W_2 A_2$$

or

$$V_{r1} = W_2 \frac{A_2}{A_1} = W_2 \frac{r_2}{r_1} = W_2 \frac{U_2}{U_1} = \left(16 \frac{m}{s}\right) \frac{\left(16 \frac{m}{s}\right)}{\left(8 \frac{m}{s}\right)} = 32 \frac{m}{s}$$

With Eq. 2, $V_{\theta 1} = (32 \frac{m}{s})(0.577) = 18.48$ m/s
and with Eq. 1 we obtain

$$w_{\substack{shaft\\ net\ in}} = \left[\left(8 \frac{m}{s}\right)\left(18.48 \frac{m}{s}\right) + \left(16 \frac{m}{s}\right)\left(16 \frac{m}{s}\right)\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) = \underline{\underline{404}}\ \frac{N \cdot m}{kg}$$

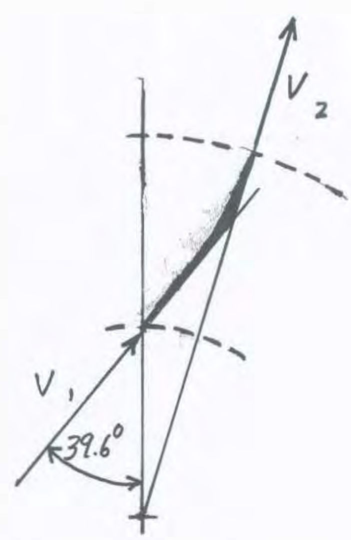
(con't)

5.81 (Con't)

(b) An appropriate blade section would be approximately tangent to the section (1) and section (2) relative velocities, W_1 and W_2. The relative flow angle from the radial direction at section (1) is

$$\beta_1 = \tan^{-1}\left[\frac{(U_1 + V_{\theta 1})}{V_{r1}}\right] = \tan^{-1}\left[\frac{(8\,\frac{m}{s} + 18.48\,\frac{m}{s})}{32\,\frac{m}{s}}\right] = 39.6^\circ$$

The relative flow angle from the radial direction at section (2) is 0°. Thus, the blade section is as sketched below.



5.82

5.82 An axial flow turbomachine rotor involves the upstream (1) and downstream (2) velocity triangles shown in Fig.P5.82. Is this turbomachine a turbine or a fan? Sketch an appropriate blade section and determine energy transferred per unit mass of fluid.

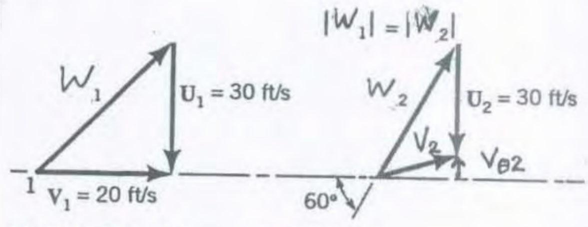


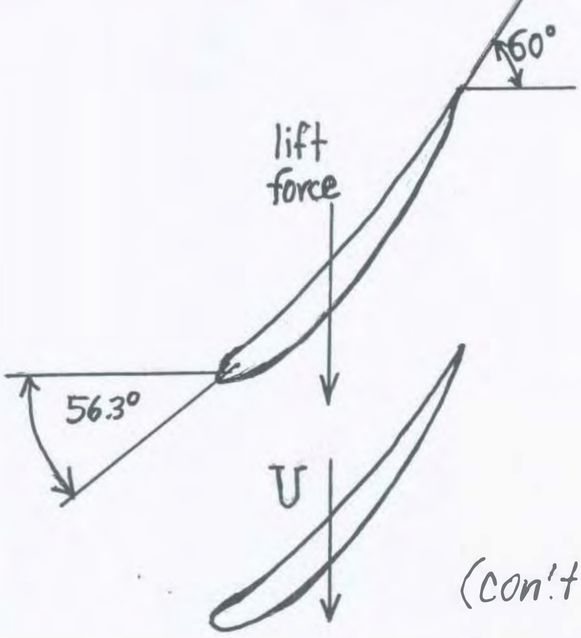
FIGURE P5.82

We can determine whether the axial flow turbomachine involved is a turbine or a fan by comparing the direction of the lift force on the rotor blade section with the direction of the blade velocity, U. If the lift force and the blade velocity are in the same direction a turbine is involved. If the lift force and blade velocity are in opposite directions, a fan is involved. The direction of the lift force can be inferred from the shape of the rotor blade section sketched to be tangent to the relative flows entering and leaving the rotor row.

The entering relative flow angle, β_1, is

$$\beta_1 = \tan^{-1}\frac{U_1}{V_1} = \tan^{-1}\frac{\left(30\,\frac{ft}{s}\right)}{\left(20\,\frac{ft}{s}\right)} = 56.3^\circ$$

Thus, the rotor blade sections sketched below are appropriate



(con't)

5.82 (con't)

Since the lift force acting on each rotor blade section is in the same direction as the blade velocity we conclude that this turbomachine is a turbine. The energy transferred per unit mass is the shaft work per unit mass, w_{shaft}, which we can determine with Eq. 5.54. Thus

$$w_{shaft} = -U_2 V_{\theta 2} \qquad (1)$$

From the velocity triangles we obtain

$$V_{\theta 2} = W_2 \sin 60° - U_2$$

and

$$W_2 = W_1 = \sqrt{V_1^2 + U_1^2}$$

Thus

$$w_{shaft} = -U_2\left(\sqrt{V_1^2 + U_1^2}\ \sin 60° - U_2\right)$$

$$w_{shaft} = -\left(30\ \frac{ft}{s}\right)\left[\sqrt{\left(20\ \frac{ft}{s}\right)^2 + \left(30\ \frac{ft}{s}\right)^2}\ \sin 60° - 30\ \frac{ft}{s}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

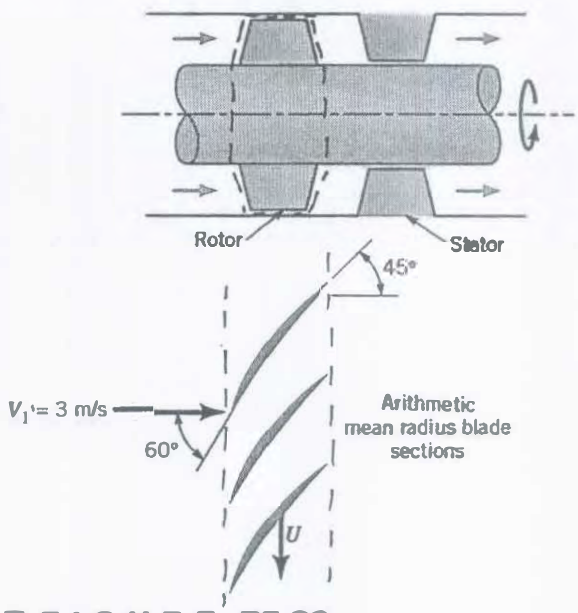
$$w_{shaft} = -\underline{\underline{36.8}}\ \frac{ft \cdot lb}{slug}$$

or

$$w_{shaft} = \frac{-36.8\ \frac{ft \cdot lb}{slug}}{\left(32.2\ \frac{lbm}{slug}\right)} = -\underline{\underline{1.14}}\ \frac{ft \cdot lb}{lbm}$$

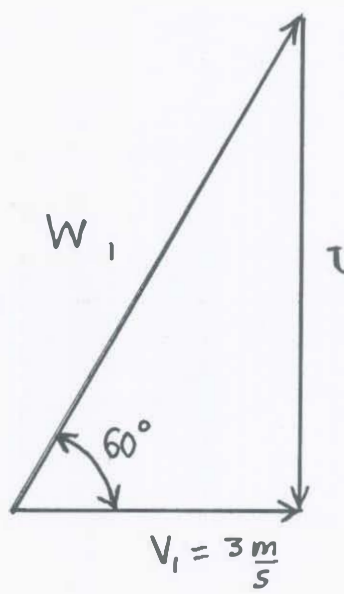
5.83

5.83 An axial flow gasoline pump (see Fig. P5.83) consists of a rotating row of blades (rotor) followed downstream by a stationary row of blades (stator). The gasoline enters the rotor axially (without any angular momentum) with an absolute velocity of 3 m/s. The rotor blade inlet and exit angles are 60° and 45° from the axial direction. The pump annulus passage cross-sectional area is constant. Consider the flow as being tangent to the blades involved. Sketch velocity triangles for flow just upstream and downstream of the rotor and just downstream of the stator where the flow is axial. How much energy is added to each kilogram of gasoline? Is this an actual or ideal amount?



■ FIGURE P5.83

The velocity triangle for flow just upstream of the rotor is sketched below for the arithmetic mean radius.



With the triangle we conclude that

$$W_1 = \frac{V_1}{\cos 60^\circ} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 60^\circ} = 6\ \frac{m}{s}$$

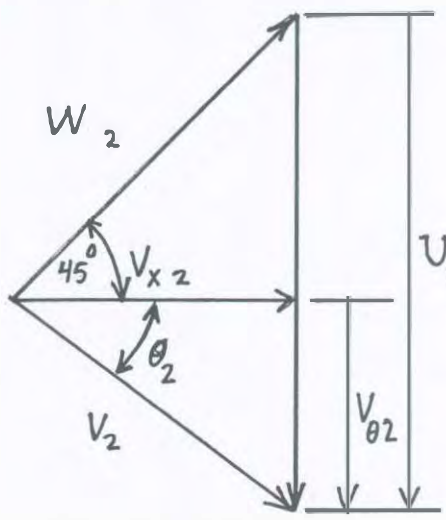
and

$$U_1 = W_1 \sin 60^\circ = \left(6\ \frac{m}{s}\right) \sin 60^\circ = 5.2\ \frac{m}{s}$$

(con't)

5.83 | (con't)

The velocity triangle for flow just downstream of the rotor is sketched below for the arithmetic mean radius. For incompressible flow $V_{x2} = V_1$. For mean radius flow $U_2 = U_1$. Thus for relative flow tangent to the blade we obtain the velocity triangle sketched below.



With the triangle we conclude that

$$V_{\theta 2} = U_2 - W_{\theta 2} = U_2 - V_{x2}\tan 45° = 5.2\ \frac{m}{s} - \left(3\ \frac{m}{s}\right)\tan 45° = 2.2\ \frac{m}{s}$$

Also

$$\theta_2 = \tan^{-1}\left(\frac{V_{\theta 2}}{V_{x2}}\right) = \tan^{-1}\left[\frac{\left(2.2\ \frac{m}{s}\right)}{\left(3\ \frac{m}{s}\right)}\right] = 36.2°$$

$$W_2 = \frac{V_{x2}}{\cos 45°} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 45°} = 4.24\ \frac{m}{s}$$

$$V_2 = \frac{V_{x2}}{\cos\theta_2} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 36.2°} = 3.72\ \frac{m}{s}$$

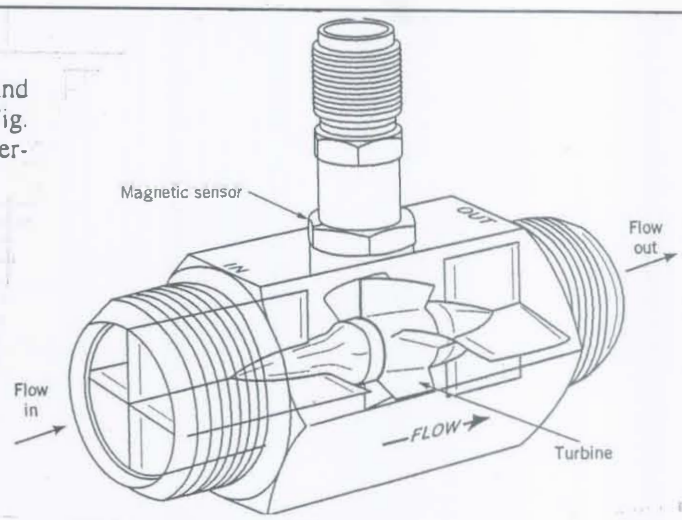
Using the stationary and non-deforming control volume shown above in the first sketch of this solution and Eq. 5.54 we can calculate the energy added to each kg of gasoline.

$$w_{shaft} = U_2 V_{\theta 2} = \left(5.2\ \frac{m}{s}\right)\left(2.2\ \frac{m}{s}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) = \underline{\underline{11.4}}\ \frac{N\cdot m}{kg}$$

This is the actual amount of energy delivered to the gasoline. However, not all of it is absorbed by the gasoline, some is lost.

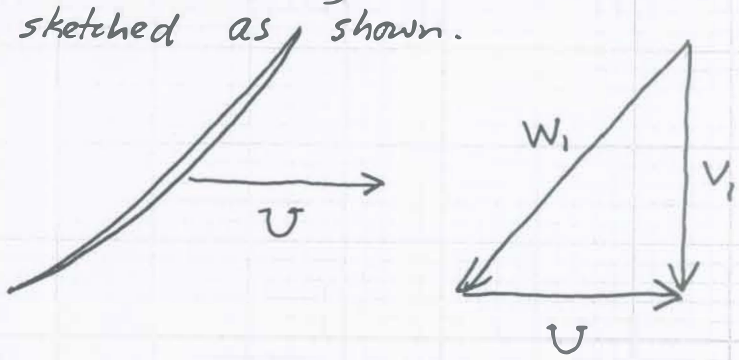
5. 84

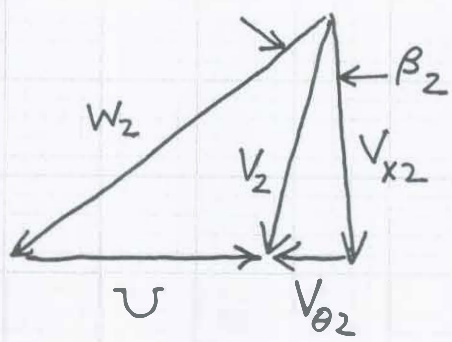
5.84 Sketch the velocity triangles for the flows entering and leaving the rotor of the turbine-type flow meter shown in Fig. P5.84. Show how rotor angular velocity is proportional to average fluid velocity.



■ FIGURE P5.84 (Courtesy of EG&G Flow Technology, Inc.)

For a section of the turbine blade at radius r, the blade moves tangentially with a velocity $U = r\omega$. The velocity triangles may be sketched as shown.





Using Eq. 5.50 we get

$$T_{shaft} = r_2 V_{\theta 2} \dot{m}_2 = r_2 \left(V_{x2} \tan \beta_2 - U \right) \dot{m}_2$$

For nearly zero T_{shaft}

$$0 = V_{x2} \tan \beta_2 - U = V_{x2} \tan \beta_2 - r\omega$$

So

$$\underline{\underline{\omega = \frac{V_{x2} \tan \beta_2}{r}}}$$

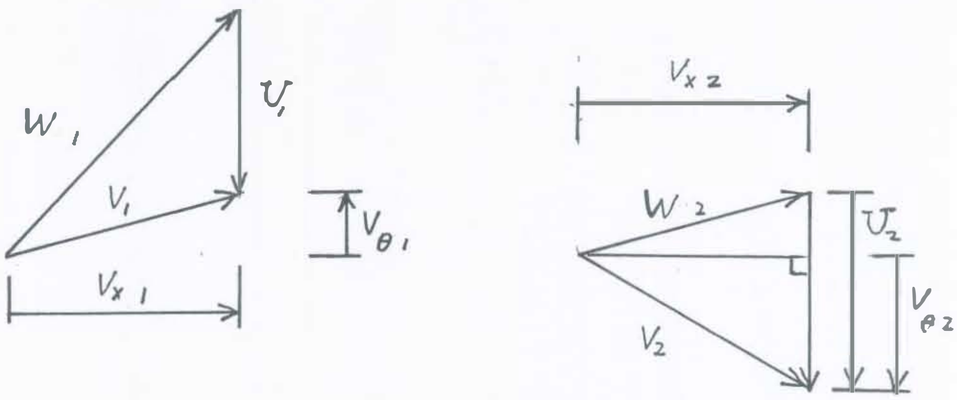
5.85

5.85 By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

where V = absolute flow velocity magnitude, W = relative flow velocity magnitude, and U = blade speed.

Any set of velocity triangle for flow through a turbomachine rotor row would give the same result. We use the triangles of Fig. P5.77.



From the inlet flow velocity triangle we get

$$V_{x1}^2 = V_1^2 - V_{\theta 1}^2 \qquad (1)$$

and

$$V_{x1}^2 = W_1^2 - (V_{\theta 1} + U_1)^2 = W_1^2 - V_{\theta 1}^2 - 2U_1V_{\theta 1} - U_1^2 \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$U_1 V_{\theta 1} = \frac{W_1^2 - V_1^2 - U_1^2}{2} \qquad (3)$$

From the outlet flow velocity triangle we get

$$V_{x2}^2 = V_2^2 - V_{\theta 2}^2 \qquad (4)$$

and

$$V_{x2}^2 = W_2^2 - (U_2 - V_{\theta 2})^2 = W_2^2 - U_2^2 + 2U_2V_{\theta 2} - V_{\theta 2}^2 \qquad (5)$$

(con't)

5.91

5.91 A 1000-m-high waterfall involves steady flow from one large body to another. Detemine the temperature rise associated with this flow.

This is like Example 5.22.

To determine the temperature change we use the relationship

$$T_2 - T_1 = \frac{\check{u}_2 - \check{u}_1}{c} \qquad (1)$$

where the specific heat, $c = 1 \frac{Btu}{lbm \cdot {}^\circ R}$. We use the energy equation (Eq. 5.70) to obtain

$$\check{u}_2 - \check{u}_1 = g(z_1 - z_2) \qquad (2)$$

Combining Eqs. 1 and 2 yields

$$T_2 - T_1 = \frac{g(z_1 - z_2)}{c}$$

or

$$T_2 - T_1 = \frac{\left(9.81 \frac{m}{s^2}\right)(1000\ m)\left(0.4536 \frac{kg}{lbm}\right)\left(0.5556 \frac{K}{{}^\circ R}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1 \frac{Btu}{lbm \cdot {}^\circ R}\right)\left(1055 \frac{N \cdot m}{Btu}\right)}$$

and

$$T_2 - T_1 = \underline{\underline{2.34 K}}$$

5.85 (Con't)

Combining Eqs. 4 and 5 we obtain

$$U_2 V_{\theta 2} = \frac{V_2^2 - W_2^2 + U_2^2}{2} \qquad (6)$$

For the set of velocity triangles

$$w_{\substack{\text{shaft}\\ \text{net in}}} = U_1 V_{\theta 1} + U_2 V_{\theta 2} \qquad (7)$$

Combining Eqs. 3, 6 and 7 we obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

5.90

5.90 Is a necessary condition associated with application of the Bernoulli equation zero heat transfer? Explain.

From Eq. 5.78, we conclude that for application of the Bernoulli equation (loss = 0)

$$q_{\underset{in}{net}} = \check{u}_{out} - \check{u}_{in}$$

Thus the heat transfer, $q_{\underset{in}{net}}$, with application of the Bernoulli equation is not necessarily zero. No.

5.92

5.92 A 100-ft-wide river with a flowrate of 2400 ft^3/s flows over a rock pile as shown in Fig. P5.92. Determine the direction of flow and the head loss associated with the flow across the rock pile.

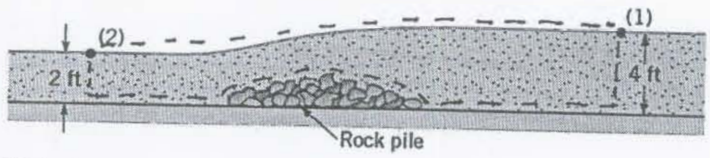


FIGURE P5.92

To determine the direction of flow we will assume a direction, use the energy equation (Eq. 5.84) and calculate the head loss. If the head loss is positive, our assumed direction of flow is correct. If the head loss is negative which is not physically possible, our assumed direction of flow is wrong.

So, assuming the flow is from right to left or from point (1) to point (2) in the sketch above, we get using Eq. 5.84

$$\overbrace{\cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\rho}}}^{\text{same pressure}} + \frac{V_1^2}{2g} + z_1 + \cancel{h_s}^{\,0,\text{ no shaft work}} - h_L$$

Now

$$V_1 = \frac{Q}{A_1} = \frac{\left(2400\,\frac{ft^3}{s}\right)}{(4\,ft)(100\,ft)} = 6\,\frac{ft}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{\left(2400\,\frac{ft^3}{s}\right)}{(2\,ft)(100\,ft)} = 12\,\frac{ft}{s}$$

So

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + z_1 - z_2 = \frac{\left(6\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} - \frac{\left(12\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} + 4\,ft - 2\,ft$$

$h_L = \underline{\underline{0.32\ ft}}$ and since h_L is positive, our assumed <u>right to left</u> flow is correct

5.93

5.93 Air steadily expands adiabatically and without friction from stagnation conditions of 690 kpa (abs) and 290 K to a static pressure of 101 kpa (abs). Determine the velocity of the expanded air assuming: **(a)** incompressible flow; **(b)** compressible flow.

This is similar to Example 5.29.

(a) For incompressible flow, the Bernoulli equation (Eq. 5.109) applied to adiabatic and frictionless flow from the stagnation state to the static state leads to

$$V = \sqrt{\frac{2(P_o - P)}{\rho}} \qquad (1)$$

where the ideal gas equation of state yields

$$\rho_o = \frac{P_o}{RT_o} \qquad (2)$$

Combining Eqs. 1 and 2 results in

$$V = \sqrt{\frac{2(P_o - P)RT_o}{P_o}}$$

or

$$V = \sqrt{\frac{2\left[690\ kPa(abs) - 101\ kPa(abs)\right]\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)(290K)}{690\ kPa(abs)\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}$$

and

$$V = \underline{\underline{377\ \frac{m}{s}}}$$

(b) For compressible flow, Eq. 5.113 applied to adiabatic and frictionless flow from the stagnation state to the static state leads to

$$V = \sqrt{\left(\frac{2k}{k-1}\right)\left(\frac{P_o}{\rho_o} - \frac{P}{\rho}\right)} \qquad (3)$$

However for this process

$$\frac{P}{\rho^k} = \text{constant}$$

(con't)

5.93 (con't)

Thus $\rho = \rho_0 \left(\dfrac{p}{p_0}\right)^{\frac{1}{k}}$ (4)

and combining Eqs. 3 and 4 leads to

$$V = \sqrt{\left(\frac{2k}{k-1}\right)\left[\frac{p_0}{\rho_0} - \frac{p}{\rho_0\left(\frac{p}{p_0}\right)^{\frac{1}{k}}}\right]} \qquad (5)$$

With the ideal equation of state (Eq. 2), Eq. 5 becomes

$$V = \sqrt{\left(\frac{2k}{k-1}\right) RT_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}\right]}$$

or

$$V = \sqrt{\frac{2(1.40)\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(290K)}{(1.40-1)\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}\left\{1 - \left[\frac{101\ kPa(abs)}{690\ kPa(abs)}\right]^{\left(\frac{1.40-1}{1.40}\right)}\right\}}$$

and

$$V = \underline{\underline{580}}\ \frac{m}{s}$$

5.94

5.94 A horizontal Venturi flow meter consists of a converging–diverging conduit as indicated in Fig. P5.94. The diameters of cross sections (1) and (2) are 6 and 4 in. The velocity and static pressure are uniformly distributed at cross sections (1) and (2). Determine the volume flowrate (ft^3/s) through the meter if $p_1 - p_2 = 3$ psi, the flowing fluid is oil ($\rho = 56$ lbm/ft^3), and the loss per unit mass from (1) to (2) is negligibly small.

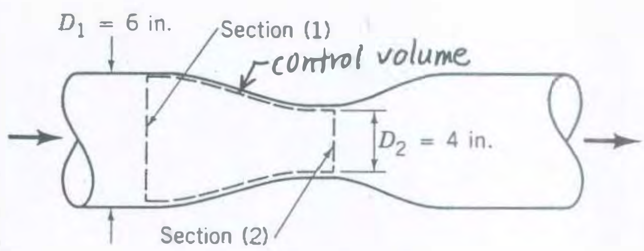


FIGURE P5.94

The control volume shown in the sketch above is used. Application of the conservation of mass equation (Eq. 5.13) to the incompressible steady flow through this control volume leads to

$$Q = A_1 V_1 = A_2 V_2 \qquad (1)$$

Application of the energy equation (Eq. 5.79) to the flow through this control volume yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} = \frac{p_1}{\rho} + \frac{V_1^2}{2} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{p_2}{\rho} + \frac{Q^2}{A_2^2\, 2} = \frac{p_1}{\rho} + \frac{Q^2}{A_1^2\, 2}$$

or

$$Q = \left\{ 2\left(\frac{p_1 - p_2}{\rho}\right)\left[\frac{1}{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)}\right]\right\}^{\frac{1}{2}} = \left\{2\left(\frac{p_1 - p_2}{\rho}\right)\left[\frac{1}{\frac{1}{\left(\frac{\pi D_2^2}{4}\right)^2} - \frac{1}{\left(\frac{\pi D_1^2}{4}\right)^2}}\right]\right\}^{\frac{1}{2}}$$

$$Q = \left\{ \frac{2\left(3\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)\left(32.2\,\frac{lbm \cdot ft}{lb\ s^2}\right)}{\left(56\,\frac{lbm}{ft^3}\right)} \left[\frac{1}{\frac{1}{\left(\frac{\pi(4\,in.)^2}{4\left(12\,\frac{in.}{ft}\right)^2}\right)^2} - \frac{1}{\left(\frac{\pi(6\,in.)^2}{4\left(12\,\frac{in.}{ft}\right)^2}\right)^2}}\right]\right\}^{\frac{1}{2}}$$

$$Q = \underline{\underline{2.17}}\ \frac{ft^3}{s}$$

5.95

5.95 Oil ($SG = 0.9$) flows downward through a vertical pipe contraction as shown in Fig. P5.95. If the mercury manometer reading, h, is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.

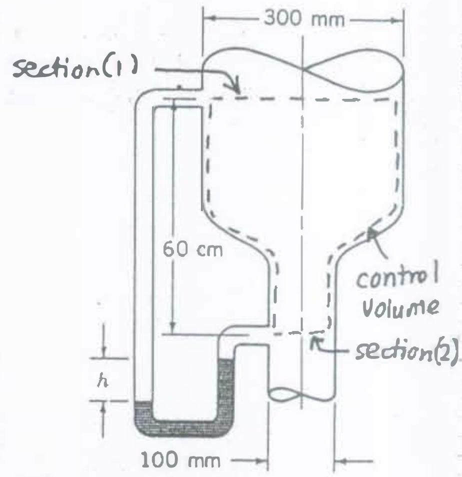


FIGURE P5.95

The volume flowrate may be obtained with

$$Q = A_1 V_1 = A_2 V_2 = \frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

To determine either V_1 or V_2 we apply the energy equation (Eq. 5.82) to the flow between sections (1) and (2). Thus,

$$\frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 + \cancel{w_{\text{shaft net in}}}^{\,0} - \cancel{\text{loss}}^{\,\text{neglect}} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{P_1 - P_2}{\rho} + g(z_1 - z_2) \qquad (3)$$

To determine $\frac{P_1 - P_2}{\rho}$ we use the manometer equation from Section 2.6 to obtain

$$\frac{P_1 - P_2}{\rho} = gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - g(z_1 - z_2) \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$V_2 = \sqrt{\frac{2gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}}$$

or

$$V_2 = \sqrt{\frac{(2)(9.81\,\frac{m}{s^2})(0.1\ m)\left(\frac{13.6}{0.9} - 1\right)}{1 - \left(\frac{100\ mm}{300\ mm}\right)^4}} = 5.29\ \frac{m}{s}$$

and from Eq. 1 we have

$$Q = \frac{\pi (0.1 m)^2}{4}\left(5.29\,\frac{m}{s}\right) = \underline{\underline{0.042\ \frac{m^3}{s}}}$$

Actual flowrate would be less than the frictionless value because the loss would be greater than the zero amount used above.

5.96

5.96 An incompressible liquid flows steadily along the pipe shown in Fig. P5.96. Determine the direction of flow and the head loss over the 6-m length of pipe.

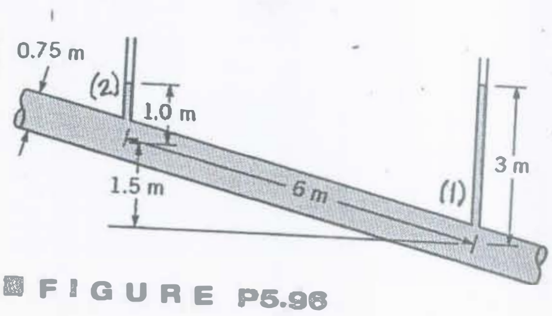


FIGURE P5.96

Assume flow from (1) to (2) and use the energy equation (Eq. 5.84) to get for the contents of the control volume shown:

$$\frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_s}^{\,0} - h_\ell$$

Thus

$$h_\ell = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + z_1 - z_2 = 3\,m - 1.0\,m - 1.5\,m = \underline{\underline{0.5\,m}}$$

and since $h_\ell > 0$, the assumed direction of flow is correct.

The flow is uphill.

5.97

5.97 Water flows through a vertical pipe, as is indicated in Fig. P5.97. Is the flow up or down in the pipe? Explain.

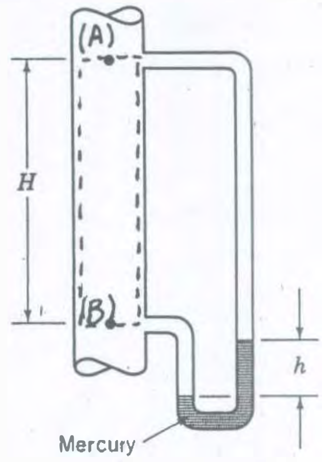


FIGURE P5.97

The control volume shown in the sketch above is used. For steady, incompressible flow downward from (A) to (B) we obtain from Eq. 5.79

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A - {}_A loss_B$$

From conservation of mass we conclude that

$$V_A = V_B$$

Thus from Eq. 1

$${}_A loss_B = gH + \frac{p_A - p_B}{\rho}$$

However the manometer equation (see Section 2.6) yields

$$\frac{p_A - p_B}{\rho} = g\left[h\left(1 - SG_{Hg}\right) - H\right]$$

and

$${}_A loss_B = gh\left(1 - SG_{Hg}\right)$$

which is a negative quantity since $SG_{Hg} = 13.6$. A negative loss is not physically possible so the flow must be upward from B to A. For upward flow the above analysis leads to

$${}_B loss_A = gh\left(SG_{Hg} - 1\right)$$

which is positive and therefore physically reasonable.

5.98

5.98 A circular disk can be lifted up by blowing on it with the device shown in Fig. P5.98. Explain why this happens.

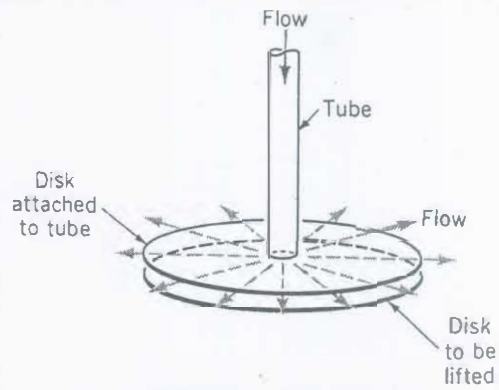


FIGURE P5.98

Applying the energy equation (Eq. 5.82) to the flow from section (1) anywhere within the space between the two circular disks to section (2) at the exit of the flow between the two disks we obtain

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} = \frac{p_1}{\rho} + \frac{V_1^2}{2} - \text{loss}$$

We note that the exit pressure, p_2, is p_{atm}. Thus, Eq. 1 becomes

$$p_1 = p_{atm} + \rho\left(\frac{V_2^2 - V_1^2}{2}\right) + \text{loss} \qquad (1)$$

With conservation of mass we conclude that

$$V_1 = V_2 \frac{A_2}{A_1} = V_2 \frac{D_2}{D_1}$$

Which when combined with Eq. 1 yields

$$p_1 = p_{atm} + \rho\frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^2\right] + \text{loss} \qquad (2)$$

We conclude with Eq. 2 that the pressures within the flow between the 2 disks are mostly less than $p_2 = p_{atm}$ since $D_1 < D_2$ and loss is small. An exception is the stagnation pressure where the tube flow impacts on the lower disk. The less than atmospheric pressure value of p_1 result in the disk being lifted up.

5.99

5.99 A siphon is used to draw water at 20 °C from a large container as indicated in Fig. P5.99. Does changing the elevation, h, of the siphon centerline above the water level in the tank vary the flowrate through the siphon? Explain. What is the maximum allowable value of h?

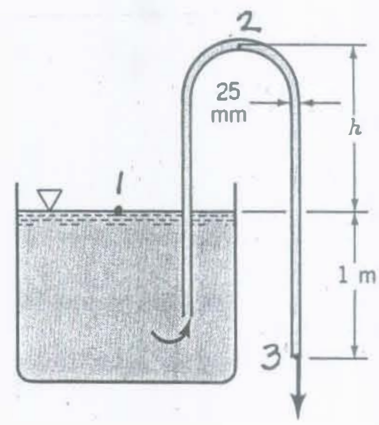


FIGURE P5.99

The volume flowrate through the siphon is related to velocity by the equation

$$Q = VA$$

where A is the constant cross section area of the siphon. Thus V is constant throughout the siphon.
Assuming steady, incompressible flow without friction allows us to use the Bernoulli equation between any two points along a path line. Thus

$$\cancel{\frac{p_1}{\rho}} + \cancel{\frac{V_1^2}{2}}^{0} + gz_1 = \cancel{\frac{p_3}{\rho}} + \frac{V_3^2}{2} + gz_3$$

or

$$V_3 = \sqrt{2g(1\,m)}$$

and it appears as if V_3 and thus Q is constant and independent of the value of h.
However, if the Bernoulli equation is written for flow between point 2 and 3 we obtain

$$\frac{p_2}{\rho} + \cancel{\frac{V_2^2}{2}} + gz_2 = \frac{p_3}{\rho} + \cancel{\frac{V_3^2}{2}} + gz_3$$

$$h = \frac{p_3 - p_2}{\rho g} - 1$$

and we conclude that since $p_3 = p_{atm}$, as h becomes larger, p_2 becomes smaller. The maximum value of h is associated with the minimum value of p_2 which is the vapor pressure of water. Thus

$$h_{max} = \frac{p_3 - p_v}{\rho g} - 1 = \frac{\left(101\times10^3\,\frac{N}{m^2}\right) - \left(2.338\times10^3\,\frac{N}{m^2}\right)^{\text{Table B.2}}}{\underbrace{\left(998.2\,\frac{kg}{m^3}\right)}_{\text{Table B.2}}\left(9.81\,\frac{m}{s^2}\right)\left(1\,\frac{N}{kg\cdot m/s^2}\right)} - 1 = \underline{\underline{9.06\,m}}$$

5.100

5.100 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.100. If the friction loss between A and B is $0.8V^2/2$, where V is the velocity of flow in the siphon, determine the flowrate involved.

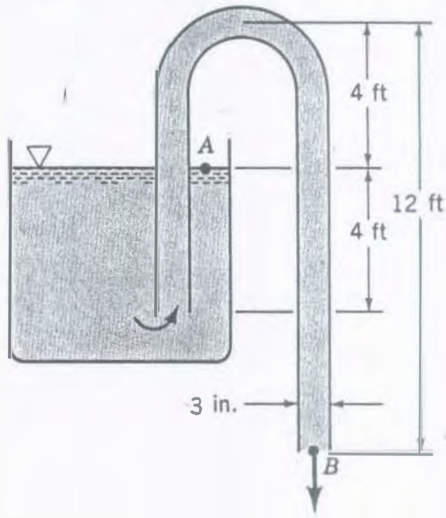


FIGURE P5.100

To determine the flowrate, Q, we use

$$Q = AV = \frac{\pi D^2}{4} V \qquad (1)$$

To obtain V we apply the energy equation (Eq. 5.82) between points A and B in the sketch above. Thus,

$$\cancel{\frac{p_B}{\rho}}^{0} + \frac{V_B^2}{2} + g z_B = \cancel{\frac{p_A}{\rho}}^{0} + \cancel{\frac{V_A^2}{2}}^{0} + g z_A + w_{\substack{shaft \\ net\ in}} - loss$$

or

$$\frac{V^2}{2} + g z_B = g z_A - 0.8 \frac{V^2}{2}$$

Thus

$$V = \sqrt{\frac{g(z_A - z_B)}{0.9}} = \sqrt{\frac{(32.2 \frac{ft}{s^2})(8\, ft)}{0.9}} = 16.9 \frac{ft}{s}$$

and with Eq. 1

$$Q = \frac{\pi (3 in.)^2}{4 \left(144 \frac{in.^2}{ft^2}\right)} \left(16.9 \frac{ft}{s}\right) = \underline{\underline{0.830 \frac{ft^3}{s}}}$$

5.101

5.101 Water flows through a valve (see Fig. P5.101) at the rate of 1000 lbm/s. The pressure just upstream of the valve is 90 psi and the pressure drop across the valve is 5 psi. The inside diameters of the valve inlet and exit pipes are 12 and 24 in. If the flow through the valve occurs in a horizontal plane, determine the loss in available energy across the valve.

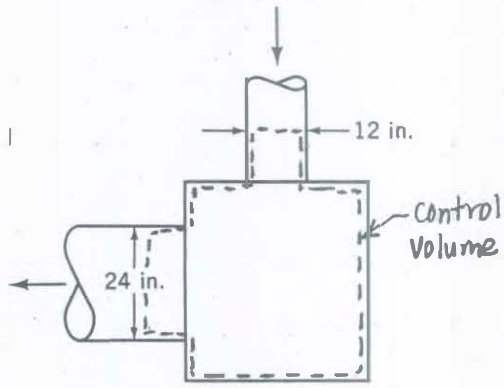


FIGURE P5.101

The control volume shown in the sketch above is used. We can use Eq. 5.79 to determine the loss in available energy associated with the incompressible, steady flow through this control volume. Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2}$$

From the conservation of mass principle

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{\dot{m}}{\rho \frac{\pi D_1^2}{4}}$$

and

$$V_2 = \frac{\dot{m}}{\rho \frac{\pi D_2^2}{4}}$$

Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{1}{2}\left(\frac{\dot{m}\,4}{\rho \pi}\right)^2 \left(\frac{1}{D_1^4} - \frac{1}{D_2^4}\right)$$

$$loss = \frac{\left(50 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{1.94 \frac{slugs}{ft^3}} + \frac{1}{2}\left[\frac{\left(1000 \frac{lbm}{s}\right)(4)}{\left(1.94 \frac{slugs}{ft^3}\right)\left(32.2 \frac{lbm}{slug}\right)\pi}\right]^2 \left[\frac{\left(12 \frac{in.}{ft}\right)^4}{(12\,in.)^4} - \frac{\left(12 \frac{in.}{ft}\right)^4}{(24\,in.)^4}\right]\left(\frac{1\,lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$loss = \underline{\underline{5660}}\ \frac{ft \cdot lb}{slug}$$

5.102

5.102 Compare the volume flowrates associated with two different vent configurations, a cylindrical hole in the wall having a diameter of 4 in. and the same diameter cylindrical hole in the wall but with a well-rounded entrance (see Fig. P5.102). The room is held at a constant pressure of 1.5 psi above atmospheric. Both vents exhaust into the atmosphere. The loss in available energy associated with flow through the cylindrical vent from the room to the vent exit is $0.5V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is $0.05V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air.

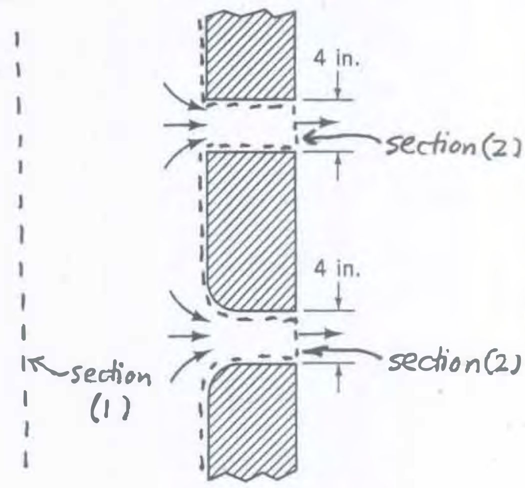


FIGURE P5.102

This is like Example 5.23.

The volume flowrate for each vent configuration is obtained with

$$Q = A_2 V_2 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

and the exit velocity of each vent is obtained with the energy equation (Eq. 5.82). Thus,

$$\frac{V_2^2}{2} = \cancel{\frac{V_1^2}{2}}^{\,0} + \frac{p_1 - p_2}{\rho} + g(\cancel{z_1 - z_2})^{\,0} - loss$$

or

$$\frac{V_2^2}{2} = \frac{p_1 - p_2}{\rho} - K_L \frac{V_2^2}{2}$$

and

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho(1 + K_L)}} \qquad (2)$$

For the cylindrical vent with an abrupt entrance, Eq. 2 leads to

$$V_2 = \sqrt{\frac{(2)(1.5\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)(1+0.5)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 347.9\ \frac{ft}{s}$$

and with Eq. 1 we obtain

$$Q_{\substack{abrupt\ entrance\\ vent}} = \frac{\pi (4\ in.)^2}{4\left(144\ \frac{in.^2}{ft^2}\right)}\left(347.9\ \frac{ft}{s}\right) = \underline{\underline{30.4\ \frac{ft^3}{s}}}$$

For the cylindrical vent with a rounded entrance, Eq. 2 leads to

$$V_2 = \frac{(2)(1.5\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)(1+0.05)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)} = 415.8\ \frac{ft}{s}$$

and with Eq. 1 we obtain

$$Q_{\substack{rounded\ entrance\\ vent}} = \frac{\pi (4\ in.)^2}{4\left(144\ \frac{in.^2}{ft^2}\right)}\left(415.8\ \frac{ft}{s}\right) = \underline{\underline{36.3\ \frac{ft^3}{s}}}$$

5.103

5.103 A gas expands through a nozzle from a pressure of 300 psia to a pressure of 5 psia. The enthalpy change involved, $\check{h}_1 - \check{h}_2$, is 150 Btu/lbm. If the expansion is adiabatic but with frictional effects and the inlet gas speed is negligibly small, determine the exit gas velocity.

Because of the appreciable pressure drop involved in this gas flow we consider this problem to involve compressible flow. From Eq. 5.71 we obtain

$$V_2 = \sqrt{2(\check{h}_1 - \check{h}_2)}$$

or

$$V_2 = \sqrt{2\left(150\,\frac{\text{Btu}}{\text{lb}_m}\right)\left(778\,\frac{\text{ft}\cdot\text{lb}}{\text{Btu}}\right)\left(32.2\,\frac{\text{lb}_m\cdot\text{ft}}{\text{lb}\cdot\text{s}^2}\right)}$$

$$V_2 = \underline{\underline{2740}}\,\frac{\text{ft}}{\text{s}}$$

5.104

5.104 For the 180° elbow and nozzle flow shown in Fig. P5.104, determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?

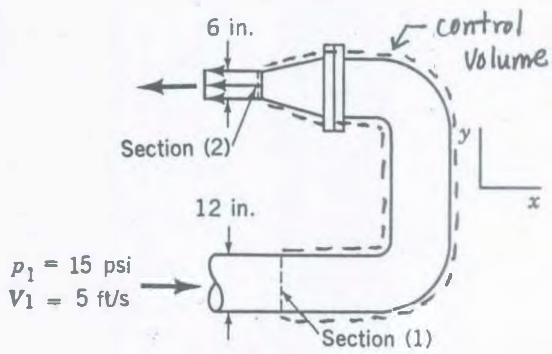


FIGURE P5.104

For solving the first part of this problem, the control volume shown in the sketch above is used. To determine the loss accompanying flow from section 1 to section 2 Eq. 5.79 can be used as follows.

$$ {}_1loss_2 = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) $$

Since x-y coordinates are specified we assume that the flow is horizontal and $z_1 - z_2 = 0$. Also, $p_2 = p_{atm} = 0$ psi.

From the conservation of mass principle we conclude that

$$ V_2 = V_1 \frac{A_1}{A_2} = V_1\left(\frac{D_1^2}{D_2^2}\right) $$

Thus

$$ {}_1loss_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1^2}{D_2^2}\right)^2\right] = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] $$

or

$$ {}_1loss_2 = \frac{\left(15\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)} + \frac{\left(5\ \frac{ft}{s}\right)^2}{2}\left[1 - \left(\frac{12\ in.}{6\ in.}\right)^4\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) $$

$$ {}_1loss_2 = \underline{\underline{926}}\ \frac{ft\cdot lb}{slug} $$

For the second part of this problem we consider the flow of a fluid particle from section 2 to a state of rest, a. Eq. 5.79 leads to

$$ {}_2loss_a = \frac{V_2^2}{2} $$

Note that we have assumed that $p_2 = p_a = p_{atm}$ and $z_2 = z_a$.

Thus

$$ {}_2loss_A = \frac{V_2^2}{2} = \frac{V_1^2}{2}\left(\frac{D_1^2}{D_2^2}\right)^2 = \frac{V_1^2}{2}\left(\frac{D_1}{D_2}\right)^4 = \frac{\left(5\ \frac{ft}{s}\right)^2\left(\frac{12\ in.}{6\ in.}\right)^4\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)}{2} $$

$$ {}_2loss_A = \underline{\underline{200}}\ \frac{ft\cdot lb}{slug} $$

5.105

5.105 An automobile engine will work best when the back pressure at the exhaust manifold, engine block interface is minimized. Show how reduction of losses in the exhaust manifold, piping, and muffler will also reduce the back pressure. How could losses in the exhaust system be reduced? What primarily limits the minimization of exhaust system losses?

We apply the energy equation (Eq. 5.83) to the flow from the engine block, exhaust manifold interface to the exhaust system exit to get

$$p_{in} = p_{out} + \rho\frac{V_{out}^2}{2} - \rho\frac{V_{in}^2}{2} + \rho(\text{loss}) \qquad (1)$$

With Eq. 1 we see that reduction of loss in the exhaust system results in a lower value of p_{in} and thus the engine back pressure. Losses in the exhaust system could be reduced by eliminati g major loss components such as the catalytic converter and the muffler as is often done in race cars. However, noise and emissions legislation limits the extent to which this kind of loss reduction can occur in conventional vehicles. Some loss reduction can also occur by configuring the exhaust system piping with few bends and appropriate area distributions. However, requirements often leads to bends and turns in the piping and costs limit the extend of optimizing area distributions.

5.107

5.107 (See Fluids in the News article titled "Smart shocks," Section 5.3.3.) A 200-lb force applied to the end of the piston of the shock absorber shown in Fig. P5.107 causes the two ends of the shock absorber to move toward each other with a speed of 5 ft/s. Determine the head loss associated with the flow of the oil through the channel. Neglect gravity and any friction force between the piston and cylinder walls.

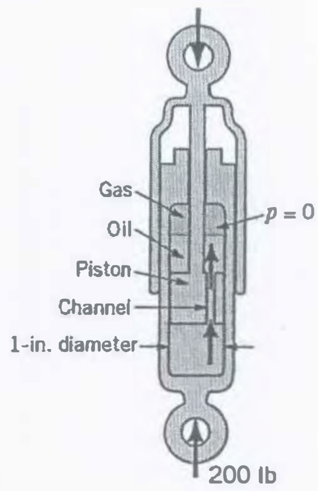


FIGURE P5.107

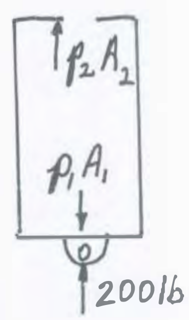
From a force balance on the cylinder

$p_1 A_1 - p_2 A_2 = 200\,lb$

or with $p_2 = 0$,

$p_1 = 200\,lb/A_1 = 200\,lb/\left(\frac{\pi}{4}\left(\frac{1}{12}ft\right)^2\right)$

$= 3.67 \times 10^4 \frac{lb}{ft^2} = 255\,psi$



From the energy equation,

$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$, where

$z_1 \approx z_2$, $V_2 = 0$, $V_1 = 5\,\frac{ft}{s}$, $p_1 = 255\,psi$, and $p_2 = 0$. Assume $\gamma = 50\,\frac{lb}{ft^3}$.

Thus,

$h_L = \frac{p_1}{\gamma} + \frac{V^2}{2g} = \frac{3.67 \times 10^4 \frac{lb}{ft^2}}{\left(50\,\frac{lb}{ft^3}\right)} + \frac{\left(5\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 734\,ft + 0.388\,ft = \underline{\underline{734\,ft}}$

5.108

5.108 What is the maximum possible power output of the hydroelectric turbine shown in Fig.P5.108?

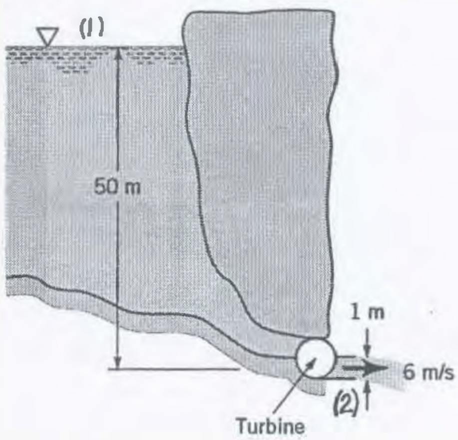


FIGURE P5.108

For flow from section (1) to section (2), Eq. 5.82 yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + w_{\substack{shaft \\ net\ in}} - loss \qquad (1)$$

Since $p_1 = p_2 = p_{atm}$, $\quad w_{\substack{shaft \\ net\ in}} = -w_{\substack{shaft \\ net\ out}}$ $\quad$ Eq. 1 can be expressed as

$$w_{\substack{shaft \\ net\ out}} = g(z_1 - z_2) - \frac{V_2^2}{2} - loss$$

The maximum work or power output is achieved when $loss = 0$.

Thus

$$\dot{W}_{\substack{shaft \\ net\ out \\ maximum}} = \dot{m}\, w_{\substack{shaft \\ net\ out \\ maximum}} = \dot{m}\left[g(z_1 - z_2) - \frac{V_2^2}{2}\right]$$

Now

$$\dot{m} = \rho V_2 A_2 = \rho V_2 \frac{\pi D_2^2}{4} = \left(999\ \frac{kg}{m^3}\right)\left(6\ \frac{m}{s}\right)\frac{\pi (1\ m)^2}{4} = 4710\ \frac{kg}{s}$$

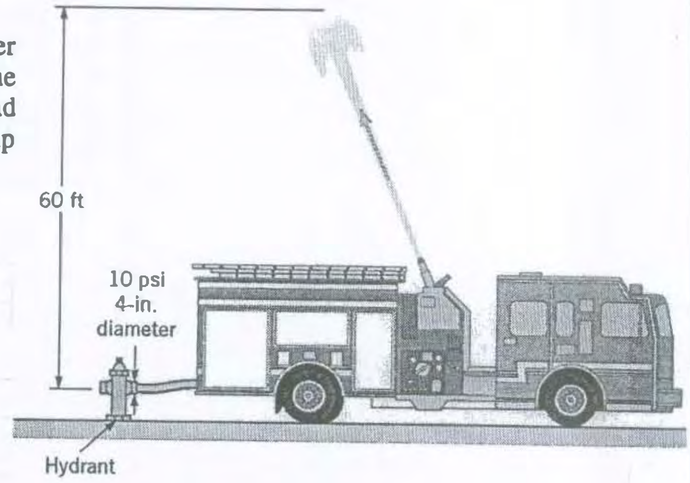
and

$$\dot{W}_{\substack{shaft \\ net\ out \\ maximum}} = \left(4710\ \frac{kg}{s}\right)\left[\left(9.81\ \frac{m}{s^2}\right)(50\ m) - \frac{\left(6\ \frac{m}{s}\right)^2}{2}\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

$$\dot{W}_{\substack{shaft \\ net\ out \\ maximum}} = \underline{\underline{2.22 \times 10^6}}\ \frac{N \cdot m}{s} = \underline{\underline{2.22 \times 10^6\ W}} = \underline{\underline{2.22}}\ MW$$

5.109

5.109 The pumper truck shown in Fig. P5.109 is to deliver 1.5 ft^3/s to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in.-diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.



■ FIGURE P5.109

To solve this problem we first use the energy equation (Eq. 5.84) for flow from the hydrant exit (1) to the maximum desired elevation of 60 ft (2) to get h_s or in this case, the pump head. With the pump head we can get the pump power from Eq. 5.85.

$$\cancel{\frac{p_2}{\rho}}^{\,0} + \cancel{\frac{V_2^2}{2g}}^{\,0} + z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 + h_s - \cancel{h_L}^{\,0}$$

$$h_s = z_2 - z_1 - \frac{p_1}{\rho} - \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{\left(1.5\ \frac{ft^3}{s}\right)(4)}{\pi\left(\frac{4\ in.}{12\ \frac{in.}{ft}}\right)^2} = 17.2\ \frac{ft}{s}$$

$$h_s = 60\ ft - \frac{\left(10\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)} - \frac{\left(17.2\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)}$$

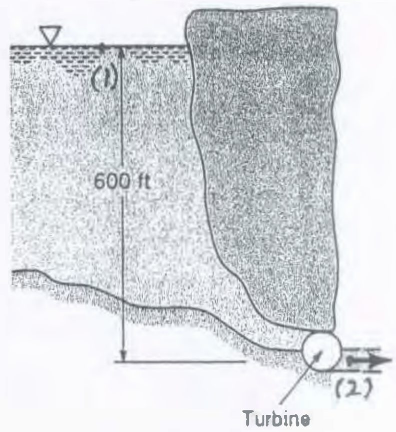
$$h_s = 32.3\ ft$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \gamma Q h_s = \frac{\left(62.4\ \frac{lb}{ft^3}\right)\left(1.5\ \frac{ft^3}{s}\right)(32.2\ ft)}{\left(550\ \frac{ft.lb}{s.hp}\right)}$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{5.48\ hp}}$$

5.110

5.110 The hydroelectric turbine shown in Fig. P5.110 passes 8 million gal/min across a head of 600 ft. What is the maximum amount of power output possible? Why will the actual amount be less?



■ FIGURE P5.110

From the energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = 0$, $p_2 = 0$, and $V_1 = 0$.

Thus,

$$h_s = (z_2 - z_1) + h_L + \frac{V_2^2}{2g}$$

And, the power is given by

$$\dot{W}_{turb} = \gamma Q h = \gamma Q\left[(z_2 - z_1) + h_L + \frac{V_2^2}{2g}\right]$$

The maximum power would occur if there were no losses ($h_L = 0$) and negligible kinetic energy at the exit ($V_2 \approx 0$; large diameter outlet).

Thus,

$$\dot{W}_{turb\,max} = \gamma Q(z_2 - z_1) = 62.4\frac{lb}{ft^3}\left(8\times10^6\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{1\,ft^3}{7.48\,gal}\right)(-600\,ft)$$

$$-6.67\times10^8\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\frac{ft\cdot lb}{s}}\right) = \underline{\underline{-1.21\times10^6\,hp}}$$

The minus sign is associated with power out.

The actual power will be less by amounts corresponding to loss and exit kinetic energy.

5.111

5.111 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.111 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.

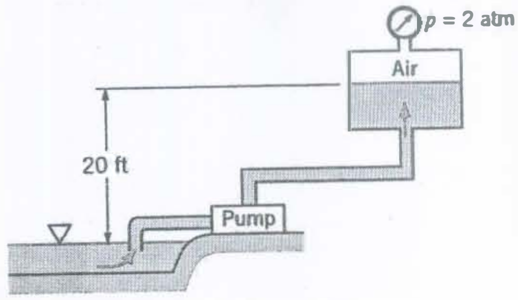


FIGURE P5.111

$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$, where $p_1 = 0$, $z_1 = 0$, $V_1 = 0$, and $z_2 = 20\,ft$.

Thus,

(1) $h_s = h_L + \frac{p_2}{\gamma} + z_2$

Also,

$Q = [(1000\,gal)/(10\,min)]\left(\frac{1\,ft^3}{7.48\,gal}\right)\left(\frac{1\,min}{60\,s}\right) = 0.223\,\frac{ft^3}{s}$

so that

$h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{(3\,hp)(550\,\frac{ft \cdot lb/s}{hp})}{(62.4\,\frac{lb}{ft^3})(0.223\,\frac{ft^3}{s})} = 119\,ft$

(a) If $p_2 = 2\,atm = 2(14.7\,\frac{lb}{in.^2})(144\,in.^2/ft^2) = 4{,}230\,\frac{lb}{ft^2}$, then from Eq. (1)

$h_s = h_L + \frac{4{,}230\,\frac{lb}{ft^2}}{(62.4\,\frac{lb}{ft^3})} + 20\,ft = h_L + 87.8\,ft$

Thus, if

$h_L \leq h_s - 87.8\,ft = 119\,ft - 87.8\,ft = 31.2\,ft$ the given pump will work for $p_2 = 2\,atm$.

(b) If $p_2 = 3\,atm = 6{,}350\,\frac{lb}{ft^2}$, then

$h_s = h_L + \frac{6{,}350\,\frac{lb}{ft^3}}{(62.4\,\frac{lb}{ft^3})} + 20\,ft = h_L + 122\,ft$

Thus, if this pump is to work

$119\,ft = h_L + 122\,ft$, or $h_L \leq -3\,ft$

Since it is not possible to have $h_L < 0$, the pump will not work for $p_2 = 3\,atm$.

5.112

5.112 A hydraulic turbine is provided with 4.25 m^3/s of water at 415 kPa. A vacuum gage in the turbine discharge 3 m below the turbine inlet centerline reads 250 mm Hg vacuum. If the turbine shaft output power is 1100 kW, calculate the power loss through the turbine. The supply and discharge pipe inside diameters are identically 80 mm.

We consider the turbine inlet and discharge to be sections (1) and (2). For flow from sections (1) to (2) Eq. 5.82 yields

$$\text{loss} = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) - w_{\substack{shaft\\ net\ out}} \qquad (1)$$

since

$$V_1 = V_2$$

and

$$w_{\substack{shaft\\ net\ out}} = -w_{\substack{shaft\\ net\ in}}$$

For power loss through the turbine we need to multiply Eq. 1 by the mass flowate, $\dot{m}$, thus

$$\text{power loss} = \dot{m}\left(\frac{p_1 - p_2}{\rho}\right) + \dot{m}\,g(z_1 - z_2) - \dot{W}_{\substack{shaft\\ net\ out}} \qquad (2)$$

However,

$$\dot{m} = \rho Q = \left(999\ \frac{kg}{m^3}\right)\left(4.25\ \frac{m^3}{s}\right) = 4246\ \frac{kg}{s}$$

Also

$$p_2 = -(0.25\ m\ Hg)(\rho_{Hg})(g) = (0.25\ m)(13.6)\left(999\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)$$

or

$$p_2 = -33{,}300\ \frac{N}{m^2}$$

With Eq. 2

$$\text{power loss} = \left(4246\ \frac{kg}{s}\right)\frac{\left(415{,}000\ \frac{N}{m^2} + 33{,}300\ \frac{N}{m^2}\right)}{\left(999\ \frac{kg}{m^3}\right)} + \left(4246\ \frac{kg}{s}\right)\left(9.81\ \frac{m}{s^2}\right)(3\ m)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) - \left(1.1\times10^6\ \frac{N\cdot m}{s}\right)$$

or

$$\text{power loss} = 930{,}000\ \frac{N\cdot m}{s} = \underline{\underline{930\ kW}}$$

5.113

5.113 Water is supplied at 150 ft^3/s and 60 psi to a hydraulic turbine through a 3-ft inside diameter inlet pipe as indicated in Fig. P5.113. The turbine discharge pipe has a 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections (1) and (2).

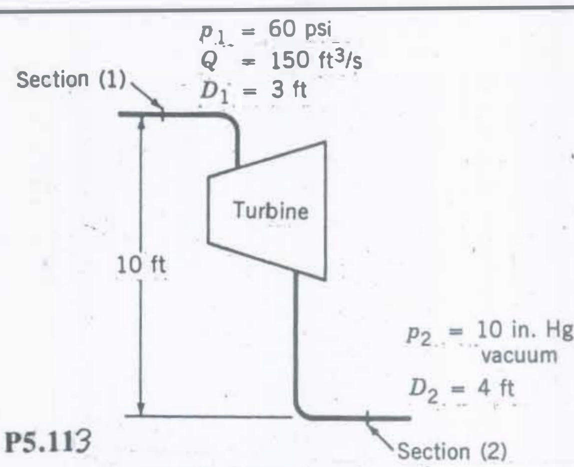


FIGURE P5.113

For flow between sections (1) and (2), Eq. 5.82 leads to

$$\text{power loss} = \rho Q\left[\left(\frac{P_1-P_2}{\rho}\right) + g(z_1-z_2) + \left(\frac{V_1^2-V_2^2}{2}\right)\right] - \dot{W}_{\substack{\text{shaft}\\ \text{net out}}} \qquad (1)$$

From given data

$$P_2 = \frac{(-10\ in.\ Hg)(13.6)\left(1.94\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right)}{\left(12\ \frac{in.}{ft}\right)}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) = -708\ \frac{lb}{ft^2}$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(4)\left(150\ \frac{ft^3}{s}\right)}{\pi\ (3ft)^2} = 21.22\ \frac{ft}{s}$$

From conservation of mass (Eq. 5.13)

$$V_2 = V_1\frac{A_1}{A_2} = V_1\frac{D_1^2}{D_2^2} = \left(21.22\ \frac{ft}{s}\right)\frac{(3ft)^2}{(4ft)^2} = 11.94\ \frac{ft}{s}$$

From Eq. 1

$$\text{power loss} = \frac{\left(1.94\ \frac{slugs}{ft^3}\right)\left(150\ \frac{ft^3}{s}\right)}{\left(550\ \frac{ft.lb}{s.hp}\right)}\left\{\frac{\left(60\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)+\left(708\ \frac{lb}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}\right.$$

$$\left. + \left(32.2\ \frac{ft}{s^2}\right)(10\ ft)\left(\frac{1\ lb}{slug.\frac{ft}{s^2}}\right) + \left[\frac{\left(21.22\ \frac{ft}{s}\right)^2-\left(11.94\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug.\frac{ft}{s^2}}\right)\right\}$$

$$-2500hp$$

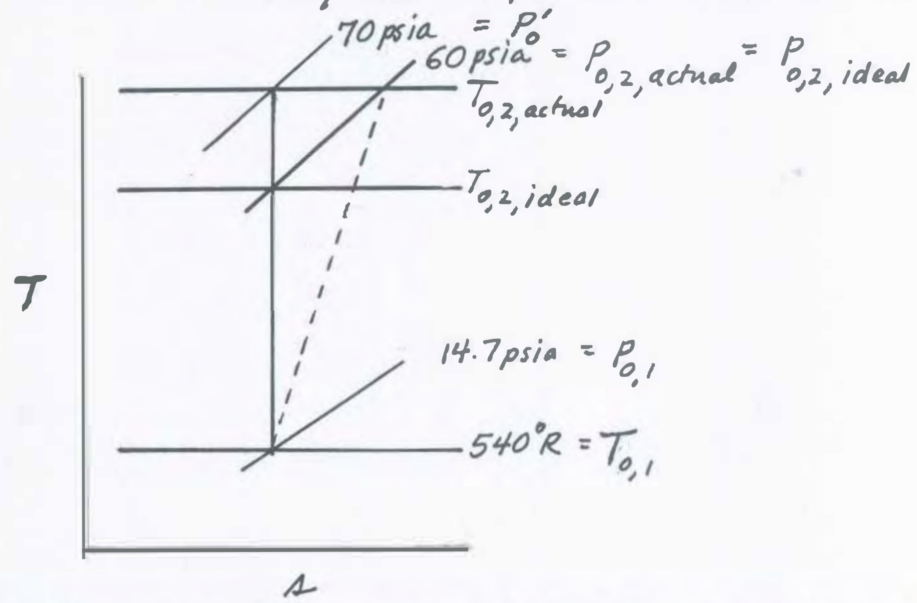
or

$$\text{power loss} = \underline{\underline{301}}\ hp$$

5.114

5.114 A centrifugal air compressor stage operates between an inlet stagnation pressure of 14.7 psia and an exit stagnation pressure of 60 psia. The inlet stagnation temperature is 80 °F. If the loss of total pressure through the compressor stage associated with irreversible flow phenomena is 10 psi, calculate the actual and ideal stagnation temperature rise through the compressor. Calculate the ratio of ideal to actual temperature rise to obtain efficiency.

We assume that the air compressor operates adiabatically. An ideal compression process is frictionless and adiabatic and thus according to Eq. 5.101, it is a constant entropy or isentropic process. With Eq. 5.101 we also conclude that an actual adiabatic compression process with friction must involve an entropy increase. On temperature-entropy coordinates, the ideal and actual compression processes appear as indicated in the sketch below. Also shown is the 10 psi loss in stagnation pressure due to friction.



We consider the air being compressed to behave as an ideal gas. Then from Eqs. 1.8 and 5.111 we obtain for the ideal processes

$$T_{0,2,ideal} = T_{0,1}\left(\frac{p_{0,2,ideal}}{p_{0,1}}\right)^{\frac{k-1}{k}} = (540°R)\left(\frac{60\text{ psia}}{14.7\text{ psia}}\right)^{\frac{1.4-1}{1.4}} = 807\ °R$$

and

$$T_{0,2,actual} = T_{0,1}\left(\frac{p_0'}{p_{0,1}}\right)^{\frac{1.4-1}{0.4}} = (540°R)\left(\frac{70\text{ psia}}{14.7\text{ psia}}\right)^{\frac{1.4-1}{1.4}} = 843\ °R$$

(con't)

5.114 (con't)

Then

actual stagnation temperature rise $= T_{0,2,actual} - T_{0,1} = 843°R - 540°R = \underline{\underline{303°R}}$

and

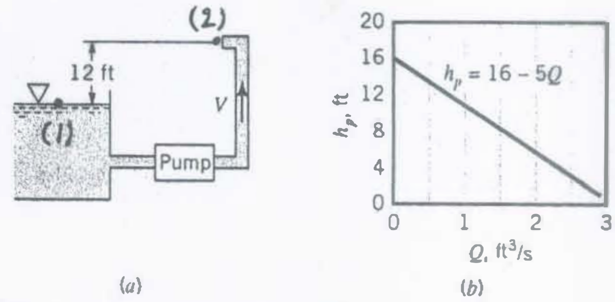
ideal stagnation temperature rise $= T_{0,2,ideal} - T_{0,1} = 807°R - 540°R = \underline{\underline{267°R}}$

Also

$$\text{efficiency} = \frac{T_{0,2,ideal} - T_{0,1}}{T_{0,2,actual} - T_{0,1}} = \frac{267°R}{303°R} = \underline{\underline{0.88}}$$

5.115

5.115 Water is pumped through a 4-in.-diameter pipe as shown in Fig. P5.115*a*. The pump characteristics (pump head versus flowrate) are given in Fig. P5.115*b*. Determine the flowrate if the head loss in the pipe is $h_L = 8V^2/2g$.



■ FIGURE P5.115

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ z_1 = 0,\ z_2 = 12\text{ ft},$$
$$V_1 = 0, \text{ and } V_2 = Q/A_2$$

Thus,

$$h_s - h_L = z_2 + \frac{V_2^2}{2g}, \text{ with}$$

$$h_s = h_p = 16 - 5Q \text{ and } h_L = 8\frac{V_2^2}{2g} = 8\frac{Q^2}{2gA_2^2}$$

There fore,

$$16 - 5Q - \frac{4Q^2}{gA_2^2} = 12 + \frac{Q^2}{2gA_2^2}$$

or

(1) $$\left(\frac{9}{2gA_2^2}\right)Q^2 + (5)Q - 4 = 0, \text{ where } g \sim \frac{\text{ft}}{\text{s}^2},\ A_2 \sim \text{ft}^2, \text{ and } Q \sim \frac{\text{ft}^3}{\text{s}}$$

Using the given data, Eq. (1) becomes

$$\left[\frac{9}{2(32.2)\left(\frac{\pi}{4}\left(\frac{4}{12}\right)^2\right)^2}\right]Q^2 + 5Q - 4 = 0$$

or

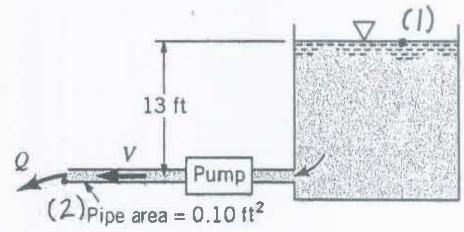
(2) $$18.35\,Q^2 + 5Q - 4 = 0$$

The positive root of Eq. (2) is $\underline{\underline{Q = 0.350 \frac{\text{ft}^3}{\text{s}}}}$

(The negative root of Eq. (2) has no physical meaning.)

5.116

5.116 Water is pumped from the large tank shown in Fig. P5.116. The head loss is known to be equal to $4V^2/2g$ and the pump head is $h_p = 20 - 4Q^2$, where h_p is in ft when Q is in ft^3/s. Determine the flowrate.



■ FIGURE P5.116

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \quad \text{where } p_1 = p_2 = 0,\ z_1 = 13\,ft,\ z_2 = 0,\ h_s = h_p \text{ and } V_1 = 0.$$

Thus,

(1) $$z_1 + h_p - h_L = \frac{V_2^2}{2g}$$

Also,

$$h_L = 4\frac{V^2}{2g} = 4\frac{V_2^2}{2g} = 4\frac{(Q/A_2)^2}{2g} \quad \text{since } V_2 = \frac{Q}{A_2}$$

Hence, Eq. (1) becomes

$$z_1 + (20 - 4Q^2) - 4\frac{(Q/A_2)^2}{2g} = \frac{(Q/A_2)^2}{2g}$$

or

$$\left[\left(\frac{5}{2g\,A_2^2}\right) + 4\right]Q^2 = 20 + z_1, \quad \text{where } g \sim \frac{ft}{s^2},\ A_2 \sim ft^2, \text{ and } Q \sim \frac{ft^3}{s}$$

Thus, with the given data

$$\left[\left(\frac{5}{2(32.2\,\frac{ft}{s^2})(0.1\,ft^2)^2}\right) + 4\right]Q^2 = 20 + 13\,ft$$

or

$$\underline{\underline{Q = 1.67\,\frac{ft^3}{s}}}$$

5.117

5.117 When a fan or pump is tested at the factory, head curves (head across the fan or pump versus volume flowrate) are often produced. A generic fan or pump head curve is shown in Fig. P5.117*a*. For any piping system, the drop in pressure or head involved because of loss can be estimated as a function of volume flowrate. A generic piping system loss curve is also shown in Fig. P5.117*b*. When the pump or fan and piping system associated with the two curves of Fig. P5.117 are combined, what will the flowrate be? Why? How can the flowrate through this combined system be varied?

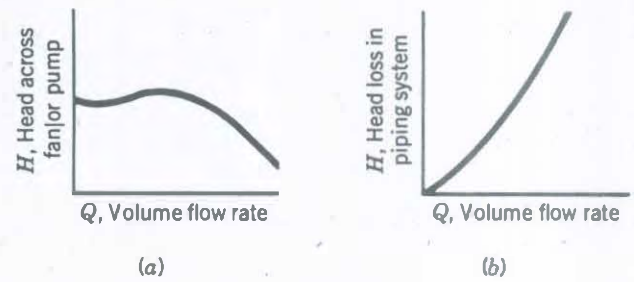
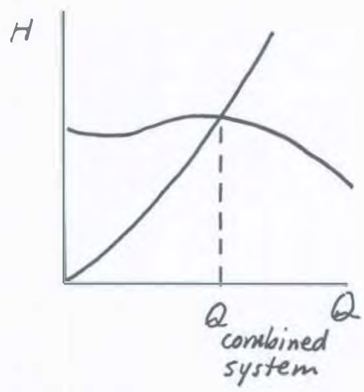
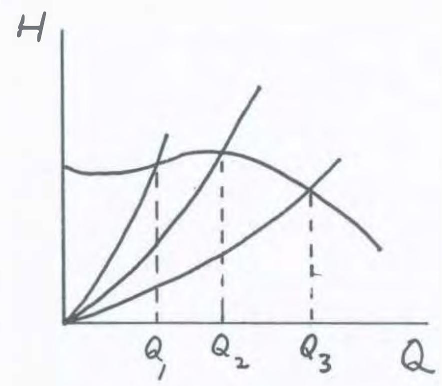


FIGURE P5.117

The flowrate of the combination of the fan or pump and the piping system represented by the two curves sketched above will correspond to the intersection of the two curves as indicated in the sketch below because this conditions satisfies both components in terms of head and flowrate.

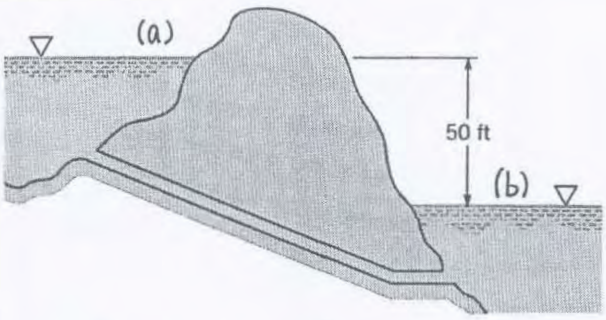


To vary the flowrate through the combined system, the piping system curve is normally altered as shown below by changing the resistance to flow of the piping system. This could be accomplished, for example with a variable area valve.



5.118

5.118 Water flows by gravity from one lake to another as sketched in Fig. P5.118 at the steady rate of 80 gpm. What is the loss in available energy associated with this flow? If this same amount of loss is associated with pumping the fluid from the lower lake to the higher one at the same flowrate, estimate the amount of pumping power required.



■ FIGURE P5.118

$$Q = \frac{80 \frac{gal}{min}}{\left(60 \frac{s}{min}\right)\left(7.48 \frac{gal}{ft^3}\right)} = 0.178 \frac{ft^3}{s}$$

For the flow from section (a) to section (b) Eq. 5.82 leads to

$$loss = g(z_a - z_b) = \left(32.2 \frac{ft}{s^2}\right)(50 ft)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{1610}}\ \frac{ft\ lb}{slug}$$

For pumped flow from section (b) to section (a) Eq. 5.82 yields

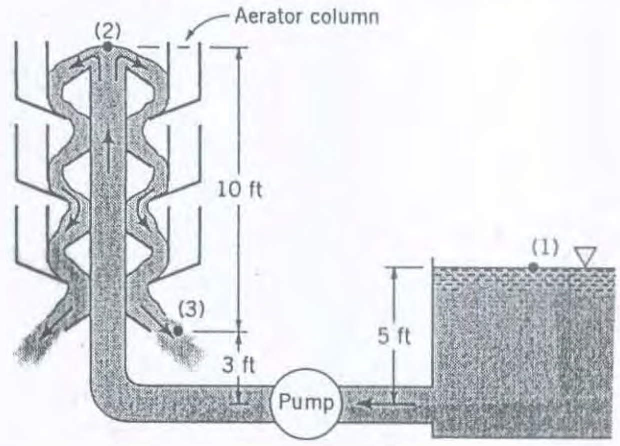
$$\dot{W}_{\substack{shaft\\net\ in}} = \rho Q\left[g(z_a - z_b) + loss\right] = \left(1.94 \frac{slugs}{ft^3}\right)\left(0.178 \frac{ft^3}{s}\right)\left[\left(32.2 \frac{ft}{s^2}\right)(50 ft)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) + 1610 \frac{ft \cdot lb}{slug}\right]$$

or

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{1110}}\ \frac{ft \cdot lb}{s} = \underline{\underline{2.02\ hp}}$$

5.119

5.119 Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in **Video V5.8** and Fig. P5.119 at a rate of 3.0 ft^3/s. **(a)** Determine the power that the pump adds to the water if the head loss from (1) to (2) where $V_2 = 0$ is 4 ft. **(b)** Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $V_3 = 2$ ft/s.



■ FIGURE P5.119

(a) The energy equation from (1) to (2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_s - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with

$p_1 = p_2 = V_1 = V_2 = 0$ gives

$$h_s = h_L + z_2 - z_1 = 4\,ft + (10+3)\,ft - 5\,ft = 12\,ft$$

Thus, the pump power is

$$\dot{W}_s = \gamma Q h_s = 62.4\,\frac{lb}{ft^3}\left(3\,\frac{ft^3}{s}\right)(12\,ft) = 2246\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{4.08\,hp}}$$

(b) The energy equation from (2) to (3)

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_s - h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

with

$p_2 = p_3 = V_2 = h_s = 0$ gives

$$h_L = z_2 - z_3 - \frac{V_3^2}{2g} = 13\,ft - 3\,ft - \frac{\left(2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 10\,ft - 0.062\,ft$$

or

$$h_L = \underline{\underline{9.94\,ft}}$$

5.120

5.120 A liquid enters a fluid machine at section (1) and leaves at sections (2) and (3) as shown in Fig. P5.120. The density of the fluid is constant at 2 slugs/ft³. All of the flow occurs in a horizontal plane and is frictionless and adiabatic. For the above-mentioned and additional conditions indicated in Fig. P5.120, determine the amount of shaft power involved.

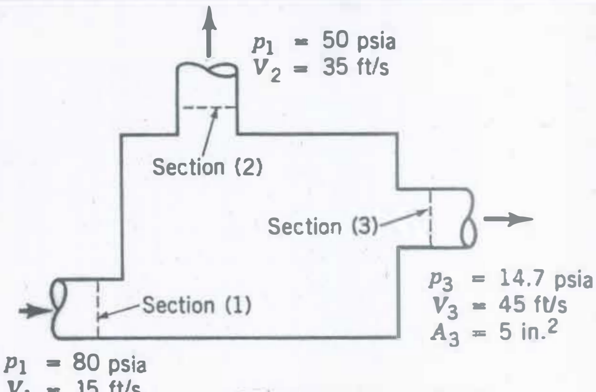


FIGURE P5.120

For the frictionless and adiabatic flow through this fluid machine Eqs. 5.64, 5.65 and 5.76 lead to

$$\dot{W}_{\substack{\text{shaft}\\\text{net in}}} = \dot{m}_3\left(\frac{p_3}{\rho} + \frac{V_3^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) \qquad (1)$$

since

$$\dot{m}_1\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = (\dot{m}_2 + \dot{m}_3)\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = \dot{m}_2(\check{u}_1 - \check{u}_2) + \dot{m}_3(\check{u}_1 - \check{u}_2) = 0$$

At section (3)

$$\dot{m}_3 = \rho A_3 V_3 = \left(2\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(\frac{5\text{ in.}^2}{144\,\frac{\text{in.}^2}{\text{ft}^2}}\right)\left(45\,\frac{\text{ft}}{\text{s}}\right) = 3.125\,\frac{\text{slugs}}{\text{s}}$$

At section (1)

$$\dot{m}_1 = \rho A_1 V_1 = \left(2\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(\frac{30\text{ in.}^2}{144\,\frac{\text{in.}^2}{\text{ft}^2}}\right)\left(15\,\frac{\text{ft}}{\text{s}}\right) = 6.25\,\frac{\text{slugs}}{\text{s}}$$

From conservation of mass

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 6.25\,\frac{\text{slugs}}{\text{s}} - 3.125\,\frac{\text{slugs}}{\text{s}} = 3.125\,\frac{\text{slugs}}{\text{s}}$$

With Eq. 1 we obtain

$$\dot{W}_{\substack{\text{shaft}\\\text{net in}}} = \left\{\left(3.125\,\frac{\text{slugs}}{\text{s}}\right)\left[\frac{\left(14.7\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(2\,\frac{\text{slugs}}{\text{ft}^3}\right)} + \frac{\left(45\,\frac{\text{ft}}{\text{s}}\right)^2}{2}\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)\right]\right.$$
$$-\left(6.25\,\frac{\text{slugs}}{\text{s}}\right)\left[\frac{\left(80\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(2\,\frac{\text{slugs}}{\text{ft}^3}\right)} + \frac{\left(15\,\frac{\text{ft}}{\text{s}}\right)^2}{2}\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)\right]$$
$$\left.+\left(3.125\,\frac{\text{slugs}}{\text{s}}\right)\left[\frac{\left(50\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(2\,\frac{\text{slugs}}{\text{ft}^3}\right)} + \frac{\left(35\,\frac{\text{ft}}{\text{s}}\right)^2}{2}\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)\right]\right\}\left(\frac{1}{550\,\frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}}\right)$$

or

$$\dot{W}_{\substack{\text{shaft}\\\text{net in}}} = \underline{\underline{-31.1\text{ hp}}}$$, the net shaft power is out (<0)

5.121

5.121 Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.121. The loss in available energy associated with 2.5 ft³/s being pumped from sections (1) to (2) is $61\overline{V}^2/2$ where $\overline{V}$ is the average velocity of water in the 8-in.-inside diameter piping involved. Determine the amount of shaft power required.

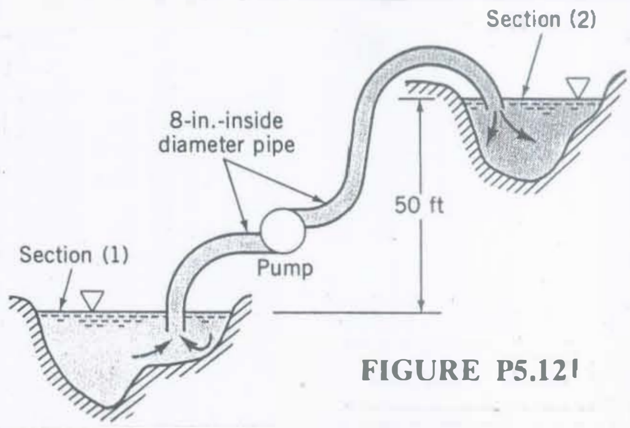


FIGURE P5.121

For the flow from section (1) to section (2) Eq. 5.82 leads to

$$\dot{W}_{\substack{shaft\\net\ in}} = \rho Q\left[g(z_2 - z_1) + loss\right] = \rho Q\left[g(z_2-z_1) + 61\frac{\overline{V}^2}{2}\right] \quad (1)$$

From the volume flowrate we obtain

$$\overline{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{\left(2.5\ \frac{ft^3}{s}\right)}{\frac{\pi}{4}\left(\frac{8\ in.}{12\ \frac{in.}{ft}}\right)^2} = 7.162\ \frac{ft}{s}$$

Thus, from Eq. 1

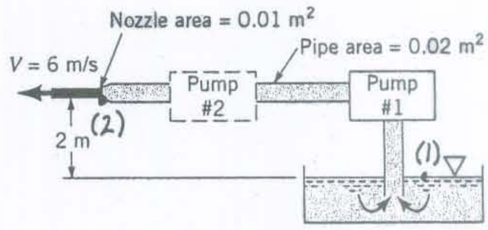
$$\dot{W}_{\substack{shaft\\net\ in}} = \left(1.94\ \frac{slugs}{ft^3}\right)\left(2.5\ \frac{ft^3}{s}\right)\left[\left(32.2\ \frac{ft}{s^2}\right)(50\ ft) + \frac{(61)\left(7.162\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\left(\frac{1}{550\ \frac{ft.lb}{s.hp}}\right)$$

or

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{28\ hp}}$$

5.122

5.122 Water is to be pumped from the large tank shown in Fig. P5.122 with an exit velocity of 6 m/s. It was determined that the original pump (pump 1) that supplies 1 kW of power to the water did not produce the desired velocity. Hence, it is proposed that an additional pump (pump 2) be installed as indicated to increase the flowrate to the desired value. How much power must pump 2 add to the water? The head loss for this flow is $h_L = 250Q^2$, where h_L is in m when Q is in m^3/s.



■ FIGURE P5.122

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0$, $V_1 = 0$, $z_1 = 0$, $z_2 = 2m$.

Thus,

$h_s = h_L + z_2 + \frac{V_2^2}{2g}$, where $V_2 = 6 m/s$ so that $Q = A_2 V_2 = 0.01 m^2 (6 m/s)$
$= 0.06 \, m^3/s$

Note: $h_s = h_{pump1} + h_{pump2}$

Thus, with $h_L = 250 Q^2 = 250 (0.06)^2 = 0.90 m$ it follows that

$$h_s = 0.90m + 2m + \frac{(6 m/s)^2}{2(9.81 \frac{m}{s^2})} = 4.73m$$

so that

$$\dot{W}_s = \gamma Q h_s = (9.80 \times 10^3 \tfrac{N}{m^3})(0.06 \tfrac{m^3}{s})(4.73m) = 2.78 \times 10^3 \tfrac{N \cdot m}{s} = 2.78 \, kW$$

Therefore,

$\dot{W}_s = \dot{W}_{pump1} + \dot{W}_{pump2} = 2.78 \, kW$, with $\dot{W}_{pump1} = 1 \, kW$

Hence,

$$\dot{W}_{pump2} = 2.78 \, kW - 1 \, kW = \underline{\underline{1.78 \, kW}}$$

5.123

5.123 (See Fluids in the News article titled **"Curtain of air,"** Section 5.3.3.) The fan shown in Fig. P5.123 produces an air curtain to separate a loading dock from a cold storage room. The air curtain is a jet of air 10 ft wide, 0.5 ft thick moving with speed $V = 30$ ft/s. The loss associated with this flow is $loss = K_L V^2/2$, where $K_L = 5$. How much power must the fan supply to the air to produce this flow?

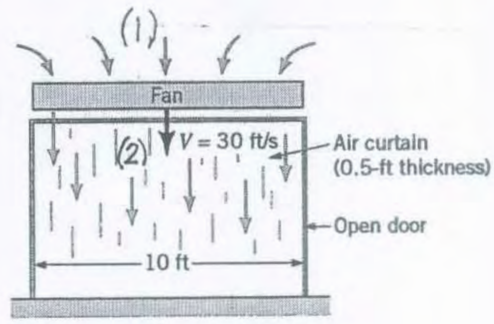


FIGURE P5.123

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g},$$

where

$$p_1 \approx p_2 \approx 0, \quad z_1 \approx z_2, \quad V_1 \approx 0, \text{ and } h_L = \frac{loss}{g} = 5\frac{V_2^2}{2g}$$

Thus,

$$h_s = h_L + \frac{V_2^2}{2g} = 5\frac{V_2^2}{2g} + \frac{V_2^2}{2g} = \frac{3V_2^2}{g} = \frac{3\left(30\frac{ft}{s}\right)^2}{\left(32.2\frac{ft}{s^2}\right)} = 83.9\ ft$$

Hence,

$$\dot{W}_s = \gamma Q h_s = \rho g A_2 V_2 h_s = \left(0.00238\frac{slugs}{ft^3}\right)\left(32.2\frac{ft}{s^2}\right)(10\ ft)(0.5\ ft)\left(30\frac{ft}{s}\right)(83.9\ ft)$$

$$= 964\ \frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{1.75\ hp}}$$

5.124

5.124 If a $\frac{3}{4}$-hp motor is required by a ventilating fan to produce a 24-in. stream of air having a velocity of 40 ft/s as shown in Fig. P5.124, estimate (a) the efficiency of the fan and (b) the thrust of the supporting member on the conduit enclosing the fan.

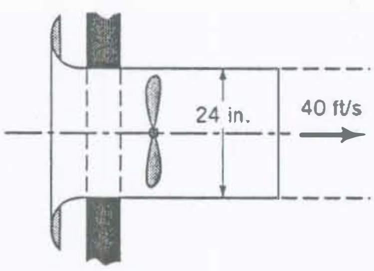


FIGURE P5.124

(a) The solution to this part of the problem is like Example 5.24. We use

$$\eta = \frac{w_{shaft} - loss}{w_{shaft}}$$

to calculate the fan efficiency.

We use the energy equation (Eq. 5.82) for flow through the control volume sketched above to calculate the loss as follows

$$\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\substack{shaft\\net\ in}} - loss$$

But $\quad P_2 = P_1$ and $z_2 = z_1$; $V_1 \simeq 0$; $w_{\substack{shaft\\net\ in}} = \dfrac{hp}{\dot{m}}$

Also $\quad \dot{m} = \rho A_2 V_2 = \dfrac{P}{RT}\dfrac{\pi d_2^2}{4} V_2$

So

$$loss = w_{\substack{shaft\\net\ in}} - \frac{V_2^2}{2} = \frac{hp}{\frac{P}{RT}\left(\frac{\pi d_2^2}{4}\right)V_2} - \frac{V_2^2}{2}$$

$$loss = \frac{\left(\frac{3}{4}\,hp\right)\left(550\,\frac{ft.lb}{s.hp}\right)}{\left\{\dfrac{\left(14.7\,\frac{lb}{in^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)\pi\left[\frac{24\,in.}{\left(12\,\frac{in.}{ft}\right)}\right]^2}{\left(53.3\,\frac{ft.lb}{lbm.^{\circ}R}\right)(530^{\circ}R)\,4}\left(40\,\frac{ft}{s}\right)\right\}} - \frac{\left(40\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{lbm.ft}{lb.s^2}\right)}$$

(con't)

5.124 (con't)

$$\text{loss} = 44\ \frac{ft \cdot lb}{lbm} - 24.8\ \frac{ft \cdot lb}{lbm} = 19.2\ \frac{ft \cdot lb}{lbm}$$

So

$$\eta = \frac{44\ \frac{ft \cdot lb}{lbm} - 19.2\ \frac{ft \cdot lb}{lbm}}{44\ \frac{ft \cdot lb}{lbm}} = \underline{\underline{0.56}}$$

For

(b) We use the horizontal component of the linear momentum equation to evaluate the anchoring force required to hold the fan in place

$$F_{AX} = V_2\,\dot{m}$$

From part (a)

$$\dot{m} = \frac{P}{RT}\frac{\pi d_2^2}{4} V_2 = \frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)\pi\left(\frac{24\ in.}{12\ \frac{in.}{ft}}\right)^2\left(40\ \frac{ft}{s}\right)}{\left(53.3\ \frac{ft \cdot lb}{lbm\ {}^\circ R}\right)(530\,{}^\circ R)\ 4}$$

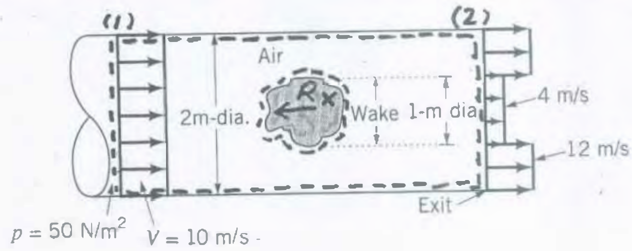
$$\dot{m} = 9.41\ \frac{lbm}{s}$$

So

$$F_{AX} = \frac{\left(40\ \frac{ft}{s}\right)\left(9.41\ \frac{lbm}{s}\right)}{\left(32.2\ \frac{lbm \cdot ft}{lb \cdot s^2}\right)} = \underline{\underline{11.7\ lb}}$$

5.125

5.125 Air flows past an object in a pipe of 2-m diameter and exits as a free jet as shown in Fig. P5.125. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m², respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. (a) Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. (b) Determine the force that the air puts on the object.



■ FIGURE P5.125

(a) To determine the loss suffered by a fluid particle as it flows from (1) to a location in the wake at (2) we apply the energy equation (Eq. 5.84) to that particle flow to get:

$$\cancel{\frac{p_2}{\gamma}}^{\,0} + \frac{V_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{\,0} - h_L \qquad (1)$$

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

and

$$h_L = \frac{\left(50\ \frac{N}{m^2}\right)}{\left(12\ \frac{N}{m^3}\right)} + \frac{\left(10\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)} - \frac{\left(4\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)} = \underline{\underline{8.45\ m}}$$

To determine the head loss associated with the entire flow across the object we use the non-uniform flow energy equation (Eq. 5.89) for flow from (1) to (2) through the control volume shown in the sketch to get:

$$\cancel{\frac{p_2}{\gamma}}^{\,0} + \frac{\alpha_2 \bar{V}_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{\alpha_1 \bar{V}_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{\,0} - h_L \qquad (2)$$

From Eq. 5.86 we get:

$$\frac{\alpha \bar{V}^2}{2g} = \frac{\int \frac{V^2}{2g} \rho \vec{V}\cdot\hat{n}\, dA}{\dot{m}} = \frac{\int \frac{V^2}{2g} \rho \vec{V}\cdot\hat{n}\, dA}{\rho \bar{V} A}$$

Eq. (2) becomes

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{\int_{A_2} \frac{V^2}{2g} \rho \vec{V}\cdot\hat{n}\, dA}{(\rho V A)_{4\frac{m}{s}} + (\rho V A)_{12\frac{m}{s}}}$$

(con't)

5.125 (Con't)

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{1}{2g}\left[\frac{V_{12\frac{m}{s}}^3 A_{12\frac{m}{s}} + V_{3\frac{m}{s}}^3 A_{3\frac{m}{s}}}{V_{4\frac{m}{s}} A_{4\frac{m}{s}} + V_{12\frac{m}{s}} A_{12\frac{m}{s}}}\right]$$

and

$$h_L = \frac{\left(50\,\frac{N}{m^2}\right)}{\left(12\,\frac{N}{m^3}\right)} + \frac{\left(10\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} - \frac{1}{2\left(9.81\,\frac{m}{s^2}\right)}\left\{\frac{\left(12\,\frac{m}{s}\right)^3 \pi\left[\frac{(2m)^2-(1m)^2}{4}\right] + \left(4\,\frac{m}{s}\right)^3 \pi\frac{(1m)^2}{4}}{\left(4\,\frac{m}{s}\right)\pi\frac{(1m)^2}{4} + \left(12\,\frac{m}{s}\right)\pi\left[\frac{(2m)^2-(1m)^2}{4}\right]}\right\}$$

$$h_L = \underline{\underline{2.58\ m}}$$

(b) To determine the force that the air puts on the object, R_x, we use the horizontal component of the linear momentum equation to get:

$$-\rho V_1^2 A_1 + \rho V_{12\frac{m}{s}}^2 A_{12\frac{m}{s}} + \rho V_{4\frac{m}{s}}^2 A_{4\frac{m}{s}} = p_1 A_1 - R_x$$

and thus

$$R_x = p_1 A_1 + \rho V_1^2 A_1 - \rho\left(V_{12\frac{m}{s}}^2 A_{12\frac{m}{s}} + V_{4\frac{m}{s}}^2 A_{4\frac{m}{s}}\right)$$

So

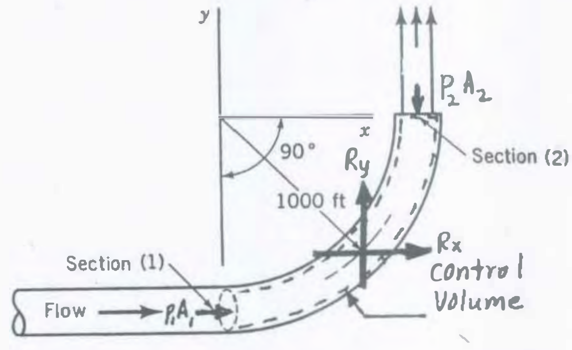
$$R_x = \left(50\,\frac{N}{m^2}\right)\frac{\pi(2m)^2}{4} + \left(1.23\,\frac{kg}{m^3}\right)\left(10\,\frac{m}{s}\right)^2\frac{\pi(2m)^2}{4}\left(1\,\frac{N\cdot s^2}{m\cdot kg}\right)$$
$$-1.23\,\frac{kg}{m^3}\left\{\left(12\,\frac{m}{s}\right)^2\frac{\pi\left[(2m)^2-(1m)^2\right]}{4} + \left(4\,\frac{m}{s}\right)^2\frac{\pi(1m)^2}{4}\right\}\left(1\,\frac{N\cdot s^2}{m\cdot kg}\right)$$

and

$$R_x = \underline{\underline{110\ N}}$$

5.126

5.126 Water flows through a 2-ft-diameter pipe arranged horizontally in a circular arc as shown in Fig. P5.126. If the pipe discharges to the atmosphere ($p = 14.7$ psia) determine the x and y components of the resultant force exerted by the water on the piping between sections (1) and (2). The steady flowrate is 3000 ft^3/min. The loss in pressure due to fluid friction between sections (1) and (2) is 60 psi.



■ FIGURE P5.126

To determine the x and y components of the resultant force exerted by the water on the piping between section (1) and (2) we use the x and y components of the linear momentum equation (Eq 5.22). For the control volume containing the water in the pipe between section (1) and (2), Eq. 22 leads to

$$R_x = -P_1A_1 - V_1\rho Q = -P_1\frac{\pi D_1^2}{4} - V_1\rho Q \qquad (1)$$

and

$$R_y = P_2A_2 + V_2\rho Q \qquad (2)$$

The resultant force components in Eqs. 1 and 2 are exerted by the pipe on the water. The resultant force of water on pipe is equal in magnitude but opposite in direction.

To determine P_1 we use the energy equation, Eq. 5.83. Thus,

$$P_1 = p(\text{loss}) = 60\,psi = 74.7\,psia \quad (\text{we need to use absolute pressures})$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{\left(3000\,\frac{ft^3}{min}\right)}{\frac{\pi(2ft)^2}{4}\left(60\,\frac{s}{min}\right)} = 15.92\,\frac{ft}{s}$$

and

$$V_2 = V_1 = 15.92\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$R_x = -(74.7\,psia)\,\pi\frac{(2ft)^2}{4}\left(144\,\frac{in.^2}{ft^2}\right) - \frac{\left(15.92\,\frac{ft}{s}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(3000\,\frac{ft^3}{min}\right)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(60\,\frac{s}{min}\right)}$$

or

$$R_x = -32{,}200\,lb$$

and the x direction component of the force exerted by the water on the pipe between sections (1) and (2) is $+\underline{\underline{32{,}200\,lb}}$.

(con't)

5.126 (con't)

With Eq. 2 we obtain

$$R_y = (14.7\,psia)\frac{\pi(2\,ft)^2}{4}\left(144\,\frac{in^2}{ft^2}\right) + \left(15.92\,\frac{ft}{s}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(3000\,\frac{ft^3}{min}\right)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)\frac{1}{\left(60\,\frac{s}{min}\right)} = 8190\,lb$$

and the y-direction component of the force exerted by the water on the pipe between sections (1) and (2) is $-$ 8190 lb.

5.127

5.127 Water flows steadily down the inclined pipe as indicated in Fig. P5.127. Determine the following: **(a)** The difference in pressure $p_1 - p_2$. **(b)** The loss per unit mass between sections (1) and (2). **(c)** The net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).

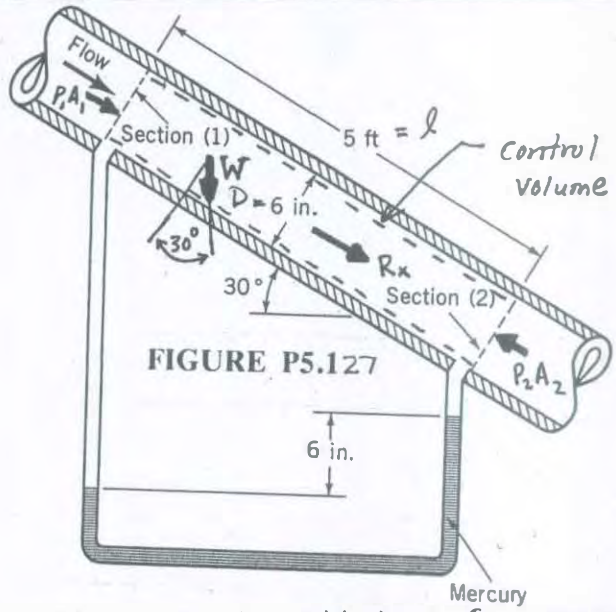


FIGURE P5.127

(a) The difference in pressure, $P_1 - P_2$, may be obtained from the manometer (see Section 2.6) with the fluid statics equation

$$P_1 - P_2 = -\gamma_{H_2O}\left[(5\,ft)\sin 30^\circ + \frac{(6\ in.)}{\left(12\frac{in.}{ft}\right)}\right] + \gamma_{Hg}\frac{(6\ in.)}{\left(12\frac{in.}{ft}\right)}$$

or

$$P_1 - P_2 = -\left(62.4\frac{lb}{ft^3}\right)\left[(5\,ft)\sin 30^\circ + (0.5\,ft)\right] + (13.6)\left(62.4\frac{lb}{ft^3}\right)(0.5\,ft) = 237\frac{lb}{ft^3}$$

and

$$P_1 - P_2 = 237\frac{lb}{ft^3}\frac{1}{\left(144\frac{in.^2}{ft^2}\right)} = \underline{\underline{1.65\ psi}}$$

(b) The loss per unit mass between sections (1) and (2) may be obtained with Eq. 5.79. Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = \left(237\frac{lb}{ft^2}\right)\frac{1}{\left(1.94\frac{slugs}{ft^3}\right)} + \left(32.2\frac{ft}{s^2}\right)(5\,ft)(\sin 30^\circ)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$loss = \underline{\underline{203\ \frac{ft\cdot lb}{slug}}}$$

(c) The net axial force exerted by the pipe wall on the flowing water may be obtained by using the axial component of the linear momentum equation (Eq. 5.22). Thus for the control volume shown above

$$R_x = -\frac{\pi D^2}{4}(P_1 - P_2) - \gamma\frac{\pi D^2}{4}(\ell)\sin 30^\circ = -\frac{\pi D^2}{4}\left[(P_1 - P_2) + \gamma\ell\sin 30^\circ\right]$$

or

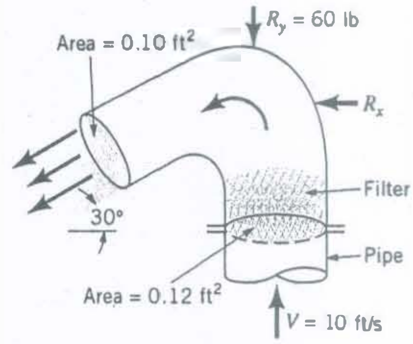
$$R_x = -\frac{\pi}{4}\left(\frac{6\ in.}{12\frac{in.}{ft}}\right)^2\left[237\frac{lb}{ft^2} + \left(62.4\frac{lb}{ft^3}\right)(5\,ft)\sin 30^\circ\right]$$

and

$$R_x = -77.2\ lb = \underline{\underline{77.2\ lb\ \text{opposite to flow direction.}}}$$

5.128

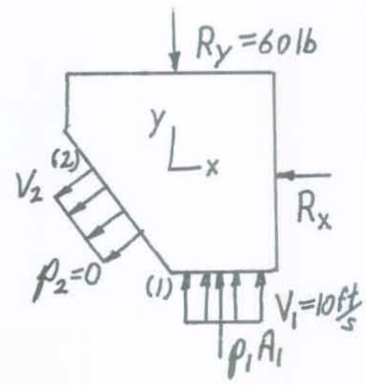
5.128 Water flows steadily in a pipe and exits as a free jet through an end cap that contains a filter as shown in Fig. P5.128. The flow is in a horizontal plane. The axial component, R_y, of the anchoring force needed to keep the end cap stationary is 60 lb. Determine the head loss for the flow through the end cap.



■ FIGURE P5.128

The y-component of the momentum equation,

$\int_{cs} v\, \rho \vec{V}\cdot\hat{n}\, dA = \sum F_y$, for the control volume shown is



(1) $V_1\rho(-V_1)A_1 + (-V_2\sin 30°)\rho V_2 A_2 = p_1 A_1 - R_y$

where $V_1 = 10\text{ ft/s}$ and

$$V_2 = \frac{A_1}{A_2}V_1 = \left(\frac{0.12\text{ ft}^2}{0.10\text{ ft}^2}\right)(10\text{ ft/s}) = 12\text{ ft/s}$$

Thus, since $\rho A_1 V_1 = \rho A_2 V_2$, Eq. (1) gives

$$p_1 A_1 = R_y - \rho V_1^2 A_1 - \rho V_2^2 \sin 30° A_2 = R_y - \rho A_1 V_1[V_1 + V_2\sin 30°]$$

$$= 60\,lb - \left(1.94\frac{slugs}{ft^3}\right)(0.12\,ft^2)\left(10\frac{ft}{s}\right)\left[10\frac{ft}{s} + 12\frac{ft}{s}\sin 30°\right] = 22.8\,lb$$

Hence,

$$p_1 = 22.8\,lb/A_1 = 22.8\,lb/(0.12\,ft^2) = 190\,lb/ft^2$$

From the energy equation for this flow,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} - h_L = \frac{V_2^2}{2g}, \text{ or}$$

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2 - V_2^2}{2g} = \frac{190\,lb/ft^2}{62.4\,lb/ft^3} + \frac{(10\,ft/s)^2 - (12\,ft/s)^2}{2\,(32.2\,ft/s^2)} = \underline{\underline{2.36\,ft}}$$

5.129

5.129 When fluid flows through an abrupt expansion as indicated in Fig. P5.1 , the loss in available energy across the expansion, loss_{ex}, is often expressed as

$$\text{loss}_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

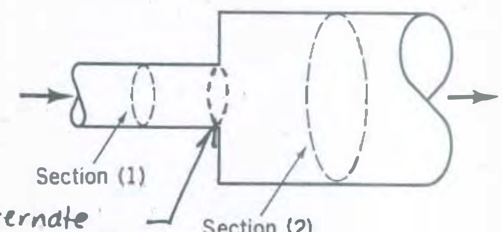


FIGURE P5.129

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$loss_{ex} = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + P_1 A_1 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + P_1 A_2 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, P_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \qquad (4)$$

Combining Eqs. 1 and 4 we obtain

$$loss_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

(con't)

5.129 (con't)

From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

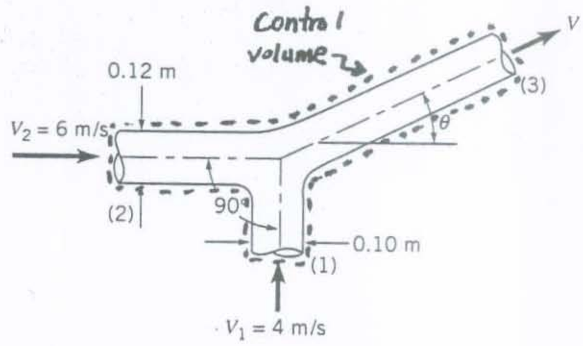
$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$

5.130

5.130 Two water jets collide and form one homogeneous jet as shown in Fig. P5.130. (a) Determine the speed, V, and direction, θ, of the combined jet. (b) Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



■ FIGURE P5.130

For the water flowing through the control volume sketched above, the x- and y-direction components of the linear momentum equation are

$$-V_2 \rho V_2 A_2 + V_3 \cos\theta \, \rho V_3 A_3 = 0 \qquad (1)$$

and

$$-V_1 \rho V_1 A_1 + V_3 \sin\theta \, \rho V_3 A_3 = 0 \qquad (2)$$

From the conservation of mass principle we get

$$-\rho V_1 A_1 - \rho V_2 A_2 + \rho V_3 A_3 = 0 \qquad (3)$$

Combining Eqs. 1 and 2 we obtain

$$\tan\theta = \frac{V_1^2 A_1}{V_2^2 A_2} = \frac{V_1 \frac{\pi d_1^2}{4}}{V_2 \frac{\pi d_2^2}{4}} = \frac{\left(4 \frac{m}{s}\right)^2 \frac{\pi (0.1m)^2}{4}}{\left(6 \frac{m}{s}\right)^2 \frac{\pi (0.12m)^2}{4}} = 0.3086$$

So

$$\theta = \tan^{-1} 0.3086 = \underline{\underline{17.2^\circ}}$$

Now, combining Eqs. 1 and 3 we get

$$-V_2^2 \rho A_2 + V_3 \cos\theta \left(\rho V_1 A_1 + \rho V_2 A_2\right) = 0$$

or

$$V_3 = \frac{V_2^2 A_2}{\cos\theta \left(V_1 A_1 + V_2 A_2\right)} = \frac{V_2^2 d_2^2}{\cos\theta \left(V_1 d_1^2 + V_2 d_2^2\right)}$$

Thus

$$V_3 = \frac{\left(6 \frac{m}{s}\right)^2 (0.12m)^2}{(\cos 17.2^\circ)\left[\left(4 \frac{m}{s}\right)(0.1m)^2 + \left(6 \frac{m}{s}\right)(0.12m)^2\right]}$$

and

$$V_3 = \underline{\underline{4.29 \frac{m}{s}}}$$

(con't)

5.130 (con't)

To determine the loss of available energy associated with the flow through this control volume we obtain by applying the energy equation (Eq. 5.64)

$$-\left(\check{u}_1 + \frac{V_1^2}{2}\right)\dot{m}_1 - \left(\check{u}_2 + \frac{V_2^2}{2}\right)\dot{m}_2 + \left(\check{u}_3 + \frac{V_3^2}{2}\right)\dot{m}_3 = 0 \qquad (4)$$

Also, the conservation of mass equation, Eq. 3, can also be written as

$$-\dot{m}_1 - \dot{m}_2 + \dot{m}_3 = 0 \qquad (5)$$

Combining Eqs. 4 and 5, we obtain

$$\dot{m}_1(\check{u}_3 - \check{u}_1) + \dot{m}_2(\check{u}_3 - \check{u}_2) = \dot{m}_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \dot{m}_2\left(\frac{V_2^2 - V_3^2}{2}\right) \qquad (6)$$

The left hand side of Eq. 6 represents the rate of available energy loss in this fluid flow. Thus rate of available energy loss is

$$\text{rate of loss} = \rho V_1 A_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \rho V_2 A_2\left(\frac{V_2^2 - V_3^2}{2}\right)$$

or

$$\text{rate of loss} = \frac{\rho\pi}{4}\left[d_1^2 V_1\left(\frac{V_1^2 - V_3^2}{2}\right) + d_2^2 V_2\left(\frac{V_2^2 - V_3^2}{2}\right)\right]$$

Thus

$$\text{rate of loss} = \frac{\left(999\,\frac{kg}{m^3}\right)(3.14)\left(1\,\frac{N\cdot s^2}{kg\cdot m}\right)}{4}\left\{(0.10\,m)^2\left(4\,\frac{m}{s}\right)\left[\frac{\left(4\,\frac{m}{s}\right)^2 - \left(4.29\,\frac{m}{s}\right)^2}{2}\right] + (0.12\,m)^2\left(6\,\frac{m}{s}\right)\left[\frac{\left(6\,\frac{m}{s}\right)^2 - \left(4.29\,\frac{m}{s}\right)^2}{2}\right]\right\}$$

and

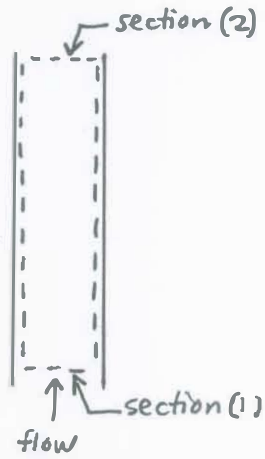
$$\text{rate of loss} = \underline{\underline{558\,\frac{N\cdot m}{s}}}$$

5.131

5.131 Water flows vertically upward in a circular cross section pipe. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R - r}{R} \right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe inside radius, and, r = radius from pipe axis. Develop an expression for the loss in available energy between sections (1) and (2).



For determining loss we use the energy equation for non-uniform flows, Eq. 5.87. Thus,

$$loss = \frac{p_1 - p_2}{\rho} + \frac{\alpha_1 \bar{V}_1^2 - \alpha_2 \bar{V}_2^2}{2} + g(z_1 - z_2) \qquad (1)$$

From conservation of mass (Eq. 5.13) we have

$$\bar{V}_1 = \bar{V}_2$$

Also, with Eq. 5.86 for the kinetic energy coefficient, α, we have

$$\alpha_1 = 1.0$$

since the velocity profile at section (1) is uniform. At section (2) we solve Eq. 5.86 (see solution for problem 5.125 (c)) and obtain

$$\alpha_2 = 1.06$$

Thus, Eq. 1 yields

$$\underline{\underline{loss = \frac{p_1 - p_2}{\rho} - 0.06 \frac{\bar{V}_1^2}{2} + g(z_1 - z_2)}}$$

5.132

5.132 The velocity profile in a turbulent pipe flow may be approximated with the expression

$$\frac{u}{u_c} = \left(\frac{R-r}{R}\right)^{1/n}$$

where u = local velocity in the axial direction, u_c = centerline velocity in the axial direction, R = pipe inner radius from pipe axis, r = local radius from pipe axis, and n = constant. Determine the kinetic energy coefficient, α, for: **(a)** $n = 5$; **(b)** $n = 6$; **(c)** $n = 7$; **(d)** $n = 8$; **(e)** $n = 9$; **(f)** $n = 10$.

For the kinetic energy coefficient, α, we may use Eq. 5.86. Thus,

$$\alpha = \frac{\int_0^R \frac{u^2}{2}\rho u\, 2\pi r\, dr}{\rho \bar{u}\, \pi R^2 \frac{\bar{u}^2}{2}} = \frac{2\int_0^1 u^3\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} = \frac{2u_c^3 \int_0^1 \left(1-\frac{r}{R}\right)^{\frac{3}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} \qquad (1)$$

For the average velocity, $\bar{u}$, we may use Eq. 5.7. Thus,

$$\bar{u} = \frac{\int_0^R \rho u\, 2\pi r\, dr}{\rho \pi R^2} = 2\int_0^1 u\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2u_c \int_0^1 \left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \qquad (2)$$

To facilitate the integrations we make the substitution

$$\beta = 1 - \frac{r}{R} \qquad (3)$$

Thus,

$$d\beta = -d\left(\frac{r}{R}\right) \qquad (4)$$

and Eq. 2 becomes

$$\bar{u} = -2u_c \int_1^0 \beta^{\frac{1}{n}}(1-\beta)\, d\beta = \frac{2n^2}{(n+1)(2n+1)}\, u_c \qquad (5)$$

Combining Eqs. 1, 3, 4 and 5 we obtain

$$\alpha = \frac{-2\int_1^0 \beta^{\frac{3}{n}}(1-\beta)\, d\beta}{\left[\frac{2n^2}{(n+1)(2n+1)}\right]^3} = \left[\frac{2n^2}{(3+n)(3+2n)}\right]\left[\frac{(n+1)(2n+1)}{2n^2}\right]^3 \qquad (6)$$

(a) For $n = 5$, Eq. 6 yields

$$\alpha = \left\{\frac{2(5)^2}{(3+5)[3+2(5)]}\right\}\left\{\frac{(5+1)[(2)(5)+1]}{2(5)^2}\right\}^3 = \underline{\underline{1.11}}$$

(b) For $n = 6$

$\alpha = \underline{\underline{1.08}}$

(c) For $n = 7$

$\alpha = \underline{\underline{1.06}}$

(d) For $n = 8$

$\alpha = \underline{\underline{1.05}}$

(e) For $n = 9$

$\alpha = \underline{\underline{1.04}}$

(f) For $n = 10$

$\alpha = \underline{\underline{1.03}}$

Note: Look at Figs. 8.17 and 8.18 for important information about these different velocity profiles.

5.133

5.133 A small fan moves air at a mass flowrate of 0.004 lbm/s. Upstream of the fan, the pipe diameter is 2.5 in., the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1, is equal to 2.0. Downstream of the fan, the pipe diameter is 1 in., the flow is turbulent, the velocity profile is quite flat, and the kinetic energy coefficient, α_2, is equal to 1.08. If the rise in static pressure across the fan is 0.015 psi and the fan shaft draws 0.00024 hp, compare the value of loss calculated: **(a)** assuming uniform velocity distributions; **(b)** considering actual velocity distributions.

(a) For uniform velocity distributions upstream and downstream of the fan, Eq. 5.82 is applicable. Thus,

$$loss = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g(\overbrace{z_{in} - z_{out}}^{0 \text{ for air}}) + w_{\substack{shaft\\ net\ in}} \qquad (1)$$

We obtain the shaft work, $w_{\substack{shaft\\ net\ in}}$ from the given shaft power, $\dot{W}_{\substack{shaft\\ net\ in}}$, with

$$w_{\substack{shaft\\ net\ in}} = \frac{\dot{W}_{\substack{shaft\\ net\ in}}}{\dot{m}} = \frac{(0.00024\ hp)\left(550\ \frac{ft \cdot lb}{s \cdot hp}\right)}{0.004\ \frac{lbm}{s}} = 33\ \frac{ft \cdot lb}{lbm}$$

For V_{in} and V_{out} we use Eq. 5.11. Thus,

$$V_{in} = \frac{\dot{m}}{\rho A_{in}} = \frac{\dot{m}}{\rho \frac{\pi D_{in}^2}{4}} = \frac{\left(0.004\ \frac{lbm}{s}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)\frac{\pi (2.5\ in.)^2}{4}} = 1.53\ \frac{ft}{s}$$

and

$$V_{out} = \frac{\dot{m}}{\rho A_{out}} = \frac{\dot{m}}{\rho \frac{\pi D_{out}^2}{4}} = \frac{\left(0.004\ \frac{lbm}{s}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)\frac{\pi (1\ in.)^2}{4}} = 9.57\ \frac{ft}{s}$$

Now from Eq. 1 we obtain

$$loss = \frac{(-0.015\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)} + \left[\frac{\left(1.53\ \frac{ft}{s}\right)^2 - \left(9.57\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} + 33\ \frac{ft \cdot lb}{lbm}$$

or

$$loss = \underline{\underline{3.43\ \frac{ft \cdot lb}{lbm}}}$$

(b) For non-uniform velocity distributions upstream and downstream of the fan Eq. 5.87 is applicable. Thus

$$loss = \frac{p_{in} - p_{out}}{\rho} + \frac{\alpha_{in} \bar{V}_{in}^2 - \alpha_{out} \bar{V}_{out}^2}{2} + g(\overbrace{z_{in} - z_{out}}^{0 \text{ for air}}) + w_{\substack{shaft\\ net\ in}}$$

or

$$loss = -28.18\ \frac{ft \cdot lb}{lbm} + \left[\frac{(2)(1.53\ \ \ )^2}{2} - \frac{(1.08)\left(9.57\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} + 33\ \frac{ft \cdot lb}{lbm}$$

and

$$loss = \underline{\underline{3.36\ \frac{ft \cdot lb}{lbm}}}$$

5.134

5.134 Air enters a radial blower with zero angular momentum. It leaves with an absolute tangential velocity, V_θ, of 200 ft/s. The rotor blade speed at rotor exit is 170 ft/s. If the stagnation pressure rise across the rotor is 0.4 psi, calculate the loss of available energy across the rotor and the rotor efficiency.

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g(\overset{0,\ \text{neglect}}{\cancel{z_{in} - z_{out}}}) + w_{\substack{shaft\\ net\ in}}$$

or

$$\text{loss} = \frac{p_{0,in} - p_{0,out}}{\rho} + w_{\substack{shaft\\ net\ in}} \qquad (1)$$

The shaft work in, $w_{\substack{shaft\\ net\ in}}$ can be obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft\\ net\ in}} = U_{out} V_{\theta\,out} \qquad (2)$$

Combining Eqs. 1 and 2 leads to

$$\text{loss} = \frac{p_{0,in} - p_{0,out}}{\rho} + U_{out} V_{\theta\,out}$$

or

$$\text{loss} = -\frac{(0.4\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)} + \left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

and

$$\text{loss} = \underline{\underline{9800}}\ \frac{ft\cdot lb}{slug} = 9800\ \frac{ft\cdot lb}{slug}\ \frac{1}{(32.174\ lbm/slug)} = \underline{\underline{305}}\ \frac{ft\cdot lb}{lbm}$$

As was done in Example 5.24, we calculate rotor efficiency from

$$\text{rotor efficiency} = \frac{w_{\substack{shaft\\ net\ in}} - \text{loss}}{w_{\substack{shaft\\ net\ in}}} = \frac{U_{out}V_{\theta\,out} - \text{loss}}{U_{out}V_{\theta\,0}}$$

$$\text{rotor efficiency} = \frac{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) - 9800\ \frac{ft\cdot lb}{slug}}{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)} = \underline{\underline{0.71}}$$

5.135

5.135 Water enters a pump impeller radially. It leaves the impeller with a tangential component of absolute velocity of 10 m/s. The impeller exit diameter is 60 mm and the impeller speed is 1800 rpm. If the stagnation pressure rise across the impeller is 45 kPa, determine the loss of available energy across the impeller and the hydraulic efficiency of the pump.

The analysis of Example 5.27 is applicable to solving this problem. Using Eq. 6 of Example 5.27 we obtain

$$\text{loss} = U_2 V_{\theta 2} - \frac{\text{actual total pressure rise across impeller}}{\rho}$$

However,

$$U_2 = r_2 \omega = \frac{(60\ \text{mm})\left(1800\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)}{(2)\left(1000\ \frac{\text{mm}}{\text{m}}\right)\left(60\ \frac{\text{s}}{\text{min}}\right)} = 5.66\ \frac{\text{m}}{\text{s}}$$

Thus

$$\text{loss} = \left(5.66\ \frac{\text{m}}{\text{s}}\right)\left(10\ \frac{\text{m}}{\text{s}}\right)\left(\frac{1\ \text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right) - \left(45\times 10^3\ \frac{\text{N}}{\text{m}^2}\right)\frac{1}{\left(999\ \frac{\text{kg}}{\text{m}^3}\right)}$$

$$\text{loss} = \underline{\underline{11.6}}\ \frac{\text{N}\cdot\text{m}}{\text{kg}}$$

From Eq. 5 of Example 5.27 we obtain

$$\eta = \frac{\dfrac{\text{actual total pressure rise across impeller}}{\rho}}{U_2 V_{\theta 2}}$$

or

$$\eta = \frac{\left[\dfrac{\left(45\times 10^3\ \frac{\text{N}}{\text{m}^2}\right)}{\left(999\ \frac{\text{kg}}{\text{m}^3}\right)}\right]}{\left(5.66\ \frac{\text{m}}{\text{s}}\right)\left(10\ \frac{\text{m}}{\text{s}}\right)\left(\frac{1\ \text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)} = \underline{\underline{0.796}}$$

5.136

5.136 Water enters an axial-flow turbine rotor with an absolute velocity tangential component, V_θ, of 15 ft/s. The corresponding blade velocity, U, is 50 ft/s. The water leaves the rotor blade row with no angular momentum. If the stagnation pressure drop across the turbine is 12 psi, determine the hydraulic efficiency of the turbine.

To determine the efficiency of the turbine we use

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} \qquad (1)$$

The actual work out, $w_{\substack{shaft \\ net\ out}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft \\ net\ out}} = -w_{\substack{shaft \\ net\ in}} = U_{in} V_{\theta\, in} \qquad (2)$$

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + \overset{\text{neglect}}{\cancel{g(z_{in} - z_{out})}} + w_{\substack{shaft \\ net\ in}} \qquad (3)$$

Combining Eqs. 2 and 3 we obtain

$$\text{loss} = \frac{p_{0,in} - p_{0,out}}{\rho} - U_{in} V_{\theta\, in} \qquad (4)$$

Combining Eqs. 1, 2 and 4 we obtain

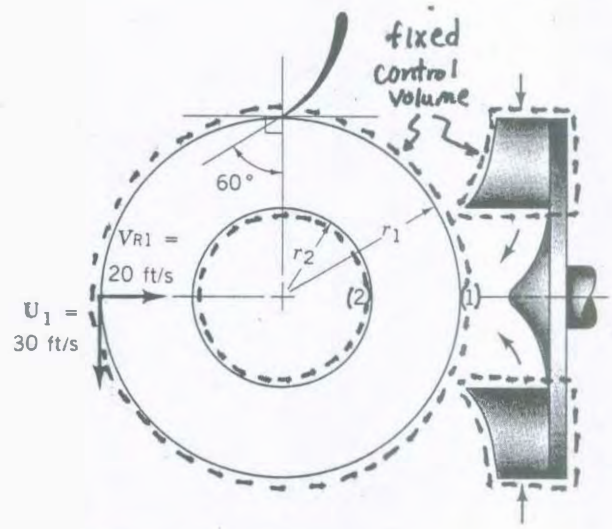
$$\eta = \frac{U_{in} V_{\theta\, in}}{U_{in} V_{\theta\, in} + \text{loss}} = \frac{U_{in} V_{\theta\, in}}{\dfrac{p_{0,in} - p_{0,out}}{\rho}} = \frac{\left(50\ \frac{ft}{s}\right)\left(15\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)}{\dfrac{(12\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}}$$

and

$$\eta = \underline{\underline{0.842}}$$

5.137

5.137 An inward flow radial turbine (see Fig. P5.137) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is water and the stagnation pressure drop across the rotor is 16 psi, determine the loss of available energy across the rotor and the hydraulic efficiency involved.



■ FIGURE P5.137

An analysis like the one of Example 5.28 would be appropriate for solving this problem. Since a turbine is involved in this problem, $w_{\substack{shaft\\ net\ in}} = -w_{\substack{shaft\\ net\ out}}$ and from Eq. 1 of Example 5.28 we can conclude that

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - w_{\substack{shaft\\ net\ out}}$$

However from Eq. 5.54 we see that

$$w_{shaft} = w_{\substack{shaft\\ net\ in}} = -U_1 V_{\theta 1} = -w_{\substack{shaft\\ net\ out}}$$

and thus

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - U_1 V_{\theta 1} \qquad (1)$$

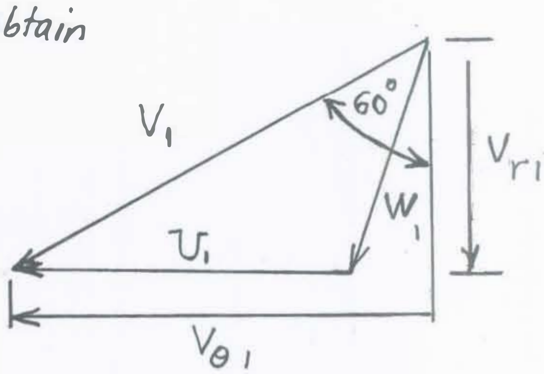
To determine the value of $V_{\theta 1}$ we examine the velocity triangle for the flow entering the rotor that is sketched below.

From the velocity triangle we obtain

$$V_{\theta 1} = V_{r1} \tan 60°$$

or

$$V_{\theta 1} = \left(20 \frac{ft}{s}\right) \tan 60° = 34.64 \frac{ft}{s}$$



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5.137 (con't)

From Eq. 1 we obtain

$$loss = \frac{\left(16 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)} - \left(30 \frac{ft}{s}\right)\left(34.64 \frac{ft}{s}\right)\left(\frac{1 \ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$loss = \underline{\underline{148}} \ \frac{ft \cdot lb}{slug}$$

From Eq. 5.82, we can conclude that

$$w_{\substack{shaft \\ net\ out}} + loss = \frac{\text{stagnation pressure drop across the rotor}}{\rho}$$

or in other words, the stagnation pressure drop across the rotor results in shaft work and loss of available energy.

Thus a meaningful efficiency is

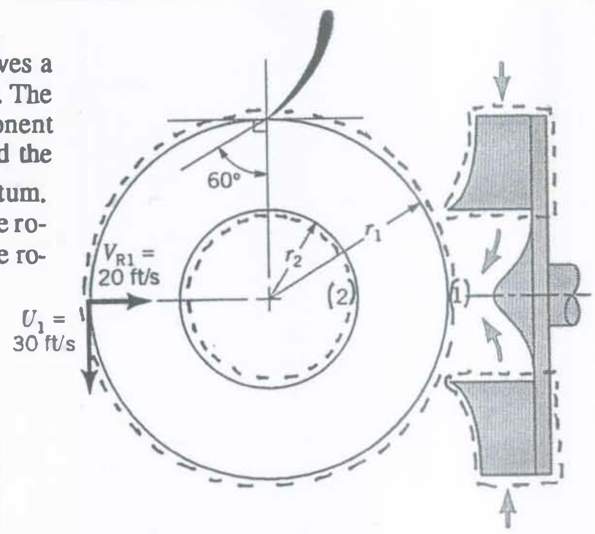
$$\eta = \frac{w_{\substack{shaft \\ net\ out}}}{\left(\frac{\text{stagnation pressure drop across the rotor}}{\rho}\right)}$$

or

$$\eta = \frac{\left(30 \frac{ft}{s}\right)\left(34.64 \frac{ft}{s}\right)\left(\frac{1 \ lb}{slug \cdot \frac{ft}{s^2}}\right)}{\frac{\left(16 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)}} = \underline{\underline{0.875}}$$

5.138

5.138 An inward flow radial turbine (see Fig. P5.137) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is air and the static pressure drop across the rotor is 0.01 psi, determine the loss of available energy across the rotor and the rotor aerodynamic efficiency.



■ FIGURE P5.137

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82). Thus,

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(\overbrace{z_1 - z_2}^{\text{neglect}}) + w_{\substack{shaft \\ net\ in}} \qquad (1)$$

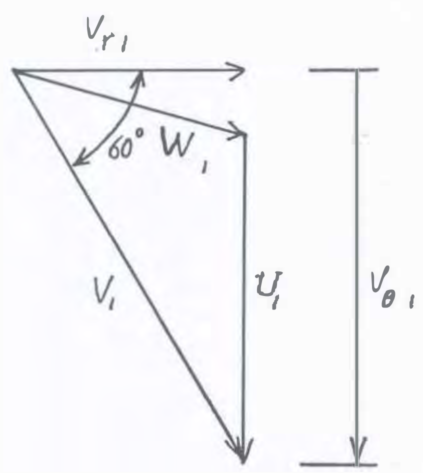
The shaft work, $w_{\substack{shaft \\ net\ in}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft \\ net\ in}} = -U_1 V_{\theta 1} = -w_{\substack{shaft \\ net\ out}} \qquad (2)$$

and combining Eqs. 1 and 2 yields

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} - U_1 V_{\theta 1} \qquad (3)$$

To determine V_1 and $V_{\theta 1}$ we construct the velocity triangle sketched below.



(con't)

5.138 (con't)

With the velocity triangle we conclude that

$$V_1 = \frac{(20\ \frac{ft}{s})}{\cos 60°} = 40\ \frac{ft}{s}$$

and

$$V_{\theta 1} = V_1 \sin 60° = \left(40\ \frac{ft}{s}\right)\sin 60° = 34.64\ \frac{ft}{s}$$

Since the flow leaving the rotor is radial, then

$$V_2 = V_{r2} = 20\ \frac{ft}{s}$$

From Eq. 3 we obtain

$$loss = \frac{\left(0.01\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)} + \frac{\left[\left(40\ \frac{ft}{s}\right)^2 - \left(20\ \frac{ft}{s}\right)^2\right]\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{2} - \left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$loss = \underline{\underline{166}}\ \frac{ft\cdot lb}{slug} = \left(166\ \frac{ft\cdot lb}{slug}\right)\frac{1}{\left(32.174\ \frac{lbm}{slug}\right)} = \underline{\underline{5.16}}\ \frac{ft\cdot lb}{lbm}$$

The efficiency may be obtained with

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} = \frac{U_1 V_{\theta 1}}{U_1 V_{\theta 1} + loss}$$

or

$$\eta = \frac{\left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right) + 166\ \frac{ft\cdot lb}{slug}} = \underline{\underline{0.86}}$$

5.140

5.140 Force from a Jet of Air Deflected by a Flat Plate

Objective: A jet of a fluid striking a flat plate as shown in Fig. P5.126 exerts a force on the plate. It is the equal and opposite force of the plate on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the plate with the experimentally measured force.

Equipment: Air source with an adjustable flowrate and a flow meter; nozzle to produce a uniform air jet; balance beam with an attached flat plate; weights; barometer; thermometer.

Experimental Procedure: Adjust the counter weight so that the beam is level when there is no mass, m, on the beam and no flow through the nozzle. Measure the diameter, d, of the nozzle outlet. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Place a known mass, m, on the flat plate and adjust the fan speed control to produce the necessary flowrate, Q, to make the balance beam level again. The flowrate is related to the flow meter manometer reading, h, by the equation $Q = 0.358\, h^{1/2}$, where Q is in ft^3/s and h is in inches of water. Repeat the measurements for various masses on the plate.

Calculations: For each flowrate, Q, calculate the weight, $W = mg$, needed to balance the beam and use the continuity equation, $Q = VA$, to determine the velocity, V, at the nozzle exit. Use the momentum equation for this problem, $W = \rho V^2 A$, to determine the theoretical relationship between velocity and weight.

Graph: Plot the experimentally measured force on the plate, W, as ordinates and air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

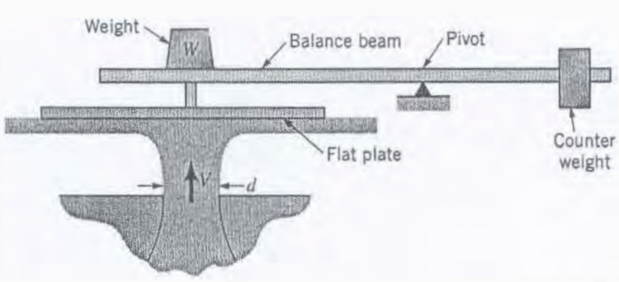


FIGURE P5.140

(con't)

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

5.140 (con't)

Solution for Problem 5.140: Force from a Jet of Air Deflected by a Flat Plate

d, in.	H_{atm}, in. Hg	T, deg F	Q = 0.358 h^0.5, with Q in cfs and h in inches of water
1.174	29.25	70	

		Experimental				Theoretical
m, kg	h, in.	Q, ft^3/s	V, ft/s	m, slug	W, lb	W, lb
0.010	0.54	0.263	35.0	0.00069	0.022	0.021
0.020	1.08	0.372	49.5	0.00137	0.044	0.042
0.030	1.52	0.441	58.7	0.00206	0.066	0.059
0.040	2.18	0.529	70.3	0.00274	0.088	0.084
0.050	2.72	0.590	78.5	0.00343	0.110	0.105
0.060	3.25	0.645	85.8	0.00411	0.132	0.126
0.070	3.81	0.699	92.9	0.00480	0.154	0.147
0.080	4.32	0.744	98.9	0.00548	0.177	0.167
0.090	4.92	0.794	105.6	0.00617	0.199	0.190
0.100	5.46	0.837	111.2	0.00685	0.221	0.211
0.150	8.13	1.021	135.7	0.01028	0.331	0.315
0.200	10.85	1.179	156.8	0.01370	0.441	0.420
0.250	13.72	1.326	176.3	0.01713	0.552	0.531

Experimental:

V = Q/A where

$A = \pi d^2/4 = \pi$*(1.174/12 ft)^2/4 = 7.52E-3 ft^2

W = mg

Theoretical:

$W = \rho V^2 A$ where

$\rho = p_{atm}/RT$ with

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2

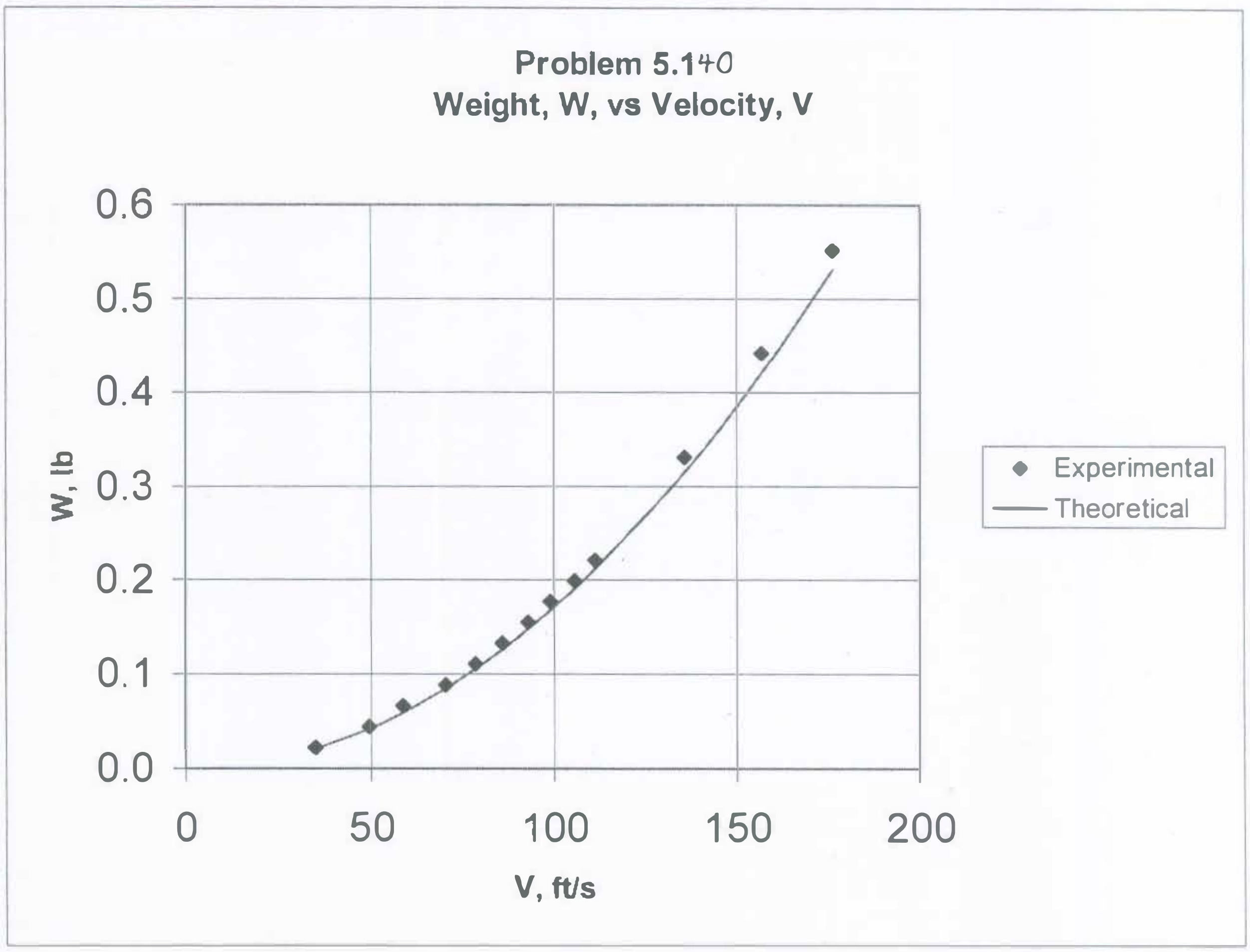
R = 1716 ft lb/slug deg R

T = 70 + 460 = 530 deg R

Thus, ρ = 0.00227 slug/ft^3

(con't)

5.140 (con't)



Problem 5.140
Weight, W, vs Velocity, V
0.6
0.5
0.4
0.3
0.2
0.1
0.0
W, lb
0
50
100
150
200
V, ft/s
Experimental
Theoretical

5.141

5.141 Pressure Distribution on a Flat Plate Due to the Deflection of an Air Jet

Objective: In order to deflect a jet of air as shown in Fig. P5.127, the flat plate must push against the air with a sufficient force to change the momentum of the air. This causes an increase in pressure on the plate. The purpose of this experiment is to measure the pressure distribution on the plate and to compare the resultant pressure force to that needed, according to the momentum equation, to deflect the air.

Equipment: Air supply with a flow meter; nozzle to produce a uniform jet of air; circular flat plate with static pressure taps at various radial locations; manometer; barometer; thermometer.

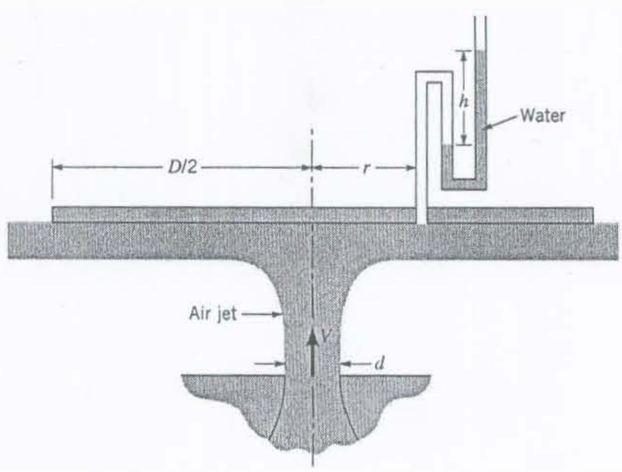
Experimental Procedure: Measure the diameters of the plate, D, and the nozzle exit, d, and the radial locations, r, of the various static pressure taps on the plate. Carefully center the plate over the nozzle exit and adjust the air flowrate, Q, to the desired constant value. Record the static pressure tap manometer readings, h, at various radial locations, r, from the center of the plate. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings, h, to determine the pressure on the plate as a function of location, r. That is, calculate $p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid.

Graph: Plot pressure, p, as ordinates and radial location, r, as abscissas.

Results: Use the experimentally determined pressure distribution to determine the net pressure force, F, that the air jet puts on the plate. That is, numerically or graphically integrate the pressure data to obtain a value for $F = \int p\, dA = \int p\,(2\pi r\, dr)$, where the limits of the integration are over the entire plate, from $r = 0$ to $r = D/2$. Compare this force obtained from the pressure measurements to that obtained from the momentum equation for this flow, $F = \rho V^2 A$, where V and A are the velocity and area of the jet, respectively.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.141

(con't)

5.141 (con't)

Solution for Problem 5.141: Pressure Distribution on a Flat Plate due to the Deflection of an Air Jet

D, in.	d, in.	H_{atm}, in. Hg	T, deg F	Q. ft^3/s
8.0	1.174	29.25	77	1.41

r, in.	h, in.	p, lb/ft^2	p, lb/in.^2	p*r, lb/in.	i	pr_i+pr_{i+1}	$r_{i+1} - r_i$
0.00	6.62	34.42	0.2391	0.0000	1	0.0834	0.39
0.39	5.92	30.78	0.2138	0.0834	2	0.1701	0.40
0.79	3.04	15.81	0.1098	0.0867	3	0.1114	0.45
1.24	0.55	2.86	0.0199	0.0246	4	0.0355	0.35
1.59	0.19	0.99	0.0069	0.0109	5	0.0205	0.45
2.04	0.13	0.68	0.0047	0.0096	6	0.0174	0.37
2.41	0.09	0.47	0.0033	0.0078	7	0.0130	0.44
2.85	0.05	0.26	0.0018	0.0051	8	0.0086	0.38
3.23	0.03	0.16	0.0011	0.0035	9	0.0035	0.44
3.67	0.00	0.00	0.0000	0.0000			

$p = \gamma_{H2O}*h$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2

R = 1716 ft lb/slug deg R

T = 77 + 460 = 537 deg R

Thus, ρ = 0.00224 slug/ft^3

Using the trapezoidal rule for integration

$F_{exp} = 2\pi*0.5*\sum_{1\,to\,9}[(pr_i + pr_{i+1})*(r_{i+1} - r_i)] = 2\pi*0.5*0.189 =$ 0.594 lb

Theory:

$F = \rho V^2 A$ where

$A = \pi d^2/4 = \pi$*(1.174/12 ft)^2/4 = 0.00752 ft^2

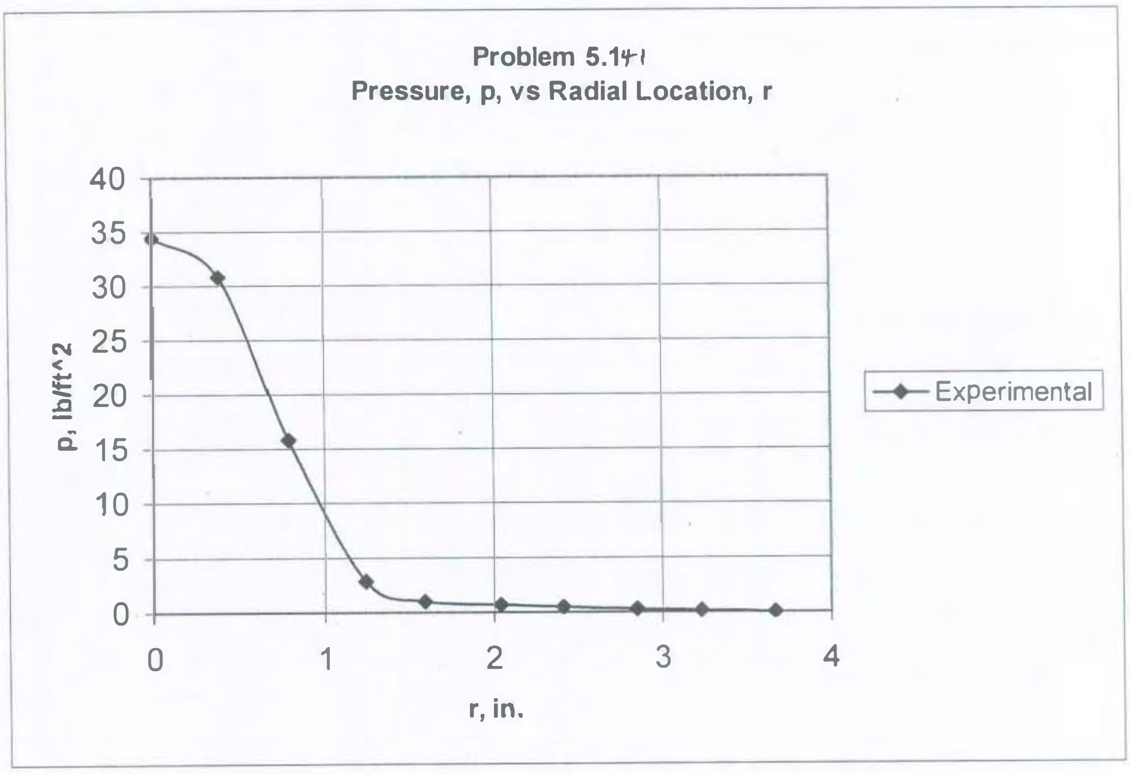
V = Q/A = (1.41 ft^3/s)/(0.00752 ft^2) = 188 ft/s

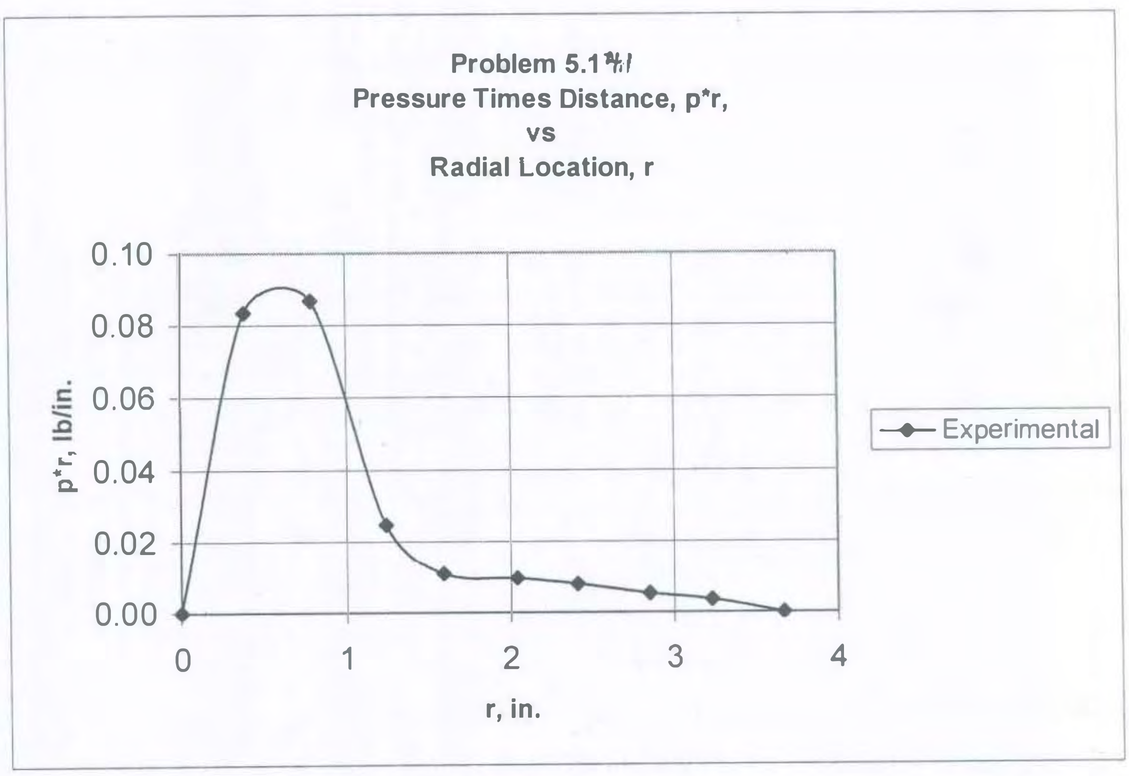
Thus,

F_{th} = 0.00224 slug/ft^3*(188 ft/s)^2*(0.00752 ft^2) = 0.595 lb

(con't)

5.141 (con't)


Problem 5.141
Pressure, p, vs Radial Location, r
p, lb/ft^2
r, in.
Experimental
0
1
2
3
4
5
10
15
20
25
30
35
40


Problem 5.141
Pressure Times Distance, p*r,
vs
Radial Location, r
p*r, lb/in.
r, in.
Experimental
0.00
0.02
0.04
0.06
0.08
0.10
0
1
2
3
4

5.142

5.142 Force from a Jet of Water Deflected by a Vane

Objective: A jet of a fluid striking a vane as shown in Fig. P5.128 exerts a force on the vane. It is the equal and opposite force of the vane on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the vane with the experimentally measured force.

Equipment: Water source; nozzle to produce a uniform jet of water; vanes to deflect the water jet; weigh tank to collect a known amount of water in a measured time period; stop watch; force balance system.

Experimental Procedure: Measure the outlet diameter, d, of the nozzle. Fasten the $\theta = 90$ degree vane to its support and adjust the balance spring to give a zero reading when there is no weight, W, on the platform and no flow through the nozzle. Place a known mass, m, on the platform and adjust the control valve on the pump to provide the necessary flowrate from the nozzle to return the platform to a zero reading. Determine the flowrate by collecting a known weight of water, W_{water}, in the weigh tank during a measured amount of time, t. Repeat the measurements for various masses, m. Repeat the experiment using a $\theta = 180$ degree vane.

Calculations: For each data set, determine the weight, $W = mg$, on the platform and the volume flowrate, $Q = W_{water}/(\gamma t)$, through the nozzle. Determine the exit velocity from the nozzle, V, by using $Q = VA$. Use the momentum equation to determine the theoretical weight that can be supported by the water jet as a function of V and θ.

Graph: For each vane, plot the experimentally determined weight, W, as ordinates and the water velocity, V, as abscissas.

Results: On the same graph plot the theoretical weight as a function of velocity for each vane.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

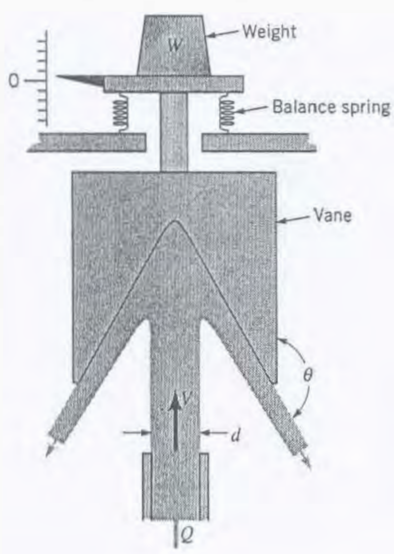


FIGURE P5.142

(con't)

5.142 (con't)

Solution for Problem 5.142 : Force from a Jet of Water Deflected by a Vane

d, in.
0.40

m, kg	W_{water}, lb	t, s	m, slug	Experimental W, lb	Q, ft^3/s	V, ft/s	Theoretical W, lb
Data for θ = 90 deg:							
0.02	7.71	29.8	0.0014	0.044	0.0041	4.7	0.038
0.07	8.66	18.2	0.0048	0.154	0.0076	8.7	0.129
0.17	8.87	10.1	0.0116	0.375	0.0141	16.1	0.440
0.12	8.92	12.6	0.0082	0.265	0.0113	13.0	0.286
0.22	9.66	10.6	0.0151	0.485	0.0146	16.7	0.474
Data for θ = 180 deg:							
0.05	6.81	24.5	0.0034	0.110	0.0045	5.1	0.088
0.10	9.02	20.8	0.0069	0.221	0.0069	8.0	0.215
0.20	8.84	13.2	0.0137	0.441	0.0107	12.3	0.512
0.25	7.88	10.9	0.0171	0.552	0.0116	13.3	0.597
0.30	8.86	11.1	0.0206	0.662	0.0128	14.7	0.727
0.35	7.97	9.5	0.0240	0.772	0.0134	15.4	0.803
0.40	6.37	7.6	0.0274	0.883	0.0134	15.4	0.802

$W = mg$
$Q = W_{water}/(\gamma * t)$
$V = Q/A$ where

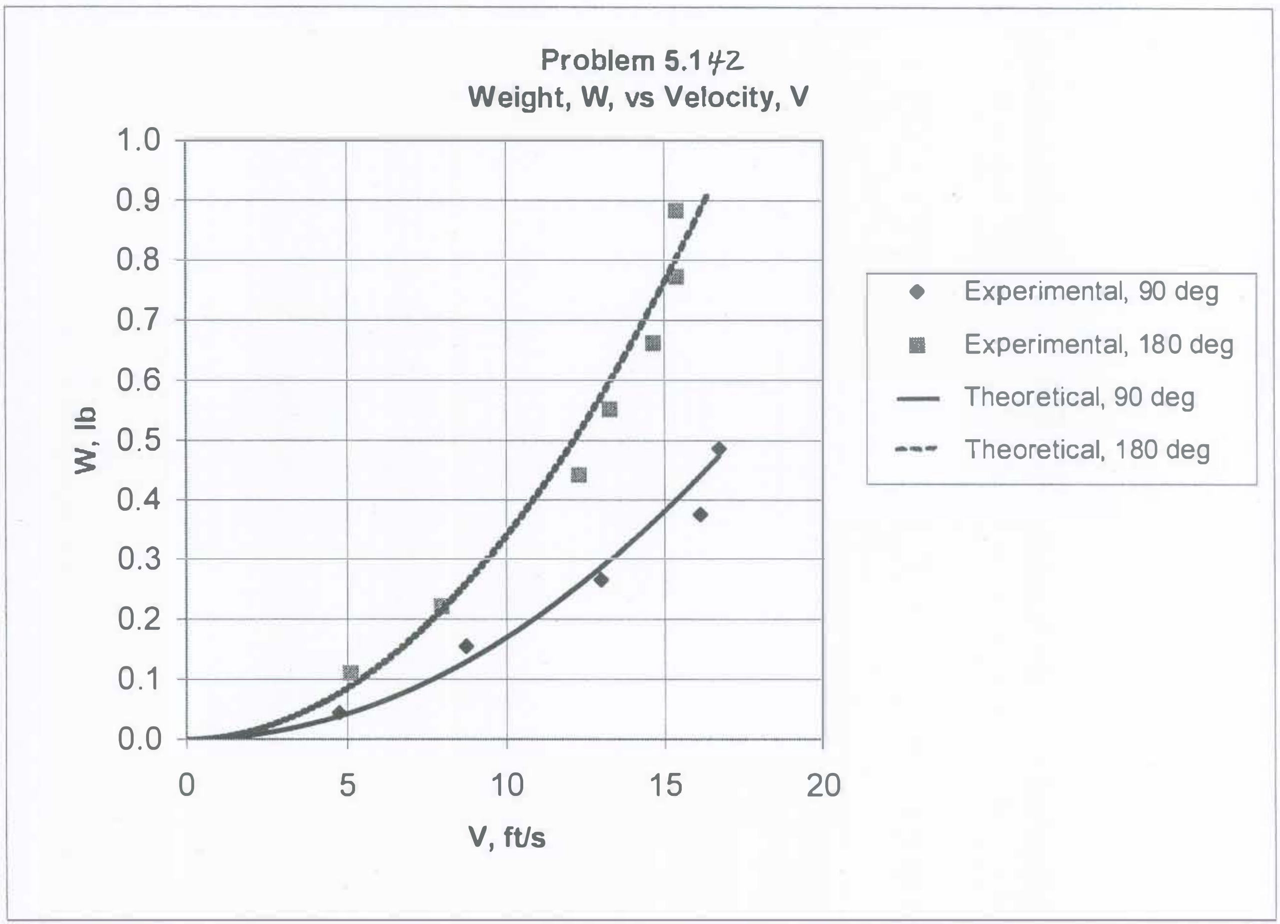
$A = \pi d^2/4 = \pi*(0.40/12\ ft)^2/4 = 0.000873\ ft^2$

Theoretical:

$W = \rho V^2 A$ for $\theta = 90$ deg
and
$W = 2\rho V^2 A$ for $\theta = 180$ deg

(Con't)

5.142 (Con't)


Problem 5.142
Weight, W, vs Velocity, V
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
W, lb
0
5
10
15
20
V, ft/s
Experimental, 90 deg
Experimental, 180 deg
Theoretical, 90 deg
Theoretical, 180 deg

5.143

5.143 Force of a Flowing Fluid on a Pipe Elbow

Objective: When a fluid flows through an elbow in a pipe system as shown in Fig. P5.129, the fluid's momentum is changed as the fluid changes direction. Thus, the elbow must put a force on the fluid. Similarly, there must be an external force on the elbow to keep it in place. The purpose of this experiment is to compare the theoretical vertical component of force needed to hold an elbow in place with the experimentally measured force.

Equipment: Variable speed fan; Pitot static tube; air speed indicator; air duct and 90-degree elbow; scale; barometer; thermometer.

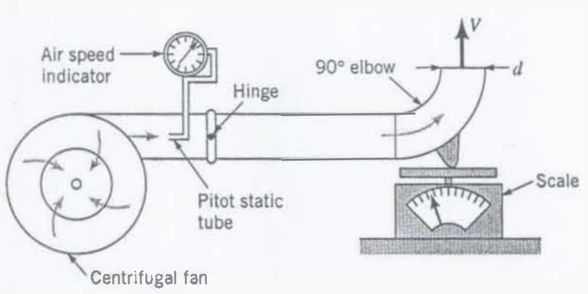
Experimental Procedure: Measure the diameter, d, of the air duct and adjust the scale to read zero when the elbow rests on it and there is no flow through it. Note that the duct is connected to the fan outlet by a pivot mechanism that is essentially friction free. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Adjust the variable speed fan to give the desired flowrate. Record the velocity, V, in the pipe as given by the Pitot static tube which is connected to an air speed indicator that reads directly in feet per minute. Record the force, F, indicated on the scale at this air speed. Repeat the measurements for various air speeds. Obtain data for two types of elbows: (1) a long radius elbow and (2) a mitered elbow (see Figs. 8.30 and 8.31).

Calculations: For a given air speed, V, use the momentum equation to calculate the theoretical vertical force, $F = \rho V^2 A$, needed to hold the elbow stationary.

Graph: Plot the experimentally measured force, F, as ordinates and the air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.143

(con't)

5.143 (con't)

Solution for Problem 5.143: Force of a Flowing Fluid on a Pipe Elbow

d, in.	H_{atm}, in. Hg	T, deg F
8.0	29.07	73

Experiment			Theory	
V, ft/min	F, lb	V, ft/s	V, ft/s	F_{th}, lb
Long Radius Elbow Data			0	0
0	0	0.0	5.0	0.02
1200	0.38	20.0	10.0	0.08
1420	0.51	23.7	15.0	0.18
1800	0.79	30.0	20.0	0.31
2160	1.05	36.0	25.0	0.49
2440	1.38	40.7	30.0	0.70
2700	1.65	45.0	35.0	0.96
2900	1.91	48.3	40.0	1.25
3100	2.19	51.7	45.0	1.58
3520	2.83	58.7	50.0	1.95
3750	3.12	62.5	55.0	2.36
3950	3.38	65.8	60.0	2.81
			65.0	3.30
Mitered Elbow Data				
1400	0.30	23.3		
1780	0.55	29.7		
2000	0.74	33.3		
2300	1.12	38.3		
2630	1.44	43.8		
2900	1.72	48.3		
3150	2.06	52.5		
3360	2.38	56.0		
3550	2.62	59.2		
3620	2.74	60.3		

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.07/12ft) = 2052 lb/ft^2

R = 1716 ft lb/slug deg R

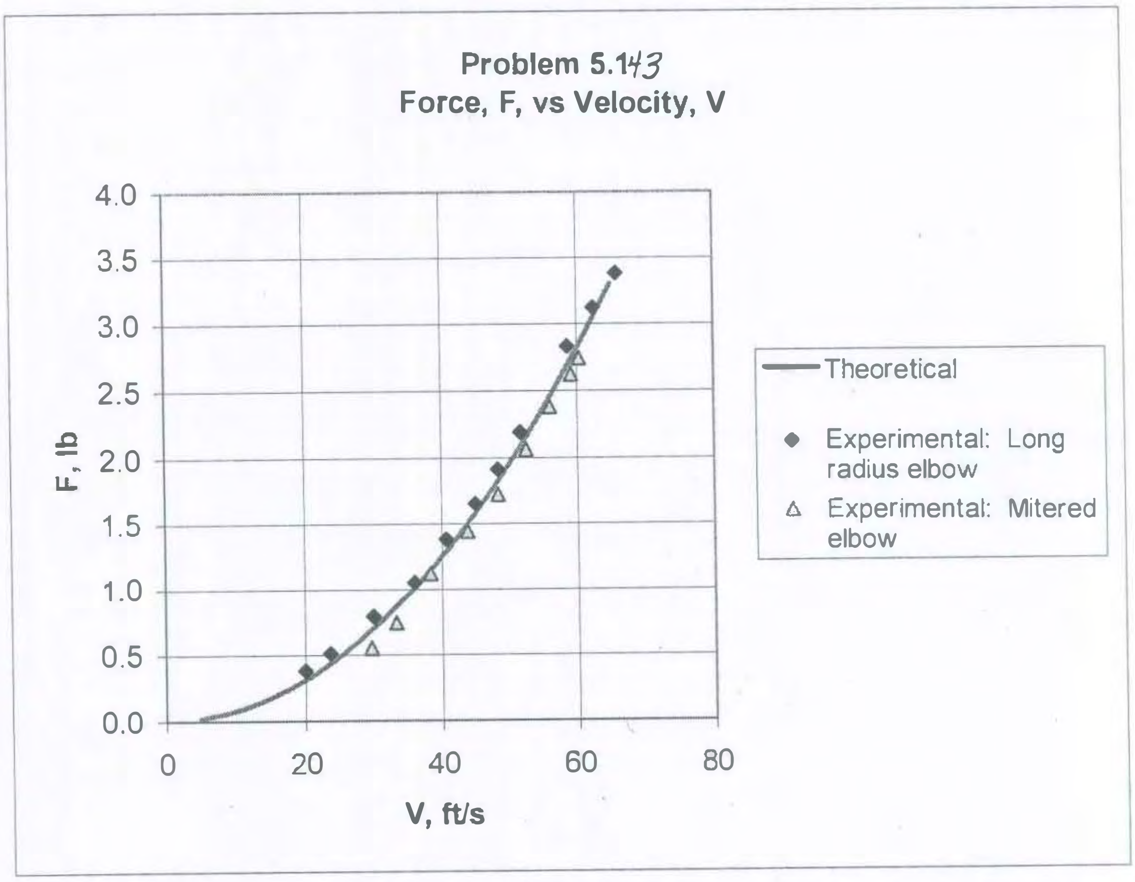
T = 73 + 460 = 533 deg R

Thus, ρ = 0.00224 slug/ft^3

A = πd^2/4 = π*(8/12)^2/4 = 0.349 ft^2

(con't)

5.143 (con't)


Problem 5.143
Force, F, vs Velocity, V
F, lb
V, ft/s
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
20
40
60
80
Theoretical
Experimental: Long radius elbow
Experimental: Mitered elbow

6.2

6.2 The velocity in a certain two-dimensional flow field is given by the equation

$$\mathbf{V} = 2xt\hat{\mathbf{i}} - 2yt\hat{\mathbf{j}}$$

where the velocity is in ft/s when x, y, and t are in feet and seconds, respectively. Determine expressions for the local and convective components of acceleration in the x and y directions. What is the magnitude and direction of the velocity and the acceleration at the point $x = y = 2$ ft at the time $t = 0$?

From expression for velocity, $u = 2xt$ and $v = -2yt$.

Since

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

then $a_x\,(\text{local}) = \dfrac{\partial u}{\partial t} = \underline{\underline{2x}}$

and

$$a_x\,(\text{conv}) = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (2xt)(2t) + (-2yt)(0)$$

$$= \underline{\underline{4xt^2}}$$

Similarly,

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

and

$$a_y\,(\text{local}) = \frac{\partial v}{\partial t} = \underline{\underline{-2y}}$$

$$a_y\,(\text{conv.}) = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (2xt)(0) + (-2yt)(-2t)$$

$$= \underline{\underline{4yt^2}}$$

At $x = y = 2\text{ ft}$ and $t = 0$

$$u = 2(2)(0) = 0 \qquad v = -2(2)(0) = 0$$

so that $\underline{\underline{\vec{V} = 0}}$

and $a_x = 2x + 4xt^2 = 2(2) + 4(2)(0) = 4\text{ ft/s}^2$

$a_y = -2y + 4yt^2 = -2(2) + 4(2)(0) = -4\text{ ft/s}^2$

Thus, $\vec{a} = \underline{\underline{4\hat{i} - 4\hat{j}\text{ ft/s}^2}}$ with $|\vec{a}| = \sqrt{(4)^2 + (-4)^2} = \underline{\underline{5.66\text{ ft/s}^2}}$

6.3

6.3 The velocity in a certain flow field is given by the equation

$$\mathbf{V} = x\hat{\mathbf{i}} + x^2z\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$$

Determine the expressions for the three rectangular components of acceleration.

From expression for velocity, $u = x$, $v = x^2z$, $w = yz$

Since

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

then

$$a_x = 0 + (x)(1) + (x^2z)(0) + (yz)(0)$$

$$= \underline{\underline{x}}$$

Similarly,

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

and

$$a_y = 0 + (x)(2xz) + (x^2z)(0) + (yz)(x^2)$$

$$= \underline{\underline{2x^2z + x^2yz}}$$

Also,

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

so that

$$a_z = 0 + (x)(0) + (x^2z)(z) + (yz)(y)$$

$$= \underline{\underline{x^2z^2 + y^2z}}$$

6.4

6.4 The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z^2$$
$$w = -3xz - z^2/2 + 4$$

(a) Determine the volumetric dilatation rate, and interpret the results. **(b)** Determine an expression for the rotation vector. Is this an irrotational flow field?

(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ (Eq. 6.9)

Thus, for velocity components given

volumetric dilatation rate $= 2x + (x+z) + (-3x-z) = \underline{\underline{0}}$

This result indicates that there is no change in the volume of a fluid element as it moves from one location to another.

(b) From Eqs. 6.12, 6.13, and 6.14 with the velocity components given:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(y - 2y\right) = -\frac{y}{2}$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}\left[0 - (y+2z)\right] = -\left(\frac{y}{2}+z\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}\left[2z - (-3z)\right] = \frac{5z}{2}$$

Thus,

$$\underline{\underline{\vec{\omega} = -\left(\frac{y}{2}+z\right)\hat{i} + \frac{5z}{2}\hat{j} - \frac{y}{2}\hat{k}}}$$

Since $\vec{\omega}$ is not zero everywhere the flow field is not irrotational. $\underline{\underline{\text{No.}}}$

6.5

6.5 Determine the vorticity field for the following velocity vector:

$$\mathbf{V} = (x^2 - y^2)\hat{\mathbf{i}} - 2xy\hat{\mathbf{j}}$$

$$\nabla \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k},$$

where

$u = x^2 - y^2$, $v = -2xy$, and $w = 0$

Thus,

$$\nabla \times \vec{V} = 0\hat{i} + 0\hat{j} + \left[\frac{\partial}{\partial x}(-2xy) - \frac{\partial}{\partial y}(x^2 - y^2)\right]\hat{k}$$

$$= [-2y - (-2y)]\hat{k} = 0\hat{k}$$

Hence,

$$\nabla \times \vec{V} = \underline{\underline{0}}$$

6.6

6.6 Determine an expression for the vorticity of the flow field described by

$$\mathbf{V} = -xy^3\,\hat{\mathbf{i}} + y^4\hat{\mathbf{j}}$$

Is the flow irrotational?

$$\vec{\zeta} = 2\vec{\omega} \qquad \text{(Eq. 6.17)}$$

From expression for velocity, $u = -xy^3$, $v = y^4$, and $w = 0$, and with

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \text{(Eq. 6.13)}$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \text{(Eq. 6.14)}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

it follows that

$$\omega_x = 0, \qquad \omega_y = 0, \quad \text{and} \quad \omega_z = \frac{1}{2}\left[0 - (-3xy^2)\right] = \frac{3}{2}xy^2$$

Thus,

$$\vec{\zeta} = 2\left(\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}\right)$$

$$= 2\left[(0)\hat{i} + (0)\hat{j} + \left(\tfrac{3}{2}xy^2\right)\hat{k}\right]$$

$$= \underline{\underline{3xy^2\,\hat{k}}}$$

Since $\vec{\zeta}$ is not zero everywhere the flow is not irrotational. <u>No.</u>

6.7

6.7 A one-dimensional flow is described by the velocity field

$$u = ay + by^2$$
$$v = w = 0$$

where a and b are constants. Is the flow irrotational? For what combination of constants (if any) will the rate of angular deformation as given by Eq. 6.18 be zero?

For irrotational flow $\vec{\omega} = 0$, and for the velocity distribution given:

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = 0$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\left(\frac{a}{2} + by\right)$$

Thus, $\vec{\omega}$ is not zero everywhere and the flow is not irrotational. $\underline{\underline{\text{No}}}$.

Since (from Eq. 6.18)

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

it follows for the velocity distribution given that

$$\dot{\gamma} = a + 2by$$

Thus, there are no values of a and b (except both equal to zero) that will give $\dot{\gamma} = 0$ for all values of y. $\underline{\underline{\text{None}}}$.

6.8

6.8 For a certain incompressible, two-dimensional flow field the velocity component in the y direction is given by the equation

$$v = 3xy + x^2y$$

Determine the velocity component in the x direction so that the volumetric dilatation rate is zero

For zero volumetric rate in a two-dimensional flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Since

$$\frac{\partial v}{\partial y} = 3x - x^2$$

Then from Eq. (1)

$$\frac{\partial u}{\partial x} = -3x + x^2 \qquad (2)$$

Equation (2) can be integrated with respect to x to obtain

$$\int du = -\int 3x\,dx + \int x^2 dx + f(y)$$

or

$$\underline{\underline{u = -\frac{3}{2}x^2 + \frac{x^3}{3} + f(y)}}$$

where $f(y)$ is an undetermined function of y.

6.9

6.9 An incompressible viscous fluid is placed between two large parallel plates as shown in Fig. P6.9. The bottom plate is fixed and the upper plate moves with a constant velocity, U. For these conditions the velocity distribution between the plates is linear, and can be expressed as

$$u = U\frac{y}{b}$$

Determine: **(a)** the volumetric dilatation rate, **(b)** the rotation vector, **(c)** the vorticity, and **(d)** the rate of angular deformation.

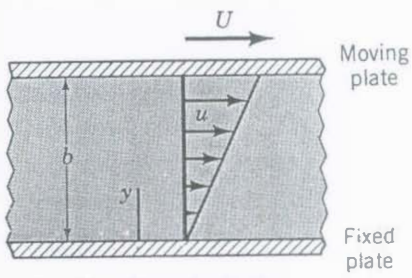


FIGURE P6.9

(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \underline{\underline{0}}$

(b) For velocity distribution given,

$$\vec{\omega} = \omega_z \hat{k}$$

and

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\frac{U}{2b}$$

Thus,

$$\underline{\underline{\vec{\omega} = -\frac{U}{2b}\hat{k}}}$$

(c)

$$\vec{\zeta} = 2\vec{\omega} = \underline{\underline{-\frac{U}{b}\hat{k}}}$$

(d)

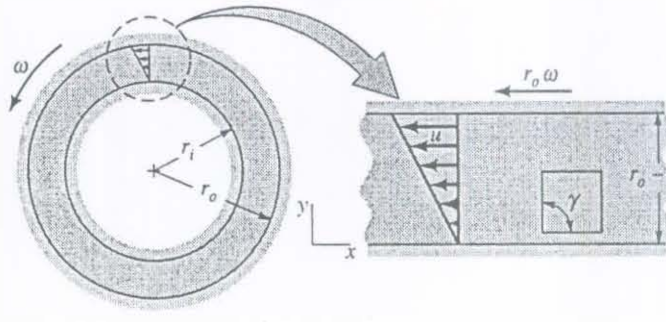
$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad \text{(Eq. 6.18)}$$

Thus,

$$\dot{\gamma} = \underline{\underline{\frac{U}{b}}}$$

6.10

6.10 A viscous fluid is contained in the space between concentric cylinders. The inner wall is fixed, and the outer wall rotates with an angular velocity ω. (See Fig. P6.10*a* and Video V6.3.) Assume that the velocity distribution in the gap is linear as illustrated in Fig. P6.10*b*. For the small rectangular element shown in Fig. P6.10*b*, determine the rate of change of the right angle γ due to the fluid motion. Express your answer in terms of r_0, r_i, and ω.



(a) (b)

■ FIGURE P6.10

From Eq. 6.18

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

For the linear distribution

$$u = -\frac{r_0 \omega}{r_0 - r_i}\, y$$

so that

$$\frac{\partial u}{\partial y} = -\frac{r_0 \omega}{r_0 - r_i}$$

and since $v = 0$

$$\dot{\gamma} = -\frac{r_0 \omega}{r_0 - r_i}$$

The negative sign indicates that the original right angle is increasing.

6.12

6.12 Verify that the stream function in cylindrical coordinates satisfies the continuity equation.

In cylindrical coordinets the continuity equation for steady, two-dimensional, incompressible flow is

(1) $$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0$$

Consider $v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ and $v_\theta = -\frac{\partial \psi}{\partial r}$ so that from Eq. (1),

$$\frac{1}{r}\frac{\partial(\partial\psi/\partial\theta)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial\theta}\left(-\frac{\partial\psi}{\partial r}\right) = \frac{1}{r}\left[\frac{\partial^2\psi}{\partial r\partial\theta} - \frac{\partial^2\psi}{\partial\theta\partial r}\right] = 0$$

Thus, any function ψ satisifies the continuity equation.

6.13

6.13 For a certain incompressible flow field it is suggested that the velocity components are given by the equations

$$u = 2xy \qquad v = -x^2y \qquad w = 0$$

Is this a physically possible flow field? Explain.

Any physically possible incompressible flow field must satisfy conservation of mass as expressed by the relationship

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

For the velocity distribution given,

$$\frac{\partial u}{\partial x} = 2y \qquad \frac{\partial v}{\partial y} = -x^2 \qquad \frac{\partial w}{\partial z} = 0$$

Substitution into Eq. (1) shows that

$$2y - x^2 + 0 \neq 0$$

Thus, this is not a physically possible flow field. No.

6.14

6.14 The velocity components of an incompressible, two-dimensional velocity field are given by the equations

$$u = y^2 - x(1 + x)$$
$$v = y(2x + 1)$$

Show that the flow is irrotational and satisfies conservation of mass.

If the two-dimensional flow is irrotational,

$$\omega_z = \frac{1}{2} \qquad \frac{\quad}{\partial x} \qquad \frac{\quad}{\partial y}$$

For the velocity distribution given,

$$\frac{\partial v}{\partial x} = 2y \qquad \frac{\partial u}{\partial y} = 2y$$

Thus,

$$\omega_z = \frac{1}{2}\left(2y - 2y\right) = 0$$

and the flow is irrotational.

To satisfy conservation of mass,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since,

$$\frac{\partial u}{\partial x} = -1 - 2x \qquad \frac{\partial v}{\partial y} = 2x + 1$$

then

$$-1 - 2x + 2x + 1 = 0$$

and

conservation of mass is satisfied.

6.15

6.15 For each of the following stream functions, with units of m^2/s, determine the magnitude and the angle the velocity vector makes with the x-axis at $x = 1$ m, $y = 2$ m. Locate any stagnation points in the flow field.
(a) $\psi = xy$
(b) $\psi = -2x^2 + y$

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

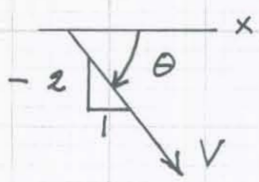
(a) For $\psi = xy$,

$$u = \frac{\partial \psi}{\partial y} = x \qquad v = -\frac{\partial \psi}{\partial x} = -y$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = -2\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2 + v^2} = \sqrt{(1m)^2 + (-2m)^2} = \underline{\underline{2.24\frac{m}{s}}}$$



$$\tan\theta = \frac{-2}{1} \qquad \underline{\underline{\theta = -63.4^\circ}}$$

Since $u = 0$ at $x = 0$ and $v = 0$ at $y = 0$, a stagnation point occurs at $x = y = 0$.

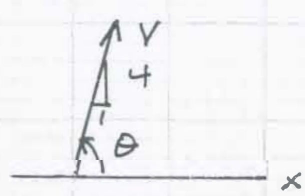
(b) For $\psi = -2x^2 + y$,

$$u = \frac{\partial \psi}{\partial y} = 1\frac{m}{s} \qquad v = -\frac{\partial \psi}{\partial x} = 4x$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = 4\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2 + v^2} = \sqrt{(1\tfrac{m}{s})^2 + (4\tfrac{m}{s})^2} = \underline{\underline{4.12\frac{m}{s}}}$$



$$\tan\theta = \frac{4}{1} \qquad \underline{\underline{\theta = 76.0^\circ}}$$

Since $u \neq 0$, there are no stagnation points.

6.16

6.16 The stream function for an incompressible, two-dimensional flow field is

$$\psi = ay - by^3$$

where a and b are constants. Is this an irrotational flow? Explain.

For the flow to be irrotational,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and for the stream function given,

$$u = \frac{\partial \psi}{\partial y} = a - 3by^2$$

$$v = -\frac{\partial \psi}{\partial x} = 0$$

Thus,

$$\frac{\partial u}{\partial y} = -6by \qquad \frac{\partial v}{\partial x} = 0$$

so that

$$\omega_z = \frac{1}{2}\left[0 - (-6by)\right] = 3by$$

Since $\omega_z \neq 0$ flow is not irrotational (unless $b=0$). No.

6.17

6.17 The stream function for an incompressible, two-dimensional flow field is

$$\psi = ay^2 - bx$$

where a and b are constants. Is this an irrotational flow? Explain.

For the flow to be irrotational (see Eq. 6.12),

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

and for the stream function given,

$$u = \frac{\partial \psi}{\partial y} = 2ay$$

$$v = -\frac{\partial \psi}{\partial x} = b$$

Thus,

$$\frac{\partial u}{\partial y} = 2a \qquad \frac{\partial v}{\partial x} = 0$$

so that

$$\omega_z = \frac{1}{2}\left[0 - (2a)\right] = -a$$

Since $\omega_z \neq 0$ flow is not irrotational (unless $a = 0$). No.

6.18

6.18 The velocity components for an incompressible, plane flow are

$$v_r = Ar^{-1} + Br^{-2}\cos\theta$$

$$v_\theta = Br^{-2}\sin\theta$$

where A and B are constants. Determine the corresponding stream function.

From the definition of the stream function,

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = -\frac{\partial\psi}{\partial r} \qquad \text{(Eq. 6.42)}$$

so that for the velocity distribution given,

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = Ar^{-1} + Br^{-2}\cos\theta \qquad (1)$$

$$\frac{\partial\psi}{\partial r} = -Br^{-2}\sin\theta \qquad (2)$$

Integrate Eq. (1) with respect to θ to obtain

$$\int d\psi = \int (A + Br^{-1}\cos\theta)\,d\theta + f_1(r)$$

or

$$\psi = A\theta + Br^{-1}\sin\theta + f_1(r) \qquad (3)$$

Similarly, integrate Eq. (2) with respect to r to obtain

$$\int d\psi = -\int Br^{-2}\sin\theta\,dr + f_2(\theta)$$

or

$$\psi = Br^{-1}\sin\theta + f_2(\theta) \qquad (4)$$

Thus, to satisfy both Eqs. (3) and (4)

$$\psi = \underline{\underline{A\theta + Br^{-1}\sin\theta + C}}$$

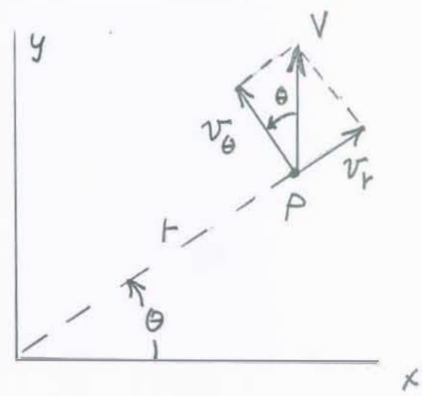
where C is an arbitrary constant.

6.19

6.19 For a certain two-dimensional flow field

$$u = 0$$
$$v = V$$

(a) What are the corresponding radial and tangential velocity components? **(b)** Determine the corresponding stream function expressed in Cartesian coordinates and in cylindrical polar coordinates.



(a) At an arbitrary point P (see figure)

$$\underline{\underline{v_r = V \sin\theta}}$$

$$\underline{\underline{v_\theta = V \cos\theta}}$$

(b) Since

$$u = \frac{\partial \psi}{\partial y} = 0 \qquad v = -\frac{\partial \psi}{\partial x} = V$$

it follows that ψ is not a function of y and

$$\underline{\underline{\psi = -Vx + C}}$$

where C is an arbitrary constant.

Also, with $x = r\cos\theta$

$$\underline{\underline{\psi = -Vr\cos\theta + C}}$$

Check this result:

$$v_\theta = -\frac{\partial \psi}{\partial r} = -(-V\cos\theta) = V\cos\theta$$

and

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r}(Vr\sin\theta) = V\sin\theta$$, which checks with part (a).

6.20

6.20 Make use of the control volume shown in Fig. P6.20 to derive the continuity equation in cylindrical coordinates (Eq. 6.33 in text).

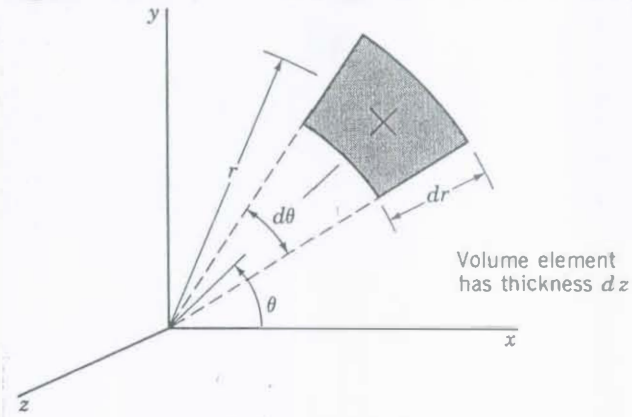


FIGURE P6.20

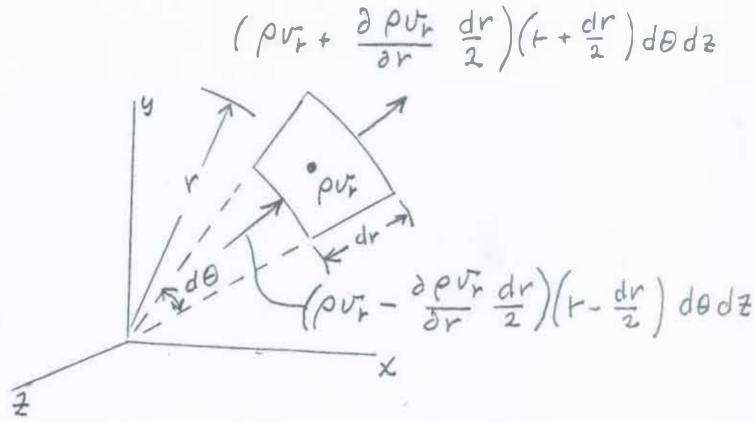
$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}\cdot\hat{n} \, dA = 0 \qquad (Eq.\ 6.19)$$

For the differential control volume shown

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall \approx \frac{\partial \rho}{\partial t} r \, d\theta \, dr \, dz \qquad (1)$$

and

$$\int_{cs} \rho \vec{V}\cdot\hat{n} \, dA = \text{net rate of mass outflow through surfaces of control volume}$$



From figure at right:

Net rate of mass outflow in r-direction =

$$\left(\rho v_r + \frac{\partial \rho v_r}{\partial r}\frac{dr}{2}\right)\left(r + \frac{dr}{2}\right) d\theta \, dz - \left(\rho v_r - \frac{\partial \rho v_r}{\partial r}\frac{dr}{2}\right)\left(r - \frac{dr}{2}\right) d\theta \, dz$$

$$= \frac{\partial \rho v_r}{\partial r} r \, dr \, d\theta \, dz + \rho v_r \, dr \, d\theta \, dz \qquad (2)$$

(con't)

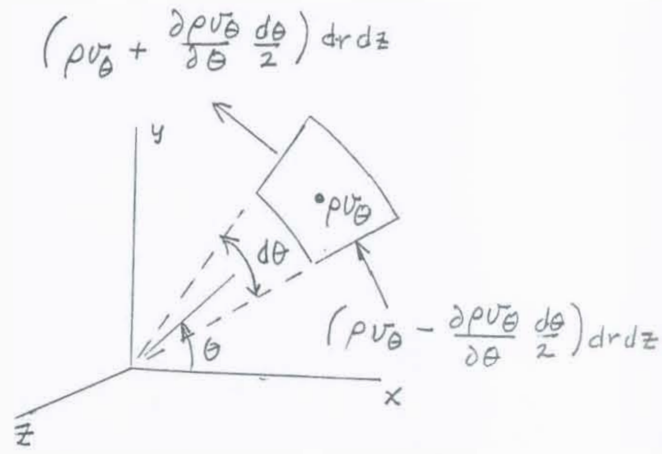
6.20 (con't)

From figure at right:

Net rate of mass outflow in θ-direction =

$$\left(\rho v_\theta + \frac{\partial \rho v_\theta}{\partial \theta}\frac{d\theta}{2}\right) dr\,dz - \left(\rho v_\theta - \frac{\partial \rho v_\theta}{\partial \theta}\frac{d\theta}{2}\right) dr\,dz$$

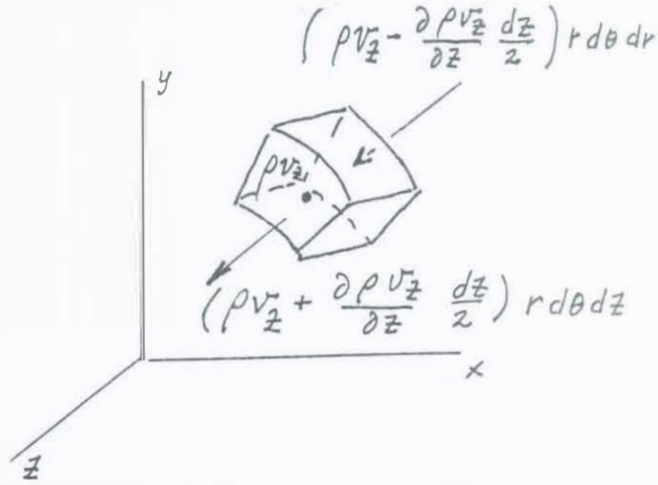
$$= \frac{\partial \rho v_\theta}{\partial \theta}\, dr\,d\theta\,dz \qquad (3)$$



From figure at right:

Net rate of mass outflow in z-direction =

$$\left(\rho v_z + \frac{\partial \rho v_z}{\partial z}\frac{dz}{2}\right) r\,d\theta\,dr - \left(\rho v_z - \frac{\partial \rho v_z}{\partial z}\frac{dz}{2}\right) r\,d\theta\,dr$$



$$= \frac{\partial \rho v_z}{\partial z}\, r\,dr\,d\theta\,dz \qquad (4)$$

Substitution of Eqs. (1) thru (4) into Eq. 6.19 yields

$$\frac{\partial \rho}{\partial t} r\,dr\,d\theta\,dz + \frac{\partial \rho v_r}{\partial r} r\,dr\,d\theta\,dz + \rho v_r\, dr\,d\theta\,dz + \frac{\partial \rho v_\theta}{\partial \theta} dr\,d\theta\,dz + \frac{\partial \rho v_z}{\partial z} r\,dr\,d\theta\,dz = 0$$

or

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_r}{\partial r} + \frac{\rho v_r}{r} + \frac{1}{r}\frac{\partial \rho v_\theta}{\partial \theta} + \frac{\partial \rho v_z}{\partial z} = 0 \qquad (5)$$

Since $\quad \dfrac{\partial \rho v_r}{\partial r} + \dfrac{\rho v_r}{r} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\rho v_r)$

Eq. (5) can be written as

$$\underline{\underline{\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0}}$$

which is Eq. 6.33.

6.21

6.21 A two-dimensional, incompressible flow is given by $u = -y$ and $v = x$. Show that the streamline passing through the point $x = 10$ and $y = 0$ is a circle centered at the origin.

For two-dimensional flow along a streamline

$$\frac{dy}{dx} = \frac{v}{u}$$

so that for the velocity components given

$$\frac{dy}{dx} = \frac{x}{-y}$$

and

$$-\int y\,dy = \int x\,dx$$

Thus,

$$-\frac{y^2}{2} = \frac{x^2}{2} + C \quad \text{(where } C \text{ is a constant)}$$

and

$$x^2 + y^2 = 2C = C' \qquad (1)$$

Equation (1) represents the equation for the family of streamlines. For a given value of C' the equation gives a circle centered at the origin with C' the square of the radius.

For $x = 10$ and $y = 0$

$$10^2 + 0 = C' = 100$$

and the equation of the streamline passing through this point is

$$x^2 + y^2 = 100$$

which is a circle of radius 10 centered at the origin.

6.22

6.22 In a certain steady, two-dimensional flow field the fluid density varies linearly with respect to the coordinate x; that is, $\rho = Ax$ where A is a constant. If the x component of velocity u is given by the equation $u = y$, determine an expression for v.

For a variable density flow,

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} \qquad (\text{Eq. } 6.29)$$

With

$$\rho u = (Ax)(y) = Axy$$

it follows that

$$\frac{\partial(\rho u)}{\partial x} = Ay$$

Thus,

$$\frac{\partial(\rho v)}{\partial y} = -Ay \qquad (1)$$

Integrate Eq. (1) with respect to y to obtain

$$\int d(\rho v) = -\int Ay\, dy + f_1(x)$$

or

$$\rho v = -\frac{Ay^2}{2} + f_1(x)$$

With $\rho = Ax$

$$v = -\left(\frac{1}{Ax}\right)\left(\frac{Ay^2}{2}\right) + \frac{f_1(x)}{Ax}$$

or

$$\underline{\underline{v = -\frac{y^2}{2x} + f(x)}}$$

where $f(x)$ is an arbitrary function of x.

6.23

6.23 In a two-dimensional, incompressible flow field, the x component of velocity is given by the equation $u = 2x$. **(a)** Determine the corresponding equation for the y component of velocity if $v = 0$ along the x axis. **(b)** For this flow field what is the magnitude of the average velocity of the fluid crossing the surface OA of Fig. P6.23? Assume that the velocities are in ft/s when x and y are in feet.

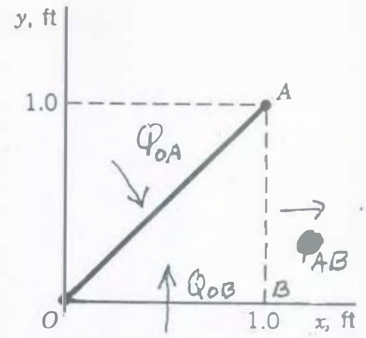


FIGURE P6.23

(a) To satisfy the continuity equation (consider a unit thickness = 1 ft)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since $\frac{\partial u}{\partial x} = 2$

it follows that

$$\frac{\partial v}{\partial y} = -2 \qquad (1)$$

Integration of Eq. (1) with respect to y yields

$$v = -2y + f(x)$$

If $v = 0$ along x-axis $(y = 0)$ then $f(x) = 0$ so that

$$\underline{\underline{v = -2y}}$$

(b) To satisfy conservation of mass

$$Q_{OA} = Q_{AB} - Q_{OB} \qquad \text{(see figure)}$$

Along AB $\quad u = 2(1) = 2 \ \frac{ft}{s}$ $\quad$ so that

$$Q_{AB} = u\, A_{AB} = (2\ ft/s)(1\ ft)(1\ ft) = 2\ \frac{ft^3}{s}$$

Along OB $\quad v = 0$ $\quad$ so that $\quad Q_{OB} = 0$.

Thus,

$$Q_{OA} = Q_{AB} = 2\ \frac{ft^3}{s}$$

and

$$V_{Av} = \frac{Q_{OA}}{\text{area}_{OA}} = \frac{2\ \frac{ft^3}{s}}{\sqrt{2}\ ft^2} = \underline{\underline{1.41\ \frac{ft}{s}}}$$

6.24

6.24 The radial velocity component in an incompressible, two-dimensional flow field ($v_z = 0$) is

$$v_r = 2r + 3r^2 \sin\theta$$

Determine the corresponding tangential velocity component, v_θ, required to satisfy conservation of mass.

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \qquad \text{(Eq. 6.35)}$$

Since $v_z = 0$,

$$\frac{\partial v_\theta}{\partial \theta} = -\frac{\partial(r v_r)}{\partial r} \qquad (1)$$

and with

$$r v_r = 2r^2 + 3r^3 \sin\theta$$

it follows that

$$\frac{\partial(r v_r)}{\partial r} = 4r + 9r^2 \sin\theta$$

Thus, Eq. (1) becomes

$$\frac{\partial v_\theta}{\partial \theta} = -(4r + 9r^2 \sin\theta) \qquad (2)$$

Equation (2) can be integrated with respect to θ to obtain

$$\int d v_\theta = -\int (4r + 9r^2 \sin\theta)\, d\theta + f(r)$$

or

$$v_\theta = \underline{\underline{-4r\theta - 9r^2\cos\theta + f(r)}}$$

where $f(r)$ is an undetermined function of r.

6.25

6.25 The stream function for an incompressible flow field is given by the equation

$$\psi = 3x^2y - y^3$$

where the stream function has the units of m^2/s with x and y in meters. **(a)** Sketch the streamline(s) passing through the origin. **(b)** Determine the rate of flow across the straight path AB shown in Fig. P6.25.

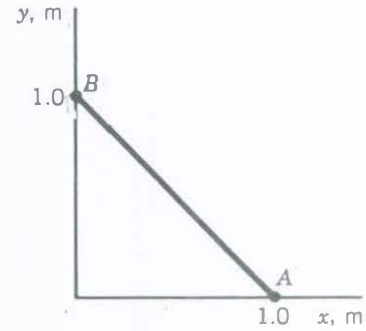


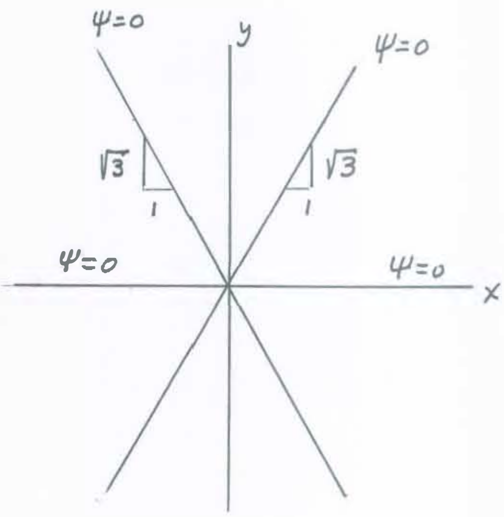
FIGURE P6.25

(a) Lines of constant ψ are streamlines. For $\psi = 3x^2y - y^3$ the streamline passing through the origin $(x=0, y=0)$ has a value $\psi = 0$. Thus, the equation for the streamlines through the origin is

$$0 = 3x^2y - y^3$$

or

$$y = \pm\sqrt{3}\, x$$



A sketch of these streamlines is shown in the figure.

(b) $q = \psi_B - \psi_A$

At B $x=0,\ y=1\,m$ so that

$$\psi_B = 3(0)^2(1) - (1)^3 = -1\ m^3/s \text{ (per unit width)}$$

At A $x = 1\,m,\ y=0$ so that

$$\psi_A = 3(1)^2(0) - (0)^3 = 0$$

Thus,

$$q = \psi_B = \underline{\underline{-1\ m^3/s \text{ (per unit width)}}}$$

The negative sign indicates that the flow is from right to left as we look from A to B.

6.26

6.26 The streamlines in a certain incompressible, two-dimensional flow field are all concentric circles so that $v_r = 0$. Determine the stream function for **(a)** $v_\theta = Ar$ and for **(b)** $v_\theta = Ar^{-1}$, where A is a constant.

From the definition of the stream function,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = -\frac{\partial \psi}{\partial r} \qquad \text{(Eq. 6.42)}$$

so that with $v_r = 0$ it follows that $\frac{\partial \psi}{\partial \theta} = 0$ and therefore

$$\psi = f(r)$$

(a) For $v_\theta = Ar$

$$\frac{\partial \psi}{\partial r} = -Ar \qquad (1)$$

Integrate Eq.(1) with respect to r to obtain

$$\int d\psi = -\int Ar\,dr$$

or

$$\psi = -\frac{Ar^2}{2} + f_1(\theta)$$

However, since ψ is not a function of θ, it follows that

$$\underline{\underline{\psi = -\frac{Ar^2}{2} + C}}$$

where C is an arbitrary constant.

(b) Similarly, for $v_\theta = Ar^{-1}$

$$\int d\psi = -\int Ar^{-1}\,dr$$

or

$$\underline{\underline{\psi = -A\ln r + C}}$$

*6.27

***6.27** The stream function for an incompressible, two-dimensional flow field is

$$\psi = 3x^2y + y$$

For this flow field plot several streamlines.

The equation for a streamline is found by setting ψ = constant in the equation for the stream function. Thus, for the given stream function

$$\psi = 3x^2y + y$$

it follows that the equation of a streamline is

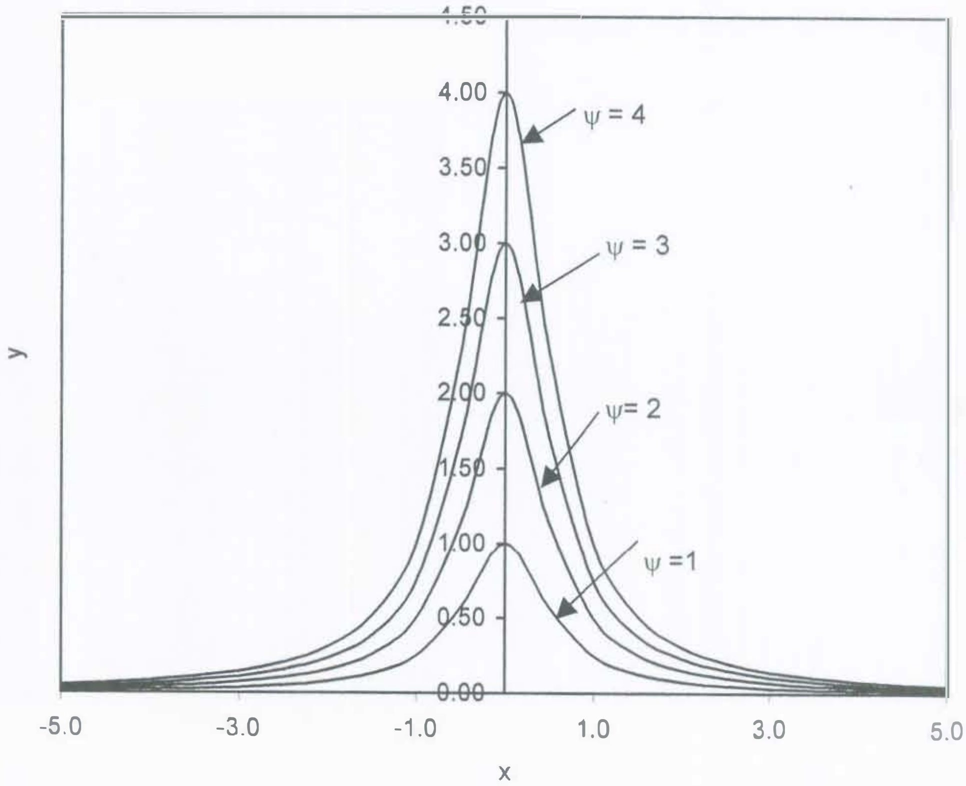
$$y = \frac{\psi}{1 + 3x^2}$$

where various constant values can be assigned to ψ to obtain a family of streamlines. Tabulated results for $\psi = 1, 2, 3, 4$, and a plot showing the streamlines are given below.

	ψ =1	ψ =2	ψ =3	ψ=4
x	y	y	y	y
-5.0	0.0132	0.0263	0.0395	0.0526
-4.5	0.0162	0.0324	0.0486	0.0648
-4.0	0.0204	0.0408	0.0612	0.0816
-3.5	0.0265	0.0530	0.0795	0.1060
-3.0	0.0357	0.0714	0.1071	0.1429
-2.5	0.0506	0.1013	0.1519	0.2025
-2.0	0.0769	0.1538	0.2308	0.3077
-1.5	0.1290	0.2581	0.3871	0.5161
-1.0	0.2500	0.5000	0.7500	1.0000
-0.5	0.5714	1.1429	1.7143	2.2857
0.0	1.0000	2.0000	3.0000	4.0000
0.5	0.5714	1.1429	1.7143	2.2857
1.0	0.2500	0.5000	0.7500	1.0000
1.5	0.1290	0.2581	0.3871	0.5161
2.0	0.0769	0.1538	0.2308	0.3077
2.5	0.0506	0.1013	0.1519	0.2025
3.0	0.0357	0.0714	0.1071	0.1429
3.5	0.0265	0.0530	0.0795	0.1060
4.0	0.0204	0.0408	0.0612	0.0816
4.5	0.0162	0.0324	0.0486	0.0648
5.0	0.0132	0.0263	0.0395	0.0526

(con't)

*6.27 (Con't)



4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
y
ψ = 4
ψ = 3
ψ= 2
ψ =1
-5.0
-3.0
-1.0
1.0
3.0
5.0
x

6.28

6.28 Consider the incompressible, two-dimensional flow of a non-viscous fluid between the boundaries shown in Fig. P6.28. The velocity potential for this flow field is

$$\phi = x^2 - y^2$$

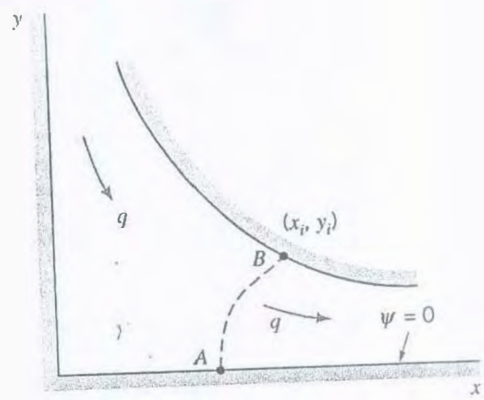


FIGURE P6.28

(a) $$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 2x$$

To determine ψ integrate with respect to y to obtain

$$\int d\psi = \int 2x\, dy$$

or

$$\psi = 2xy + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -2y$$

so that

$$\int d\psi = \int 2y\, dx$$

or

$$\psi = 2xy + f_2(y) \qquad (2)$$

To satisfy both Eqs. (1) and (2)

$$\psi = 2xy + C$$

where C is an arbitrary constant. Since $\psi = 0$ along $y = 0$
$C = 0$ and

$$\underline{\underline{\psi = 2xy}} \qquad (3)$$

(b) The discharge, q, passing through any surface connecting the two walls, such as AB (see figure), is

$$q = \psi_B - \psi_A$$

From Eq. (3), $\psi_A = 0$ and $\psi_B = 2x_i y_i$. It follows that

$$q = \underline{\underline{2x_i y_i}}$$

6.31

6.31 Given the streamfunction for a flow as $\psi = 4x^2 - 4y^2$, show that the Bernoulli equation can be applied between any two points in the flow field.

For the Bernoulli equation to be applied between any two points in the flow field (as opposed to only points along a streamline), the flow must be irrotational. That is, $\nabla \times \vec{V} = 0$, which for two-dimensional flow can be written as

(1) $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

For the given flow,

$$u = \frac{\partial \psi}{\partial y} = -8y \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} = -8x$$

Thus,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}(-8x) - \frac{\partial}{\partial y}(-8y) = -8 + 8 = 0$$

Hence, Eq. (1) is satisfied, the flow is irrotational, and the Bernoulli equation can be applied between any two points.

6.32

6.32 A two-dimensional flow field for a nonviscous, incompressible fluid is described by the velocity components

$$u = U_0 + 2y$$
$$v = 0$$

where U_0 is a constant. If the pressure at the origin (Fig. P6.32) is p_0, determine an expression for the pressure at **(a)** point A, and **(b)** point B. Explain clearly how you obtained your answer. Assume the units are consistent and body forces may be neglected.

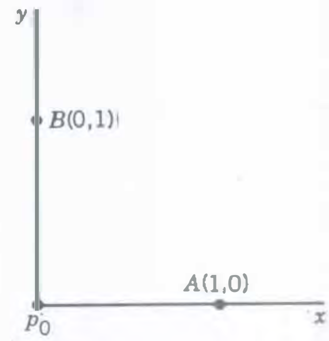


FIGURE P6.32

Check to see if flow is irrotational. Since

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and for the given velocity distribution, $\frac{\partial v}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 2$, it follows that $\omega_z \neq 0$. Since flow is not irrotational cannot apply the Bernoulli equation between any two points in the flow field.

(a) Since $v = 0$, the origin and point A are on the same streamline. Thus,

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} \qquad (1)$$

At the origin $V_0 = U_0$ and at A $V_A = U_0$ so that from Eq. (1)

$$\underline{\underline{p_A = p_0}}$$

(b) Point B is not on same streamline as origin so cannot apply Bernoulli equation between B and O. To find p_B use the y-component of Euler's equations:

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] \qquad \text{(Eq. 6.51b)}$$

Since $v = 0$ and $g_y = 0$,

$$\frac{\partial p}{\partial y} = 0$$

so that

$$\underline{\underline{p_B = p_0}}$$

6.33

6.33 In a certain two-dimensional flow field the velocity is constant with components $u = -4$ ft/s and $v = -2$ ft/s. Determine the corresponding stream function and velocity potential for this flow field. Sketch the equipotential line $\phi = 0$ which passes through the origin of the coordinate system.

From the definition of the stream function

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

so that for the velocity components given

$$\frac{\partial \psi}{\partial y} = -4 \qquad (1)$$

$$\frac{\partial \psi}{\partial x} = 2 \qquad (2)$$

Integrate Eq. (1) with respect to y to obtain

$$\int d\psi = \int -4\, dy + f_1(x)$$

or

$$\psi = -4y + f_1(x) \qquad (3)$$

Similarly, integrate Eq. (2) with respect to x to obtain

$$\int d\psi = \int 2\, dx + f_2(y)$$

or

$$\psi = 2x + f_2(y) \qquad (4)$$

Thus, to satisfy both Eqs. (3) and (4)

$$\psi = \underline{\underline{2x - 4y + C}}$$

where C is an arbitrary constant.

From the definition of the velocity potential

$$u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y} \qquad \text{(Eqs. 6.64)}$$

so that for the velocity components given

$$\frac{\partial \phi}{\partial x} = -4 \qquad (5)$$

$$\frac{\partial \phi}{\partial y} = -2 \qquad (6)$$

(con't)

6.33 (Con't)

Integrate Eq. (5) with respect to x to obtain

$$\int d\phi = \int -4\,dx + f_3(y)$$

or

$$\phi = -4x + f_3(y) \qquad (7)$$

Integrate Eq. (6) with respect to y to obtain

$$\int d\phi = \int -2\,dy + f_4(x)$$

or

$$\phi = -2y + f_4(x) \qquad (8)$$

Thus, to satisfy both Eqs. (7) and (8)

$$\phi = \underline{\underline{-4x - 2y + C}} \qquad (9)$$

where C is an arbitrary constant.

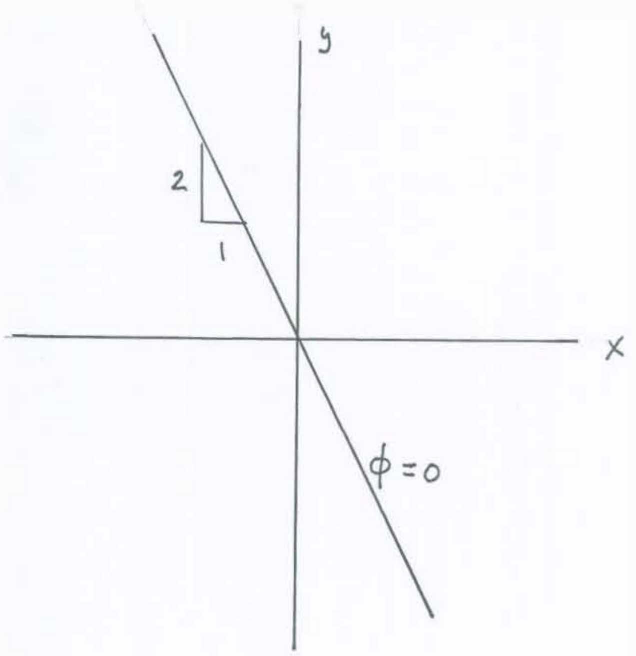
Since the equipotential line, $\phi = 0$, passes through the origin ($x = y = 0$), then $C = 0$ in Eq. (9) so that the equation of the $\phi = 0$ equipotential line is

$$2y = -4x$$

or

$$y = -2x$$

A sketch of this line is shown in the figure.



6.34

6.34 The stream function for a given two-dimensional flow field is

$$\psi = 5x^2y - (5/3)y^3$$

Determine the corresponding velocity potential.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 5x^2 - 5y^2 \qquad (1)$$

Integrate with respect to x to obtain

$$\int d\phi = \int (5x^2 - 5y^2)\,dx$$

or

$$\phi = \frac{5}{3}x^3 - 5xy^2 + f_1(y) \qquad (2)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -10xy \qquad (3)$$

and

$$\int d\phi = -\int 10xy\,dy$$

or

$$\phi = -5xy^2 + f_2(x) \qquad (4)$$

To satisfy both Eqs. (2) and (4)

$$\underline{\phi = \left(\frac{5}{3}\right)x^3 - 5xy^2 + C}$$

where C is an arbitrary constant.

6.35

6.35 Determine the stream function corresponding to the velocity potential

$$\phi = x^3 - 3xy^2$$

Sketch the streamline $\psi = 0$, which passes through the origin.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 3x^2 - 3y^2$$

Integrate with respect to y to obtain

$$\int d\psi = \int (3x^2 - 3y^2)\, dy$$

or

$$\psi = 3\left(x^2 y - \frac{y^3}{3}\right) + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -6xy$$

and integrating with respect to x yields

$$\int d\psi = \int 6xy\, dx$$

or

$$\psi = 3x^2 y + f_2(y) \qquad (2)$$

To satisfy both Eqs. (1) and (2)

$$\psi = 3x^2 y - y^3 + C$$

where C is an arbitrary constant. Since the streamline $\psi = 0$ passes through the origin $(x=0, y=0)$ it follows that $C = 0$ and

$$\underline{\underline{\psi = 3x^2 y - y^3}} \qquad (3)$$

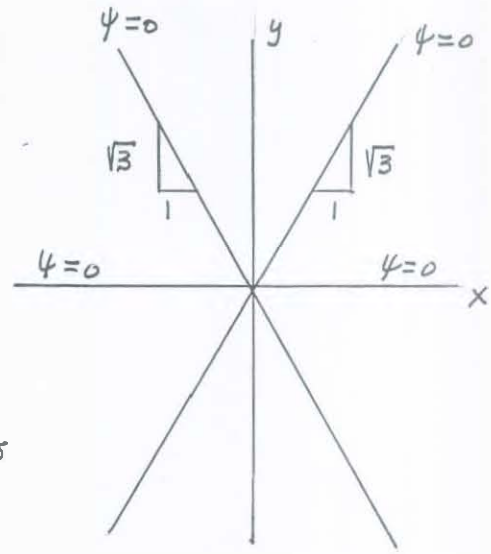
The equation of the streamline passing through the origin is found by setting $\psi = 0$ in Eq. (3) to yield

$$y(3x^2 - y^2) = 0$$

which is satisfied for $y = 0$ and

$$y = \pm\sqrt{3}\, x$$

A sketch of the $\psi = 0$ streamlines are shown in the figure.



6.36

6.36 A certain flow field is described by the stream function

$$\psi = A\theta + Br\sin\theta$$

where A and B are positive constants. Determine the corresponding velocity potential and locate any stagnation points in this flow field.

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} = \frac{A}{r} + B\cos\theta \qquad (1)$$

Integrate with respect to r to obtain

$$\int d\phi = \int \left(\frac{A}{r} + B\cos\theta\right) dr$$

or

$$\phi = A\ln r + Br\cos\theta + f_1(\theta) \qquad (2)$$

Similarly,

$$v_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -B\sin\theta \qquad (3)$$

and

$$\int d\phi = -\int Br\sin\theta \, d\theta$$

or

$$\phi = Br\cos\theta + f_2(r) \qquad (4)$$

To satisfy both Eqs. (2) and (4)

$$\underline{\underline{\phi = A\ln r + Br\cos\theta + C}}$$

where C is an arbitrary constant.

Stagnation points occur where $v_r = 0$ and $v_\theta = 0$. From Eq. (3) $v_\theta = 0$ at $\theta = 0$ and $\theta = \pi$. From Eq. (1) with $\theta = 0$

$$v_r = \frac{A}{r} + B$$

so that $v_r = 0$ for $r = -\frac{A}{B}$. However, since A and B are both positive constants this result indicates a negative value for r which is not defined.

At $\theta = \pi$

$$v_r = \frac{A}{r} + B\cos\pi = \frac{A}{r} - B$$

so that $v_r = 0$ for $r = \frac{A}{B}$. Thus, a stagnation point occurs at

$$\underline{\underline{\theta = \pi \quad \text{and} \quad r = \frac{A}{B}}}$$

6.37

6.37 It is known that the velocity distribution for two-dimensional flow of a viscous fluid between wide parallel plates (Fig. P6.37) is parabolic; that is

$$u = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]$$

with $v = 0$. Determine, if possible, the corresponding stream function and velocity potential.

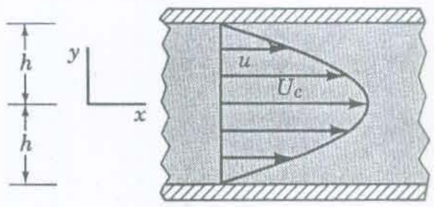


FIGURE P6.37

To determine the stream function let

$$u = \frac{\partial \psi}{\partial y} = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]$$

and integrate with respect to y to obtain

$$\int d\psi = \int U_c\left[1 - \left(\frac{y}{h}\right)^2\right] dy$$

or

$$\psi = U_c\left[y - \frac{y^3}{3h^2}\right] + f_1(x)$$

Since $v = -\frac{\partial \psi}{\partial x} = 0$, ψ is not a function of x so that

$$\underline{\underline{\psi = U_c\, y\left[1 - \frac{1}{3}\left(\frac{y}{h}\right)^2\right] + C}}$$

where C is an arbitrary constant.

To determine the velocity potential let

$$u = \frac{\partial \phi}{\partial x} = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]$$

and integrate with respect to x to obtain

$$\int d\phi = \int U_c\left[1 - \left(\frac{y}{h}\right)^2\right] dx$$

or

$$\phi = U_c\left[x - \left(\frac{y}{h}\right)^2 x\right] + f_2(y)$$

However,

$$v = \frac{\partial \phi}{\partial y} = 0 = -\frac{2U_c\, x\, y}{h^2} + \frac{\partial f_2(y)}{\partial y}$$

and this relationship cannot be satisfied for all values of x and y. Thus, there is not a velocity potential that describes this flow (the flow is not irrotational).

6.38

6.38 The velocity potential for a certain inviscid flow field is

$$\phi = -(3x^2y - y^3)$$

where ϕ has the units of ft^2/s when x and y are in feet. Determine the pressure difference (in psi) between the points (1, 2) and (4, 4), where the coordinates are in feet, if the fluid is water and elevation changes are negligible.

Since the flow field is described by a velocity potential the flow is irrotational and the Bernoulli equation can be applied between any two points. Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \qquad (1)$$

Also,

$$u = \frac{\partial \phi}{\partial x} = -6xy \qquad v = \frac{\partial \phi}{\partial y} = -3x^2 + 3y^2$$

At $x = 1\ ft$, $y = 2\ ft$

$$u_1 = -6(1)(2) = -12\ \frac{ft}{s}$$

$$v_1 = -3(1)^2 + 3(2)^2 = 9\ \frac{ft}{s}$$

So that $V_1^2 = u_1^2 + v_1^2 = \left(-12\ \frac{ft}{s}\right)^2 + \left(9\ \frac{ft}{s}\right)^2 = 225\left(\frac{ft}{s}\right)^2$

At $x = 4\ ft$, $y = 4\ ft$

$$u_2 = -6(4)(4) = -96\ \frac{ft}{s}$$

$$v_2 = -3(4)^2 + 3(4)^2 = 0$$

So that $V_2^2 = \left(-96\ \frac{ft}{s}\right)^2$

Thus, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\frac{\gamma}{g}\left[V_2^2 - V_1^2\right]$$

$$= \frac{1}{2}\frac{\left(62.4\ \frac{lb}{ft^3}\right)}{\left(32.2\ \frac{ft}{s^2}\right)}\left[\left(-96\ \frac{ft}{s}\right)^2 - 225\left(\frac{ft}{s}\right)^2\right]$$

$$= 8710\ \frac{lb}{ft^2} = \left(8710\ \frac{lb}{ft^2}\right)\left(\frac{ft^2}{144\ in.^2}\right) = \underline{\underline{60.5\ psi}}$$

6.39

6.39 The velocity potential for a flow is given by

$$\phi = \frac{a}{2}(x^2 - y^2)$$

where a is a constant. Determine the corresponding stream function and sketch the flow pattern.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = ax$$

To determine ψ integrate with respect to y to obtain

$$\int d\psi = \int ax\,dy$$

or

$$\psi = axy + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -ay$$

so that

$$\int d\psi = \int ay\,dx$$

or

$$\psi = axy + f_2(y) \qquad (2)$$

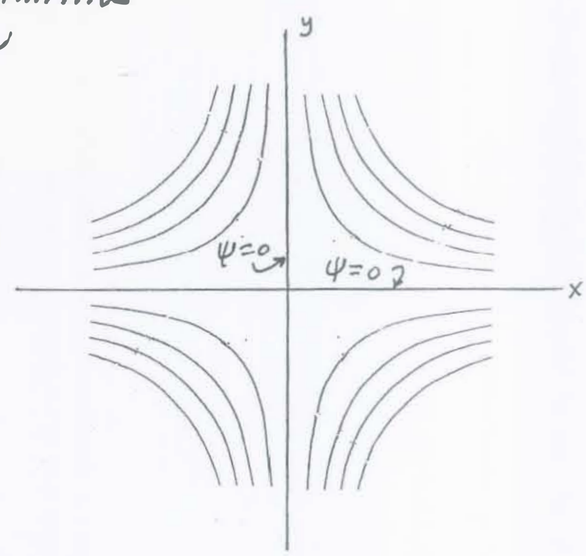
To satisfy both Eqs. (1) and (2)

$$\psi = \underline{\underline{axy + C}}$$

where C is an arbitrary constant. Let $C = 0$ so that

$$\frac{\psi}{a} = xy \qquad (3)$$

For a given a the streamline pattern is obtained by setting ψ equal to various constants. For $\psi = 0$ the x and y axes are streamlines and for other values of ψ the streamlines are rectangular hyperbolas as shown in the sketch.



6.40

6.40 The stream function for a two-dimensional, nonviscous, incompressible flow field is given by the expression

$$\psi = -2(x - y)$$

where the stream function has the units of ft^2/s with x and y in feet. (a) Is the continuity equation satisfied? (b) Is the flow field irrotational? If so, determine the corresponding velocity potential. (c) Determine the pressure gradient in the horizontal x direction at the point $x = 2$ ft, $y = 2$ ft.

(a) To satisfy the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

For the stream function given,

$$u = \frac{\partial \psi}{\partial y} = 2 \ \frac{ft}{s} \qquad v = -\frac{\partial \psi}{\partial x} = 2 \ \frac{ft}{s}$$

so that

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial y} = 0$$

and the continuity equation is satisfied. Yes.

(Note: When a flow field is defined by a stream function the continuity equation is always identically satisfied.)

(b) Since

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and

$$\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial v}{\partial x} = 0$$

it follows that $\omega_z = 0$ and the flow field is irrotational. Yes.

Thus,

$$u = \frac{\partial \phi}{\partial x} = 2 \qquad v = \frac{\partial \phi}{\partial y} = 2$$

and integration yields

$$\underline{\phi = 2(x + y) + C}$$

Where C is an arbitrary constant.

(c) With the x-axis horizontal, $g_x = 0$, and

$$-\frac{\partial p}{\partial x} = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.51a)}$$

and at $x = 2ft$, $y = 2ft$ $\quad \frac{\partial p}{\partial x} = -\rho\left[2\ \frac{ft}{s}(0) + 2\ \frac{ft}{s}(0)\right] = \underline{0}$

6.41

6.41 The velocity potential for a certain inviscid, incompressible flow field is given by the equation

$$\phi = 2x^2y - (\tfrac{2}{3})y^3$$

where ϕ has the units of m^2/s when x and y are in meters. Determine the pressure at the point $x = 2$ m, $y = 2$ m if the pressure at $x = 1$ m, $y = 1$ m is 200 kPa. Elevation changes can be neglected and the fluid is water.

Since the flow is irrotational,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \qquad (1)$$

with $V^2 = u^2 + v^2$. For the velocity potential given,

$$u = \frac{\partial \phi}{\partial x} = 4xy \qquad v = \frac{\partial \phi}{\partial y} = 2x^2 - 2y^2$$

At point 1 let $x = 1\,m$ and $y = 1\,m$ so that

$$u_1 = 4(1)(1) = 4\,\frac{m}{s} \qquad v_1 = 2(1)^2 - 2(1)^2 = 0$$

and

$$V_1^2 = \left(4\,\frac{m}{s}\right)^2 = 16\,\frac{m^2}{s^2}$$

At point 2 $x = 2m$ and $y = 2m$ so that

$$u_2 = 4(2)(2) = 16\,\frac{m}{s} \qquad v_2 = 2(2)^2 - 2(2)^2 = 0$$

and

$$V_2^2 = \left(16\,\frac{m}{s}\right)^2 = 256\,\frac{m^2}{s^2}$$

Thus, from Eq. (1)

$$p_2 = p_1 + \frac{\gamma}{2g}\left(V_1^2 - V_2^2\right)$$

$$= 200 \times 10^3\,\frac{N}{m^2} + \frac{\left(9.80 \times 10^3\,\frac{N}{m^3}\right)}{2\left(9.81\,\frac{m}{s^2}\right)}\left(16\,\frac{m^2}{s^2} - 256\,\frac{m^2}{s^2}\right)$$

$$= \underline{\underline{80.1\text{ kPa}}}$$

6.42

6.42 A steady, uniform, incompressible, inviscid, two-dimensional flow makes an angle of 30° with the horizontal x axis. **(a)** Determine the velocity potential and the stream function for this flow. **(b)** Determine an expression for the pressure gradient in the vertical y direction. What is the physical interpretation of this result?

(a) From Eqs. 6.80 and 6.81

$$\phi = U(x\cos\alpha + y\sin\alpha) \qquad \text{(Eq. 6.80)}$$

and for $\alpha = 30°$

$$\phi = U(x\cos 30° + y\sin 30°) = \underline{\underline{U(0.866x + 0.500y)}}$$

Similarly,

$$\psi = U(y\cos\alpha - x\sin\alpha) \qquad \text{(Eq. 6.81)}$$

and for $\alpha = 30°$

$$\psi = U(y\cos 30° - x\sin 30°) = \underline{\underline{U(0.866y - 0.500x)}}$$

(b) Since

$$u = \frac{\partial\phi}{\partial x} \quad \text{and} \quad v = \frac{\partial\phi}{\partial y}$$

it follows that

$$u = 0.866U \quad \text{and} \quad v = 0.500U$$

From the Euler equation in the vertical y-direction

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) \qquad \text{(Eq. 6.51b)}$$

and with $v = \text{constant}$ and $g_y = -g$

$$\frac{\partial p}{\partial y} = -\rho g$$

or

$$\underline{\underline{\frac{\partial p}{\partial y} = -\gamma}}$$

This result indicates that the pressure distribution is hydrostatic. This is not a surprising result since the Bernoulli equation indicates that if there is no change in velocity the change in pressure is simply due to the weight of the fluid, i.e., a hydrostatic variation.

6.43

6.43 The streamlines for an incompressible, inviscid, two-dimensional flow field are all concentric circles and the velocity varies directly with the distance from the common center of the streamlines; that is

$$v_\theta = Kr$$

where K is a constant. **(a)** For this *rotational* flow determine, if possible, the stream function. **(b)** Can the pressure difference between the origin and any other point be determined from the Bernoulli equation? Explain.

(a) $$v_\theta = -\frac{\partial \psi}{\partial r} = Kr \qquad (1)$$

Integrate Eq. (1) with respect to r to obtain

$$\int d\psi = -\int Kr\,dr$$

or

$$\psi = -\frac{Kr^2}{2} + f_1(\theta)$$

Since

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0$$

it follows that ψ is not a function of θ and therefore

$$\underline{\underline{\psi = -\frac{Kr^2}{2} + C}}$$

where C is an arbitrary constant.

(b) The flow is rotational and therefore the Bernoulli equation cannot be applied between the origin and any point, since these points are not on the same streamline. <u>No.</u>
(Refer to discussion associated with derivation of Eq. 6.57.)

6.44

6.44 The velocity potential

$$\phi = -k(x^2 - y^2) \qquad (k = \text{constant})$$

may be used to represent the flow against an infinite plane boundary as illustrated in Fig. P6.44. For flow in the vicinity of a stagnation point it is frequently assumed that the pressure gradient along the surface is of the form

$$\frac{\partial p}{\partial x} = Ax$$

where A is a constant. Use the given velocity potential to show that this is true.

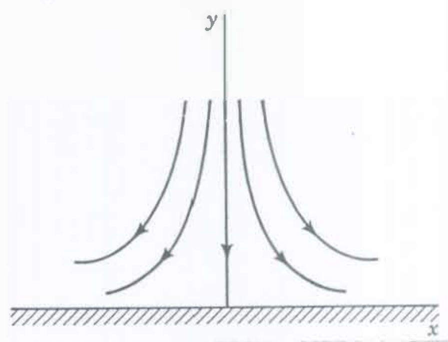


FIGURE P6.44

For the velocity potential given

$$u = \frac{\partial \phi}{\partial x} = -2kx \qquad (1)$$

$$v = \frac{\partial \phi}{\partial y} = -2ky \qquad (2)$$

and the stagnation point occurs at the origin.
For this steady, two-dimensional flow

$$-\frac{\partial p}{\partial x} = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \qquad (\text{Eq. 6.51a})$$

and along the surface $(y=0)$ $v=0$ so that

$$\frac{\partial p}{\partial x} = \rho u \frac{\partial u}{\partial x} \qquad (3)$$

From Eq. (1) $u = -2kx$ and therefore

$$\frac{\partial u}{\partial x} = -2k$$

and Eq. (3) becomes

$$\frac{\partial p}{\partial x} = \rho(-2kx)(-2k) = 4k^2 x$$

or

$$\underline{\underline{\frac{\partial p}{\partial x} = A x}}$$

where $A = 4k^2$.

6.45

6.45 Water is flowing between wedge shaped walls into a small opening as shown in Fig. P6.45. The velocity potential with units m²/s for this flow is $\phi = -2 \ln r$ with r in meters. Determine the pressure differential between points A and B.

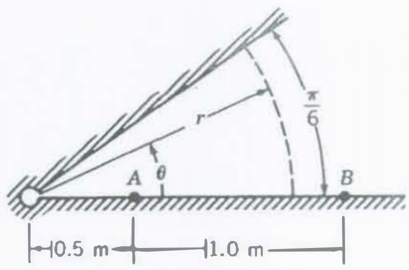


FIGURE P6.45

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} \qquad (1)$$

Along the horizontal surface, $v_\theta = 0$, and

$$v_r = \frac{\partial \phi}{\partial r} = -\frac{2}{r}$$

so that

$$V = v_r = -\frac{2}{r}$$

Thus,

$$V_A = -\frac{2}{0.5} = -4\ \frac{m}{s} \qquad V_B = -\frac{2}{1.5} = -\frac{4}{3}\ \frac{m}{s}$$

and from Eq.(1)

$$p_A - p_B = \frac{\gamma}{2g}\left[V_B^2 - V_A^2\right]$$

$$= \frac{9.80 \times 10^3 \frac{N}{m^3}}{2\left(9.81 \frac{m}{s^2}\right)}\left[\left(-\frac{4}{3}\ \frac{m}{s}\right)^2 - \left(-4\ \frac{m}{s}\right)^2\right]$$

$$= \underline{\underline{-7.10\ kPa}}$$

6.46

6.46 An ideal fluid flows between the inclined walls of a two-dimensional channel into a sink located at the origin (Fig. P6.46). The velocity potential for this flow field is

$$\phi = \frac{m}{2\pi} \ln r$$

where m is a constant. **(a)** Determine the corresponding stream function. Note that the value of the stream function along the wall OA is zero. **(b)** Determine the equation of the streamline passing through the point B, located at $x = 1$, $y = 4$.

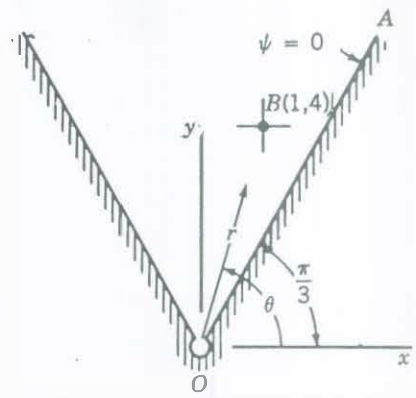


FIGURE P6.46

(a) $$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} = \frac{m}{2\pi r} \qquad (1)$$

Integrate Eq. (1) with respect to θ to obtain

$$\int d\psi = \int \frac{m}{2\pi}\, d\theta$$

or

$$\psi = \frac{m\theta}{2\pi} + f_1(r)$$

Since

$$v_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = 0 \qquad (2)$$

ψ is not a function of r so Eq. (2) becomes

$$\psi = \frac{m\theta}{2\pi} + C$$

where C is a constant. Also, $\psi = 0$ for $\theta = \frac{\pi}{3}$ so that

$$C = -\frac{m}{6}$$

and

$$\underline{\underline{\psi = m\left(\frac{\theta}{2\pi} - \frac{1}{6}\right)}} \qquad (3)$$

(b) At B $\tan\theta = \frac{4}{1}$ so that $\theta = 1.33$ rad. From Eq. (3) the value of ψ passing through this point is

$$\psi = m\left(\frac{1.33}{2\pi} - \frac{1}{6}\right) = 0.0450\,m$$

and therefore the equation of the streamline passing through B is

$$0.0450\,m = m\left(\frac{\theta}{2\pi} - \frac{1}{6}\right)$$

or

$$\underline{\underline{\theta = 1.33 \text{ rad}}}$$

(Note: It can be seen from Eq. (3) that the streamlines are all straight lines passing through the origin.)

6.47

6.47 It is suggested that the velocity potential for the flow of an incompressible, nonviscous, two-dimensional flow along the wall shown in Fig. P6.47 is

$$\phi = r^{4/3} \cos \tfrac{4}{3}\theta$$

Is this a suitable velocity potential for flow along the wall? Explain.

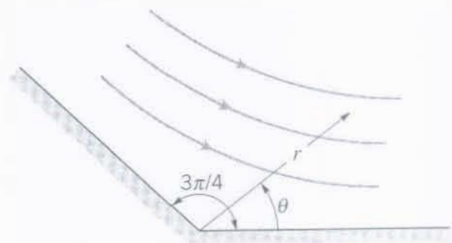


FIGURE P6.47

If this is a suitable ϕ the corresponding ψ must have a constant value along the wall (since the wall must correspond to a streamline).

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} = \frac{4}{3} r^{1/3} \cos \frac{4}{3}\theta \qquad (1)$$

Integrate Eq. (1) with respect to θ to obtain

$$\int d\psi = \int \frac{4}{3} r^{4/3} \cos \frac{4}{3}\theta$$

or

$$\psi = r^{4/3} \sin \frac{4}{3}\theta + f_1(r) \qquad (2)$$

Similarly,

$$v_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{4}{3} r^{1/3} \sin \frac{4}{3}\theta$$

and

$$\int d\psi = \int \frac{4}{3} r^{1/3} \sin \frac{4}{3}\theta \, dr$$

or

$$\psi = r^{\frac{4}{3}} \sin \frac{4}{3}\theta + f_2(\theta) \qquad (3)$$

To satisfy both Eqs. (2) and (3)

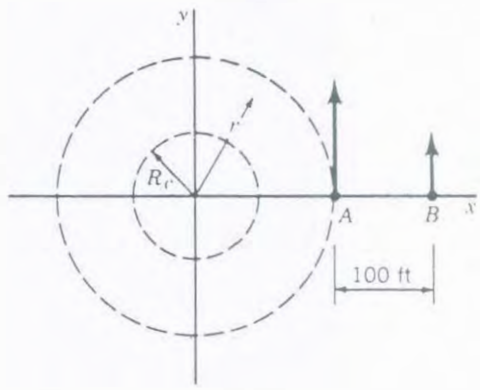
$$\psi = r^{4/3} \sin \frac{4}{3}\theta + C$$

where C is an arbitrary constant.

Along one section of the wall, $\theta = 0$, and $\psi = C$. Along the other section $\theta = \frac{3\pi}{4}$ and $\psi = C$. Thus, ψ has a constant value along the wall and the given velocity potential can be used to represent flow along the wall. Yes.

6.49

6.49 As illustrated in Fig. P6.49 a tornado can be approximated by a free vortex of strength Γ for $r > R_c$, where R_c is the radius of the core. Velocity measurements at points A and B indicate that $V_A = 125$ ft/s and $V_B = 60$ ft/s. Determine the distance from point A to the center of the tornado. Why can the free vortex model not be used to approximate the tornado throughout the flow field ($r \geq 0$)?



■ FIGURE P6.49

For a free vortex

$$v_\theta = \frac{K}{r} \qquad \text{(Eq. 6.86)}$$

Thus, at r_A, $v_\theta = 125 \frac{ft}{s}$, so that $K = 125\, r_A$

and at r_B, $v_\theta = 60 \frac{ft}{s}$, so that $K = 60\, r_B$.

Therefore,

$$125\, r_A = 60\, r_B$$

and since

$$r_B - r_A = 100 \text{ ft}$$

it follows that

$$125\, r_A = 60\,(100 + r_A)$$

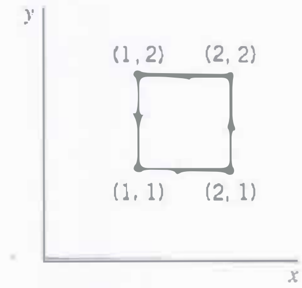
or

$$\underline{\underline{r_A = 92.3 \text{ ft}}}$$

The free vortex cannot be used to approximate a tornado throughout the flow field since at $r=0$ the velocity becomes infinite.

6.50

6.50 If the velocity field is given by $\mathbf{V} = ax\hat{\mathbf{i}} - ay\hat{\mathbf{j}}$, and a is a constant, find the circulation around the closed curve shown in Fig. P6.50.



■ FIGURE P6.50

The circulation is given by

$$\Gamma = \oint \vec{V}\cdot d\hat{s} = \oint (ax\hat{\imath} - ay\hat{\jmath})\cdot d\hat{s}$$, where $d\hat{s}$ is an element along the line.

Thus,

$$\Gamma = \int_{x=1}^{2} (ax\hat{\imath} - ay\hat{\jmath})\cdot(dx\,\hat{\imath}) + \int_{y=1}^{2} (ax\hat{\imath} - ay\hat{\jmath})\cdot(dy\,\hat{\imath})$$

$$+ \int_{x=2}^{1} (ax\hat{\imath} - ay\hat{\jmath})\cdot(-dx\,\hat{\imath}) + \int_{y=2}^{1} (ax\hat{\imath} - ay\hat{\jmath})\cdot(-dy\,\hat{\jmath})$$

$$= \int_{1}^{2} ax\,dx + \int_{1}^{2} (-ay)dy + \int_{2}^{1} (-ax)dx + \int_{2}^{1} (ay)dy$$

$$= \int_{1}^{2} ax\,dx - \int_{1}^{2} ay\,dy + \int_{1}^{2} ax\,dx - \int_{1}^{2} ay\,dy$$

$$= 2a\int_{1}^{2} x\,dx - 2a\int_{1}^{2} y\,dy$$

$$= 2a\left(\tfrac{3}{2}\right) - 2a\left(\tfrac{3}{2}\right) = 0$$

Thus,

$$\Gamma = \underline{\underline{0}}$$

Note: This flow is irrotational. That is $\nabla \times \vec{V} \equiv 0$. For any irrotational flow the circulation is identically zero.

6.51

6.51 The streamlines in a particular two-dimensional flow field are all concentric circles, as shown in Fig. P6.51. The velocity is given by the equation $v_\theta = \omega r$ where ω is the angular velocity of the rotating mass of fluid. Determine the circulation around the path $ABCD$.

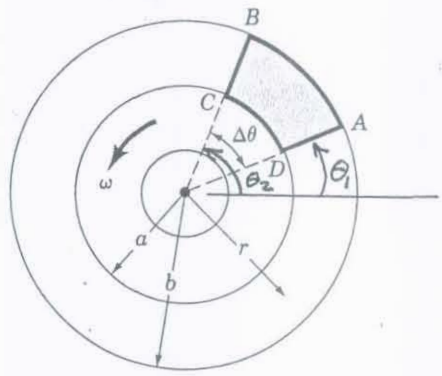


FIGURE P6.51

$$\Gamma = \oint_{ABCD} \vec{V} \cdot d\vec{s}$$

$$= \int_{AB} v_\theta \, b \, d\theta + \int_{BC} v_r \, dr + \int_{CD} v_\theta \, a \, d\theta + \int_{DA} v_r \, dr \qquad (1)$$

Since $v_r = 0$ and $v_\theta = \omega r$, Eq. (1) becomes

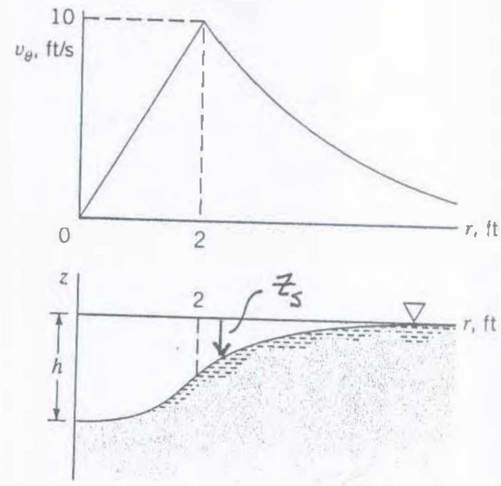
$$\Gamma = \int_{\theta_1}^{\theta_2} \omega b^2 d\theta + 0 + \int_{\theta_2}^{\theta_1} \omega a^2 d\theta + 0$$

$$= \omega b^2 (\theta_2 - \theta_1) + \omega a^2 (\theta_1 - \theta_2)$$

or

$$\Gamma = \omega (\theta_2 - \theta_1)(b^2 - a^2) = \underline{\underline{\omega \Delta\theta \, (b^2 - a^2)}}$$

6.52



■ FIGURE P6.52

6.52 The motion of a liquid in an open tank is that of a combined vortex consisting of a forced vortex for $0 \le r \le 2$ ft and a free vortex for $r > 2$ ft. The velocity profile and the corresponding shape of the free surface are shown in Fig. P6.52. The free surface at the center of the tank is a depth h below the free surface at $r = \infty$. Determine the value of h. Note that $h = h_{forced} + h_{free}$, where h_{forced} and h_{free} are the corresponding depths for the forced vortex and the free vortex, respectively. (See Section 2.12.2 for further discussion regarding the forced vortex.)

For forced vortex

$$z = \frac{\omega^2 r^2}{2g} + C \qquad \text{(Eq. 2.32)}$$

and with $z = 0$ at $r = 0$ it follows that $C = 0$.
Also, $v_\theta = r\omega$ and since $v_\theta = 10$ ft/s at $r = 2$ ft

$$\omega = \frac{10\,\frac{ft}{s}}{2\,ft} = 5\,\frac{rad}{s}$$

Thus, at $r = 2$ ft

$$z = \frac{\omega^2 r^2}{2g} = \frac{\left(5\,\frac{rad}{s}\right)^2 (2\,ft)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 1.55\,ft$$

For free vortex (see Example 6.6)

$$z_s = \frac{\Gamma^2}{8\pi^2 r^2 g}$$

where $\Gamma = 2\pi r v_\theta$

so that

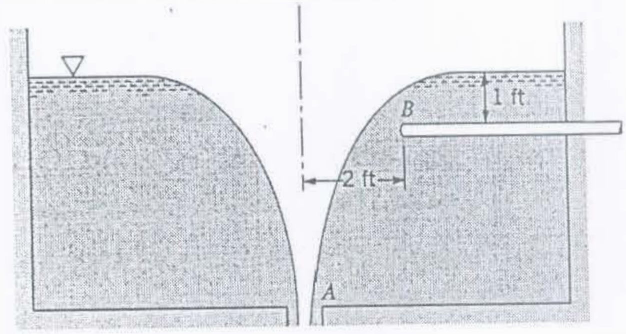
$$z_s = \frac{(2\pi r v_\theta)^2}{8\pi^2 r^2 g} = \frac{4\pi^2 (2\,ft)^2 \left(10\,\frac{ft}{s}\right)^2}{8\pi^2 (2\,ft)^2 \left(32.2\,\frac{ft}{s^2}\right)} = 1.55\,ft$$

Thus,

$$h = h_{forced} + h_{free} = 1.55\,ft + 1.55\,ft = \underline{\underline{3.10\,ft}}$$

6.53

6.53 When water discharges from a tank through an opening in its bottom, a vortex may form with a curved surface profile as shown in Fig. P6.53 and Video V6.4. Assume that the velocity distribution in the vortex is the same as that for a free vortex. At the same time the water is being discharged from the tank at point A it is desired to discharge a small quantity of water through the pipe B. As the discharge through A is increased, the strength of the vortex, as indicated by its circulation, is increased. Determine the maximum strength that the vortex can have in order that no air is sucked in at B. Express your answer in terms of the circulation. Assume that the fluid level in the tank at a large distance from the opening at A remains constant and viscous effects are negligible.



■ FIGURE P6.53

From Example 6.6,

$$z_s = -\frac{\Gamma^2}{8\pi^2 r^2 g}$$

Air will be sucked into pipe when $z_s = -1\ ft$ for $r = 2\ ft$.

Thus,

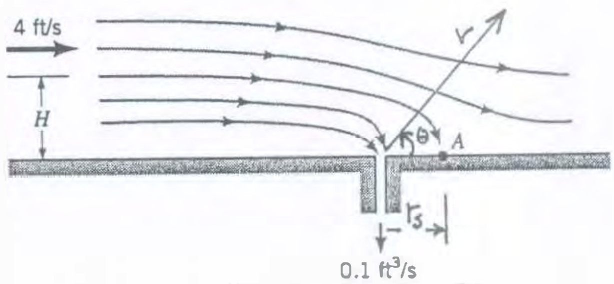
$$\Gamma^2 = -8\pi^2 r^2 g\, z_s = -8\pi^2 (2\ ft)^2 \left(32.2\ \frac{ft}{s^2}\right)(-1\ ft)$$

or

$$|\Gamma| = \underline{\underline{101\ \frac{ft^2}{s}}}$$

6.54

6.54 Water flows over a flat surface at 4 ft/s as shown in Fig. P6.54. A pump draws off water through a narrow slit at a volume rate of 0.1 ft^3/s per foot length of the slit. Assume that the fluid is incompressible and inviscid and can be represented by the combination of a uniform flow and a sink. Locate the stagnation point on the wall (point A) and determine the equation for the stagnation streamline. How far above the surface, H, must the fluid be so that it does not get sucked into the slit?



■ FIGURE P6.54

$$\psi = \psi_{\substack{uniform\\flow}} + \psi_{sink} = Ur\sin\theta - \frac{m}{2\pi}\theta \qquad (1)$$

Thus,

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\cos\theta - \frac{m}{2\pi r} \qquad (2)$$

and

$$v_\theta = -\frac{\partial\psi}{\partial r} = -U\sin\theta$$

Along the wall $v_\theta = 0$, and the stagnation point occurs where $v_r = 0$, so that from Eq. (2)

$$0 = U\cos(0^\circ) - \frac{m}{2\pi r_s}$$

and therefore

$$r_s = \frac{m}{2\pi U}$$

For $U = 4\,\frac{ft}{s}$ and $m = 0.2\,\frac{ft^2}{s}$ (note that a source strength of $0.2\,\frac{ft^2}{s}$ must be used to obtain $0.1\,\frac{ft^2}{s}$ through slit which is only one half of a "full" sink). Thus,

$$r_s = \frac{0.2\,\frac{ft^2}{s}}{2\pi\left(4\,\frac{ft}{s}\right)} = 0.00796\ ft$$

and the stagnation point is on the wall $\underline{\underline{0.00796\ ft}}$ $\underline{\underline{\text{to the right of slit}}}$.

(con't)

6.54 (con't)

The value of ψ at the stagnation point ($r = 0.00796$ ft, $\theta = 0°$) is zero (Eq. 1) so that the equation of the stagnation streamline is

$$0 = U r \sin\theta - \frac{m}{2\pi}\theta$$

or

$$r\sin\theta = \frac{m}{2\pi U}\theta$$

Since $y = r\sin\theta$ the equation of the stagnation streamline can be written as

$$\underline{\underline{y = \frac{m}{2\pi U}\theta}}$$

Fluid above the stagnation streamline will not be sucked into slit. The maximum distance, H, for the stagnation streamline occurs as $\theta \rightarrow \pi$ so that

$$H = \frac{m\pi}{2\pi U} = \frac{0.2\ \frac{ft^2}{s}}{2\left(4\ \frac{ft}{s}\right)} = \underline{\underline{0.0250\ ft}}$$

(Note: All the fluid below the stagnation streamline must pass through the slit. Thus, from conservation of mass

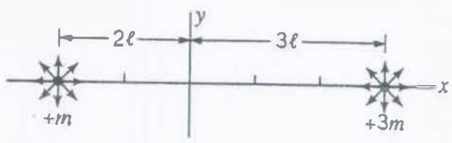
$$HU = \text{flow into slit}$$

or
$$H = \frac{0.1\ \frac{ft^2}{s}}{4\ \frac{ft}{s}} = 0.0250\ ft$$

which checks with the answer above.)

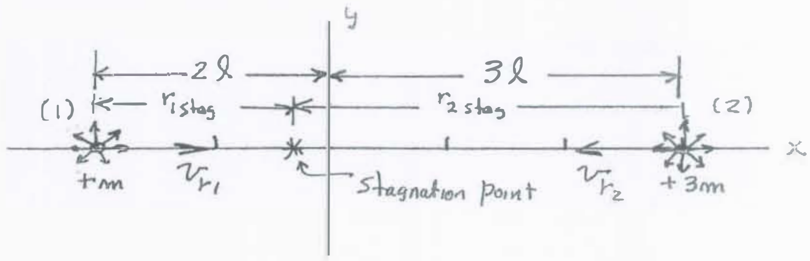
6.55

6.55 Two sources, one of strength m and the other with strength $3m$, are located on the x axis as shown in Fig. P6.55. Determine the location of the stagnation point in the flow produced by these sources.



■ FIGURE P6.55

Since the flow from each source is in the radial direction, it is only along the x-axis that the two radial components can cancel and create a stagnation point.



For source (1)

$$v_{r_1} = \frac{m}{2\pi r_1}$$

and for source (2)

$$v_{r2} = \frac{3m}{2\pi r_2}$$

The stagnation point occurs where $v_{r_1} = v_{r2}$ so that

$$\frac{m}{2\pi\, r_{1\,stag}} = \frac{3m}{2\pi\, r_{2\,stag}}$$

and

$$\frac{r_{2\,stag}}{r_{1\,stag}} = 3$$

Also,

$$r_{1\,stag} + r_{2\,stag} = 2\ell + 3\ell = 5\ell$$

so that

$$r_{1\,stag} + 3r_{1\,stag} = 5\ell$$

$$r_{1\,stag} = \frac{5}{4}\ell$$

Thus,

$$x_{stag} = -\left(2\ell - \frac{5}{4}\ell\right) = \underline{\underline{-0.75\ell}}$$

6.56

6.56 The velocity potential for a spiral vortex flow is given by $\phi = (\Gamma/2\pi)\,\theta - (m/2\pi)\ln r$, where Γ and m are constants. Show that the angle, α, between the velocity vector and the radial direction is constant throughout the flow field (see Fig. P6.56).

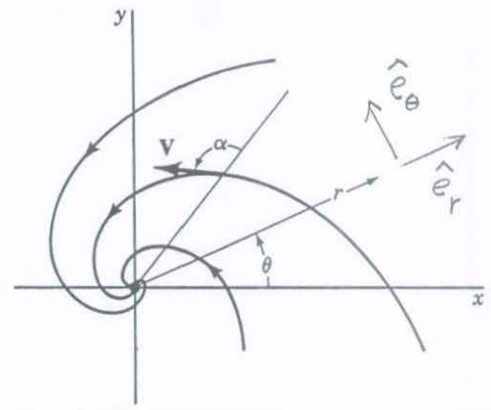


FIGURE P6.56

For the velocity potential given,

$$v_r = \frac{\partial \phi}{\partial r} = -\frac{m}{2\pi r} \qquad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{\Gamma}{2\pi r}$$

Since $\quad \vec{V}\cdot\hat{e}_r = |\vec{V}|\cos\alpha$

and $\quad \vec{V} = v_r\,\hat{e}_r + v_\theta\,\hat{e}_\theta$

then

$$\cos\alpha = \frac{\vec{V}\cdot\hat{e}_r}{|\vec{V}|} = \frac{v_r}{\sqrt{v_r^2 + v_\theta^2}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{v_\theta}{v_r}\right)^2}} = \frac{1}{\sqrt{1 + \dfrac{\left(\frac{\Gamma}{2\pi r}\right)^2}{\left(-\frac{m}{2\pi r}\right)^2}}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\Gamma}{m}\right)^2}}$$

Thus, for a given Γ and m the angle α is a constant.

6.57

6.57 For a free vortex (see Video V6.4) determine an expression for the pressure gradient (a) along a streamline, and (b) normal to a streamline. Assume the streamline is in a horizontal plane, and express your answer in terms of the circulation.

For a free vortex

$$\psi = -\frac{\Gamma}{2\pi} \ln r \qquad \text{(Eq. 6.91)}$$

so that

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0 \qquad v_\theta = -\frac{\partial \psi}{\partial r} = \frac{\Gamma}{2\pi r}$$

Since the free vortex represents an irrotational flow field, the Bernoulli equation

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant} \qquad (1)$$

is valid between any two points.

(a) Along a streamline (r = constant), v_θ is constant and $v_r = 0$ so that from Eq. (1) with z constant the pressure is constant, i.e.,

$$\underline{\underline{\frac{\partial p}{\partial \theta} = 0}}$$

(b) Normal to the streamline with $v_r = 0$ and z = constant

$$\frac{p}{\gamma} + \frac{v_\theta^2}{2g} + z = \text{constant}$$

so that

$$\frac{\partial p}{\partial r} = -\frac{\gamma}{2g}\frac{\partial (v_\theta^2)}{\partial r} = -\rho v_\theta \frac{\partial v_\theta}{\partial r}$$

$$= -\rho\left(\frac{\Gamma}{2\pi r}\right)\left(-\frac{\Gamma}{2\pi r^2}\right)$$

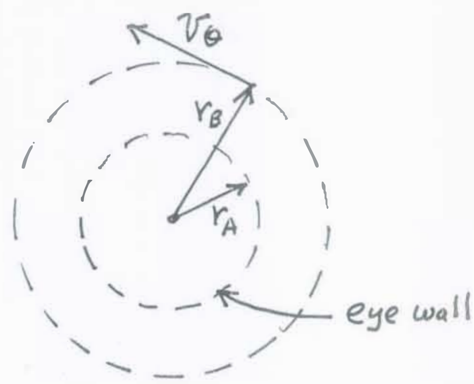
$$= \underline{\underline{\frac{\rho \Gamma^2}{4\pi^2 r^3}}}$$

6.58

6.58 (See Fluids in the News article titled "Some hurricanes facts," Section 6.5.3.) Consider a category five hurricane that has a maximum wind speed of 160 mph at the eye wall, 10 miles from the center of the hurricane. If the flow in the hurricane outside of the hurricane's eye is approximated as a free vortex, determine the wind speeds at locations 20 mi, 30 mi, and 40 mi from the center of the storm.

For free vortex

$$v_\theta = \frac{K}{r} \qquad (\text{Eq. } 6.86)$$



Thus, at eye wall

$$160 \text{ mph} = \frac{K}{10 \text{ mi}}$$

so that

$$K = (160 \text{ mph})(10 \text{ mi})$$

and

$$v_\theta = \frac{(160 \text{ mph})(10 \text{ mi})}{r_B}$$

For,

$r_B = 20 \text{ mi}$ $\qquad v_\theta = \dfrac{(160 \text{ mph})(10 \text{ mi})}{20 \text{ mi}} = \underline{\underline{80.0 \text{ mph}}}$

$r_B = 30 \text{ mi}$ $\qquad v_\theta = \dfrac{(160 \text{ mph})(10 \text{ mi})}{30 \text{ mi}} = \underline{\underline{53.3 \text{ mph}}}$

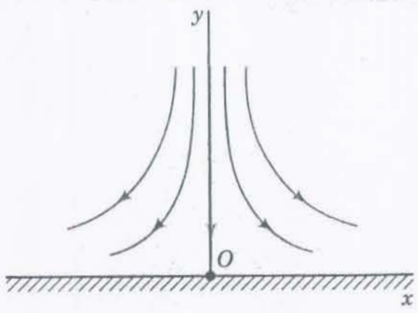
$r_B = 40 \text{ mi}$ $\qquad v_\theta = \dfrac{(160 \text{ mph})(10 \text{ mi})}{40 \text{ mi}} = \underline{\underline{40.0 \text{ mph}}}$

6.60

6.60 Potential flow against a flat plate (Fig. P6.60*a*) can be described with the stream function

$$\psi = Axy$$

where A is a constant. This type of flow is commonly called a "stagnation point" flow since it can be used to describe the flow in the vicinity of the stagnation point at O. By adding a source of strength, m, at O, stagnation point flow against a flat plate with a "bump" is obtained as illustrated in Fig. P6.60*b*. Determine the relationship between the bump height, h, the constant, A; and the source strength, m.



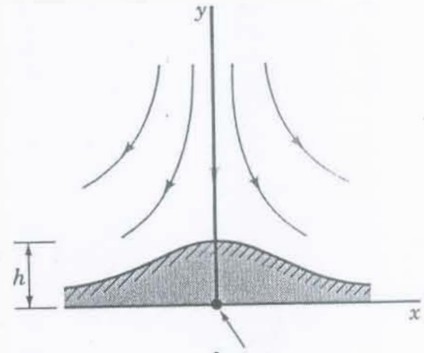


FIGURE P6.60

$$\psi = Axy + \frac{m}{2\pi}\theta = \frac{A}{2}r^2 \sin 2\theta + \frac{m}{2\pi}\theta$$

For the bump the stagnation point will occur at $x=0$, $y=h$ $(\theta = \frac{\pi}{2}, r = h)$. For the given stream function,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = Ar\cos 2\theta + \frac{m}{2\pi r} \qquad (1)$$

and

$$v_\theta = -\frac{\partial \psi}{\partial r} = Ar \sin 2\theta$$

The point, $\theta = \frac{\pi}{2}$, $r = h$, will be a stagnation point if $v_r = 0$ since $v_\theta = 0$ at this point. Thus, from Eq. (1)

$$0 = Ah\cos\pi + \frac{m}{2\pi h}$$

or

$$Ah = \frac{m}{2\pi h}$$

and therefore

$$\underline{\underline{h^2 = \frac{m}{2\pi A}}}$$

6.61

6.61 The combination of a uniform flow and a source can be used to describe flow around a streamlined body called a half-body. (See Video V6.5.) Assume that a certain body has the shape of a half-body with a thickness of 0.5 m. If this body is placed in an air stream moving at 15 m/s, what source strength is required to simulate flow around the body?

The width of half-body $= 2\pi b$ (See Fig. 6.24)

so that

$$b = \frac{(0.5\,m)}{2\pi}$$

From Eq. 6.99

$$b = \frac{m}{2\pi U}$$

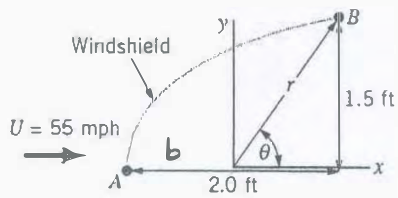
where m is the source strength, and therefore

$$m = 2\pi U b = 2\pi\left(15\,\frac{m}{s}\right)\left(\frac{0.5\,m}{2\pi}\right)$$

$$= \underline{\underline{7.50\,\frac{m^2}{s}}}$$

6.62

6.62 A vehicle windshield is to be shaped as a portion of a half-body with the dimensions shown in Fig. P6.62. (a) Make a scale drawing of the windshield shape. (b) For a free stream velocity of 55 mph, determine the velocity of the air at points A and B.



■ FIGURE P6.62

(a) From the figure

$$b + r\cos\theta = 2\text{ ft} \qquad (1)$$

$$r\sin\theta = 1.5\text{ft} \qquad (2)$$

and for a half-body

$$r = \frac{b(\pi - \theta)}{\sin\theta} \qquad (\text{Eq. } 6.100)$$

The above equations can be combined to give

$$\frac{1}{\pi - \theta} + \frac{1}{\tan\theta} = \frac{2}{1.5}$$

and a trial and error solution for θ gives

$$\theta = 0.839 \text{ rad} \quad (48.1°)$$

so that

$$b = \frac{r\sin\theta}{\pi - \theta} = \frac{1.5\text{ft}}{\pi - 0.839\text{ rad}} = 0.651\text{ ft}$$

Thus,

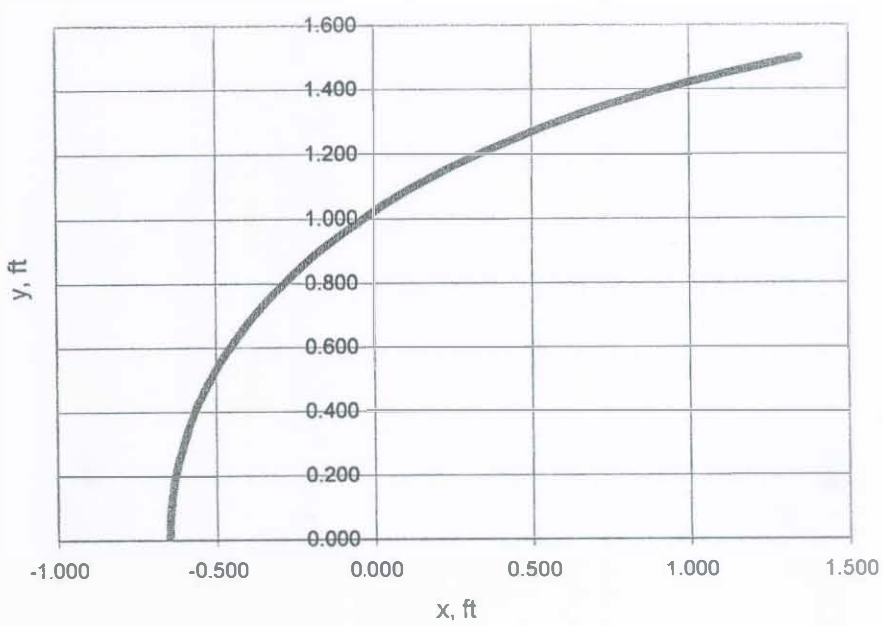
$$r = \frac{0.651\text{ ft}(\pi - \theta)}{\sin\theta} \qquad (3)$$

Equation (3) gives the profile of the windshield and with $x = r\cos\theta$ and $y = r\sin\theta$ the x and y coordinates can be obtained. Tabulated data and a plot of the data follows.

(con't)

6.62 (Con't)

Theta, rad	r, ft	x, ft	y, ft
3.142	0.651	-0.651	0.000
3.042	0.652	-0.649	0.065
2.942	0.655	-0.642	0.130
2.842	0.661	-0.631	0.195
2.742	0.669	-0.616	0.260
2.642	0.679	-0.596	0.326
2.542	0.692	-0.571	0.391
2.442	0.707	-0.541	0.456
2.342	0.726	-0.506	0.521
2.242	0.748	-0.465	0.586
2.142	0.774	-0.418	0.651
2.042	0.804	-0.364	0.716
1.942	0.838	-0.304	0.781
1.842	0.878	-0.235	0.846
1.742	0.925	-0.157	0.911
1.642	0.979	-0.069	0.977
1.542	1.042	0.030	1.042
1.442	1.116	0.144	1.107
1.342	1.203	0.273	1.172
1.242	1.307	0.423	1.237
1.142	1.432	0.596	1.302
1.042	1.584	0.800	1.367
0.942	1.771	1.042	1.432
0.839	2.015	1.346	1.499



(b) $V^2 = U^2\left(1 + 2\frac{b}{r}\cos\theta + \frac{b^2}{r^2}\right)$ (Eq. 6.101)

Point A is a stagnation point so that $\underline{V_A = 0}$.

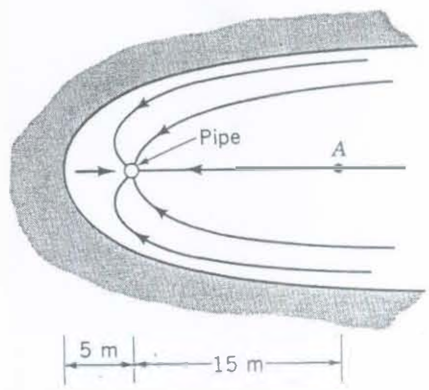
At the top of the windshield (point B) $\theta = 0.839$ rad and $r = 2.01$ ft so that

$$V_B^2 = (55\text{ mph})^2\left[1 + 2\left(\frac{0.651\text{ ft}}{2.01\text{ ft}}\right)\cos(0.839\text{ rad}) + \left(\frac{0.651\text{ ft}}{2.01\text{ ft}}\right)^2\right]$$

$V_B = \underline{68.2\text{ mph}}$

6.63

6.63 One end of a pond has a shoreline that resembles a half-body as shown in Fig. P6.63. A vertical porous pipe is located near the end of the pond so that water can be pumped out. When water is pumped at the rate of 0.08 m^3/s through a 3-m-long pipe, what will be the velocity at point A? *Hint*: Consider the flow *inside* a half-body. (See Video V6.5.)



■ FIGURE P6.63

For a half-body,

$$\psi = U r \sin\theta + \frac{m}{2\pi}\theta \qquad \text{(Eq. 6.97)}$$

so that

$$v_\theta = -\frac{\partial \psi}{\partial r} = U \sin\theta$$

and

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U\cos\theta + \frac{m}{2\pi r}$$

Thus, at point A, $\theta = 0$, $r = 15\,m$ and

$$v_\theta = 0$$

$$v_r = V_A = U + \frac{m}{2\pi(15)} \qquad (1)$$

For a flowrate of $0.06\,\frac{m^3}{s}$ in a 3-m long pipe, the source strength is $\frac{0.06}{3}\,\frac{m^2}{s}$. Since

$$b = \frac{m}{2\pi U} \qquad \text{(Eq. 6.99)}$$

then with $b = 5m$

$$U = \frac{m}{2\pi b} = \frac{\left(\frac{0.06}{3}\,\frac{m^2}{s}\right)}{2\pi(5m)} = 6.37 \times 10^{-4}\,\frac{m}{s}$$

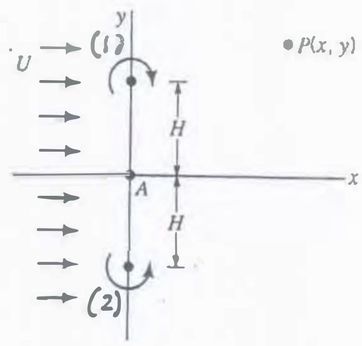
From Eq. (1)

$$V_A = 6.37 \times 10^{-4}\,\frac{m}{s} + \frac{\left(\frac{0.06}{3}\,\frac{m^2}{s}\right)}{2\pi\,(15m)}$$

$$= \underline{\underline{8.49 \times 10^{-4}\,\frac{m}{s}}}$$

6.64

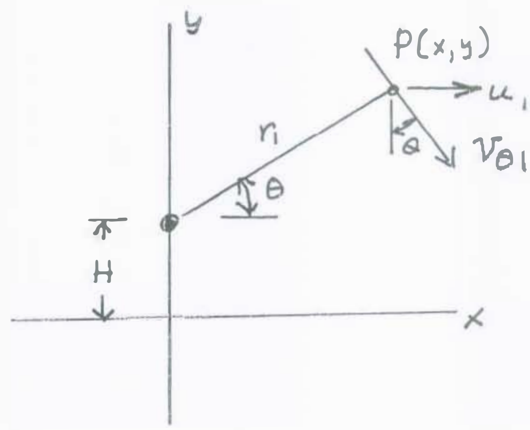
6.64 Two free vortices of equal strength, but opposite direction of rotation, are superimposed with a uniform flow as shown in Fig. P6.64. The stream functions for these two vorticies are $\psi = -[\pm\Gamma/(2\pi)] \ln r$. (a) Develop an equation for the x-component of velocity, u, at point $P(x,y)$ in terms of Cartesian coordinates x and y. (b) Compute the x-component of velocity at point A and show that it depends on the ratio Γ/H.



■ FIGURE P6.64

(a) For vortex (1), $\psi_1 = \dfrac{\Gamma}{2\pi} \ln r_1$

and $v_{\theta 1} = -\dfrac{\Gamma}{2\pi r_1}$ as shown.



$$u_1 = v_{\theta 1} \sin\theta$$

where $\sin\theta = \dfrac{y-H}{\left[(y-H)^2 + x^2\right]^{1/2}}$

and

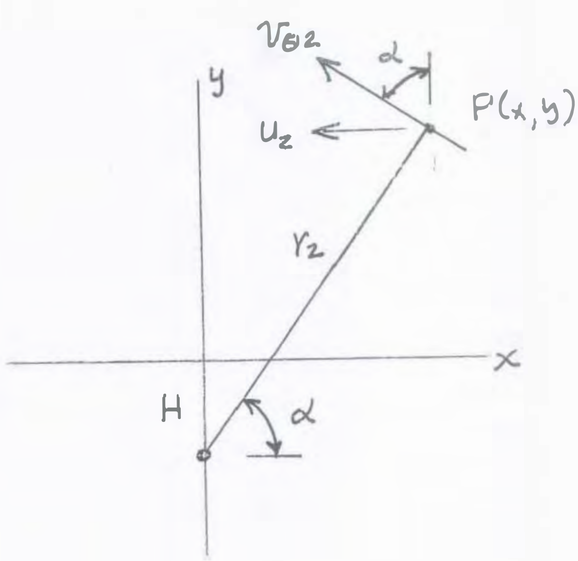
$$r_1 = \left[(y-H)^2 + x^2\right]^{1/2}$$

so that

$$u_1 = \left(\frac{\Gamma}{2\pi}\right)\left(\frac{y-H}{(y-H)^2 + x^2}\right)$$

For vortex (2), $\psi_2 = -\dfrac{\Gamma}{2\pi} \ln r_2$

and $v_{\theta 2} = \dfrac{\Gamma}{2\pi r_2}$ as shown.



$$u_2 = -v_{\theta 2} \sin\alpha$$

where $\sin\alpha = \dfrac{y+H}{\left[(y+H)^2 + x^2\right]^{1/2}}$

and

$$r_2 = \left[(y+H)^2 + x^2\right]^{1/2}$$

so that

$$u_2 = -\left(\frac{\Gamma}{2\pi}\right)\left(\frac{y+H}{(y+H)^2 + x^2}\right)$$

(con't)

6.64 (con't)

Thus, combining the two vortices with the uniform flow gives the x-component of velocity

$$u = u_1 + u_2 + U$$

$$= \frac{\Gamma}{2\pi}\left[\frac{y-H}{(y-H)^2+x^2} - \frac{y+H}{(y+H)^2+x^2}\right] + U \qquad (1)$$

(b) At point A where $x = y = 0$, Eq. (1) gives

$$u_A = U - \frac{\Gamma}{\pi H}$$

6.65

6.65 A Rankine oval is formed by combining a source–sink pair, each having a strength of 36 ft^2/s, and separated by a distance of 12 ft along the x axis, with a uniform velocity of 10 ft/s (in the positive x direction). Determine the length and thickness of the oval.

$$\frac{\ell}{a} = \left[\frac{m}{\pi U a} + 1\right]^{1/2} \qquad \text{(Eq. 6.107)}$$

$$\frac{h}{a} = \frac{1}{2}\left[\left(\frac{h}{a}\right)^2 - 1\right] \tan\left[2\left(\frac{\pi U a}{m}\right)\frac{h}{a}\right] \qquad \text{(Eq. 6.109)}$$

For $m = 36\ \frac{ft^2}{s}$, $a = 6\ ft$, and $U = 10\ \frac{ft}{s}$,

$$\frac{\pi U a}{m} = \frac{\pi\left(10\frac{ft}{s}\right)(6\ ft)}{36\ \frac{ft^2}{s}} = 5.24$$

Thus, length $= 2\ell$ and from Eq. 6.107

$$\underline{\underline{\text{length}}} = 2\,(6\ ft)\left[\frac{1}{5.24} + 1\right]^{1/2} = \underline{\underline{13.1\ ft}}$$

The thickness, $2h$, can be determined from Eq. 6.109 by trial and error. Assume value for h/a and compare with right hand side of Eq. 6.109. (See table below.)

$\frac{h}{a}$	$\frac{1}{2}\left[\left(\frac{h}{a}\right)^2 - 1\right]\tan\left[2\,(5.24)\frac{h}{a}\right]$
0.250	0.269
0.251	0.262
0.252	0.256
0.253	0.250 ← use

Thus, $\frac{h}{a} \approx 0.253$

and $\underline{\underline{\text{thickness}}} = 2h = 2\,(6\ ft)(0.253) = \underline{\underline{3.04\ ft}}$

*6.66

***6.66** Make use of Eqs. 6.107 and 6.109 to construct a table showing how ℓ/a, h/a, and ℓ/h for Rankine ovals depend on the parameter $\pi Ua/m$. Plot ℓ/h versus $\pi Ua/m$ and describe how this plot could be used to obtain the required values of m and a for a Rankine oval having a specific value of ℓ and h when placed in a uniform fluid stream of velocity, U.

For a Rankine oval

$$\frac{\ell}{a} = \left[\frac{m}{\pi U a} + 1\right]^{1/2} \qquad \text{(Eq. 6.107)}$$

and

$$\frac{h}{a} = \frac{1}{2}\left[\left(\frac{h}{a}\right)^2 - 1\right] \tan\left[2\left(\frac{\pi U a}{m}\right)\frac{h}{a}\right] \qquad \text{(Eq. 6.109)}$$

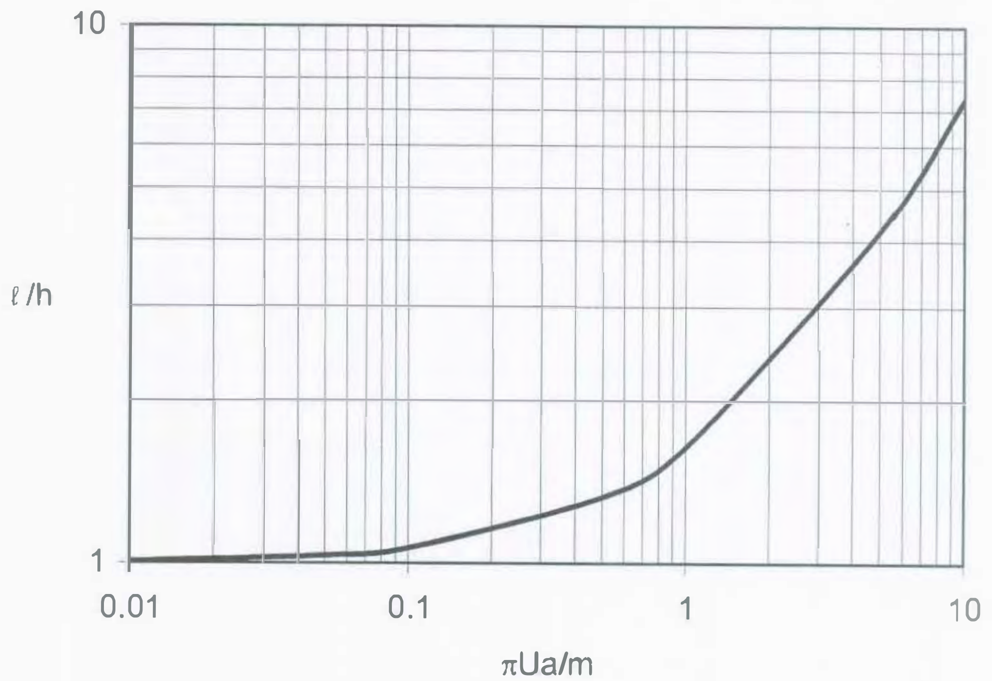
where the length of the body is 2ℓ and the width is $2h$. For a given value of $\pi Ua/m$, Eq. 6.107 can be solved for ℓ/a, and Eq. 6.109 can be solved (using an iteration procedure) for h/a. The ratio ℓ/h can then be determined. Tabulated data are given below.

$\pi Ua/m$	ℓ/a	h/a	ℓ/h
10	1.049	0.143	7.342
5	1.095	0.263	4.169
1	1.414	0.860	1.644
0.5	1.732	1.306	1.326
0.1	3.317	3.111	1.066
0.05	4.583	4.435	1.033
0.01	10.050	9.983	1.007

A plot of the data is shown on the next page.

(con't)

*6.66 (con't)



For a Rankine oval with ℓ and h specified the following steps could be followed to determine m and a:

(1) For a given ℓ/h determine the required value of $\pi U a/m$ from the graph.

(2) Using this value of $\pi U a/m$ calculate ℓ/a from Eq. 6.107.

(3) With the value of ℓ/a determined, and ℓ specified, determine the value of a.

(4) With $\pi U a/m$ and a determined, the value of U/m is known, and for a given U the value of m is fixed.

6.67

6.67 An ideal fluid flows past an infinitely long semicircular "hump" located along a plane boundary as shown in Fig. P6.67. Far from the hump the velocity field is uniform, and the pressure is p_0. (a) Determine expressions for the maximum and minimum values of the pressure along the hump, and indicate where these points are located. Express your answer in terms of ρ, U, and p_0. (b) If the solid surface is the $\psi = 0$ streamline, determine the equation of the streamline passing through the point $\theta = \pi/2$, $r = 2a$.

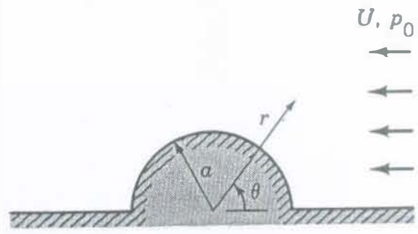


FIGURE P6.67

(a) On the surface of the hump,

$$p_s = p_0 + \frac{1}{2}\rho U^2 \left(1 - 4\sin^2\theta\right) \qquad \text{(Eq. 6.116)}$$

The maximum pressure occurs where $\sin\theta = 0$, or at $\theta = 0, \pi$, and at these points

$$\underline{\underline{p_s(\text{max}) = p_0 + \frac{1}{2}\rho U^2}} \qquad \underline{\underline{(\text{at } \theta = 0 \text{ and } \pi)}}$$

The minimum pressure occurs where $\sin\theta = 1$, or at $\theta = \frac{\pi}{2}$, and at this point

$$\underline{\underline{p_s(\text{min}) = p_0 - \frac{3}{2}\rho U^2}} \qquad \underline{\underline{(\text{at } \theta = \frac{\pi}{2})}}$$

(b) For uniform flow in the negative x-direction,

$$\psi = -Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$$

(refer to discussion associated with the derivation of Eq. 6.112).
At $\theta = \frac{\pi}{2}$, $r = 2a$

$$\psi = -2aU\left(1 - \frac{a^2}{(2a)^2}\right)\sin\frac{\pi}{2} = -\frac{3}{2}aU$$

and thus the equation of the streamline passing through this point is

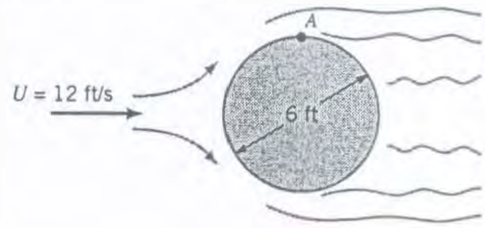
$$-\frac{3}{2}aU = -Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$$

or

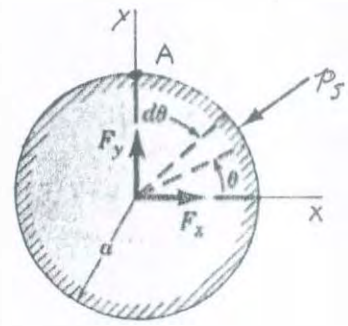
$$\underline{\underline{\frac{2}{3}\frac{r}{a}\left(1 - \frac{a^2}{r^2}\right)\sin\theta = 1}}$$

6.68

6.68 Water flows around a 6-ft diameter bridge pier with a velocity of 12 ft/s. Estimate the force (per unit length) that the water exerts on the pier. Assume that the flow can be approximated as an ideal fluid flow around the front half of the cylinder, but due to flow separation (see Video V6.8), the average pressure on the rear half is constant and approximately equal to ½ the pressure at point A (see Fig. P6.68).



■ FIGURE P6.68



From Fig. 6.28 it follows that the drag on a section (between $\theta = 0$ and $\theta = \alpha$) of a circular cylinder is given by the equation

$$Drag = F_x = -\int_0^{\alpha} p_s \cos\theta \, a \, d\theta$$

For the force on the front half of the cylinder (per unit length)

$$F_{x_1} = -2\int_{\pi/2}^{\pi} p_s \cos\theta \, a \, d\theta \qquad (1)$$

and due to symmetry $F_y = 0$. From Eq. 6.116

$$p_s = p_0 + \frac{1}{2}\rho U^2 \left(1 - 4\sin^2\theta\right) \qquad (Eq.\ 6.116)$$

and since we are only interested in the force due to the flowing fluid we will let $p_0 = 0$. Thus, from Eq. (1)

$$F_{x_1} = -2\int_{\pi/2}^{\pi} \frac{1}{2}\rho U^2 \left(1 - 4\sin^2\theta\right)\cos\theta \, a \, d\theta \qquad (2)$$

Since

$$\int_{\pi/2}^{\pi} \cos\theta \, d\theta = \sin\theta \Big]_{\pi/2}^{\pi} = -1$$

and

$$\int_{\pi/2}^{\pi} \sin^2\theta \cos\theta \, d\theta = \frac{\sin^3\theta}{3}\Big]_{\pi/2}^{\pi} = -\frac{1}{3}$$

(con't)

6.68 (con't)

It follows from Eq.(2) that

$$F_{x_1} = -\frac{\rho U^2 a}{3}$$

Note that the negative sign indicates that the water is actually "pulling" on the cylinder (front half) in the upstream direction. However, when the effect of the rear half of the cylinder is taken into account (in a real fluid) there will be a net drag in the direction of flow.

The pressure at the top of the cylinder (point A) is given by

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta) \qquad \text{(Eq. 6.116)}$$

and with $\theta = \pi/2$

$$p_A = p_0 - \frac{3}{2}\rho U^2$$

Since $p_0 = 0$

$$p_A = -\frac{3}{2}\rho U^2$$

Note that the negative pressure will give a positive F_x and

$$F_{x_2} = -\frac{p_A}{2} \times \text{projected area} = -\frac{p_A}{2} \times 2a(1)$$

So that

$$F_{x_2} = \frac{3}{4}\rho U^2 (2a)(1) = \frac{3}{2}\rho U^2 a$$

Thus,

$$F_x = F_{x_1} + F_{x_2}$$

$$= -\frac{\rho U^2 a}{3} + \frac{3\rho U^2 a}{2}$$

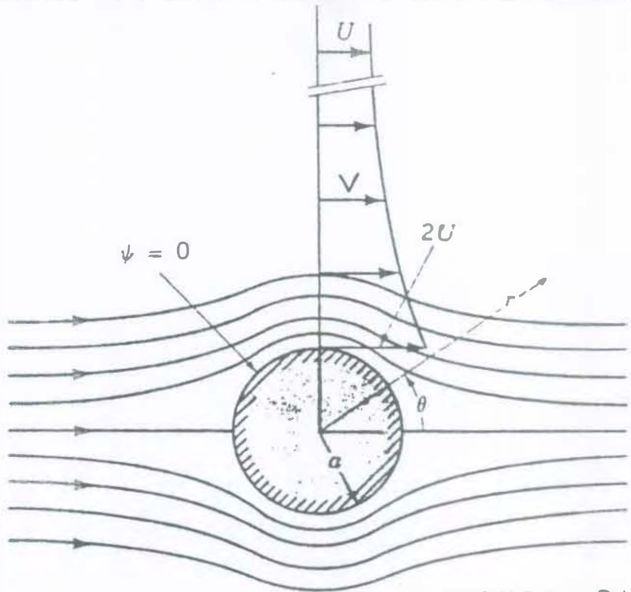
$$= \frac{7}{6}\rho U^2 a$$

With the data given,

$$F_x = \frac{7}{6}\left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(12\ \frac{\text{ft}}{\text{s}}\right)^2 (3\ \text{ft}) = \underline{\underline{978\ \frac{\text{lb}}{\text{ft}}}}$$

*6.69

***6.69** Consider the steady potential flow around the circular cylinder shown in Fig. 6.26. Show on a plot the variation of the magnitude of the dimensionless fluid velocity, V/U, along the positive y axis. At what distance, y/a (along the y axis), is the velocity within 1% of the free-stream velocity?



■ FIGURE 6.26

Along the y-axis $v_r = 0$ so that the magnitude of the velocity, V, is equal to $|v_\theta|$. Since

$$v_\theta = -U\left(1+\frac{a^2}{r^2}\right)\sin\theta \qquad \text{(Eq. 6.115)}$$

it follows that along the positive y-axis ($\theta = \frac{\pi}{2}$, $r = y$)

$$V = |v_\theta| = U\left(1+\frac{a^2}{y^2}\right)$$

or

$$\frac{V}{U} = 1+\frac{a^2}{y^2} = 1+\frac{1}{\left(\frac{y}{a}\right)^2} \qquad (1)$$

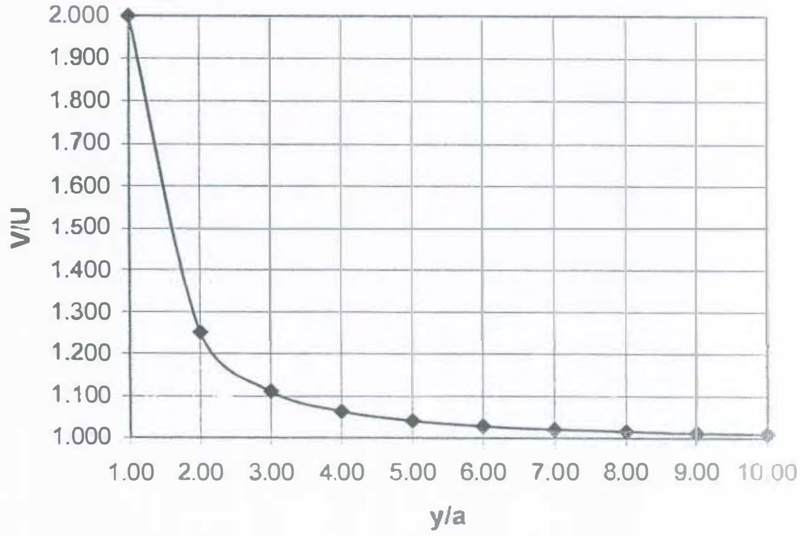
Tabulated data and a plot of the data are given below. It can be seen from these results that for

$$\underline{\underline{\frac{y}{a} \geq 10}}$$

the velocity V is within 1% of the free-stream velocity U.

y/a	V/U
1.00	2.000
2.00	1.250
3.00	1.111
4.00	1.063
5.00	1.040
6.00	1.028
7.00	1.020
8.00	1.016
9.00	1.012
10.00	1.010

Calculated from Eq. (1)



6.70

6.70 The velocity potential for a cylinder (Fig. P6.70) rotating in a uniform stream of fluid is

$$\phi = Ur\left(1 + \frac{a^2}{r^2}\right)\cos\theta + \frac{\Gamma}{2\pi}\theta$$

where Γ is the circulation. For what value of the circulation will the stagnation point be located at: **(a)** point A, **(b)** point B?

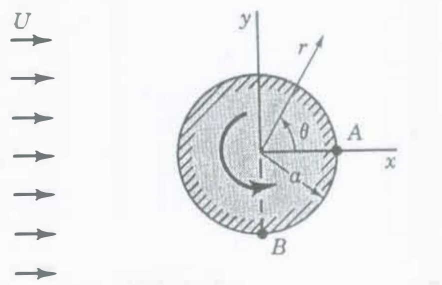


FIGURE P6.70

(a) $$\sin\theta_{stag} = \frac{\Gamma}{4\pi U a} \quad \text{(Eq. 6.122)}$$

At point A, $\theta_{stag} = 0$ and it follows that $\underline{\underline{\Gamma = 0}}$.

(b) At point B, $\theta_{stag} = \frac{3\pi}{2}$, and from Eq. 6.122

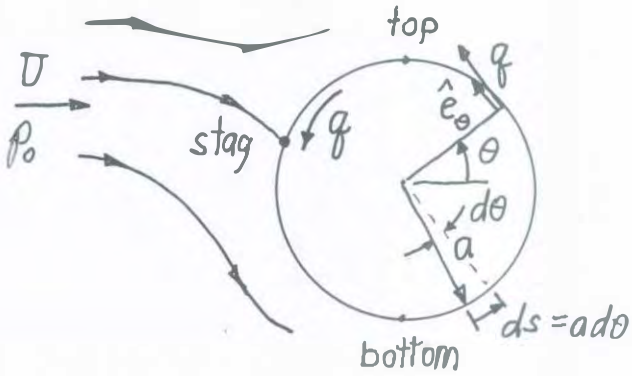
$$\Gamma = 4\pi U a \sin\frac{3\pi}{2} = \underline{\underline{-4\pi U a}}$$

6.71

6.71 Show that for a rotating cylinder in a uniform flow, the following pressure ratio equation is true.

$$\frac{p_{top} - p_{bottom}}{p_{stagnation}} = \frac{8q}{U}$$

Here U is the velocity of the uniform flow and q is the surface speed of the rotating cylinder.



From Eq. 6.123 the pressure on the surface is

(1) $$p_s = p_o + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta + \frac{2\Gamma\sin\theta}{\pi a U} - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\right)$$

where $\Gamma = \oint_C \vec{V}\cdot d\hat{s}$ is the circulation produced by the rotating cylinder

Thus, for the curve C = cylinder surface where $\vec{V}\cdot d\hat{s} = (q\hat{e}_\theta)\cdot(a\,d\theta\,\hat{e}_\theta) = aq\,d\theta$

we obtain

(2) $$\Gamma = \int_{\theta=0}^{2\pi} aq\,d\theta = 2\pi aq$$

From Eq.(1), at the top ($\theta = 90°$):

$$p_{top} = p_o + \frac{1}{2}\rho U^2\left(1 - 4 + \frac{2\Gamma}{\pi a U} - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\right)$$

and at the bottom ($\theta = 270°$):

$$p_{bottom} = p_o + \frac{1}{2}\rho U^2\left(1 - 4 - \frac{2\Gamma}{\pi a U} - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\right)$$

so that

$$p_{top} - p_{bottom} = \frac{1}{2}\rho U^2\left(\frac{4\Gamma}{\pi a U}\right), \quad \text{where } \frac{1}{2}\rho U^2 = p_{stagnation}$$

Note: The stagnation point has $V = 0$, but does not occur at the front "edge" of the rotating cylinder.

Thus, $$\frac{p_{top} - p_{bottom}}{p_{stagnation}} = \frac{4\Gamma}{\pi a U}$$

or using Eq.(2),

$$\frac{p_{top} - p_{bottom}}{p_{stagnation}} = \frac{4}{\pi a U}(2\pi aq) = \underline{\underline{\frac{8q}{U}}}$$

6.72

6.72 (See Fluids in the News article titled "A sailing ship without sails," Section 6.6.3.) Determine the magnitude of the total force developed by the two rotating cylinders on the Flettner "rotor-ship" due to the Magnus effect. Assume a wind speed relative to the ship of **(a)** 10 mph and **(b)** 30 mph. Each cylinder has a diameter of 9 ft, a length of 50 ft, and rotates at 750 rev/min. Use Eq. 6.124 and calculate the circulation by assuming the air sticks to the rotating cylinders. *Note*: This calculated force is at right angles to the direction of the wind and it is the component of this force in the direction of motion of the ship that gives the propulsive thrust. Also, due to viscous effects, the actual propulsive thrust will be smaller than that calculated from Eq. 6.124 which is based on inviscid flow theory.

$F_y = -\rho U \Gamma$ (force per unit length) (Eq. 6.124)

$\Gamma = \oint \vec{V} \cdot \vec{ds}$ (Eq. 6.89)

On the cylinder surface

$$\vec{V} = r\omega\, \hat{e}_\theta \quad \text{and} \quad \vec{ds} = r\,d\theta\, \hat{e}_\theta$$

so that

$$\Gamma = \int_0^{2\pi} (r\omega)(r\,d\theta)\, \hat{e}_\theta \cdot \hat{e}_\theta = 2\pi r^2 \omega$$

$$= (2\pi)(4.5\,ft)^2 \left(750\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{1\,min}{60\,s}\right)$$

$$= 9990\,\frac{ft^2}{s}$$

and

$$F_y = -\left(0.00238\,\frac{slugs}{ft^3}\right)\left(9990\,\frac{ft^2}{s}\right) U = -23.8\,U$$

(a) For a cylinder with length = 50 ft and number of cylinders = 2 and wind speed = 10 mph,

$$|F_y| = \left(23.8\,\frac{lb\cdot s}{ft^2}\right)\left(10\,\frac{mi}{hr}\right)\left(5280\,\frac{ft}{mi}\right)\left(\frac{1\,hr}{3600\,s}\right)(50\,ft)(2)$$

$$= \underline{\underline{34{,}900\,lb}}$$

(b) At 30 mph

$$|F_y| = 3 \times (F_y\text{ @ }10\,mph) = \underline{\underline{105{,}000\,lb}}$$

6.73

6.73 A fixed circular cylinder of infinite length is placed in a steady, uniform stream of an incompressible, nonviscous fluid. Assume that the flow is irrotational. Prove that the drag on the cylinder is zero. Neglect body forces.

$$Drag = F_x = -\int_0^{2\pi} p_s \cos\theta \, a \, d\theta \qquad \text{(Eq. 6.117)}$$

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta) \qquad \text{(Eq. 6.116)}$$

Thus,

$$Drag = -\left\{ a p_0 \int_0^{2\pi} \cos\theta \, d\theta + \frac{a}{2}\rho U^2 \int_0^{2\pi} \cos\theta \, d\theta - 2a\rho U^2 \int_0^{2\pi} \sin^2\theta \cos\theta \, d\theta \right\}$$

Since,

$$\int_0^{2\pi} \cos\theta \, d\theta = \sin\theta \Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi} \sin^2\theta \cos\theta \, d\theta = \frac{\sin^3\theta}{3} \Big]_0^{2\pi} = 0$$

it follows that

$$\underline{\underline{Drag = 0}}$$

6.74

6.74 Repeat Problem 6.73 for a rotating cylinder for which the stream function and velocity potential are given by Eqs. 6.119 and 6.120, respectively. Verify that the lift is not zero and can be expressed by Eq. 6.124.

$$\text{Drag} = F_x = -\int_0^{2\pi} p_s \cos\theta \, a\, d\theta \qquad (\text{Eq. } 6.117)$$

$$p_s = p_0 + \tfrac{1}{2}\rho U^2\left(1 - 4\sin^2\theta + \frac{2\Gamma\sin\theta}{\pi a U} - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\right) \quad (\text{Eq. } 6.123)$$

Thus,

$$\text{Drag} = -\left\{ a p_0 \int_0^{2\pi}\cos\theta\, d\theta + \frac{a}{2}\rho U^2\left[\int_0^{2\pi}\cos\theta\, d\theta - 4\int_0^{2\pi}\sin^2\theta\cos\theta\, d\theta \right.\right.$$
$$\left.\left. + \frac{2\Gamma}{\pi a U}\int_0^{2\pi}\cos\theta\sin\theta\, d\theta - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\int_0^{2\pi}\cos\theta\, d\theta\right]\right\}$$

Since,

$$\int_0^{2\pi}\cos\theta\, d\theta = \sin\theta\Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi}\sin^2\theta\cos\theta\, d\theta = \frac{\sin^3\theta}{3}\Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi}\cos\theta\sin\theta\, d\theta = \frac{\sin^2\theta}{2}\Big]_0^{2\pi} = 0$$

it follows that

$$\underline{\underline{\text{Drag} = 0}}$$

(con't)

6.74 (Con't)

$$\text{Lift} = F_y = -\int_0^{2\pi} p_s \sin\theta \; a\,d\theta \qquad \text{(Eq. 6.118)}$$

with p_s given by Eq. 6.123 it follows that

$$\text{Lift} = -\left\{ a\,p_0 \int_0^{2\pi} \sin\theta\,d\theta + \frac{a}{2}\rho U^2 \left[\int_0^{2\pi} \sin\theta\,d\theta - 4\int_0^{2\pi} \sin^3\theta\,d\theta + \frac{2\Gamma}{\pi a U}\int_0^{2\pi} \sin^2\theta\,d\theta - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\int_0^{2\pi} \sin\theta\,d\theta \right] \right\}$$

Since,

$$\int_0^{2\pi} \sin\theta\,d\theta = -\cos\theta \Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi} \sin^3\theta\,d\theta = -\frac{\cos\theta}{3}\left(\sin^2\theta + 2\right)\Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi} \sin^2\theta\,d\theta = \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\Big]_0^{2\pi} = \pi$$

it follows that

$$\text{Lift} = -\frac{a}{2}\rho U^2 \left(\frac{2\Gamma}{\pi a U}\right)(\pi)$$

Thus,

$$\underline{\underline{\text{Lift} = -\rho U \Gamma}}$$

(which is Eq. 6.124).

6.75

6.75 At a certain point at the beach, the coast line makes a right angle bend as shown in Fig. 6.75*a*. The flow of salt water in this bend can be approximated by the potential flow of an incompressible fluid in a right angle corner. (**a**) Show that the stream function for this flow is $\psi = A r^2 \sin 2\theta$, where A is a positive constant. (**b**) A fresh water reservoir is located in the corner. The salt water is to be kept away from the reservoir to avoid any possible seepage of salt water into the fresh water (Fig. 6.75*b*). The fresh water source can be approximated as a line source having a strength m, where m is the volume rate of flow (per unit length) emanating from the source. Determine m if the salt water is not to get closer than a distance L to the corner. *Hint:* Find the value of m (in terms of A and L) so that a stagnation point occurs at $y = L$. (**c**) The streamline passing through the stagnation point would represent the line dividing the fresh water from the salt water. Plot this streamline.

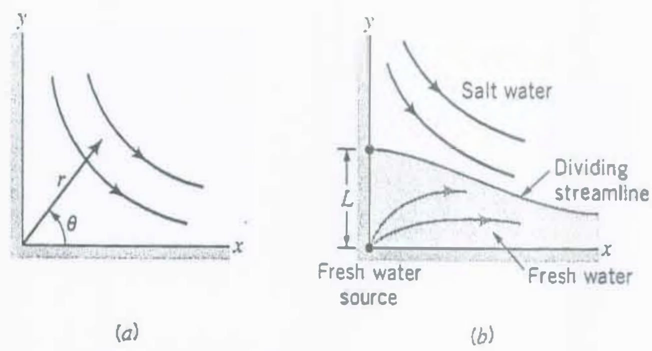


FIGURE P6.75

(a) For the given stream function,

$$\psi = A r^2 \sin 2\theta$$

along $\theta = 0$ $\psi = 0$ and $\theta = \pi/2$ $\psi = 0$.

Thus, the rays $\theta = 0$ and $\theta = \pi/2$ can be replaced with a solid boundary along which the stream function must be constant. This boundary forms a right angle and therefore this stream function can be used to represent flow in a right angle corner.

(b) Since

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 2 A r \cos 2\theta$$

at $\theta = \pi/2$

$$v_r = 2 A r \cos \pi = -2Ar$$

For a source located at the origin

$$\psi = \frac{m}{2\pi}\theta$$

and $$v_{r_s} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{m}{2\pi r}$$

To create a stagnation point at $r = L$ and $\theta = \frac{\pi}{2}$ let $v_r = v_{r_s}$.

(con't)

6.75 (con't)

Thus,

$$2AL = \frac{m}{2\pi L}$$

and

$$\underline{\underline{m = 4\pi A L^2}}$$

gives a stagnation point at $r = L$, $\theta = \pi/2$.

(c) The combined stream function is

$$\psi = Ar^2 \sin 2\theta + \frac{m}{2\pi}\theta$$

and with $m = 4\pi A L^2$

$$\psi = Ar^2 \sin 2\theta + 2AL^2\theta$$

The value of ψ at the stagnation point ($r = L$, $\theta = \pi/2$) is

$$\psi_{stag} = AL^2 \sin\pi + 2AL^2\left(\frac{\pi}{2}\right)$$

$$= AL^2\pi$$

Thus, the equation for the streamline passing through the stagnation point is

$$AL^2\pi = Ar^2 \sin 2\theta + 2AL^2\theta$$

or

$$r = \sqrt{\frac{\pi L^2 - 2L^2\theta}{\sin 2\theta}}$$

and

$$r' = \frac{r}{L} = \sqrt{\frac{\pi - 2\theta}{\sin 2\theta}} \qquad (1)$$

For plotting let

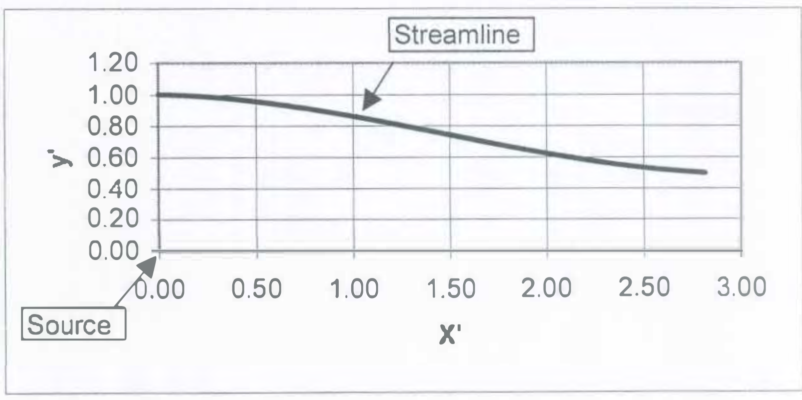
$$x' = r'\cos\theta \quad \text{and} \quad y' = r'\sin\theta$$

and a plot of the dividing streamline from Eq. (1) is shown on the following page.

(con't)

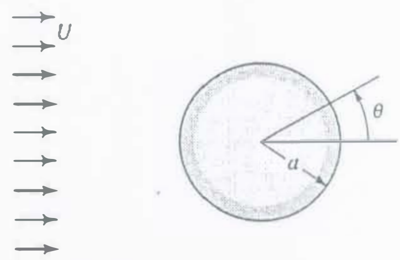
6.75 (con't)

Theta(deg)	Theta(rad)	r/L	x'	y'
10	0.175	2.857	2.814	0.496
20	0.349	1.950	1.832	0.667
30	0.524	1.555	1.347	0.778
40	0.698	1.331	1.020	0.856
50	0.873	1.191	0.765	0.912
60	1.047	1.100	0.550	0.952
70	1.222	1.042	0.356	0.979
80	1.396	1.010	0.175	0.995
90	1.571	1.000	0.000	1.000



6.76

6.76 Typical inviscid flow solutions for flow around bodies indicate that the fluid flows smoothly around the body, even for blunt bodies as shown in Video V6.10. However, experience reveals that due to the presence of viscosity, the main flow may actually separate from the body creating a wake behind the body. As discussed in a later section (Section 9.2.6), whether or not separation takes place depends on the pressure gradient along the surface of the body, as calculated by inviscid flow theory. If the pressure decreases in the direction of flow (a *favorable* pressure gradient), no separation will occur. However, if the pressure increases in the direction of flow (an *adverse* pressure gradient), separation may occur. For the circular cylinder of Fig. P6.76 placed in a uniform stream with velocity, U, determine an expression for the pressure gradient in the direction flow on the surface of the cylinder. For what range of values for the angle θ will an adverse pressure gradient occur?



■ FIGURE P6.76

From Eq. 6.116

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta)$$

Thus,

$$\frac{\partial p_s}{\partial \theta} = \underline{\underline{4\rho U^2 \sin\theta \cos\theta}} \qquad (1)$$

Since an adverse pressure gradient occurs for a positive $\partial p_s/\partial\theta$, it follows from Eq. (1) that θ falls in the range of $\pm 90°$ for an adverse pressure gradient. This range corresponds to the rear half of the cylinder.

6.78

6.78 For a steady, two-dimensional, incompressible flow, the velocity is given by $\mathbf{V} = (ax - cy)\hat{\mathbf{i}} + (-ay + cx)\hat{\mathbf{j}}$, where a and c are constants. Show that this flow can be considered inviscid.

For a two-dimensional flow the shearing stress is

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

With $u = ax - cy$ and $v = -ay + cx$ we obtain

$$\tau_{xy} = \mu(-c + c) = 0$$

Thus, $\tau_{xy} = \underline{\underline{0}}$ for all x, y and the flow can be considered inviscid with no shearing stress.

6.79

6.79 Determine the shearing stress for an incompressible Newtonian fluid with a velocity distribution of $\mathbf{V} = (3xy^2 - 4x^3)\hat{\mathbf{i}} + (12x^2y - y^3)\hat{\mathbf{j}}$.

The shearing stress for an incompressible* Newtonian fluid is

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$
$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$
$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

For the given velocity vector,

$$u = 3xy^2 - 4x^3$$
$$v = 12x^2y - y^3$$
$$w = 0$$

Thus,

$$\tau_{xy} = \mu(6xy + 24xy) = \underline{\underline{30\mu xy}}$$

$$\tau_{yz} = \mu(0+0) = \underline{\underline{0}}$$

and

$$\tau_{zx} = \mu(0+0) = \underline{\underline{0}}$$

* Note: For the given velocity

$$\nabla\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = (3y^2 - 12x^2) + (12x^2 - 3y^2) + (0) = 0$$

Hence,

$\nabla\cdot\vec{V} = 0$, the flow is incompressible.

6.80

6.80 The two-dimensional velocity field for an incompressible, Newtonian fluid is described by the relationship

$$\mathbf{V} = (12xy^2 - 6x^3)\hat{\mathbf{i}} + (18x^2y - 4y^3)\hat{\mathbf{j}}$$

where the velocity has units of m/s when x and y are in meters. Determine the stresses σ_{xx}, σ_{yy}, and τ_{xy} at the point $x = 0.5$ m, $y = 1.0$ m if pressure at this point is 6 kPa and the fluid is glycerin at 20 °C. Show these stresses on a sketch.

$$\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x} \qquad \text{(Eq. 6.125a)}$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y} \qquad \text{(Eq. 6.125b)}$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \text{(Eq. 6.125d)}$$

For the given velocity distribution, with $x = 0.5\,m$ and $y = 1.0\,m$:

$$\frac{\partial u}{\partial x} = 12y^2 - 18x^2 = 12(1.0)^2 - 18(0.5)^2 = 7.50\,\frac{1}{s}$$

$$\frac{\partial u}{\partial y} = 24xy = 24(0.5)(1.0) = 12.0\,\frac{1}{s}$$

$$\frac{\partial v}{\partial x} = 36xy = 36(0.5)(1.0) = 18.0\,\frac{1}{s}$$

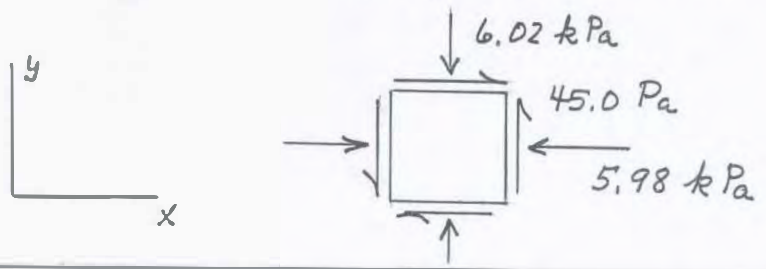
$$\frac{\partial v}{\partial y} = 18x^2 - 12y^2 = 18(0.5)^2 - 12(1.0)^2 = -7.50\,\frac{1}{s}$$

Thus, for $p = 6 \times 10^3\,\frac{N}{m^2}$ and $\mu = 1.50\,\frac{N\cdot s}{m^2}$,

$$\sigma_{xx} = -6\times 10^3\,\frac{N}{m^2} + 2\left(1.50\,\frac{N\cdot s}{m^2}\right)\left(7.50\,\frac{1}{s}\right) = \underline{\underline{-5.98\,kPa}}$$

$$\sigma_{yy} = -6\times 10^3\,\frac{N}{m^2} + 2\left(1.50\,\frac{N\cdot s}{m^2}\right)\left(-7.50\,\frac{1}{s}\right) = \underline{\underline{-6.02\,kPa}}$$

$$\tau_{xy} = \left(1.50\,\frac{N\cdot s}{m^2}\right)\left(12.0\,\frac{1}{s} + 18.0\,\frac{1}{s}\right) = \underline{\underline{45.0\,Pa}}$$



6.81

6.81 For a two-dimensional incompressible flow in the x–y plane show that the z component of the vorticity, ζ_z, varies in accordance with the equation

$$\frac{D\zeta_z}{Dt} = \nu \nabla^2 \zeta_z$$

What is the physical interpretation of this equation for a nonviscous fluid? *Hint:* This *vorticity transport equation* can be derived from the Navier–Stokes equations by differentiating and eliminating the pressure between Eqs. 6.127a and 6.127b.

For two-dimensional flow with $w=0$, Eq. 6.127a reduces to

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (1)$$

and Eq. 6.127b reduces to

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (2)$$

Differentiate Eq. (1) with respect to y and Eq. (2) with respect to x, and subtract Eq. (1) from Eq. (2) to obtain

$$\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) =$$

$$\frac{\mu}{\rho}\left[\frac{\partial}{\partial x}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right] \qquad (3)$$

By definition (see Eq. 6.17)

$$\zeta_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Re-write Eq. (3) to obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) =$$

$$\frac{\mu}{\rho}\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right] \qquad (4)$$

(con't)

6.81 (con't)

Since each term in parenthesis in Eq. (4) is ζ_z it follows that

$$\frac{\partial \zeta_z}{\partial t} + u \frac{\partial \zeta_z}{\partial x} + v \frac{\partial \zeta_z}{\partial y} = \frac{\mu}{\rho}\left(\frac{\partial^2 \zeta_z}{\partial x^2} + \frac{\partial^2 \zeta_z}{\partial y^2} \right) \qquad (5)$$

The left side of Eq. (5) can be expressed as (see Eq. 4.5) $\frac{D\zeta_z}{Dt}$ where the operator $\frac{D(\ )}{Dt}$ is the material derivative. The right hand side of Eq. (5) can be expressed as

$$\nu \nabla^2 \zeta_z$$

where $\nu = \mu/\rho$ so that Eq. (5) can be written as

$$\underline{\underline{\frac{D\zeta_z}{Dt} = \nu \nabla^2 \zeta_z}}$$

For a nonviscous fluid, $\nu = 0$, and in this case

$$\frac{D\zeta_z}{Dt} = 0$$

Thus, for a two-dimensional flow of an incompressible, nonviscous fluid, the change in the vorticity of a fluid particle as it moves through the flow field is zero.

6.82

6.82 The velocity of a fluid particle moving along a horizontal streamline that coincides with the x axis in a plane, two-dimensional incompressible flow field was experimentally found to be described by the equation $u = x^2$. Along this streamline determine an expression for: **(a)** the rate of change of the v-component of velocity with respect to y; **(b)** the acceleration of the particle; and **(c)** the pressure gradient in the x direction. The fluid is Newtonian.

(a) From the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

so that with $u = x^2$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \underline{\underline{-2x}} \qquad (1)$$

Also, Eq. (1) can be integrated with respect to y to obtain

$$\int dv = \int -2x\, dy$$

or

$$v = -2xy + f(x)$$

Since the x-axis is a streamline, $v=0$ along this axis and therefore $f(x)=0$ so that

$$v = -2xy$$

(b)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (x^2)(2x) + (-2xy)(0) = 2x^3$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (x^2)(-2y) + (-2xy)(-2x) = 2x^2 y$$

Along x-axis, $y=0$, and therefore $a_y=0$. Thus,

$$\vec{a} = \underline{\underline{2x^3\,\hat{i}}}$$

(c) From Eq. 6.127a (with $g_x = 0$),

$$a_x = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

so that

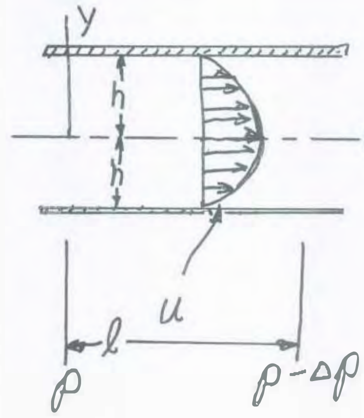
$$2x^3 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}(2+0)$$

and

$$\frac{\partial p}{\partial x} = \underline{\underline{2\mu - 2\rho x^3}}$$

6.84

6.84 Oil ($\mu = 0.4\ N{\cdot}s/m^2$) flows between two fixed horizontal infinite parallel plates with a spacing of 5 mm. The flow is laminar and steady with a pressure gradient of -900 (N/m^2) per unit meter. Determine the volume flowrate per unit width and the shear stress on the upper plate.



From Eq. (6.136)

q = volume flowrate per unit width out of the paper

$$= \frac{2h^3 \Delta p}{3\mu l} \quad \text{where } \frac{\partial p}{\partial x} = -\frac{\Delta p}{l}$$

For this flow $2h = 5\ mm$ or $h = 2.5\ mm = 2.5\times10^{-3}\ m$
and $\Delta p/l = (+900\ N/m^2)/m = +900\ N/m^3$

Thus,

$$q = \frac{2(2.5\times10^{-3}\ m)^3(900\ \frac{N}{m^3})}{3\ (0.4\ N{\cdot}s/m^2)} = \underline{\underline{2.34\times10^{-5}\ \frac{m^2}{s}}}$$

The shear stress is $\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$
where

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - h^2) = -\frac{\Delta p}{2\mu l}(y^2 - h^2)$$

and

$$v = 0$$

Hence,

$$\tau_{xy} = -\frac{\Delta p}{2\mu l}(2y)\mu = -\frac{\Delta p}{l}y$$

On the upper plate $y = h$ so that

τ_{upper} = magnitude of shear stress on upper plate

$$= \frac{\Delta p}{l}h = (900\ \frac{N}{m^3})(2.5\times10^{-3}\ m) = \underline{\underline{2.25\ \frac{N}{m^2}}}$$ acting in the positive x-direction (the direction of flow).

6.85

6.85 Two fixed, horizontal, parallel plates are spaced 0.4 in. apart. A viscous liquid ($\mu = 8 \times 10^{-3}$ lb · s/ft², $SG = 0.9$) flows between the plates with a mean velocity of 0.5 ft/s. The flow is laminar. Determine the pressure drop per unit length in the direction of flow. What is the maximum velocity in the channel?

$$V = \frac{h^2}{3\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.137)}$$

Thus,

$$\frac{\Delta p}{\ell} = \frac{3\mu V}{h^2} = \frac{3\left(8\times 10^{-3}\ \frac{lb\cdot s}{ft^2}\right)\left(0.5\ \frac{ft}{s}\right)}{\left(\frac{0.2\ in.}{12\ \frac{in.}{ft}}\right)^2} = \underline{\underline{43.2\ \frac{lb}{ft^2}\ \text{per ft}}}$$

$$u_{max} = \frac{3}{2}V \qquad \text{(Eq. 6.138)}$$

$$= \frac{3}{2}\left(0.5\ \frac{ft}{s}\right) = \underline{\underline{0.75\ \frac{ft}{s}}}$$

6.86

6.86 A viscous, incompressible fluid flows between the two infinite, vertical, parallel plates of Fig. P6.86 Determine, by use of the Navier–Stokes equations, an expression for the pressure gradient in the direction of flow. Express your answer in terms of the mean velocity. Assume that the flow is laminar, steady, and uniform.

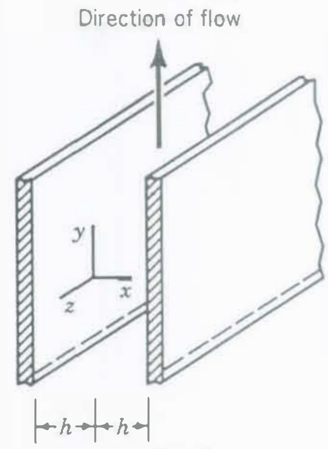


FIGURE P6.86

With the coordinate system shown $u=0$, $w=0$ and from the continuity equation $\frac{\partial v}{\partial y}=0$. Thus, from the y-component of the Navier-Stokes equations (Eq. 6.127b), with $g_y=-g$,

$$0 = -\frac{\partial p}{\partial y} - \rho g + \mu \frac{d^2 v}{dx^2} \qquad (1)$$

Since the pressure is not a function of x, Eq. (1) can be written as

$$\frac{d^2 v}{dx^2} = \frac{P}{\mu}$$

(where $P = \frac{\partial p}{\partial y} + \rho g$) and integrated to obtain

$$\frac{dv}{dx} = \frac{P}{\mu} x + C_1 \qquad (2)$$

From symmetry $\frac{dv}{dx}=0$ at $x=0$ so that $C_1=0$. Integration of Eq. (2) yields

$$v = \frac{P}{\mu}\frac{x^2}{2} + C_2$$

Since at $x=\pm h$, $v=0$ it follows that $C_2 = -\frac{P}{2\mu}(h^2)$ and therefore

$$v = \frac{P}{2\mu}(x^2-h^2)$$

The flowrate per unit width in the z-direction can be expressed as

$$q = \int_{-h}^{h} v\,dx = \int_{-h}^{h} \frac{P}{2\mu}(x^2-h^2)\,dx = -\frac{2}{3}\frac{P h^3}{\mu}$$

Thus, with V (mean velocity) given by the equation

$$V = \frac{q}{2h} = -\frac{1}{3}\frac{P h^2}{\mu}$$

it follows that

$$\underline{\underline{\frac{\partial p}{\partial y} = -\frac{3\mu V}{h^2} - \rho g}}$$

6.87

6.87 A fluid is initially at rest between two horizontal, infinite, parallel plates. A constant pressure gradient in a direction parallel to the plates is suddenly applied and the fluid starts to move. Determine the appropriate differential equation(s), initial condition, and boundary conditions that govern this type of flow. You need not solve the equation(s).

Differential equations are the same as Eqs. 6.129, 6.130, and 6.131 except that $\frac{\partial u}{\partial t} \neq 0$ (since the flow is unsteady). Thus, Eq. 6.129 must include the local acceleration term, $\frac{\partial u}{\partial t}$, and the governing differential equations are:

(x-direction) $\underline{\underline{\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}}}$ (with $\frac{\partial p}{\partial x}$ = constant)

(y-direction) $\underline{\underline{0 = -\frac{\partial p}{\partial y} - \rho g}}$

(z-direction) $\underline{\underline{0 = -\frac{\partial p}{\partial z}}}$

Initial condition: $\underline{\underline{u = 0 \text{ for } t = 0 \text{ for all } y}}$.

Boundary conditions: $\underline{\underline{u = 0 \text{ for } y = \pm h \text{ for } t \geq 0}}$.

6.88

6.88 (See Fluids in the News article titled "10 tons on 8 psi," Section 6.9.1.) A massive, precisely machined, 6-ft-diameter granite sphere rests upon a 4-ft-diameter cylindrical pedestal as shown in Fig. P6.88. When the pump is turned on and the water pressure within the pedestal reaches 8 psi, the sphere rises off the pedestal, creating a 0.005-in. gap through which the water flows. The sphere can then be rotated about any axis with minimal friction. (a) Estimate the pump flowrate, Q_0, required to accomplish this. Assume the flow in the gap between the sphere and the pedestal is essentially viscous flow between fixed, parallel plates. (b) Describe what would happen if the pump flowrate were increased to $2Q_0$.

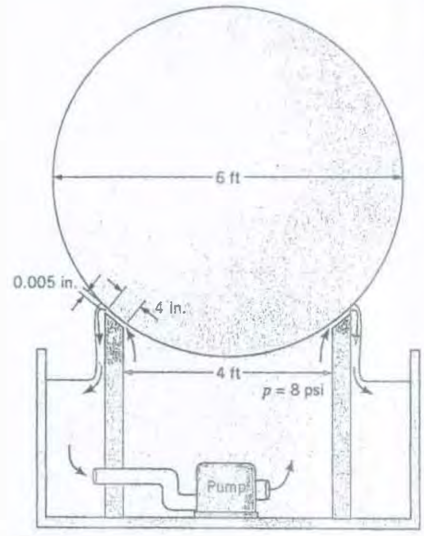
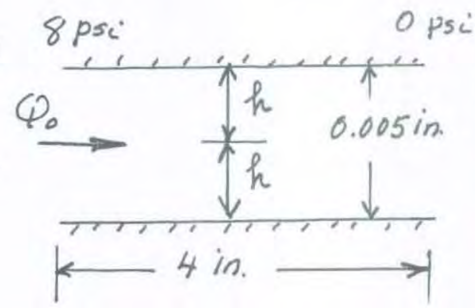


FIGURE P6.

(a) $q = \dfrac{2h^3 \Delta p}{3\mu \ell}$ where $q = \dfrac{\text{flowrate}}{\text{unit width}}$ (Eq. 6.136)



width $= \pi\,(4 ft)$

$h = \dfrac{0.005 \text{ in.}}{2} = 0.0025 \text{ in.} = 2.08 \times 10^{-4} ft$

$$q = \frac{(2)(2.08 \times 10^{-4} ft)^3 \left(8 \frac{lb}{in.^2}\right)\left(\frac{144 \text{ in.}^2}{ft^2}\right)}{3\left(2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2}\right)\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right)}$$

$= 8.86 \times 10^{-4} \dfrac{ft^3}{s}$ per unit width

Thus,

$$Q_0 = \left(8.86 \times 10^{-4} \frac{ft^2}{s}\right)(4\pi \, ft) = \underline{\underline{0.0111 \frac{ft^3}{s}}} \quad \left(4.98 \frac{\text{gallons}}{\text{min}}\right)$$

(b) Since 8 psi supports the sphere it is expected that this pressure remains approximately the same as the flowrate increases. To maintain this pressure the distance h would have to increase as Q_0 (or q) is increased. Thus, from Eq. 6.136,

$$\frac{q_{new}}{q_{old}} = \left(\frac{h_{new}}{h_{old}}\right)^3$$

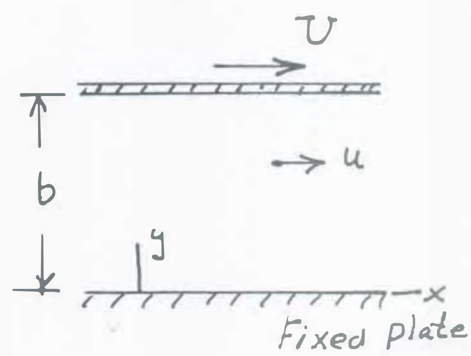
$$2 = \left(\frac{h_{new}}{h_{old}}\right)^3$$

$$h_{new} = (2)^{1/3}(0.0025 \text{ in.}) = 0.00315 \text{ in.}$$

Thus, the gap width would increase to approximately $\underline{\underline{0.00630 \text{ in.}}}$

6.89

6.89 Two horizontal, infinite, parallel plates are spaced a distance b apart. A viscous liquid is contained between the plates. The bottom plate is fixed and the upper plate moves parallel to the bottom plate with a velocity U. Because of the no-slip boundary condition (see Video V6.11), the liquid motion is caused by the liquid being dragged along by the moving boundary. There is no pressure gradient in the direction of flow. Note that this is a so-called simple *Couette flow* discussed in Section 6.9.2. (a) Start with the Navier–Stokes equations and determine the velocity distribution between the plates. (b) Determine an expression for the flowrate passing between the plates (for a unit width). Express your answer in terms of b and U.



(a) For steady flow with $v = w = 0$ it follows that the Navier-Stokes equations reduce to (in direction of flow)

$$0 = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{(Eq. 6.129)}$$

Thus, for zero pressure gradient

$$\frac{\partial^2 u}{\partial y^2} = 0$$

so that

$$u = C_1 y + C_2$$

At $y = 0$ $u = 0$ and it follows that $C_2 = 0$. Similarly, at $y = b$ $u = U$ and $C_1 = \frac{U}{b}$

Therefore,

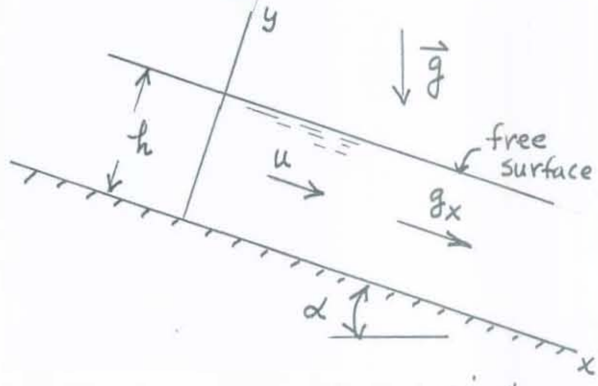
$$\underline{\underline{u = \frac{U}{b}\, y}}$$

(b) $$q = \int_0^b u(1)\,dy = \frac{U}{b}\int_0^b y\,dy = \frac{U}{b}\,\frac{y^2}{2}\Big|_0^b = \underline{\underline{\frac{Ub}{2}}}$$

where q is the flowrate per unit width.

6.90

6.90 A layer of viscous liquid of constant thickness (no velocity perpendicular to plate) flows steadily down an infinite, inclined plane. Determine, by means of the Navier–Stokes equations, the relationship between the thickness of the layer and the discharge per unit width. The flow is laminar, and assume air resistance is negligible so that the shearing stress at the free surface is zero.



$g_x = g \sin\alpha$

With the coordinate system shown in the figure $v=0$, $w=0$, and from the continuity equation $\frac{\partial u}{\partial x}=0$. Thus, from the x-component of the Navier-Stokes equations (Eq. 6.127a),

$$0 = -\frac{\partial p}{\partial x} + \rho g \sin\alpha + \mu \frac{d^2u}{dy^2} \qquad (1)$$

Also, since there is a free surface, there cannot be a pressure gradient in the x-direction so that $\frac{\partial p}{\partial x}=0$ and Eq. (1) can be written as

$$\frac{d^2u}{dy^2} = -\frac{\rho g}{\mu}\sin\alpha$$

Integration yields

$$\frac{du}{dy} = -\left(\frac{\rho g}{\mu}\sin\alpha\right)y + C_1 \qquad (2)$$

Since the shearing stress

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

equals zero at the free surface ($y=h$) it follows that

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y=h$$

so that the constant in Eq. (2) is

$$C_1 = \frac{\rho g}{\mu}\sin\alpha$$

Integration of Eq. (2) yields

$$u = -\left(\frac{\rho g}{\mu}\sin\alpha\right)\frac{y^2}{2} + \left(\frac{\rho g}{\mu}\sin\alpha\right)y + C_2$$

Since $u=0$ at $y=0$, it follows that $C_2=0$, and therefore

$$u = \frac{\rho g}{\mu}\sin\alpha\left(hy - \frac{y^2}{2}\right)$$

The flowrate per unit width can be expressed as $q = \int_0^h u\,dy$ so that

$$q = \int_0^h \frac{\rho g}{\mu}\sin\alpha\left(hy - \frac{y^2}{2}\right)dy = \underline{\underline{\frac{\rho g h^3 \sin\alpha}{3\mu}}}$$

6.91

6.91 Due to the no-slip condition, as a solid is pulled out of a viscous liquid some of the liquid is also pulled along as described in Example 6.9 and shown in Video V6.11. Based on the results given in Example 6.9, show on a dimensionless plot the velocity distribution in the fluid film (v/V_0 vs. x/h) when the average film velocity, V, is 10% of the belt velocity, V_0.

From Example 6.9, the average velocity is given by the equation

$$V = V_0 - \frac{\gamma h^2}{3\mu} \qquad (1)$$

with the velocity distribution

$$v = \frac{\gamma}{2\mu} x^2 - \frac{\gamma h}{\mu} x + V_0 \qquad (2)$$

If $V = 0.1 V_0$, then from Eq. (1)

$$0.1 V_0 = V_0 - \frac{\gamma h^2}{3\mu}$$

or

$$V_0 = \frac{\gamma h^2}{2.7\mu} \qquad (3)$$

In dimensionless form Eq. (2) becomes

$$\frac{v}{V_0} = \frac{\gamma h^2}{2\mu V_0}\left(\frac{x}{h}\right)^2 - \frac{\gamma h^2}{\mu V_0}\left(\frac{x}{h}\right) + 1 \qquad (4)$$

From Eq. (3)

$$\frac{\gamma h^2}{\mu V_0} = 2.7$$

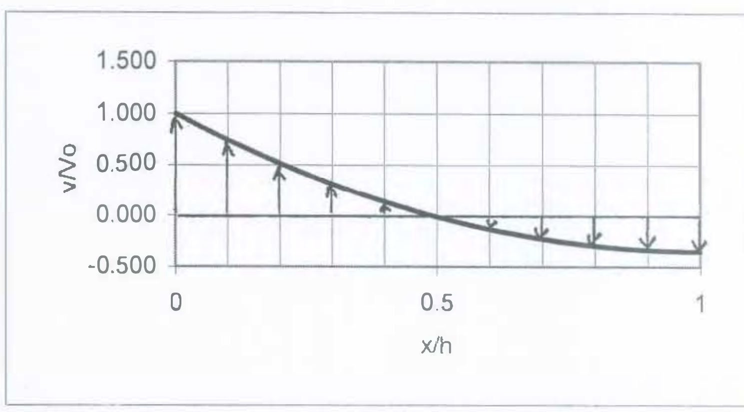
and Eq. (4) can be written as

$$\frac{v}{V_0} = 1.35\left(\frac{x}{h}\right)^2 - 2.7\left(\frac{x}{h}\right) + 1 \qquad (5)$$

A plot of the velocity distribution is shown below.

x/h	v/Vo
0	1.000
0.1	0.744
0.2	0.514
0.3	0.312
0.4	0.136
0.5	-0.013
0.6	-0.134
0.7	-0.229
0.8	-0.296
0.9	-0.337
1	-0.350

Calculated from Eq. (5)



6.92

6.92 An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in Fig. P6.92. The two plates move in opposite directions with constant velocities, U_1 and U_2, as shown. The pressure gradient in the x direction is zero and the only body force is due to the fluid weight. Use the Navier–Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar flow.

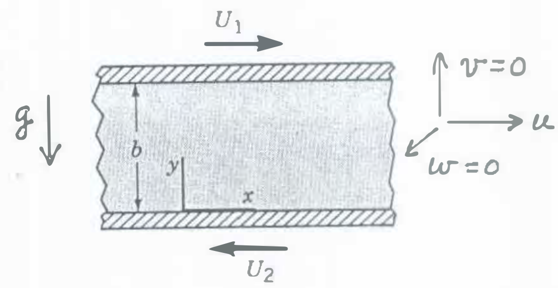


FIGURE P6.92

For the specified conditions, $v=0$, $w=0$, $\frac{\partial p}{\partial x}=0$, and $g_x=0$, so that the x-component of the Navier-Stokes equations (Eq. 6.127a) reduces to

$$\frac{d^2u}{dy^2}=0 \tag{1}$$

Integration of Eq. (1) yields

$$u = C_1 y + C_2 \tag{2}$$

For $y=0$, $u=-U_2$ and therefore from Eq. (2)

$$C_2 = -U_2$$

For $y=b$, $u=U_1$ so that

$$U_1 = C_1 b - U_2$$

or

$$C_1 = \frac{U_1+U_2}{b}$$

Thus,

$$\underline{\underline{u = \left(\frac{U_1+U_2}{b}\right) y - U_2}}$$

6.93

6.93 Two immiscible, incompressible, viscous fluids having the same densities but different viscosities are contained between two infinite, horizontal, parallel plates (Fig. P6.93. The bottom plate is fixed and the upper plate moves with a constant velocity U. Determine the velocity at the interface. Express your answer in terms of U, μ_1, and μ_2. The motion of the fluid is caused entirely by the movement of the upper plate; that is, there is no pressure gradient in the x direction. The fluid velocity and shearing stress is continuous across the interface between the two fluids. Assume laminar flow.

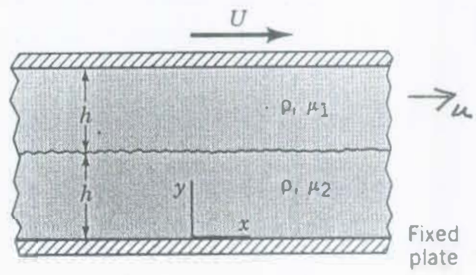


FIGURE P6.93

For the specified conditions, $v=0$, $w=0$, $\frac{\partial p}{\partial x}=0$, and $g_x=0$, so that the x-component of the Navier-Stokes equations (Eq. 6.127a) for either the upper or lower layer reduces to

$$\frac{d^2u}{dy^2}=0 \qquad (1)$$

Integration of Eq. (1) yields

$$u = Ay + B$$

which gives the velocity distribution in either layer.
In the upper layer at $y=2h$, $u=U$ so that

$$B_1 = U - A_1(2h)$$

where the subscript 1 refers to the upper layer.
For the lower layer at $y=0$, $u=0$ so that

$$B_2 = 0$$

where the subscript 2 refers to the lower layer. Thus,

$$u_1 = A_1(y-2h)+U$$

and

$$u_2 = A_2 y$$

At $y=h$, $u_1 = u_2$ so that

$$A_1(h-2h)+U = A_2 h$$

or

$$A_2 = -A_1 + \frac{U}{h} \qquad (2)$$

(con't)

6.93 (con't)

Since the velocity distribution is linear in each layer the shearing stress

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\frac{du}{dy}$$

is constant throughout each layer. For the upper layer

$$\tau_1 = \mu_1 A_1$$

and for the lower layer

$$\tau_2 = \mu_2 A_2$$

At the interface $\tau_1 = \tau_2$ so that

$$\mu_1 A_1 = \mu_2 A_2$$

or

$$\frac{A_1}{A_2} = \frac{\mu_2}{\mu_1} \qquad (3)$$

Substitution of Eq. (3) into Eq. (2) yields

$$A_2 = -\frac{\mu_2}{\mu_1}A_2 + \frac{U}{h}$$

or

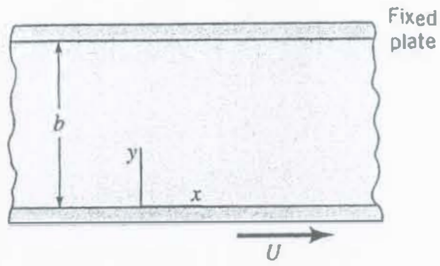
$$A_2 = \frac{U/h}{1 + \mu_2/\mu_1}$$

Thus, velocity at the interface is

$$u_2(y=h) = A_2 h = \underline{\underline{\frac{U}{1 + \frac{\mu_2}{\mu_1}}}}$$

6.94

6.94 The viscous, incompressible flow between the parallel plates shown in Fig. P6.94 is caused by both the motion of the bottom plate and a pressure gradient, $\partial p/\partial x$. As noted in Section 6.9.2, an important dimensionless parameter for this type of problem is $P = -(b^2/2\,\mu U)(\partial p/\partial x)$ where μ is the fluid viscosity. Make a plot of the dimensionless velocity distribution (similar to that shown in Fig. 6.32*b*) for $P = 3$. For this case where does the maximum velocity occur?



■ FIGURE P6.94

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)y^2 + C_1 y + C_2 \qquad (\text{Eq. } 6.133)$$

At $y=0$, $u=U$ so that $C_2 = U$. At $y=b$, $u=0$ and therefore

$$0 = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)b^2 + C_1 b + U$$

or

$$C_1 = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)b - \frac{U}{b}$$

Thus,

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by) + U\left(1 - \frac{y}{b}\right)$$

or in dimensionless form

$$\frac{u}{U} = \frac{b^2}{2\mu U}\left(\frac{\partial p}{\partial x}\right)\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \frac{y}{b} + 1 \qquad (1)$$

Since,

$$P = -\frac{b^2}{2\mu U}\left(\frac{\partial p}{\partial x}\right)$$

Eq. (1) can be written as

$$\frac{u}{U} = -P\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \frac{y}{b} + 1 \qquad (2)$$

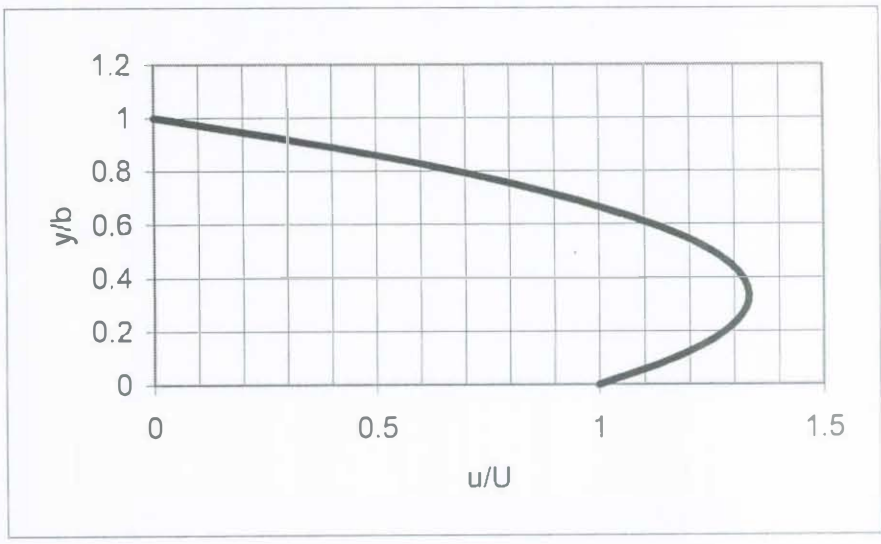
A plot of this velocity distribution for $P = 3$ is shown on the following page.

(con't)

6.94 (con't)

u/U	y/b
1	0
1.17	0.1
1.28	0.2
1.33	0.3
1.32	0.4
1.25	0.5
1.12	0.6
0.93	0.7
0.68	0.8
0.37	0.9
0	1

Calculated from Eq. (2) with P =3.



To determine where the maximum velocity occurs differentiate Eq. (2) and set equal to zero. Thus,

$$\frac{d(u/U)}{dy} = -P\left[2\left(\frac{y}{b^2}\right) - \frac{1}{b}\right] - \frac{1}{b} = 0$$

and with $P = 3$

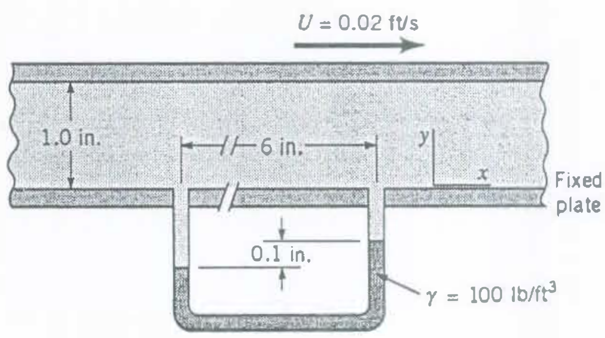
$$\frac{d(u/U)}{dy} = -3\left[\frac{1}{b}\left(2\frac{y}{b} - 1\right)\right] - \frac{1}{b} = 0$$

so that

$$\underline{\underline{\frac{y}{b} = \frac{1}{3}}}$$

6.95

6.95 A viscous fluid (specific weight $= 80\ \text{lb/ft}^3$; viscosity $= 0.03\ \text{lb}\cdot\text{s/ft}^2$) is contained between two infinite, horizontal parallel plates as shown in Fig. P6.95. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.1 in. If the upper plate moves with a velocity of 0.02 ft/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.



■ FIGURE P6.95

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left(y^2 - by\right) \qquad (\text{Eq. } 6.140)$$

Maximum velocity will occur at distance y_m where $\frac{du}{dy} = 0$.

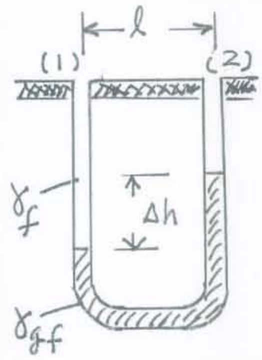
Thus,

$$\frac{du}{dy} = \frac{U}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(2y - b)$$

and for $\frac{du}{dy} = 0$

$$y_m = -\frac{\mu U}{b\left(\frac{\partial p}{\partial x}\right)} + \frac{b}{2} \qquad (1)$$

For manometer (see figure to right),



$$p_1 + \gamma_f\,\Delta h - \gamma_{gf}\,\Delta h = p_2$$

or

$$p_1 - p_2 = (\gamma_{gf} - \gamma_f)\,\Delta h$$

$$= \left(100\ \frac{\text{lb}}{\text{ft}^3} - 80\ \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{0.1\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right) = 0.167\ \frac{\text{lb}}{\text{ft}^2}$$

Also,

$$-\frac{\partial p}{\partial x} = \frac{p_1 - p_2}{\ell} = \frac{0.167\ \frac{\text{lb}}{\text{ft}^2}}{\left(\frac{6\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)} = 0.334\ \frac{\text{lb}}{\text{ft}^3}$$

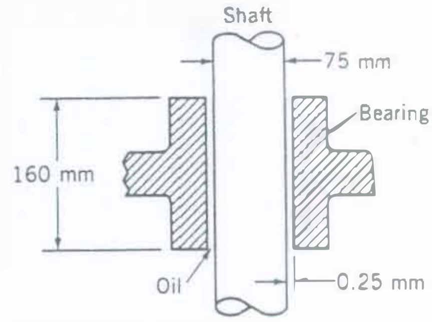
Thus, from Eq. (1)

$$y_m = -\frac{\left(0.03\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(0.02\ \frac{\text{ft}}{\text{s}}\right)}{\left(\frac{1.0\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)\left(-0.334\ \frac{\text{lb}}{\text{ft}^3}\right)} + \frac{\frac{1.0\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}}{2}$$

$$= 0.0632\ \text{ft}\left(\frac{12\ \text{in.}}{\text{ft}}\right) = \underline{\underline{0.759\ \text{in.}}}$$

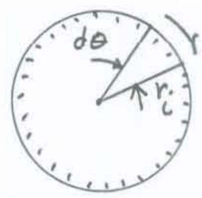
6.96

6.96 A vertical shaft passes through a bearing and is lubricated with an oil having a viscosity of 0.2 N·s/m² as shown in Fig. P6.96. Assume that the flow characteristics in the gap between the shaft and bearing are the same as those for laminar flow between infinite parallel plates with zero pressure gradient in the direction of flow. Estimate the torque required to overcome viscous resistance when the shaft is turning at 80 rev/min.



■ FIGURE P6.96

The torque due to force dF acting on a differential area, $dA = r_i\, \ell\, d\theta$, is (see figure at right)



$$dF = \tau\, dA = \tau r_i \ell\, d\theta$$

$\ell \sim$ shaft length

$$d\mathcal{T} = r_i\, dF = r_i^2 \tau \ell\, d\theta$$

where τ is the shearing stress. Thus,

$$\mathcal{T} = r_i^2 \tau \ell \int_0^{2\pi} d\theta = 2\pi r_i^2 \tau \ell \qquad (1)$$

In the gap,

$$u = U \frac{y}{b} \qquad \text{(Eq. 6.142)}$$

where $U = r_i \omega$ and b is the gap width. Also,

$$\tau = \mu \frac{du}{dy} = \frac{\mu U}{b}$$

Thus, from Eq. (1)

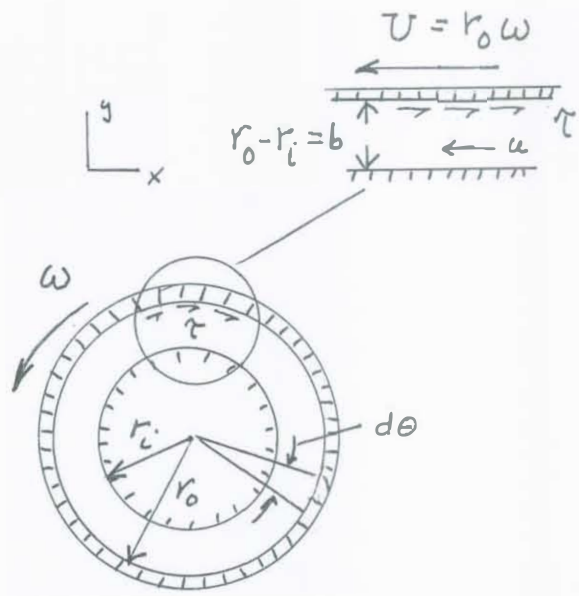
$$\mathcal{T} = 2\pi r_i^2 \left(\mu \frac{U}{b}\right) \ell = 2\pi r_i^3 \mu \omega \frac{\ell}{b}$$

$$= 2\pi \left(\frac{0.075}{2}\ m\right)^3 \left(0.2\ \frac{N \cdot s}{m^2}\right) \left[\left(80\ \frac{rev}{min}\right)\left(\frac{2\pi\ rad}{rev}\right)\left(\frac{min}{60\ s}\right)\right] \frac{(0.160\ m)}{(0.25 \times 10^{-3}\ m)}$$

$$= \underline{\underline{0.355\ N \cdot m}}$$

6.97

6.97 A viscous fluid is contained between two long concentric cylinders. The geometry of the system is such that the flow between the cylinders is approximately the same as the laminar flow between two infinite parallel plates. **(a)** Determine an expression for the torque required to rotate the outer cylinder with an angular velocity ω. The inner cylinder is fixed. Express your answer in terms of the geometry of the system, the viscosity of the fluid, and the angular velocity. **(b)** For a small rectangular element located at the fixed wall determine an expression for the rate of angular deformation of this element. (See Video V6.3 and Fig. P6.10.)



$\ell \sim$ cylinder length
$\tau \sim$ shearing stress

(a) The torque which must be applied to outer cylinder to overcome the force due to the shearing stress is (see figure)

$$d\mathcal{T} = r_o\, dF = r_o\,(\tau\, r_o\, \ell\, d\theta) = r_o^2\, \tau\, \ell\, d\theta$$

so that

$$\mathcal{T} = r_o^2\, \tau\, \ell \int_0^{2\pi} d\theta = 2\pi r_o^2\, \tau\, \ell \qquad (1)$$

In the gap

$$u = U\frac{y}{b} \qquad \text{(Eq. 6.142)}$$

Since,

$$\tau = \mu\frac{du}{dy} = \frac{\mu U}{b}$$

and $b = r_o - r_i$, $U = r_o\omega$ (see figure), it follows from Eq. (1) that

$$\mathcal{T} = 2\pi r_o^2\left(\frac{\mu\, r_o\omega}{r_o - r_i}\right)\ell = \underline{\underline{\frac{2\pi r_o^3\, \mu\omega\ell}{r_o - r_i}}}$$

(con't)

6.97 (con't)

(b) From Eq. 6.18

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

For the linear distribution

$$u = -\frac{r_o \omega}{r_o - r_i}\, y = -\frac{U y}{b}$$

so that

$$\frac{\partial u}{\partial y} = -\frac{U}{b}$$

and s $v = 0$

$$\underline{\underline{\dot{\gamma} = -\frac{U}{b}}}$$

The negative sign indicates that the original right angle shown in Fig. P6.10b is increasing.

*6.98

*6.98 Oil (SAE 30) flows between parallel plates spaced 5 mm apart. The bottom plate is fixed but the upper plate moves with a velocity of 0.2 m/s in the positive x direction. The pressure gradient is 60 kPa/m, and is negative. Compute the velocity at various points across the channel and show the results on a plot. Assume laminar flow.

The velocity distribution is given by the equation

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left(y^2 - by\right) \qquad \text{(Eq. 6.140)}$$

and for the given data,

$$u = \frac{\left(0.2\,\frac{m}{s}\right)}{(0.005\,m)}\,y + \frac{1}{2\left(0.38\,\frac{N\cdot s}{m^2}\right)}\left(-60\times10^3\,\frac{N}{m^3}\right)\left[y^2 - (0.005\,m)\,y\right]$$

so that

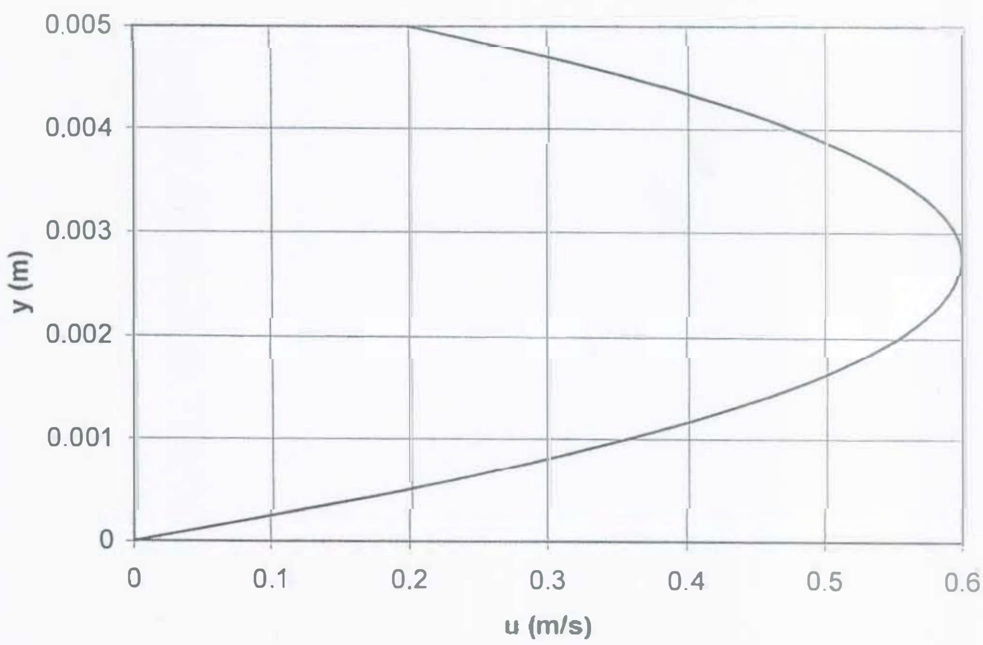
$$u = 40y + 7.89\times10^4\left(0.005y - y^2\right) \qquad (1)$$

with u in m/s when y is in m.

Tabulated data and a plot of the data are given below.

y, m	u, m/s
0	0
0.0005	0.1975
0.0010	0.3556
0.0015	0.4742
0.0020	0.5534
0.0025	0.5931
0.0030	0.5934
0.0035	0.5542
0.0040	0.4756
0.0045	0.3575
0.0050	0.2000

Calculated from Eq. (1)



6.99

6.99 Consider a steady, laminar flow through a straight horizontal tube having the constant elliptical cross section given by the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The streamlines are all straight and parallel. Investigate the possibility of using an equation for the z component of velocity of the form

$$w = A\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)$$

as an exact solution to this problem. With this velocity distribution what is the relationship between the pressure gradient along the tube and the volume flowrate through the tube?

From the description of the problem, $u=0$, $v=0$, $g_z=0$, $w \neq f(t)$, and the continuity equation indicates that $\frac{\partial w}{\partial z}=0$. With these conditions the z-component of the Navier-Stokes equations (Eq. 6.127c) reduces to

$$\frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \qquad (1)$$

Due to the no-slip boundary condition, $w=0$ on the elliptical boundary

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Thus, the proposed velocity distribution satisfies this condition since on the boundary

$$w = A\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = A\left[1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)\right] = A\left[1 - (1)\right] = 0$$

This result indicates that the proposed velocity distribution can be used as a solution. Substitution of the velocity distribution into Eq. (1) gives the relationship between the pressure gradient, $\frac{\partial p}{\partial z}$, and the velocity. Since,

$$\frac{\partial^2 w}{\partial x^2} = -\frac{2A}{a^2} \qquad \frac{\partial^2 w}{\partial y^2} = -\frac{2A}{b^2}$$

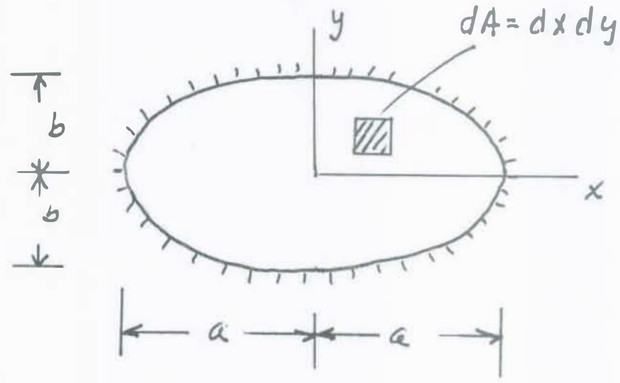
it follows that

$$\frac{\partial p}{\partial z} = -2A\mu\left(\frac{1}{a^2} + \frac{1}{b^2}\right) \qquad (2)$$

(con't)

6.99 (con't)

The volume flowrate, Q, through the tube is given by the equation



$$Q = \int_{area} w \, dA$$

$$= 4 \int_0^b \int_0^{a\sqrt{1-\frac{y^2}{b^2}}} w \, dx \, dy$$

Thus,

$$Q = 4A \int_0^b \int_0^{a\sqrt{1-\frac{y^2}{b^2}}} \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) dx \, dy$$

$$= 4A \int_0^b \left[x - \frac{x^3}{3a^2} - \frac{y^2}{b^2} x \right]_0^{a\sqrt{1-\frac{y^2}{b^2}}} dy$$

$$= 4A \int_0^b \left[a\sqrt{1-\frac{y^2}{b^2}} \left(1 - \frac{y^2}{b^2}\right) - \frac{1}{3} a \sqrt{1-\frac{y^2}{b^2}} \left(1 - \frac{y^2}{b^2}\right) \right] dy$$

$$= \frac{8Aa}{3} \int_0^b \left(1 - \frac{y^2}{b^2}\right)^{3/2} dy = \frac{8Aa}{3} \left(\frac{3b\pi}{16}\right) = \frac{A\pi a b}{2}$$

and therefore

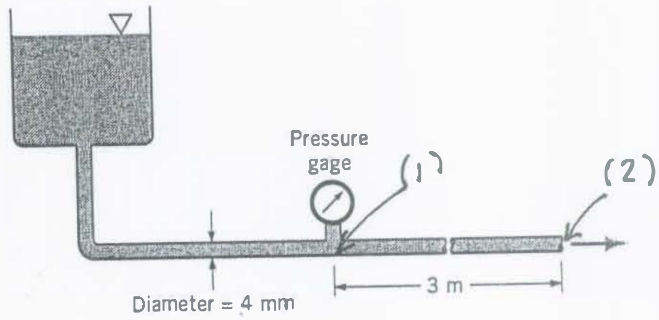
$$A = \frac{2Q}{\pi a b}$$

From Eq. (2)

$$\frac{\partial p}{\partial z} = -\frac{4\mu Q}{\pi a b} \left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

6.100

6.100 A simple flow system to be used for steady flow tests consists of a constant head tank connected to a length of 4-mm-diameter tubing as shown in Fig. P6.100. The liquid has a viscosity of 0.015 N · s/m², a density of 1200 kg/m³, and discharges into the atmosphere with a mean velocity of 2 m/s. (a) Verify that the flow will be laminar. (b) The flow is fully developed in the last 3 m of the tube. What is the pressure at the pressure gage? (c) What is the magnitude of the wall shearing stress, τ_{rz}, in the fully developed region?



■ FIGURE P6.100

(a) Check Reynolds number to determine if flow is laminar:

$$Re = \frac{\rho V (2R)}{\mu} = \frac{(1200 \frac{kg}{m^3})(2 \frac{m}{s})(0.004 m)}{0.015 \frac{N \cdot s}{m^2}} = 640$$

Since the Reynolds number is well below 2100 the flow is laminar.

(b) For laminar flow,

$$V = \frac{R^2}{8\mu} \frac{\Delta p}{\ell} \qquad \text{(Eq. 6.152)}$$

Since $\Delta p = p_1 - p_2 = p_1 - 0$ (see figure)

$$p_1 = \frac{8 \mu V \ell}{R^2} = \frac{8 (0.015 \frac{N \cdot s}{m^2})(2 \frac{m}{s})(3 m)}{(\frac{0.004}{2} m)^2} = \underline{\underline{180 \, kPa}}$$

(c)

$$\tau_{rz} = \mu \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right) \qquad \text{(Eq. 6.126f)}$$

For fully developed pipe flow, $v_r = 0$, so that

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

Also,

$$v_z = v_{max} \left[1 - \left(\frac{r}{R} \right)^2 \right] \qquad \text{(Eq. 6.154)}$$

and with $v_{max} = 2V$, where V is the mean velocity

$$\tau_{rz} = 2V\mu \left(-\frac{2r}{R^2} \right)$$

Thus, at the wall, $r = R$,

$$\left| (\tau_{rz})_{wall} \right| = \left| -\frac{4V\mu}{R} \right| = \left| -\frac{4 (2 \frac{m}{s})(0.015 \frac{N \cdot s}{m^2})}{(\frac{0.004}{2} m)} \right| = \underline{\underline{60.0 \frac{N}{m^2}}}$$

6.101

6.101 **(a)** Show that for Poiseuille flow in a tube of radius R the magnitude of the wall shearing stress, τ_{rz}, can be obtained from the relationship

$$|(\tau_{rz})_{\text{wall}}| = \frac{4\mu Q}{\pi R^3}$$

for a Newtonian fluid of viscosity μ. The volume rate of flow is Q. **(b)** Determine the magnitude of the wall shearing stress for a fluid having a viscosity of 0.004 N·s/m² flowing with an average velocity of 130 mm/s in a 2-mm-diameter tube.

(a) $$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(Eq. 6.126 f)}$$

For Poiseuille flow in a tube, $v_r = 0$, and therefore

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

Since, $$v_z = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \text{(Eq. 6.154)}$$

and $v_{max} = 2V$, where V is the mean velocity, it follows that

$$\frac{\partial v_z}{\partial r} = -\frac{4Vr}{R^2}$$

Thus, at the wall $(r = R)$,

$$(\tau_{rz})_{wall} = -\frac{4\mu V}{R}$$

and with $Q = \pi R^2 V$

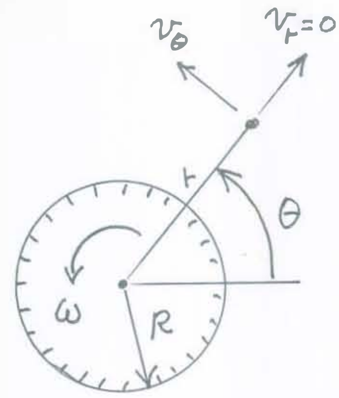
$$\left|(\tau_{rz})_{wall}\right| = \underline{\underline{\frac{4\mu Q}{\pi R^3}}}$$

(b) $$\left|(\tau_{rz})_{wall}\right| = \frac{4\mu V}{R} = \frac{4\left(0.004\ \frac{N\cdot s}{m^2}\right)\left(0.130\ \frac{m}{s}\right)}{\left(\frac{0.002}{2}\ m\right)}$$

$$= \underline{\underline{2.08\ Pa}}$$

6.102

6.102 An infinitely long, solid, vertical cylinder of radius R is located in an infinite mass of an incompressible fluid. Start with the Navier–Stokes equation in the θ direction and derive an expression for the velocity distribution for the steady flow case in which the cylinder is rotating about a fixed axis with a constant angular velocity ω. You need not consider body forces. Assume that the flow is axisymmetric and the fluid is at rest at infinity.



For this flow field, $v_r=0$, $v_z=0$, and from the continuity equation,

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \qquad \text{(Eq. 6.35)}$$

it follows that

$$\frac{\partial v_\theta}{\partial \theta} = 0 \qquad \text{(See figure for notation.)}$$

Thus, the Navier-Stokes equation in the θ-direction (Eq. 6.128b) for steady flow reduces to

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2}\right]$$

Due to the symmetry of the flow,

$$\frac{\partial p}{\partial \theta} = 0$$

so that

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} = 0$$

or

$$\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} = 0 \qquad (1)$$

Since v_θ is a function of only r, Eq. (1) can be expressed as an ordinary differential equation, and re-written as

$$\frac{d^2 v_\theta}{dr^2} + \frac{d}{dr}\left(\frac{v_\theta}{r}\right) = 0 \qquad (2)$$

Equation (2) can be integrated to yield

$$\frac{dv_\theta}{dr} + \frac{v_\theta}{r} = C_1$$

or

$$r\frac{dv_\theta}{dr} + v_\theta = C_1 r \qquad (3)$$

(con't)

6.102 (con't)

Equation (3) can be expressed as

$$\frac{d(rv_\theta)}{dr} = c_1 r$$

and a second integration yields

$$r v_\theta = \frac{c_1 r^2}{2} + c_2$$

or

$$v_\theta = \frac{c_1 r}{2} + \frac{c_2}{r}$$

As $r \rightarrow \infty$, $v_\theta \rightarrow 0$, (since fluid is at rest at infinity) so that $c_1 = 0$. Thus,

$$v_\theta = \frac{c_2}{r}$$

and since at $r = R$, $v_\theta = R\omega$, it follows that $c_2 = R^2\omega$ and

$$\underline{\underline{v_\theta = \frac{R^2\omega}{r}}}$$

*6.103

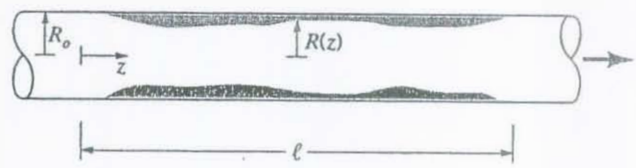
*6.103 As is shown by Eq. 6.150 the pressure gradient for laminar flow through a tube of constant radius is given by the expression:

$$\frac{\partial p}{\partial z} = -\frac{8\mu Q}{\pi R^4}$$

For a tube whose radius is changing very gradually, such as the one illustrated in Fig. P6.103 it is expected that this equation can be used to approximate the pressure change along the tube if the actual radius, $R(z)$, is used at each cross section. The following measurements were obtained along a particular tube.

z/ℓ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$R(z)/R_o$	1.00	0.73	0.67	0.65	0.67	0.80	0.80	0.71	0.73	0.77	1.00

Compare the pressure drop over the length ℓ for this nonuniform tube with one having the constant radius R_o. *Hint:* To solve this problem you will need to numerically integrate the equation for the pressure gradient given above.



■ FIGURE P6.103

From the equation given for the pressure gradient,

$$\int_{p_1}^{p_2} dp = -\int_0^{\ell} \frac{8\mu Q}{\pi [R(z)]^4} \, dz$$

Since $p_1 - p_2 = \Delta p$ (the pressure drop) it follows that

$$\Delta p = \frac{8\mu Q}{\pi} \int_0^{\ell} [R(z)]^{-4} dz$$

or, with $z^* = z/\ell$ and $R^* = R/R_o$,

$$\Delta p = \frac{8\mu Q \ell}{\pi R_o^4} \int_0^1 (R^*)^{-4} dz^*$$

For a constant radius tube (see Eq. 6.151),

$$\Delta p_{R=R_o} = \frac{8\mu Q \ell}{\pi R_o^4}$$

so that

$$\frac{\Delta p \text{ (nonuniform tube)}}{\Delta p \text{ (uniform tube)}} = \int_0^1 (R^*)^{-4} dz^*$$

This integral can be evaluated numerically using the trapezoidal rule, i.e.,

$$I = \frac{1}{2} \sum_{i=1}^{n-1} (y_i + y_{i+1})(x_{i+1} - x_i)$$ where

$y \sim (R^*)^{-4}$ and $x \sim z^*$.

(con't)

6.103 * (con't)

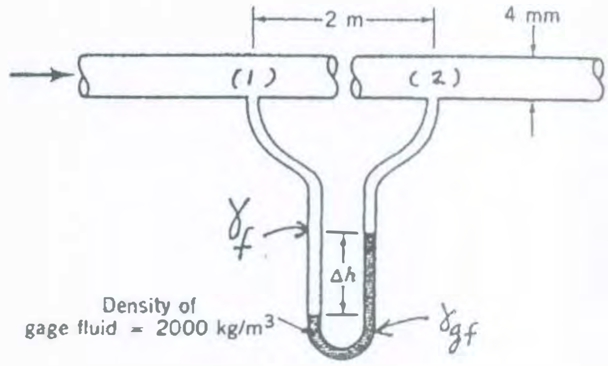
z/ℓ	R/Ro	(R/Ro)^-4
0.0	1.00	1.00
0.1	0.73	3.52
0.2	0.67	4.96
0.3	0.65	5.60
0.4	0.67	4.96
0.5	0.80	2.44
0.6	0.80	2.44
0.7	0.71	3.94
0.8	0.73	3.52
0.9	0.77	2.84
1.0	1.00	1.00

Using the tabulated data above, the approximate value of the integral is 3.52. Thus,

$$\frac{\Delta p\ (\text{nonuniform tube})}{\Delta p\ (\text{uniform tube})} = \underline{\underline{3.52}}$$

6.104

6.104 A liquid (viscosity = 0.002 N·s/m^2; density = 1000 kg/m^3) is forced through the circular tube shown in Fig. P6. A differential manometer is connected to the tube as shown to measure the pressure drop along the tube. When the differential reading, Δh, is 9 mm, what is the mean velocity in the tube?



■ FIGURE P6.

Assume laminar flow so that

$$V = \frac{R^2}{8\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.145)}$$

For manometer (see figure),

$$p_1 + \gamma\,\Delta h - \gamma_{gf}\,\Delta h = p_2$$

or

$$p_1 - p_2 = \Delta p = \Delta h\,(\gamma_{gf} - \gamma) = \Delta h\,(g)(\rho_{gf} - \rho_f)$$

$$= (0.009\,m)\left(9.81\,\frac{m}{s^2}\right)\left(2000\,\frac{kg}{m^3} - 1000\,\frac{kg}{m^3}\right)$$

$$= 88.3\,\frac{N}{m^2}$$

Thus,

$$V = \frac{\left(\frac{0.004}{2}\,m\right)^2\left(88.3\,\frac{N}{m^2}\right)}{8\left(0.002\,\frac{N\cdot s}{m^2}\right)(2\,m)} = \underline{\underline{1.10\times10^{-2}\,\frac{m}{s}}}$$

Check Reynolds number to confirm that flow is laminar:

$$R_e = \frac{\rho V(2R)}{\mu} = \frac{\left(10^3\,\frac{kg}{m^3}\right)\left(1.10\times10^{-2}\,\frac{m}{s}\right)(0.004\,m)}{0.002\,\frac{N\cdot s}{m^2}}$$

$$= 22.0 < 2100$$

Since $R_e < 2100$ flow is laminar.

6.105

6.105 An incompressible Newtonian fluid flows steadily between two infinitely long, concentric cylinders as shown in Fig. P6.105. The outer cylinder is fixed, but the inner cylinder moves with a longitudinal velocity V_0 as shown. The pressure gradient in the axial direction is $-\Delta p/\ell$. For what value of V_0 will the drag on the inner cylinder be zero? Assume that the flow is laminar, axisymmetric, and fully developed.

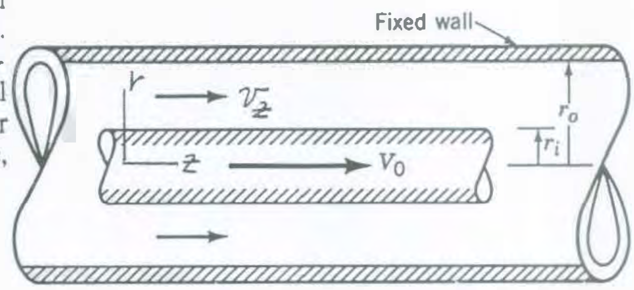


FIGURE P6.105

Equation 6.147, which was developed for flow in circular tubes, applies in the annular region. Thus,

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r^2 + c_1 \ln r + c_2 \qquad (1)$$

With boundary conditions, $r = r_o$, $v_z = 0$, and $r = r_i$, $v_z = V_0$, it follows that:

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_o^2 + c_1 \ln r_o + c_2 \qquad (2)$$

$$V_0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_i^2 + c_1 \ln r_i + c_2 \qquad (3)$$

Subtract Eq. (2) from Eq. (3) to obtain

$$V_0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right) + c_1 \ln \frac{r_i}{r_o}$$

so that

$$c_1 = \frac{V_0 - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right)}{\ln \frac{r_i}{r_o}}$$

The drag on the inner cylinder will be zero if

$$\left(\tau_{rz}\right)_{r=r_i} = 0$$

Since,

$$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(Eq. 6.126f)}$$

and with $v_r = 0$, it follows that

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

(con't)

6.105 (Con't)

Differentiate Eq. (1) with respect to r to obtain

$$\frac{\partial v_z}{\partial r} = \frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right) r + \frac{c_1}{r}$$

so that at $r = r_i$

$$(\tau_{rz})_{r=r_i} = \mu\left[\frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right) r_i + \frac{V_o - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)(r_i^2 - r_o^2)}{r_i \ln \frac{r_i}{r_o}}\right]$$

Thus, in order for the drag to be zero,

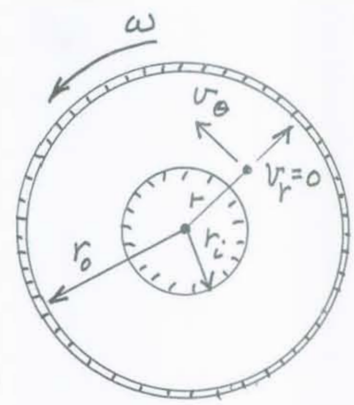
$$\frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right) r_i + \frac{V_o - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)(r_i^2 - r_o^2)}{r_i \ln \frac{r_i}{r_o}} = 0$$

or

$$\underline{\underline{V_o = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left[2 r_i^2 \ln \frac{r_i}{r_o} - (r_i^2 - r_o^2)\right]}}$$

6.106

6.106 A viscous fluid is contained between two infinitely long vertical concentric cylinders. The outer cylinder has a radius r_o and rotates with an angular velocity ω. The inner cylinder is fixed and has a radius r_i. Make use of the Navier–Stokes equations to obtain an exact solution for the velocity distribution in the gap. Assume that the flow in the gap is axisymmetric (neither velocity nor pressure are functions of angular position θ within gap) and that there are no velocity components other than the tangential component. The only body force is the weight.



The velocity distribution in the annular space is given by the equation

$$v_\theta = \frac{c_1 r}{2} + \frac{c_2}{r} \qquad (1)$$

(See solution to Problem 6.94 for derivation.)

With the boundary conditions $r = r_i$, $v_\theta = 0$, and $r = r_o$, $v_\theta = r_o \omega$ (see figure for notation), it follows from Eq. (1) that:

$$0 = \frac{c_1 r_i}{2} + \frac{c_2}{r_i}$$

$$r_o \omega = \frac{c_1 r_o}{2} + \frac{c_2}{r_o}$$

Therefore,

$$c_1 = \frac{2\omega}{1 - \frac{r_i^2}{r_o^2}}$$

and

$$c_2 = \frac{-r_i^2 \omega}{1 - \frac{r_i^2}{r_o^2}}$$

so that

$$v_\theta = \frac{r\omega}{1 - \frac{r_i^2}{r_o^2}} - \frac{r_i^2 \omega}{r\left(1 - \frac{r_i^2}{r_o^2}\right)}$$

or

$$v_\theta = \frac{r\omega}{\left(1 - \frac{r_i^2}{r_o^2}\right)}\left[1 - \frac{r_i^2}{r^2}\right]$$

6.107

6.107 For flow between concentric cylinders, with the outer cylinder rotating at an angular velocity ω and the inner cylinder fixed, it is commonly assumed that the tangential velocity (v_θ) distribution in the gap between the cylinders is linear. Based on the exact solution to this problem (see Problem 6.106 the velocity distribution in the gap is not linear. For an outer cylinder with radius $r_o = 2.00$ in. and an inner cylinder with radius $r_i = 1.80$ in., show, with the aid of a plot, how the dimensionless velocity distribution, $v_\theta / r_o \omega$, varies with the dimensionless radial position, r/r_o, for the exact and approximate solutions.

For a linear velocity distribution (approximate solution)

$$v_\theta = (r_o \omega)\left(\frac{r - r_i}{r_o - r_i}\right)$$

and in nondimensional form

$$\frac{v_\theta}{r_o \omega} = \frac{\frac{r}{r_o} - \frac{r_i}{r_o}}{1 - \frac{r_i}{r_o}} \qquad (1)$$

For the exact solution (see Problem 6.106)

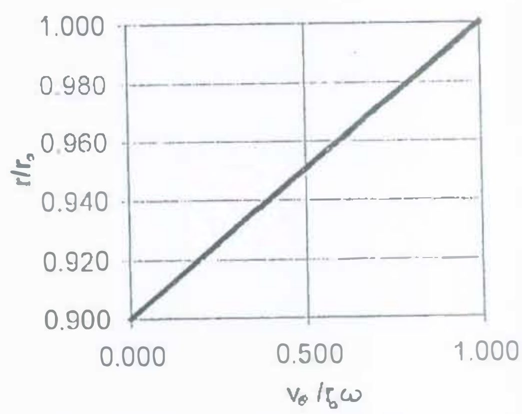
$$v_\theta = \frac{r\omega}{\left(1 - \frac{r_i^2}{r_o^2}\right)}\left[1 - \frac{r_i^2}{r^2}\right]$$

and in nondimensional form

$$\frac{v_\theta}{r_o \omega} = \frac{\frac{r}{r_o}}{\left(1 - \frac{r_i^2}{r_o^2}\right)}\left[1 - \frac{r_i^2}{r_o^2}\left(\frac{r}{r_o}\right)^{-2}\right] \qquad (2)$$

For $r_i = 1.80$ in. and $r_o = 2.00$ in., some tabulated values and a graph are shown below. Note that there is little difference between the exact and approximate solutions for this small gap width. For all practical purposes both solutions fall on the single curve shown.

Linear	Exact	
$v_\theta / r_o \omega$	$v_\theta / r_o \omega$	r/r_o
0.000	0.000	0.900
0.125	0.131	0.913
0.250	0.280	0.925
0.375	0.387	0.938
0.500	0.512	0.950
0.825	0.837	0.983
0.750	0.759	0.975
0.875	0.880	0.988
1.000	1.000	1.000



6.108

6.108 A viscous liquid ($\mu = 0.012\ \text{lb}\cdot\text{s/ft}^2$, $\rho = 1.79$ slugs/ft^3) flows through the annular space between two horizontal, fixed, concentric cylinders. If the radius of the inner cylinder is 1.5 in. and the radius of the outer cylinder is 2.5 in., what is the pressure drop along the axis of the annulus per foot when the volume flowrate is 0.14 ft^3/s?

Check Reynolds number to determine if flow is laminar:

$$Re = \frac{\rho V D_h}{\mu}$$

where $D_h = 2(r_o - r_i)$ and $V = \dfrac{Q}{\pi(r_o^2 - r_i^2)}$.

Thus,

$$Re = \frac{2\rho Q}{\pi\mu(r_o + r_i)} = \frac{2\left(1.79\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(0.14\ \frac{\text{ft}^3}{\text{s}}\right)}{\pi\left(0.012\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(\dfrac{2.5\text{ in.} + 1.5\text{ in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)}$$

$$= 39.9 < 2100$$

Since the Reynolds number is well below 2100 the flow is laminar and

$$Q = \frac{\pi}{8\mu}\frac{\Delta p}{\ell}\left[r_o^4 - r_i^4 - \frac{(r_o^2 - r_i^2)^2}{\ln\frac{r_o}{r_i}}\right] \qquad \text{(Eq. 6.156)}$$

so that

$$\frac{\Delta p}{\ell} = \frac{\dfrac{8\mu Q}{\pi}}{r_o^4 - r_i^4 - \dfrac{(r_o^2 - r_i^2)^2}{\ln\frac{r_o}{r_i}}}$$

$$= \frac{8\left(0.012\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(0.14\ \frac{\text{ft}^3}{\text{s}}\right)/\pi}{\left(\dfrac{2.5\text{ in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)^4 - \left(\dfrac{1.5\text{ in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)^4 - \dfrac{\left[\left(\dfrac{2.5\text{ in}}{12\ \frac{\text{in.}}{\text{ft}}}\right)^2 - \left(\dfrac{1.5\text{ in}}{12\ \frac{\text{in.}}{\text{ft}}}\right)^2\right]^2}{\ln\dfrac{2.5\text{ in.}}{1.5\text{ in.}}}}$$

$$= \underline{\underline{33.1\ \frac{\text{lb}}{\text{ft}^2}\ \text{per ft}}}$$

6.109

6.109 Show how Eq. 6.155 is obtained.

From Eq. 6.147

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r^2 + c_1 \ln r + c_2 \qquad \text{(Eq. 6.147)}$$

For flow in an annulus, $v_z = 0$ at $r = r_o$ and $v_z = 0$ at $r = r_i$. Thus, from Eq. 6.147

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_o^2 + c_1 \ln r_o + c_2$$

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_i^2 + c_1 \ln r_i + c_2$$

and solving for c_1 and c_2 we have

$$c_1 = \frac{-\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)(r_o^2 - r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)} \qquad (1)$$

$$c_2 = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_o^2 + \frac{r_o^2 - r_i^2}{\ln\left(\frac{r_o}{r_i}\right)} \ln r_o\right) \qquad (2)$$

Substitution of Eqs. (1) and (2) into Eq. 6.147 gives

$$\underline{\underline{v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left[r^2 - r_o^2 + \frac{r_i^2 - r_o^2}{\ln\left(\frac{r_o}{r_i}\right)} \ln \frac{r}{r_o}\right]}}$$

which is the desired equation (Eq. 6.155).

6.110

6.110 A wire of diameter d is stretched along the centerline of a pipe of diameter D. For a given pressure drop per unit length of pipe, by how much does the presence of the wire reduce the flowrate if (a) $d/D = 0.1$; (b) $d/D = 0.01$?

The volume flowrate is given by Eq. 6.156

$$Q = \frac{\pi \Delta p}{8\mu \ell}\left[r_o^4 - r_i^4 - \frac{(r_o^2 - r_i^2)^2}{\ln\left(\frac{r_o}{r_i}\right)}\right] \qquad \text{(Eq. 6.156)}$$

which can be written as

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - \left(\frac{r_i}{r_o}\right)^4 + \frac{\left[1-\left(\frac{r_i}{r_o}\right)^2\right]^2}{\ln\left(\frac{r_i}{r_o}\right)}\right\} \qquad (1)$$

Since $\frac{r_i}{r_o} = \frac{d}{D}$, Eq. (1) can also be written as

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - \left(\frac{d}{D}\right)^4 + \frac{\left[1-\left(\frac{d}{D}\right)^2\right]^2}{\ln\left(\frac{d}{D}\right)}\right\} \qquad (2)$$

Note that for $\frac{d}{D} = 0$ (no wire)

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}$$

which corresponds to Poiseuille's Law (Eq. 6.151).

(a) For $\frac{d}{D} = 0.1$, Eq. (2) gives

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - (0.1)^4 + \frac{\left[1-(0.1)^2\right]^2}{\ln(0.1)}\right\} = 0.574$$

Thus, for the same Δp the flowrate is reduced by

$$\% \text{ reduction in } Q = (1 - 0.574) \times 100 = \underline{\underline{42.6\%}}$$

(b) Similarly, for $\frac{d}{D} = 0.01$ Eq. (2) gives

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - (0.01)^4 + \frac{\left[1-(0.01)^2\right]^2}{\ln(0.01)}\right\} = 0.783$$

and

$$\% \text{ reduction in } Q = (1 - 0.783) \times 100 = \underline{\underline{21.7\%}}$$

Note that the presence of even a very small wire along the tube centerline has a significant effect on the flowrate.

7.2

7.2 Verify the left-hand side of Eq. 7.2 is dimensionless using the MLT system.

$$\frac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right), \text{ where } D \doteq L,\ \Delta p_\ell \doteq \frac{M}{L^2T^2},\ \rho \doteq \frac{M}{L^3}, \text{ and } V \doteq \frac{L}{T}$$

Thus,

$$\frac{D\,\Delta p_\ell}{\rho V^2} \doteq L\,\frac{M}{L^2T^2}\Big/\left(\frac{M}{L^3}\,\frac{L^2}{T^2}\right) = \underline{\underline{M^0L^0T^0}}$$

That is $\frac{D\,\Delta p_\ell}{\rho V^2}$ is dimensionless.

7.3

7.3 The Reynolds number, $\rho VD/\mu$, is a very important parameter in fluid mechanics. Verify that the Reynolds number is dimensionless, using both the *FLT* system and the *MLT* system for basic dimensions, and determine its value for ethyl alcohol flowing at a velocity of 3 m/s through a 2-in.-diameter pipe.

$$\text{Reynolds number} = \frac{\rho V D}{\mu} \doteq \frac{(FL^{-4}T^{2})(LT^{-1})(L)}{FL^{-2}T} \doteq \underline{\underline{F^0L^0T^0}}$$

$$\doteq \frac{(ML^{-3})(LT^{-1})(L)}{ML^{-1}T^{-1}} \doteq \underline{\underline{M^0L^0T^0}}$$

For ethyl alcohol, $\mu = 1.19 \times 10^{-3}\ \frac{N \cdot s}{m^2}$ and $\rho = 789\ \frac{kg}{m^3}$.

Thus,

$$\frac{\rho V D}{\mu} = \frac{\left(789\ \frac{kg}{m^3}\right)\left(3\ \frac{m}{s}\right)\left(\frac{2}{12}\ ft\right)\left(0.3048\ \frac{m}{ft}\right)}{1.19 \times 10^{-3}\ \frac{N \cdot s}{m^2}}$$

$$= \underline{\underline{1.01 \times 10^5}}$$

7.4

7.4 What are the dimensions of acceleration of gravity, density, dynamic viscosity, kinematic viscosity, specific weight, and speed of sound in (**a**) the *FLT* system, and (**b**) the *MLT* system? Compare your results with those given in Table 1.1 in Chapter 1.

$$g = \text{acceleration of gravity} = \frac{\text{velocity}}{\text{time}} \doteq \frac{L}{T^2}$$

$$\rho = \text{density} = \frac{\text{mass}}{\text{unit volume}} \doteq \frac{M}{L^3} \doteq \frac{FT^2}{L^4} \quad (\text{since } F \doteq MLT^{-2})$$

$$\mu = \text{dynamic viscosity} = \frac{\text{stress}}{\text{velocity gradient}} \doteq \frac{FL^{-2}}{T^{-1}} \doteq \frac{M}{LT}$$

$$\nu = \text{kinematic viscosity} = \frac{\text{dynamic viscosity}}{\text{density}} \doteq \frac{FL^{-2}T}{FT^2L^{-4}} \doteq \frac{L^2}{T}$$

$$\gamma = \text{specific weight} = \frac{\text{weight}}{\text{unit volume}} \doteq \frac{F}{L^3} \doteq \frac{(MLT^{-2})}{L^3} \doteq \frac{MT^{-2}}{L^2}$$

$$c = \text{speed of sound} = \frac{\text{length}}{\text{time}} = \frac{L}{T}$$

Thus,

(a) in the FLT system,

$g \doteq \underline{\underline{LT^{-2}}}$

$\rho \doteq \underline{\underline{FL^{-4}T^{2}}}$

$\mu \doteq \underline{\underline{FL^{-2}T}}$

$\nu \doteq \underline{\underline{L^{2}T^{-1}}}$

$\gamma \doteq \underline{\underline{FL^{-3}}}$

$c \doteq \underline{\underline{LT^{-1}}}$

(b) in the MLT system,

$g \doteq \underline{\underline{LT^{-2}}}$

$\rho \doteq \underline{\underline{ML^{-3}}}$

$\mu \doteq \underline{\underline{ML^{-1}T^{-1}}}$

$\nu \doteq \underline{\underline{L^{2}T^{-1}}}$

$\gamma \doteq \underline{\underline{ML^{-2}T^{-2}}}$

$c \doteq \underline{\underline{LT^{-1}}}$

7.5

7.5 For the flow of a thin film of a liquid with a depth h and a free surface, two important dimensionless parameters are the Froude number, $V/\sqrt{gh}$, and the Weber number, $\rho V^2 h/\sigma$. Determine the value of these two parameters for glycerin (at 20 °C) flowing with a velocity of 0.7 m/s at a depth of 3 mm.

$$\frac{V}{\sqrt{gh}} = \frac{0.7 \frac{m}{s}}{\sqrt{\left(9.81 \frac{m}{s^2}\right)(0.003\,m)}} = \underline{\underline{4.08}}$$

$$\frac{\rho V^2 h}{\sigma} = \frac{\left(1260 \frac{kg}{m^3}\right)\left(0.7 \frac{m}{s}\right)^2 (0.003\,m)}{6.33 \times 10^{-2} \frac{N}{m}} = \underline{\underline{29.3}}$$

7.6

7.6 The Mach number for a body moving through a fluid with velocity V is defined as V/c, where c is the speed of sound in the fluid. This dimensionless parameter is usually considered to be important in fluid dynamics problems when its value exceeds 0.3. What would be the velocity of a body at a Mach number of 0.3 if the fluid is: **(a)** air at standard atmospheric pressure and 20 °C, and **(b)** water at the same temperature and pressure?

(a) $$\frac{V}{c} = 0.3$$

For air at 20°C, $c = 343.3 \frac{m}{s}$ (Table B.4 in Appendix B) so that

$$V = 0.3\left(343.3 \frac{m}{s}\right) = \underline{\underline{103 \frac{m}{s}}}$$

(b) For water at 20°C, $c = 1481 \frac{m}{s}$ (Table B.2 in Appendix B) so that

$$V = 0.3\left(1481 \frac{m}{s}\right) = \underline{\underline{444 \frac{m}{s}}}$$

7.8

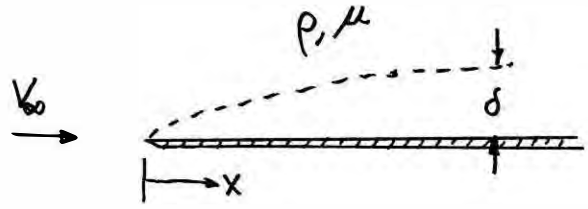
7.8 The power, $\mathcal{P}$, required to run a pump that moves fluid within a piping system is dependent upon the volume flowrate, Q, density, ρ, impeller diameter, d, angular velocity, ω, and fluid viscosity, μ. Find the number of pi terms for this relationship.

Given that $\mathcal{P} = f(Q, \rho, d, \omega, \mu)$
where (see Table 1.1),
$\mathcal{P} \doteq FL/T$, $\rho = M/L^3 \doteq FT^2/L^4$, $d \doteq L$, $\omega \doteq 1/T$, and
$\mu \doteq FT/L^2$

Thus, $k - r = 6 - 3 = 3$ since there are $k = 6$ variables and $r = 3$ basic dimensions (MLT)
Hence, it takes $\underline{\underline{3 \text{ pi terms}}}$: $\Pi_1 = \phi(\Pi_2, \Pi_3)$

7.9

7.9 For low speed flow over a flat plate, one measure of the boundary layer is the resulting thickness, δ, at a given downstream location. The boundary layer thickness is a function of the free stream velocity, V_∞, fluid density and viscosity ρ and μ, and the distance from the leading edge, x. Find the number of pi terms for this relationship.



Given that

$\delta = f(V_\infty, \rho, \mu, x)$

where

$\delta \doteq L,\ V_\infty \doteq \frac{L}{T},\ \rho \doteq \frac{M}{L^3},\ \mu \doteq \frac{M}{LT},$ and $x \doteq L$

Thus, there are 5 variables and 3 basic dimensions (MLT) so that

$k - r = 5 - 3 = 2$

Hence, <u>2 pi terms</u> are needed: $\Pi_1 = \phi(\Pi_2)$

7.10

7.10 The excess pressure inside a bubble (discussed in Chapter 1) is known to be dependent on bubble radius and surface tension. After finding the pi terms, determine the variation in excess pressure if we (a) double the radius and (b) double the surface tension.

Given $\Delta p = f(R, \sigma)$, where $\Delta p \doteq \frac{F}{L^2} = \frac{M}{LT^2}$, $R \doteq L$, and $\sigma \doteq \frac{F}{L} = \frac{M}{T^2}$

Consider the (MLT) units so that
$k - r = 3 - 3 = 0$ since there 3 variables and 3 dimensions.
According to this, there should be $k-r=0$ pi terms!?
However, if we consider the (FLT) units we see that it takes only F and L, T is not needed, so that $r = 2$.
Hence, $k - r = 3 - 2 = 1$, so only 1 pi term is needed.

That is, $\Pi_1 = \text{constant}$
To determine Π_1, consider

$\Pi_1 = \Delta p \, R^a \sigma^b$ or

$$\Delta p \, R^a \sigma^b \doteq \frac{F}{L^2} L^a \left(\frac{F}{L}\right)^b = F^{1+b} \, L^{-2+a-b}$$

Thus:

$F: \quad 1 + b = 0$

$L: \quad a - b - 2 = 0$

or $b = -1$ and $a = b + 2 = 1$

Hence $\underline{\underline{\Pi_1 = \frac{\Delta p \, R}{\sigma}}}$ or $\frac{\Delta p \, R}{\sigma} = C$, where $C = \text{constant}$.

or
$$\Delta p = \frac{C\sigma}{R} \qquad (1)$$

(a) If R is doubled, $\underline{\underline{\Delta p \text{ is reduced by half.}}}$ (See Eq. (1))

(b) If σ is doubled, $\underline{\underline{\Delta p \text{ is doubled.}}}$ (See Eq. (1))

7.11

7.11 It is known that the variation of pressure, Δp, within a static fluid is dependent upon the specific weight of the fluid and the elevation difference, Δz. Using dimensional analysis, find the form of the hydrostatic equation for pressure variation.

Given $\Delta p = f(\gamma, \Delta z)$

where

$\Delta p \doteq \frac{F}{L^2}$, $\gamma \doteq \frac{F}{L^3}$, and $\Delta z \doteq L$ so that $k=3$ and $r=2$ (i.e. (F, L))

Thus,

$k-r = 3-2 = 1$ so that there is only 1 pi term: $\Pi_1 = C =$ constant

Consider

$$\Pi_1 = \Delta p\, \gamma^a \Delta z^b \doteq \left(\frac{F}{L^2}\right)\left(\frac{F}{L^3}\right)^a (L)^b = F^{1+a} L^{(-2-3a+b)} = F^0 L^0$$

Hence,

$F: \; 1+a=0$

$L: \; -2-3a+b=0$

so that $a=-1$ and $b=2+3a=2-3=-1$

Therefore,

$$\Pi_1 = \Delta p\, \gamma^{-1} \Delta z^{-1} = C$$

or

$$\underline{\underline{\Delta p = C\gamma\Delta z}} \quad \text{where } C = \text{constant}$$

Through experimentation it is found that $C=1$. Note that this agrees with the material in Chapter 2.

7.12

7.12 At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity, V, in the larger pipe, and the fluid density, ρ, and viscosity, μ. Use D_1, V, and μ as repeating variables to determine a suitable set of dimensionless parameters. Why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

$$\Delta p = f(D_1, D_2, V, \rho, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D_1 \doteq L$ $\quad D_2 \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $6-3=3$ dimensionless parameters required. Use D_1, V, and μ as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D_1^a V^b \mu^c$$

and

$$(FL^{-2})(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$-2 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=1$, $b=-1$, $c=-1$, and therefore

$$\Pi_1 = \frac{\Delta p\, D_1}{V\mu}$$

Check dimensions using MLT system:

$$\frac{\Delta p\, D_1}{V\mu} \doteq \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = D_2 D_1^a V^b \mu^c$$

$$L\,(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{D_2}{D_1}$$

(con't)

7.12 (Con't)

Π_2 is obviously dimensionless.

For Π_3:

$$\Pi_3 = \rho D_1^{a} V^{b} \mu^{c}$$

$$(FL^{-4}T^{2})(L)^{a}(LT^{-1})^{b}(FL^{-2}T)^{c} \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-4 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$

$$2 - b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 1$, $b = 1$, $c = -1$ and therefore

$$\Pi_3 = \frac{\rho D_1 V}{\mu}$$

Check dimensions using MLT system:

$$\frac{\rho D_1 V}{\mu} \doteq \frac{(ML^{-3})(L)(LT^{-1})}{ML^{-1}T^{-1}} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Thus,

$$\underline{\underline{\frac{\Delta p\, D_1}{V\mu} = \phi\left(\frac{D_2}{D_1}, \frac{\rho D_1 V}{\mu}\right)}}$$

From the continuity equation,

$$V \frac{\pi}{4} D_1^{2} = V_s \frac{\pi}{4} D_2^{2}$$

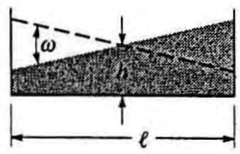
where V_s is the velocity in the smaller pipe. Since

$$V_s = \left(\frac{D_1}{D_2}\right)^2 V$$

V_s is not independent of D_1, D_2, and V and therefore should <u>not</u> be included as an independent variable.

7.13

7.13 Water sloshes back and forth in a tank as shown in Fig. P7.13. The frequency of sloshing, ω, is assumed to be a function of the acceleration of gravity, g, the average depth of the water, h, and the length of the tank, ℓ. Develop a suitable set of dimensionless parameters for this problem using g and ℓ as repeating variables.



■ FIGURE P7.13

$$\omega = f(g, h, \ell)$$

$$\omega \doteq T^{-1} \quad g \doteq LT^{-2} \quad h \doteq L \quad \ell \doteq L$$

From the pi theorem, $4-2=2$ dimensionless parameters required. Use g and ℓ as repeating variables, Thus,

$$\Pi_1 = \omega g^a \ell^b$$

and

$$(T^{-1})(LT^{-2})^a (L)^b \doteq L^0 T^0$$

so that

$$a + b = 0 \qquad \text{(for } L\text{)}$$

$$-1 - 2a = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1/2$, $b = 1/2$, and therefore

$$\Pi_1 = \omega \sqrt{\frac{\ell}{g}}$$

Check dimensions:

$$\omega \sqrt{\frac{\ell}{g}} \doteq \frac{1}{T}\sqrt{\frac{L}{LT^{-2}}} \doteq L^0 T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = h g^a \ell^b$$

$$L(LT^{-2})^a (L)^b \doteq L^0 T^0$$

$$1 + a + b = 0 \qquad \text{(for } L\text{)}$$

$$-2a = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 0$, $b = -1$, and therefore

$$\Pi_2 = \frac{h}{\ell}$$

and Π_2 is obviously dimensionless. Thus,

$$\omega \sqrt{\frac{\ell}{g}} = \phi\left(\frac{h}{\ell}\right)$$

7.14

7.14 **Assume that the power, $\mathcal{P}$, required to drive a fan is a function of the fan diameter, D, the fluid density, ρ, the rotational speed, ω, and the flowrate, Q. Use D, ω, and ρ as repeating variables to determine a suitable set of pi terms.**

$$\mathcal{P} = f(D, \rho, \omega, Q)$$

$\mathcal{P} \doteq FLT^{-1}$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \omega \doteq T^{-1}$ $\quad Q \doteq L^{3}T^{-1}$

From the pi theorem, $5-3=2$ pi terms required. Use D, ω, and ρ as repeating variables. Thus,

$$\Pi_1 = \mathcal{P}\, D^{a}\omega^{b}\rho^{c}$$

and

$$(FLT^{-1})(L)^{a}(T^{-1})^{b}(FL^{-4}T^{2})^{c} \doteq F^{0}L^{0}T^{0}$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-1 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -5$, $b = -3$, $c = -1$, and therefore

$$\Pi_1 = \frac{\mathcal{P}}{\rho D^{5}\omega^{3}}$$

Check dimensions using MLT system:

$$\frac{\mathcal{P}}{\rho D^{5}\omega^{3}} \doteq \frac{ML^{2}T^{-3}}{(ML^{-3})(L)^{5}(T^{-1})^{3}} \doteq M^{0}L^{0}T^{0} \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = Q\, D^{a}\omega^{b}\rho^{c}$$

$$(L^{3}T^{-1})(L)^{a}(T^{-1})^{b}(FL^{-4}T^{2})^{c} \doteq F^{0}L^{0}T^{0}$$

$$c = 0 \qquad \text{(for } F\text{)}$$
$$3 + a - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-1 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

(con't)

7.14 (con't)

It follows that $a=-3$, $b=-1$, $c=0$, and therefore

$$\Pi_2 = \frac{Q}{D^3\omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3\omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

Thus,

$$\underline{\underline{\frac{\mathcal{P}}{\rho D^5\omega^3} = \phi\left(\frac{Q}{D^3\omega}\right)}}$$

7.15

7.15 Assume that the flowrate, Q, of a gas from a smokestack is a function of the density of the ambient air, ρ_a, the density of the gas, ρ_g, within the stack, the acceleration of gravity, diameter of the stack, h and d, respectively. Use ρ_a, d, and g as repeating variables to develop a set of pi terms that could be used to describe this problem.

$$Q = f(\rho_a, \rho_g, g, h, d)$$

$Q \doteq L^3T^{-1}$ $\rho_a \doteq ML^{-3}$ $\rho_g \doteq ML^{-3}$ $g \doteq LT^{-2}$ $h \doteq L$ $d \doteq L$

From the pi theorem, $6-3 = 3$ pi terms required. Use ρ_a, d, and g as repeating variables. Thus,

$$\Pi_1 = Q\,\rho_a^a\, d^b\, g^c$$

and

$$(L^3T^{-1})(ML^{-3})^a(L)^b(LT^{-2})^c \doteq M^0L^0T^0$$

so that

$$a = 0 \qquad \text{(for } M\text{)}$$
$$3 - 3a + b + c = 0 \qquad \text{(for } L\text{)}$$
$$-1 - 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=0$, $b = -\frac{5}{2}$, $c = -\frac{1}{2}$, and therefore

$$\Pi_1 = \frac{Q}{d^{\frac{5}{2}}\, g^{1/2}}$$

Check dimensions using FLT system:

$$\frac{Q}{d^{5/2} g^{1/2}} \doteq \frac{L^3T^{-1}}{(L)^{5/2}(LT^{-2})^{1/2}} \doteq F^0L^0T^0 \qquad \therefore \text{OK}$$

(con't)

7.15 (Con't)

For Π_2:

$$\Pi_2 = \rho_g \rho_a^a d^b g^c$$

$$(ML^{-3})(ML^{-3})^a (L)^b (LT^{-2})^c \doteq M^0 L^0 T^0$$

$$1 + a = 0 \quad \text{(for } M\text{)}$$

$$-3 - 3a + b + c = 0 \quad \text{(for } L\text{)}$$

$$-2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{\rho_g}{\rho_a}$$

which is obviously dimensionless.

For Π_3:

$$\Pi_3 = h \rho_a^a d^b g^c$$

$$(L)(ML^{-3})^a (L)^b (LT^{-2})^c \doteq M^0 L^0 T^0$$

$$a = 0 \quad \text{(for } M\text{)}$$

$$1 - 3a + b + c = 0 \quad \text{(for } L\text{)}$$

$$-2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = 0$, $b = -1$, $c = 0$, and therefore

$$\Pi_3 = \frac{h}{d}$$

which is obviously dimensionless.

Thus,

$$\underline{\underline{\frac{Q}{d^{5/2} g^{1/2}} = \phi\left(\frac{\rho_g}{\rho_a}, \frac{h}{d}\right)}}$$

7.16

7. The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \omega \doteq T^{-1}$ $\quad Q \doteq L^{3}T^{-1}$

From the pi theorem, 5-3 = 2 pi terms required. Use D, ρ, and ω as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D^a \rho^b \omega^c$$

and so that

$$(FL^{-2})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$$1 + b = 0 \qquad \text{(for } F\text{)}$$

$$-2 + a - 4b = 0 \qquad \text{(for } L\text{)}$$

$$2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -2$, $b = -1$, $c = -2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2\rho\omega^2} \doteq \frac{ML^{-1}T^{-2}}{(L)^2(ML^{-3})(T^{-1})^2} \doteq M^0L^0T^0 \qquad \therefore \text{ok}$$

For Π_2:

$$\Pi_2 = Q\, D^a \rho^b \omega^c$$

$$(L^3T^{-1})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$$b = 0 \qquad \text{(for } F\text{)}$$

$$3 + a - 4b = 0 \qquad \text{(for } L\text{)}$$

$$-1 + 2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -3$, $b = 0$, $c = -1$, and therefore

$$\Pi_2 = \frac{Q}{D^3\omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3\omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \qquad \therefore \text{ok}$$

Thus,

$$\frac{\Delta p}{D^2\rho\omega^2} = \phi\left(\frac{Q}{D^3\omega}\right)$$

7.17

7.17 A thin elastic wire is placed between rigid supports. A fluid flows past the wire, and it is desired to study the static deflection, δ, at the center of the wire due to the fluid drag. Assume that

$$\delta = f(\ell, d, \rho, \mu, V, E)$$

where ℓ is the wire length, d the wire diameter, ρ the fluid density, μ the fluid viscosity, V the fluid velocity, and E the modulus of elasticity of the wire material. Develop a suitable set of pi terms for this problem.

$\delta \doteq L \quad \ell \doteq L \quad d \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad \mu \doteq FL^{-2}T \quad V \doteq LT^{-1} \quad E \doteq FL^{-2}$

From the pi theorem, $7-3=4$ pi terms required. Use d, V, and E as repeating variables. Thus,

$$\Pi_1 = \delta\, d^a V^b E^c$$

and

$$(L)(L)^a(LT^{-1})^b(FL^{-2})^c \doteq F^0L^0T^0$$

so that

$$c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$-b = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_1 = \frac{\delta}{d}$$

which is obviously dimensionless.

For Π_2:

$$\Pi_2 = \ell\, d^a V^b E^c$$

and as for Π_1, $a=-1$, $b=0$, $c=0$ so that

$$\Pi_2 = \frac{\ell}{d}$$

For Π_3:

$$\Pi_3 = \rho\, d^a V^b E^c$$

$$(FL^{-4}T^{2})(L)^a(LT^{-1})^b(FL^{-2})^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$-4 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$2 - b = 0 \qquad \text{(for } T\text{)}$$

(Con't)

7.17 (con't.)

It follows that $a=0$, $b=2$, $c=-1$, and therefore

$$\Pi_3 = \frac{\rho V^2}{E}$$

Check dimensions using MLT system:

$$\frac{\rho V^2}{E} \doteq \frac{(ML^{-3})(LT^{-1})^2}{ML^{-1}T^{-2}} \doteq M^0L^0T^0 \qquad \therefore \text{ OK}$$

For Π_4:

$$\Pi_4 = \mu\, d^a V^b E^c$$

$$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-2})^c \doteq F^0L^0T^0$$

$$1+c=0 \qquad \text{(for } F\text{)}$$

$$-2+a+b-2c=0 \qquad \text{(for } L\text{)}$$

$$1-b=0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=1$, $c=-1$, and therefore

$$\Pi_4 = \frac{\mu V}{d E}$$

Check dimensions using MLT system:

$$\frac{\mu V}{dE} \doteq \frac{(ML^{-1}T^{-1})(LT^{-1})}{(L)(ML^{-1}T^{-2})} \doteq M^0L^0T^0 \qquad \therefore \text{ OK}$$

Thus,

$$\underline{\underline{\frac{\delta}{d} = \phi\left(\frac{\ell}{d}, \frac{\rho V^2}{E}, \frac{\mu V}{dE}\right)}}$$

7.18

7.18 Because of surface tension, it is possible, with care, to support an object heavier than water on the water surface as shown in Fig. P7.18. (See Video V1.9.) The maximum thickness, h, of a square of material that can be supported is assumed to be a function of the length of the side of the square, ℓ, the density of the material, ρ, the acceleration of gravity, g, and the surface tension of the liquid, σ. Develop a suitable set of dimensionless parameters for this problem.

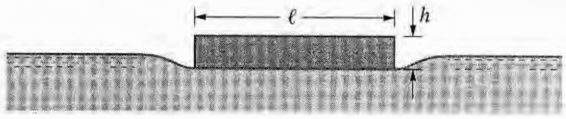


FIGURE P7.18

$$h = f(\ell, \rho, g, \sigma)$$

$h \doteq L \quad \ell \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad g \doteq LT^{-2} \quad \sigma \doteq FL^{-1}$

From the pi theorem, $5-3=2$ pi terms required. Use ℓ, g, and ρ as repeating variables. Thus,

$$\Pi_1 = h\,\ell^a g^b \rho^c$$

and

$$(L)(L)^a(LT^{-2})^b(FL^{-4}T^{2})^c \doteq F^0L^0T^0$$

So that

$$c = 0 \qquad \text{(for } F\text{)}$$

$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-2b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_1 = \frac{h}{\ell}$$

which is obviously dimensionless.

For Π_2:

$$\Pi_2 = \sigma\,\ell^a g^b \rho^c$$

$$(FL^{-1})(L)^a(LT^{-2})^b(FL^{-4}T^{2})^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-2b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -2$, $b = -1$, $c = -1$, and therefore

$$\Pi_2 = \frac{\sigma}{\ell^2 g \rho}$$

Check dimensions using MLT system:

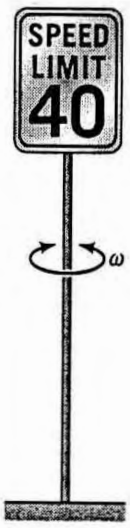
$$\frac{\sigma}{\ell^2 g\rho} \doteq \frac{(MT^{-2})}{(L^2)(LT^{-2})(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

Thus,

$$\underline{\underline{\frac{h}{\ell} = \phi\left(\frac{\sigma}{\ell^2 g\rho}\right)}}$$

7.19

7.19 Under certain conditions, wind blowing past a rectangular speed limit sign can cause the sign to oscillate with a frequency ω. (See Fig. P7.19 and Video V9.9.) Assume that ω is a function of the sign width, b, sign height, h, wind velocity, V, air density, ρ, and an elastic constant, k, for the supporting pole. The constant, k, has dimensions of FL. Develop a suitable set of pi terms for this problem.



■ FIGURE P7.19

$$\omega = f(b, h, V, \rho, k)$$

$\omega \doteq T^{-1}$ $b \doteq L$ $h \doteq L$ $V \doteq LT^{-1}$ $\rho \doteq FL^{-4}T^{2}$ $k \doteq FL$

From the pi theorem $6-3=3$ pi terms required. Use b, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \omega b^a V^b \rho^c$$

and

$$(T^{-1})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$$c = 0 \quad \text{(for } F\text{)}$$
$$a + b - 4c = 0 \quad \text{(for } L\text{)}$$
$$-1 - b + 2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = 1$, $b = -1$, $c = 0$, and therefore

$$\Pi_1 = \frac{\omega b}{V}$$

Check dimensions:

$$\frac{\omega b}{V} \doteq \frac{(T^{-1})(L)}{(LT^{-1})} \doteq L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = h\, b^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \quad \text{(for } F\text{)}$$
$$1 + a + b - 4c = 0 \quad \text{(for } L\text{)}$$
$$-b + 2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{h}{b}$$

which is obviously dimensionless. (con't)

7.19 (Con't)

For Π_3:

$$\Pi_3 = k\, b^a V^b \rho^c$$

$$(FL)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F)$$

$$1 + a + b - 4c = 0 \qquad \text{(for } L)$$

$$-b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a = -3$, $b = -2$, $c = -1$, and therefore

$$\Pi_3 = \frac{k}{b^3 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{k}{b^3V^2\rho} \doteq \frac{ML^2T^{-2}}{(L^3)(LT^{-1})^2(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Thus,

$$\frac{\omega b}{V} = \phi\left(\frac{h}{b}, \frac{k}{b^3V^2\rho}\right)$$

7.20

7.20 The height, h, that a liquid will rise in a capillary tube is a function of the tube diameter, D, the specific weight of the liquid, γ, and the surface tension, σ. Perform a dimensional analysis using both the FLT and MLT systems for basic dimensions. Note: The results should obviously be the same regardless of the system of dimensions used. If your analysis indicates otherwise, go back and check your work giving particular attention to the required number of reference dimensions.

$$h = f(D, \gamma, \sigma)$$

Using FLT system:

$h \doteq L \quad D \doteq L \quad \gamma \doteq FL^{-3} \quad \sigma \doteq FL^{-1}$

From the pi theorem, $4-2 = 2$ pi terms required.

By inspection, for Π_1 (containing h):

$$\Pi_1 = \frac{h}{D}$$

which is obviously dimensionless.

For Π_2 (containing γ and σ):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} \doteq \frac{FL^{-1}}{(FL^{-3})(L)^2} = F^0L^0$$

Thus,

$$\underline{\underline{\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)}}$$

Using MLT system:

$h \doteq L \quad D \doteq L \quad \gamma \doteq ML^{-2}T^{-2} \quad \sigma \doteq MT^{-2}$

Although there appears to be 3 reference dimensions, only 2 reference dimensions are actually required (L and MT^{-2}) to describe the variables. By inspection, for Π_1 (see above)

$$\Pi_1 = \frac{h}{D}$$

and for Π_2 (containing γ and σ):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} = \frac{MT^{-2}}{(ML^{-2}T^{-2})(L)^2} = M^0L^0T^0$$

Thus, (as above)

$$\underline{\underline{\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)}}$$

7.21

7.21 A cone and plate viscometer consists of a cone with a very small angle α which rotates above a flat surface as shown in Fig. P7.21. The torque, $\mathcal{T}$, required to rotate the cone at an angular velocity, ω, is a function of the radius, R, the cone angle, α, and the fluid viscosity, μ, in addition to ω. With the aid of dimensional analysis, determine how the torque will change if both the viscosity and angular velocity are doubled.

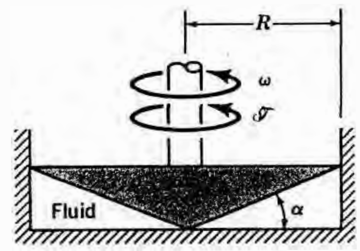


FIGURE P7.21

$$\mathcal{T} = f(R, \alpha, \mu, \omega)$$

$\mathcal{T} \doteq FL \qquad R \doteq L \qquad \alpha \doteq F^0L^0T^0 \qquad \mu \doteq FL^{-2}T \qquad \omega \doteq T^{-1}$

From the pi theorem, 5 − 3 = 2 pi terms required.

By inspection, for Π_1 (containing $\mathcal{T}$):

$$\Pi_1 = \frac{\mathcal{T}}{\mu\omega R^3} \doteq \frac{FL}{(FL^{-2}T)(T^{-1})(L)^3} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\mathcal{T}}{\mu\omega R^3} \doteq \frac{ML^2T^{-2}}{(ML^{-1}T^{-1})(T^{-1})(L)^3} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

The angle, α, can be used as Π_2 since it is dimensionless. Thus,

$$\frac{\mathcal{T}}{\mu\omega R^3} = \phi(\alpha)$$

or

$$\mathcal{T} = \mu\omega R^3 \phi(\alpha)$$

It follows that if both μ and ω are doubled $\mathcal{T}$ will increase by a factor of 4.

7.22

7.22 The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, \ell, V, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \ell \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p\, D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\Delta p\, D}{\mu V} \doteq \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} \doteq M^0 L^0 T^0 \quad \therefore \text{ok}$$

For Π_2 (containing ℓ):

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p\, D}{\mu V} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

From the statement of the problem, $\Delta p \propto \ell$ so that Eq. (1) must be of the form

$$\frac{\Delta p\, D}{\mu V} = K \frac{\ell}{D}$$

where K is some constant. It thus follows that

$$\underline{\underline{\Delta p \propto \frac{1}{D^2}}}$$

for a given velocity.

7.23

7.23 A cylinder with a diameter, D, floats upright in a liquid as shown in Fig. P7.23. When the cylinder is displaced slightly along its vertical axis it will oscillate about its equilibrium position with a frequency, ω. Assume that this frequency is a function of the diameter, D, the mass of the cylinder, m, and the specific weight, γ, of the liquid. Determine, with the aid of dimensional analysis, how the frequency is related to these variables. If the mass of the cylinder were increased, would the frequency increase or decrease?

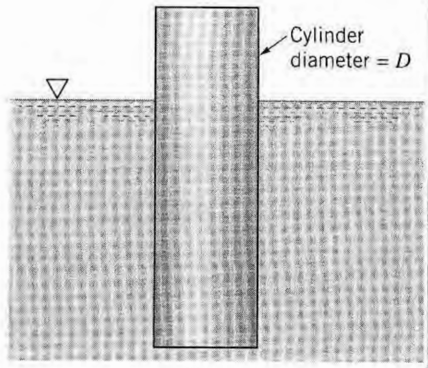


FIGURE P7.23

$$\omega = f(D, m, \gamma)$$

$$\omega \doteq T^{-1} \qquad D \doteq L \qquad m \doteq FL^{-1}T^2 \qquad \gamma = FL^{-3}$$

From the pi theorem, $4-3 = 1$ pi term required.

By inspection:

$$\Pi_1 = \frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{FL^{-1}T^2}{FL^{-3}}} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{M}{ML^{-2}T^{-2}}} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Since there is only 1 pi term, it follows that

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} = C$$

where C is a constant. Thus,

$$\underline{\underline{\omega = CD\sqrt{\frac{\gamma}{m}}}}$$

From this result it follows that if m is increased ω will <u>decrease</u>.

7.24

7.24 A liquid spray nozzle is designed to produce a specific size droplet with diameter, d. The droplet size depends on the nozzle diameter, D, nozzle velocity, V, and the liquid properties ρ, μ, σ. Using the common dimensionless terms found in Table 7.1, determine the functional relationship for the dependent diameter ratio of d/D.

Given $d = f(D, V, \rho, \mu, \sigma)$ so that $k = 6$ (there 6 variables) and $r = 3$ (it takes MLT or FLT to describe them).

Hence, $k - r = 6 - 3 = 3$ which means that 3 pi terms are needed.

$\Pi_1 = \phi(\Pi_2, \Pi_3)$, where $\Pi_1 = \frac{d}{D}$ is clearly dimensionless.

With the independent variables (i.e. D, V, ρ, μ, σ) it is clear that the Reynolds number can be one of the π terms.

Hence, set $\Pi_2 = \rho V D/\mu$.

Π_3 must include the surface tension, σ, since it does not appear in Π_1 or Π_2. Based on the information in Table 7.1 it is seen that the Weber number, We, can be the other π term.

Hence, set $\Pi_3 = \rho V^2 D/\sigma$

Thus,

$$\frac{d}{D} = \phi\left(\frac{\rho V D}{\mu}, \frac{\rho V^2 D}{\sigma}\right)$$

or

$$\frac{d}{D} = \phi(Re, We)$$

7.25

7.25 The velocity, c, at which pressure pulses travel through arteries (pulse-wave velocity) is a function of the artery diameter, D, and wall thickness, h, the density of blood, ρ, and the modulus of elasticity, E, of the arterial wall. Determine a set of nondimensional parameters that can be used to study experimentally the relationship between the pulse-wave velocity and the variables listed. Form the nondimensional parameters by inspection.

$$c = f(D, h, \rho, E)$$

$c \doteq LT^{-1}$ $\quad D \doteq L \quad h \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad E \doteq FL^{-2}$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing c):

$$\Pi_1 = \frac{c\sqrt{\rho}}{\sqrt{E}} \doteq \frac{(LT^{-1})(FL^{-4}T^{2})^{1/2}}{(FL^{-2})^{1/2}} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{c\sqrt{\rho}}{\sqrt{E}} \doteq \frac{(LT^{-1})(ML^{-3})^{1/2}}{(ML^{-1}T^{-2})^{1/2}} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 let

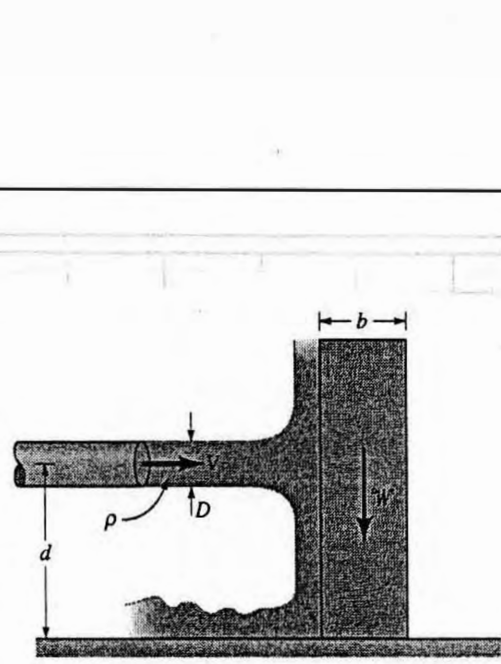
$$\Pi_2 = \frac{h}{D}$$

which is obviously dimensionless. Thus,

$$\underline{\underline{c\sqrt{\frac{\rho}{E}} = \phi\left(\frac{h}{D}\right)}}$$

7.26

7.26 As shown in Fig. P7.26 and Video V5.6, a jet of liquid directed against a block can tip over the block. Assume that the velocity, V, needed to tip over the block is a function of the fluid density, ρ, the diameter of the jet, D, the weight of the block, $\mathcal{W}$, the width of the block, b, and the distance, d, between the jet and the bottom of the block. **(a)** Determine a set of dimensionless parameters for this problem. Form the dimensionless parameters by inspection. **(b)** Use the momentum equation to determine an equation for V in terms of the other variables. **(c)** Compare the results of parts (a) and (b).



■ FIGURE P7.26

(a) $$V = f(\rho, D, \mathcal{W}, b, d)$$

$$V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad D \doteq L \quad \mathcal{W} \doteq F \quad b \doteq L \quad d \doteq L$$

From the pi theorem, $6-3 = 3$ pi terms required.
By inspection for Π_1 (containing V)

$$\Pi_1 = VD\sqrt{\frac{\rho}{\mathcal{W}}} \doteq (LT^{-1})(L)\left(\sqrt{\frac{FL^{-4}T^{2}}{F}}\right) \doteq F^0L^0T^0$$

Check using MLT:

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = (LT^{-1})(L)\left(\sqrt{\frac{ML^{-3}}{MLT^{-2}}}\right) \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2 let

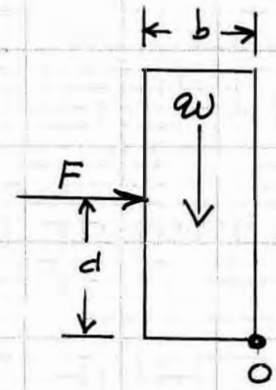
$$\Pi_2 = \frac{b}{d}$$

and for Π_3

$$\Pi_3 = \frac{d}{D}$$

and both Π_2 and Π_3 are obviously dimensionless.
Thus,

$$\underline{\underline{VD\sqrt{\frac{\rho}{\mathcal{W}}} = \phi\left(\frac{b}{d}, \frac{d}{D}\right)}}$$



(b) For impending tipping around O

$$\Sigma M_O = 0$$

so that

$$Fd = \mathcal{W}\left(\frac{b}{2}\right) \qquad (1)$$

(con't)

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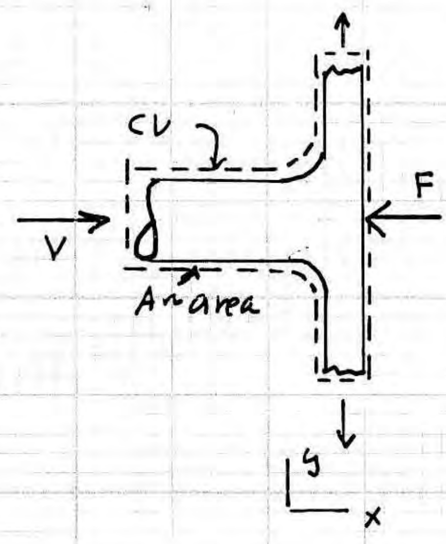
7.26 (con't)

From momentum considerations using the CV shown

$$\int \rho u \vec{V}\cdot\hat{n}\, dA = \sum F_x$$

$$\rho V^2 A = F$$



Thus, from Eq. (1)

$$(\rho V^2 A)(d) = \mathcal{W}\left(\frac{b}{2}\right)$$

so that

$$V = \sqrt{\frac{\mathcal{W}(b)}{2\rho A d}}$$

and with $A = \pi/4\, D^2$

$$\underline{\underline{V = \sqrt{\frac{2\mathcal{W} b}{\pi \rho d D^2}}}} \qquad (2)$$

(c) From part (a)

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}}\ \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$

Eq. (2) can be written as

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}}\left(\sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}\right) \qquad (3)$$

It follows by comparing Eqs. (2) and (3) that

$$\phi\left(\frac{b}{d}, \frac{d}{D}\right) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}$$

so that $\phi\left(\frac{b}{d}, \frac{d}{D}\right)$ is actually independent of $\frac{d}{D}$.

7.27

7.27 Assume that the drag, $\mathfrak{D}$, on an aircraft flying at supersonic speeds is a function of its velocity, V, fluid density, ρ, speed of sound, c, and a series of lengths, $\ell_1, \ldots, \ell_i$, which describe the geometry of the aircraft. Develop a set of pi terms that could be used to investigate experimentally how the drag is affected by the various factors listed. Form the pi terms by inspection.

$$\mathcal{D} = f(V, \rho, c, \ell_1, \ldots \ell_i)$$

$\mathcal{D} \doteq F \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^2 \quad c \doteq LT^{-1} \quad$ all lengths, $\ell_i \doteq L$

From the pi theorem, $(4+i)-3 = 1+i$ pi terms required, where i is the number of length terms ($i = 1, 2, 3,$ etc.).

By inspection, for Π_1 (containing $\mathcal{D}$):

$$\Pi_1 = \frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2(L)^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing c):

$$\Pi_2 = \frac{c}{V} \quad \text{or} \quad \frac{V}{c}$$

and both are obviously dimensionless.

For all other pi terms containing ℓ_i

$$\Pi_i = \frac{\ell_i}{\ell_1}$$

and these terms involving the ℓ's are obviously dimensionless.

Thus,

$$\frac{\mathcal{D}}{\rho V^2 \ell_1^2} = \phi\left(\frac{V}{c}, \frac{\ell_i}{\ell_1}\right)$$

where $\frac{\ell_i}{\ell_1}$ is a series of pi terms, $\frac{\ell_2}{\ell_1}, \frac{\ell_3}{\ell_1}$, etc.

7.29*

*7.29 The pressure drop, Δp, over a certain length of horizontal pipe is assumed to be a function of the velocity, V, of the fluid in the pipe, the pipe diameter, D, and the fluid density and viscosity, ρ and μ. **(a)** Show that this flow can be described in dimensionless form as a "pressure coefficient," $C_p = \Delta p/(0.5\,\rho V^2)$ that depends on the Reynolds number, $\mathrm{Re} = \rho VD/\mu$. **(b)** The following data were obtained in an experiment involving a fluid with $\rho = 2$ slugs/ft³, $\mu = 2 \times 10^{-3}$ lb · s/ft², and $D = 0.1$ ft. Plot a dimensionless graph and use a power law equation to determine the functional relationship between the pressure coefficient and the Reynolds number.

V, ft/s	Δp, lb/ft²
3	192
11	704
17	1088
20	1280

(c) What are the limitations on the applicability of your equation obtained in part **(b)**?

(a) $\Delta p = f(V, D, \rho, \mu)$

$$\Delta p \doteq FL^{-2} \quad V \doteq LT^{-1} \quad D \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad \mu \doteq FL^{-2}T$$

From the pi theorem, 5 − 3 = 2 pi terms required.
By inspection for Π_1,

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^{2})(LT^{-1})^2} \doteq F^0L^0T^0 \quad \therefore \text{ OK}$$

Check using MLT system:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2:

$$\Pi_2 = \frac{\rho VD}{\mu} \doteq \frac{(FL^{-4}T^{2})(LT^{-1})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0 \quad \therefore \text{ OK}$$

Check using MLT system:

$$\frac{\rho VD}{\mu} \doteq \frac{(ML^{-3})(LT^{-1})(L)}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

Thus,

$$\frac{\Delta p}{\rho V^2} = \tilde{\phi}\left(\frac{\rho VD}{\mu}\right)$$

Since $\tilde{\phi}$ is an unknown function, a factor of 0.5 can be included in Π_1 (if desired) so that

$$\frac{\Delta p}{0.5\rho V^2} = \phi\left(\frac{\rho VD}{\mu}\right)$$

Thus, $C_p = \phi(\mathrm{Re})$

where C_p is the pressure coefficient and Re the Reynolds number.

(con't)

7.29* (Con't)

(b) Using the data given,

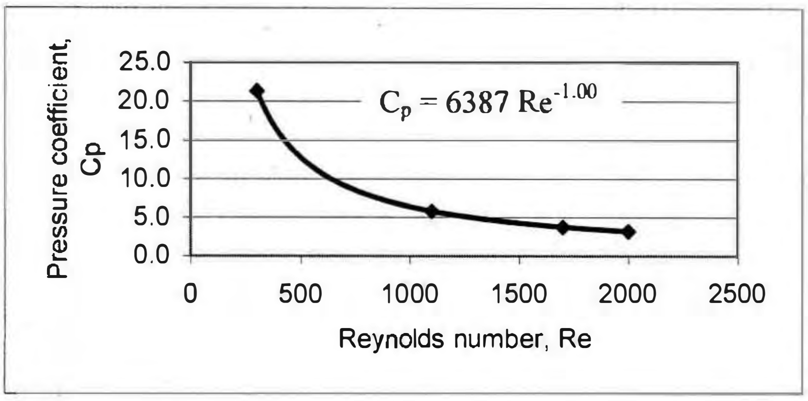
$$C_p = \frac{\Delta p}{0.5\rho V^2} = \frac{\Delta p}{(0.5)(2\ \frac{slugs}{ft^3})V^2} = \frac{\Delta p}{V^2}$$

and

$$Re = \frac{\rho V D}{\mu} = \frac{2(\frac{slugs}{ft^3})(V)(0.1 ft)}{2 \times 10^{-3}\ \frac{lb \cdot s}{ft^2}} = 100V$$

Tabulated values for C_p and Re and a plot of the data are shown below.

V, ft/s	Δ p, psf	Re	Cp
3	192	300	21.3
11	704	1100	5.82
17	1090	1700	3.77
20	1280	2000	3.20



The power law relationship is

$$\underline{\underline{C_p = \frac{6387}{Re}}} \qquad (1)$$

(c) Based on the variables used and the given data, the empirical relationship, Eq. (1), would only be applicable in the Reynolds number range

$$300 \le Re \le 2000$$

Note: Although the equation might be valid outside this range, results should not be extrapolated beyond the range of data used.

7.30 *

*7.30 The pressure drop across a short hollowed plug placed in a circular tube through which a liquid is flowing (see Fig. P7.30) can be expressed as

$$\Delta p = f(\rho, V, D, d)$$

where ρ is the fluid density, and V is the mean velocity in the tube. Some experimental data obtained with $D = 0.2$ ft, $\rho = 2.0$ slugs/ft³, and $V = 2$ ft/s are given in the following table:

d (ft)	0.06	0.08	0.10	0.15
Δp (lb/ft²)	493.8	156.2	64.0	12.6

Plot the results of these tests, using suitable dimensionless parameters, on log–log graph paper. Use a standard curve-fitting technique to determine a general equation for Δp. What are the limits of applicability of the equation?

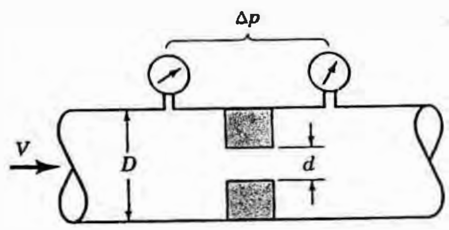


FIGURE P7.30

$\Delta p \doteq FL^{-2} \quad \rho \doteq FL^{-4}T^{2} \quad V \doteq LT^{-1} \quad D \doteq L \quad d \doteq L$

From the pi theorem, 5−3 = 2 pi terms required. By inspection for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^{2})(LT^{-1})^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing D and d):

$$\Pi_2 = \frac{D}{d}$$

(which is obviously dimensionless). Thus,

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{D}{d}\right)$$

For the data given:

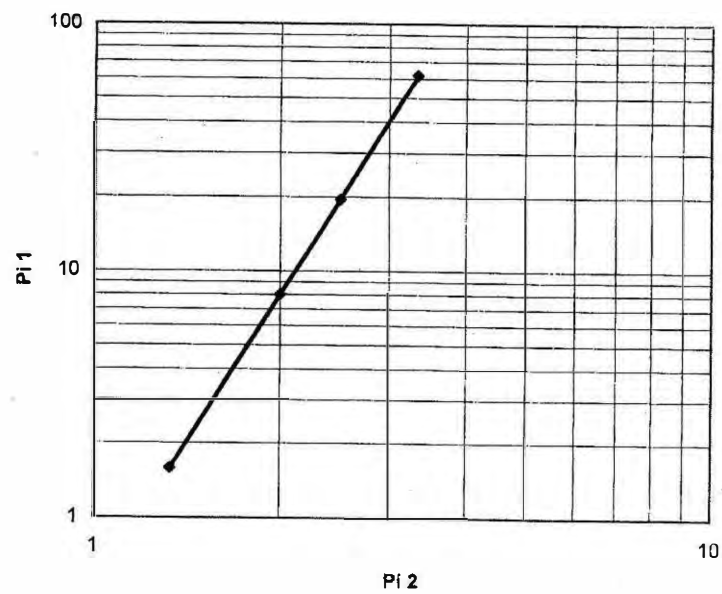
D/d	3.33	2.50	2.00	1.33
$\Delta p/\rho V^2$	61.7	19.5	8.00	1.58

A log-log plot of these data is shown on the following page.

(con't)

7.30 *

(con't)



Since the data plot as a straight line on a log-log plot, the equation for the data is of the form

$$\Pi_1 = a\,\Pi_2^{\,b}$$

where $\Pi_1 = \Delta p/\rho V^2$ and $\Pi_2 = D/d$. A power law fit of the data gives

$$a = 0.505 \quad \text{and} \quad b = 3.99$$

Thus,

$$\underline{\frac{\Delta p}{\rho V^2} = 0.505\left(\frac{D}{d}\right)^{3.99}}$$

This equation is applicable over the range of data $\underline{1.33 \leq \frac{D}{d} \leq 3.33}$.

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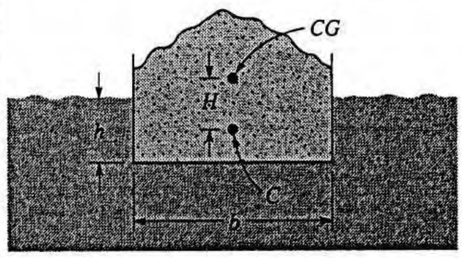
7.32 *

*7.32 As shown in Fig. 2.26, Fig. P7.32, and Video V2.10, a rectangular barge floats in a stable configuration provided the distance between the center of gravity, *CG*, of the object (boat and load) and the center of buoyancy, *C*, is less than a certain amount, *H*. If this distance is greater than *H* the boat will tip over. Assume *H* is a function of the boat's width, *b*, length, ℓ, and draft, *h*. (a) Put this relationship into dimensionless form. (b) The results of a set of experiments with a model barge with a width of 1.0 m is shown in the table. Plot this data in dimensionless form and determine a power-law equation relating the dimensionless parameters.

ℓ, m	h, m	H, m
2.0	0.10	0.833
4.0	0.10	0.833
2.0	0.20	0.417
4.0	0.20	0.417
2.0	0.35	0.238
4.0	0.35	0.238



■ FIGURE P7.32

(a) $$H = f(b, \ell, h)$$

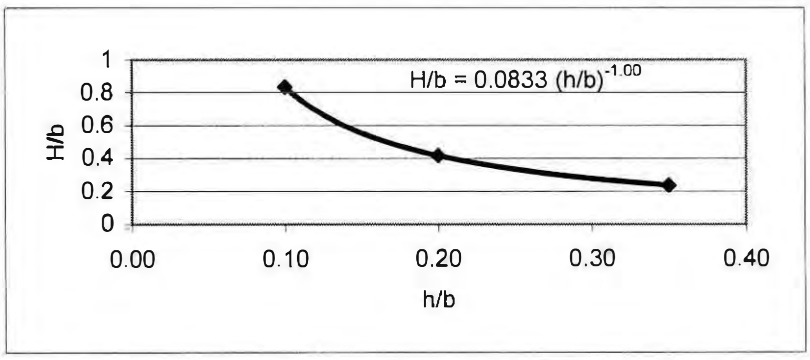
From the pi theorem, 4-1 = 3 pi terms required. By inspection:

$$\frac{H}{b} = \phi\left(\frac{h}{b}, \frac{\ell}{b}\right)$$

All of the pi terms are obviously dimensionles

(b) For the data given, tabulated values for H/b, h/b, and ℓ/b are shown below.

h/b	H/b	ℓ/b
0.10	0.833	2.0
0.10	0.833	4.0
0.20	0.417	2.0
0.20	0.417	4.0
0.35	0.238	2.0
0.35	0.238	4.0



An inspection of these data reveals that H/b does not depend on ℓ/b, i.e., the same value of H/b is obtained for different values of ℓ/b. Thus,

$$\frac{H}{b} = \phi\left(\frac{h}{b}\right)$$

and from the plot of the data, using a power-law equation

$$\frac{H}{b} = 0.0833\left(\frac{h}{b}\right)^{-1.00}$$

7.33

7.33 The time, t, it takes to pour a certain volume of liquid from a cylindrical container depends on several factors, including the viscosity of the liquid. (See Video V1.3) Assume that for very viscous liquids the time it takes to pour out 2/3 of the initial volume depends on the initial liquid depth, ℓ, the cylinder diameter, D, the liquid viscosity, μ, and the liquid specific weight, γ. The data shown in the following table were obtained in the laboratory. For these tests $\ell = 45$ mm, $D = 67$ mm, and $\gamma = 9.60$ kN/m^3. (a) Perform a dimensional analysis and based on the data given, determine if variables used for this problem appear to be correct. Explain how you arrived at your answer. (b) If possible, determine an equation relating the pouring time and viscosity for the cylinder and liquids used in these tests. If it is not possible, indicate what additional information is needed.

μ (N·s/m^2)	11	17	39	61	107
t(s)	15	23	53	83	145

$$t = f(\ell, D, \mu, \gamma)$$

(a) $t \doteq T \quad \ell \doteq L \quad D \doteq L \quad \mu \doteq FL^{-2}T \quad \gamma \doteq FL^{-3}$

From the pi theorem $5-3=2$ pi terms required.

By inspection, for Π_1 (containing t)

$$\Pi_1 = \frac{t\gamma D}{\mu} \doteq \frac{(T)(FL^{-3})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0$$

Check using MLT system:

$$\frac{t\gamma D}{\mu} \doteq \frac{(T)(ML^{-2}T^{-2})(L)}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing ℓ)

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{t\gamma D}{\mu} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

For the data given $\frac{\ell}{D} = \frac{45\,mm}{67\,mm} = 0.672$ (a constant).

Thus, from Eq. (1) with ℓ/D a constant it follows that $\frac{t\gamma D}{\mu}$ = constant. For the data given:

(con't)

7.33 (con't)

$\frac{t\gamma D}{\mu}$	877	870	874	875	872

Since Π_1 is essentially constant over the range of the experimental data the variables used for the problem appear to be correct.

(b) The average value for Π_1 is 874 so that

$$\frac{t\gamma D}{\mu} = 874$$

and therefore

$$t = \frac{874}{\gamma D}\mu = \frac{874\,\mu}{\left(9.6\times10^{3}\frac{N}{m^3}\right)\left(67\times10^{-3}\,m\right)}$$

$$\underline{t = 1.36\,\mu}$$

with t in seconds when μ is in units of $N\cdot s/m^2$. Note that this restricted equation is only valid for $\ell/D = 0.672$, $D = 67\,mm$, and $\gamma = 9.60\,kN/m^3$ with $2/3$ of the initial volume being poured.

7.34

7.34 In order to maintain uniform flight, smaller birds must beat their wings faster than larger birds. It is suggested that the relationship between the wingbeat frequency, ω, beats per second, and the bird's wingspan, ℓ, is given by a power law relationship, $\omega \sim \ell^n$. (a) Use dimensional analysis with the assumption that the wingbeat frequency is a function of the wingspan, the specific weight of the bird, γ, the acceleration of gravity, g, and the density of the air, ρ_a, to determine the value of the exponent n. (b) Some typical data for various birds are given in the table below. Does this data support your result obtained in part (a)? Provide appropriate analysis to show how you arrived at your conclusion.

Bird	Wingspan, m	Wingbeat frequency, beats/s
purple martin	0.28	5.3
robin	0.36	4.3
mourning dove	0.46	3.2
crow	1.00	2.2
Canada goose	1.50	2.6
great blue heron	1.80	2.0

(a) Given $\omega = f(\ell, \gamma, g, \rho_a)$ so that $k - r = 5 - 3 = 2$ or $\Pi_1 = \phi(\Pi_2)$

$\omega \doteq T^{-1}$, $\ell \doteq L$, $\gamma \doteq ML^{-2}T^{-2}$, $g \doteq LT^{-2}$, $\rho_a \doteq ML^{-3}$

Thus, consider

$$\Pi_1 = \omega \rho_a^a g^b \ell^c \doteq T^{-1}(ML^{-3})^a (LT^{-2})^b L^c$$
$$= M^a L^{-3a+b+c} T^{-1-2b} = M^0 L^0 T^0$$

so that

M: $a = 0$

L: $-3a + b + c = 0$

T: $-1 - 2b = 0$, which gives $a = 0$, $b = -\frac{1}{2}$, $c = \frac{1}{2}$

Thus:

$$\Pi_1 = \omega \sqrt{\frac{\ell}{g}}$$

For Π_2:

$$\Pi_2 = \gamma \rho_a^a g^b \ell^c \doteq (ML^{-2}T^{-2})(ML^{-3})^a (LT^{-2})^b L^c$$
$$= M^{1+a} L^{-2-3a+b+c} T^{-2-2b}$$

so that

M: $1 + a = 0$

L: $-2 - 3a + b + c = 0$

T: $-2 - 2b = 0$

(con't)

7.34 (con't)

These equations give $a=-1$, $b=-1$, and $c=0$

Thus,

$$\Pi_2 = \frac{\gamma}{g\rho_a}$$

Therefore, $\omega\sqrt{\dfrac{\ell}{g}} = \phi\left(\dfrac{\gamma}{g\rho_a}\right)$ or $\omega = \sqrt{\dfrac{g}{\ell}}\,\phi\left(\dfrac{\gamma}{g\rho_a}\right)$

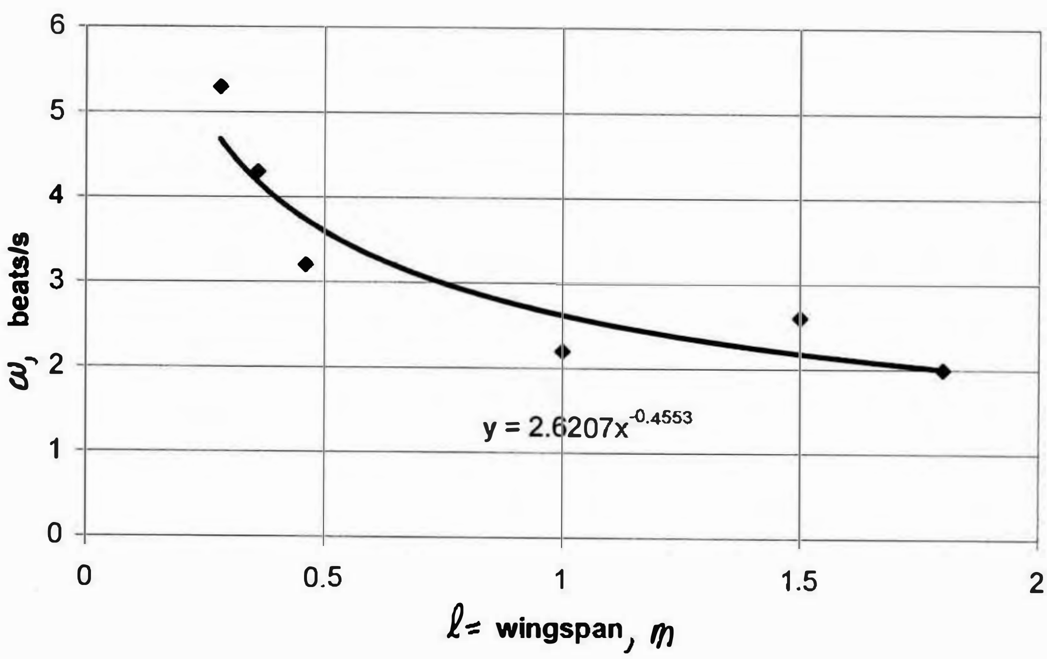
which indicates that

$\omega \sim \ell^{-1/2}$. That is $\omega \sim \ell^n$ where $\underline{\underline{n = -\frac{1}{2}}}$

(b) The given data is plotted below and a power law curve fit is applied, with the results

$$\omega = 2.62\,\ell^{-0.455}, \text{ where } \omega \sim \text{beats/s when } \ell \sim \text{m}.$$

The obtained power, -0.455, is very close to that predicted by dimensional methods, -0.500.



*7.35

*7.35 The concentric cylinder device of the type shown in Fig. P7.35 is commonly used to measure the viscosity, μ, of liquids by relating the angle of twist, θ, of the inner cylinder to the angular velocity, ω, of the outer cylinder. Assume that

$$\theta = f(\omega, \mu, K, D_1, D_2, \ell)$$

where K depends on the suspending wire properties and has the dimensions FL. The following data were obtained in a series of tests for which $\mu = 0.01 \text{ lb} \cdot \text{s/ft}^2$, $K = 10 \text{ lb} \cdot \text{ft}$, $\ell = 1 \text{ ft}$, and D_1 and D_2 were constant.

θ (rad)	ω (rad/s)
0.89	0.30
1.50	0.50
2.51	0.82
3.05	1.05
4.28	1.43
5.52	1.86
6.40	2.14

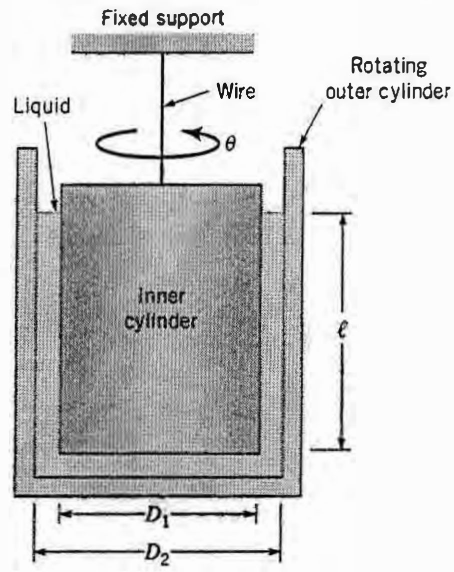


FIGURE P7.35

Determine from these data, with the aid of dimensional analysis, the relationship between θ, ω, and μ for this particular apparatus. *Hint:* Plot the data using appropriate dimensionless parameters, and determine the equation of the resulting curve using a standard curve-fitting technique. The equation should satisfy the condition that $\theta = 0$ for $\omega = 0$.

$\theta \doteq F^0L^0T^0 \qquad \omega \doteq T^{-1} \qquad \mu \doteq FL^{-2}T$

$K \doteq FL \qquad D_1 \doteq L \qquad D_2 \doteq L \qquad \ell \doteq L$

From the pi theorem, $7-3 = 4$ pi terms required. By inspection,

$$\Pi_1 = \theta$$

For Π_2 (containing ω):

$$\Pi_2 = \frac{\omega \mu \ell^3}{K} \doteq \frac{(T^{-1})(FL^{-2}T)(L)^3}{FL} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\omega \mu \ell^3}{K} \doteq \frac{(T^{-1})(ML^{-1}T^{-1})(L)^3}{ML^2T^{-2}} \doteq M^0L^0T^0 \quad \therefore OK$$

For Π_3 and Π_4 (containing D_1 and D_2):

$$\Pi_3 = \frac{D_1}{\ell} \qquad \Pi_4 = \frac{D_2}{\ell}$$

(which are obviously dimensionless).

(con't)

*7.35 (con't)

Thus, the dimensional analysis yields,

$$\theta = \phi\left(\frac{\omega\mu\ell^3}{K}, \frac{D_1}{\ell}, \frac{D_2}{\ell}\right)$$

For a given device, D_1/ℓ and D_2/ℓ are constant so that

$$\theta = \phi_1\left(\frac{\omega\mu\ell^3}{K}\right)$$

and with the data given

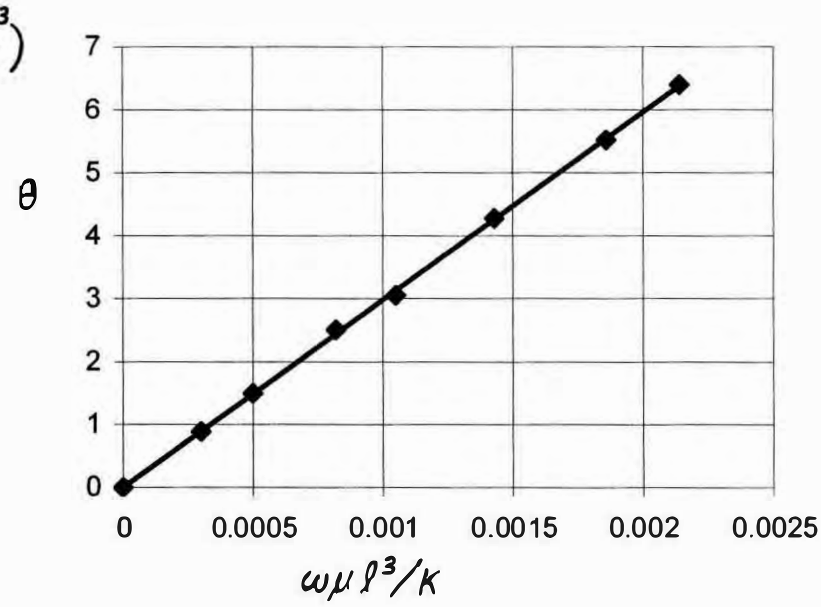
$$\frac{\omega\mu\ell^3}{K} = \frac{\omega\left(\frac{rad}{s}\right)\left(0.01\,\frac{lb\cdot s}{ft^2}\right)(1\,ft)^3}{10\ lb\cdot ft} = \omega \times 10^{-3}$$

Using the θ vs. ω data supplied, it follows that:

$\frac{\omega\mu\ell^3}{K}$	0.30×10^{-3}	0.50×10^{-3}	0.82×10^{-3}	1.05×10^{-3}	1.43×10^{-3}	1.86×10^{-3}	2.14×10^{-3}
θ	0.89	1.50	2.51	3.05	4.28	5.52	6.40

These data are plotted below. The best linear curve fit gives

$$\theta = 2.98\times10^3\left(\frac{\omega\mu\ell^3}{K}\right)$$



Hence, for the particular device with $\ell = 1\,ft$ and $K = 10\ lb\cdot ft$,

$$\theta = \frac{(2.98\times10^3)(1\,ft)^3}{10\ lb\cdot ft}\left[\omega\,(rad/s)\times\mu\ (lb\cdot s/ft^2)\right]$$

so that

$$\underline{\underline{\theta = 298\,\omega\mu}}$$

with θ in rad for ω in rad/s and μ in $lb\cdot s/ft^2$.

7.37

7.37 Air at 80 °F is to flow through a 2-ft pipe at an average velocity of 6 ft/s. What size pipe should be used to move water at 60 °F and average velocity of 3 ft/s if Reynolds number similarity is enforced?

For Reynolds number similarity,

$$Re_{air} = Re_{water}\ , \text{ or}$$

$$\left(\frac{VD}{\nu}\right)_{air} = \left(\frac{VD}{\nu}\right)_{water}$$

Thus,

$$D_{water} = \left(\frac{\nu_{water}}{\nu_{air}}\right)\left(\frac{V_{air}}{V_{water}}\right) D_{air}$$, where from Tables B.1 and B.2

$$\underset{60°F}{\nu_{water}} = 1.210 \times 10^{-5}\ \frac{ft^2}{s} \quad \text{and} \quad \underset{80°F}{\nu_{air}} = 1.69 \times 10^{-4}\ \frac{ft^2}{s}$$

Hence,

$$D_{water} = \left(\frac{1.210 \times 10^{-5}\ \frac{ft^2}{s}}{1.69 \times 10^{-4}\ \frac{ft^2}{s}}\right)\left(\frac{6\ ft/s}{3\ ft/s}\right)(2\ ft) = \underline{\underline{0.286\ ft}}$$

7.38

7.38 To test the aerodynamics of a new prototype automobile, a scale model will be tested in a wind tunnel. For dynamic similarity, it will be required to match Reynolds number between model and prototype. Assuming that you will be testing a one-tenth-scale model and both model and prototype will be exposed to standard air pressure, will it be better for the wind tunnel air to be colder or hotter than standard sea-level air temperature of 15 °C? Why?

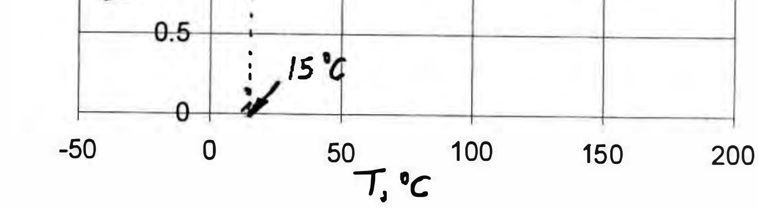
Let $(\ )_m$ and $(\ )_p$ denote model and prototype, respectively.
Thus, $Re_m = Re_p$, or

$$\left(\frac{V\ell}{\nu}\right)_m = \left(\frac{V\ell}{\nu}\right)_p, \text{ or}$$

$$V_m = \frac{\nu_m}{\nu_p}\frac{\ell_p}{\ell_m}V_p = 10\frac{\nu_m}{\nu_p}V_p \quad \text{since } \ell_m = \frac{1}{10}\ell_p$$

If the windtunnel air is at standard sea-level conditions, then $\nu_m = \nu_p$ and

$V_m = 10V_p$. Hence, if $V_p = 55$ mph, then $V_m = 550$ mph, which is too large for simple tests. For one this, at 550mph compressibility effects become important. At 55mph they are not. Assume the test are conducted with $\nu_m/\nu_p < 1$ so that more realistic wind tunnel velocities are prescribed. From the data in Table B.4, at $T = 15°C$, $\nu_p = 1.47 \times 10^{-5}\ m^2/s$ and the data shown below are obtained.



For $\nu_m/\nu_p < 1$ it follows that $T < 15°C$. Hence, it would be better to have a <u>cold windtunnel</u>. However, even with $T = -40°C$, which gives $\nu_m/\nu_p = 0.707$, the $V_p = 55$ mph would require $V_m = 10(0.707)\,55\text{mph} = 389\text{mph}$.

7.39

7.39 You are to conduct wind tunnel testing of a new football design that has a smaller lace height than previous designs (see Videos V6.1 and V6.2). It is known that you will need to maintain Re and St similarity for the testing. Based on standard college quarterbacks, the prototype parameters are set at V = 40 mph and ω = 300 rpm. The prototype football has a 7-in. diameter. Due to instrumentation required to measure pressure and shear stress on the surface of the football, the model will require a length scale of 2:1 (the model will be larger than the prototype). Determine the required model freestream velocity and model angular velocity.

Let $(\ )_m$ and $(\ )_p$ denote the model and prototype, respectively.

For Reynolds number similarity, $Re_m = Re_p$, or

$$\frac{V_m D_m}{\nu_m} = \frac{V_p D_p}{\nu_p}, \text{ so that with } \nu_m = \nu_p \text{ (i.e. same air properties)}$$

$$V_m = \frac{D_p}{D_m} V_p = \left(\frac{1}{2}\right)(40\,mph) = 20\,mph, \text{ since } D_m = 2D_p.$$

Thus,

$$V_m = 20\frac{mi}{hr}\left(\frac{5280\,ft}{mi}\right)\left(\frac{1\,hr}{3600\,s}\right) = \underline{\underline{29.3\,\frac{ft}{s}}}$$

For Strouhal number similarity, $St_m = St_p$, or

$$\frac{\omega_m D_m}{V_m} = \frac{\omega_p D_p}{V_p}, \text{ where } \omega_p = 300\,rpm$$

Hence,

$$\omega_m = \frac{V_m}{V_p}\frac{D_p}{D_m}\omega_p = \left(\frac{20\,mph}{40\,mph}\right)\left(\frac{1}{2}\right)(300\,rpm) = \underline{\underline{75\,rpm}}$$

7.40

7.40 A model of a submarine, 1 : 15 scale, is to be tested at 180 ft/s in a wind tunnel with standard sea-level air, while the prototype will be operated in seawater. Determine the speed of the prototype to ensure Reynolds number similarity.

Let $(\ )_m$ and $(\ )_p$ denote model and prototype, respectively. Thus, $Re_m = Re_p$, or

$$\frac{V_m \ell_m}{\nu_m} = \frac{V_p \ell_p}{\nu_p}, \text{ where } \ell_m = \frac{1}{15}\ell_p$$

Hence,

$$V_m = \left(\frac{\nu_m}{\nu_p}\right)\left(\frac{\ell_p}{\ell_m}\right)V_p = 15\left(\frac{\nu_m}{\nu_p}\right)V_p$$

Also,

$$\nu_m = 1.57\times10^{-4}\,\frac{ft^2}{s} \text{ and } \nu_p = 1.26\times10^{-5}\,\frac{ft^2}{s} \text{ so that}$$

$$V_m = 15\left(\frac{1.57\times10^{-4}\,ft^2/s}{1.26\times10^{-5}\,ft^2/s}\right)V_p = 187\,V_p$$

Thus,

$$V_p = \frac{V_m}{187} = \frac{180\,\frac{ft}{s}}{187} = \underline{\underline{0.963\,\frac{ft}{s}}}$$

7.41

7.41 SAE 30 oil at 60 °F is pumped through a 3-ft-diameter pipeline at a rate of 6400 gal/min. A model of this pipeline is to be designed using a 3-in.-diameter pipe and water at 60 °F as the working fluid. To maintain Reynolds number similarity between these two systems, what fluid velocity will be required in the model?

For Reynolds number similarity,

$$\frac{V_m D_m}{\nu_m} = \frac{V D}{\nu}$$

or

$$V_m = \frac{\nu_m}{\nu} \frac{D}{D_m} V \qquad (1)$$

Since,

$$V = \frac{Q}{\text{area}}$$

and

$$Q = \frac{\left(6400 \frac{\text{gal}}{\text{min}}\right)\left(231 \frac{\text{in.}^3}{\text{gal}}\right)\left(\frac{1 \text{ ft}^3}{1728 \text{ in.}^3}\right)}{60 \frac{\text{s}}{\text{min}}} = 14.3 \frac{\text{ft}^3}{\text{s}}$$

then

$$V = \frac{14.3 \frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(3 \text{ ft})^2} = 2.02 \frac{\text{ft}}{\text{s}}$$

Thus, from Eq. (1)

$$V_m = \frac{\left(1.21 \times 10^{-5} \frac{\text{ft}^2}{\text{s}}\right)(3 \text{ ft})}{\left(4.5 \times 10^{-3} \frac{\text{ft}^2}{\text{s}}\right)\left(\frac{3}{12} \text{ ft}\right)}\left(2.02 \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{6.52 \times 10^{-2} \frac{\text{ft}}{\text{s}}}}$$

7.42

7.42 The water velocity at a certain point along a 1 : 10 scale model of a dam spillway is 3 m/s. What is the corresponding prototype velocity if the model and prototype operate in accordance with Froude number similarity?

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

so that

$$V = \sqrt{\left(\frac{g}{g_m}\right)\left(\frac{\ell}{\ell_m}\right)}\; V_m$$

and with $g = g_m$, $\ell/\ell_m = 10$, $V_m = 3\ m/s$

$$V = \sqrt{10}\left(3\,\frac{m}{s}\right) = \underline{\underline{9.49\ \frac{m}{s}}}$$

7.43

7.43 The drag characteristics of a torpedo are to be studied in a water tunnel using a 1:5 scale model. The tunnel operates with freshwater at 20 °C, whereas the prototype torpedo is to be used in seawater at 15.6 °C. To correctly simulate the behavior of the prototype moving with a velocity of 30 m/s, what velocity is required in the water tunnel?

For dynamic similarity, the Reynolds number must be the same for model and prototype. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{V D}{\nu}$$

so that

$$V_m = \frac{\nu_m}{\nu} \frac{D}{D_m} V$$

Since, ν_m (water @ 20°C) $= 1.004 \times 10^{-6}\ m^2/s$ (Table B.2), ν (seawater @ 15.6°C) $= 1.17 \times 10^{-6}\ m^2/s$ (Table 1.6), and $D/D_m = 5$, it follows that

$$V_m = \frac{(1.004 \times 10^{-6} \frac{m^2}{s})}{(1.17 \times 10^{-6} \frac{m^2}{s})} (5)(30 \tfrac{m}{s}) = \underline{\underline{129 \tfrac{m}{s}}}$$

7.44

7.44 For a certain fluid flow problem it is known that both the Froude number and the Weber number are important dimensionless parameters. If the problem is to be studied by using a 1:15 scale model, determine the required surface tension scale if the density scale is equal to 1. The model and prototype operate in the same gravitational field.

For dynamic similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}} \qquad \text{(Froude number similarity)}$$

and

$$\frac{\rho_m V_m^2 \ell_m}{\sigma_m} = \frac{\rho V^2 \ell}{\sigma} \qquad \text{(Weber number similarity)}$$

To satisfy Froude number similarity (with $g = g_m$),

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

and therefore for Weber number similarity

$$\frac{\sigma_m}{\sigma} = \frac{\rho_m}{\rho}\left(\frac{V_m}{V}\right)^2 \frac{\ell_m}{\ell} = \frac{\rho_m}{\rho}\left(\frac{\ell_m}{\ell}\right)\frac{\ell_m}{\ell} = \frac{\rho_m}{\rho}\left(\frac{\ell_m}{\ell}\right)^2$$

Thus, with $\ell_m/\ell = 1/15$ and $\rho_m/\rho = 1$,

$$\frac{\sigma_m}{\sigma} = (1)\left(\frac{1}{15}\right)^2 = \underline{\underline{4.44 \times 10^{-3}}}$$

7.45

7.45 The fluid dynamic characteristics of an airplane flying at 240 mph at 10,000 ft are to be investigated with the aid of a 1:20 scale model. If the model tests are to be performed in a wind tunnel using standard air, what is the required air velocity in the wind tunnel? Is this a realistic velocity?

For dynamic similarity, the Reynolds number must be the same for model and prototype. Thus,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$V_m = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{\ell}{\ell_m} V \qquad (1)$$

Since,

$$\mu = 3.534 \times 10^{-7}\ \frac{lb\cdot s}{ft^2}\ ; \quad \rho = 1.756 \times 10^{-3}\ \frac{slugs}{ft^3} \quad (\text{Table C.1})$$

$$\mu_m = 3.74 \times 10^{-7}\ \frac{lb\cdot s}{ft^2}\ ; \quad \rho_m = 2.38 \times 10^{-3}\ \frac{slugs}{ft^3} \quad (\text{Table 1.7})$$

and $\ell/\ell_m = 20$, it follows from Eq. (1) that

$$V_m = \frac{\left(3.74 \times 10^{-7}\ \frac{lb\cdot s}{ft^2}\right)}{\left(3.534 \times 10^{-7}\ \frac{lb\cdot s}{ft^2}\right)} \frac{\left(1.756 \times 10^{-3}\ \frac{slugs}{ft^3}\right)}{\left(2.38 \times 10^{-3}\ \frac{slugs}{ft^3}\right)} (20)(240\ mph)$$

$$= \underline{\underline{3750\ mph}}$$

No, it is not a realistic velocity — much too high.

7.46

7.46 If an airplane travels at a speed of 1120 km/hr at an altitude of 15 km, what is the required speed at an altitude of 8 km to satisfy Mach number similarity? Assume the air properties correspond to those for the U.S. standard atmosphere.

For Mach number similarity,

$$\left(\frac{V}{c}\right)_{15\,km} = \left(\frac{V}{c}\right)_{8\,km} \qquad (1)$$

The speed of sound can be calculated from the equation

$$c = \sqrt{kRT} \qquad (\text{Eq. } 1.20)$$

and for air, $k = 1.40$, $R = 286.9\ J/kg \cdot K$.

At 15 km altitude,

$$T = -56.50\,°C + 273.15 = 216.7\ K \qquad (\text{Table C.2})$$

and at 8 km

$$T = -36.94\,°C + 273.15 = 236.2\ K \qquad (\text{Table C.2})$$

Thus, at 15 km altitude

$$c_{15\,km} = \sqrt{(1.40)\left(286.9\ \frac{J}{kg \cdot K}\right)(216.7\ K)} = 295\ \frac{m}{s}$$

and at 8 km

$$c_{8\,km} = \sqrt{(1.40)\left(286.9\ \frac{J}{kg \cdot K}\right)(236.2\ K)} = 308\ \frac{m}{s}$$

From Eq. (1)

$$V_{8\,km} = \frac{c_{8\,km}}{c_{15\,km}} V_{15\,km} = \left(\frac{308\ \frac{m}{s}}{295\ \frac{m}{s}}\right)\left(1120\ \frac{km}{hr}\right)$$

$$= \underline{\underline{1170\ \frac{km}{hr}}}$$

7.47

7.47 (See Fluids in the News article "Modeling parachutes in a water tunnel," Section 7.8.1.) Flow characteristics for a 30-ft-diameter prototype parachute are to be determined by tests of a 1-ft-diameter model parachute in a water tunnel. Some data collected with the model parachute indicate a drag of 17 lb when the water velocity is 4 ft/s. Use the model data to predict the drag on the prototype parachute falling through air at 10 ft/s. Assume the drag to be a function of the velocity, V, the fluid density, ρ, and the parachute diameter, D.

$$\mathcal{D} = f(V, \rho, D)$$

$$\mathcal{D} \doteq F \qquad V \doteq LT^{-1} \qquad \rho \doteq FL^{-4}T^{2} \qquad D \doteq L$$

From the pi theorem, 4-3=1 pi term required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 D^2} = C$$

where C is a constant. Thus, for similarity between model and prototype

$$\frac{\mathcal{D}}{\rho V^2 D^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 D_m^2}$$

So that

$$\mathcal{D} = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{D}{D_m}\right)^2 \mathcal{D}_m$$

$$= \left(\frac{2.38 \times 10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}}{1.94\ \frac{\text{slugs}}{\text{ft}^3}}\right)\left(\frac{10\ \frac{\text{ft}}{\text{s}}}{4\ \frac{\text{ft}}{\text{s}}}\right)^2\left(\frac{30\ \text{ft}}{1\ \text{ft}}\right)^2 (17\ \text{lb})$$

$$= \underline{\underline{117\ \text{lb}}}$$

7.48

7.48 The lift and drag developed on a hydrofoil are to be determined through wind tunnel tests using standard air. If full scale tests are to be run, what is the required wind tunnel velocity corresponding to a hydrofoil velocity in seawater of 15 mph? Assume Reynolds number similarity is required.

For Reynolds number similarity,

$$\frac{V_m \ell_m}{\nu_m} = \frac{V \ell}{\nu}$$

where ℓ is some characteristic length of the hydrofoil. Thus,

$$V_m = \frac{\nu_m}{\nu} \frac{\ell}{\ell_m} V$$

and with $\ell/\ell_m = 1$ (full scale test)

$$V_m = \frac{\nu_m}{\nu} V = \frac{\left(1.57 \times 10^{-4} \frac{ft^2}{s}\right)}{\left(1.26 \times 10^{-5} \frac{ft^2}{s}\right)} (15\ mph)$$

$$= \underline{\underline{187\ mph}}$$

7.49

7.4 A 1/50 scale model is to be used in a towing tank to study the water motion near the bottom of a shallow channel as a large barge passes over. (See Video V7.16) Assume that the model is operated in accordance with the Froude number criteria for dynamic similitude. The prototype barge moves at a typical speed of 15 knots. **(a)** At what speed (in ft/s) should the model be towed? **(b)** Near the bottom of the model channel a small particle is found to move 0.15 ft in one second so that the fluid velocity at that point is approximately 0.15 ft/s. Determine the velocity at the corresponding point in the prototype channel.

(a) For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

Thus,

$$V_m = \sqrt{\frac{1}{50}}\,(15\text{ knots}) = 2.12\text{ knots}$$

From Table A.1 $\quad 1\text{ knot} = \left(0.514\,\frac{m}{s}\right)\left(3.281\,\frac{\frac{ft}{s}}{\frac{m}{s}}\right) = 1.69\,\frac{ft}{s}$

so that

$$V_m = (2.12\text{ knots})\left(1.69\,\frac{ft/s}{knot}\right) = \underline{\underline{3.58\,\frac{ft}{s}}}$$

(b) Since from Eq. (1)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\frac{1}{50}}$$

so that

$$V = \sqrt{50}\left(0.15\,\frac{ft}{s}\right) = \underline{\underline{1.06\,\frac{ft}{s}}}$$

7.50

7.50 A solid sphere having a diameter d and specific weight γ_s is immersed in a liquid having a specific weight $\gamma_f (\gamma_f > \gamma_s)$ and then released. It is desired to use a model system to determine the maximum height, h, above the liquid surface that the sphere will rise upon release from a depth H. It can be assumed that the important liquid properties are the density, γ_f / g, specific weight, γ_f, and viscosity, μ_f. Establish the model design conditions and the prediction equation, and determine whether the same liquid can be used in both the model and prototype systems.

Assume that $h = f(d, H, \gamma_s, \gamma_f, g, \mu_f)$. Note that by including γ_s, γ_f, and g, both the mass and weight of the fluid and sphere are taken into account. This follows since ρ (density) $= \gamma / g$. It would be incorrect to list γ_f, ρ_f, and g as independent variables. We expect the mass of the sphere to be important since the sphere will have accelerated motion. Since,

$$h \doteq L \quad d \doteq L \quad H \doteq L \quad \gamma_s \doteq FL^{-3} \quad \gamma_f \doteq FL^{-3} \quad g \doteq LT^{-2} \quad \mu_f \doteq FL^{-2}T$$

the pi theorem indicates that $7-3 = 4$ pi terms required. A dimensional analysis yields,

$$\frac{h}{d} = \phi\left(\frac{H}{d}, \frac{\gamma_s}{\gamma_f}, \frac{\mu_f}{\gamma_f}\sqrt{\frac{g}{d^3}}\right)$$

Thus, the model design conditions are

$$\underline{\underline{\frac{H_m}{d_m} = \frac{H}{d}}} \qquad \underline{\underline{\frac{\gamma_{sm}}{\gamma_{fm}} = \frac{\gamma_s}{\gamma_f}}} \qquad \underline{\underline{\frac{\mu_{fm}}{\gamma_{fm}}\sqrt{\frac{g_m}{d_m^3}} = \frac{\mu_f}{\gamma_f}\sqrt{\frac{g}{d^3}}}}$$

and the prediction equation is

$$\underline{\underline{\frac{h}{d} = \frac{h_m}{d_m}}}$$

From the last model design condition (with $g = g_m$),

$$\frac{\mu_{fm}}{\mu_f} = \frac{\gamma_{fm}}{\gamma_f}\sqrt{\frac{d_m^3}{d^3}} \qquad (1)$$

Since d_m / d is the length scale, and is presumably not equal to one, Eq. (1) will not be satisfied if the same liquid is used. Thus, $\underline{\underline{\text{the same liquid cannot be used.}}}$

7.51

7.51 A thin layer of an incompressible fluid flows steadily over a horizontal smooth plate as shown in Fig. P7.51. The fluid surface is open to the atmosphere, and an obstruction having a square cross section is placed on the plate as shown. A model with a length scale of $\frac{1}{4}$ and a fluid density scale of 1.0 is to be designed to predict the depth of fluid, y, along the plate. Assume that inertial, gravitational, surface tension, and viscous effects are all important. What are the required viscosity and surface tension scales?

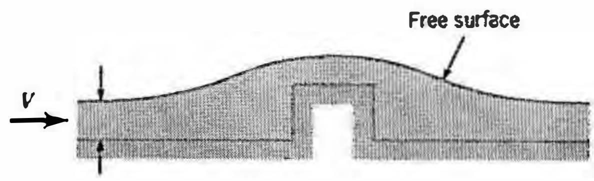


FIGURE P7.51

A fluid dynamics problem for which inertial, gravitational, surface tension, and viscous effects are all important requires Froude, Reynolds, and Weber number similarity (see Table 7.1). Thus, for

$$\frac{V_m}{\sqrt{g_m d_m}} = \frac{V}{\sqrt{g d}}$$

(Froude number similarity), it follows that (with $g = g_m$)

$$\frac{V_m}{V} = \sqrt{\frac{d_m}{d}}$$

For Reynolds number similarity,

$$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho V d}{\mu}$$

and

$$\frac{\mu_m}{\mu} = \frac{\rho_m}{\rho}\frac{V_m}{V}\frac{d_m}{d} = \frac{\rho_m}{\rho}\sqrt{\frac{d_m}{d}}\,\frac{d_m}{d} = \frac{\rho_m}{\rho}\left(\frac{d_m}{d}\right)^{3/2}$$

$$= (1.0)\left(\frac{1}{4}\right)^{3/2} = \frac{1}{8} = \underline{\underline{0.125}}$$

For Weber number similarity,

$$\frac{\rho_m V_m^2 d_m}{\sigma_m} = \frac{\rho V^2 d}{\sigma}$$

and

$$\frac{\sigma_m}{\sigma} = \frac{\rho_m}{\rho}\frac{V_m^2}{V^2}\frac{d_m}{d} = \frac{\rho_m}{\rho}\left(\sqrt{\frac{d_m}{d}}\right)^2\frac{d_m}{d} = \frac{\rho_m}{\rho}\left(\frac{d_m}{d}\right)^2$$

$$= (1.0)\left(\frac{1}{4}\right)^2 = \frac{1}{16} = \underline{\underline{0.0625}}$$

7.52

7.52 The drag on a 2-m-diameter satellite dish due to an 80-km/hr wind is to be determined through a wind tunnel test using a geometrically similar 0.4-m-diameter model dish. Assume standard air for both model and prototype. (a) At what air speed should the model test be run? (b) With all similarity conditions satisfied, the measured drag on the model was determined to be 170 N. What is the predicted drag on the prototype dish?

(a) From Eq. 7.19, Reynolds number similarity is required. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{VD}{\nu}$$

where D is the dish diameter. It follows that

$$V_m = \frac{\nu_m}{\nu} \frac{D}{D_m} V$$

and with $\nu_m / \nu = 1$

$$V_m = \left(\frac{2\ m}{0.4\ m}\right)\left(80\ \frac{km}{hr}\right) = \underline{\underline{400\ \frac{km}{hr}}}$$

(b) From Eq. 7.19,

$$\frac{\mathscr{D}_m}{\frac{1}{2}\rho_m V_m^2 D_m^2} = \frac{\mathscr{D}}{\frac{1}{2}\rho V^2 D^2}$$

so that (with $\rho_m = \rho$)

$$\mathscr{D} = \frac{V^2}{V_m^2} \frac{D^2}{D_m^2} \mathscr{D}_m$$

$$= \frac{\left(80\ \frac{km}{hr}\right)^2}{\left(400\ \frac{km}{hr}\right)^2} \frac{(2\ m)^2}{(0.4\ m)^2} (170\ N) = \underline{\underline{170\ N}}$$

(Note that $\mathscr{D} = \mathscr{D}_m$ in this problem, since from the condition of Reynolds number similarity, $V^2/V_m^2 = D_m^2/D^2$. This is not true in general.)

7.53

7.53 A large, rigid, rectangular billboard is supported by an elastic column as shown in Fig. P7.53. There is concern about the deflection, δ, of the top of the structure during a high wind of velocity V. A wind tunnel test is to be conducted with a 1 : 15 scale model. Assume the pertinent column variables are its length and cross-sectional dimensions, and the modulus of elasticity of the material used for the column. The only important "wind" variables are the air density and velocity. (a) Determine the model design conditions and the prediction equation for the deflection. (b) If the same structural materials are used for the model and prototype, and the wind tunnel operates under standard atmospheric conditions, what is the required wind tunnel velocity to match an 80 km/hr wind?

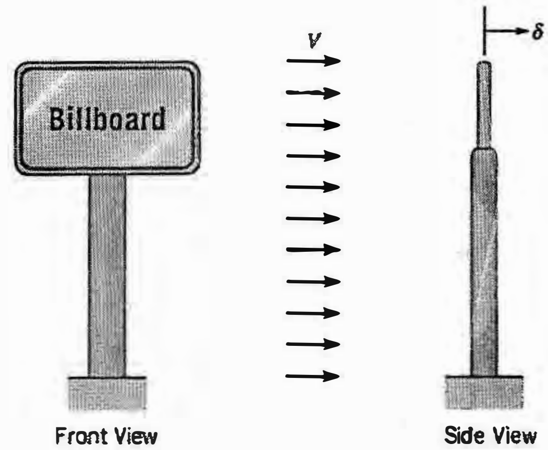


FIGURE P7.53

Assume $\delta = f(\ell, \ell_i, \rho, V, E)$

where: $\delta \sim$ deflection $\doteq L$, $\ell \sim$ column length $\doteq L$, $\ell_i \sim$ other lengths $\doteq L$ ($i = 1, 2, \ldots$ etc.); $\rho \sim$ air density $\doteq FL^{-4}T^2$, $V \sim$ wind velocity $\doteq LT^{-1}$; $E \sim$ modulus of elasticity $\doteq FL^{-2}$. From the pi theorem, $(5+i) - 3 = 2+i$ pi terms required, and a dimensional analysis yields

$$\frac{\delta}{\ell} = \phi\left(\frac{\ell_i}{\ell}, \frac{\rho V^2}{E}\right)$$

(a) The model design conditions are

$$\underline{\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell}} \qquad \underline{\frac{\rho_m V_m^2}{E_m} = \frac{\rho V^2}{E}}$$

and the prediction equation is

$$\frac{\delta}{\ell} = \frac{\delta_m}{\ell_m}$$

or with a length scale of 1:15

$$\underline{\delta = 15\,\delta_m}$$

(b) From the second model design condition,

$$V_m^2 = \frac{E_m}{E}\frac{\rho}{\rho_m} V^2$$

so that with $E = E_m$ and $\rho = \rho_m$

$$V_m^2 = V^2$$

or

$$V_m = V = \underline{80\ \frac{km}{hr}}$$

7.54

7.54 A thin flat plate having a diameter of 0.3 ft is towed through a tank of oil (γ = 53 lb/ft³) at a velocity of 5 ft/s. The plane of the plate is perpendicular to the direction of motion, and the plate is submerged so that wave action is negligible. Under these conditions the drag on the plate is 1.4 lb. If viscous effects are neglected, predict the drag on a geometrically similar, 2-ft-diameter plate that is towed with a velocity of 3 ft/s through water at 60 °F under conditions similar to those for the smaller plate.

If viscous and wave effects are neglected,

$$\mathscr{D} = f(d, \rho, V)$$

where: $\mathscr{D} \sim$ drag $\doteq F$, $d \sim$ plate diameter $\doteq L$, $\rho \sim$ fluid density $\doteq FL^{-4}T^2$, and $V \sim$ velocity $\doteq LT^{-1}$. From the pi theorem, $4-3=1$ pi term required, and a dimensional analysis yields

$$\Pi_1 = \frac{\mathscr{D}}{\rho V^2 d^2}$$

Since there is only one pi term

$$\frac{\mathscr{D}_m}{\rho_m V_m^2 d_m^2} = \frac{\mathscr{D}}{\rho V^2 d^2} = \text{constant}$$

where m refers to the smaller, 0.3-ft-diameter plate.

Thus,

$$\mathscr{D} = \frac{\rho}{\rho_m} \frac{V^2}{V_m^2} \frac{d^2}{d_m^2} \mathscr{D}_m \qquad (1)$$

From the data given:

$\rho = 1.94 \text{ slugs/ft}^3$; $d = 2 \text{ ft}$; $V = 3 \text{ ft/s}$

$\rho_m = \dfrac{53 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}$; $d_m = 0.3 \text{ ft}$; $V_m = 5 \text{ ft/s}$; $\mathscr{D}_m = 1.4 \text{ lb}$

Therefore, from Eq. (1),

$$\mathscr{D} = \frac{\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)}{\left(\frac{53 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)} \frac{\left(3 \frac{\text{ft}}{\text{s}}\right)^2}{\left(5 \frac{\text{ft}}{\text{s}}\right)^2} \frac{(2 \text{ ft})^2}{(0.3 \text{ ft})^2} (1.4 \text{ lb}) = \underline{\underline{26.4 \text{ lb}}}$$

7.55

7.55 For a certain model study involving a 1:5 scale model it is known that Froude number similarity must be maintained. The possibility of cavitation is also to be investigated, and it is assumed that the cavitation number must be the same for model and prototype. The prototype fluid is water at 30 °C, and the model fluid is water at 70 °C. If the prototype operates at an ambient pressure of 101 kPa (abs), what is the required ambient pressure for the model system?

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

so that (with $g = g_m$)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

For cavitation number similarity,

$$\frac{(p_r - p_v)_m}{\frac{1}{2}\rho_m V_m^2} = \frac{(p_r - p_v)}{\frac{1}{2}\rho V^2}$$

It follows that

$$(p_r - p_v)_m = \frac{\rho_m}{\rho}\frac{V_m^2}{V^2}(p_r - p_v)$$

and making use of Eq. (1)

$$(p_r - p_v)_m = \frac{\rho_m}{\rho}\frac{\ell_m}{\ell}(p_r - p_v) \qquad (2)$$

For water (from Table B.2):

@ 70°C $\rho_m = 977.8\ kg/m^3$; $p_{vm} = 3.116 \times 10^4\ N/m^2$ (abs)

@ 30°C $\rho = 995.7\ kg/m^3$; $p_v = 4.243 \times 10^3\ N/m^2$ (abs)

Thus, from Eq. (2)

$$p_{rm} = \frac{\left(977.8\ \frac{kg}{m^3}\right)}{\left(995.7\ \frac{kg}{m^3}\right)}\left(\frac{1}{5}\right)\left(101 \times 10^3\ \frac{N}{m^2} - 4.243 \times 10^3\ \frac{N}{m^2}\right) + 3.116 \times 10^4\ \frac{N}{m^2}$$

$$= \underline{\underline{50.2\ kPa\ (abs)}}$$

7.56

7.56 A thin layer of spherical particles rests on the bottom of a horizontal tube as shown in Fig. P7.56. When an incompressible fluid flows through the tube, it is observed that at some critical velocity the particles will rise and be transported along the tube. A model is to be used to determine this critical velocity. Assume the critical velocity, V_c to be a function of the pipe diameter, D, particle diameter, d, the fluid density, ρ, and viscosity, μ, the density of the particles, ρ_p, and the acceleration of gravity, g. **(a)** Determine the similarity requirements for the model, and the relationship between the critical velocity for model and prototype (the prediction equation). **(b)** For a length scale of $\frac{1}{2}$ and a fluid density scale of 1.0, what will be the critical velocity scale (assuming all similarity requirements are satisfied)?

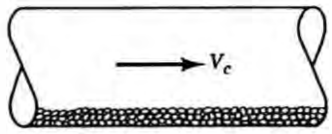


FIGURE P7.56

(a) $$V_c = f(D, d, \rho, \mu, \rho_p, g)$$

$V_c \doteq LT^{-1}$ $D \doteq L$ $d \doteq L$ $\rho \doteq FL^{-4}T^2$ $\mu \doteq FL^{-2}T$ $\rho_p \doteq FL^{-4}T^2$ $g \doteq LT^{-2}$

From the pi theorem, $7-3=4$ pi terms required, and a dimensional analysis yields

$$\frac{\rho V_c D}{\mu} = \phi\left(\frac{d}{D}, \frac{\rho}{\rho_p}, \frac{g d^3 \rho^2}{\mu^2}\right)$$

Thus, the similarity requirements are

$$\underline{\underline{\frac{d_m}{D_m} = \frac{d}{D}}} \qquad \underline{\underline{\frac{\rho_m}{\rho_{pm}} = \frac{\rho}{\rho_p}}} \qquad \underline{\underline{\frac{g_m d_m^3 \rho_m^2}{\mu_m^2} = \frac{g d^3 \rho^2}{\mu^2}}}$$

The prediction equation is

$$\underline{\underline{\frac{\rho V_c D}{\mu} = \frac{\rho_m V_{cm} D_m}{\mu_m}}}$$

(b) If all similarity requirements are satisfied, the prediction equation indicates that

$$\frac{V_{cm}}{V_c} = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{D}{D_m} = (1.0)\left(\frac{\mu_m}{\mu}\right)(2) = 2\,\frac{\mu_m}{\mu} \qquad (1)$$

From the third similarity requirement (with $g = g_m$),

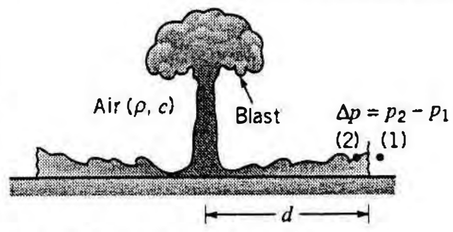
$$\frac{\mu_m}{\mu} = \sqrt{\left(\frac{d_m}{d}\right)^3\left(\frac{\rho_m}{\rho}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^3(1.0)^2} = \sqrt{\frac{1}{8}}$$

Thus, from Eq. (1)

$$\frac{V_{cm}}{V_c} = 2\sqrt{\frac{1}{8}} = \underline{\underline{0.707}}$$

7.57

7.57 The pressure rise, Δp, across a blast wave, as shown in Fig. P7.57 and Video V11.7 is assumed to be a function of the amount of energy released in the explosion, E, the air density, ρ, the speed of sound, c, and the distance from the blast, d. **(a)** Put this relationship in dimensionless form. **(b)** Consider two blasts: the prototype blast with energy release E and a model blast with 1/1000th the energy release ($E_m = 0.001\ E$). At what distance from the model blast will the pressure rise be the same as that at a distance of 1 mile from the prototype blast?



■ FIGURE P7.57

(a)
$$\Delta p = f(E, \rho, c, d)$$

$\Delta p \doteq FL^{-2}$ $\quad E \doteq FL$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad c \doteq LT^{-1}$ $\quad d \doteq L$

From the pi theorem, 5−3 = 2 pi terms required, and a dimensional analysis yields

$$\underline{\underline{\frac{\Delta p}{\rho c^2} = \phi\left(\frac{E}{\rho c^2 d^3}\right)}}$$

(b) For similarity,

$$\frac{E_m}{\rho_m c_m^2 d_m^3} = \frac{E}{\rho c^2 d^3}$$

and with $\rho_m = \rho$, $c_m = c$, it follows that

$$d_m^3 = \frac{E_m}{E} d^3$$

For $E_m/E = 0.001$ and $d = 1\ mi$

$$d_m^3 = (0.001)(1\ mi)^3$$

$$d_m = 0.100\ mi$$

With this similarity requirement satisfied, the prediction equation is

$$\frac{\Delta p_m}{\rho_m c_m^2} = \frac{\Delta p}{\rho c^2}$$

and therefore

$$\Delta p_m = \Delta p$$

at

$$\underline{\underline{d_m = 0.100\ mi}}$$

7.58

7.58 The drag, $\mathfrak{D}$, on a sphere located in a pipe through which a fluid is flowing is to be determined experimentally (see Fig. P7.58). Assume that the drag is a function of the sphere diameter, d, the pipe diameter, D, the fluid velocity, V, and the fluid density, ρ. **(a)** What dimensionless parameters would you use for this problem? **(b)** Some experiments using water indicate that for $d = 0.2$ in., $D = 0.5$ in., and $V = 2$ ft/s, the drag is 1.5×10^{-3} lb. If possible, estimate the drag on a sphere located in a 2-ft-diameter pipe through which water is flowing with a velocity of 6 ft/s. The sphere diameter is such that geometric similarity is maintained. If it is not possible, explain why not.

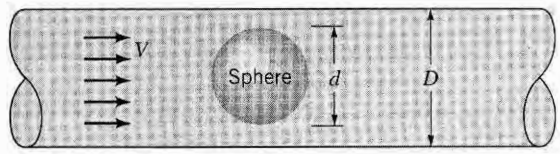


FIGURE P7.58

(a)
$$\mathcal{D} = f(d, D, V, \rho)$$

$$\mathcal{D} \doteq F \quad d \doteq L \quad D \doteq L \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2}$$

From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 D^2} = \phi\left(\frac{d}{D}\right)$$

(b) The similarity requirement is

$$\frac{d_m}{D_m} = \frac{d}{D}$$

so that
$$\frac{0.2\ in.}{0.5\ in.} = \frac{d\ (ft)}{2\ ft}$$

and $d = 0.8\ ft$ (required diameter).

Thus, the prediction equation is

$$\frac{\mathcal{D}}{\rho V^2 D^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 D_m^2}$$

So that
$$\mathcal{D} = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2\left(\frac{D}{D_m}\right)^2 \mathcal{D}_m \quad (\text{and with } \rho = \rho_m)$$

$$\mathcal{D} = \left(\frac{6\ \frac{ft}{s}}{2\ \frac{ft}{s}}\right)^2\left(\frac{2\ ft}{0.5/12\ ft}\right)^2\left(1.5\times10^{-3}\ lb\right) = \underline{\underline{31.1\ lb}}$$

7.59

7.59 An incompressible fluid oscillates harmonically ($V = V_0 \sin \omega t$, where V is the velocity) with a frequency of 10 rad/s in a 4-in.-diameter pipe. A $\frac{1}{4}$ scale model is to be used to determine the pressure difference per unit length, Δp_ℓ (at any instant) along the pipe. Assume that

$$\Delta p_\ell = f(D, V_0, \omega, t, \mu, \rho)$$

where D is the pipe diameter, ω the frequency, t the time, μ the fluid viscosity, and ρ the fluid density. **(a)** Determine the similarity requirements for the model and the prediction equation for Δp_ℓ. **(b)** If the same fluid is used in the model and the prototype, at what frequency should the model operate?

$\Delta p_\ell \doteq FL^{-3}$ $\quad D \doteq L$ $\quad V_0 \doteq LT^{-1}$ $\quad \omega \doteq T^{-1}$ $\quad t \doteq T$ $\quad \mu \doteq FL^{-2}T$ $\quad \rho \doteq FL^{-4}T^2$

From the pi theorem, 7-3 = 4 pi terms required, and a dimensional analysis yields

$$\frac{D\,\Delta p_\ell}{\rho V_0^2} = \phi\left(\frac{V_0 t}{D},\ \omega t,\ \frac{\rho V_0 D}{\mu}\right)$$

(a) Thus, the similarity requirements are

$$\underline{\underline{\frac{V_{0m} t_m}{D_m} = \frac{V_0 t}{D}}} \qquad \underline{\underline{\omega_m t_m = \omega t}} \qquad \underline{\underline{\frac{\rho_m V_{0m} D_m}{\mu_m} = \frac{\rho V_0 D}{\mu}}}$$

and the prediction equation is

$$\underline{\underline{\frac{D\,\Delta p_\ell}{\rho V_0^2} = \frac{D_m\,\Delta p_{\ell m}}{\rho_m V_{0m}^2}}}$$

(b) For Reynolds number similarity (the last similarity requirement), with the same fluid in model and prototype,

$$\frac{V_{0m}}{V_0} = \frac{D}{D_m}$$

so that from the first similarity requirement

$$\frac{t_m}{t} = \frac{D_m}{D}\frac{V_0}{V_{0m}} = \left(\frac{D_m}{D}\right)\left(\frac{D_m}{D}\right) = \left(\frac{D_m}{D}\right)^2$$

Thus, to satisfy the remaining similarity requirement

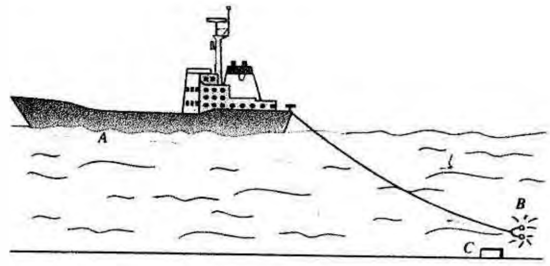
$$\omega_m t_m = \omega t$$

or

$$\omega_m = \frac{t}{t_m}\omega = \left(\frac{D}{D_m}\right)^2 \omega = (4)^2\left(10\ \frac{\text{rad}}{\text{s}}\right) = \underline{\underline{160\ \frac{\text{rad}}{\text{s}}}}$$

7.60

7.60 As shown in Fig. P7.60, a "noisemaker" B is towed behind a minesweeper A to set off enemy acoustic mines such as at C. The drag force of the noisemaker is to be studied in a water tunnel at a ¼ scale model (model ¼ the size of the prototype). The drag force is assumed to be a function of the speed of the ship, the density and viscosity of the fluid, and the diameter of the noisemaker. (a) If the prototype towing speed in 3 m/s, determine the water velocity in the tunnel for the model tests. (b) If the model tests of part (a) produced a model drag of 900 N, determine the drag expected on the prototype.



■ FIGURE P7.60

(a) $\mathscr{D} = f(V, \rho, \mu, D)$, where $\mathscr{D} \doteq F \doteq \frac{ML}{T^2}$, $V \doteq \frac{L}{T}$, $\rho \doteq \frac{M}{L^3}$, $\mu \doteq \frac{M}{LT}$, and $D \doteq L$

Thus, $k-r = 5-3 = 2$ so that $\Pi_1 = \phi(\Pi_2)$,

where by inspection there are the ingredients for a Reynolds number, $Re = \rho V D/\mu$, and a drag coefficient, $C_D = \mathscr{D}/(\frac{1}{2}\rho V^2 D^2)$.

Hence,

$$C_D = \phi(Re)$$

For similarity, $Re_m = Re$, or

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho V D}{\mu}$$ so that with $\rho_m = \rho$ and $\mu_m = \mu$,

$V_m D_m = VD$ or with $D_m = \frac{1}{4}D$,

$$V_m = \frac{D}{D_m}V = 4V = 4(3\,\tfrac{m}{s}) = \underline{\underline{12\,\tfrac{m}{s}}}$$

(b) With $Re_m = Re$ it follows that $C_{D_m} = C_D$, or

$$\frac{\mathscr{D}_m}{\frac{1}{2}\rho_m V_m^2 D_m^2} = \frac{\mathscr{D}}{\frac{1}{2}\rho V^2 D^2}$$

Thus, since $\rho_m = \rho$,

$$\frac{\mathscr{D}_m}{V_m^2 D_m^2} = \frac{\mathscr{D}}{V^2 D^2}$$ or

$$\mathscr{D} = \left(\frac{V}{V_m}\right)^2 \left(\frac{D}{D_m}\right)^2 \mathscr{D}_m = \left(\frac{3\,m/s}{12\,m/s}\right)^2 (4)^2 (900N) = \underline{\underline{900\,N}}$$

Note: The prototype has the same drag as the model.

7.61

7.61 The drag characteristics for a newly designed automobile having a maximum characteristic length of 20 ft are to be determined through a model study. The characteristics at both low speed (approximately 20 mph) and high speed (90 mph) are of interest. For a series of projected model tests an unpressurized wind tunnel that will accommodate a model with a maximum characteristic length of 4 ft is to be used. Determine the range of air velocities that would be required for the wind tunnel if Reynolds number similarity is desired. Are the velocities suitable? Explain.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$V_m = \frac{\mu_m}{\mu} \frac{\rho}{\rho_m} \frac{\ell}{\ell_m} V \qquad (1)$$

Since the wind tunnel is unpressurized, the air properties will be approximately the same for model and prototype. Thus, Eq. (1) reduces to

$$V_m = \frac{\ell}{\ell_m} V$$

and for the data given

$$V_m = \frac{(20\,ft)}{(4\,ft)} V = 5V$$

Therefore, at low speed

$$V_m = 5\,(20\,mph) = 100\,mph$$

and at high speed

$$V_m = 5\,(90\,mph) = 450\,mph$$

So that the model velocity range is $\underline{\underline{100\,mph \text{ to } 450\,mph}}$.

At the high velocity in the wind tunnel, compressibility of the air would start to become an important factor, whereas compressibility is not important for the prototype. Thus, the higher velocity required for the model would not be suitable. <u>No</u>.

7.62

7.62 The drag characteristics of an airplane are to be determined by model tests in a wind tunnel operated at an absolute pressure of 1300 kPa. If the prototype is to cruise in standard air at 385 km/hr, and the corresponding speed of the model is not to differ by more than 20% from this (so that compressibility effects may be ignored), what range of length scales may be used if Reynolds number similarity is to be maintained? Assume the viscosity of air is unaffected by pressure, and the temperature of the air in the tunnel is equal to the temperature of the air in which the airplane will fly.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$\frac{\ell_m}{\ell} = \frac{\mu_m}{\mu} \frac{\rho}{\rho_m} \frac{V}{V_m} \qquad (1)$$

For an ideal gas, $p = \rho R T$, and with constant temperature,

$$\frac{p}{\rho} = \text{constant}$$

or

$$\frac{p}{p_m} = \frac{\rho}{\rho_m}$$

and Eq. (1) can be written as (with $\mu_m = \mu$)

$$\frac{\ell_m}{\ell} = \frac{p}{p_m} \frac{V}{V_m}$$

For the data given

$$\frac{\ell_m}{\ell} = \frac{(101 \text{ kPa})}{(1300 \text{ kPa})} \frac{V}{V_m}$$

and with $V_m = (1 \pm 0.2) V$, it follows that

$$\frac{\ell_m}{\ell} = \frac{(101 \text{ kPa})}{(1300 \text{ kPa})} \frac{1}{(1 \pm 0.2)}$$

Thus, the range of length scales is $\underline{\underline{0.0647 \text{ to } 0.0971}}$.

7.63

7.63 Wind blowing past a flag causes it to "flutter in the breeze." The frequency of this fluttering, ω, is assumed to be a function of the wind speed, V, the air density, ρ, the acceleration of gravity, g, the length of the flag, ℓ, and the "area density," ρ_A, (with dimensions of ML^{-2}) of the flag material. It is desired to predict the flutter frequency of a large $\ell = 40$ ft flag in a $V = 30$ ft/s wind. To do this a model flag with $\ell = 4$ ft is to be tested in a wind tunnel. **(a)** Determine the required area density of the model flag material if the large flag has $\rho_A = 0.006$ slugs/ft^2. **(b)** What wind tunnel velocity is required for testing the model? **(c)** If the model flag flutters at 6 Hz, predict the frequency for the large flag.

$$\omega = f(V, \rho, g, \ell, \rho_A)$$

$$\omega \doteq T^{-1} \quad V \doteq LT^{-1} \quad \rho \doteq ML^{-3} \quad g \doteq LT^{-2} \quad \ell \doteq L \quad \rho_A \doteq ML^{-2}$$

From the pi Theorem, $6-3=3$ pi terms required, and a dimensional analysis yields

$$\omega\sqrt{\frac{\ell}{g}} = \phi\left(\frac{V}{\sqrt{g\ell}}, \frac{\rho_A}{\rho\ell}\right)$$

(a) For similarity

$$\frac{\rho_{Am}}{\rho_m \ell_m} = \frac{\rho_A}{\rho \ell}$$

and since $\rho_m = \rho$

$$\rho_{Am} = \frac{\ell_m}{\ell}\rho_A = \left(\frac{4\ \text{ft}}{40\ \text{ft}}\right)\left(0.006\ \frac{\text{slugs}}{\text{ft}^2}\right) = \underline{\underline{0.0006\ \frac{\text{slugs}}{\text{ft}^2}}}$$

(b) For similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

and with $g_m = g$

$$V_m = \sqrt{\frac{\ell_m}{\ell}}\, V = \sqrt{\frac{4\ \text{ft}}{40\ \text{ft}}}\left(30\ \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{9.49\ \frac{\text{ft}}{\text{s}}}}$$

(c) With the similarity requirements satisfied the prediction equation is

$$\omega\sqrt{\frac{\ell}{g}} = \omega_m\sqrt{\frac{\ell_m}{g_m}}$$

so that

$$\omega = \sqrt{\frac{g}{g_m}}\sqrt{\frac{\ell_m}{\ell}}\,\omega_m = \sqrt{\frac{4\ \text{ft}}{40\ \text{ft}}}\,(6\ \text{Hz}) = \underline{\underline{1.90\ \text{Hz}}}$$

7.65

7.65 The drag on a sphere moving in a fluid is known to be a function of the sphere diameter, the velocity, and the fluid viscosity and density. Laboratory tests on a 4-in.-diameter sphere were performed in a water tunnel and some model data are plotted in Fig. P7.65. For these tests the viscosity of the water was 2.3×10^{-5} lb·s/ft² and the water density was 1.94 slugs/ft³. Estimate the drag on an 8-ft diameter balloon moving in air at a velocity of 3 ft/s. Assume the air to have a viscosity of 3.7×10^{-7} lb·s/ft² and a density of 2.38×10^{-3} slugs/ft³.

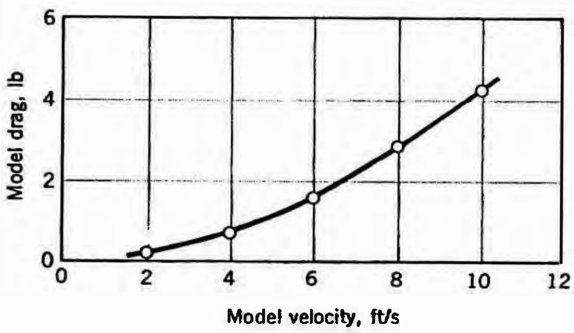


FIGURE P7.65

$$\mathcal{D} = f(d, V, \rho, \mu)$$

where: $\mathcal{D} \sim$ drag $\doteq F$, $d \sim$ sphere diameter $\doteq L$, $V \sim$ velocity $\doteq LT^{-1}$, $\rho \sim$ fluid density $\doteq FL^{-4}T^{2}$, $\mu \sim$ fluid viscosity $\doteq FL^{-2}T$.

From the pi theorem, $5-3 = 2$ pi terms required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \phi\left(\frac{\rho V d}{\mu}\right)$$

Thus, Reynolds number similarity is required so that

$$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho V d}{\mu}$$

or

$$V_m = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{d}{d_m}V$$

$$= \frac{\left(2.3\times10^{-5}\ \frac{lb\cdot s}{ft^2}\right)}{\left(3.7\times10^{-7}\ \frac{lb\cdot s}{ft^2}\right)}\frac{\left(2.38\times10^{-3}\ \frac{slugs}{ft^3}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}\frac{(8\ ft)}{\left(\frac{4}{12}\ ft\right)}\left(3\ \frac{ft}{s}\right)$$

$$= 5.49\ \frac{ft}{s}$$

From the graph, for $V_m = 5.49$ ft/s, $\mathcal{D}_m = 1.30$ lb. Since

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 d_m^2}$$

or

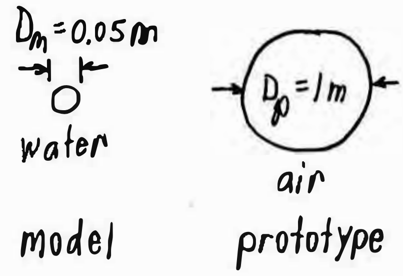
$$\mathcal{D} = \frac{\rho}{\rho_m}\frac{V^2}{V_m^2}\frac{d^2}{d_m^2}\mathcal{D}_m$$

so that

$$\mathcal{D} = \frac{\left(2.38\times10^{-3}\ \frac{slugs}{ft^3}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}\frac{\left(3\ \frac{ft}{s}\right)^2}{\left(5.49\ \frac{ft}{s}\right)^2}\frac{(8\ ft)^2}{\left(\frac{4}{12}\ ft\right)^2}(1.30\ lb) = \underline{\underline{0.274\ lb}}$$

7.67

7.67 Drag measurements were taken for a sphere, with a diameter of 5 cm, moving at 4 m/s in water at 20 °C. The resulting drag on the sphere was 10 N. For a balloon with 1-m diameter rising in air with standard temperature and pressure, determine (a) the velocity if Reynolds number similarity is enforced and (b) the drag force if the drag coefficient (Eq. 7.19) is the dependent pi term.



(a) For Reynolds number similarity, $Re_p = Re_m$, where $(\ )_p$ and $(\ )_m$ refer to prototype and model, respectively. Thus,

$$\frac{V_p D_p}{\nu_p} = \frac{V_m D_m}{\nu_m} \quad \text{or}$$

$$V_p = \left(\frac{\nu_p}{\nu_m}\right)\left(\frac{D_m}{D_p}\right) V_m$$

$$= \frac{(1.46\times10^{-5}\,m^2/s)}{(1.004\times10^{-6}\,m^2/s)}\,\frac{(0.05\,m)}{(1\,m)}\,(4\,m/s) = \underline{\underline{2.91\,\frac{m}{s}}}$$

(b) $C_{D_p} = C_{D_m}$, since $C_D = \phi(Re)$ and $Re_m = Re_p$,

$$\frac{\mathscr{D}_p}{\frac{1}{2}\rho_p V_p^2 D_p^2} = \frac{\mathscr{D}_m}{\frac{1}{2}\rho_m V_m^2 D_m^2}$$

Thus,

$$\mathscr{D}_p = \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{D_p}{D_m}\right)^2 \mathscr{D}_m$$

$$= \frac{(1.23\,kg/m^3)}{(998.2\,kg/m^3)}\left(\frac{2.91\,m/s}{4\,m/s}\right)^2\left(\frac{1\,m}{0.05\,m}\right)^2(10\,N) = \underline{\underline{2.61\,N}}$$

7.68

7.68 A prototype automobile is designed to travel at 65 km/hr. A model of this design is tested in a wind tunnel with identical standard sea-level air properties at a 1:5 scale. The measured model drag is 400 N, enforcing dynamic similarity. Determine (a) the drag force on the prototype and (b) the power required to overcome this drag. See Eq. 7.19.

For this model, $(\ )_m$, and prototype, $(\ )_p$, assume

$C_D = \phi(Re)$, where $Re = V\ell/\nu$ and $C_D = \mathcal{D}/(\frac{1}{2}\rho V^2 \ell^2)$

so that if $Re_m = Re_p$,

then $C_{D_m} = C_{D_p}$

(a) $V_m \ell_m / \nu_m = V_p \ell_p / \nu_p$, where $\nu_m = \nu_p$.

Hence,

$$V_m = \frac{\ell_p}{\ell_m} V_p = \left(\frac{5}{1}\right) 65 \frac{km}{hr} = 325 \frac{km}{hr} \times \left(\frac{1000\,m}{1\,km}\ \frac{1\,hr}{3600\,s}\right) = 90.3 \frac{m}{s}$$

Also, with $C_{D_m} = C_{D_p}$,

$$\frac{\mathcal{D}_m}{\frac{1}{2}\rho_m V_m^2 \ell_m^2} = \frac{\mathcal{D}_p}{\frac{1}{2}\rho_p V_p^2 \ell_p^2}, \text{ or since } \rho_m = \rho_p,$$

$$\mathcal{D}_p = \left(\frac{V_p}{V_m}\right)^2 \left(\frac{\ell_p}{\ell_m}\right)^2 \mathcal{D}_m = \left(\frac{65\,km/hr}{325\,km/hr}\right)^2 (5)^2 (400\,N) = \underline{\underline{400\,N}}$$

(b) $\mathcal{P}$ = power $= \mathcal{D}V$ so that

$$\mathcal{P}_p = \mathcal{D}_p V_p = 400\,N\left(65 \frac{km}{hr}\right)\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{1000\,m}{1\,km}\right) = 7{,}220 \frac{N \cdot m}{s}$$

$$= \underline{\underline{7{,}220\,W}}$$

Note: $\mathcal{P}_p = 7{,}220\,W\left(1.341 \times 10^{-3} \frac{hp}{W}\right) = 9.68\,hp$

7.69

7.69 A new blimp will move at 6 m/s in 20 °C air, and we want to predict the drag force. Using a 1 : 13-scale model in water at 20 °C and measuring a 2500-N drag force on the model, determine (a) the required water velocity, (b) the drag on the prototype blimp and, (c) the power that will be required to propel it through the air.

For this model and prototype, assume (see Eq. 7.19)

$C_D = \phi(Re)$, where $Re = V\ell/\nu$ = Reynolds number, and

$C_D = \mathscr{D}/(\frac{1}{2}\rho V^2 \ell^2)$ = drag coefficient

(a) Thus, with $(\ )_m$ and $(\ )_p$ denoting model and prototype, respectively,

$Re_m = Re_p$, or

$$\frac{V_m \ell_m}{\nu_m} = \frac{V_p \ell_p}{\nu_p}$$

Hence,*

$$V_m = \left(\frac{\ell_p}{\ell_m}\right)\left(\frac{\nu_m}{\nu_p}\right)V_p = \left(\frac{13}{1}\right)\left(\frac{1.004\times10^{-6}\ m^2/s}{1.51\times10^{-5}\ m^2/s}\right)\left(6\ \frac{m}{s}\right) = \underline{\underline{5.2\ \frac{m}{s}}}$$

(b) $C_{D_m} = C_{D_p}$, or

$$\frac{\mathscr{D}_m}{\frac{1}{2}\rho_m V_m^2 \ell_m^2} = \frac{\mathscr{D}_p}{\frac{1}{2}\rho_p V_p^2 \ell_p^2}$$

Hence,*

$$\mathscr{D}_p = \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{\ell_p}{\ell_m}\right)^2 \mathscr{D}_m$$

$$= \frac{1.204\ kg/m^3}{998.2\ kg/m^3}\left(\frac{6\ m/s}{5.2\ m/s}\right)^2(13)^2(2500N) = \underline{\underline{678N}}$$

(c) $\mathcal{P}_p = \mathscr{D}_p V_p = 678N\left(6\ \frac{m}{s}\right) = 4070\ \frac{N\cdot m}{s} = \underline{\underline{4070\ W}}$

* Fluid properties are from Tables B.2 and B.4.

7.70

7.70 At a large fish hatchery the fish are reared in open, water-filled tanks. Each tank is approximately square in shape with curved corners, and the walls are smooth. To create motion in the tanks, water is supplied through a pipe at the edge of the tank. The water is drained from the tank through an opening at the center. (See Video V7.9.) A model with a length scale of 1:13 is to be used to determine the velocity, V, at various locations within the tank. Assume that $V = f(\ell, \ell_i, \rho, \mu, g, Q)$ where ℓ is some characteristic length such as the tank width, ℓ_i represents a series of other pertinent lengths, such as inlet pipe diameter, fluid depth, etc., ρ is the fluid density, μ is the fluid viscosity, g is the acceleration of gravity, and Q is the discharge through the tank. **(a)** Determine a suitable set of dimensionless parameters for this problem and the prediction equation for the velocity. If water is to be used for the model, can all of the similarity requirements be satisfied? Explain and support your answer with the necessary calculations. **(b)** If the flowrate into the full-sized tank is 250 gpm, determine the required value for the model discharge assuming Froude number similarity. What model depth will correspond to a depth of 32 in. in the full-sized tank?

(a) $$V = f(\ell, \ell_i, \rho, \mu, g, Q)$$

From the pi theorem, 7−3 = 4 pi terms required and a dimensional analysis yields

$$\underline{\underline{\frac{V\ell^2}{Q} = \phi\left(\frac{\ell_i}{\ell}, \frac{Q^2}{\ell^5 g}, \frac{\rho Q}{\ell \mu}\right)}}$$

Thus, the similarity requirements are

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell} \qquad \frac{Q_m^2}{\ell_m^5 g_m} = \frac{Q^2}{\ell^5 g} \qquad \frac{\rho_m Q_m}{\ell_m \mu_m} = \frac{\rho Q}{\ell \mu}$$

and the prediction equation is

$$\frac{V\ell^2}{Q} = \frac{V_m \ell_m^2}{Q_m}$$

From the last similarity requirement with $\rho_m = \rho$ and $\mu_m = \mu$

$$\frac{Q_m}{Q} = \frac{\rho}{\rho_m}\frac{\mu}{\mu_m}\frac{\ell_m}{\ell} = \frac{\ell_m}{\ell}$$

However, from the second similarity requirement with $g_m = g$

$$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2}$$

Since these two requirements are in conflict it follows that the similarity requirements cannot be satisfied. $\underline{\underline{\text{No}}}$.

(con't)

7.70 (Con't)

(b) For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Thus, from the prediction equation

$$\frac{V\ell^2}{Q} = \frac{V_m \ell_m^2}{Q_m}$$

it follows that

$$\frac{Q_m}{Q} = \frac{V_m}{V}\left(\frac{\ell_m}{\ell}\right)^2 = \sqrt{\frac{\ell_m}{\ell}}\left(\frac{\ell_m}{\ell}\right)^2 = \left(\frac{\ell_m}{\ell}\right)^{5/2}$$

so that with $\ell_m/\ell = 1/13$

$$Q_m = \left(\frac{1}{13}\right)^{5/2}(250\ gpm) = \underline{\underline{0.410\ gpm}}$$

Note that this same result can be obtained from the second similarity requirement (which corresponds to Froude number similarity) since

$$\frac{Q_m^2}{\ell_m^5 g_m} = \frac{Q^2}{\ell^5 g}$$

and therefore

$$Q_m = \left(\frac{\ell_m}{\ell}\right)^{5/2} Q$$

Geometric similarity requires that

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell}$$

or

$$\frac{\ell_{im}}{\ell_i} = \frac{\ell_m}{\ell} = \frac{1}{13}$$

so that all lengths scale as the length scale. Thus,

$$(depth)_{model} = \left(\frac{1}{13}\right)(depth)_{prototype}$$

$$= \left(\frac{1}{13}\right)(32\ in.) = \underline{\underline{2.46\ in.}}$$

7.71

7.71 Flow patterns that develop as winds blow past a vehicle, such as a train, are often studied in low-speed environmental (meteorological) wind tunnels. (See **Video V7.16**.) Typically, the air velocities in these tunnels are in the range of 0.1 m/s to 30 m/s. Consider a cross wind blowing past a train locomotive. Assume that the local wind velocity, V, is a function of the approaching wind velocity (at some distance from the locomotive), U, the locomotive length, ℓ, height, h, and width, b, the air density, ρ, and the air viscosity, μ. (a) Establish the similarity requirements and prediction equation for a model to be used in the wind tunnel to study the air velocity, V, around the locomotive. (b) If the model is to be used for cross winds gusting to $U = 25$ m/s, explain why it is not practical to maintain Reynolds number similarity for a typical length scale 1:50.

(a)
$$V = f(U, \ell, h, b, \rho, \mu)$$

$V \doteq LT^{-1}$ $U \doteq LT^{-1}$ $\ell \doteq L$ $h \doteq L$ $b \doteq L$ $\rho \doteq FL^{-4}T^{2}$ $\mu \doteq FL^{-2}T$

From the pi theorem, $7-3=4$ pi terms required, and a dimensional analysis yields

$$\frac{V}{U} = \phi\left(\frac{\ell}{h}, \frac{b}{h}, \frac{\rho h U}{\mu}\right)$$

Thus, the similarity requirements are

$$\underline{\underline{\frac{\ell_m}{h_m} = \frac{\ell}{h}}} \qquad \underline{\underline{\frac{b_m}{h_m} = \frac{b}{h}}} \qquad \underline{\underline{\frac{\rho_m h_m U_m}{\mu_m} = \frac{\rho h U}{\mu}}}$$

The prediction equation is

$$\underline{\underline{\frac{V}{U} = \frac{V_m}{U_m}}}$$

(b) Since the density and viscosity of the air flowing around the train and the air in the wind tunnel would be practically the same ($\rho_m \simeq \rho$, $\mu_m \simeq \mu$), it follows from the last similarity requirement (which is the Reynolds number) that

$$U_m = \left(\frac{h}{h_m}\right) U$$

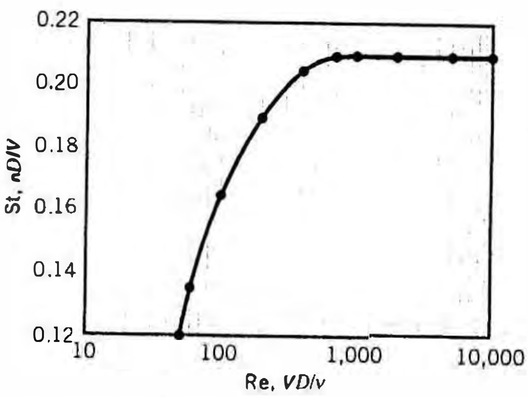
Thus, with a length scale of 1:50 and with $U = 25\,m/s$

$$U_m = (50)(25\,m/s) = 1{,}250\ m/s$$

This required model velocity is much higher than can be acheived in the wind tunnel and therefore it is not practical to maintain Reynolds number similarity. <u>The required model velocity is too high.</u>

7.72

7.72 (See "Galloping Gertie," Section 7.8.2.) The Tacoma Narrows bridge failure is a dramatic example of the possible serious effects of wind-induced vibrations. As a fluid flows around a body, vortices may be created which are shed periodically creating an oscillating force on the body. If the frequency of the shedding vortices coincides with the natural frequency of the body, large displacements of the body can be induced as was the case with the Tacoma Narrows bridge. To illustrate this type of phenomenon, consider fluid flow past a circular cylinder. Assume the frequency, n, of the shedding vortices behind the cylinder is a function of the cylinder diameter, D, the fluid velocity, V, and the fluid kinematic viscosity, ν. **(a)** Determine a suitable set of dimensionless variables for this problem. One of the dimensionless variables should be the Strouhal number, nD/V. **(b)** Some results of experiments in which the shedding frequency of the vortices (in Hz) was measured, using a particular cylinder and Newtonian, incompressible fluid, are shown in Fig. P7.7. Is this a "universal curve" that can be used to predict the shedding frequency for any cylinder placed in any fluid? Explain. **(c)** A certain structural component in the form of a 1-in.-diameter, 12-ft-long rod acts as a cantilever beam with a natural frequency of 19 Hz. Based on the data in Fig. P7.7, estimate the wind speed that may cause the rod to oscillate at its natural frequency. *Hint:* Use a trial and error solution.



■ FIGURE P7.72

(a) $$n = f(D, V, \nu)$$

$$n \doteq T^{-1} \qquad D \doteq L \qquad V \doteq LT^{-1} \qquad \nu \doteq L^2T^{-1}$$

From the pi theorem, $4 - 2 = 2$ pi terms required, and a dimensional analysis yields

$$\underline{\underline{\frac{nD}{V} = \phi\left(\frac{VD}{\nu}\right)}}$$

(b) Yes. If the variables of part (a) are correct then this is a "universal" or general relationship between the Strouhal number and the Reynolds number. It is valid over the range of Reynolds numbers covered in the experiment.

(con't)

7.72 (con't)

(c) For $n = 19\ Hz$ and $D = \frac{1\ in.}{12\ \frac{in.}{ft}} = \frac{1}{12}\ ft$

$$S_t = \frac{nD}{V} = \frac{(19\ Hz)(\frac{1}{12}\ ft)}{V} \qquad (1)$$

From Fig. P7.72, assume $S_t = 0.21$ and from Eq. (1)

$$0.21 = \frac{(19\ Hz)(\frac{1}{12}\ ft)}{V}$$

so that $\underline{\underline{V = 7.54\ \frac{ft}{s}}}$ (5.14 mph)

Check Re: $Re = \frac{VD}{\nu} = \frac{(7.54\ \frac{ft}{s})(\frac{1}{12}\ ft)}{1.57\times10^{-4}\ \frac{ft^2}{s}} = 4000$

From Fig. P7.72 at $Re = 4000$, $St = 0.21$ and therefore assumed value of S_t OK.

7.73

7.73 (See "Ice engineering," Section 7.9.3.) A model study is to be developed to determine the force exerted on bridge piers due to floating chunks of ice in a river. The piers of interest have square cross sections. Assume that the force, R, is a function the pier width, b, the depth of the ice, d, the velocity of the ice, V, the acceleration of gravity, g, the density of the ice, ρ_i, and a measure of the strength of the ice, E_i, where E_i has the dimensions FL^{-2}. (a) Based on these variables determine a suitable set of dimensionless variables for this problem. (b) The prototype conditions of interest include an ice thickness of 12 in. and an ice velocity of 6 ft/s. What model ice thickness and velocity would be required if the length scale is to be 1/10? (c) If the model and prototype ice have the same density can the model ice have the same strength properties as that of the prototype ice? Explain.

(a) $$R = f(b, d, V, g, \rho_i, E_i)$$

$R \doteq F \quad b \doteq L \quad d \doteq L \quad V \doteq LT^{-1} \quad g \doteq LT^{-2} \quad \rho_i \doteq FL^{-4}T^{2} \quad E_i \doteq FL^{-2}$

From the pi theorem, $7-3=4$ pi terms required, and a dimensional analysis yields

$$\frac{R}{E_i b^2} = \phi\left(\frac{b}{d}, \frac{V^2}{gd}, \frac{\rho_i V^2}{E_i}\right)$$

(b) For similarity,

$$\frac{b_m}{d_m} = \frac{b}{d} \quad \text{or} \quad \frac{d_m}{d} = \frac{b_m}{b} = \frac{1}{10}$$

so that

$$d_m = \frac{1}{10}(12\text{ in.}) = \underline{1.20\text{ in.}}$$

Also,

$$\frac{V_m^2}{g_m d_m} = \frac{V^2}{gd} \qquad (1)$$

and with $g_m = g$

$$V_m = \sqrt{\left(\frac{g_m}{g}\right)\left(\frac{d_m}{d}\right)}\,V = \sqrt{(1)\left(\frac{1}{10}\right)}\left(6\,\frac{\text{ft}}{\text{s}}\right) = \underline{1.90\,\frac{\text{ft}}{\text{s}}}$$

(c) For similarity,

$$\frac{\rho_{im} V_m^2}{E_{im}} = \frac{\rho V^2}{E_i}$$

Thus,

$$\frac{E_{im}}{E_i} = \left(\frac{\rho_{im}}{\rho_i}\right)\left(\frac{V_m}{V}\right)^2 = \frac{d_m}{d}$$

since $\rho_{im} = \rho_i$ and from Eq. (1) $\left(\frac{V_m}{V}\right)^2 = \frac{d_m}{d}$

Since $d_m/d = 1/10$, $E_{im} \neq E_i$ and model ice cannot have same strength properties. $\underline{No}$.

7.74

7.74 As illustrated in Video V7.9, models are commonly used to study the dispersion of a gaseous pollutant from an exhaust stack located near a building complex. Similarity requirements for the pollutant source involve the following independent variables: the stack gas speed, V, the wind speed, U, the density of the atmospheric air, ρ, the difference in densities between the air and the stack gas, $\rho - \rho_s$, the acceleration of gravity, g, the kinematic viscosity of the stack gas, ν_s, and the stack diameter, D. **(a)** Based on these variables, determine a suitable set of similarity requirements for modeling the pollutant source. **(b)** For this type of model a typical length scale might be 1:200. If the same fluids were used in model and prototype, would the similarity requirements be satisfied? Explain and support your answer with the necessary calculations.

(a) Since $V \doteq LT^{-1}$ $U \doteq LT^{-1}$ $\rho \doteq FL^{-4}T^2$ $\rho - \rho_s \doteq FL^{-4}T^2$ $g \doteq LT^{-2}$ $\nu_s \doteq L^2T^{-1}$ $D \doteq L$, it follows from the pi theorem that 7-3 = 4 pi terms are required. A dimensional analysis yields $\frac{V}{U}$, $\frac{VD}{\nu_s}$, $\frac{V^2}{gD}$, and $\frac{\rho-\rho_s}{\rho}$ as a possible set of pi terms. Thus, the similarity requirements would be:

$$\underline{\underline{\frac{V_m}{U_m} = \frac{V}{U}}} \qquad \underline{\underline{\frac{V_m D_m}{\nu_{sm}} = \frac{VD}{\nu_s}}} \qquad \underline{\underline{\frac{V_m^2}{g_m D_m} = \frac{V^2}{gD}}} \qquad \underline{\underline{\frac{(\rho-\rho_s)_m}{\rho_m} = \frac{(\rho-\rho_s)}{\rho}}}$$

(b) For $\frac{D_m}{D} = \frac{1}{200}$ and $\nu_{s_m} = \nu_s$ the second similarity requirement is

$$\frac{V_m}{V} = \frac{\nu_{sm}}{\nu_s}\frac{D}{D_m} = 200 \quad \text{(see above)}$$

However, from the third similarity requirement with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{D_m}{D}} = \sqrt{\frac{1}{200}}$$

This result conflicts with that from the second similarity requirement, and therefore the similarity requirements cannot be satisfied under the stated conditions. $\underline{\underline{\text{No}}}$.

7.75

7.75 River models are used to study many different types of flow situations. (See, for example, Video V7.12) A certain small river has an average width and depth of 60 ft and 4 ft, respectively, and carries water at a flowrate of 700 ft^3/s. A model is to be designed based on Froude number similarity so that the discharge scale is 1/250. At what depth and flowrate would the model operate?

For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Since the flowrate is $Q = VA$, where A is the appropriate cross sectional area,

$$\frac{Q_m}{Q} = \frac{V_m A_m}{V A} = \sqrt{\frac{\ell_m}{\ell}}\,\frac{A_m}{A}$$

Also,

$$\frac{A_m}{A} = \left(\frac{\ell_m}{\ell}\right)^2$$

so that

$$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2} = \frac{1}{250} \qquad (1)$$

Thus,

$$\frac{\ell_m}{\ell} = 0.110$$

and for a prototype depth of 4 ft the corresponding model depth is

$$\ell_m = (0.110)(4\,ft) = \underline{\underline{0.440\,ft}}$$

The model flowrate is obtained from Eq. (1):

$$Q_m = \left(\frac{1}{250}\right)\left(700\,\frac{ft^3}{s}\right) = \underline{\underline{2.80\,\frac{ft^3}{s}}}$$

7.76

7.76 As winds blow past buildings, complex flow patterns can develop due to various factors such as flow separation and interactions between adjacent buildings. (See Video V7.13) Assume that the local gage pressure, p, at a particular location on a building is a function of the air density, ρ, the wind speed, V, some characteristic length, ℓ, and all other pertinent lengths, ℓ_i, needed to characterize the geometry of the building or building complex. **(a)** Determine a suitable set of dimensionless parameters that can be used to study the pressure distribution. (b) An eight-story building that is 100 ft tall is to be modeled in a wind tunnel. If a length scale of 1:300 is to be used, how tall should the model building be? (c) How will a measured pressure in the model be related to the corresponding prototype pressure? Assume the same air density in model and prototype. Based on the assumed variables, does the model wind speed have to be equal to the prototype wind speed? Explain.

(a) $$p = f(\rho, V, \ell, \ell_i)$$

$$p \doteq FL^{-2} \qquad \rho \doteq FL^{-4}T^{2} \qquad V \doteq LT^{-1} \qquad \ell \doteq L \qquad \ell_i \doteq L$$

From the pi theorem, 5-3=2 pi terms required, and a dimensional analysis yields

$$\underline{\underline{\frac{p}{\rho V^2} = \phi\left(\frac{\ell}{\ell_i}\right)}}$$

(b) For geometric similarity

$$\frac{\ell_m}{\ell_{im}} = \frac{\ell}{\ell_i}$$

So that

$$\frac{\ell_m}{\ell} = \frac{\ell_{im}}{\ell_i}$$

and it follows that all pertinent lengths are scaled with the length scale ℓ_m/ℓ. Thus, with $\ell_m/\ell = 1/300$

$$\text{model height} = \frac{100\text{ ft}}{300} = \underline{\underline{0.333\text{ ft}}}$$

(c) With geometric similarity satisfied it follows that

$$\frac{p}{\rho V^2} = \frac{p_m}{\rho_m V_m^2}$$

Thus, with $\rho_m = \rho$

$$\underline{\underline{p = \left(\frac{V}{V_m}\right)^2 p_m}}$$

With the set of given variables there is no requirement for the velocity scale, V_m/V, so the model wind speed does not have to be equal tho the prototype wind speed. <u>No.</u>

7.77

7.77 Start with the two-dimensional continuity equation and the Navier–Stokes equations (Eqs. 7.35, 7.36, and 7.37) and verify the nondimensional forms of these equations (Eqs. 7.38, 7.41, and 7.42).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{(Eq. 7.35)}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{(Eq. 7.36)}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-\rho g+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad \text{(Eq. 7.37)}$$

As indicated in Section 7.10 let

$$u^*=\frac{u}{V} \qquad v^*=\frac{v}{V} \qquad p^*=\frac{p}{p_0}$$

$$x^*=\frac{x}{\ell} \qquad y^*=\frac{y}{\ell} \qquad t^*=\frac{t}{\tau}$$

The various transformations can be made as follows:

$$\frac{\partial u}{\partial x}=\frac{\partial(Vu^*)}{\partial x^*}\frac{\partial x^*}{\partial x}=\frac{V}{\ell}\frac{\partial u^*}{\partial x^*}$$

and similarly,

$$\frac{\partial v}{\partial x}=\frac{V}{\ell}\frac{\partial v^*}{\partial x^*} \qquad \frac{\partial u}{\partial y}=\frac{V}{\ell}\frac{\partial u^*}{\partial y^*} \qquad \frac{\partial v}{\partial y}=\frac{V}{\ell}\frac{\partial v^*}{\partial y^*}$$

Also,

$$\frac{\partial^2 u}{\partial x^2}=\frac{V}{\ell}\frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x}=\frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial x^{*2}}$$

and similarly,

$$\frac{\partial^2 v}{\partial x^2}=\frac{V}{\ell^2}\frac{\partial^2 v^*}{\partial x^{*2}} \qquad \frac{\partial^2 u}{\partial y^2}=\frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial y^{*2}} \qquad \frac{\partial^2 v}{\partial y^2}=\frac{V}{\ell^2}\frac{\partial^2 v^*}{\partial y^{*2}}$$

For the local acceleration,

$$\frac{\partial u}{\partial t}=\frac{\partial(Vu^*)}{\partial t^*}\frac{\partial t^*}{\partial t}=\frac{V}{\tau}\frac{\partial u^*}{\partial t^*}$$

and similarly,

$$\frac{\partial v}{\partial t}=\frac{V}{\tau}\frac{\partial v^*}{\partial t^*}$$

(con't)

7.77 (con't)

For the pressure terms,

$$\frac{\partial p}{\partial x} = \frac{\partial\, p_0 p^*}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{p_0}{\ell}\frac{\partial p^*}{\partial x^*}$$

and similarly,

$$\frac{\partial p}{\partial y} = \frac{p_0}{\ell}\frac{\partial p^*}{\partial y^*}$$

Substitution of the various terms, expressed in terms of the dimensionless variables, can be made into the original differential equations (Eqs. 7.35, 7.36, and 7.37) to yield Eqs. 7.38, 7.39, and 7.40. To obtain the final form for Eqs. 7.41 and 7.42 divide each term by $\rho V^2/\ell$.

7.78

7.78 A viscous fluid is contained between wide, parallel plates spaced a distance h apart as shown in Fig. P7.78. The upper plate is fixed, and the bottom plate oscillates harmonically with a velocity amplitude U and frequency ω. The differential equation for the velocity distribution between the plates is

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$

where u is the velocity, t is time, and ρ and μ are fluid density and viscosity, respectively. Rewrite this equation in a suitable nondimensional form using h, U, and ω as reference parameters.

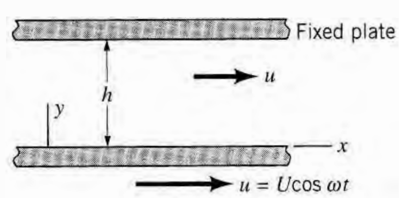


FIGURE P7.78

Let $y^* = \frac{y}{h}$, $u^* = \frac{u}{U}$, and $t^* = \omega t$ so that:

$$\frac{\partial u}{\partial t} = \frac{\partial (U u^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = U \frac{\partial u^*}{\partial t^*}(\omega) = U\omega \frac{\partial u^*}{\partial t^*}$$

$$\frac{\partial u}{\partial y} = \frac{\partial (U u^*)}{\partial y^*}\frac{\partial y^*}{\partial y} = U \frac{\partial u^*}{\partial y^*}\left(\frac{1}{h}\right) = \frac{U}{h}\frac{\partial u^*}{\partial y^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U}{h}\frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \frac{U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Thus, the original differential equation becomes

$$\rho U\omega \frac{\partial u^*}{\partial t^*} = \frac{\mu U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

or

$$\underline{\underline{\left[\frac{\rho\omega h^2}{\mu}\right]\frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}}}}$$

7.79

7.79 The deflection of the cantilever beam of Fig. P7.79 is governed by the differential equation

$$EI\frac{d^2y}{dx^2} = P(x - \ell)$$

where E is the modulus of elasticity and I is the moment of inertia of the beam cross section. The boundary conditions are $y = 0$ at $x = 0$ and $dy/dx = 0$ at $x = 0$. **(a)** Rewrite the equation and boundary conditions in dimensionless form using the beam length, ℓ, as the reference length. **(b) Based on the results of part (a) what are the similarity requirements and the prediction equation for a model to predict deflections?**

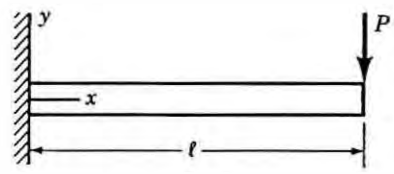


FIGURE P7.79

(a) Let $y^* = \frac{y}{\ell}$ and $x^* = \frac{x}{\ell}$ so that

$$\frac{dy}{dx} = \frac{d(\ell y^*)}{dx^*}\frac{dx^*}{dx} = \ell\,\frac{dy^*}{dx^*}\left(\frac{1}{\ell}\right) = \frac{dy^*}{dx^*}$$

and

$$\frac{d^2y}{dx^2} = \frac{d}{dx^*}\left(\frac{dy^*}{dx^*}\right)\frac{dx^*}{dx} = \frac{1}{\ell}\,\frac{d^2y^*}{dx^{*2}}$$

Thus, the original differential equation becomes

$$\left[\frac{EI}{\ell}\right]\frac{d^2y^*}{dx^{*2}} = P(\ell x^* - \ell)$$

or

$$\underline{\underline{\frac{d^2y^*}{dx^{*2}} = \left[\frac{P\ell^2}{EI}\right](x^* - 1)}}$$

and the boundary conditions are

$$\underline{\underline{y^* = 0 \text{ at } x^* = 0}} \quad \text{and} \quad \underline{\underline{\frac{dy^*}{dx^*} = 0 \text{ at } x^* = 0.}}$$

(b) The similarity requirements are

$$x_m^* = x^* \quad \text{or} \quad \underline{\underline{\frac{x_m}{\ell_m} = \frac{x}{\ell}}} \quad \text{and} \quad \underline{\underline{\frac{P_m \ell_m^2}{E_m I_m} = \frac{P\ell^2}{EI}}}$$

The prediction equation is

$$y^* = y_m^*$$

or

$$\underline{\underline{\frac{y}{\ell} = \frac{y_m}{\ell_m}}}$$

7.80

7.80 A liquid is contained in a pipe that is closed at one end as shown in Fig. P7.80. Initially the liquid is at rest, but if the end is suddenly opened the liquid starts to move. Assume the pressure p_1 remains constant. The differential equation that describes the resulting motion of the liquid is

$$\rho \frac{\partial v_z}{\partial t} = \frac{p_1}{\ell} + \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right)$$

where v_z is the velocity at any radial location, r, and t is time. Rewrite this equation in dimensionless form using the liquid density, ρ, the viscosity, μ, and the pipe radius, R, as reference parameters.

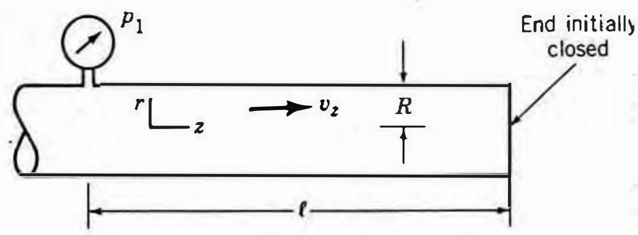


FIGURE P7.80

Let $r^* = \frac{r}{R}$, $t^* = \frac{t}{\tau}$, and $v_z^* = \frac{v_z}{V}$ where τ is some combinations of the parameters ρ, μ, and R having the dimensions of time, and V is some combination of the same parameters having the dimensions of a velocity. Let

$$\tau = \frac{\rho R^2}{\mu} \doteq \frac{(FL^{-4}T^2)(L)^2}{FL^{-2}T} \doteq T$$

and

$$V = \frac{\mu}{\rho R} \doteq \frac{FL^{-2}T}{(FL^{-4}T^2)(L)} \doteq LT^{-1}$$

With these dimensionless variables:

$$\frac{\partial v_z}{\partial t} = \frac{\partial (V v_z^*)}{\partial t^*} \frac{\partial t^*}{\partial t} = V \frac{\partial v_z^*}{\partial t^*}\left(\frac{1}{\tau}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{\mu}{\rho R^2}\right)\frac{\partial v_z^*}{\partial t^*} = \left(\frac{\mu}{\rho}\right)^2 \frac{1}{R^3} \frac{\partial v_z^*}{\partial t^*}$$

$$\frac{\partial v_z}{\partial r} = \frac{\partial (V v_z^*)}{\partial r^*} \frac{\partial r^*}{\partial r} = V \frac{\partial v_z^*}{\partial r^*}\left(\frac{1}{R}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{1}{R}\right)\frac{\partial v_z^*}{\partial r^*} = \frac{\mu}{\rho R^2} \frac{\partial v_z^*}{\partial r^*}$$

$$\frac{\partial^2 v_z}{\partial r^2} = \frac{\mu}{\rho R^2} \frac{\partial}{\partial r^*}\left(\frac{\partial v_z^*}{\partial r^*}\right)\frac{\partial r^*}{\partial r} = \frac{\mu}{\rho R^2} \frac{\partial^2 v_z^*}{\partial r^{*2}}\left(\frac{1}{R}\right) = \frac{\mu}{\rho R^3} \frac{\partial^2 v_z^*}{\partial r^{*2}}$$

The original differential equation can now be expressed as

$$\left[\rho \left(\frac{\mu}{\rho}\right)^2 \frac{1}{R^3}\right] \frac{\partial v_z^*}{\partial t^*} = \frac{p_1}{\ell} + \left[\mu \left(\frac{\mu}{\rho R^3}\right)\right]\left(\frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_z^*}{\partial r^*}\right)$$

or

$$\frac{\partial v_z^*}{\partial t^*} = \frac{p_1 \rho R^3}{\ell \mu^2} + \frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_z^*}{\partial r^*}$$

7.81

7.81 An incompressible fluid is contained between two infinite parallel plates as illustrated in Fig. P7.81. Under the influence of a harmonically varying pressure gradient in the x direction, the fluid oscillates harmonically with a frequency ω. The differential equation describing the fluid motion is

$$\rho \frac{\partial u}{\partial t} = X \cos \omega t + \mu \frac{\partial^2 u}{\partial y^2}$$

where X is the amplitude of the pressure gradient. Express this equation in nondimensional form using h and ω as reference parameters.

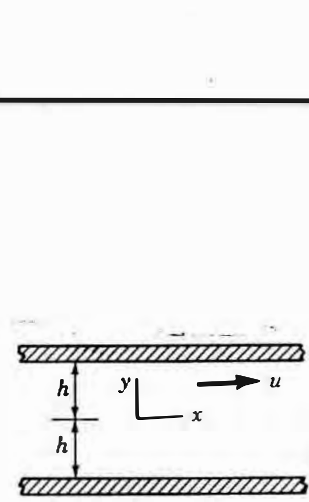


FIGURE P7.81

Let $y^* = \frac{y}{h}$, $t^* = \omega t$, and $u^* = \frac{u}{h\omega}$ so that:

$$\frac{\partial u}{\partial t} = \frac{\partial (h\omega u^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = h\omega \frac{\partial u^*}{\partial t^*}(\omega) = h\omega^2 \frac{\partial u^*}{\partial t^*}$$

$$\frac{\partial u}{\partial y} = \frac{\partial (h\omega u^*)}{\partial y^*}\frac{\partial y^*}{\partial y} = h\omega \frac{\partial u^*}{\partial y^*}\left(\frac{1}{h}\right) = \omega \frac{\partial u^*}{\partial y^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \omega \frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \omega \frac{\partial^2 u^*}{\partial y^{*2}}\left(\frac{1}{h}\right) = \frac{\omega}{h}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The original differential equation can now be expressed as

$$\left[\rho h \omega^2\right] \frac{\partial u^*}{\partial t^*} = X \cos t^* + \left[\frac{\mu\omega}{h}\right]\frac{\partial^2 u^*}{\partial y^{*2}}$$

or

$$\underline{\underline{\frac{\partial u^*}{\partial t^*} = \left[\frac{X}{\rho h \omega^2}\right]\cos t^* + \left[\frac{\mu}{\rho h^2 \omega}\right]\frac{\partial^2 u^*}{\partial y^{*2}}}}$$

7.82

7.82 Flow from a Tank

Objective: When the drain hole in the bottom of the tank shown in Fig. P7.82 is opened, the liquid will drain out at a rate which is a function of many parameters. The purpose of this experiment is to measure the liquid depth, h, as a function of time, t, for two geometrically similar tanks and to learn how dimensional analysis can be of use in situations such as this.

Equipment: Two geometrically similar cylindrical tanks; stop watch; thermometer; ruler.

Experimental Procedure: Make appropriate measurements to show that the two tanks are geometrically similar. That is, show that the large tank is twice the size of the small tank (twice the height; twice the diameter; twice the hole diameter in the bottom). Fill the large tank with cold water of a known temperature, T, and determine the water depth, h, in the tank as a function of time, t, after the drain hole is opened. Thus, obtain $h = h(t)$. Note that t ranges from $t = 0$ when $h = H$ (where H is the initial depth of the water), to $t = t_{\text{final}}$ then the tank is completely drained ($h = 0$). Repeat the measurements using the small tank with the same temperature water. To ensure geometric similarity, the initial water level in the small tank must be one-half of what it was in the large tank. Repeat the experiment for each tank with hot water. Thus you will have a total of four sets of $h(t)$ data.

Calculations: Assume that the depth, h, of water in the tank is a function of its initial depth, H, the diameter of the tank, D, the diameter of the drain hole in the bottom of the tank, d, the time, t, after the drain is opened, the acceleration of gravity, g, and the fluid density, ρ, and viscosity, μ. Develop a suitable set of dimensionless parameters for this problem using H, g, and ρ as repeating variables. Use t as the dependent parameter. For each of the four conditions tested, calculate the dimensionless time, $tg^{1/2}/H^{1/2}$, as a function of the dimensionless depth, h/H.

Graph: On a single graph, plot the depth, h, as ordinates and time, t, as abscissas for each of the four sets of data.

Results: On another graph, plot the dimensionless water depth, h/H, as a function of dimensionless time, $tg^{1/2}/H^{1/2}$, for each of the four sets of data. Based on your results, comment on the importance of density and viscosity for your experiment and on the usefulness of dimensional analysis.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

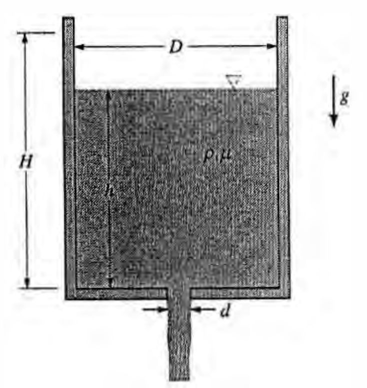


FIGURE P7.82

(con't)

7.82 (Con't)

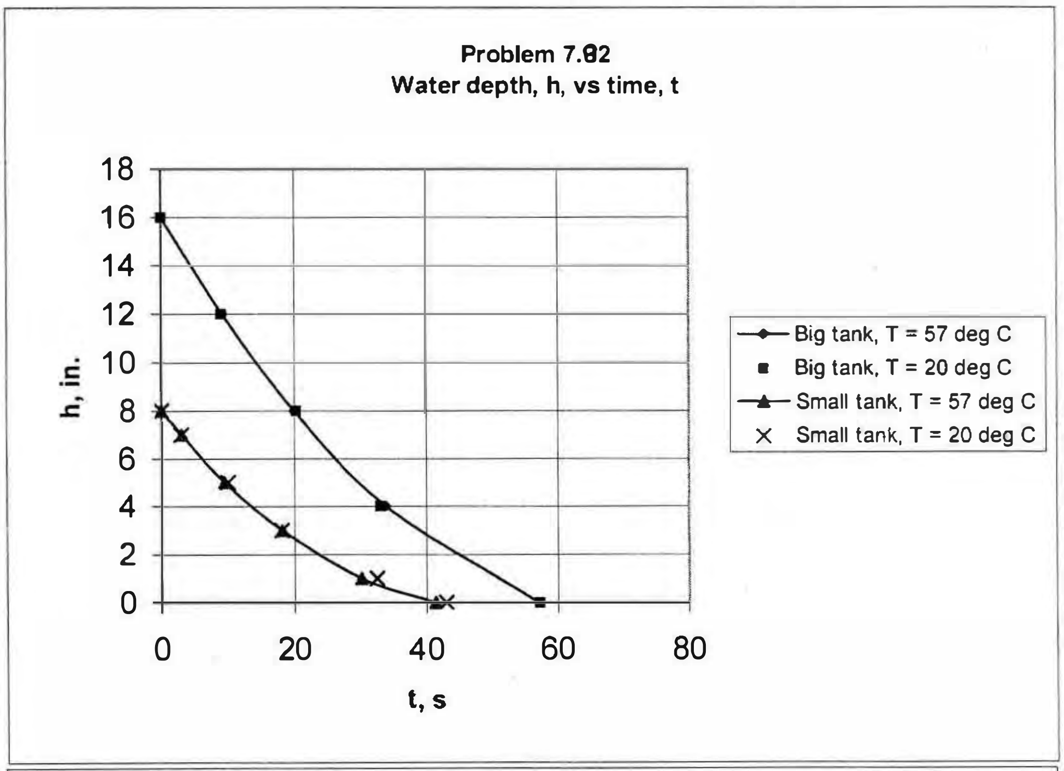
Solution for Problem 7.82: Flow from a Tank

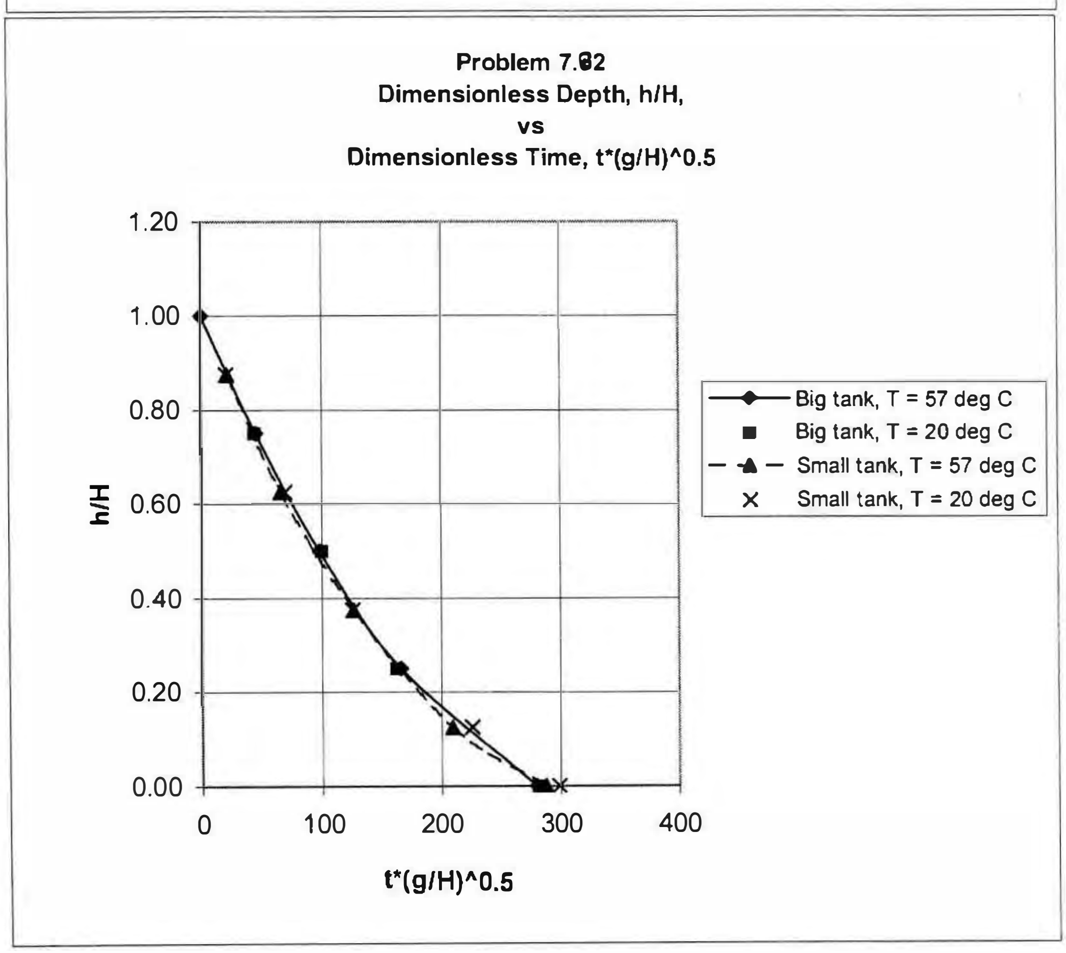
H for big tank, in.	H for small tank, in.
16.0	8.0

	h, in.	t, s	$tg^{1/2}/H^{1/2}$	h/H
Big Tank with T = 57 deg C				
	16.0	0.0	0.0	1.000
	12.0	9.2	45.2	0.750
	8.0	20.0	98.3	0.500
	4.0	33.8	166.1	0.250
	0.0	57.0	280.1	0.000
Big Tank with T = 20 deg C				
	16.0	0.0	0.0	1.000
	12.0	9.0	44.2	0.750
	8.0	20.3	99.8	0.500
	4.0	33.0	162.2	0.250
	0.0	57.2	281.1	0.000
Small Tank with T = 57 deg C				
	8.0	0.0	0.0	1.000
	7.0	3.1	21.5	0.875
	5.0	9.5	66.0	0.625
	3.0	18.2	126.5	0.375
	1.0	30.1	209.2	0.125
	0.0	41.4	287.7	0.000
Small Tank with T = 20 deg C				
	8.0	0.0	0.0	1.000
	7.0	3.0	20.8	0.875
	5.0	10.0	69.5	0.625
	3.0	18.1	125.8	0.375
	1.0	32.5	225.9	0.125
	0.0	43.0	298.8	0.000

(Con't)

7.82 (con't)


Problem 7.82
Water depth, h, vs time, t
h, in.
t, s
Big tank, T = 57 deg C
Big tank, T = 20 deg C
Small tank, T = 57 deg C
Small tank, T = 20 deg C


Problem 7.82
Dimensionless Depth, h/H,
vs
Dimensionless Time, t*(g/H)^0.5
h/H
t*(g/H)^0.5
Big tank, T = 57 deg C
Big tank, T = 20 deg C
Small tank, T = 57 deg C
Small tank, T = 20 deg C

7.83

7.83 Vortex Shedding from a Circular Cylinder

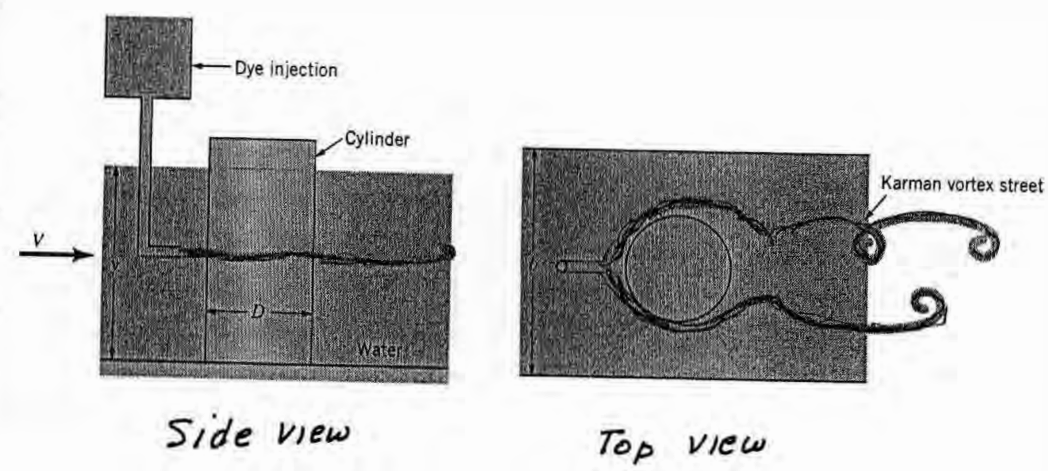
Objective: Under certain conditions, the flow of fluid past a circular cylinder will produce a Karman vortex street behind the cylinder. As shown in Fig. P7.83, this vortex street consists of a set of vortices (swirls) that are shed alternately from opposite sides of the cylinder and then swept downstream with the fluid. The purpose of this experiment is to determine the shedding frequency, ω cycles (vortices) per second, of these vortices as a function of the Reynolds number, Re, and to compare the measured results with published data.

Equipment: Water channel with an adjustable flowrate; flow meter; set of four different diameter cylinders; dye injection system; stopwatch.

Experimental Procedure: Insert a cylinder of diameter D into the holder on the bottom of the water channel. Adjust the control valve and the downstream gate on the channel to produce the desired flowrate, Q, and velocity, V. Make sure that the flow-straightening screens (not shown in the figure) are in place to reduce unwanted turbulence in the flowing water. Measure the width, b, of the channel and the depth, y, of the water in the channel so that the water velocity in the channel, $V = Q/(by)$, can be determined. Carefully adjust the control valve on the dye injection system to inject a thin stream of dye slightly upstream of the cylinder. By viewing down onto the top of the water channel, observe the vortex shedding and measure the time, t, that it takes for N vortices to be shed from the cylinder. For a given velocity, repeat the experiment for different diameter cylinders. Repeat the experiment using different velocities. Measure the water temperature so that the viscosity can be looked up in Table B.1.

Calculations: For each of your data sets calculate the vortex shedding frequency, $\omega = N/t$, which is expressed as vortices (or cycles) per second. Also calculate the dimensionless frequency called the Strouhl number, $\text{St} = \omega D/V$, and the Reynolds number, $\text{Re} = \rho VD/\mu$.

Graph: On a single graph, plot the vortex shedding frequency, ω, as ordinates and the water velocity, V, as abscissas for each of the four cylinders you tested. On another graph, plot the Strouhl number as ordinates and the Reynolds number as abscissas for each of the four sets of data.



■ FIGURE P7.83

(con't)

7.83 (con't)

Results: On your Strouhl number verses Reynolds number graph, plot the results taken from the literature and shown in the following table.

St	Re
0	<50
0.16	100
0.18	150
0.19	200
0.20	300
0.21	400
0.21	600
0.21	800

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

Solution for Problem 7.83: Vortex Shedding from a Circular Cylinder

T, deg F	b, ft
70	0.50

Q, ft^3/s	y, ft	D, ft	N	t, s	ω, cycles/s	V, ft/s	Re	St	Data from Literature Re	St
0.036	0.82	0.0202	10.0	13.2	0.758	0.0878	169	0.174	50	0.00
0.036	0.82	0.0314	10.0	19.9	0.503	0.0878	263	0.180	100	0.16
0.036	0.82	0.0421	10.0	24.5	0.408	0.0878	352	0.196	150	0.18
0.036	0.82	0.0518	10.0	30.1	0.332	0.0878	433	0.196	200	0.19
									300	0.20
									400	0.21
0.062	0.79	0.0202	10.0	6.3	1.587	0.1570	302	0.204	600	0.21
0.062	0.79	0.0314	10.0	9.6	1.042	0.1570	469	0.208	800	0.21
0.062	0.79	0.0421	10.0	12.5	0.800	0.1570	629	0.215		
0.062	0.79	0.0518	10.0	15.1	0.662	0.1570	774	0.219		
0.029	0.86	0.0202	10.0	19.2	0.521	0.0674	130	0.156		
0.029	0.86	0.0314	10.0	28.2	0.355	0.0674	202	0.165		
0.029	0.86	0.0421	10.0	33.1	0.302	0.0674	270	0.189		
0.029	0.86	0.0518	10.0	36.7	0.272	0.0674	333	0.209		
0.018	0.92	0.0202	10.0	31.2	0.321	0.0391	75	0.165		
0.018	0.92	0.0314	10.0	41.3	0.242	0.0391	117	0.194		
0.018	0.92	0.0421	10.0	52.2	0.192	0.0391	157	0.206		
0.018	0.92	0.0518	10.0	65.3	0.153	0.0391	193	0.203		

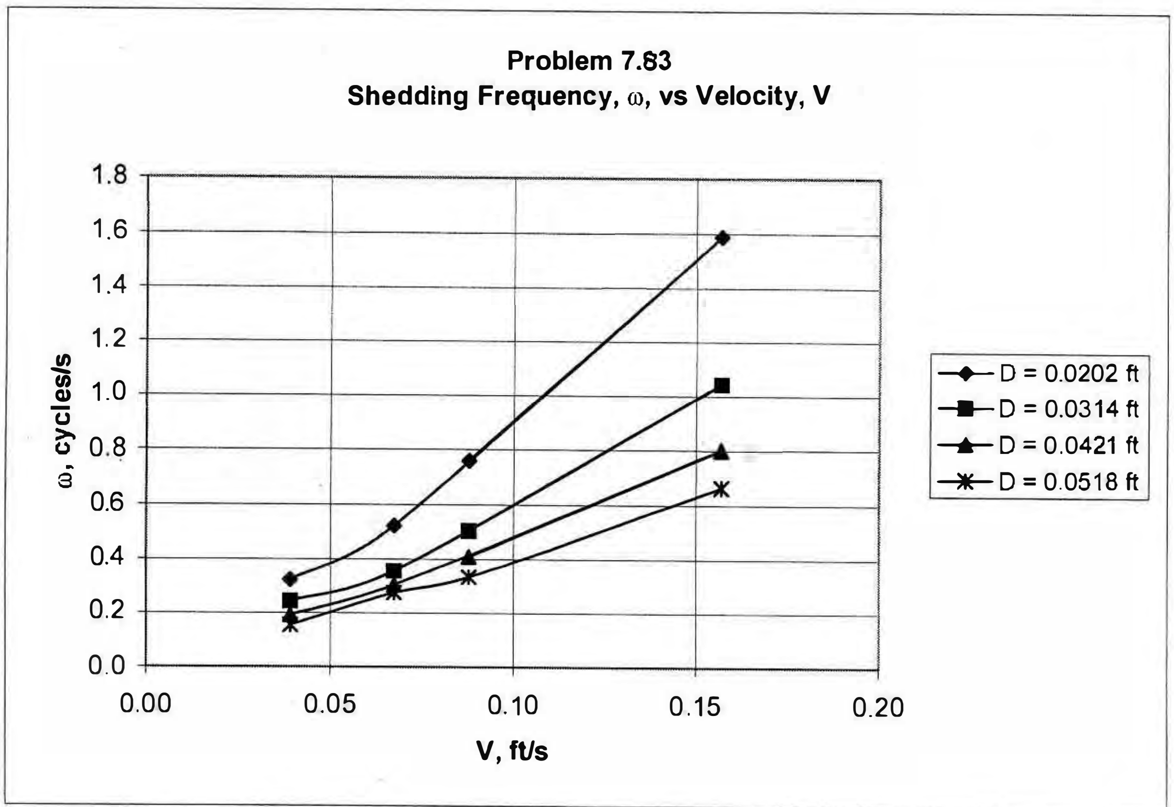
ω = N/t
V = Q/(by)

St = ωD/V and Re = DV/ν, where

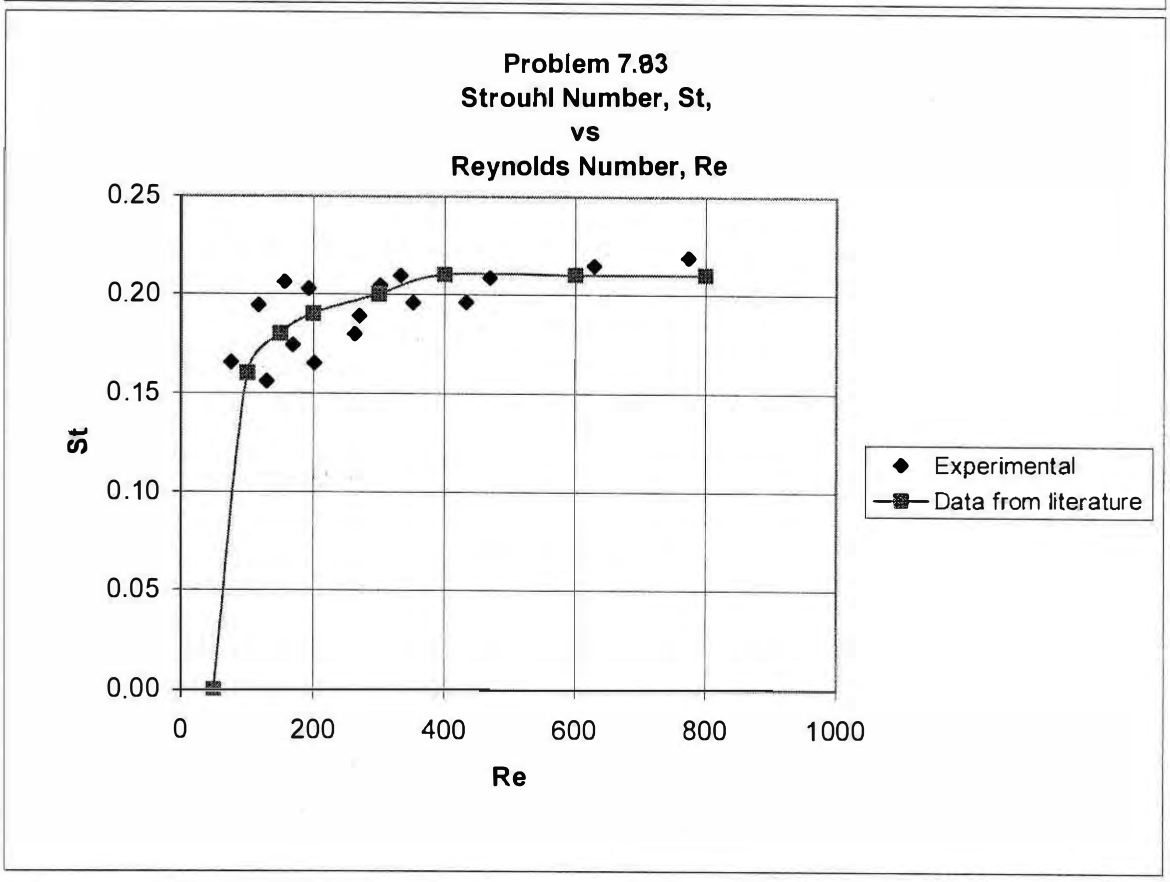
ν = 1.052E-5 ft^2/s

(con't)

7.83 (con't)



Problem 7.83
Shedding Frequency, ω, vs Velocity, V
ω, cycles/s
V, ft/s
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
0.20
D = 0.0202 ft
D = 0.0314 ft
D = 0.0421 ft
D = 0.0518 ft



Problem 7.83
Strouhl Number, St,
vs
Reynolds Number, Re
St
Re
0.25
0.20
0.15
0.10
0.05
0.00
0
200
400
600
800
1000
Experimental
Data from literature

7.84

7.84 Head Loss across a Valve

Objective: A valve in a pipeline like that shown in Fig. P7.84 acts like a variable resistor in an electrical circuit. The amount of resistance or head loss across a valve depends on the amount that the valve is open. The purpose of this experiment is to determine the head loss characteristics of a valve by measuring the pressure drop, Δp, across the valve as a function of flowrate, Q, and to learn how dimensional analysis can be of use in situations such as this.

Equipment: Air supply with flow meter; valve connected to a pipe; manometer connected to a static pressure tap upstream of the valve; barometer; thermometer.

Experimental Procedure: Measure the pipe diameter, D. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Completely close the valve and then open it N turns from its closed position. Adjust the air supply to provide the desired flowrate, Q, of air through the valve. Record the manometer reading, h, so that the pressure drop, Δp, across the valve can be determined. Repeat the measurements for various flowrates. Repeat the experiment for various valve settings, N, ranging from barely open to wide open.

Calculations: For each data set calculate the average velocity in the pipe, $V = Q/A$, where $A = \pi D^2/4$ is the pipe area. Also calculate the pressure drop across the valve, $\Delta p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. For each data set also calculate the loss coefficient, K_L, where the head loss is given by $h_L = \Delta p/\gamma = K_L V^2/2g$ and γ is the specific weight of the flowing air.

Graph: On a single graph, plot the pressure drop, Δp, as ordinates and the flowrate, Q, as abscissas for each of the valve settings, N, tested.

Results: On another graph, plot the loss coefficient, K_L, as a function of valve setting, N, for all of the data sets.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

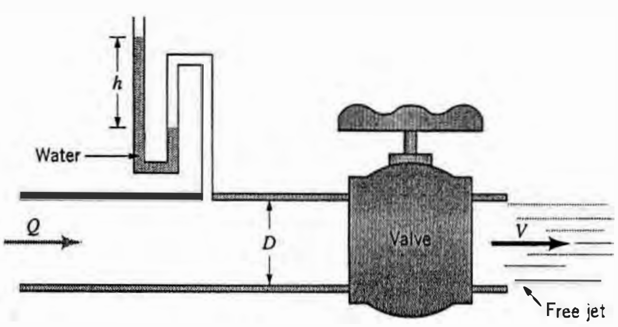


FIGURE P7.84

(con't)

7.84 (con't)

Solution for Problem 7.84: Head Loss across a Valve

D, in.	H_{atm}, in. Hg	T, deg F
0.81	28.7	70

h, in.	Q, ft^3/s	Δp, lb/ft^2	V, ft/s	N	K_L
N = 2 Turns Open Data					
9.20	0.235	47.8	65.7	2	9.95
6.50	0.195	33.8	54.5	2	10.21
5.04	0.169	26.2	47.2	2	10.54
N = 3 Turns Open Data					
9.40	0.479	48.9	133.9	3	2.45
6.33	0.386	32.9	107.9	3	2.54
5.01	0.341	26.1	95.3	3	2.57
3.62	0.289	18.8	80.8	3	2.59
1.92	0.214	10.0	59.8	3	2.50
N = 4 Turns Open Data					
9.35	0.827	48.6	231.1	4	0.816
7.65	0.767	39.8	214.3	4	0.777
6.01	0.691	31.3	193.1	4	0.752
4.32	0.578	22.5	161.5	4	0.772
3.24	0.504	16.8	140.8	4	0.762
2.62	0.456	13.6	127.4	4	0.752
1.85	0.391	9.6	109.3	4	0.723
0.98	0.283	5.1	79.1	4	0.731
N = 5 Turns Open Data					
3.03	0.897	15.8	250.7	5	0.225
2.37	0.799	12.3	223.3	5	0.222
1.79	0.701	9.3	195.9	5	0.218
1.39	0.618	7.2	172.7	5	0.217
0.97	0.517	5.0	144.5	5	0.217
0.64	0.426	3.3	119.0	5	0.211

$\Delta p = \gamma_{H2O}*h$

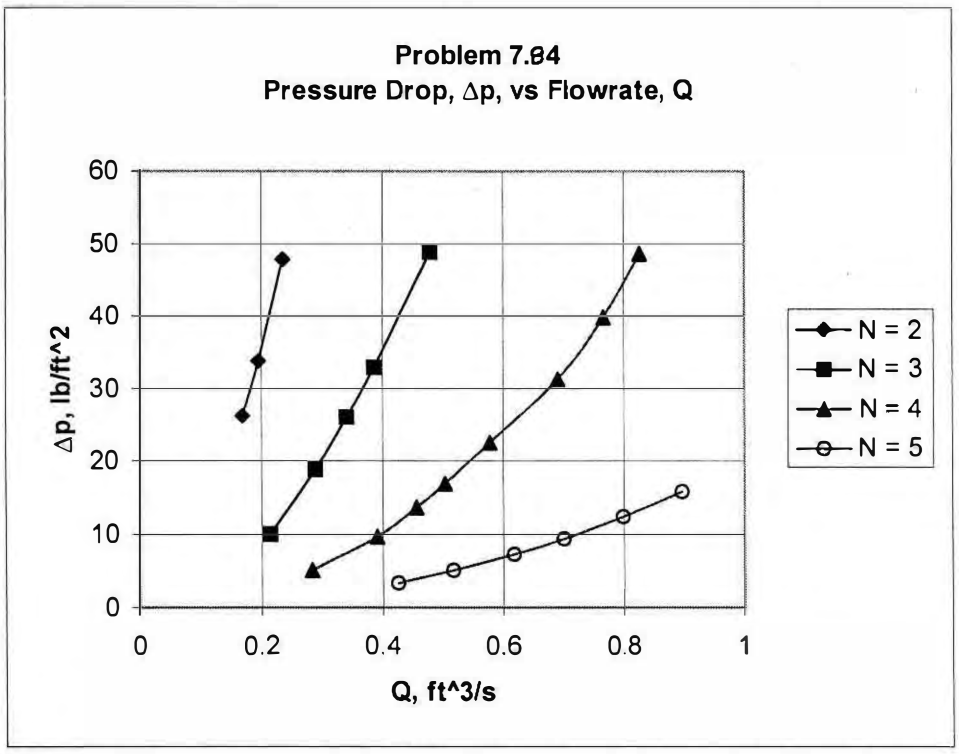
$K_L = \Delta p/(\rho V^2/2)$ where
$V = Q/A = Q/(\pi*D^2/4)$
and
$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(28.7/12 ft) = 2026 lb/ft^2
R = 1716 ft lb/slug deg R
T = 70 + 460 = 530 deg R

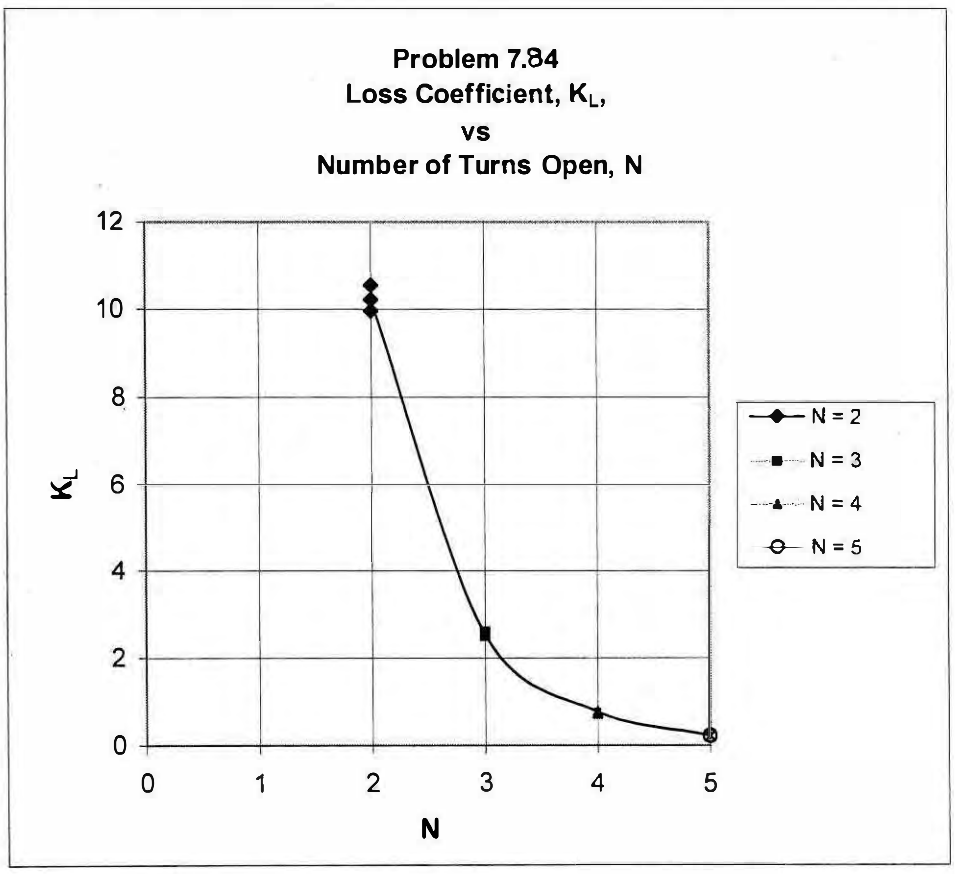
Thus, ρ = 0.00223 slug/ft^3

(con't)

7.84 (con't)



Problem 7.84
Pressure Drop, Δp, vs Flowrate, Q
Δp, lb/ft^2
Q, ft^3/s
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1
N = 2
N = 3
N = 4
N = 5



Problem 7.84
Loss Coefficient, K_L,
vs
Number of Turns Open, N
K_L
N
12
10
8
6
4
2
0
0
1
2
3
4
5
N = 2
N = 3
N = 4
N = 5

7.85

7.85 Calibration of a Rotameter

Objective: The flowrate, Q, through a rotameter can be determined from the scale reading, SR, which indicates the vertical position of the float within the tapered tube of the rotameter as shown in Fig. P7.85. Clearly, for a given scale reading, the flowrate depends on the density of the flowing fluid. The purpose of this experiment is to calibrate a rotameter so that it can be used for both water and air.

Equipment: Rotameter, air supply with a calibrated flow meter, water supply, weighing scale, stop watch, thermometer, barometer.

Experimental Procedure: Connect the rotameter to the water supply and adjust the flowrate, Q, to the desired value. Record the scale reading, SR, on the rotameter and measure the flowrate by collecting a given weight, W, of water that passes through the rotameter in a given time, t. Repeat for several flow rates.

Connect the rotameter to the air supply and adjust the flowrate to the desired value as indicated by the flow meter. Record the scale reading on the rotameter. Repeat for several flowrates. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For the water portion of the experiment, use the weight, W, and time, t, data to determine the volumetric flowrate, $Q = W/\gamma t$. The equilibrium position of the float is a result of a balance between the fluid drag force on the float, the weight of the float, and the buoyant force on the float. Thus, a typical dimensionless flowrate can be written as $Q/[d(\rho/Vg(\rho_f - \rho))^{1/2}]$, where d is the diameter of the float, V is the volume of the float, g is the acceleration of gravity, ρ is the fluid density, and ρ_f is the float density. Determine this dimensionless flowrate for each condition tested.

Graph: On a single graph, plot the flowrate, Q, as ordinates and scale reading, SR, as abscissas for both the water and air data.

Results: On another graph, plot the dimensionless flowrate as a function of scale reading for both the water and air data. Note that the scale reading is a percent of full scale and, hence, is a dimensionless quantity. Based on your results, comment on the usefulness of dimensional analysis.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

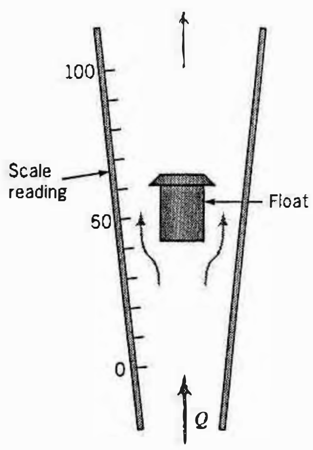


FIGURE P7.85

(con't)

7.85 (con't)

Solution for Problem 7.85: Calibration of a Rotameter

d, in.	**V**, in.^3	ρ_f, slug/ft^3	H_{atm}, in.	T, deg F
1.40	1.50	15.1	29.05	78

Air Flow Data

SR	Q, ft^3/s	$(Q/d)[\rho/(\mathbf{V}g(\rho_f-\rho))]1/2$
14.6	0.229	0.142
21.5	0.321	0.200
28.1	0.413	0.257
33.6	0.491	0.305
39.2	0.564	0.351
44.8	0.644	0.400
50.2	0.714	0.444
55.9	0.798	0.496
63.1	0.888	0.552
68.6	0.973	0.605
73.5	1.05	0.653
76.2	1.08	0.671

Water Flow Data

SR	W, lb	t, s	Q, ft^3/s	$(Q/d)[\rho/(\mathbf{V}g(\rho_f-\rho))]1/2$
13.1	6.52	19.9	0.0053	0.103
18.5	8.01	17.7	0.0073	0.143
24.2	7.02	10.4	0.0108	0.213
28.2	7.81	10.1	0.0124	0.244
37.1	8.20	8.4	0.0156	0.308
45.7	9.21	7.5	0.0197	0.387
52.6	8.19	5.7	0.0230	0.453

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.05/12 ft) = 2050 lb/ft^2

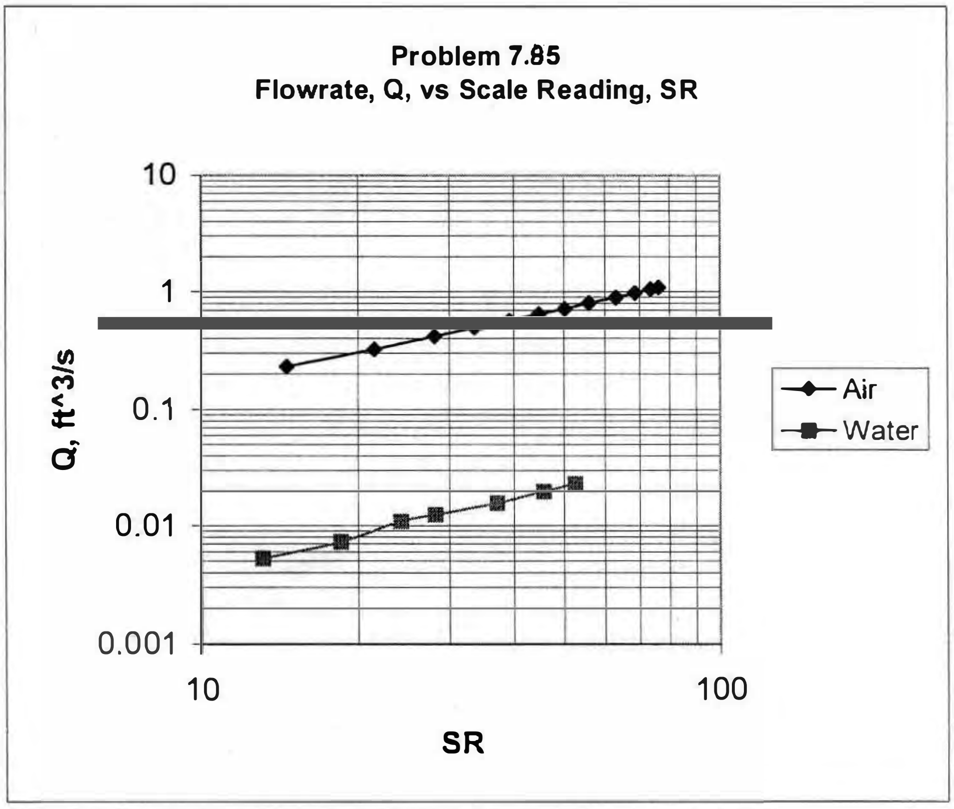
R = 1716 ft lb/slug deg R

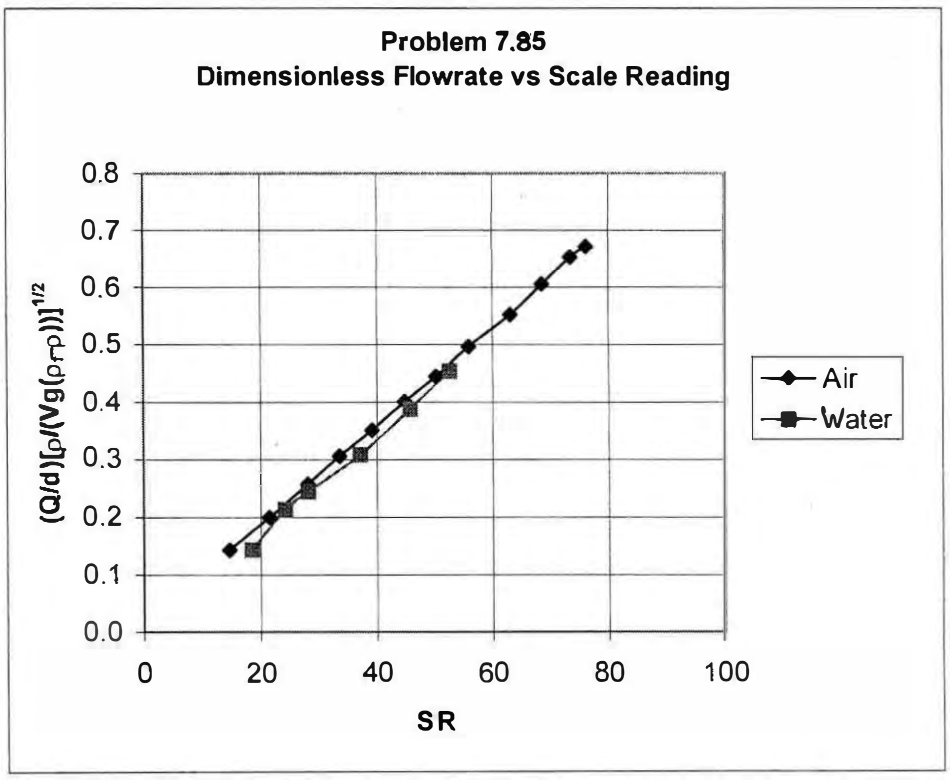
T = 78 + 460 = 538 deg R

Thus, ρ = 0.00222 slug/ft^3

(con't)

7.85 (con't)


Problem 7.85
Flowrate, Q, vs Scale Reading, SR
10
1
0.1
0.01
0.001
10
100
Q, ft^3/s
SR
Air
Water


Problem 7.85
Dimensionless Flowrate vs Scale Reading
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
20
40
60
80
100
(Q/d)[ρ/(Vg(ρf-ρ))]^1/2
SR
Air
Water

8.2

8.2 Water flows through a 50-ft pipe with a 0.5-in. diameter at 5 gal/min. What fraction of this pipe can be considered an entrance region?

Based on Tables 1.3 & 1.4

$$5\ \text{gal/min} = 1.11\times10^{-2}\ \text{ft}^3/\text{s}$$

Determine Re

$$V = Q/A = \frac{1.11\times10^{-2}}{\frac{\pi}{4}\left(\frac{0.5}{12}\right)^2} = 8.17\ \text{ft/s}$$

$$Re = \frac{VD}{\nu} = \frac{(8.17)(0.5/12)}{1.21\times10^{-5}} = 2.81\times10^4$$

For turbulent flow

$$\frac{l_e}{D} = 4.4(Re)^{1/6}$$

$$l_e = (0.5/12)\,4.4\,(2.81\times10^4)^{1/6}$$

$$l_e = \underline{\underline{1.0\ \text{ft}}}$$

8.3

8.3 Rainwater runoff from a parking lot flows through a 3-ft-diameter pipe, completely filling it. Whether flow in a pipe is laminar or turbulent depends on the value of the Reynolds number. (See Video V8.2) Would you expect the flow to be laminar or turbulent? Support your answer with appropriate calculations.

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$ If $Re > 4000$ the flow is turbulent. The corresponding velocity is

$$V = \frac{Re\,\nu}{D} = \frac{(4000)(1.21 \times 10^{-5} \frac{ft^2}{s})}{3\,ft} = 0.0161 \frac{ft}{s}$$

Most likely the velocity will be greater than this, i.e., turbulent flow.

8.4

8.4 Blue and yellow streams of paint at 60 °F (each with a density of 1.6 slugs/ft^3 and a viscosity 1000 times greater than water) enter a pipe with an average velocity of 4 ft/s as shown in Fig. P8.4. Would you expect the paint to exit the pipe as green paint or separate streams of blue and yellow paint? Explain. Repeat the problem if the paint were "thinned" so that it is only 10 times more viscous than water. Assume the density remains the same.

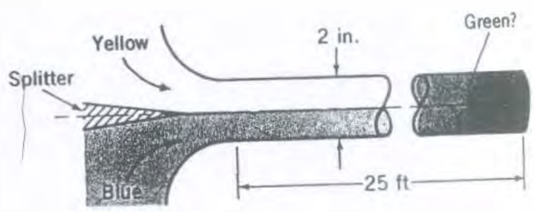


FIGURE P8.4

If the flow is laminar the paint would exit as separate blue and yellow streams.

$$Re = \frac{\rho V D}{\mu} = \frac{\rho V D}{1000\,\mu_{H_2O}} = \frac{1.6\,\frac{slugs}{ft^3}\left(4\,\frac{ft}{s}\right)\left(\frac{2}{12}\,ft\right)}{1000\left(2.34\times10^{-5}\,\frac{lb\cdot s}{ft^2}\right)} = 45.6 < 2100$$

Thus, laminar flow so blue and yellow streams.

If use $\mu = 10\,\mu_{H_2O}$ obtain

$Re = 4560 > 4000$ so have turbulent flow with natural mixing and green paint.

Note: Check to determine if the 25 ft length is greater than the entrance length, ℓ_e.

For laminar flow $\frac{\ell_e}{D} = 0.06\,Re$, or $\ell_e = 0.06(45.6)\left(\frac{2}{12}\,ft\right) = 0.456\,ft < 25\,ft$

For turbulent flow $\frac{\ell_e}{D} = 4.4\,Re^{1/6}$, or $\ell_e = 4.4(4560)^{\frac{1}{6}}\left(\frac{2}{12}\,ft\right) = 2.99\,ft < 25\,ft$

8.5

8.5 Air at 200 °F flows at standard atmospheric pressure in a pipe at a rate of 0.08 lb/s. Determine the minimum diameter allowed if the flow is to be laminar.

Maximum $Re = \frac{\rho V D}{\mu}$ for laminar flow is $Re = 2100$.

or with

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}, \quad Re = \frac{\rho\left(\frac{4Q}{\pi D^2}\right)D}{\mu} = \frac{4\rho Q}{\pi \mu D} = 2100$$

Hence,

$$Q = \frac{2100 \pi \mu D}{4\rho} \qquad (1)$$

Given $\gamma Q = 0.08 \frac{lb}{s}$, where $\gamma = g\rho$ and $\rho = \frac{p}{RT}$

Thus,

$$\rho = \frac{(14.7 \times 144 \frac{lb}{ft^2})}{(1716 \frac{ft \cdot lb}{slug \cdot °R})(460+200)°R} = 0.00187 \frac{slugs}{ft^3}$$

so that

$$Q = \frac{0.08 \frac{lb}{s}}{(32.2 \frac{ft}{s^2})(0.00187 \frac{slugs}{ft^3})} = 1.33 \frac{ft^3}{s}$$

Hence, with $\mu = 4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2}$ (see Table B.3), Eq. (1) gives

$$D = \frac{4\rho Q}{2100 \pi \mu} = \frac{4(0.00187 \frac{slugs}{ft^3})(1.33 \frac{ft^3}{s})}{2100\pi(4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2})} = \underline{\underline{3.36 \, ft}}$$

8.6

8.6 To cool a given room it is necessary to supply 4 ft^3/s of air through an 8-in.-diameter pipe. Approximately how long is the entrance length in this pipe?

$$V = \frac{Q}{A} = \frac{4\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{8}{12}\,ft\right)^2} = 11.5\,\frac{ft}{s}$$ Thus, with $\nu = 1.57 \times 10^{-4}\,\frac{ft^2}{s}$ (see Table 1.6)

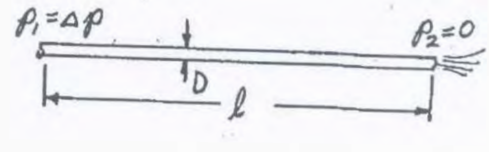
$$Re = \frac{VD}{\nu} = \frac{11.5\,\frac{ft}{s}\left(\frac{8}{12}\,ft\right)}{1.57 \times 10^{-4}\,\frac{ft^2}{s}} = 48{,}800 > 4000$$ so the flow is turbulent.

Hence,

$$\frac{\ell_e}{D} = 4.4\,Re^{1/6}, \text{ or } \ell_e = 4.4\,(48{,}800)^{1/6}\left(\frac{8}{12}\right) = \underline{\underline{17.7\,ft}}$$

8.7

8.7 A long small-diameter tube is to be used as a viscometer by measuring the flowrate through the tube as a function of the pressure drop along the tube. The calibration constant, $K = Q/\Delta p$, is calculated by assuming the flow is laminar. For tubes of diameter 0.5, 1.0, and 2.0 mm, determine the maximum flowrate allowed (in cm^3/s) if the fluid is (a) 20 °C water, or (b) standard air.



$Re = \frac{VD}{\nu}$ where $Q = VA = \frac{\pi}{4}D^2 V$

Thus,

$$Re = \frac{4QD}{\pi D^2 \nu} = \frac{4Q}{\pi D \nu}, \text{ or } Q = \frac{\pi D \nu Re}{4}$$

Maximum Q occurs with maximum Re for laminar flow: $Re = 2100$

Thus, $Q_{max} = 1650\,\nu D$

a) For 20°C water $\nu = 1.004 \times 10^{-6} \frac{m^2}{s}$

Hence, $Q_{max} = 1650\left(1.004 \times 10^{-6} \frac{m^2}{s}\right) D = 1.66 \times 10^{-3} D \frac{m^3}{s}$ with $D \sim m$

b) For standard air $\nu = 1.46 \times 10^{-5} \frac{m^2}{s}$

Hence, $Q_{max} = 1650\left(1.46 \times 10^{-5} \frac{m^2}{s}\right) D = 2.41 \times 10^{-2} D \frac{m^3}{s}$ with $D \sim m$

Thus, the following values are obtained:

	D, m	Q_{max}, $\frac{m^3}{s}$	Q_{max}, $\frac{cm^3}{s}$
(a) water	0.0005	8.30×10^{-7}	0.83
	0.0010	1.66×10^{-6}	1.66
	0.0020	3.32×10^{-6}	3.32
(b) air	0.0005	1.21×10^{-5}	12.1
	0.001	2.41×10^{-5}	24.1
	0.002	4.82×10^{-5}	48.2

Note: $1\,cm^3 = 10^{-6} m^3$

8.8

8.8 Carbon dioxide at 20 °C and a pressure of 550 kPa (abs) flows in a pipe at a rate of 0.04 N/s. Determine the maximum diameter allowed if the flow is to be turbulent.

For turbulent flow, $Re = \frac{\rho V D}{\mu} > 4000$, where $Q = VA = \frac{\pi}{4} D^2 V$

or $Re = \frac{4\rho Q D}{\pi \mu D^2} = \frac{4\rho Q}{\pi \mu D} = 4000$

Thus, $D = \frac{4\rho Q}{4000 \pi \mu}$, where $g\rho Q = 0.04 \frac{N}{s}$ and $\mu = 1.4 \times 10^{-5} \frac{N \cdot s}{m^2}$ (Table 1.8)

Hence,

$$D = \frac{4\left(0.04 \frac{N}{s}\right)\left(\frac{1}{9.81 \frac{m}{s^2}}\right)}{4000\,\pi \left(1.47 \times 10^{-5} \frac{N \cdot s}{m^2}\right)} = \underline{\underline{0.0883 \text{ m}}}$$

8.9

8.9 The pressure distribution measured along a straight, horizontal portion of a 50-mm-diameter pipe attached to a tank is shown in the table below. Approximately how long is the entrance length? In the fully developed portion of the flow, what is the value of the wall shear stress?

x (m) (± 0.01 m)	p (mm H_2O) (± 5 mm)
0 (tank exit)	520
0.5	427
1.0	351
1.5	288
2.0	236
2.5	188
3.0	145
3.5	109
4.0	73
4.5	36
5.0 (pipe exit)	0

The entrance length extends to the fully developed portion in which $\frac{\partial p}{\partial x} = \text{constant}$. Approximate $\frac{\partial p}{\partial x} \approx \frac{\delta p}{\delta x}$ to obtain the following:

From $x=$	to $x=$( )m	δp, mm H_2O	δx	$\frac{\partial p}{\partial x}$, $\frac{\text{mm}\,H_2O}{m}$
0	0.5	−93	0.5	−186
0.5	1.0	−76	0.5	−152
1.0	1.5	−63	0.5	−126
1.5	2.0	−52	0.5	−104
2.0	2.5	−48	0.5	−96
2.5	3.0	−43	0.5	−86
3.0	3.5	−36	0.5	−72
3.5	4.0	−36	0.5	−72
4.0	4.5	−37	0.5	−74
4.5	5.0	−36	0.5	−72

Within the error on δp, the pressure gradient is constant for $x \gtrsim 3.\,m$ Thus, $\underline{\underline{\ell_e \approx 3m.}}$

For $x > 3\,m$, $\frac{\Delta p}{\ell} = 72\ \frac{mm\,H_2O}{m}$ Since $1\ mm\,H_2O \times \gamma_{H_2O} = (1 \times 10^{-3}m)(9800\frac{N}{m^3})$ $= 9.80\frac{N}{m^2}$, then

$$\frac{\Delta p}{\ell} = 72\ \frac{mm\,H_2O}{m}\left(\frac{9.80\frac{N}{m^2}}{mm\,H_2O}\right) = 706\ \frac{N}{m^3}$$

Since $\Delta p = \frac{4\tau_w \ell}{D}$ it follows that

$$\tau_w = \frac{D}{4}\frac{\Delta p}{\ell} = \frac{0.050m}{4}\left(706\frac{N}{m^3}\right) = \underline{\underline{8.83\frac{N}{m^2}}}$$

8.10

8.10 (See Fluids in the News article titled "Nanoscale flows," Section 8.1.1.) (a) Water flows in a tube that has a diameter of $D = 0.1$ m. Determine the Reynolds number if the average velocity is 10 diameters per second. (b) Repeat the calculations if the tube is a nanoscale tube with a diameter of $D = 100$ nm.

(a) $Re = \frac{VD}{\nu}$, where $D = 0.1 m$, $V = 10(0.1m)/s = 1\frac{m}{s}$, and $\nu = 1.12 \times 10^{-6}\frac{m^2}{s}$

Thus,

$$Re = \frac{(1\frac{m}{s})(0.1m)}{1.12\times10^{-6}\frac{m^2}{s}} = \underline{\underline{89{,}300}}$$

(b) $Re = \frac{VD}{\nu}$, where $D = 100\,nm\left(\frac{1\,m}{10^9 nm}\right) = 10^{-7}m$, $V = 10(10^{-7}m)/s = 10^{-6}\frac{m}{s}$, and $\nu = 1.12\times10^{-6}\frac{m^2}{s}$

Thus,

$$Re = \frac{(10^{-6}\frac{m}{s})(10^{-7}m)}{1.12\times10^{-6}\frac{m^2}{s}} = \underline{\underline{8.93\times10^{-8}}}$$

8.12

8.12 For fully developed laminar pipe flow in a circular pipe, the velocity profile is given by $u(r) = 2\,(1 - r^2/R^2)$ in m/s, where R is the inner radius of the pipe. Assuming that the pipe diameter is 4 cm, find the maximum and average velocities in the pipe as well as the volume flow rate.

$$u(r) = 2(1 - r^2/R^2)$$

Based on Eq. (8.7),

Maximum velocity, $\underline{\underline{V_c = 2\ m/s}}$

We could also use the fact that the maximum velocity occurs at the centerline of the pipe, $r=0$

$$u(0) = 2(1 - 0/R^2) = \underline{\underline{2\ m/s}}$$

Average velocity, V

$$V = V_c/2 = 2/2 = \underline{\underline{1\ m/s}}$$

Volume flowrate, Q

$$Q = VA = (1)\,\frac{\pi}{4}(0.04)^2 = \underline{\underline{1.26 \times 10^{-3}\ m^3/s}}$$

8.13

8.13 The wall shear stress in a fully developed flow portion of a 12-in.-diameter pipe carrying water is 1.85 lb/ft². Determine the pressure gradient, $\partial p/\partial x$, where x is in the flow direction, if the pipe is **(a)** horizontal, **(b)** vertical with flow up, or **(c)** vertical with flow down.

In general, $\dfrac{\Delta p - \gamma \ell \sin\theta}{\ell} = \dfrac{2\tau}{r}$

Thus, with $\tau = \tau_w$ at $r = \frac{D}{2}$ and $\frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell}$ this becomes

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} - \gamma \sin\theta$$

a) For a horizontal pipe $\theta = 0$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} = \underline{\underline{-7.40\,\frac{lb}{ft^3}}}$$

b) For vertical flow up $\theta = 90°$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} - \gamma = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} - 62.4\,\frac{lb}{ft^3} = \underline{\underline{-69.8\,\frac{lb}{ft^3}}}$$

and

c) For vertical flow down $\theta = -90°$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} + \gamma = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} + 62.4\,\frac{lb}{ft^3} = \underline{\underline{55.0\,\frac{lb}{ft^3}}}$$

8.14

8.14 The pressure drop needed to force water through a horizontal 1-in.-diameter pipe is 0.60 psi for every 12-ft length of pipe. Determine the shear stress on the pipe wall. Determine the shear stress at distances 0.3 and 0.5 in. away from the pipe wall.

For a horizontal pipe $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ or $\tau = \frac{r}{2}\frac{\Delta p}{\ell}$

Thus,

$$\tau = r\,\frac{(0.6 \times 144\,\frac{lb}{ft^2})}{2(12\ ft)} = 3.6\,r\,\frac{lb}{ft^2}\text{, where } r \sim ft$$

Hence,

$$\tau_w = 3.6\left(\frac{0.5}{12}\right) = \underline{\underline{0.15\,\frac{lb}{ft^2}}}$$

and with $r = (0.5 - 0.3)\ in. = 0.2\ in.$,

$$\tau = 3.6\left(\frac{0.2}{12}\right) = \underline{\underline{0.06\,\frac{lb}{ft^2}}}$$

Finally, with $r = (0.5 - 0.5)\ in. = 0\ in.$ $\quad \underline{\underline{\tau = 0}}$

8.15

8.15 Repeat Problem 8.14 if the pipe is on a 20° hill. Is the flow up or down the hill? Explain.

For a pipe on a hill $\frac{\Delta p}{\ell} = \frac{2\tau}{r} + \gamma \sin\theta$, where $\theta = \pm 20°$

Assume the flow is uphill: $\theta = +20°$

Thus, $\tau = \frac{r}{2}\left[\frac{\Delta p}{\ell} - \gamma\sin\theta\right]$ or $\tau_w = \frac{1}{2}\left(\frac{0.5}{12}\,ft\right)\left[\frac{0.6 \times 144\,\frac{lb}{ft^2}}{12\ ft} - 62.4\,\frac{lb}{ft^3}\sin 20°\right]$

or $\tau_w = -0.295\,\frac{lb}{ft^2}$ Since we must have $\tau_w > 0$, the flow must not be uphill.

Assume the flow is downhill: $\theta = -20°$

Thus, $\tau = \frac{r}{2}\left[\frac{\Delta p}{\ell} - \gamma\sin\theta\right]$ or $\tau = \frac{r}{2}\left[\frac{0.6 \times 144\,\frac{lb}{ft^2}}{12\ ft} + 62.4\,\frac{lb}{ft^3}\sin 20°\right]$

$= 14.3\,r\,\frac{lb}{ft^2}$, where $r \sim ft$. The <u>flow is downhill</u>

Hence, with $r = \frac{D}{2}$

$$\tau_w = 14.3\left(\frac{0.5}{12}\right) = \underline{\underline{0.596\,\frac{lb}{ft^2}}}$$

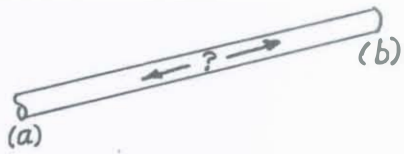
With $r = (0.5 - 0.3)\ in. = 0.2\ in.$,

$$\tau = 14.3\left(\frac{0.2}{12}\right) = \underline{\underline{0.238\,\frac{lb}{ft^2}}}$$

With $r = (0.5 - 0.5)\ in. = 0$, $\quad \underline{\underline{\tau = 0}}$

8.16

8.16 Water flows in a constant diameter pipe with the following conditions measured: At section (a) p_a = 32.4 psi and z_a = 56.8 ft; at section (b) p_b = 29.7 psi and z_b = 68.2 ft. Is the flow from (a) to (b) or from (b) to (a)? Explain.



Assume the flow is uphill. Thus, $\frac{p_a}{\gamma} + \frac{V_a^2}{2g} + z_a = \frac{p_b}{\gamma} + \frac{V_b^2}{2g} + z_b + h_L$

or with $V_a = V_b$,

$$h_L = \frac{p_a}{\gamma} + z_a - \frac{p_b}{\gamma} - z_b = \frac{(32.4\,psi - 29.7\,psi)(144\,\frac{in^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + 56.8\,ft - 68.2\,ft$$

or

$h_L = -5.17\,ft < 0$, which is impossible. Thus, the flow is downhill, from (b) to (a).

*8.17

*8.17 Some fluids behave as a non-Newtonian power-law fluid characterized by $\tau = -C(du/dr)^n$, where $n = 1, 3, 5$, and so on, and C is a constant. (If $n = 1$, the fluid is the customary Newtonian fluid.) (a) For flow in a round pipe of a diameter D, integrate the force balance equation (Eq. 8.3) to obtain the velocity profile

$$u(r) = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{1/n}\left[r^{(n+1)/n} - \left(\frac{D}{2}\right)^{(n+1)/n}\right]$$

(b) Plot the dimensionless velocity profile u/V_c, where V_c is the centerline velocity (at $r = 0$), as a function of the dimensionless radial coordinate $r/(D/2)$, where D is the pipe diameter. Consider values of $n = 1, 3, 5$, and 7.

(a) For any fluid $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ so that with $\tau = -C\left(\frac{du}{dr}\right)^n$ we obtain

$$\frac{\Delta p}{\ell} = -\frac{2C}{r}\left(\frac{du}{dr}\right)^n \quad \text{or} \quad \frac{du}{dr} = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}} r^{\frac{1}{n}} \quad *$$

or

$$-\int du = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\int r^{\frac{1}{n}}dr$$ which integrates to give

$$u = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\frac{n}{(n+1)}r^{\left(\frac{n+1}{n}\right)} + C_1, \text{ where } C_1 \text{ is a constant.} \qquad (1)$$

The fluid sticks to the pipe so that $u = 0$ at $r = \frac{D}{2}$.

Hence, from Eq. (1)

$$C_1 = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\frac{n}{(n+1)}\left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}$$

so that

$$u = \frac{n}{(n+1)}\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\left[-r^{\frac{(n+1)}{n}} + \left(\frac{D}{2}\right)^{\frac{(n+1)}{n}}\right]$$

*Note: Since we are considering only odd integer values for n we can use the fact that if

$$\left(\frac{du}{dr}\right)^n = -K, \text{ where } K > 0, \text{ then } \frac{du}{dr} = -K^{\frac{1}{n}}$$

so that $\frac{du}{dr} < 0$.

(b) From part (a):

$$u(r) = \frac{n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left[-r^{\left(\frac{n+1}{n}\right)} + \left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}\right] \qquad (2)$$

Let $V_c = u(r=0)$, or $V_c = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}$ (3)

Note: For $\tau = C\left(\frac{du}{dr}\right)^n$ with $\frac{du}{dr} < 0$ and n an odd integer, to have $\tau > 0$, we must have $C < 0$. Thus, from Eq. (2), $V_c > 0$ as it must.

By dividing Eq. (2) by Eq. (3) we obtain

(con't)

*8.17 (con't)

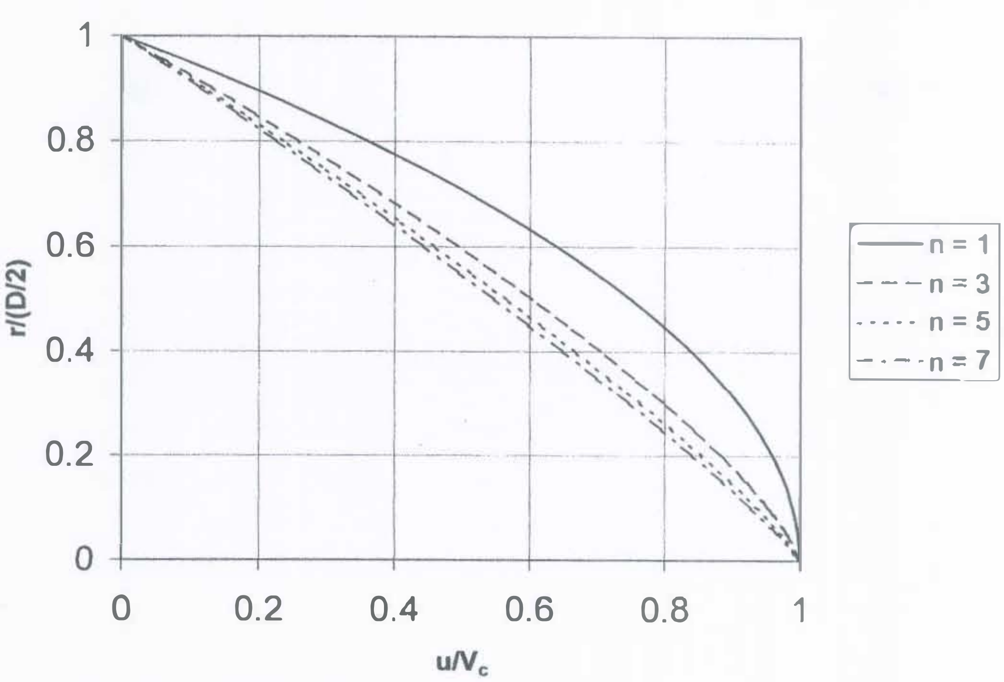
$$\frac{u}{V_c} = 1 - \left[\frac{r}{\left(\frac{D}{2}\right)}\right]^{\left(\frac{n+1}{n}\right)}$$

This result is plotted below for $n = 1, 3, 5$, and 7, with $0 \le \frac{r}{\left(\frac{D}{2}\right)} \le 1$.

An EXCEL program was used to do the calculations and plotting.

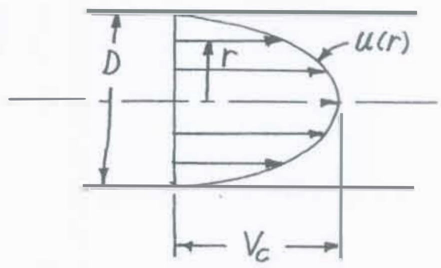
	n = 1	n = 3	n = 5	n = 7
r/(D/2)	u/V$_c$	u/V$_c$	u/V$_c$	u/V$_c$
0	1	1	1	1
0.05	0.998	0.982	0.973	0.967
0.1	0.990	0.954	0.937	0.928
0.15	0.978	0.920	0.897	0.886
0.2	0.960	0.883	0.855	0.841
0.25	0.938	0.843	0.811	0.795
0.3	0.910	0.799	0.764	0.747
0.35	0.878	0.753	0.716	0.699
0.4	0.840	0.705	0.667	0.649
0.45	0.798	0.655	0.616	0.599
0.5	0.750	0.603	0.565	0.547
0.55	0.698	0.549	0.512	0.495
0.6	0.640	0.494	0.458	0.442
0.65	0.578	0.437	0.404	0.389
0.7	0.510	0.378	0.348	0.335
0.75	0.438	0.319	0.292	0.280
0.8	0.360	0.257	0.235	0.225
0.85	0.278	0.195	0.177	0.170
0.9	0.190	0.131	0.119	0.113
0.95	0.097	0.066	0.060	0.057
1	0.000	0.000	0.000	0.000

r/(D/2) vs u/V$_c$



8.18

8.18 For laminar flow in a round pipe of diameter D, at what distance from the centerline is the actual velocity equal to the average velocity?



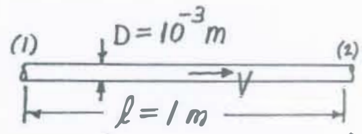
For laminar flow

$$u = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$$

Thus, if $u = \frac{V_c}{2} = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$, $r = \frac{D}{2\sqrt{2}} = \underline{\underline{0.354\,D}}$

8.19

8.19 Water at 20 °C flows through a horizontal 1-mm-diameter tube to which are attached two pressure taps a distance 1 m apart. **(a)** What is the maximum pressure drop allowed if the flow is to be laminar? **(b)** Assume the manufacturing tolerance on the tube diameter is $D = 1.0 \pm 0.1$ mm. Given this uncertainty in the tube diameter, what is the maximum pressure drop allowed if it must be assured that the flow is laminar?



From Table B.2 $\nu = 1.00 \times 10^{-6} \frac{m^2}{s}$

$\mu = 1.00 \times 10^{-3} \frac{N \cdot s}{m^2}$

a) Maximum Δp corresponds to maximum V, or

$$Re = \frac{VD}{\nu} = 2100$$

Thus, $V = \frac{2100\,\nu}{D} = \frac{2100(1 \times 10^{-6} \frac{m^2}{s})}{10^{-3}\,m} = 2.10 \frac{m}{s}$

For laminar flow

$$V = \frac{\Delta p D^2}{32 \mu \ell}, \text{ or } \Delta p = \frac{32 \mu \ell V}{D^2} = \frac{32(1 \times 10^{-3} \frac{N \cdot s}{m^2})(1m)(2.10 \frac{m}{s})}{(10^{-3} m)^2}$$

Thus,

$$\underline{\underline{\Delta p = 6.72 \times 10^4 \frac{N}{m^2}}}$$

b) Since $V = \frac{2100\,\nu}{D}$ and $\Delta p = \frac{32 \mu \ell V}{D^2}$ it follows that

$$\Delta p = \frac{32 \mu \ell (2100 \nu)}{D^3}$$

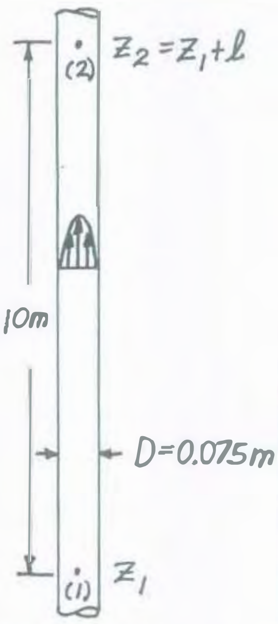
Thus, the larger the diameter, the smaller the Δp allowed to maintain laminar flow.

Thus, consider $D = 1.1\,mm = 1.1 \times 10^{-3}\,m$, or

$$\Delta p = \frac{32(1 \times 10^{-3} \frac{N \cdot s}{m^2})(1m)(2100)(1 \times 10^{-6} \frac{m^2}{s})}{(1.1 \times 10^{-3} m)^3} = \underline{\underline{5.05 \times 10^4 \frac{N}{m^2}}}$$

8.20

8.20 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.



$\rho = 1260 \frac{kg}{m^3}$
$\mu = 1.50 \frac{N \cdot s}{m^2}$

For laminar flow in a pipe,

$V =$ average velocity $= \frac{1}{2} V_{max} = \frac{1}{2}(1 \frac{m}{s}) = 0.5 \frac{m}{s}$

Thus,

$$Re = \frac{\rho V D}{\mu} = \frac{(1260 \frac{kg}{m^3})(0.5 \frac{m}{s})(0.075\,m)}{1.50 \frac{N \cdot s}{m^2}} = 31.5 < 2100$$

The flow is laminar so that

$$V = \frac{(\Delta p - \gamma \ell \sin\theta) D^2}{32 \mu \ell}, \text{ where } \theta = 90^\circ$$

Thus,

$$\Delta p = \frac{32 \mu \ell V}{D^2} + \gamma \ell = \frac{32\,(1.50 \frac{N \cdot s}{m^2})(10m)(0.5 \frac{m}{s})}{(0.075\,m)^2} + (9.81 \frac{m}{s^2})(1260 \frac{kg}{m^3})(10m)$$

$$= 1.66 \times 10^5 \frac{N}{m^2}, \text{ or } \underline{\underline{\Delta p = 166\ kPa}}$$

Also,

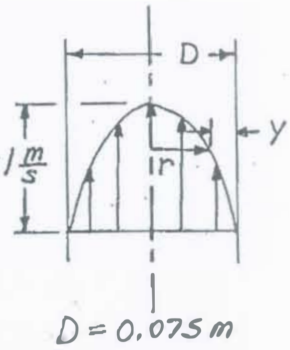
$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L$, or with $V_1 = V_2$, $z_2 - z_1 = \ell$, and

$p_1 = p_2 + \Delta p$ this gives

$$h_L = \frac{\Delta p}{\gamma} - \ell = \frac{1.66 \times 10^5 \frac{N}{m^2}}{(9.81 \frac{m}{s^2})(1260 \frac{kg}{m^3})} - 10m = \underline{\underline{3.43\,m}}$$

8.21

8.21 Determine the magnitude of the velocity gradient at points 10, 20, and 30 mm from the pipe wall for the flow in Problem 8.20.*



For laminar flow in a round pipe

$$u(r) = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$$

or

$$\frac{du}{dr} = -2V_c\left(\frac{2r}{D}\right)\left(\frac{2}{D}\right) = -\frac{8V_c}{D^2}r$$

Thus,

$$\frac{du}{dr} = \frac{-8(1\frac{m}{s})r}{(0.075\,m)^2} = -1422r\ \frac{1}{s},\ \text{where } r \sim m$$

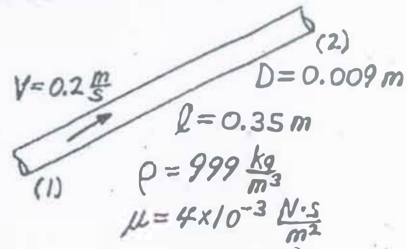
Also, $y = \text{distance from wall} = \frac{D}{2} - r = 0.0375 - r$

y, m	r, m	$\frac{du}{dr}$, $\frac{1}{s}$
0.01	0.0275	-39.1
0.02	0.0175	-24.9
0.03	0.0075	-10.7

* 8.20 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

8.22

8.22 A large artery in a person's body can be approximated by a tube of diameter 9 mm and length 0.35 m. Also assume that blood has a viscosity of approximately $4 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$, a specific gravity of 1.0, and that the pressure at the beginning of the artery is equivalent to 120 mm Hg. If the flow were steady (it is not) with $V = 0.2$ m/s, determine the pressure at the end of the artery if it is oriented (a) vertically up (flow up) or (b) horizontal.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}\,,\quad \text{where } V_1=V_2=V \qquad (1)$$

and

$$p_1=\gamma_{Hg}h=133\ \frac{kN}{m^3}(0.120\,m)=15.96\ \frac{kN}{m^2}$$

Also, $Re=\dfrac{\rho VD}{\mu}=\dfrac{(999\frac{kg}{m^3})(0.2\frac{m}{s})(0.009\,m)}{4\times10^{-3}\frac{N\cdot s}{m^2}}=450<2100$ Thus the flow is laminar so that

$$f=\frac{64}{Re}=\frac{64}{450}=0.142$$

Hence, from Eq.(1), $p_2=p_1-\gamma(z_2-z_1)-f\frac{\ell}{D}\frac{1}{2}\rho V^2$

a) For flow vertically up, $z_2-z_1=\ell$ so that

$$p_2=p_1-\gamma\ell-f\frac{\ell}{D}\frac{1}{2}\rho V^2=15.96\ \frac{kN}{m^2}-(9.81\times10^3\ \frac{N}{m^3})(0.35\,m)$$
$$-0.142\ \frac{0.35\,m}{0.009\,m}\left(\frac{1}{2}\right)\left(999\ \frac{kg}{m^3}\right)\left(0.2\ \frac{m}{s}\right)^2$$

or

$$p_2=15.96\ \frac{kN}{m^2}-3.43\ \frac{kN}{m^2}-0.110\ \frac{kN}{m^2}=\underline{\underline{12.42\ kPa}}$$

b) For horizontal flow $z_1=z_2$ so that

$$p_2-p_1=15.96\ \frac{kN}{m^2}-0.142\ \frac{0.35\,m}{0.009\,m}\left(\frac{1}{2}\right)\left(999\ \frac{kg}{m^3}\right)\left(0.2\ \frac{m}{s}\right)^2$$
$$=15.96\ \frac{kN}{m^2}-0.110\ \frac{kN}{m^2}=\underline{\underline{15.85\ kPa}}$$

Note the gravitational effects are considerably more important than viscous effects (3.43 kPa compared to 0.110 kPa).

8.23

8.23 At time $t = 0$ the level of water in tank A shown in Fig. P8.23 is 2 ft above that in tank B. Plot the elevation of the water in tank A as a function of time until the free surfaces in both tanks are at the same elevation. Assume quasisteady conditions—that is, the steady pipe flow equations are assumed valid at any time, even though the flowrate does change (slowly) in time. Neglect minor losses. *Note:* Verify and use the fact that the flow is laminar.

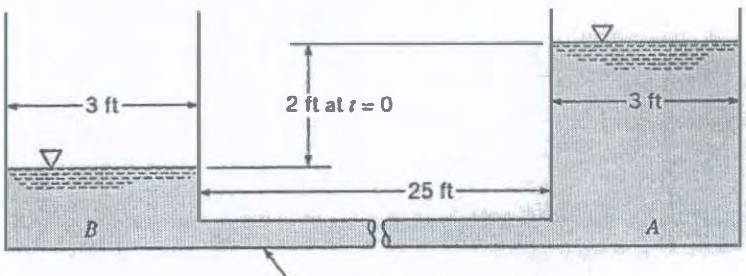
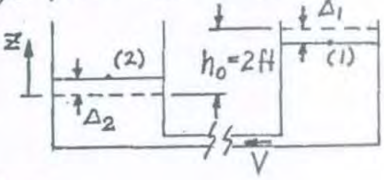


FIGURE P8.23

$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}$, where $p_1=p_2=0$ and $V_1=V_2\approx 0$ (1)

At $t=0$, $z_2=0$ and $z_1=h_0=2\text{ ft}$



Because the tanks are the same diameter

$\Delta_1=\Delta_2$ and with $z_2=\Delta_2$, $z_1=h_0-\Delta_2$

we obtain $z_1=h_0-z_2$ Thus, Eq.(1) becomes

$z_1=z_2+f\frac{\ell}{D}\frac{V^2}{2g}$ or $2z_1-h_0=f\frac{\ell}{D}\frac{V^2}{2g}$ (2)

Also, $A_1\left(-\frac{dz_1}{dt}\right)=Q=\frac{\pi}{4}D^2V$, where $A_1=\frac{\pi}{4}D_T^2$ with $D_T=3\text{ft}=$ tank diameter

Thus, $V=-\left(\frac{D_T}{D}\right)^2\frac{dz_1}{dt}$ (3)

The maximum $Re=\frac{\rho VD}{\mu}$ occurs when the head, z_1-z_2, is greatest.

From Eq.(2) (with $z_1=h_0$), $h_0=f\frac{\ell}{D}\frac{V_{max}^2}{2g}$

Assume laminar flow so that $f=\frac{64}{Re}$ or $f=\frac{64\mu}{\rho VD}$ (4)

Thus, from Eq.(4)

$$h_0=\frac{64\mu}{\rho V_{max}D}\frac{\ell}{D}\frac{V_{max}^2}{2g}=\frac{32\mu\ell V_{max}}{\gamma D^2}\text{, or } V_{max}=\frac{\gamma D^2h_0}{32\mu\ell}=\frac{(62.4\frac{lb}{ft^3})(\frac{0.1}{12}ft)^2(2ft)}{32(2.34\times10^{-5}\frac{lb\cdot s}{ft^2})(25ft)}$$

$$=0.462\frac{ft}{s}$$

or $Re_{max}=\frac{(1.94\frac{slugs}{ft^3})(0.462\frac{ft}{s})(\frac{0.1}{12}ft)}{2.34\times10^{-5}\frac{lb\cdot s}{ft^2}}=319<2100$ The flow remains laminar.

Thus, Eqs.(2) and (4) give

$2z_1-h_0=\frac{64\mu}{\rho VD}\frac{\ell}{D}\frac{V^2}{2g}=\frac{32\mu\ell V}{\gamma D^2}$, or by using Eq.(3)

$$2z_1-h_0=-\left(\frac{D_T}{D}\right)^2\frac{32\mu\ell}{\gamma D^2}\frac{dz_1}{dt} \quad (5)$$

Let $F\equiv z_1-\frac{h_0}{2}$ so that $\frac{dF}{dt}=\frac{dz_1}{dt}$ and Eq.(5) becomes

$$2F=-\left(\frac{D_T}{D}\right)^2\frac{32\mu\ell}{\gamma D^2}\frac{dF}{dt}$$

(con't)

8.23 (con't)

or $\alpha \frac{dF}{dt} + F = 0$, where $\alpha = \frac{16 \mu \ell}{\gamma D^2} \left(\frac{D_T}{D}\right)^2$

Thus,

$$\alpha \int \frac{dF}{F} = -\int dt \quad \text{or} \quad \alpha \ln F = -t + \tilde{C}, \text{ where } \tilde{C} = \text{constant}$$

Hence,

$F = C e^{-(t/\alpha)}$ That is, $z_1 - \frac{h_o}{2} = C e^{-(t/\alpha)}$ with the initial condition $z_1 = h_o$ when $t = 0$, or $C = \frac{h_o}{2}$

Thus, $z_1 - \frac{h_o}{2} = \frac{h_o}{2} e^{-(t/\alpha)}$

or

$z_1 = \frac{h_o}{2}\left[1 + e^{-(t/\alpha)}\right]$ Note: As $t \to \infty$, $z_1 \to \frac{h_o}{2}$

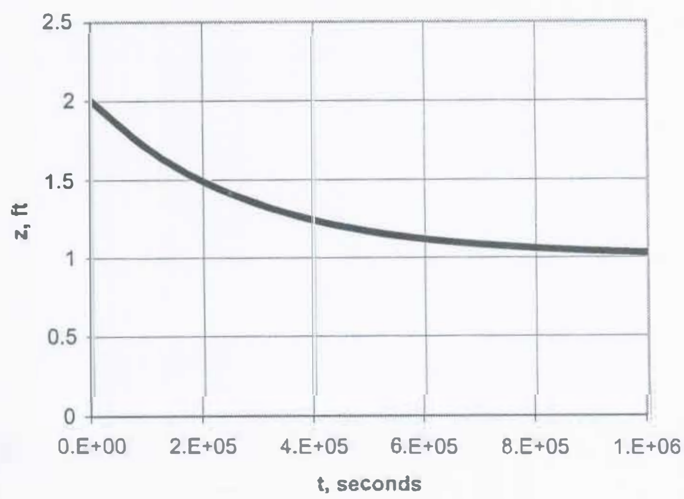
For the conditions given, $h_o = 2$ ft and

$$\alpha = \frac{16(2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2})(25 ft)}{(62.4 \frac{lb}{ft^3})(\frac{0.1}{12} ft)^2} \left(\frac{3 ft}{(\frac{0.1}{12} ft)}\right)^2 = 2.80 \times 10^5 s$$

Hence,

$$\underline{\underline{z_1 = 1 + e^{-\left(\frac{t}{2.8 \times 10^5 s}\right)}}}, \text{ where } z_1 \sim ft \text{ and } t \sim s$$

This result is plotted below. (Note: $\lim_{t \to \infty} z_1 = 1$ ft)



8.24

8.24 A fluid flows through a horizontal 0.1-in.-diameter pipe. When the Reynolds number is 1500, the head loss over a 20-ft length of the pipe is 6.4 ft. Determine the fluid velocity.

$h_L = f\frac{\ell}{D}\frac{V^2}{2g}$, where since $Re = 1500 < 2100$ the flow is laminar.

Thus, $f = 64/Re = 64/1500 = 0.0427$ so that

$$6.4\,ft = 0.0427\,\frac{20\,ft}{(0.1/12\,ft)}\,\frac{V^2}{2(32.2\,ft/s^2)}$$

or

$V = \underline{\underline{2.01\,\frac{ft}{s}}}$

8.25

8.25 A viscous fluid flows in a 0.10-m-diameter pipe such that its velocity measured 0.012 m away from the pipe wall is 0.8 m/s. If the flow is laminar, determine the centerline velocity and the flowrate.

For laminar flow in a pipe

$u(r) = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$, where $D = 0.1\,m$ and $u = 0.8\,\frac{m}{s}$ at $r = \frac{0.1m}{2} - 0.012m = 0.038m$

Thus,

$0.8\,\frac{m}{s} = V_c\left[1 - \left(\frac{2(0.038m)}{0.10m}\right)^2\right]$ or $V_c = \underline{\underline{1.89\,\frac{m}{s}}}$

so that

$Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}D^2(0.5V_c) = \frac{\pi}{4}(0.1m)^2(0.5)(1.89\,\frac{m}{s}) = \underline{\underline{7.42\times10^{-3}\,\frac{m^3}{s}}}$

8.26

8.26 Oil flows through the horizontal pipe shown in Fig. P8.20 under laminar conditions. All sections are the same diameter except one. Which section of the pipe (A, B, C, D, or E) is slightly smaller diameter than the others? Explain.

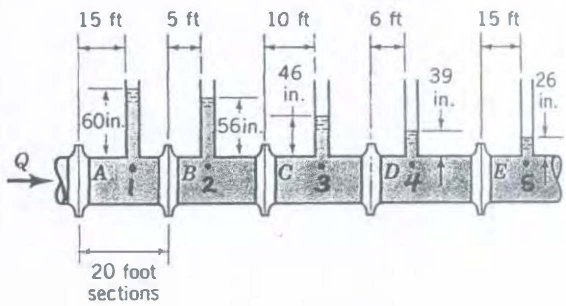
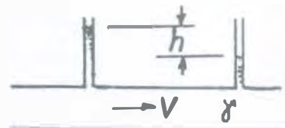


FIGURE P8.26

For laminar flow in a horizontal pipe $Q = \frac{\pi D^4}{128\mu}\frac{\Delta p}{\ell}$, where $Q_A = Q_B = Q_C = Q_D = Q_E$. Thus $\frac{\Delta p}{\ell} \sim \frac{1}{D^4}$ The smallest diameter pipe has the largest $\frac{\Delta p}{\ell}$, where $\Delta p = \gamma h$



Let $a = \left.\frac{\Delta p}{\ell}\right)_{\text{pipe } A}$, $b = \left.\frac{\Delta p}{\ell}\right)_{\text{pipe } B}$, etc.

Hence, from the data in the figure for the section between (1) and (2):

$$5a + 5b = \gamma\frac{(60-56)}{12}, \text{ where } a \text{ and } b \sim \frac{lb}{ft^3} \text{ and } \gamma \sim \frac{lb}{ft^3}. \quad (1)$$

Similarly, from (2) to (3)

$$15b + 10c = \gamma\frac{(56-46)}{12}, \quad (2)$$

from (3) to (4)

$$10c + 6d = \gamma\frac{(46-39)}{12}, \quad (3)$$

and from (4) to (5)

$$14d + 15e = \gamma\frac{(39-26)}{12} \quad (4)$$

Eqs. (1) through (4) can be written as

(5) $a + b = 0.0667\gamma$

(6) $1.5b + c = 0.0833\gamma$

(7) $c + 0.6d = 0.0583\gamma$

(8) $d + 1.071e = 0.0774\gamma$

From the problem statement, 4 pipes are the same diameter, one is smaller diameter. Thus, 4 of the 5 variables (a, b, c, d, e) should be equal, one larger than the others.

Assume $a > b = c = d = e$ From Eq. (6), $1.5b + b = 0.0833\gamma$ or $b = 0.0333\gamma$ but from Eq. (7), $b + 0.6b = 0.0583\gamma$ or $b = 0.0364\gamma$ which is not the same as that from Eq. (6).

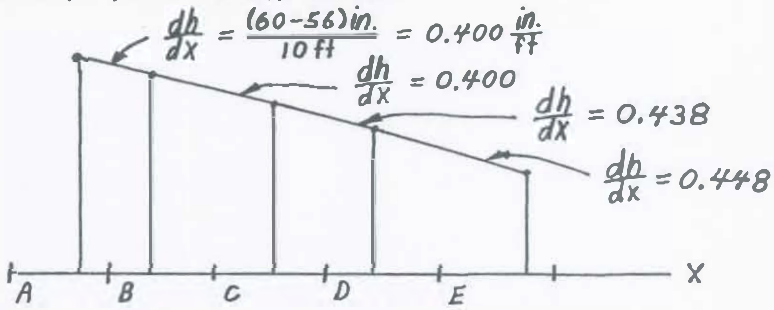
Assuming $b > a = c = d = e$, or $c > a = b = d = e$, or $e > a = b = c = d$ lead to similar inconsistencies. However, if we assume $d > a = b = c = e$ we obtain from Eq (5): $a = 0.0333\gamma$; from Eq. (6): the same value of a; from Eq. (7): $d = 0.0417\gamma$; the same value of d from Eq. (8).

(con't)

8.26 (con't)

Thus, $a=b=c=e$ and $d>a$. That is, the small pipe is pipe D.

Note: This result can also be obtained as follows. From the given data the pressure gradient (average) between piezometer tube locations is as shown below.



Given that all section have the same diameter except for one, it follows (based on the different $\frac{dh}{dx}$ values) that the diameter of section D is less than that of the others.

8.27

8.27 Asphalt at 120 °F, considered to be a Newtonian fluid with a viscosity 80,000 times that of water and a specific gravity of 1.09, flows through a pipe of diameter 2.0 in. If the pressure gradient is 1.6 psi/ft determine the flowrate assuming the pipe is **(a)** horizontal; **(b)** vertical with flow up.

If the flow is laminar, then $Q = \dfrac{\pi(\Delta p - \gamma \ell \sin\theta)D^4}{128\mu\ell}$ (1)

where $\gamma = SG\,\gamma_{H_2O} = 1.09(62.4\,\frac{lb}{ft^3}) = 68.0\,\frac{lb}{ft^3}$

and

$$\mu = 80{,}000\,\mu_{H_2O} = 8\times10^4\left(1.164\times10^{-5}\,\frac{lb\cdot s}{ft^2}\right) = 0.931\,\frac{lb\cdot s}{ft^2}$$

a) For horizontal flow, $\theta = 0$

Thus, from Eq.(1)

$$Q = \frac{\pi(1.6\times144\,\frac{lb}{ft^2})(\frac{2}{12}\,ft)^4}{128(0.931\,\frac{lb\cdot s}{ft^2})(1\,ft)} = \underline{\underline{4.69\times10^{-3}\,\frac{ft^3}{s}}}$$

b) For vertical flow up, $\theta = 90$

Thus, from Eq.(1)

$$Q = \frac{\pi(1.6\times144\,\frac{lb}{ft^2} - 68\,\frac{lb}{ft^3}(1\,ft))(\frac{2}{12}\,ft)^4}{128(0.931\,\frac{lb\cdot s}{ft^2})(1\,ft)} = \underline{\underline{3.30\times10^{-3}\,\frac{ft^3}{s}}}$$

Note: We must check to see if our assumption of laminar flow is correct.

Since $V = \frac{Q}{A} = \dfrac{4.69\times10^{-3}\,\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{2}{12})^2} = 0.215\,\frac{ft}{s}$ it follows that

$$Re = \frac{\rho V D}{\mu} = \frac{1.09(1.94\,\frac{slug}{ft^3})(0.215)(\frac{2}{12}\,ft)}{0.931\,\frac{lb\cdot s}{ft^2}} = 0.0814 < 2100$$

The flow is laminar.

8.28

8.28 Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4}\ m^2/s$ flows through the vertical pipe shown in Fig. P8.28 at a rate of $4 \times 10^{-4}\ m^3/s$. Determine the manometer reading, h.

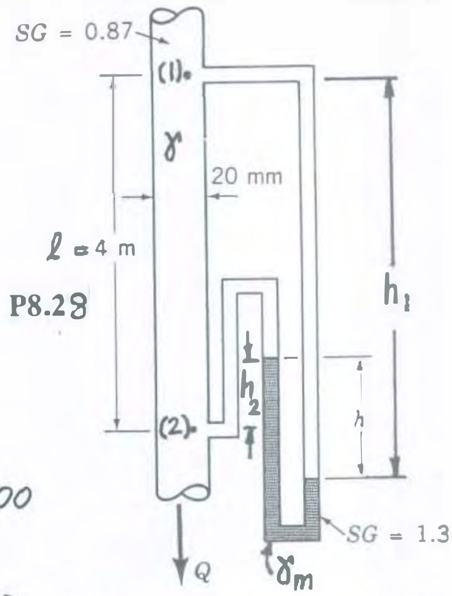


FIGURE P8.28

$$V = \frac{Q}{A} = \frac{4 \times 10^{-4} \frac{m^3}{s}}{\frac{\pi}{4}(0.02 m)^2} = 1.27 \frac{m}{s} \text{ so that}$$

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(1.27 \frac{m}{s})(0.02 m)}{2.2 \times 10^{-4} \frac{m^2}{s}} = 115 < 2100$$

The flow is laminar with

$$Q = \frac{\pi(\Delta p + \gamma \ell) D^4}{128 \mu \ell}, \text{ or } \Delta p = p_1 - p_2 = \frac{128 \mu \ell Q}{\pi D^4} - \gamma \ell \quad (1)$$

Hence, with $\gamma = SG\, \gamma_{H_2O} = 0.87(9.81 \frac{kN}{m^3}) = 8.53 \frac{kN}{m^3}$ and

$$\mu = \nu \rho = \nu\, SG\, \rho_{H_2O} = (2.2 \times 10^{-4} \frac{m^2}{s})(0.87)(1000 \frac{kg}{m^3}) = 0.191 \frac{N \cdot s}{m^2}$$

Eq. (1) gives

$$\Delta p = \frac{128(0.191 \frac{N \cdot s}{m^2})(4 m)(4 \times 10^{-4} \frac{m^3}{s})}{\pi (0.020 m)^4} - (8.53 \frac{kN}{m^3})(4 m)(10^3 \frac{N}{kN})$$

or $\Delta p = 4.37 \times 10^4 \frac{N}{m^2} = 43.7 \frac{kN}{m^2}$ (2)

From manometer considerations

$p_1 + \gamma h_1 - \gamma_m h + \gamma h_2 = p_2$, where $\gamma_m = SG_m \gamma_{H_2O} = 1.3(9.81 \frac{kN}{m^3}) = 12.74 \frac{kN}{m^3}$

and $h_1 = h - h_2 + \ell$, or $h_2 + h_1 = h + \ell$

Thus,

$$p_1 - p_2 = \Delta p = -\gamma(h_2 + h_1) + \gamma_m h = (\gamma_m - \gamma) h - \gamma \ell \quad (3)$$

Combine Eqs. (2) and (3) to give

$$43.7 \frac{kN}{m^2} = (12.74 - 8.53) \frac{kN}{m^3} h - (8.53 \frac{kN}{m^3})(4 m)$$

or

$$\underline{\underline{h = 18.5 m}}$$

8.29

8.29 Determine the manometer reading, h, for Problem 8.28 if the flow is up rather than down the pipe. Note: The manometer reading will be reversed.

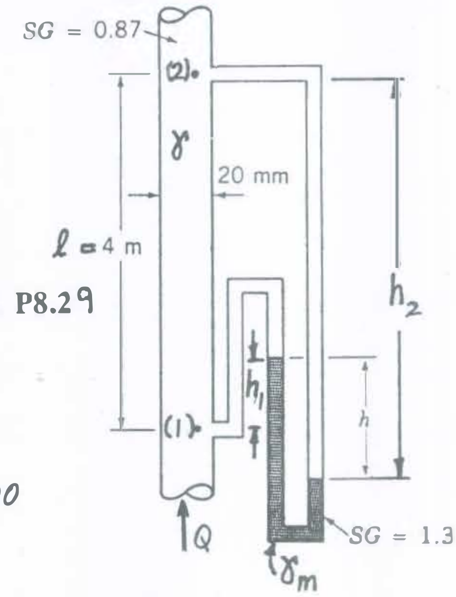


FIGURE P8.29

$$V = \frac{Q}{A} = \frac{4\times10^{-4}\,\frac{m^3}{s}}{\frac{\pi}{4}(0.02m)^2} = 1.27\,\frac{m}{s} \text{ so that}$$

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(1.27\frac{m}{s})(0.02m)}{2.2\times10^{-4}\,\frac{m^2}{s}} = 115 < 2100$$

The flow is laminar with

$$Q = \frac{\pi(\Delta p - \gamma\ell)D^4}{128\mu\ell}, \text{ or } \Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4} + \gamma\ell \tag{1}$$

Hence, with $\gamma = SG\,\gamma_{H_2O} = 0.87(9.81\frac{kN}{m^3}) = 8.53\,\frac{kN}{m^3}$ and

$\mu = \nu\rho = \nu\,SG\,\rho_{H_2O} = (2.2\times10^{-4}\frac{m^2}{s})(0.87)(1000\frac{kg}{m^3}) = 0.191\ \frac{N\cdot s}{m^2}$

Eq. (1) gives

$$\Delta p = \frac{128(0.191\frac{N\cdot s}{m^2})(4m)(4\times10^{-4}\frac{m^3}{s})}{\pi(0.020m)^4} + (8.53\frac{kN}{m^3})(4m)(10^3\frac{N}{kN})$$

or

$$\Delta p = 1.119\times10^5\,\frac{N}{m^2} = 111.9\,\frac{kN}{m^2} \tag{2}$$

From manometer considerations

$p_1 - \gamma h_1 + \gamma_m h - \gamma h_2 = p_2$, where $\gamma_m = SG_m\,\gamma_{H_2O} = 1.3(9.81\frac{kN}{m^3}) = 12.74\,\frac{kN}{m^3}$

and $h_2 = \ell + h - h_1$ or $h_2 + h_1 = \ell + h$

Thus,

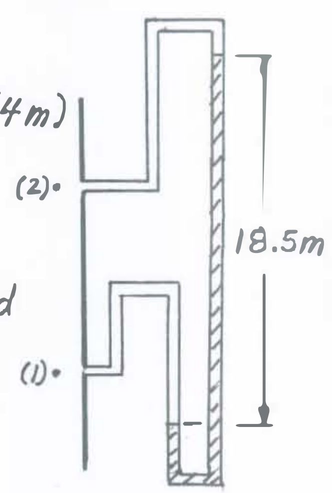
$$p_1 - p_2 = \Delta p = \gamma(h_2 + h_1) - \gamma_m h = -(\gamma_m - \gamma)h + \gamma\ell \tag{3}$$

Combine Eqs. (2) and (3) to give

$$111.9\,\frac{kN}{m^2} = -(12.74 - 8.53)\frac{kN}{m^3}h + 8.53\,\frac{kN}{m^3}(4m)$$

or

$$\underline{\underline{h = -18.5m}}$$



Note: Since $h<0$ the manometer is displaced in the direction opposite that shown in the original figure.

8.30

8.30 A liquid with $SG = 0.96$, $\mu = 9.2 \times 10^{-4}\ \mathrm{N \cdot s/m^2}$, and vapor pressure $p_v = 1.2 \times 10^4\ \mathrm{N/m^2(abs)}$ is drawn into the syringe as is indicated in Fig. P8.30. What is the maximum flowrate if cavitation is not to occur in the syringe?

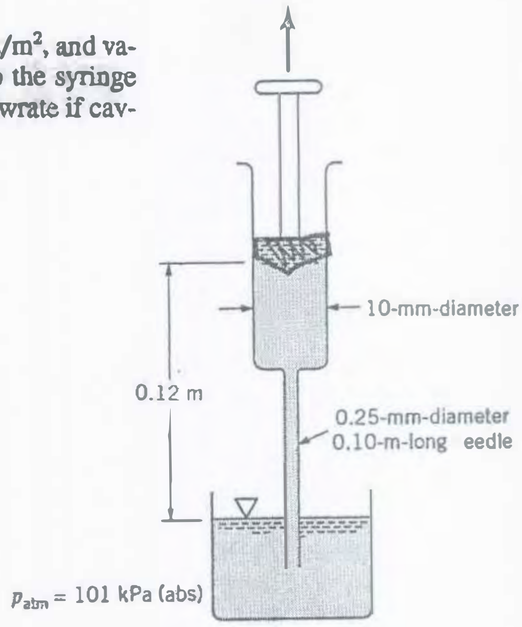


FIGURE P8.30

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g}$, where $p_1 = 101\ kPa$, $z_1 = 0$, $V_1 = 0$, $z_2 = 0.12\ m$. The maximum flowrate will occur when p_2 is the minimum allowed: $p_2 = p_v = 1.2 \times 10^4 \frac{N}{m^2}$.

Thus, $\frac{p_1}{\gamma} = \frac{p_v}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + K_{L\,entrance} + K_{L\,exit}\right)\frac{V^2}{2g}$, (1)

where $V_2 = \frac{VA}{A_2} = V\left(\frac{D}{D_2}\right)^2 = V\left(\frac{0.25mm}{10\,mm}\right)^2 = 0.000625\,V$ Thus, $V_2 \approx 0$ and Eq. (1) becomes

$$\frac{(101 \times 10^3 - 1.2 \times 10^4)\frac{N}{m^2}}{0.96(9.80 \times 10^3 \frac{N}{m^3})} = 0.12m + \left(f\left(\frac{0.1\,m}{0.25 \times 10^{-3}\,m}\right) + 0.5 + 1\right)\frac{V^2}{2(9.81\frac{m}{s^2})}$$

or

$$122 = (267f + 1)V^2 \qquad (2)$$

Assume (because of the small diameter) that the flow is laminar.

Thus, $f = \frac{64}{Re} = \frac{64\mu}{\rho V D}$

or

$$f = \frac{64\,(9.2 \times 10^{-4} \frac{N \cdot s}{m^2})}{0.96(999\frac{kg}{m^3})V(0.25 \times 10^{-3}m)} = \frac{0.246}{V}$$

Hence, from Eq. (2)

$$122 = \left(267\,\frac{0.246}{V} + 1\right)V^2 \quad \text{or} \quad 122\,V = (65.7 + V)V^2$$

Thus, $V^2 + 65.7\,V - 122 = 0$, which has solutions

$$V = \frac{-65.7 \pm \sqrt{65.7^2 + 4(122)}}{2} = 1.81\frac{m}{s}, \text{ or } -67.5\frac{m}{s} \text{ (neglect the } V<0 \text{ root)}$$

Hence,

$$Q = AV = \frac{\pi}{4}(0.25 \times 10^{-3}m)^2(1.81\frac{m}{s}) = \underline{\underline{8.88 \times 10^{-8}\frac{m^3}{s}}}$$

Check if laminar flow:

$$Re = \frac{\rho V D}{\mu} = \frac{0.96(999\frac{kg}{m^3})(1.81\frac{m}{s})(0.25 \times 10^{-3}m)}{9.2 \times 10^{-4}\frac{N \cdot s}{m^2}} = 472 < 2100 \text{ (laminar)}$$

8.32

8.32 For oil ($SG = 0.86$, $\mu = 0.025\ \text{Ns/m}^2$) flow of $0.3\ \text{m}^3/\text{s}$ through a round pipe with diameter of 500 mm, determine the Reynolds number. Is the flow laminar or turbulent?

$$SG = \rho/\rho_{H_2O} = 0.86$$

$$\rho_{oil} = 0.86(\rho_{H_2O}) = 0.86\,(999) = 859\ \text{kg/m}^3$$

$$V = Q/A = 0.3/\tfrac{\pi}{4}(0.5)^2 = 1.53\ \text{m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(859)(1.53)(0.5)}{0.025} = 2.63 \times 10^4$$

Based on the criterion that $Re < 2100$ represents laminar flow, this flow is <u>turbulent</u>.

8.33

8.33 For air at a pressure of 200 kPa (abs) and temperature of 15 °C, determine the maximum laminar volume flowrate for flow through a 2.0-cm-diameter tube.

For laminar flow, the maximum Re value is 2100

$$Re = \frac{\rho V D}{\mu} = 2100$$

$$V = \frac{2100\,\mu}{\rho D}$$

To determine air density, make use of ideal gas law

$$P = \rho R T \quad \text{or} \quad \rho = P/RT$$

$$\rho = \frac{200 \times 10^3}{(286.9)(273+15)} = 2.42 \ \mathrm{kg/m^3}$$

Viscosity has little variation with pressure, so it is reasonable to assume the use of the standard value for air, $\mu = 1.79 \times 10^{-5}$

$$V = \frac{2100\,(1.79 \times 10^{-5})}{(2.42)\,(0.02)} = 0.78 \ \mathrm{m/s}$$

Maximum laminar volume flowrate

$$Q = VA = (0.78)\left(\frac{\pi}{4}(0.02)^2\right)$$

$$\underline{\underline{Q = 2.4 \times 10^{-4} \ \mathrm{m^3/s}}}$$

8.34

8.34 Show that the power law approximation for the velocity profile in turbulent pipe flow (Eq. 8.31) cannot be accurate at the centerline or at the pipe wall because the velocity gradients at these locations are not correct. Explain.

If $\bar{u} = V_o\left[1-\frac{r}{R}\right]^{\frac{1}{n}}$, then $\frac{d\bar{u}}{dr} = \frac{V_o}{n}\left[1-\frac{r}{R}\right]^{\left(\frac{1}{n}-1\right)}\left(-\frac{1}{R}\right)$

or

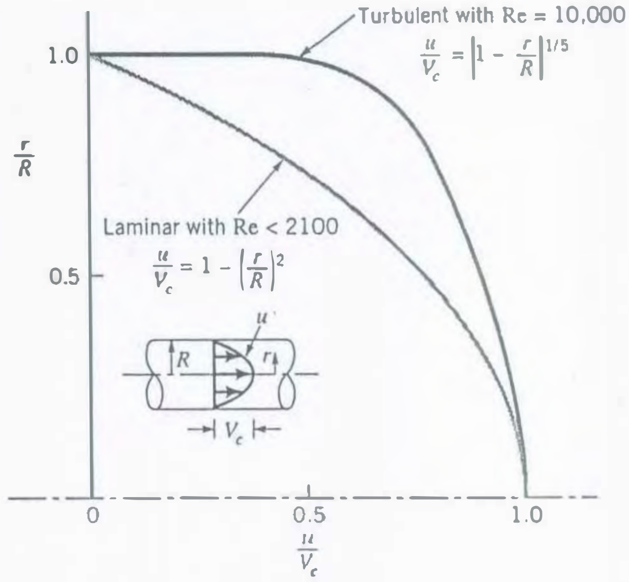
$\frac{d\bar{u}}{dr} = -\frac{V_o}{nR}\left[1-\frac{r}{R}\right]^{\left(\frac{1-n}{n}\right)}$ Thus, $\left.\frac{d\bar{u}}{dr}\right|_{r=0} = \underline{\underline{-\frac{V_o}{nR}}}$, <u>but by symmetry it must be zero.</u>

Also, $\left.\frac{d\bar{u}}{dr}\right|_{r=R} = -\frac{V_o}{nR}\left[1-1\right]^{\left(\frac{1-n}{n}\right)} = \underline{\underline{-\infty}}$ since $\left(\frac{1-n}{n}\right) < 0$ for $n > 1$

Physically, the velocity gradient must be finite.

8.35

8.35 As shown in Video V8.9 and Fig. P8.35 the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at Re = 10,000 the velocity profile can be approximated by the power-law profile shown in the figure. (a) For laminar flow, determine at what radial loaction you would place a Pitot tube if it is to measure the average velocity in the pipe. (b) Repeat part (a) for turbulent flow with Re = 10,000.



■ FIGURE P8.35

For laminar or turbulent flow,

$$Q = AV = \pi R^2 V = \int u\, dA = \int u(2\pi r\, dr) = 2\pi \int_{r=0}^{R} u\, r\, dr$$

a) Laminar flow:

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \left(\frac{r}{R}\right)^2\right] dr = 2\pi V_c \left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi \frac{R^2}{2} V_c$$

Thus, $V = \frac{1}{2} V_c$ For $u = V = \frac{V_c}{2}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{1}{2} = 1 - \left(\frac{r}{R}\right)^2, \text{ or } \left(\frac{r}{R}\right)^2 = \frac{1}{2} \quad \text{Thus, } r = \frac{1}{\sqrt{2}} R = \underline{\underline{0.707R}}$$

b) Turbulent flow

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \frac{r}{R}\right]^{1/5} dr = 2\pi R^2 V_c \int_0^1 \left(\frac{r}{R}\right)\left[1 - \left(\frac{r}{R}\right)\right]^{1/5} d\left(\frac{r}{R}\right)$$

Let $y \equiv 1 - \left(\frac{r}{R}\right)$ so that $\left(\frac{r}{R}\right) = 1 - y$ and $d\left(\frac{r}{R}\right) = -dy$

Thus,

$$\pi R^2 V = 2\pi R^2 V_c \int_{y=1}^{y=0} (1-y)\, y^{1/5} (-dy) = 2\pi R^2 V_c \int_0^1 \left(y^{1/5} - y^{6/5}\right) dy$$

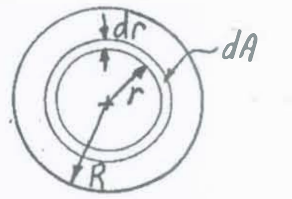
$$= 2\pi R^2 V_c \left[\frac{5}{6} - \frac{5}{11}\right] = 2\pi R^2 V_c \left(\frac{25}{66}\right)$$

or $V = \frac{50}{66} V_c$ For $u = V = \frac{50}{60}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{50}{66} = \left[1 - \frac{r}{R}\right]^{1/5} \quad \text{or} \quad \frac{r}{R} = 0.750 \text{ so that } r = \underline{\underline{0.750R}}$$

8.36

8.36 The kinetic energy coefficient, α, is defined in Eq. 5.86. Show that its value for a power-law turbulent velocity profile (Eq. 8.31) is given by $\alpha = (n+1)^3(2n+1)^3/[4n^4(n+3)(2n+3)]$.



From Eq. 5.86, $\alpha = \dfrac{\rho\int \bar{u}^3 dA}{\rho A V V^2}$ where V = average velocity, $A = \pi R^2$, and $\bar{u} = V_c\left[1 - \frac{r}{R}\right]^{\frac{1}{n}}$. From Example 8.4, $V = \dfrac{2n^2 V_c}{(n+1)(2n+1)}$ (0)

Thus, with $dA = 2\pi r\, dr$

$$\alpha = \frac{\int \bar{u}^3 dA}{AV^3}, \text{ where } \int \bar{u}^3 dA = 2\pi\int_{r=0}^{R} V_c^3\left[1-\frac{r}{R}\right]^{\frac{3}{n}} r\,dr = 2\pi R^2 V_c^3 \int_{y=0}^{1}[1-y]^{\frac{3}{n}}y\,dy \quad (1)$$

where $y = \frac{r}{R}$.

Let $x = 1-y$ so that $y = 1-x$ and $dy = -dx$

Hence,

$$\int_{y=0}^{1}[1-y]^{\frac{3}{n}}y\,dy = -\int_{x=1}^{0} x^{3/n}(1-x)\,dx = \int_0^1\left(x^{\frac{3}{n}} - x^{\frac{3}{n}+1}\right)dx$$

$$= \frac{n}{n+3}x^{\frac{n+3}{n}} - \frac{n}{2n+3}x^{\frac{2n+3}{n}}\Bigg|_{x=0}^{x=1}$$

Thus,

$$\int_{y=0}^{1}[1-y]^{\frac{3}{n}}y\,dy = \frac{n^2}{(n+3)(2n+3)} \quad (2)$$

From Eqs. (0), (1), and (2):

$$\alpha = \frac{2\pi R^2 V_c^3 \dfrac{n^2}{(n+3)(2n+3)}}{\pi R^2\left[\dfrac{2n^2 V_c}{(n+1)(2n+1)}\right]^3} = \underline{\underline{\frac{(n+1)^3(2n+1)^3}{4n^4(n+3)(2n+3)}}}$$

8.38

8.38 Determine the thickness of the viscous sublayer in a smooth 8-in.-diameter pipe if the Reynolds number is 25,000.

$\delta_s = \frac{5\nu}{u^*}$, where $u^* = \left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}}$ and $\tau_w = \frac{D\Delta p}{4\ell}$. Since $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$

we obtain $\tau_w = \frac{\rho f V^2}{8}$ and $u^* = \sqrt{\frac{f}{8}}\,V$

Thus,

$$\delta_s = \frac{5\,\nu}{\sqrt{\frac{f}{8}}\,V} = \frac{5\,\nu D}{\sqrt{\frac{f}{8}}\,VD}, \text{ or } \delta_s = \frac{5D}{Re\sqrt{\frac{f}{8}}} \qquad (1)$$

From Fig. 8.20, for a smooth pipe with $Re = 2.5\times10^4$, $f = 0.024$

Thus, from Eq. (1)

$$\delta_s = \frac{5\sqrt{8}\left(\frac{8}{12}\text{ft}\right)}{2.5\times10^4\sqrt{0.024}} = \underline{\underline{0.00243\text{ ft}}}$$

8.39

8.39 Water at 60 °F flows through a 6-in.-diameter pipe with an average velocity of 15 ft/s. Approximately what is the height of the largest roughness element allowed if this pipe is to be classified as smooth?

Let h = roughness height. Thus, $h = \delta_s$, where $\delta_s = \frac{5\nu}{u^*}$

with $u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2}$ and $\tau_w = \frac{D\Delta p}{4\ell}$. Since $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ we obtain

$\tau_w = \frac{\rho f V^2}{8}$ or $u^* = \sqrt{\frac{f}{8}}\,V$

For a smooth pipe with $Re = \frac{VD}{\nu} = \frac{(15\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}} = 6.19\times10^5$ we obtain from Fig. 8.20 $f = 0.0125$

Thus, $u^* = \left(\frac{0.0125}{8}\right)^{1/2}(15\frac{ft}{s}) = 0.593\frac{ft}{s}$

or

$$\delta_s = \frac{5\nu}{u^*} = \frac{5(1.21\times10^{-5}\frac{ft^2}{s})}{0.593\frac{ft}{s}} = \underline{\underline{1.02\times10^{-4}\,ft}}$$

8.41

8.41 A person with no experience in fluid mechanics wants to estimate the friction factor for 1-in.-diameter galvanized iron pipe at a Reynolds number of 8,000. They stumble across the simple equation of $f = 64/\mathrm{Re}$ and use this to calculate the friction factor. Explain the problem with this approach and estimate their error.

For $Re = 8000$ under standard conditions, the pipe flow will be turbulent.

<u>f - laminar</u>

$$f = 64/Re = 64/8000 = 8 \times 10^{-3}$$

<u>f - turbulent</u>

for galvinized iron pipe, $\varepsilon = 0.0005\ \mathrm{ft}$

so, $\varepsilon/D = 0.0005/(1/12) = 6 \times 10^{-3}$

Making use of the Moody chart

$f \simeq 0.04$

The error is in using the laminar equation to calculate the friction factor when the flow is turbulent.

$$\frac{f_{actual}}{f_{laminar}} = \frac{f_{turbulent}}{f_{laminar}} = \frac{0.04}{0.008} = 5$$

That is, the friction factor is 5 time greater than if the flow were laminar.

8.42

8.42 Water flows through a horizontal plastic pipe with a diameter of 0.2 m at a velocity of 10 cm/s. Determine the pressure drop per meter of pipe using the Moody chart.

The pressure drop in the pipe can be found from

$$\Delta P = f \frac{l}{D} \frac{\rho V^2}{2}$$

The friction factor is determined from the Moody chart.

$$Re = \frac{\rho V D}{\mu} = \frac{(999)(0.1)(0.2)}{1.12 \times 10^{-3}} = 1.8 \times 10^4$$

For plastic pipe, $\varepsilon = 0.0\,mm$

$\varepsilon / D = 0.0/0.2 = 0.0$

From the Moody chart

$f = 0.026$

So ΔP per meter ($l = 1m$)

$$\Delta P = (0.026)\left(\frac{1}{0.2}\right)\left[\frac{999(0.1)^2}{2}\right]$$

$\underline{\Delta P = 0.649}$ Pa per meter

8.43

8.43 For Problem 8.42, calculate the power lost to the friction per meter of pipe.

$\Delta P = 0.649$ Pa per meter of pipe, $V = 0.1 \, m/s$, $D = 0.2 \, m$

Based on equations in Ch.5, power can be found from

$\mathcal{P} = (\Delta P) Q$

$Q = VA = (0.1)\left(\frac{\pi}{4}(0.2)^2\right) = 3.14 \times 10^{-3} \, m^3/s$

$\mathcal{P} = (0.649)\left(3.14 \times 10^{-3}\right) = 2.04 \times 10^{-3} \, N \cdot m/s = \underline{\underline{2.04 \times 10^{-3} \, W}}$

8.44

8.44 Oil ($SG = 0.9$), with a kinematic viscosity of 0.007 ft^2/s, flows in a 3-in.-diameter pipe at 0.01 ft^3/s. Determine the head loss per unit length of this flow.

$h_L = f \frac{l}{D} \frac{V^2}{2g}$ where $l = 1\,ft$ for "per unit length of pipe".

Determine friction factor based on Re & ε/D

$Q = 0.01\ ft^3/s = VA$

$V = \dfrac{0.01}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 0.20\ ft/s$

$Re = \dfrac{VD}{\nu} = \dfrac{0.20(3/12)}{0.007} = 7.14$

Since Re is below 2100, the flow is laminar.

The friction factor can be determined from

$f = 64/Re = 64/7.14 = 8.96$

$h_L = (8.96)\dfrac{1}{(3/12)}\dfrac{(0.2)^2}{2(32.2)} = \underline{\underline{0.022\ ft}}$ per ft of pipe

8.45

8.45 Water flows through a 6-in.-diameter horizontal pipe at a rate of 2.0 cfs and a pressure drop of 4.2 psi per 100 ft of pipe. Determine the friction factor.

For a horizontal pipe $\Delta p = f \frac{\ell}{D} \frac{1}{2} \rho V^2$,

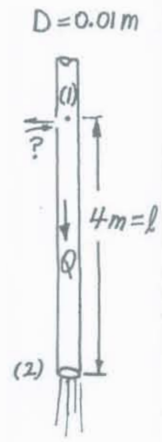
where $V = \frac{Q}{A} = \frac{2.0 \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{6}{12} ft\right)^2} = 10.2 \frac{ft}{s}$

Thus,

$$f = \frac{2 D \Delta p}{\rho \ell V^2} = \frac{2\left(\frac{6}{12} ft\right)\left(4.2 \times 144 \frac{lb}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)(100\ ft)\left(10.2 \frac{ft}{s}\right)^2} = \underline{\underline{0.0300}}$$

8.46

8.46 Water flows downward through a vertical 10-mm-diameter galvanized iron pipe with an average velocity of 5.0 m/s and exits as a free jet. There is a small hole in the pipe 4 m above the outlet. Will water leak out of the pipe through this hole, or will air enter into the pipe through the hole? Repeat the problem if the average velocity is 0.5 m/s.



$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}$, where $p_2=0$, $z_2=0$,

$z_1=4m$, $V_1=V_2=V$. Thus,

$\frac{p_1}{\gamma}=f\frac{\ell}{D}\frac{V^2}{2g}-z_1$, or $p_1=f\frac{\ell}{D}\frac{1}{2}\rho V^2-\gamma\ell$ With ε from Table 8.1, (1)

$\frac{\varepsilon}{D}=\frac{0.15\,mm}{10\,mm}=0.015$ so that with $Re=\frac{VD}{\nu}=\frac{(5\frac{m}{s})(0.01m)}{1.12\times10^{-6}\frac{m^2}{s}}=4.46\times10^4$

we obtain $f=0.045$ (see Fig. 8.20).

Thus, from Eq. (1)

$p_1=0.045\left(\frac{4m}{0.01\,m}\right)\frac{1}{2}(999\frac{kg}{m^3})(5\frac{m}{s})^2-9800\frac{N}{m^3}(4m)=1.86\times10^5\frac{N}{m^2}$

Since $p_1>0$, <u>water will leak out of the pipe when $V=5\frac{m}{s}$</u>

If $V=0.5\frac{m}{s}$, then $Re=4.46\times10^3$ and $f=0.052$

Thus, from Eq. (1)

$p_1=0.052\left(\frac{4m}{0.01m}\right)\frac{1}{2}(999\frac{kg}{m^3})(0.5\frac{m}{s})^2-9800\frac{N}{m^3}(4m)=-3.66\times10^4\frac{N}{m^2}$

Since $p_1<0$, <u>air will enter the pipe when $V=0.5\frac{m}{s}$</u>

Note: The above conclusion is valid regardless of the length, ℓ.

8.47

8.47 Air at standard conditions flows through an 8-in.-diameter, 14.6 ft-long, straight duct with the velocity versus pressure drop data indicated in the following table. Determine the average friction factor over this range of data.

V (ft/min)	Δp (in. water)
3950	0.35
3730	0.32
3610	0.30
3430	0.27
3280	0.24
3000	0.20
2700	0.16

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{where } V_1 = V_2 = V,\ \Delta p = p_1 - p_2,\ z_1 = z_2$$

Thus, $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ of $f = \frac{2\Delta p D}{\rho \ell V^2}$ where $\Delta p = \gamma_{H_2O} h$

or

$$f = \frac{2\left(\frac{h}{12}\text{ft}\right)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\text{ft}\right)}{\left(0.00238\frac{\text{slugs}}{\text{ft}^3}\right)(14.6\text{ft})\left(\frac{V}{60}\frac{\text{ft}}{\text{s}}\right)^2} = 7.18\times 10^5\frac{h}{V^2}, \quad \text{where } h \sim \text{in. of water},\ V \sim \frac{\text{ft}}{\text{min}}$$

Calculated values are given below:

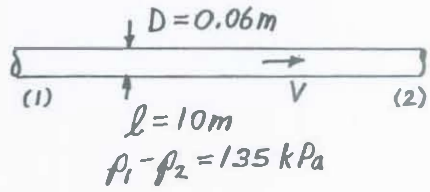
V, ft/min	h, in. water	f
3950	0.35	0.0161
3730	0.32	0.0165
3610	0.30	0.0165
3430	0.27	0.0165
3280	0.24	0.0160
3000	0.20	0.0160
2700	0.16	0.0158
	Average f =	0.0162

The average value of f is

$$f_{ave} = \underline{\underline{0.0162}}$$

8.48

8.48 Water flows through a horizontal 60-mm-diameter galvanized iron pipe at a rate of 0.02 m^3/s. If the pressure drop is 135 kPa per 10 m of pipe, do you think this pipe is **(a)** a new pipe, **(b)** an old pipe with a somewhat increased roughness due to aging, or **(c)** a very old pipe that is partially clogged by deposits? Justify your answer.



For the horizontal pipe ($z_1 = z_2$) with $V_1 = V_2$ the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \text{ reduces to } p_1 - p_2 = f\frac{\ell}{D}\frac{1}{2}\rho V^2$$

or

$$135\times10^3 \frac{N}{m^2} = f\frac{10m}{0.06m}\,\frac{1}{2}\left(999\frac{kg}{m^3}\right)\left(7.07\frac{m}{s}\right)^2, \text{ or } f = 0.0324$$

where we have used $V = \frac{Q}{A} = \frac{0.02\frac{m^3}{s}}{\frac{\pi}{4}(0.06m)^2} = 7.07\frac{m}{s}$

With $Re = \frac{VD}{\nu} = \frac{(7.07\frac{m}{s})(0.06m)}{1.12\times10^{-6}\frac{m^2}{s}} = 3.79\times10^5$ and $\frac{\varepsilon}{D} = \frac{0.15\text{ mm}}{60\text{ mm}} = 2.5\times10^{-3}$

for a new galvanized iron pipe (see Table 8.1), the friction factor should be (see Fig. 8.20) $f = 0.0255$. Since this is less than the actual value $f = 0.0324$, the pipe is not a new pipe.

With $Re = 3.79\times10^5$ and $f = 0.0324$ we obtain from Fig. 8.20 a relative roughness of $\frac{\varepsilon}{D} = 0.006$. This is approximately twice the roughness of a new pipe — certainly quite possible. A very old partially clogged pipe would have considerably greater head loss. Thus, the pipe is an old pipe with somewhat increased roughness.

8.49

8.49 Water flows at a rate of 10 gallons per minute in a new horizontal 0.75-in.-diameter galvanized iron pipe. Determine the pressure gradient, $\Delta p/\ell$, along the pipe.

$$Q = 10\,\frac{gal}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{231\,in.^3}{1\,gal}\right)\left(\frac{1\,gal}{1728\,in.^3}\right) = 0.0223\,\frac{ft^3}{s}$$

Thus,

$$V = \frac{Q}{A} = \frac{0.0223\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.75}{12}\,ft\right)^2} = 7.27\,\frac{ft}{s}$$

Now, for a horizontal pipe

$$\Delta p = f\,\frac{\ell}{D}\,\frac{1}{2}\rho V^2 \quad \text{where since}$$

$$Re = \frac{VD}{\nu} = \frac{7.27\,\frac{ft}{s}\left(\frac{0.75}{12}\,ft\right)}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 3.76\times10^4$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}\,ft\right)} = 0.008$$

it follows from Fig. 8.20 that $f = 0.037$

Thus,

$$\frac{\Delta p}{\ell} = \frac{0.037\,(1.94\,slugs/ft^3)\,(7.27\,ft/s)^2}{\left(\frac{0.75}{12}\,ft\right)(2)} = 30.4\,\frac{lb}{ft^3}\left(\frac{1\,ft^2}{144\,in.^2}\right)$$

$$= \underline{\underline{0.211\,psi/ft}}$$

8.50

8.50 Two equal length, horizontal pipes, one with a diameter of 1 in., the other with a diameter of 2 in., are made of the same material and carry the same fluid at the same flow rate. Which pipe produces the larger head loss? Justify your answer.

For either pipe $h_L = f\frac{\ell}{D}\frac{V^2}{2g}$, where $V = Q/A = Q/(\frac{\pi}{4}D^2)$.

Thus,

$$h_L = f\frac{\ell}{D}\left[4Q/(\pi D^2)\right]^2/2g = \frac{8}{\pi^2}f\frac{\ell}{D^5}Q^2/g$$

or

$$h_L = \left[\frac{8}{\pi^2}\frac{\ell Q^2}{g}\right]\frac{f}{D^5} \qquad (1)$$

Let $(\ )_1$ and $(\ )_2$ denote the 1 in. and 2 in. diameter pipes, respectively. Thus, with $Q_1 = Q_2$ and $\ell_1 = \ell_2$, Eq. (1) gives

$$\frac{h_{L1}}{h_{L2}} = \frac{(f_1/D_1^5)}{(f_2/D_2^5)} = \left(\frac{f_1}{f_2}\right)\left(\frac{D_2}{D_1}\right)^5 = \left(\frac{f_1}{f_2}\right)\left(\frac{2\text{ in.}}{1\text{ in.}}\right)^5$$

or

$$\frac{h_{L1}}{h_{L2}} = 32\left(\frac{f_1}{f_2}\right) \qquad (2)$$

Although $f_1 \neq f_2$ (because $Re_1 \neq Re_2$ and $\varepsilon/D_1 \neq \varepsilon/D_2$) the ratio f_1/f_2 would not be significantly different than 1, especially compared to the factor of 32 in Eq. (2). For example, assume $Re_1 = 10{,}000$ and $\varepsilon/D_1 = 0.001$ so that $f_1 = 0.033$ (see Fig. 8.20). Thus, since

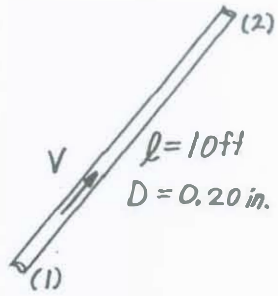
$Re = VD/\nu = (Q/\frac{\pi}{4}D^2)D/\nu = \frac{4Q}{\pi\nu}/D$ it follows that if $Re_1 = 10{,}000$, then $Re_2 = 5{,}000$ and $\varepsilon/D_2 = 0.0005$ if $\varepsilon/D_1 = 0.001$. Hence, $f_2 = 0.037$ so that $h_{L1}/h_{L2} = 32\,(0.033/0.037) = 28.5 \gg 1$.

Similar results would be true for other Re, ε/D values.

Thus, $h_{L1}/h_{L2} = 32\,(f_1/f_2) > 1$. The <u>smaller pipe</u> has the larger head loss.

8.52

8.52 Blood (assume $\mu = 4.5 \times 10^{-5}$ lb·s/ft^2, $SG = 1.0$) flows through an artery in the neck of a giraffe from its heart to its head at a rate of 2.5×10^{-4} ft^3/s. Assume the length is 10 ft and the diameter is 0.20 in. If the pressure at the beginning of the artery (outlet of the heart) is equivalent to 0.70 ft Hg, determine the pressure at the end of the artery when the head is **(a)** 8 ft above the heart, or **(b)** 6 ft below the heart. Assume steady flow. How much of this pressure difference is due to elevation effects, and how much is due to frictional effects?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } V_1 = V_2 = V \qquad (1)$$

and

$$V = \frac{Q}{A} = \frac{2.5\times10^{-4}\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.2}{12}ft\right)^2} = 1.146\,\frac{ft}{s} \qquad \text{Thus, } Re = \frac{\rho V D}{\mu}, \text{ or}$$

$$Re = \frac{(1.94\,\frac{slugs}{ft^3})(1.146\,\frac{ft}{s})(\frac{0.2}{12}ft)}{4.5\times10^{-5}\,\frac{lb\cdot s}{ft^2}} = 823$$ Hence, the flow is laminar with

$$f = \frac{64}{Re} = \frac{64}{823} = 0.0778$$

Also, $p_1 = \gamma_{Hg} h = (847\,\frac{lb}{ft^3})(0.70ft) = 593\,\frac{lb}{ft^2}$

Hence, from Eq. (1)

$$p_2 = p_1 - \gamma(z_2 - z_1) - f\frac{\ell}{D}\frac{1}{2}\rho V^2$$

a) With $z_2 - z_1 = 8ft$,

$$p_2 = 593\,\frac{lb}{ft^2} - (62.4\,\frac{lb}{ft^3})(8ft) - 0.0778\,\frac{10ft}{(\frac{0.2}{12}ft)}(\tfrac{1}{2})(1.94\,\frac{slugs}{ft^3})(1.146\,\frac{ft}{s})^2$$

$$= 593\,\frac{lb}{ft^2} - 499\,\frac{lb}{ft^2} - 59.5\,\frac{lb}{ft^2} = \underline{\underline{34.5\,\frac{lb}{ft^2}}}$$

Note: $-499\,\frac{lb}{ft^2}$ is due to elevation, -59.5 is due to friction.

b) With $z_2 - z_1 = -6ft$,

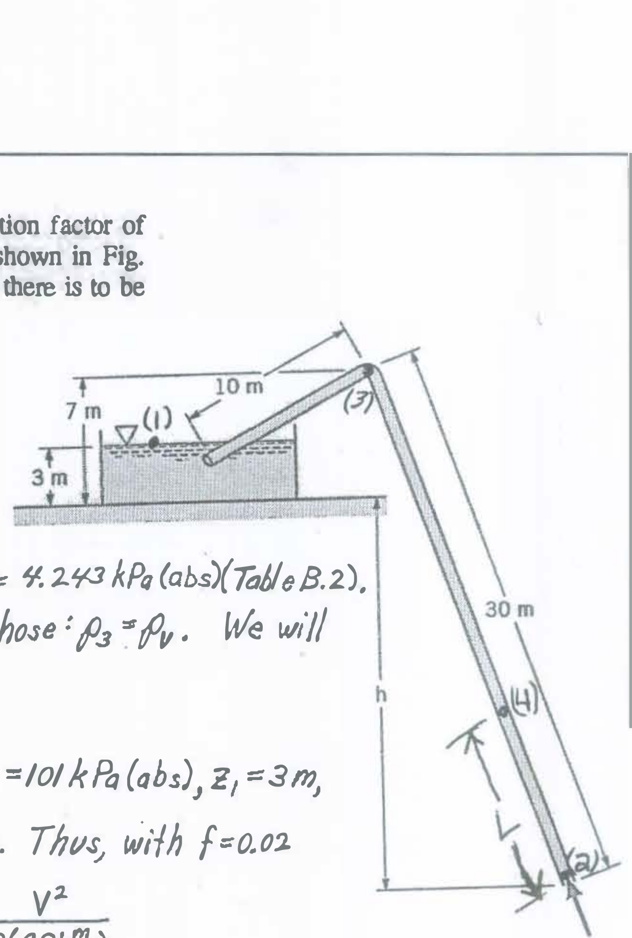
$$p_2 = 593\,\frac{lb}{ft^2} - (62.4\,\frac{lb}{ft^3})(-6ft) - 0.0778\,\frac{10ft}{(\frac{0.2}{12}ft)}(\tfrac{1}{2})(1.94\,\frac{slugs}{ft^3})(1.146\,\frac{ft}{s})^2$$

$$= 593\,\frac{lb}{ft^2} + 374\,\frac{lb}{ft^2} - 59.5\,\frac{lb}{ft^2} = \underline{\underline{908\,\frac{lb}{ft^2}}}$$

Note: $374\,\frac{lb}{ft^2}$ is due to elevation, $-59.5\,\frac{lb}{ft}$ is due to friction.

8.53

8.53 A 40-m-long, 12-mm-diameter pipe with a friction factor of 0.020 is used to siphon 30 °C water from a tank as shown in Fig. P8.53. Determine the maximum value of h allowed if there is to be no cavitation within the hose. Neglect minor losses.



The minimum pressure is the vapor pressure $p_v = 4.243\ kPa\,(abs)$ (Table B.2). Assume the minimum pressure is at the top of the hose: $p_3 = p_v$. We will check this assumption after we obtain h.

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 + f\frac{\ell}{D}\frac{V^2}{2g},$$ where $p_1 = 101\ kPa\,(abs)$, $z_1 = 3\ m$, $z_3 = 7\ m$, $V_1 = 0$, $V_3 = V$, and $p_3 = 4.243\ kPa\,(abs)$. Thus, with $f = 0.02$

$$\frac{(101 - 4.243)\frac{kN}{m^2}}{9.77\frac{kN}{m^3}} + 3\ m = 7\ m + \left(1 + 0.02\left(\frac{10\ m}{0.012\ m}\right)\right)\frac{V^2}{2(9.81\frac{m}{s^2})}$$

or

$V = 2.56\ \frac{m}{s}$

Obtain h from

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g},$$ where $p_2 = 0$, $V_2 = V = 2.56\ \frac{m}{s}$, $z_2 = -h$, and $\ell = 40\ m$. That is, with $p_1 = p_2 = 0$

$$3\ m = -h + \left(1 + 0.02\left(\frac{40\ m}{0.012\ m}\right)\right)\frac{(2.56\frac{m}{s})^2}{2(9.81\frac{m}{s^2})},$$ or $\underline{\underline{h = 19.6\ m}}$

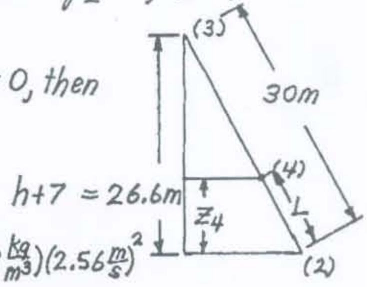
Check if minimum pressure occurs at (3). Consider point (4).

From $\frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{L}{D}\frac{V^2}{2g}$ with $p_2 = 0$, $V_2 = V_4 = V$

we obtain

$p_4 = \gamma(z_2 - z_4) + f\frac{L}{D}\frac{1}{2}\rho V^2$ If we use $z_2 = 0$, then

from the figure: $\frac{L}{z_4} = \frac{30}{26.6}$, or $L = 1.128\, z_4$



Thus,

$$p_4 = 9.80\frac{kN}{m^3}(-z_4) + (0.02)\left(\frac{1.128\, z_4}{0.012}\right)\left(\frac{1}{2}\right)\left(999\frac{kg}{m^3}\right)\left(2.56\frac{m}{s}\right)^2$$

or

$p_4 = (-9.80\times10^3 + 6.15\times10^3)\, z_4 = -3650\, z_4$

Thus, p_4 decreases as z_4 increases. That is, the minimum pressure occurs at section (3) as assumed.

8.54

8.54 Gasoline flows in a smooth pipe of 40-mm diameter at a rate of 0.001 m³/s. If it were possible to prevent turbulence from occurring, what would be the ratio of the head loss for the actual turbulent flow compared to that if it were laminar flow?

Let $(\ )_t$ denote the turbulent flow and $(\ )_\ell$ the laminar flow.

Thus, $h_{Lt} = f_t \frac{\ell}{D}\frac{V^2}{2g}$ and $h_{L\ell} = f_\ell \frac{\ell}{D}\frac{V^2}{2g}$ (1)

where

$$V = V_t = V_\ell = \frac{Q}{A} = \frac{0.001\,\frac{m^3}{s}}{\frac{\pi}{4}(0.04m)^2} = 0.796\,\frac{m}{s}$$

From Table 1.6 $\rho = 680\,\frac{kg}{m^3}$ and $\mu = 3.1 \times 10^{-4}\,\frac{N\cdot s}{m^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(680\frac{kg}{m^3})(0.796\frac{m}{s})(0.04m)}{3.1\times10^{-4}\,\frac{N\cdot s}{m^2}} = 6.98\times10^4$$

Hence, from Fig. 8.20, for a smooth pipe $f_t = 0.0192$

while for laminar flow $f_\ell = \frac{64}{Re} = \frac{64}{6.98\times10^4} = 9.16\times10^{-4}$

Thus, from Eq. (1)

$$\frac{h_{Lt}}{h_{L\ell}} = \frac{f_t}{f_\ell} = \frac{0.0192}{9.16\times10^{-4}} = \underline{\underline{21.0}}$$

8.55

8.55 A 3-ft-diameter duct is used to carry ventilating air into a vehicular tunnel at a rate of 9000 ft³/min. Tests show that the pressure drop is 1.5 in. of water per 1500 ft of duct. What is the approximate size of the equivalent roughness of the surface of the duct?

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1=z_2,\ V_1=V_2, \text{ and} \qquad (1)$$

$$p_1-p_2=\gamma_{H_2O}\,h=\left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{1.5}{12}\,ft\right)=7.80\,\frac{lb}{ft^2}$$

Also, $V=\frac{Q}{A}=\frac{\left(9000\,\frac{ft^3}{min}\right)\left(\frac{1\,min}{60\,s}\right)}{\frac{\pi}{4}(3\,ft)^2}=21.2\,\frac{ft}{s}$

Thus, from Eq. (1) $\quad p_1-p_2=f\frac{\ell}{D}\frac{1}{2}\rho V^2$ or

$$f=\frac{2D(p_1-p_2)}{\rho\ell V^2}=\frac{2(3\,ft)\left(7.80\,\frac{lb}{ft^2}\right)}{\left(2.38\times10^{-3}\,\frac{slugs}{ft^3}\right)(1500\,ft)\left(21.2\,\frac{ft}{s}\right)^2}=\underline{\underline{0.0292}}$$

From Fig. 8.20 with $f=0.0292$ and $Re=\frac{VD}{\nu}=\frac{\left(21.2\,\frac{ft}{s}\right)(3\,ft)}{1.57\times10^{-4}\,\frac{ft^2}{s}}=4.05\times10^5$

we obtain $\frac{\varepsilon}{D}=0.0044$ $\quad$ Thus, $\varepsilon=0.0044\,(3\,ft)=\underline{\underline{0.0132\ ft}}$

8.57

8.57 An optional method of stating minor losses from pipe components is to express the loss in terms of equivalent length; the head loss from the component is quoted as the length of straight pipe with the same diameter that would generate an equivalent loss. Develop an equation for the equivalent length, ℓ_{eq}.

$$h_{L\,minor} = K_L \, V^2/2g$$

$$h_{L\,major} = f \frac{\ell}{D} \, V^2/2g$$

The pipe length from the major loss can be used to represent the equivalent length, ℓ_{eq}.

$$f \frac{\ell_{eq}}{D} \, V^2/2g = K_L \, V^2/2g$$

$$f \frac{\ell_{eq}}{D} = K_L$$

$$\ell_{eq} = \frac{K_L D}{f}$$

8.58

8.58 Given 90° threaded elbows used in conjunction with copper pipe (drawn tubing) of 0.75-in. diameter, convert the loss for a single elbow to equivalent length of copper pipe for wholly turbulent flow.

$$\ell_{eq} = \frac{K_L D}{f}$$

For 90° threaded elbow, $K_L = 1.5$

For copper pipe (drawn tubing), $\varepsilon = 0.000005\ ft$

So

$$\frac{\varepsilon}{D} = \frac{0.000005}{\left(0.75/12\right)} = 8 \times 10^{-5}$$

From Moody chart (wholly turbulent flow)

$$f \cong 0.0115$$

$$\ell_{eq} = \frac{(1.5)\left(0.75/12\right)}{0.0115} = \underline{\underline{8.15\ ft}}$$

8.59

8.59 Based on Problem 8.57, develop a graph to predict equivalent length, ℓ_{eq}, as a function of pipe diameter for a 45° threaded elbow connecting copper piping (drawn tubing) for wholly turbulent flow.

$$\ell_{eq} = \frac{K_L D}{f}$$

For 45° threaded elbow, $K_L = 0.4$

For copper tubing (drawn tubing), $\varepsilon = 0.0015$ mm

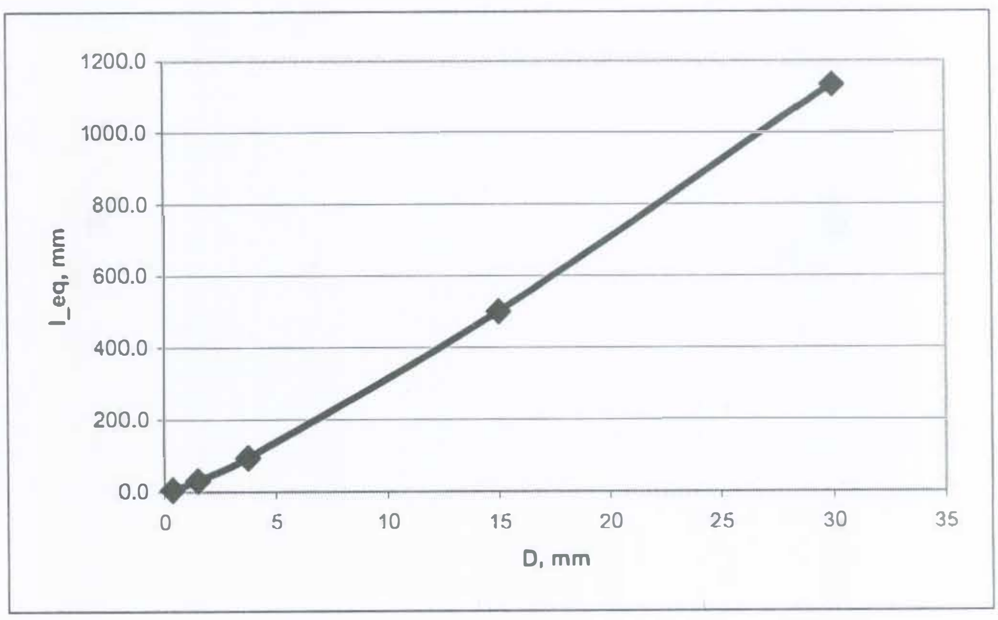
To calculate f, use alternate form

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

For wholly turbulent flow, assume $Re = 8 \times 10^7$

This is large enough Re to make f essentially independent of Re (see Moody chart, Fig. 8.20).

D(mm)	e/D	f	l_eq (mm)
30	0.00005	0.010602	1131.8
15	0.0001	0.012025	499.0
3.75	0.0004	0.015932	94.2
1.5	0.001	0.019678	30.5
0.375	0.004	0.028474	5.3



8.60

8.60 A regular 90° threaded elbow is used to connect two straight portions of 4-in.-diameter galvanized iron pipe. (a) If the flow is assumed to be wholly turbulent, determine the equivalent length of straight pipe for this elbow. (b) Does a pipe fitting such as this elbow have a significant or negligible effect on the flow? Explain.

(a) $h_L = K_L \frac{V^2}{2g}$, where from Table 8.2 $K_L = 1.5$ for a 90° threaded elbow.

Also,

$\ell_{eq} = \frac{K_L D}{f}$, where from Table 8.1 $\varepsilon = 0.0005\text{ ft}$ for a galvanized iron pipe. Thus, with $\varepsilon/D = 0.0005\text{ ft}/(4/12)\text{ft} = 0.0015$ and a very large Reynolds number (i.e. wholly turbulent flow) it follows from Fig. 8.20 that $f = 0.021$.

Thus,

$$\ell_{eq} = \frac{1.5(4/12)\text{ft}}{0.021} = \underline{\underline{23.8\text{ ft}}}$$

(b) In general $h_L = K_L \frac{V^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} = \left(K_L + f\frac{\ell}{D}\right)\frac{V^2}{2g}$

or

$h_L = f\frac{(\ell + \ell_{eq})}{D}\frac{V^2}{2g}$ since $K_L = f\frac{\ell_{eq}}{D}$

Thus, whether or not a pipe filling such as this elbow has a significant effect on the flow depends on the relative size of ℓ_{eq} (= 23.8 ft for this case) and the pipe length, ℓ. If $\ell_{eq} \ll \ell$, then the fitting is negligible.

8.61

8.61 To conserve water and energy, a "flow reducer" is installed in the shower head as shown in Fig. P8.61 If the pressure at point (1) remains constant and all losses except for that in the "flow reducer" are neglected, determine the value of the loss coefficient (based on the velocity in the pipe) of the "flow reducer" if its presence is to reduce the flowrate by a factor of 2. Neglect gravity.

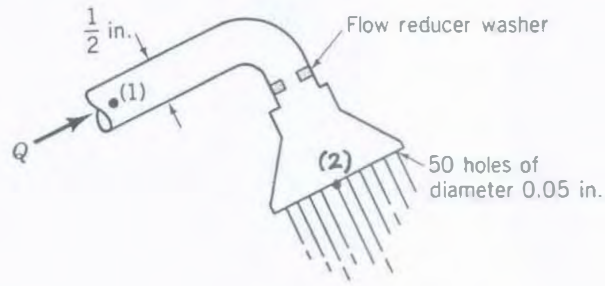


FIGURE P8.61

Without the reducer $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$, where $p_2 = 0$, $z_1 = z_2$ and

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4Q}{\pi\left(\frac{0.5}{12}\,ft\right)^2} = 733Q$$

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{50\pi\left(\frac{0.05}{12}\,ft\right)^2} = 1467Q \quad (V_1 \text{ and } V_2 \sim \tfrac{ft}{s} \text{ and } Q \sim \tfrac{ft^3}{s})$$

Thus, $p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2}\rho(1467^2Q^2 - 733Q^2) = 8.07\times10^5\rho Q^2 \ \frac{lb}{ft^2}$, where $\rho \sim \frac{slug}{ft^3}$, $Q \sim \frac{ft^3}{s}$ (1)

With the flow reducer the flowrate is reduced by a factor of two.

Thus, $V_1 = \frac{1}{2}(733Q)$ and $V_2 = \frac{1}{2}(1467Q)$ with (2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + K_L\frac{V_1^2}{2g} \quad \text{or} \quad p_1 = \frac{1}{2}\rho\left(V_2^2 + (K_L - 1)V_1^2\right) \qquad (3)$$

Hence, by combing Eqs. (1), (2), and (3) we obtain

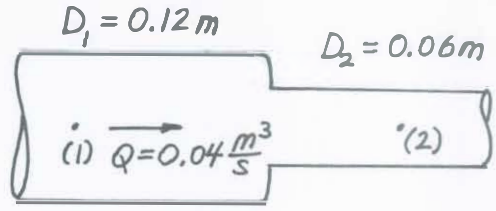
$$8.07\times10^5\rho Q^2 = \frac{1}{2}\rho\left[\left(\frac{1467}{2}Q\right)^2 + (K_L - 1)\left(\frac{733}{2}Q\right)^2\right]$$

or

$$\underline{\underline{K_L = 9.00}}$$

8.62

8.62 Water flows at a rate of 0.040 m^3/s in a 0.12-m-diameter pipe that contains a sudden contraction to a 0.06-m-diameter pipe. Determine the pressure drop across the contraction section. How much of this pressure difference is due to losses and how much is due to kinetic energy changes?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + K_L \frac{V_2^2}{2g}, \quad \text{where } z_1 = z_2 \qquad (1)$$

and

$$V_1 = \frac{Q}{A_1} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.12\,m)^2} = 3.54\,\frac{m}{s}, \quad V_2 = \frac{Q}{A_2} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.06\,m)^2} = 14.1\,\frac{m}{s}$$

Thus, with $\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{0.06\,m}{0.12\,m}\right)^2 = 0.25$ we obtain from Fig. 8.30

$K_L = 0.40$

Hence, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\rho\left[K_L V_2^2 + V_2^2 - V_1^2\right] = \frac{1}{2}\left(999\,\frac{kg}{m^3}\right)\left[0.40\left(14.1\,\frac{m}{s}\right)^2 + \left(14.1\,\frac{m}{s}\right)^2 - \left(3.54\,\frac{m}{s}\right)^2\right]$$

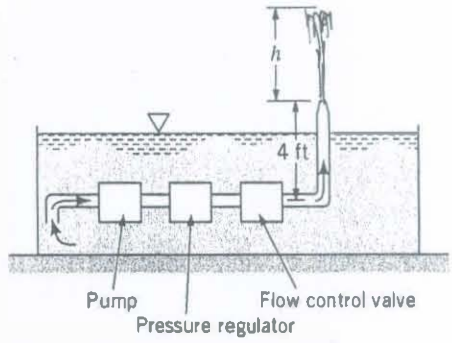
or

$$p_1 - p_2 = 39.7 \times 10^{3}\,\frac{N}{m^2} + 93.0 \times 10^3\,\frac{N}{m^2} = \underline{\underline{133\,kPa}}$$

This represents a $\underline{\underline{39.7\text{ kPa}}}$ drop from losses and a $\underline{\underline{93.0\text{ kPa}}}$ drop due to an increase in kinetic energy.

8.64

8.64 (See "New hi-tech fountains," Section 8.5.) The fountain shown in Fig. P8.64 is designed to provide a stream of water that rises $h = 10$ ft to $h = 20$ ft above the nozzle exit in a periodic fashion. To do this the water from the pool enters a pump, passes through a pressure regulator that maintains a constant pressure ahead of the flow control valve. The valve is electronically adjusted to provide the desired water height. With $h = 10$ ft the loss coefficient for the valve is $K_L = 50$. Determine the valve loss coefficient needed for $h = 20$ ft. All losses except for the flow control valve are negligible. The area of the pipe is 5 times the area of the exit nozzle.



■ FIGURE P8.64

For any height h,

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}, \text{ where } z_1=0,\ z_2=h+4\text{ft},\ p_2=0,\ V_2=0,$$
$$\text{and } h_L=K_L V_1^2/2$$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}-K_L\frac{V_1^2}{2g}=z_2 \qquad (1)$$

For $h=10\text{ft}$: $(K_L=50)$

$$\frac{p_1}{\gamma}=z_2-\frac{V_1^2}{2g}+K_L\frac{V_1^2}{2g}=(10\text{ft}+4\text{ft})+(50-1)\frac{V_1^2}{2g} \qquad (2)$$

Also, from (3) to (2):

$$\frac{p_3}{\gamma}+z_3+\frac{V_3^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}, \text{ where } p_2=p_3=0,\ z_2-z_3=h, \text{ and } V_2=0$$

Thus,

$$\frac{V_3^2}{2g}=h \quad \text{or} \quad V_3=\sqrt{2gh} \qquad (3)$$

so for $h=10\text{ft}$, $V_3=\sqrt{2(32.2\frac{\text{ft}}{\text{s}^2})(10\text{ft})}=25.4\frac{\text{ft}}{\text{s}}$

Also, $V_1A_1=V_3A_3$ so that $V_1=\frac{A_3}{A_1}V_3=(\frac{1}{5})(25.4\frac{\text{ft}}{\text{s}})=5.08\frac{\text{ft}}{\text{s}}$

Hence, Eq. (2) gives

$$\frac{p_1}{\gamma}=14\text{ft}+49(5.08\tfrac{\text{ft}}{\text{s}})^2/(2(32.2\tfrac{\text{ft}}{\text{s}^2}))=33.6\text{ft}$$

For $h=20\text{ft}$, from Eq. (3): $V_3=\sqrt{(2)(32.2\frac{\text{ft}}{\text{s}^2})(20\text{ft})}=35.9\frac{\text{ft}}{\text{s}}$

Hence, $V_1=\frac{A_3}{A_1}V_3=(\frac{1}{5})(35.9\frac{\text{ft}}{\text{s}})=7.18\frac{\text{ft}}{\text{s}}$

Since p_1 is constant (independent of h), the value $\frac{p_1}{\gamma}=33.6$ ft obtained above for $h=10$ft is also valid for $h=20$ft. Thus, with $z_2=h+4\text{ft}=20\text{ft}+4\text{ft}=24\text{ft}$, Eq. (1) is:

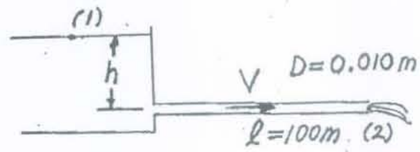
$$33.6\text{ft}+(7.18\tfrac{\text{ft}}{\text{s}})^2/(2(32.2\text{ft/s}^2))-K_L(7.18\tfrac{\text{ft}}{\text{s}})^2/(2(32.2\text{ft/s}^2))=24\text{ft}$$

or

$$K_L=\underline{\underline{13.0}}$$

8.65

***8.65** Water flows from a large open tank through a sharp-edged entrance and into a galvanized iron pipe of length 100 m and diameter 10 mm. The water exits the pipe as a free jet at a distance h below the free surface of the tank. Plot a log–log graph of the flowrate, Q, as a function of h for $0.1 \le h \le 10$ m.



$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+K_L\right)\frac{V^2}{2g}$, where $p_1=0$, $V_1=0$, $z_1=h$,
$p_2=0$, $V_2=V$, and $z_2=0$ Thus,

$h=\left(1+f\frac{\ell}{D}+K_L\right)\frac{V^2}{2g}$ where from Fig. 8.25 $K_L=0.5$

Hence,

$h=\left(1+f\left(\frac{100\,m}{0.01\,m}\right)+0.5\right)\frac{V^2}{2(9.81\frac{m}{s^2})}$ or $19.6h=(1.5+10{,}000f)V^2$, with (1)
$h\sim m$, $V\sim\frac{m}{s}$

Also, $\frac{\varepsilon}{D}=\frac{0.15\,mm}{10\,mm}=0.015$ (see Table 8.2)

and

$Re=\frac{VD}{\nu}=\frac{(0.01m)V}{1.12\times10^{-6}\frac{m^2}{s}}$ or $Re=8930V$ (2)

If the flow is laminar, then $f=\frac{64}{Re}$ (i.e. $Re<2100$) (3)

If the flow is turbulent, then from Eq. 8.35b

$$\frac{1}{\sqrt{f}}=-1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right]$$

That is

$$\frac{1}{\sqrt{f}}=-1.8\log\left[4.05\times10^{-3}+\frac{6.9}{Re}\right] \qquad (4)$$

The maximum h for laminar flow occurs when $Re=2100$, or from Eq. (2)

$V=\frac{2100}{8930}=0.235\frac{m}{s}$ and $f=\frac{64}{2100}=0.0304$. Thus, from Eq. (1)

$h=\frac{(1.5+10{,}000(0.0304))(0.235\frac{m}{s})^2}{19.6\frac{m}{s^2}}=0.861\,m$

Thus, for $h<0.861\,m$ the flow is laminar. For $h>0.861$ assume flow is turbulent.

For $0.1\,m\le h\le 10\,m$ solve Eqs. (1), (2), and (3) or (4) depending if $h<0.861\,m$ or $h>0.861\,m$ to obtain V. Then

$Q=\frac{\pi}{4}D^2V=\frac{\pi}{4}(0.01m)^2V=7.85\times10^{-5}V\ \frac{m^3}{s}$, where $V\sim\frac{m}{s}$ (5)

For laminar flow (i.e., $h\le0.861\,m$) Eqs. (1), (2), and (3) give

$19.6h=\left[1.5+10^4\frac{64}{8930V}\right]V^2$

or

$V^2+47.8V-13.1h=0$, which can be solved using the quadratic equation to give

$V=-23.9\pm\left[571+13.1h\right]^{1/2}$ Since $V>0$ we can disregard the "–" root.

(con't)

Thus, using Eq. (5)

$$Q = 7.85\times10^{-5}\left[-23.9+(571+13.1h)^{1/2}\right] \text{ for } 0 \le h \le 0.861\,\text{m} \quad (6)$$

This equation was used in a MS Excel spreadsheet to find Q as a function of h for laminar flow.

NOTE: The coefficients of Eq. (6) must be very precisely given because for small values of h, $(571+13.1h)^{1/2} \cong -23.9$

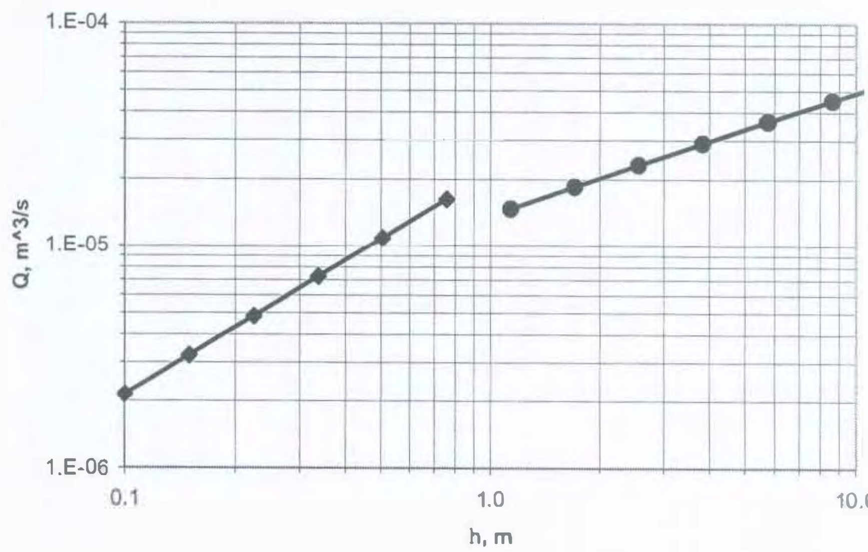
So in the spreadsheet

$$Q = 7.85\times10^{-5}\left[-23.88951+(570.7087+13.066667h)^{1/2}\right]$$

For $h > 0.861\,\text{m}$, Eqns. (1), (2), and (4) were used in the spreadsheet to manually iterate on f. Eqn (5) was used to find Q(h).

Insert a guess value (e.g. f = 0.02) in the f(guess) cell. A new f value will be calculated in the f(new) cell. Use this new value as the updated f(guess). Continue until f(guess) = f(new).

h, m	f(guess)	V, m/s	Re	f(new)	Q, m^3/s
0.100					2.15E-06
0.150					3.22E-06
0.225					4.82E-06
0.338					7.23E-06
0.506					1.08E-05
0.759					1.62E-05
1.139	0.0639	1.87E-01	1667	0.0639	1.47E-05
1.709	0.0604	2.35E-01	2100	0.0604	1.85E-05
2.563	0.0575	2.95E-01	2636	0.0575	2.32E-05
3.844	0.0551	3.69E-01	3298	0.0551	2.90E-05
5.767	0.0531	4.61E-01	4114	0.0531	3.62E-05
8.650	0.0515	5.73E-01	5116	0.0515	4.50E-05
12.975	0.0501	7.11E-01	6353	0.0501	5.58E-05



8.66

8.66 Air flows through the mitered bend shown in Fig. P8.66 at a rate of 5.0 cfs. To help straighten the flow after the bend, a set of 0.25-in.-diameter drinking straws is placed in the pipe as shown. Estimate the extra pressure drop between points (1) and (2) caused by these straws.

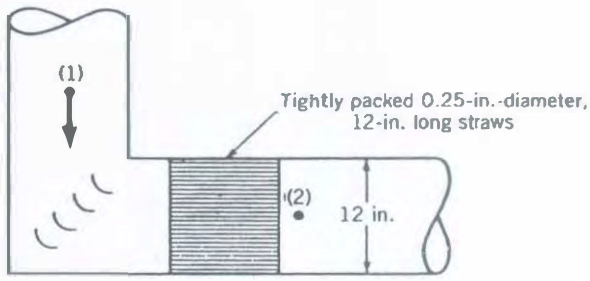


FIGURE P8.66

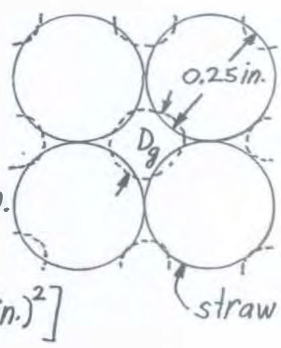
The extra pressure drop, Δp, is equal to the pressure drop through the length of the straws minus the pressure drop in that 12 in. length of the pipe without the straws. That is

$\Delta p = \Delta p_s - \Delta p_{ns}$, where $\Delta p_{ns} = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ with $V = \frac{Q}{A} = \frac{5\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{12}{12}ft)^2} = 6.37\frac{ft}{s}$

Also, $Re = \frac{VD}{\nu} = \frac{(6.37\frac{ft}{s})(1ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 4.06\times10^4$. If we assume the pipe is smooth, it follows from Fig. 8.20 that $f = 0.0215$. Thus,

$$\Delta p_{ns} = 0.0215\left(\frac{12\ in.}{12\ in.}\right)\left(\frac{1}{2}\right)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right)\left(6.37\frac{ft}{s}\right)^2 = 1.04\times10^{-3}\frac{lb}{ft^2} \qquad (1)$$

With the straws in place, $\Delta p_s = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ where the values of f, D, and V and different than those used above. In general the flow geometry is quite complex — flow through the straws and flow in the gaps between the straws. For simplicity, assume the gaps act as a circular flow area of diameter $D_g = \frac{3}{8}D = \frac{3}{8}(0.25\,in.) = 0.0938\,in.$



Thus, in each 0.5 in. by 0.5 in. cross section there are 4 straws, or a total of $N = 4\frac{straws}{(0.25\,in^2)}\left[\frac{\pi}{4}(12\,in.)^2\right]$

i.e. $N = 1810$ straws.

If the flow is laminar, then $Q \sim D^4$ so that $\frac{Q_{gap}}{Q_{straw}} = \left(\frac{0.0938\,in.}{0.25\,in.}\right)^4 = 0.0198$

That is, only about 2% of the flow is in the gap region — neglect this amount.

Thus, $V = \frac{Q}{NA} = \frac{5\frac{ft^3}{s}}{1810\left(\frac{\pi}{4}\left(\frac{0.25}{12}ft\right)^2\right)} = 8.10\frac{ft}{s}$

Hence,

$$Re = \frac{VD}{\nu} = \frac{(8.10\frac{ft}{s})(\frac{0.25}{12}ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 1070 < 2100,$$ the flow is laminar with

$f = \frac{64}{Re} = \frac{64}{1070} = 0.0598$, or $\Delta p_s = 0.0598\left(\frac{12\,in.}{0.25\,in.}\right)\left(\frac{1}{2}\right)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right)\left(8.10\frac{ft}{s}\right)^2$

or $\Delta p_s = 0.224\frac{lb}{ft^2}$ Hence, when combined with result (1)

$$\Delta p = \Delta p_s - \Delta p_{ns} = (0.224 - 0.00104)\frac{lb}{ft^2} = \underline{\underline{0.223\frac{lb}{ft^2}}}$$

8.67

8.67 Repeat Problem 8.66 if the straws are replaced by a piece of porous foam rubber that has a loss coefficient equal to 5.4.

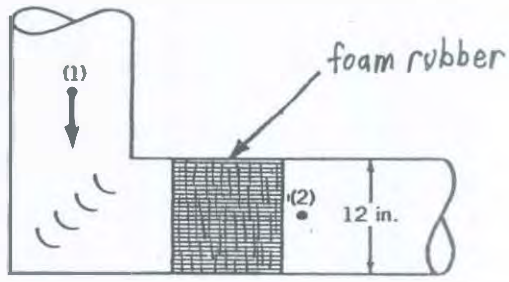


FIGURE P8.67

The extra pressure drop, Δp, is equal to the pressure drop through the length of foam rubber minus the pressure drop in that 12 in. length of the pipe without the foam. That is,

$\Delta p = \Delta p_f - \Delta p_{nf}$, where $\Delta p_{nf} = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ with $V = \frac{Q}{A} = \frac{5\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{12}{12}ft)^2} = 6.37\frac{ft}{s}$

Also, $Re = \frac{VD}{\nu} = \frac{(6.37\frac{ft}{s})(1ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 4.06\times10^4$. If we assume the pipe is smooth, it follows from Fig. 8.20 that $f = 0.0215$. Thus,

$$\Delta p_{nf} = 0.0215\left(\frac{12\text{ in.}}{12\text{ in.}}\right)\left(\frac{1}{2}\right)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right)\left(6.37\frac{ft}{s}\right)^2 = 1.04\times10^{-3}\frac{lb}{ft^2} \qquad (1)$$

The pressure drop due to the foam is

$$\Delta p_f = K_L\frac{1}{2}\rho V^2$$

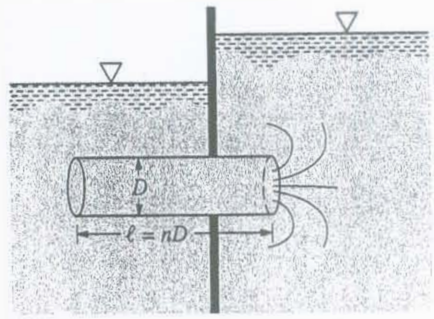
$$= 5.4\left(\frac{1}{2}\right)\left(0.00238\frac{slugs}{ft^3}\right)\left(6.37\frac{ft}{s}\right)^2 = 0.261\frac{lb}{ft^2}$$

Thus,

$$\Delta p = \Delta p_f - \Delta p_{nf} = 0.261\frac{lb}{ft^2} - 0.00104\frac{lb}{ft^2} = \underline{\underline{0.260\frac{lb}{ft^2}}}$$

8.68

8.68 As shown in Fig. P8.68, water flows from one tank to another through a short pipe whose length is n times the pipe diameter. Head losses occur in the pipe and at the entrance and exit. (See Video V8.10) Determine the maximum value of n if the major loss is to be no more than 10% of the minor loss and the friction factor is 0.02.



■ FIGURE P8.68

If $h_{L_{major}} = 10\% h_{L_{minor}}$, then

$$10 f \frac{\ell}{D} \frac{V^2}{2g} = \Sigma K_L \frac{V^2}{2g} \quad \text{or} \quad \frac{\ell}{D} = \frac{\Sigma K_L}{10 f} \qquad (1)$$

where $\Sigma K_L = K_{L_{entrance}} + K_{L_{exit}} = 0.8 + 1 = 1.8$

Thus, with $f = 0.02$ and $\ell = nD$ Eq. (1) becomes

$$\frac{nD}{D} = \frac{1.8}{10(0.02)}$$

or

$$\underline{\underline{n = 9}}$$

8.69

8.69 Air flows through the fine mesh gauze shown in Fig. P8.69 with an average velocity of 1.50 m/s in the pipe. Determine the loss coefficient for the gauze.

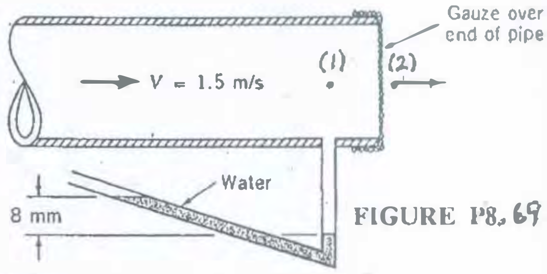


FIGURE P8.69

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+K_L\frac{V^2}{2g}\text{, where } z_1=z_2\text{, } V_1=V_2=V=1.5\,\frac{m}{s}$$

Thus, $K_L=\frac{2(p_1-p_2)}{\rho V^2}$ where $p_2=0$ and $p_1=8\,mm$ water

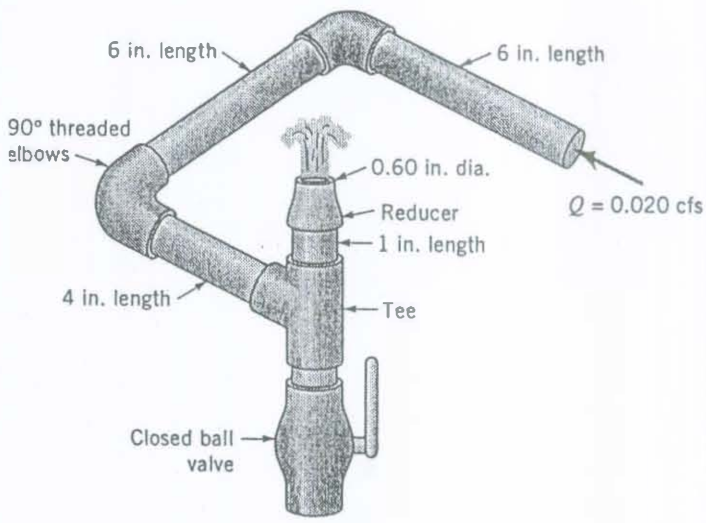
or $p_1=(8\times10^{-3}\,m)(9.80\times10^3\,\frac{N}{m^3})=78.4\,\frac{N}{m^2}$

Hence,

$$K_L=\frac{2\,(78.4\,\frac{N}{m^2})}{(1.23\,\frac{kg}{m^3})(1.5\,\frac{m}{s})^2}=\underline{\underline{56.7}}$$

8.70

8.70 Water flows steadily through the 0.75-in. diameter galvanized iron pipe system shown in Video V8.14 and Fig. P8.70 at a rate of 0.020 cfs. Your boss suggests that friction losses in the straight pipe sections are negligible compared to losses in the threaded elbows and fittings of the system. Do you agree or disagree with your boss? Support your answer with appropriate calculations.



■ FIGURE P8.70

Major loss $= f\frac{\ell}{D}\frac{V^2}{2g}$ where

$\ell = (6+6+4+1)\,\text{in.} = 17\,\text{in.},\quad D = 0.75\,\text{in.}$

and

$$V = \frac{Q}{A} = \frac{0.02\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.75/12)^2\,ft^2} = 6.52\,\frac{ft}{s}$$

Thus, with

$$Re = \frac{VD}{\nu} = \frac{6.52\,\frac{ft}{s}\left(\frac{0.75}{12}ft\right)}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 3.37\times10^4 \text{ and}$$

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}ft\right)} = 8\times10^{-3}$$ (see Table 8.1) we obtain (see Fig. 8.20)

$$f = 0.038 \text{ so that } f\frac{\ell}{D}\frac{V^2}{2g} = 0.038\,\frac{17\,in.}{0.75\,in.}\frac{V^2}{2g} = 0.861\,\frac{V^2}{2g} \qquad (1)$$

Also,

$$\text{Minor loss} = \sum K_L\frac{V^2}{2g} = \left[2(1.5) + 2 + 0.15\right]\frac{V^2}{2g} = 5.15\,\frac{V^2}{2g} \qquad (2)$$

90° elbow   tee   reducer with $\frac{A_2}{A_1} = \left(\frac{0.6\,in.}{0.75\,in.}\right)^2 = 0.64$

(see Fig. 8.26)

Thus, from Eqs. (1) and (2):

$$\frac{\text{major loss}}{\text{minor loss}} = \frac{0.861\,\frac{V^2}{2g}}{5.15\,\frac{V^2}{2g}} = 0.167 = 16.7\%$$

Probably <u>disagree</u> with boss because pipe friction is about 17% of other losses.

8.72

8.72 Given two rectangular ducts with equal cross-sectional area, but different aspect ratios (width/height) of 2 and 4, which will have the greater frictional losses? Explain your answer.

The duct with the greater losses is the one with the largest headloss per length, h_L/ℓ, where $h_L = f\frac{\ell}{D_h}\frac{V^2}{2g}$. If the areas are equal, then the velocities are equal since $V = Q/A$.

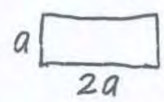
Let $(\ )_2$ and $(\ )_4$ denote ducts with aspect ratios of 2 and 4, respectively. Thus,

$$(h_L/\ell)_4 = \frac{f_4}{D_{h4}}\frac{V_4^2}{2g} \quad \text{and} \quad (h_L/\ell)_2 = \frac{f_2}{D_{h2}}\frac{V_2^2}{2g}, \quad \text{where } V_2 = V_4.$$

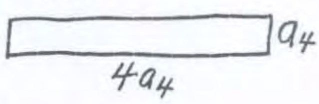
Hence,

$$(h_L/\ell)_4 / (h_L/\ell)_2 = \frac{f_4}{D_{h4}} \Big/ \frac{f_2}{D_{h2}} = \frac{f_4}{f_2}\frac{D_{h2}}{D_{h4}} \qquad (1)$$

Let $A_2 = (2a)a$



and $A_4 = (4a_4)a_4$



Thus, since $A_2 = A_4$,

$$2a^2 = 4a_4^2, \quad \text{or} \quad a_4 = \frac{1}{\sqrt{2}}a$$

and

$$D_{h2} = 4A_2/P_2 = 4(2a^2)/[4a+2a] = \frac{4}{3}a = 1.33a \qquad (2)$$

and

$$D_{h4} = 4A_4/P_4 = 4(2a^2)/\left[\frac{2}{\sqrt{2}}a + \frac{8}{\sqrt{2}}a\right] = \frac{4\sqrt{2}}{5}a = 1.13a \qquad (3)$$

so that

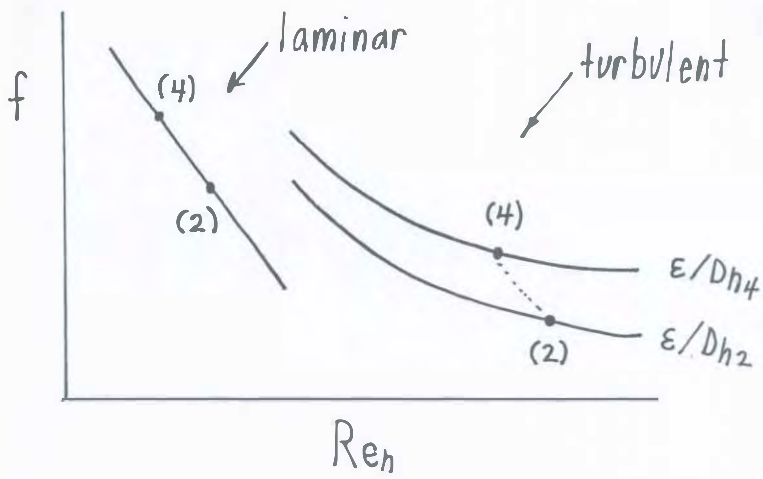
$$\frac{D_{h2}}{D_{h4}} = \frac{\frac{4}{3}a}{\frac{4\sqrt{2}}{5}a} = \frac{5}{3\sqrt{2}} = 1.179 \quad \text{so that Eq. (1) becomes}$$

$$(h_L/\ell)_4 / (h_L/\ell)_2 = 1.179\frac{f_4}{f_2} \qquad (4)$$

In general, $f = f(Re, \frac{\varepsilon}{D})$ in such a way that if $\frac{\varepsilon}{D}$ increases, f increases and if Re decreases, f increases. This is seen from the Moody chart as indicated below.

(con't)

8.72 (con't)



For a given ε, $\left(\frac{\varepsilon}{D_h}\right)_4 > \left(\frac{\varepsilon}{D_h}\right)_2$ since $D_{h_4} < D_{h_2}$ (see Eqs. (2) and (3)).

Also, since $Re_h = VD_h/\nu$ it follows that

$Re_{h_4} < Re_{h_2}$ since $D_{h_4} < D_{h_2}$ and $V_2 = V_4$.

Thus, whether the flow is laminar or turbulent it follows that $f_4 > f_2$. It follows from Eq. (4) that

$(h_L/\ell)_4 / (h_L/\ell_2) > 1$

That is, the duct with the $\underline{\underline{\text{aspect ratio of 4}}}$ has the greater head loss.

8.73

8.73 Air at standard temperature and pressure flows at a rate of 7.0 cfs through a horizontal, galvanized iron duct that has a rectangular cross-sectional shape of 12 in. by 6 in. Estimate the pressure drop per 200 ft of duct.

For a horizontal duct $\Delta p = \gamma h_L = f \frac{\ell}{D_h} \frac{1}{2} \rho V^2$, where $V = \frac{Q}{A}$ (1)

or

$$V = \frac{7 \frac{ft^3}{s}}{(12\,in.)(6\,in.)\left(\frac{1\,ft^2}{144\,in.^2}\right)} = 14.0 \frac{ft}{s} \quad \text{and} \quad Re_h = \frac{V D_h}{\nu}$$

with

$$D_h = \frac{4A}{P} = \frac{4\,(0.5\,ft^2)}{(2+1)\,ft} = 0.667\,ft$$

Thus,

$$Re_h = \frac{(14.0 \frac{ft}{s})(0.667\,ft)}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 5.95 \times 10^4$$

Also, for galvanized iron $\varepsilon = 0.0005\,ft$, or $\frac{\varepsilon}{D_h} = \frac{0.0005\,ft}{0.667\,ft} = 0.000750$

From Fig. 8.20 we obtain $f = 0.0227$

Thus, from Eq. (1) with $\ell = 200\,ft$,

$$\Delta p = (0.0227) \frac{200\,ft}{0.667\,ft} \frac{1}{2} (2.38 \times 10^{-3} \frac{slug}{ft^3})(14.0 \frac{ft}{s})^2 = 1.59 \frac{lb}{ft^2} = \underline{\underline{0.0110\ psi}}$$

8.74

8.74 Air flows through a rectangular galvanized iron duct of size 0.30 m by 0.15 m at a rate of 0.068 m^3/s. Determine the head loss in 12 m of this duct.

$$h_L = f\frac{\ell}{D_h}\frac{V^2}{2g}\text{, where } D_h = \frac{4A}{P} = \frac{4(0.3\,m)(0.15\,m)}{2[0.3\,m+0.15\,m]} = 0.2\,m$$

and

$$V = \frac{Q}{A} = \frac{0.068\frac{m^3}{s}}{(0.3\,m)(0.15\,m)} = 1.51\frac{m}{s} \quad \text{Also, } Re_h = \frac{VD_h}{\nu} = \frac{(1.51\frac{m}{s})(0.2\,m)}{1.46\times10^{-5}\frac{m^2}{s}} = 20{,}700$$

and from Table 8.1,

$$\frac{\varepsilon}{D_h} = \frac{0.15\times10^{-3}\,m}{0.2\,m} = 7.5\times10^{-4} \quad \text{Hence, from Fig. 8.20 } f = 0.027$$

so that

$$h_L = (0.027)\left(\frac{12\,m}{0.2\,m}\right)\frac{(1.51\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} = \underline{\underline{0.188\,m}}$$

8.75

8.75 Air at standard conditions flows through a horizontal 1 ft by 1.5 ft rectangular wooden duct at a rate of 5000 ft^3/min. Determine the head loss, pressure drop, and power supplied by the fan to overcome the flow resistance in 500 ft of the duct.

$$h_L = f\frac{\ell}{D_h}\frac{V^2}{2g}\text{, where } V = \frac{Q}{A} = \frac{(5000\frac{ft^3}{min})(\frac{1\,min}{60\,s})}{(1\,ft)(1.5\,ft)} = 55.6\frac{ft}{s}$$

$$\text{and } D_h = \frac{4A}{P} = \frac{4(1\,ft)(1.5\,ft)}{2[1\,ft+1.5\,ft]} = 1.2\,ft$$

Also,

$$Re_h = \frac{VD_h}{\nu} = \frac{(55.6\frac{ft}{s})(1.2\,ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 4.25\times10^5 \quad \text{and from Table 8.1}$$

$\varepsilon \approx 0.0006\,ft$ to $0.003\,ft$. Use an "average" $\varepsilon = 0.0018\,ft$ so that

$$\frac{\varepsilon}{D_h} = \frac{0.0018\,ft}{1.2\,ft} = 0.0015 \quad \text{Thus, from Fig. 8.20 } f = 0.022\text{, or}$$

$$h_L = (0.022)\left(\frac{500\,ft}{1.2\,ft}\right)\frac{(55.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} = \underline{\underline{440\,ft}}$$

For this horizontal pipe $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$, where $z_1 = z_2$ and $V_1 = V_2$.

Thus, $p_1 - p_2 = \gamma h_L = (7.65\times10^{-2}\frac{lb}{ft^3})(440\,ft) = 33.7\frac{lb}{ft^2} = \underline{\underline{0.234\,psi}}$

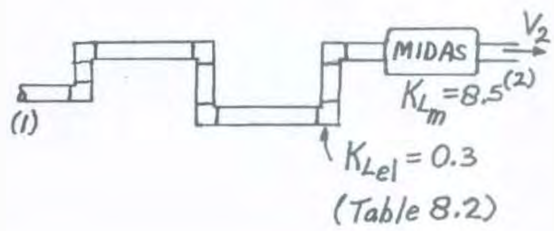
$$\mathcal{P} = \gamma Q h_L = Q(p_1 - p_2) = (5000\frac{ft^3}{min})(\frac{1\,min}{60\,s})(33.7\frac{lb}{ft^2}) = (2810\frac{ft\cdot lb}{s})\left[\frac{1\,hp}{(550\frac{ft\cdot lb}{s})}\right]$$

or

$$\mathcal{P} = \underline{\underline{5.11\,hp}}$$

8.76

8.76 Assume a car's exhaust system can be approximated as 14 ft of 0.125-ft-diameter cast-iron pipe with the equivalent of six 90° flanged elbows and a muffler. (See Video V8.12) The muffler acts as a resistor with a loss coefficient of $K_L = 8.5$. Determine the pressure at the beginning of the exhaust system if the flowrate is 0.10 cfs, the temperature is 250 °F, and the exhaust has the same properties as air.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g},$$ where $z_1=z_2$, $p_2=0$, and

$$V=V_1=V_2=\frac{Q}{A}=\frac{0.1\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.125\,ft)^2}=8.15\,\frac{ft}{s}$$

Thus,

$$p_1=\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{1}{2}\rho V^2,$$ where $$\rho=\frac{p}{RT}=\frac{(14.7\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1716\,\frac{ft\cdot lb}{slug\cdot{}^\circ R})(460+250)^\circ R}=1.74\times10^{-3}\,\frac{slug}{ft^3}$$

Also, $\frac{\varepsilon}{D}=\frac{0.00085\,ft}{0.125\,ft}=0.0068$ (Table 8.1)

so that with $Re=\frac{\rho VD}{\mu}=\frac{(1.74\times10^{-3}\,\frac{slug}{ft^3})(8.15\,\frac{ft}{s})(0.125\,ft)}{4.7\times10^{-7}\,\frac{lb\cdot s}{ft^2}}=3770$ we obtain from Fig. 8.20, $f=0.047$

Hence,

$$p_1=\left(0.047\left(\frac{14\,ft}{0.125\,ft}\right)+6(0.3)+8.5\right)\left(\frac{1}{2}\right)\left(1.74\times10^{-3}\,\frac{slug}{ft^3}\right)\left(8.15\,\frac{ft}{s}\right)^2$$

$$=\underline{\underline{0.899\,\frac{lb}{ft^2}}}$$

8.77

8.77 The pressure at section (2) shown in Fig. P8.77 is not to fall below 60 psi when the flowrate from the tank varies from 0 to 1.0 cfs and the branch line is shut off. Determine the minimum height, h, of the water tank under the assumption that (a) minor losses are negligible, (b) minor losses are not negligible.

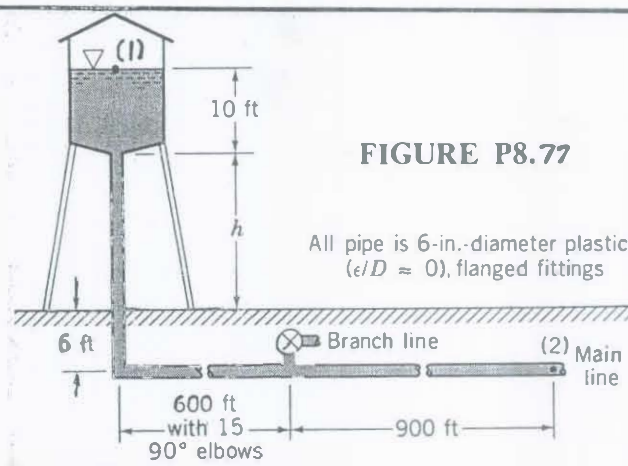


FIGURE P8.77

All pipe is 6-in.-diameter plastic ($\varepsilon/D \approx 0$), flanged fittings

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}\text{, where } p_1=0,\ V_1=0,\ z_1=16\text{ ft}+h,$$

and $z_2=0$ Thus, with $V=V_2$

$$16+h=\frac{p_2}{\gamma}+\frac{V^2}{2g}+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}$$. Note: h must be no less than that with $p_{2\,min}=60\text{psi}$ and $Q_{max}=1\text{ cfs}$, or

$$V_2=V=\frac{Q}{A_2}=\frac{1\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{6}{12}ft\right)^2}=5.09\frac{ft}{s}$$

Hence,

$$h=-16ft+\frac{(60\frac{lb}{in^2})(144\frac{in^2}{ft^2})}{62.4\frac{lb}{ft^3}}+\left(1+f\left(\frac{h+6+600+900}{\frac{6}{12}}\right)+\Sigma K_L\right)\frac{(5.09\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$h=122.5+\left(1+f\left(\frac{1506+h}{0.5}\right)+\Sigma K_L\right)(0.402)\ \text{ft, where } h\sim\text{ft} \qquad (1)$$

With $\frac{\varepsilon}{D}=0$ and $Re=\frac{VD}{\nu}=\frac{(5.09\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}}=2.10\times10^5$ we obtain

$f=0.0155$ (see Fig. 8.20)

a) Neglect minor losses ($\Sigma K_L=0$):

From Eq. (1)

$$h=122.5+\left(1+(0.0155)\left(\frac{1506+h}{0.5}\right)\right)(0.402)$$

or $\underline{\underline{h=143\text{ ft}}}$

b) Include minor losses:

$$\Sigma K_L=K_{L\,entrance}+15K_{L\,elbow}+K_{L\,tee}=0.5+15(0.3)+0.2=5.2$$

(see Table 8.2, assume flanged fittings)

Thus, from Eq. (1)

$$h=122.5+\left(1+(0.0155)\left(\frac{1506+h}{0.5}\right)+5.2\right)(0.402)$$

or

$\underline{\underline{h=146\text{ ft}}}$

Note: For this case minor losses are not very important.

8.78

8.78 Repeat Problem 8.77 with the assumption that the branch line is open so that half of the flow from the tank goes into the branch, and half continues in the main line.

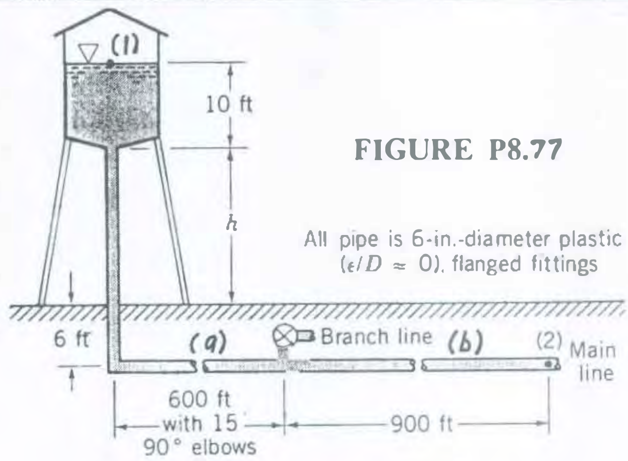


FIGURE P8.77

For the flow from (1) to (2):

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f_a\frac{\ell_a}{D_a}+\Sigma K_{L_a}\right)\frac{V_a^2}{2g}+\left(f_b\frac{\ell_b}{D_b}+\Sigma K_{L_b}\right)\frac{V_b^2}{2g} \quad (1)$$

where $(\ )_a$ and $(\ )_b$ denote pipes "a" and "b" as indicated in the figure.

Thus, with $p_1=0$, $V_1=0$, $z_1=16\,ft+h$, $z_2=0$, and $p_2=60\,psi$. Also,

$$V_a=\frac{Q_a}{A_a}=\frac{1\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=5.09\frac{ft}{s},\quad V_b=\frac{Q_b}{A_b}=\frac{0.5\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=2.55\frac{ft}{s},\ \text{Eq.(1) becomes}$$

$$16+h=\frac{(60\frac{lb}{in^2})(144\frac{in^2}{ft^2})}{62.4\frac{lb}{ft^3}}+\left(1+f_a\left(\frac{h+6+600}{\frac{6}{12}}\right)+\Sigma K_{L_a}\right)\frac{(5.09\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$
$$+\left(f_b\left(\frac{900}{\frac{6}{12}}\right)+\Sigma K_{L_b}\right)\frac{(2.55\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$h=122.5+\left(1+f_a\left(\frac{606+h}{0.5}\right)+\Sigma K_{L_a}\right)(0.402)+(1800f_b+\Sigma K_{L_b})(0.101),\ \text{where } h\sim ft \quad (2)$$

With $\frac{\varepsilon}{D}=0$, $Re_a=\frac{V_a D_a}{\nu}=\frac{(5.09\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}}=2.10\times10^5$, and

$Re_b=\frac{V_b D_b}{\nu}=\frac{1}{2}Re_a=1.05\times10^5$ we obtain $f_a=0.0155$ and $f_b=0.0175$ (Fig. 8.20)

a) Neglect minor losses ($\Sigma K_{L_a}=\Sigma K_{L_b}=0$):

From Eq. (2)

$$h=122.5+\left(1+(0.0155)\left(\frac{606+h}{0.5}\right)\right)(0.402)+(1800(0.0175)(0.101)$$

or

$$\underline{\underline{h=135\ ft}}$$

b) Include minor losses:

$\Sigma K_{L_a}=K_{L_{entrance}}+15K_{L_{elbow}}=0.5+15(0.3)=5.0$ (see Table 8.2; assume flanged fittings)

and

$\Sigma K_{L_b}=K_{L_{tee}}=0.2$

From Eq. (2)

$$h=122.5+\left(1+(0.0155)\left(\frac{606+h}{0.5}\right)+5.0\right)(0.402)+(1800(0.0175)+0.2)(0.101)$$

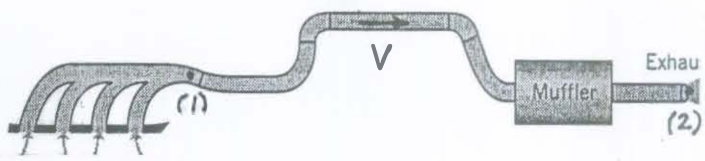
or

$$\underline{\underline{h=137\,ft}}$$

Note: For this case minor losses are not very important.

8.79

8.79 The exhaust from your car's engine flows through a complex pipe system as shown in Fig. P8.79 and **Video V8.5**. Assume that the pressure drop through this system is Δp_1 when the engine is idling at 1000 rpm at a stop sign. Estimate the pressure drop (in terms of Δp_1) with the engine at 3000 rpm when you are driving on the highway. List all the assumptions that you made to arrive at your answer.



■ FIGURE P8.79

For steady flow,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

Assume $z_1 = z_2$ and $V_1 = V_2$ so that with $h_L = \left[f\frac{\ell}{D} + K_L \right] \frac{V^2}{2g}$ and $\Delta p \equiv p_1 - p_2$ we obtain

$$\Delta p = \gamma h_L = \gamma \left(f\frac{\ell}{D} + K_L \right) \frac{V^2}{2g} = \frac{1}{2} \rho V^2 \left(f\frac{\ell}{D} + K_L \right)$$

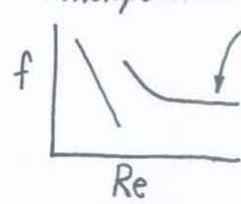
Hence,

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = \frac{\frac{1}{2} \rho_{3000} V_{3000}^2 \left(f_{3000} \frac{\ell}{D} + K_L \right)}{\frac{1}{2} \rho_{1000} V_{1000}^2 \left(f_{1000} \frac{\ell}{D} + K_L \right)}$$

Assume $\rho_{1000} = \rho_{3000}$ and $f_{1000} = f_{3000}$ (i.e. f independent of Re)

Thus,

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = \left(\frac{V_{3000}}{V_{1000}} \right)^2$$



But $V = \frac{Q}{A}$ where Q is assumed proportional to engine rpm.

That is $V_{3000} = 3\, V_{1000}$ so that

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = (3)^2 = \underline{\underline{9}}$$

8.80

8.80 According to fire regulations in a town, the pressure drop in a commercial steel horizontal pipe must not exceed 1.0 psi per 150 ft of pipe for flowrates up to 500 gal/min. If the water temperature is above 50° F, can a 6-in-diameter pipe be used?

Determine the pressure drop in a 6in. diameter pipe.

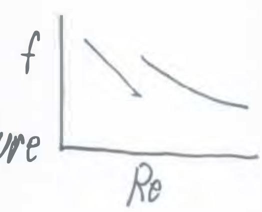
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } V_1 = V_2 \text{ and } z_1 = z_2.$$

Thus

$$\frac{p_1 - p_2}{\gamma} = f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } f = f(Re, \tfrac{\varepsilon}{D}). \qquad (1)$$

From Table 8.1, $\varepsilon = 0.00015\,ft$ so that $\frac{\varepsilon}{D} = \frac{1.5\times10^{-4}}{(6/12\,ft)} = 3\times10^{-4}$

The largest $p_1 - p_2$ will occur with the largest f, which occurs with the smallest Re, or largest ν. Since the viscosity of water increases a the temperature decreases, we consider the coldest case — $T = 50°F$.



From Table B.1, at $50°F$, $\gamma = 62.41\,lb/ft^3$ and $\nu = 1.407\times10^{-5}\,\frac{ft^2}{s}$

Also,

$$V = \frac{Q}{A} = \frac{(500\,\frac{gal}{min})(\frac{1\,min}{60\,s})(231\,\frac{in.^3}{gal})(\frac{1\,ft^3}{1728\,in^3})}{\frac{\pi}{4}(\frac{6}{12}\,ft)^2} = 5.67\,\frac{ft}{s}$$

Thus,

$$Re = \frac{VD}{\nu} = \frac{(5.67\,\frac{ft}{s})(6/12\,ft)}{1.407\times10^{-5}\,\frac{ft^2}{s}} = 2.01\times10^5$$

Hence, with $Re = 2.01\times10^5$ and $\frac{\varepsilon}{D} = 3\times10^{-4}$ we obtain from Fig. 8.20,

$f = 0.018$

Therefore, from Eq.(1),

$$\frac{p_1 - p_2}{\gamma} = 0.018\,\frac{(150\,ft)}{(6/12\,ft)}\,\frac{(5.67\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = 2.70\,ft$$

so that

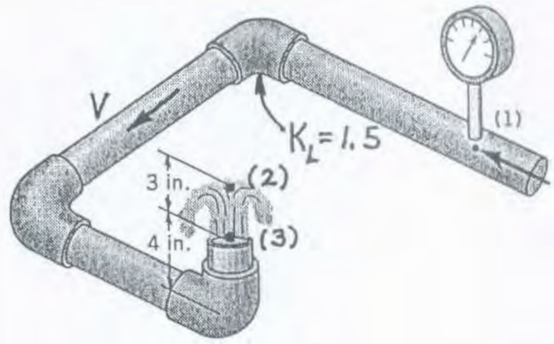
$$p_1 - p_2 = (2.70\,ft)(62.4\,\frac{lb}{ft^3}) = 168\,\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) = 1.17\,psi > 1.0\,psi$$

A 6in. diameter pipe requires slightly more than the allowed 1.0psi per 150ft.

Thus, $\underline{\underline{\text{no}}}$, a 6in. pipe cannot be used. The minimum diameter can be shown to be $D = 0.513\,ft = 6.37\,in.$

8.81

8.81 As shown in Video V8.14 and Fig. P8.81 water "bubbles up" 3 in. above the exit of the vertical pipe attached to three horizontal pipe segments. The total length of the 0.75-in.-diameter galvanized iron pipe between point (1) and the exit is 21 in. Determine the pressure needed at point (1) to produce this flow.



■ FIGURE P8.81

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = 0$, $p_2 = 0$, $V_2 = 0$ Thus,

(1) $$\frac{p_1}{\gamma} = z_2 + h_L - \frac{V_1^2}{2g} \quad \text{where } V_1 = V_3 = V$$

With no head loss from (3) to (2) and $p_2 = p_3 = V_2 = 0$ we obtain

$$\frac{V_3^2}{2g} + z_3 = z_2, \text{ or } V_3 = \sqrt{2g(z_2 - z_3)} = \sqrt{2(32.2\tfrac{ft}{s^2})\left(\tfrac{3}{12}ft\right)} = 4.01\tfrac{ft}{s}$$

Thus,

$$Re = \frac{VD}{\nu} = \frac{V_3 D}{\nu} = \frac{4.01\frac{ft}{s}\left(\frac{0.75}{12}ft\right)}{1.21\times10^{-5}\frac{ft^2}{s}} = 2.07\times10^4$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}\right)ft} = 0.008 \quad \text{(see Table 8.1), so that (see Fig. 8.20)}$$

$$f = 0.039$$

Also, $h_L = f\frac{\ell}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$ where $\sum K_L = 3(1.5) = 4.5$

Hence, Eq. (1) becomes

$$\frac{p_1}{\gamma} = z_2 + \left[f\frac{\ell}{D} + \sum K_L\right]\frac{V^2}{2g} - \frac{V_1^2}{2g} \quad \text{where } V_1 = V$$

or

$$\frac{p_1}{\gamma} = \frac{7}{12}ft + \left[0.039\,\frac{21\,in.}{0.75\,in.} + 4.5 - 1\right]\frac{\left(4.01\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = (0.583 + 1.147)ft$$

$$= 1.73\,ft$$

Thus,

$$p_1 = (62.4\tfrac{lb}{ft^3})(1.73\,ft) = 108\tfrac{lb}{ft^2} = \underline{0.750\,psi}$$

8.82

8.82 Water at 10 °C is pumped from a lake as shown in Fig. P8.82. If the flowrate is 0.011 m³/s, what is the maximum length inlet pipe, ℓ, that can be used without cavitation occurring?

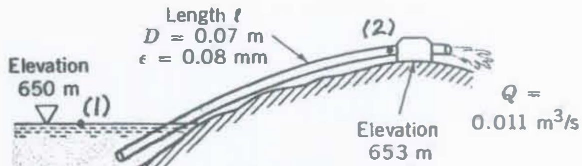


FIGURE P8.82

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}, \text{ where } p_1=101\text{ kPa}, z_1=650\text{m} \quad (1)$$

$V_1=0$, $V_2=V$, $z_2=653\text{m}$, and from Table B.2 $p_2=p_v=1.228\text{ kPa}$

Also, $V=\frac{Q}{A}=\frac{0.011\frac{m^3}{s}}{\frac{\pi}{4}(0.07m)^2}=2.86\frac{m}{s}$ so that

$Re=\frac{VD}{\nu}=\frac{(2.86\frac{m}{s})(0.07m)}{1.307\times10^{-6}\frac{m^2}{s}}=1.53\times10^5$. With this Re and from Table 8.1 with

$\frac{\epsilon}{D}=\frac{0.08mm}{70mm}=0.00114$ we obtain $f=0.0216$ (see Fig. 8.20)

Hence, with $\Sigma K_L=0.8$ for the entrance, Eq. (1) becomes

$$\frac{(101-1.228)\times10^3\frac{N}{m^2}}{9.80\times10^3\frac{N}{m^3}}+650m=653m+\left(1+(0.0216)\left(\frac{\ell}{0.07m}\right)+0.8\right)\frac{(2.86\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}$$

or $\underline{\underline{\ell=50.0\text{ m}}}$

8.83

8.83 Water flows through the pipe system shown in Fig. P8.83 at a rate of 0.30 ft^3/s. The pipe diameter is 2 in., and its roughness is 0.002 in. The loss coefficient for each of the five filters is 6.0, and all other minor losses are negligible. Determine the power added to the water by the pump if the pressure immediately before the pump is to be the same as that immediately after the last filter. The length of the pipe between these two locations is 80 ft.

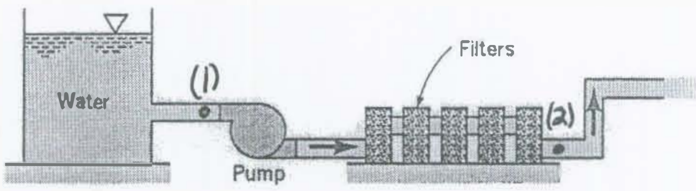


FIGURE P8.83

From the energy equation

$$h_p = h_L = \left(f\frac{l}{D} + K_L\right)V^2/2g$$

$$Q = VA, \quad V = 0.30 \Big/ \left(\frac{\pi}{4}(2/12)^2\right) = 13.75\ \text{ft/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(1.94)(13.75)(2/12)}{2.34\times10^{-5}} = 1.9\times10^5$$

$$\varepsilon/D = \frac{0.002}{2} = 1\times10^{-3}$$

From the Moody chart, $f \cong 0.0215$

So

$$h_p = \left[0.0215\frac{80}{2/12} + (5)(6)\right]\frac{(13.75)^2}{2(32.2)}$$

$$= [10.32 + 30](2.94)$$

(Note: the filters produce ~3x the pipe loss)

$$h_p = 118.54\ \text{ft}$$

Calculate the power

$$\dot{W} = \gamma Q h_p = (62.4)(0.3)(118.54)$$

$$= 2219.1\ \frac{\text{ft lb}}{\text{s}} \times \frac{1\ \text{hp}}{550\ \frac{\text{ft lb}}{\text{s}}} = \underline{4.03\ \text{hp}}$$

8.84

8.84 Water at 40 °F flows through the coils of the heat exchanger as shown in Fig. P8.84 at a rate of 0.9 gal/min. Determine the pressure drop between the inlet and outlet of the horizontal device.

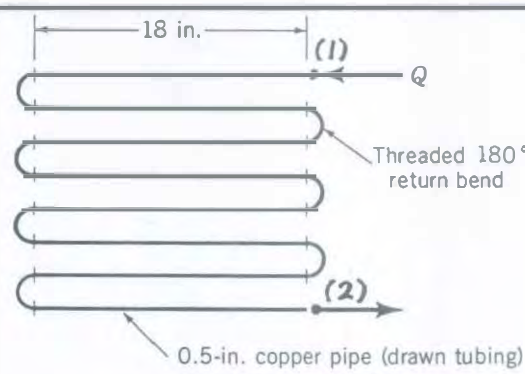


FIGURE P8.84

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1=z_2,$$

$$V=V_1=V_2=\frac{Q}{A}=\frac{(0.9\frac{gal}{min})(231\frac{in.^3}{gal})(\frac{1\,ft^3}{1728\,in.^3})(\frac{1\,min}{60\,s})}{\frac{\pi}{4}(\frac{0.5}{12}\,ft)^2}=1.47\,\frac{ft}{s}$$

Thus,

$$p_1-p_2=\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{1}{2}\rho V^2, \text{ with } \ell=8\left(\frac{18}{12}\,ft\right)=12\,ft \qquad (1)$$

and $\Sigma K_L=7(1.5)=10.5$ (see Table 8.2)

Also, from Table 8.1 $\frac{\varepsilon}{D}=(0.000005\,ft/(0.5/12\,ft))=1.2\times10^{-4}$

and $Re=\frac{VD}{\nu}=\frac{(1.47\frac{ft}{s})(\frac{0.5}{12}\,ft)}{1.66\times10^{-5}\frac{ft^2}{s}}=3690$ (see Table 8.1 for ν)

Hence, from Fig. 8.20

$f=0.041$

and from Eq. (1)

$$p_1-p_2=\left(0.041\left(\frac{12\,ft}{\frac{0.5}{12}\,ft}\right)+10.5\right)\left(\frac{1}{2}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(1.47\,\frac{ft}{s}\right)^2$$

or

$$p_1-p_2=46.8\,\frac{lb}{ft^2}=\underline{\underline{0.325\,psi}}$$

8.85

8.85 For the flow in Problem 8.84, ethylene glycol is added to the water for freeze protection if the temperature drops below the freezing point. The density is unchanged, and all flow conditions are the same except that the viscosity of the mixture has changed to 0.01 Ns/m² at the given temperature. Recalculate the pressure drop between inlet and outlet. Discuss how this loss will change if the fluid temperature does drop below freezing.

First, convert the viscosity to BG units using Table 1.4

$$\mu = 0.01 \frac{N \cdot s}{m^2} (2.089 \times 10^{-2}) = 2.09 \times 10^{-4} \frac{lb\,s}{ft^2}$$

From Table B.1, $\rho = 1.94$ slugs/ft³

So,

$$\nu = \frac{\mu}{\rho} = \frac{2.09 \times 10^{-4} \frac{lb \cdot s}{ft^2}}{1.94 \text{ slugs}/ft^3} = 1.077 \times 10^{-4} \text{ ft}^2/s$$

Calculate an updated Reynolds number with $V = 1.47$ ft/s (see Prob. 8.85)

$$Re = \frac{VD}{\nu} = \frac{(1.47 \text{ ft/s})(0.5/12 \text{ ft})}{1.077 \times 10^{-4} \text{ ft}^2/s} = 569$$

Therefore, the new flow is laminar

$$f = 64/Re = 64/569 = 0.112$$

From Problem 8.84

$$P_1 - P_2 = \left(f \frac{\ell}{D} + \Sigma K_L\right) \frac{1}{2} \rho V^2$$

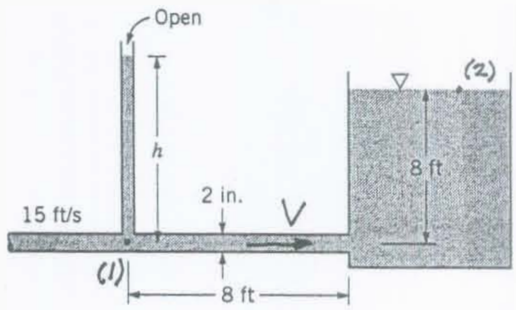
$$= \left(0.112 \left(\frac{12 \text{ ft}}{0.5/12 \text{ ft}}\right) + 10.5\right) \left(\tfrac{1}{2}\right) \left(1.94 \tfrac{slugs}{ft^3}\right) \left(1.47 \tfrac{ft}{s}\right)^2$$

$$P_1 - P_2 = 89.6 \text{ lb/ft}^2 = \underline{\underline{0.622 \text{ psi}}}$$

This addition approximately doubles the pressure drop. If the fluid temperature does drop below freezing, there will be a further increase in viscosity and the pressure drop.

8.86

8.86 Water flows through a 2-in.-diameter pipe with a velocity of 15 ft/s as shown in Fig. P8.86. The relative roughness of the pipe is 0.004, and the loss coefficient for the exit is 1.0. Determine the height, h, to which the water rises in the piezometer tube.



■ FIGURE P8.86

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$\frac{p_1}{\gamma} = h$, $z_1 = 0$, $p_2 = 0$, $z_2 = 8\text{ ft}$, $V_2 = 0$ and

$h_L = \left(f\frac{\ell}{D} + K_L\right)\frac{V^2}{2g}$ with $V = V_1$ and $K_L = 1$

Thus,

(1) $$h + \frac{V^2}{2g} - \left(f\frac{\ell}{D} + K_L\right)\frac{V^2}{2g} = z_2$$

But $Re = \frac{\rho V D}{\mu} = \frac{1.94 \frac{\text{slug}}{\text{ft}^3}\left(15\frac{\text{ft}}{\text{s}}\right)\left(\frac{2}{12}\text{ft}\right)}{2.34\times10^{-5}\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}} = 2.07\times10^{5}$

Hence from Fig. 8.20 with $\varepsilon/D = 0.004$ we obtain $f = 0.029$

so that Eq. (1) becomes

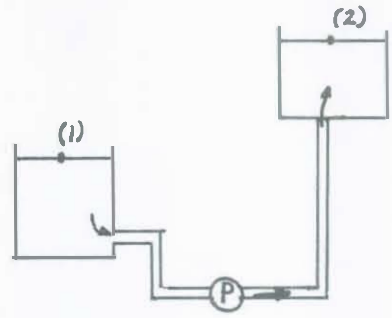
$$h + \left[1 - 0.029\frac{14\text{ ft}}{\left(\frac{2}{12}\text{ft}\right)} - 1\right]\frac{\left(15\frac{\text{ft}}{\text{s}}\right)^2}{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)} = 8\text{ ft}$$

or

$$h = \underline{\underline{16.5\text{ ft}}}$$

8.87

8.87 Water is pumped through a 60-m-long, 0.3-m-diameter pipe from a lower reservoir to a higher reservoir whose surface is 10 m above the lower one. The sum of the minor loss coefficients for the system is $K_L = 14.5$. When the pump adds 40 kW to the water the flowrate is 0.20 m³/s. Determine the pipe roughness.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 = 0,$$
and $z_2 = 10\,m$

Thus, $h_p - h_L = z_2$, where

$$h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{40\times10^3\,N\cdot m/s}{(9.80\times10^3\,N/m^3)(0.2\,m^3/s)} = 20.4\,m$$

Hence,

$$20.4\,m - \left[f\frac{\ell}{D} + \Sigma K_L\right]\frac{V^2}{2g} = 10\,m \qquad (1)$$

with

$$V = \frac{Q}{A} = (0.2\,m^3/s)/\left(\frac{\pi}{4}(0.3\,m)^2\right) = 2.82\,m/s$$

Thus, from Eq. (1)

$$20.4\,m - \left[f\left(\frac{60\,m}{0.3\,m}\right) + 14.5\right]\frac{(2.82\,m/s)^2}{2(9.81\,m/s^2)}$$

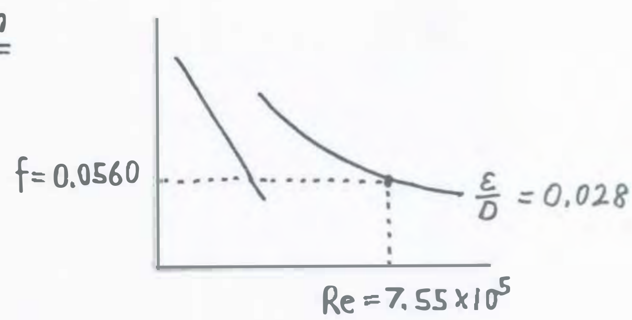
or

$f = 0.0560$

Also, $Re = \frac{\rho V D}{\mu} = \frac{999\,kg/m^3\,(2.82\,m/s)(0.3\,m)}{1.12\times10^{-3}\,N\cdot s/m^2} = 7.55\times10^5$

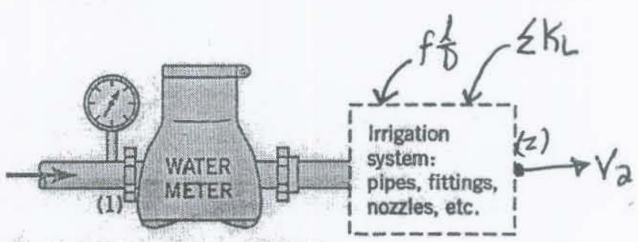
Thus, from the Moody chart (Fig. 8.35), with $Re = 7.55\times10^5$ and $f = 0.0560$ it follows that $\varepsilon/D = 0.028$, or

$\varepsilon = 0.028\,(0.3\,m) = \underline{\underline{0.0084\,m}}$



8.89

8.89 As shown in Fig. P8.89, a standard household water meter is incorporated into a lawn irrigation system to measure the volume of water applied to the lawn. Note that these meters measure volume, not volume flowrate. (See Video V8.15.) With an upstream pressure of $p_1 = 50$ psi the meter registered that 120 ft^3 of water was delivered to the lawn during an "on" cycle. Estimate the upstream pressure, p_1, needed if it is desired to have 150 ft^3 delivered during an "on" cycle. List any assumptions needed to arrive at your answer.



■ FIGURE P8.89

The energy equation for this flow is

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - \left[f\frac{\ell}{D} + \Sigma K_L\right]\frac{V^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $z_1 = z_2$, $p_2 = 0$, $V_1 = V$, and $V_2 = \frac{A_1}{A_2}V_1$

Thus, from Eq. (1)

$$p_1 = \frac{1}{2}\rho V_1^2\left[f\frac{\ell}{D} + \Sigma K_L + \left(\frac{A_1}{A_2}\right)^2 - 1\right] \qquad (2)$$

But $Q = A_1V_1 = \frac{\forall}{t}$, where $\forall$ is the volume of water supplied during an "on" cycle and t is the length of the cycle.

For a given system ΣK_L is independent of Q. Similarly, for large Re pipe flow, f is independent of Re (or Q). Thus,

$\left[f\frac{\ell}{D} + \Sigma K_L + \left(\frac{A_1}{A_2}\right)^2 - 1\right]$ is constant, independent of Q.

Hence, from Eq. (2), if the length of the cycle is constant,

$$\frac{p_1)_{150ft^3}}{p_1)_{120ft^3}} = \frac{\frac{1}{2}\rho V_1^2)_{150}}{\frac{1}{2}\rho V_1^2)_{120}} = \left[\frac{V_1)_{150}}{V_1)_{120}}\right]^2 = \left(\frac{\forall_{150}}{\forall_{120}}\right)^2 = \left(\frac{150}{120}\right)^2 = 1.563$$

or

$$p_1)_{150} = 1.563\,p_1)_{120} = 1.563(50\,psi) = \underline{\underline{78.1\,psi}}$$

8.90

8.90 A fan is to produce a constant air speed of 40 m/s throughout the pipe loop shown in Fig. P8.90. The 3-m-diameter pipes are smooth, and each of the four 90-degree elbows has a loss coefficient of 0.30. Determine the power that the fan adds to the air.

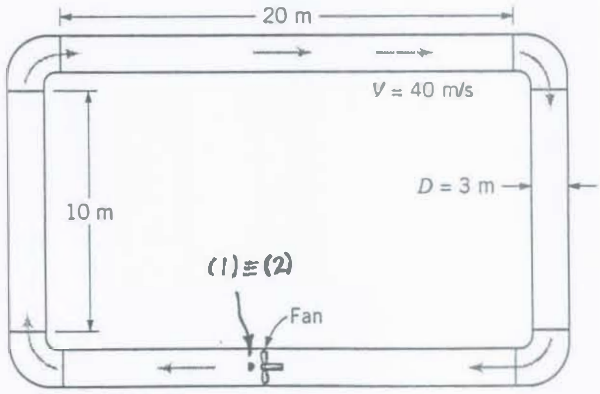


FIGURE P8.90

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_s = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

If we locate (1) and (2) at the same place it follows that

$p_1 = p_2$, $V_1 = V_2$, and $z_1 = z_2$.

Thus,

$$h_s = h_L = \left(f\frac{\ell}{D} + \sum_i K_{L_i}\right)\frac{V^2}{2g} \quad \text{where } \sum_i K_{L_i} = 4(0.30) = 1.2$$

Also, $Re = \dfrac{\rho V D}{\mu} = \dfrac{1.23\,\frac{kg}{m^3}(40\,\frac{m}{s})(3\,m)}{1.79 \times 10^{-5}\,\frac{N \cdot s}{m^2}} = 8.25 \times 10^6$

and $\frac{\varepsilon}{D} = 0$ so that from Fig. 8.20, $f = 0.0083$

Hence,

$$h_s = \left(0.0083\,\frac{(20+20+10+10)\,m}{3\,m} + 1.2\right)\frac{(40\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})} = 111\,m$$

so that

$$\dot{W}_s = \gamma Q h_s = \rho g Q h_s = (1.23\,\tfrac{kg}{m^3})(9.81\,\tfrac{m}{s^2})\left[\tfrac{\pi}{4}(3\,m)^2(40\,\tfrac{m}{s})\right]111\,m$$

$$= 3.79 \times 10^5\,\frac{N \cdot m}{s} = \underline{\underline{379\,kW}}$$

8.91

8.91 The turbine shown in Fig. P8.91 develops 400 kW. Determine the flowrate if **(a)** head losses are negligible or **(b)** head loss due to friction in the pipe is considered. Assume $f = 0.02$. *Note*: There may be more than one solution or there may be no solution to this problem.

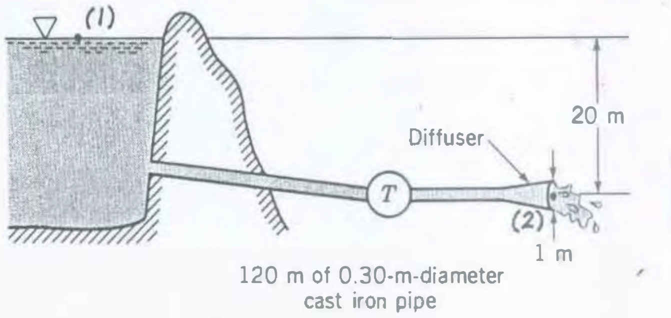


FIGURE P8.91

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}+h_T\text{, where } p_1=p_2=0,\ z_1=20\,m,$$

$z_2=0$ Thus, $z_1=\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g}+h_T$ (1)

a) Neglect head losses $(f=0)$:

$$z_1=\frac{V_2^2}{2g}+h_T\text{, where } h_T=\frac{P}{\gamma Q}=\frac{400\times10^3\,\frac{N\cdot m}{s}}{(9.80\times10^3\frac{N}{m^3})\frac{\pi}{4}(1m)^2V_2}=\frac{52.0}{V_2}\ m$$

Thus,

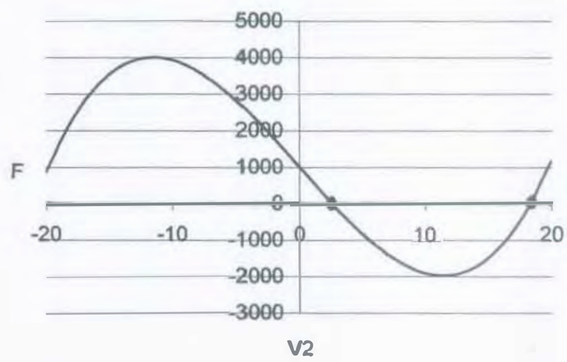
$$20m=\frac{V_2^2}{2(9.81\frac{m}{s^2})}+\frac{52.0}{V_2}\quad\text{or}\quad V_2^3-392V_2+1020=0 \qquad (2)$$

Determine the roots of this cubic equation. Let $V_2^3-392V_2+1020\equiv F$

As indicated by the graph, there at two real, positive roots to $F=0$: $V_2=2.65\,\frac{m}{s}$ or $V_2=18.3\,\frac{m}{s}$ Thus, $Q=A_2V_2=\frac{\pi}{4}(1m)^2V_2$, or

$\underline{Q=2.08\frac{m^3}{s}\text{ or }Q=14.4\frac{m^3}{s}}$

The negative root $(V_2<0)$ has no physical meaning.



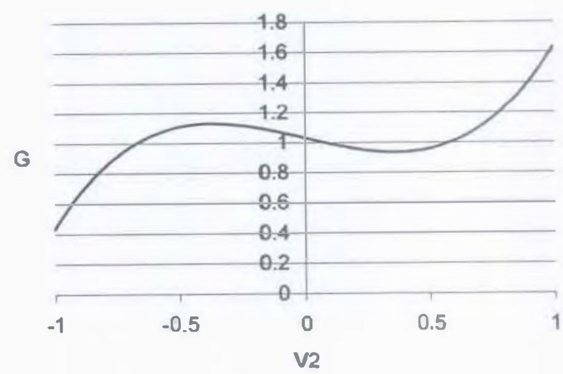
b) Include head loss $(f=0.02)$: From Eq. (1) $V=\frac{V_2A_2}{A}=V_2\left(\frac{D_2}{D}\right)^2=V_2\left(\frac{1m}{0.3m}\right)^2=11.1V_2$

or

$$20m=\left(1+0.02\left(\frac{120m}{0.3m}\right)(11.1)^2\right)\frac{V_2^2}{2(9.81\frac{m}{s^2})}+\frac{52.0}{V_2}\ m$$

Thus, $V_2^3-0.398V_2+1.034=0$ Let $G\equiv V_2^3-0.398V_2+1.034$; determine V_2 that gives $G=0$.

As indicated by the graph, there is no positive real root. Hence, $\underline{\text{the flow cannot occur with}}$ $\underline{\dot{W}_s=400kW.}$



8.92

*8.92 In some locations with very "hard" water, a scale can build up on the walls of pipes to such an extent that not only does the roughness increases with time, but the diameter significantly decreases with time. Consider a case for which the roughness and diameter vary as $\varepsilon = 0.02 + 0.01t$ mm, $D = 50\ (1 - 0.02t)$ mm, where t is in years. Plot the flowrate as a function of time for $t = 0$ to $t = 10$ years if the pressure drop per 12 m of horizontal pipe remains constant at $\Delta p = 1.3$ kPa.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1 = z_2,\ V_1 = V_2 = V, \text{ and}$$

Thus,

$$\Delta p = p_1 - p_2 = 1.3\ kPa$$

$$\Delta p = f\frac{\ell}{D}\frac{V^2}{2g}\gamma, \text{ or } 1.3\times10^3\frac{N}{m^2} = f\left(\frac{12m}{0.05(1-0.02t)}\right)\frac{V^2}{2(9.81\frac{m}{s^2})}(9.80\times10^3\frac{N}{m^3})$$

or

$$fV^2 = 0.0108(1-0.02t), \text{ where } t \sim yr,\ V \sim \frac{m}{s} \qquad (1)$$

Also, $Re = \frac{VD}{\nu} = \frac{V[0.05(1-0.02t)]}{1.12\times10^{-6}\frac{m^2}{s}}$, or $Re = 4.46\times10^4(1-0.02t)V$ (2)

and

$$\frac{\varepsilon}{D} = \frac{(0.02+0.01t)}{50(1-0.02t)} \qquad (3)$$

Finally, from the alternate formula, Eq. 8.35b,

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right] \qquad (4)$$

For $0 \le t \le 10$ yr, obtain $\frac{\varepsilon}{D}$ from Eq.(3) and solve Eqs. (1), (2), and (4) for f, V, and Re. Then $Q = VA = V\frac{\pi}{4}(0.05(1-0.02t))^2$

or

$$Q = 1.96\times10^{-3}(1-0.02t)^2 V \text{ where } Q \sim \frac{m^3}{s},\ V \sim \frac{m}{s},\ t \sim yr \qquad (5)$$

Eqns (1)-(5) were used in a MS Excel spreadsheet to manually iterate on the solution.

$$f_{guess} \xrightarrow{(1)} V \xrightarrow{(2)} Re \xrightarrow{(4)} f_{new} \stackrel{?}{=} f_{guess}$$

No

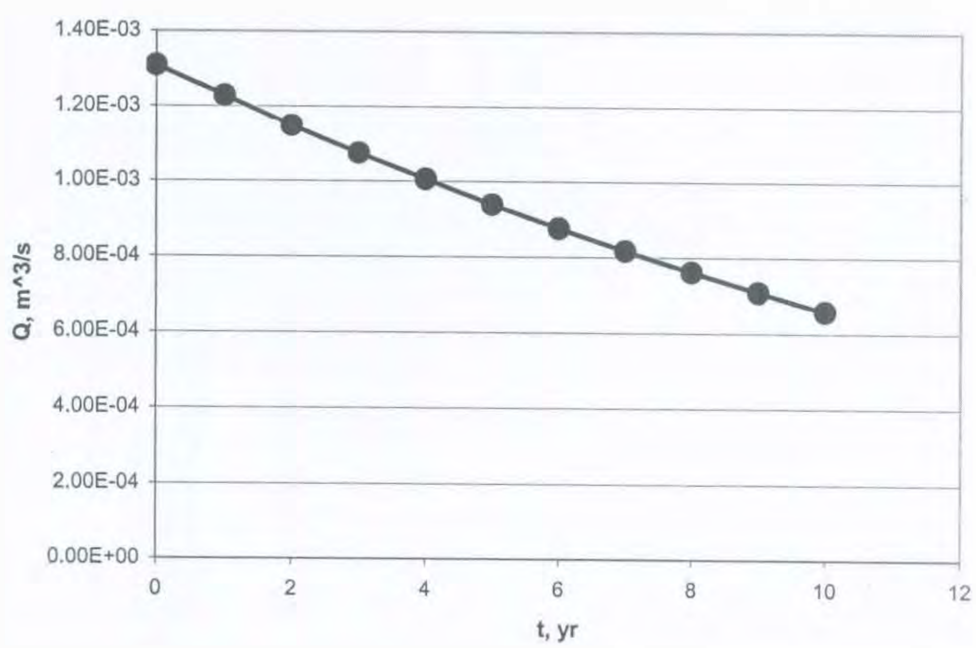
The spreadsheet results are shown below along with a plot of the data.

(cont)

8.92 (cont.)

Insert a guess value (e.g. f = 0.02) in the f(guess) cell. A new f value will be calculated in the f(new) cell. Use this new value as the updated f(guess). Continue until f(guess) = f(new).

t, yr	e/D	f(guess)	V, m/s	Re	f(new)	Q, m^3/s
0	4.00E-04	0.0243	6.67E-01	29758	0.0243	1.31E-03
1	6.12E-04	0.0249	6.52E-01	28496	0.0250	1.23E-03
2	8.33E-04	0.0257	6.35E-01	27195	0.0257	1.15E-03
3	1.06E-03	0.0264	6.20E-01	25998	0.0264	1.07E-03
4	1.30E-03	0.0271	6.06E-01	24845	0.0271	1.00E-03
5	1.56E-03	0.0279	5.90E-01	23692	0.0279	9.37E-04
6	1.82E-03	0.0286	5.76E-01	22625	0.0286	8.75E-04
7	2.09E-03	0.0293	5.63E-01	21595	0.0293	8.16E-04
8	2.38E-03	0.0300	5.50E-01	20602	0.0300	7.61E-04
9	2.68E-03	0.0307	5.37E-01	19643	0.0307	7.08E-04
10	3.00E-03	0.0315	5.24E-01	18686	0.0315	6.57E-04



8.93

8.93 Water flows from the nozzle attached to the spray tank shown in Fig. P8.93. Determine the flowrate if the loss coefficient for the nozzle (based on upstream conditions) is 0.75 and the friction factor for the rough hose is 0.11.

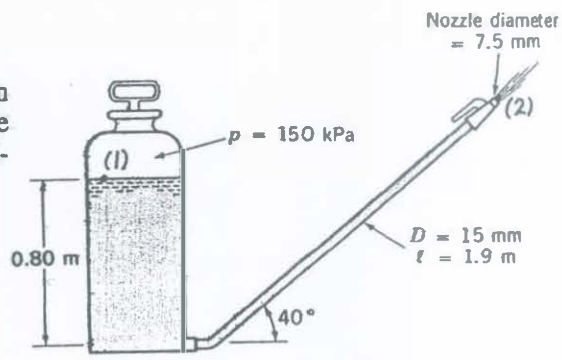


FIGURE P8.93

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+K_L\right)\frac{V^2}{2g}, \text{ where } p_1=150\,kPa,\ p_2=0, \quad (1)$$

$z_1=0.8\,m$, $z_2=\ell\sin 40^\circ=(1.9\,m)\sin 40^\circ=1.22\,m$, $V_1=0$,

$V=\frac{Q}{A}$, and $V_2=\frac{Q}{A_2}=\left(\frac{A}{A_2}\right)V=\left(\frac{D}{D_2}\right)^2V=\left(\frac{15\,mm}{7.5\,mm}\right)^2V=4V$

Thus, with $f=0.11$ and $K_L=0.75$ Eq.(1) gives

$$\frac{150\times10^3\,\frac{N}{m^2}}{9.80\times10^3\,\frac{N}{m^3}}+0.8\,m=1.22\,m+\left(4^2+0.11\left(\frac{1.9\,m}{0.015\,m}\right)+0.75\right)\frac{V^2}{2(9.81\,\frac{m}{s^2})}$$

or

$V=3.09\,\frac{m}{s}$

Thus, $Q=AV=\frac{\pi}{4}(0.015\,m)^2(3.09\,\frac{m}{s})=\underline{\underline{5.46\times10^{-4}\,\frac{m^3}{s}}}$

8.94

8.94 When the pump shown in Fig. P8.94 adds 0.2 horsepower to the flowing water, the pressures indicated by the two gages are equal. Determine the flowrate.

Length of pipe between gages = 60 ft
Pipe diameter = 0.1 ft
Pipe friction factor = 0.03
Filter loss coefficient = 12

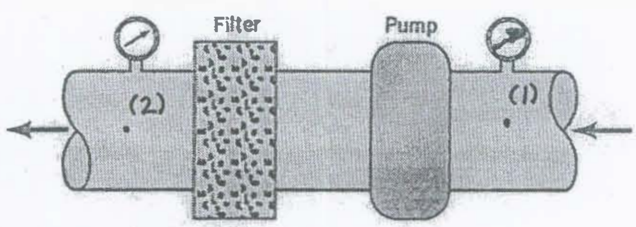


FIGURE P8.94

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + h_p = \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + h_L$$

$p_1 = p_2$, $z_1 = z_2$, $V_1 = V_2$

So, $h_p = h_L$ (1)

The pump adds 0.2 hp of power.

$$\dot{W} = 0.2\ hp \times \frac{550\ \frac{ft\cdot lb}{s}}{1\ hp} = 110\ \frac{ft\cdot lb}{s}$$

Convert to head by:

$$h_p = \frac{\dot{W}}{\gamma Q} = \frac{110\ \frac{ft\cdot lb}{s}}{62.4\frac{lb}{ft^3}\ Q} = \frac{1.76}{Q}$$

Sub into (1)

$$\frac{1.76}{Q} = \left(f\frac{l}{D} + \Sigma K_L\right)\frac{V^2}{2g} = \left(f\frac{l}{D} + \Sigma K_L\right)\frac{(Q/A)^2}{2g}$$

or $$Q^3 = \frac{1.76(2)(g)A^2}{\left(f\frac{l}{D} + \Sigma K_L\right)}$$ where $A = \frac{\pi}{4}(0.1\ ft)^2 = 7.85\times10^{-3} ft^2$

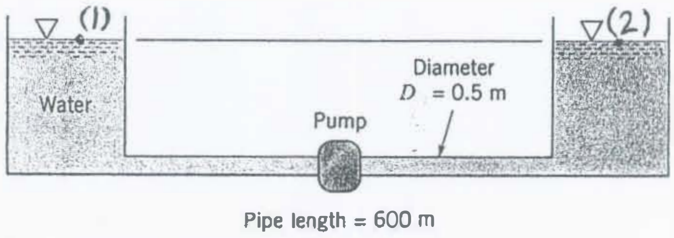
$$= \frac{1.76(64.4)(7.85\times10^{-3})^2}{\left(0.03\frac{60}{0.1} + 12\right)}$$

$$Q^3 = 2.328\times10^{-4}$$

$$Q = 0.0615\ ft^3/s$$

8.95

8.95 Water is pumped between two large open tanks as shown in Fig. P8.95. If the pump adds 50 kW of power to the fluid, what is the flowrate passing between the tanks? Assume the friction factor to be equal to 0.02 and minor losses to be negligible.



■ FIGURE P8.95

With $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_1 = z_2$

$$h_p = h_L = f \frac{\ell}{D} \frac{V^2}{2g} \qquad (1)$$

With the pump adding 50 kW of power

$$\dot{W} = 50 \times 10^3 \, W = h_p Q \gamma$$

$$h_p = \frac{50 \times 10^3}{Q(9.8 \times 10^3)} = \frac{5.10}{Q}$$

Sub into (1)

$$\frac{5.10}{Q} = f \frac{\ell}{D} \frac{V^2}{2g} = f \frac{\ell}{D} \frac{Q^2}{2gA^2}$$

$$Q^3 = \frac{5.10(2)(D g A^2)}{f \ell} \quad \text{where } A = \frac{\pi}{4}(0.5)^2 = 0.196 \, m^2$$

Thus, $$Q^3 = \frac{5.10(2)(0.5)(9.81)(0.196)^2}{(0.02)(600)}$$

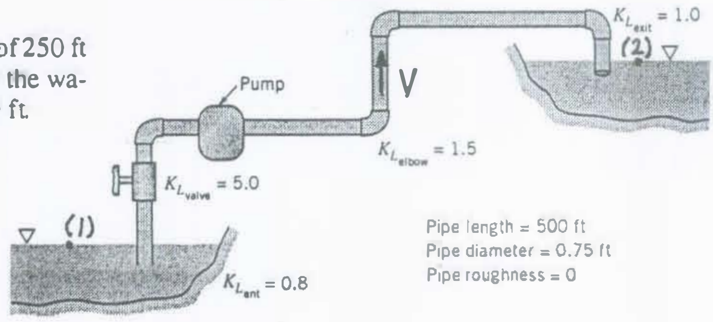
$$= 0.1602$$

or

$$Q = 0.543 \, m^3/s$$

8.97

8.97 The pump shown in Fig. P8.97 delivers a head of 250 ft to the water. Determine the power that the pump adds to the water. The difference in elevation of the two ponds is 200 ft.



■ FIGURE P8.97

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = 0$, $z_2 = 200\ ft$, $h_p = 250\ ft$

Thus,

$$-f\frac{\ell}{D}\frac{V^2}{2g} - \sum_i K_{L_i}\frac{V^2}{2g} + h_p = z_2 \quad \text{so that with } \sum_i K_{L_i}\frac{V^2}{2g} = (0.8 + 4(1.5) + 5.0 + 1)\frac{V^2}{2g} = 12.8\frac{V^2}{2g}$$

$$\left[-f\left(\frac{500}{0.75}\right) - 12.8\right]\frac{V^2}{2(32.2)} + 250 = 200$$

or

(1) $(667 f + 12.8)V^2 = 3220$

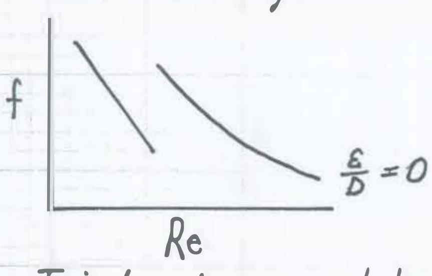
Also, $Re = \dfrac{\rho V D}{\mu} = \dfrac{(1.94\ \frac{slugs}{ft^3})\,V\,(0.75\ ft)}{2.34 \times 10^{-5}\ \frac{lb \cdot s}{ft^2}}$

or

(2) $Re = 6.22 \times 10^4\, V$

and from Fig. 8.20:

(3)



Trial and error solution. Assume $f = 0.02 \xrightarrow{(1)} V = 11.1\ \frac{ft}{s} \xrightarrow{(2)} Re = 6.9 \times 10^5$ $\xrightarrow{(3)} f = 0.012 \neq 0.02$

Assume $f = 0.012 \xrightarrow{(1)} V = 12.4\ \frac{ft}{s} \xrightarrow{(2)} Re = 7.7 \times 10^5 \xrightarrow{(3)} f = 0.0121 \approx 0.012$

Thus, $V = 12.4\ \frac{ft}{s}$ and

$$\dot{W}_s = \gamma Q h_p = (62.4\ \tfrac{lb}{ft^3})\tfrac{\pi}{4}(0.75\ ft)^2(12.4\ \tfrac{ft}{s})(250\ ft) = 8.55 \times 10^4\ \tfrac{ft \cdot lb}{s}$$

$$= 8.55 \times 10^4\ \tfrac{ft \cdot lb}{s} \times 1\,\frac{hp}{550\ \frac{ft \cdot lb}{s}} = \underline{\underline{155\ hp}}$$

Alternatively, we could replace Eq. (3) (the Moody chart) by Eq 8.35

(con't)

8.97 (con't)

(the Colebrook equation) and obtain V as follows.

From Eq. (1),

$V=\left[3220/(667f+12.8)\right]^{1/2}$, which when combined with Eq. (2) gives

(4) $Re = 6.22\times10^{4}\left[3220/(667f+12.8)\right]^{\frac{1}{2}} = 3.53\times10^{6}/(667f+12.8)^{\frac{1}{2}}$

Also, the Colebrook equation with $\varepsilon/D=0$ is

(5) $\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.51}{Re\sqrt{f}}\right)$

By combining Eqs (4) and (5) we obtain a single equation involving only f:

$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{2.51(667f+12.8)^{\frac{1}{2}}}{3.53\times10^{6}\sqrt{f}}\right]$$

Using a compute root-finding program to solve Eq (6) gives $f = 0.0123$, consistent with the above trial and error method.

8.98

8.98 Water flows through two sections of the vertical pipe shown in Fig. P8.98. The bellows connection cannot support any force in the vertical direction. The 0.4-ft-diameter pipe weighs 0.2 lb/ft and the friction factor is assumed to be 0.02. At what velocity will the force, F, required to hold the pipe be zero?

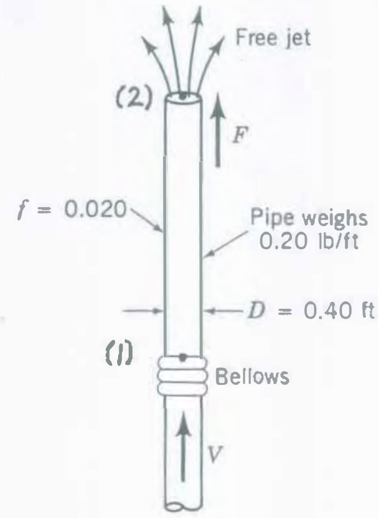
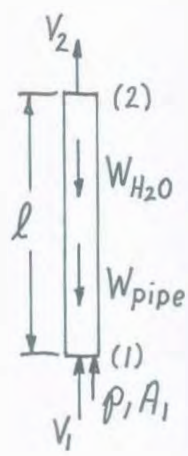


FIGURE P8.98

From the momentum equation applied to the control volume indicated



$$p_1 A_1 - W_{H_2O} - W_{pipe} = \dot{m}(V_2 - V_1) = 0 \text{ since } V_1 = V_2$$

Thus, $p_1 = \dfrac{W_{H_2O} + W_{pipe}}{A_1} = \dfrac{\gamma \ell A_1 + \ell\left(\frac{W_{pipe}}{\ell}\right)}{A_1}$

or

$$p_1 = \gamma \ell + \frac{(0.20\frac{lb}{ft})\ell}{\frac{\pi}{4}(0.4ft)^2} = \gamma\ell + 1.59\ell, \text{ where } p_1 \sim \frac{lb}{ft^2},\ \ell \sim ft$$

Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_2 = 0,$$

$V_1 = V_2 = V$, $z_1 = 0$, and $z_2 = \ell$

Thus, $p_1 = \gamma z_2 + f\frac{\ell}{D}\frac{1}{2}\rho V^2,$

or when combined with the above force balance result

$$p_1 = \gamma\ell + f\frac{\ell}{D}\frac{1}{2}\rho V^2 = \gamma\ell + 1.59\ell$$

That is, $\dfrac{f\rho V^2}{2D} = 1.59$ or $V = \sqrt{\dfrac{2D(1.59)}{\rho f}} = \sqrt{\dfrac{2(0.4)(1.59)}{(1.94)(0.02)}} = \underline{\underline{5.73\frac{ft}{s}}}$

Note: This answer is independent of the pipe length, ℓ.

8.99

8.99 Water is circulated from a large tank, through a filter, and back to the tank as shown in Fig. P8.99. The power added to the water by the pump is 200 ft · lb/s. Determine the flowrate through the filter.

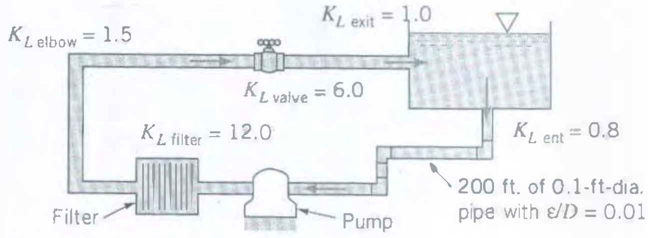


FIGURE P8.99

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \left(f\frac{\ell}{D} + \sum_i K_{L_i}\right)\frac{V^2}{2g} \qquad (1)$$

where

$p_1 = p_2$, $V_1 = V_2 = 0$, and $z_1 = z_2$

Also, $\dot{W}_p = \gamma Q h_p$ or

$$h_p = \frac{200\,\frac{ft\cdot lb}{s}}{62.4\,\frac{lb}{ft^3}\left(\frac{\pi}{4}(0.1ft)^2\right)V} = \frac{408}{V}$$

Thus, Eq. (1) becomes

$$\frac{408}{V} = \left(\frac{200\,ft}{0.1\,ft}f + (0.8 + 5(1.5) + 12 + 6 + 1)\right)\frac{V^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$V^3 = \frac{13.13}{(f + 0.01365)} \qquad (2)$$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{1.94\,\frac{slugs}{ft^3}(V\,\frac{ft}{s})(0.1ft)}{2.34\times10^{-5}\,\frac{lb\cdot s}{ft^2}} \quad \text{or} \quad Re = 8290V \qquad (3)$$

Trial and error solution:

Assume $f = 0.04$. From Eq. (2), $V = 6.26\,\frac{ft}{s}$; from Eq. (3), $Re = 5.20\times10^4$. Thus, from Fig. 8.20, $f = 0.039 \neq 0.04$

Assume $f = 0.039$, or $V = 6.29\,\frac{ft}{s}$ and $Re = 5.21\times10^4$ and $f = 0.039$ (Checks)

Thus, $Q = AV = \frac{\pi}{4}(0.1ft)^2(6.29\,\frac{ft}{s}) = \underline{\underline{0.0494\,\frac{ft^3}{s}}}$

Alternatively, the Colebrook equation (Eq. 8.35) could be used rather than the Moody chart. Thus,

(con't)

8.99 (con't)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right), \text{ where from Eq.(2),} \qquad (4)$$

$$f = (13.13/V^3) - 0.01365 \qquad (5)$$

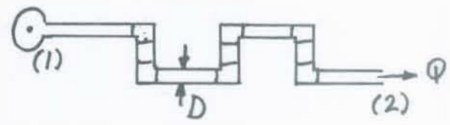
Thus, by combining Eqs. (3), (4), and (5) we obtain the following equation for V:

$$1/\left[(13.13/V^3) - 0.01365\right]^{\frac{1}{2}} = -2.0 \log\left\{\frac{0.01}{3.7} + 2.51/\left[(8290V)((13.13/V^3) - 0.01365)^{\frac{1}{2}}\right]\right\} \qquad (6)$$

Using a computer root-finding program gives the solution to Eq.(6) as $V = 6.29 \frac{ft}{s}$, the same as obtained by the above trial and error method.

8.100

8.100 A certain process requires 2.3 cfs of water to be delivered at a pressure of 30 psi. This water comes from a large-diameter supply main in which the pressure remains at 60 psi. If the galvanized iron pipe connecting the two locations is 200 ft long and contains six threaded 90° elbows, determine the pipe diameter. Elevation differences are negligible.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}\text{, where } p_2=30\,psi,\; p_1=60\,psi,$$

$$z_1=z_2,\; V_1=0,\; V_2=V=\frac{Q}{A}=\frac{2.3\,\frac{ft^3}{s}}{\frac{\pi}{4}D^2}=\frac{2.93}{D^2}\,\frac{ft}{s}\text{, with } D\sim ft$$

Thus,

$$p_1-p_2=\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{1}{2}\rho V^2$$

or

$$(60-30)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)=\left(1+f\left(\frac{200\,ft}{D}\right)+6(1.5)+0.5\right)\left(\frac{2.93}{D^2}\frac{ft}{s}\right)^2\left(\frac{1}{2}\right)\left(1.94\frac{slugs}{ft^3}\right)$$

where we have used

$$\Sigma K_L=6K_{L_{elbow}}+K_{L_{entrance}}=6(1.5)+0.5$$

Thus,

$$49.4=\left(1+\frac{19.0f}{D}\right)\frac{1}{D^4} \qquad (1)$$

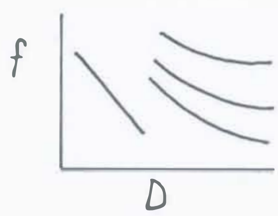
Also,

$$Re=\frac{VD}{\nu}=\frac{\left(\frac{2.93}{D^2}\right)D}{\nu}=\frac{2.93\,\frac{ft^2}{s}}{1.21\times10^{-5}\frac{ft^2}{s}D}\text{, or } Re=2.42\times10^5\,\frac{1}{D} \qquad (2)$$

and from Table 8.1

$$\frac{\varepsilon}{D}=\frac{0.0005\,ft}{D} \qquad (3)$$

Finally, from Fig. 8.20:



(4)

Trial and error solution of Eqs. (1), (2), (3), and (4) for f, D, $\frac{\varepsilon}{D}$, and Re.

Normally it is easiest to guess a value of f, calculate D, etc. In this case (because of minor losse), Eq. (1) is not easy to use in this fashion. Thus, assume D, calculate f (Eq. (1)), Re (Eq. (2)), and $\frac{\varepsilon}{D}$ (Eq. (3)). Look up f in Fig. 8.20 (Eq. (4)) and compare with that from Eq. (1).

Assume $D=0.4\,ft$. Thus, $f=0.00557$, $Re=6.05\times10^5$, $\frac{\varepsilon}{D}=0.00125$
or from Fig. 8.20 $f=0.021\neq0.00557$

Assume $D=0.5\,ft$; $f=0.0551$, $Re=4.84\times10^5$, $\frac{\varepsilon}{D}=0.001$ or $f=0.0203\neq0.0551$

Assume $D=0.45\,ft$; $f=0.0243$, $Re=5.38\times10^5$, $\frac{\varepsilon}{D}=0.00111$ or $f=0.0205\neq0.0243$

Assume $D=0.44\,ft$; $f=0.0197$, $Re=5.50\times10^5$, $\frac{\varepsilon}{D}=0.00114$ or $f=0.0205\neq0.0197$

After enough trials obtain $D=\underline{\underline{0.442\,ft}}$

Note: If Fig. 8.20 (Eq. (4)) is replaced by the Colebrook equation,

(con't)

8.100 (con't)

this problem can be solved as follows.

Thus, from Eq. (1),

$f = (49.4D^5 - D)/19$ so that with the Colebrook equation (Eq. 8.35), when combined with Eqs. (2) and (3), gives

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

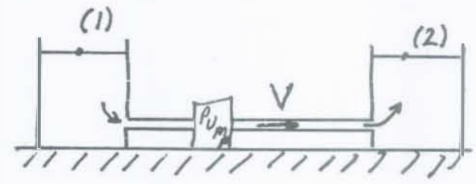
or

$$\left[\frac{19}{(49.4D^5-D)}\right]^{1/2} = -2.0 \log\left[\frac{0.0005}{3.7\,D} + \frac{2.51\,D\,\sqrt{19}}{2.42\times10^5\,(49.4\,D^5-D)^{1/2}}\right] \qquad (5)$$

Using a computer root-finding routine gives the solution to Eq. (5) as $D = 0.442$ ft which is the same as that obtained by the trial and error method above.

8.101

8.101 Water is pumped between two large open reservoirs through 1.5 km of smooth pipe. The water surfaces in the two reservoirs are at the same elevation. When the pump adds 20 kW to the water the flowrate is 1 m³/s. If minor losses are negligible, determine the pipe diameter.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 = z_2$$

Thus,

(1) $h_s = h_L$ where $h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{20\times10^3\,N\cdot m/s}{(9.80\times10^3\frac{N}{m^3})(1\frac{m^3}{s})} = 2.04\,m$

and

$h_L = f\frac{\ell}{D}\frac{V^2}{2g}$ with $V = \frac{Q}{A} = \frac{1\,m^3/s}{\frac{\pi}{4}D^2} = \frac{1.273}{D^2}\ m/s$ with $D \sim m$

Hence,

(2) $h_L = f\,\frac{1.5\times10^3\,m}{D}\,\frac{(1.23/D^2)^2\,m^2/s^2}{2(9.81\,m/s^2)} = 123.9\,f/D^5\ m$

From Eqs (1) and (2), $2.04 = 123.9\,f/D^5$ or $f = 0.0165\,D^5$

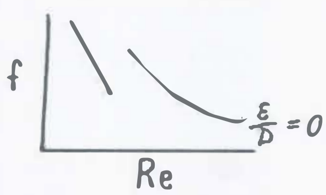
(3) $D = 2.27\,f^{1/5}$

Also,

$Re = \frac{\rho V D}{\mu} = \frac{999\frac{kg}{m^3}(1.273/D^2)m\,D\,m}{1.12\times10^{-3}\frac{N\cdot s}{m^2}}$ or

(4) $Re = 1.14\times10^6/D$

Finally, with $\varepsilon/D = 0$ the Moody chart (Fig. 8.20) is the final equation.

(5) 

Trial and error solution of Eqs. (3), (4), and (5) for f, Re, and D:
Assume $f = 0.02$ so Eq (3) gives $D = 2.27(0.02)^{1/5} = 1.04\,m$ and Eq (4) gives $Re = 1.14\times10^6/1.04 = 1.10\times10^6$. Thus, from Eq (5), $f = 0.0115$ which is not equal to the assumed $f = 0.02$. Try again with $f = 0.0115$ which gives $D = 0.931\,m$, $Re = 1.22\times10^4$, and $f = 0.0113 \neq 0.0115$. One final try with $f = 0.0113$ gives $D = 0.927\,m$, $Re = 1.23\times10^4$, and $f = 0.0113$ as assumed. Thus, $\underline{\underline{D = 0.927\,m}}$.

An alternate method is to use the Colebrook formula (Eq (8.35)) rather than the Moody chart (Eq (5)). Thus, with $\varepsilon/D = 0$,

(con't)

8.101 (con't)

Eq (8.35) is

$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{Re\sqrt{f}}\right)$ which, when combined with Eqs. (3) and (4), gives

$$(6) \quad \frac{1}{(0.0165 D^5)^{1/2}} = -2.0 \log\left[\frac{2.51\, D}{1.14\times10^6 (0.0165 D^5)^{1/2}}\right]$$

Using a computer root-finding program to solve Eq. (6) gives $D = 0.926$, which is consistent with the trial and error solution given above.

8.102

8.102 Determine the diameter of a steel pipe that is to carry 2,000 gal/min of gasoline with a pressure drop of 5 psi per 100 ft of horizontal pipe.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1 = z_2 \text{ and } V_1 = V_2$$

Thus,

$$p_1 - p_2 = f\frac{\ell}{D}\frac{1}{2}\rho V^2 \text{ with } p_1 - p_2 = 5\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right),\ \ell = 100\,ft, \tag{1}$$

$\rho = 1.32\,\frac{slugs}{ft^3}$, $\mu = 6.5\times10^{-6}\,\frac{lb\cdot s}{ft^2}$, and

$$V = \frac{Q}{A} = \frac{\left(2000\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(231\frac{in.^3}{gal}\right)\left(\frac{1}{1728}\frac{ft^3}{in.^3}\right)}{\frac{\pi}{4}D^2}, \text{ or } V = \frac{5.67}{D^2}\,\frac{ft}{s} \text{ with } D \sim ft$$

Hence, Eq. (1) gives:

$$5(144)\frac{lb}{ft^2} = f\left(\frac{100\,ft}{D\,ft}\right)\frac{1}{2}\left(1.32\frac{slugs}{ft^3}\right)\left(\frac{5.67}{D^2}\frac{ft}{s}\right)^2$$

or

$$D = 1.24\,f^{1/5} \tag{2}$$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{\left(1.32\frac{slugs}{ft^3}\right)\left(\frac{5.67}{D^2}\frac{ft}{s}\right)D\,ft}{6.5\times10^{-6}\,\frac{lb\cdot s}{ft^2}}, \text{ or } Re = \frac{1.15\times10^6}{D} \tag{3}$$

and

$$\frac{\varepsilon}{D} = \frac{0.00015}{D}, \text{ where } D \sim ft \tag{4}$$

Finally, the fourth equation is the Moody chart (or the Colebrook equation) (5)

Note: 4 equations ((2), (3), (4), and (5)) and 4 unknowns (f, $\frac{\varepsilon}{D}$, D, Re)

Trial and error solution:

Guess $f = 0.02 \xrightarrow{(2)} D = 0.567\,ft$; $\xrightarrow{(3)} Re = 2.03\times10^6$, $\xrightarrow{(4)} \frac{\varepsilon}{D} = 0.000265$; $\xrightarrow{(5)} f = 0.0148 \neq 0.02$

Thus, the guessed value is not correct.

Guess $f = 0.0148 \xrightarrow{(2)} D = 0.534\,ft$; $\xrightarrow{(3)} Re = 2.15\times10^6$, $\xrightarrow{(4)} \frac{\varepsilon}{D} = 0.000281$; $\xrightarrow{(5)} f = 0.0150 \approx 0.0148$

Thus, $D = 1.24(0.0150)^{1/5} = \underline{\underline{0.535\,ft}}$

By using the Colebrook equation, Eq. 8.35, rather than the Moody chart, Eq. (5), we have

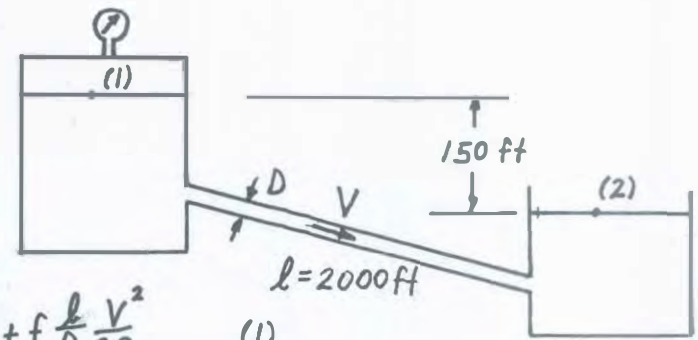
$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right] \text{ which, using Eqs (2), (3), and (4) is,}$$

$$\frac{1}{(D/1.24)^{5/2}} = -2.0\log\left[\frac{0.00015}{3.7D} + \frac{2.51\,D}{1.15\times10^6\,(D/1.24)^{5/2}}\right]$$

Using a computer root-finding program to solve Eq. (6) gives $D = 0.536\,ft$ which is consistent with the above trial and error solution.

8.103

8.103 Water is to be moved from a large, closed tank in which the air pressure is 20 psi into a large, open tank through 2000 ft of smooth pipe at the rate of 3 ft^3/s. The fluid level in the open tank is 150 ft below that in the closed tank. Determine the required diameter of the pipe. Neglect minor losses.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

where

$V_1 = V_2 = 0$, $z_1 - z_2 = 150$ ft, and $p_1 = 20$ psi, $p_2 = 0$

Also,

$$V = \frac{Q}{A} = \frac{3\,\frac{ft^3}{s}}{\frac{\pi}{4}D^2} = \frac{3.82}{D^2}, \text{ where } V \sim \frac{ft}{s},\ D \sim ft$$

Thus, Eq. (1) becomes

$$\frac{(20\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + 150\,ft = f\,\frac{2000\,ft}{D}\,\frac{\left(\frac{3.82}{D^2}\right)^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$D = 1.18\,f^{1/5} \qquad (2)$$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{\rho\left(\frac{3.82}{D^2}\right)D}{\mu} = \frac{1.94\,(3.82)}{2.34\times10^{-5}\,D}, \text{ or } Re = \frac{3.17\times10^5}{D} \qquad (3)$$

Trial and error solution:

Assume $f = 0.02$ so from Eq. (2), $D = 0.540$ ft and from Eq. (3) $Re = 5.87\times10^5$. Thus, from Fig. 8.20 (with $\frac{\varepsilon}{D} = 0$) $f = 0.013 \neq 0.02$

Assume $f = 0.013$ which gives $D = 0.495$ ft, $Re = 6.40\ 10^5$, and $f = 0.0125$

Assume $f = 0.0125$, so $D = 0.491$ ft, $Re = 6.46\times10^5$, $f = 0.0125$ (Checks)

Thus, $\underline{\underline{D = 0.491\ ft}}$

Alternately, the Colebrook equation, Eq. 8.35, rather than the Moody chart, Fig. 8.20, could be used as follows:

(con't)

8.103 (con't)

With $\varepsilon/D = 0$, Eq. 8.35 is

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(2.51/(Re\sqrt{f})\right) \quad \text{where} \qquad (4)$$

from Eq. (2), $f = (D/1.18)^5$ (5)

Thus, combining Eqs. (3), (4), and (5) gives

$$1/(D/1.18)^{5/2} = -2.0 \log\left[2.51/\left((3.17\times10^5/D)(D/1.18)^{5/2}\right)\right] \qquad (6)$$

Using a computer root-finding technique gives the solution to Eq. (6) as $D = 0.492\,ft$, which is consistent with the above trial and error solution.

8.104

8.104 Rainwater flows through the galvanized iron downspout shown in Fig. P8.104 at a rate of 0.006 m³/s. Determine the size of the downspout cross section if it is a rectangle with an aspect ratio of 1.7 to 1 and it is completely filled with water. Neglect the velocity of the water in the gutter at the free surface and the head loss associated with the elbow.

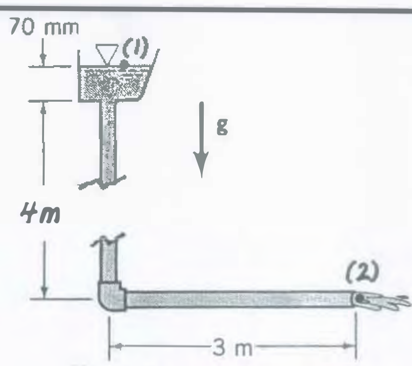
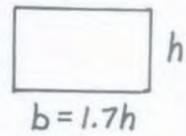


FIGURE P8.104

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D_h}\frac{V^2}{2g}\text{, where } p_1=p_2=0,\ V_1=0,\ V_2=V, \quad (1)$$

$z_1=4.07\,m$, and $z_2=0$

Also, $D_h=\frac{4A}{P}=\frac{4(1.7h^2)}{2(1.7h+h)}=1.26h$



and
$$V=\frac{Q}{A}=\frac{0.006\frac{m^3}{s}}{1.7h^2}=0.00353\,h^{-2}\ \frac{m}{s}\text{, where } h\sim m$$

Thus, from Eq.(1)

$$4.07\,m=\left(1+f\left(\frac{7\,m}{1.26h\,m}\right)\right)\left(\frac{3.53\times10^{-3}}{h^2}\right)^2\frac{m^2}{s^2}\left(\frac{1}{2(9.81\frac{m}{s^2})}\right)$$

or

$$6.41\times10^6\,h^4=1+5.55\frac{f}{h} \quad (2)$$

From Table 8.1 $\frac{\varepsilon}{D_h}=\frac{0.15\times10^{-3}\,m}{1.26h\,m}=\frac{1.19\times10^{-4}}{h}$, where $h\sim m$ (3)

and
$$Re_h=\frac{VD_h}{\nu}=\frac{(0.00353h^{-2}\frac{m}{s})(1.26h\,m)}{1.12\times10^{-6}\frac{m^2}{s}}\text{ or } Re_h=\frac{3970}{h} \quad (4)$$

Finally, from Fig. 8.20:

Trial and error solution of Eqs. (2),(3),(4), and (5) for $f, h, Re_h, \frac{\varepsilon}{D_h}$.

(5)

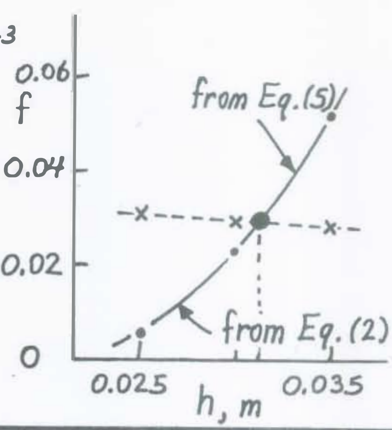
Assume $h=0.04\,m$; from (2) $f=0.111$, from (3) $\frac{\varepsilon}{D_h}=1.07\times10^{-3}$, and from (4) $Re_h=9.93\times10^4$. Hence, from (4) $f=0.0223\neq0.111$

Assume $h=0.03\,m$; from (2) $f=0.0227$, $\frac{\varepsilon}{D_h}=4.0\times10^{-3}$ and $Re_h=1.32\times10^5$
Hence, from (5) $f=0.0290\neq0.0227$

Assume $h=0.025\,m$; or $f=0.00677$, $\frac{\varepsilon}{D_h}=4.76\times10^{-3}$ and $Re_h=1.59\times10^5$
Hence, from (5) $f=0.0303\neq0.00677$

Assume $h=0.035\,m$; or $f=0.0544$; $\frac{\varepsilon}{D_h}=3.40\times10^{-3}$
$Re_h=1.13\times10^5$. Hence from (5) $f=0.0280$

Plot f from Eq.(2) and f from Eq.(5) as a function of h. Solution is where the two curves intersect.



Thus $h=0.031\,m$ and $b=1.7(0.031\,m)$
or $\underline{\underline{0.031\,m\text{ by }0.053\,m}}$

(con't)

8.104 (con't)

This problem can be solved using the Colebrook equation, Eq. 8.35, rather than the Moody chart, Fig. 8.20, as follows:

From Eq. 8.35,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{Re_h\sqrt{f}}\right) \qquad (6)$$

where, from Eq. (2),

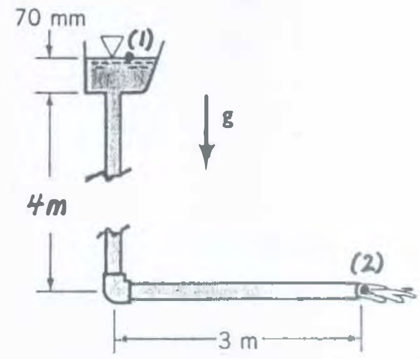
$$f = (6.41\times10^{6} h^{5} - h)/5.55 \qquad (7)$$

Combining Eqs. (3), (4), (6), and (7) gives a single equation for h:

$$\frac{1}{\left[(6.41\times10^{6}h^{5}-h)/5.55\right]} = -2.0\log\left\{\frac{1.19\times10^{-4}}{3.7h} + \frac{2.51h}{3970\left[(6.41\times10^{6}h^{5}-h)/5.55\right]}\right\} \qquad (8)$$

Using a computer root-finding program gives $h = 0.0313\ m$, which is the same as that obtained by the above trial and error method.

*8.105



*8.105 Repeat Problem 8.104 if the downspout is circular.

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_1=p_2=0,\ V_1=0,\ V_2=V,$$

$z_1 = 4.07\ m$, and $z_2=0$ Thus, $z_1=(1+f\frac{\ell}{D})\frac{V^2}{2g}$ or

$$(4.07\,m)(2)(9.81\tfrac{m}{s^2})=(1+f(\tfrac{7m}{D}))V^2 \qquad (1)$$

Hence, with $V=\frac{Q}{\frac{\pi}{4}D^2}$ or $V=\frac{4(0.006\frac{m^3}{s})}{\pi D^2}=\frac{0.00764}{D^2}$, Eq.(1) becomes

$$79.9=\left(1+\frac{7f}{D}\right)\left(\frac{0.00764}{D^2}\right)^2$$

or $f=1.956\times10^5D^5-0.1429D$, where $D \sim m$ (2)

Also, $Re=\frac{VD}{\nu}=\frac{(\frac{0.00764}{D^2})D}{\nu}=\frac{0.00764\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})D}$

or

$$Re=\frac{6.82\times10^3}{D} \qquad (3)$$

From Table 8.1 $\frac{\varepsilon}{D}=\frac{0.15\times10^{-3}}{D}$ so that Eq. 8.35 becomes (4)

$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{\frac{\varepsilon}{D}}{3.7}+\frac{2.51}{Re\sqrt{f}}\right]$$ or when combined with Eqs. (3) and (4)

$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{4.05\times10^{-5}}{D}+\frac{3.68\times10^{-4}D}{\sqrt{f}}\right] \qquad (5)$$

Solve Eqs. (2) and (5) for f and D as follows: Substitute f from Eq. (2) into Eq. (5) to obtain a single equation for D:

$$\frac{1}{(1.956\times10^5D^5-0.1429D)^{1/2}}=-2.0\log\left[\frac{4.05\times10^{-5}}{D}+\frac{3.68\times10^{-4}D}{(1.956\times10^5D^5-0.1429D)^{1/2}}\right] \qquad (6)$$

Using a computer root-finding technique gives

$\underline{\underline{D=0.0445\ m}}$

8.107

8.107 Air, assumed incompressible, flows through the two pipes shown in Fig. P8.107. Determine the flowrate if minor losses are neglected and the friction factor in each pipe is 0.015. Determine the flowrate if the 0.5-in.-diameter pipe were replaced by a 1-in.-diameter pipe. Comment on the assumption of incompressibility.

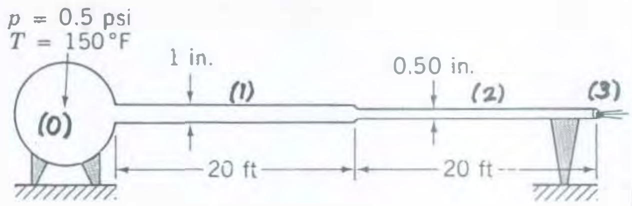


FIGURE P8.107

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=h_{L_1}+h_{L_2}+\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3, \text{ where } V_0=0,\ z_0=z_2,\ p_3=0, \tag{1}$$

$$V_2=V_3,\ h_{L_1}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g},\ h_{L_2}=f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}, \text{ and } V_1=V_2\frac{A_2}{A_1}=V_2\left(\frac{D_2}{D_1}\right)^2=\left(\frac{0.5\text{ in.}}{1.0\text{ in.}}\right)^2V_2 = 0.25V_2$$

Thus, Eq. (1) becomes

$$\frac{p_0}{\gamma}=f_1\frac{\ell_1}{D_1}\frac{(0.25V_2)^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}+\frac{V_2^2}{2g}$$

or

$$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}(0.25)^2+f_2\frac{\ell_2}{D_2}+1\right] \tag{2}$$

With $p_0=\rho_0RT_0$ or $\rho_0=\frac{p_0}{RT_0}=\frac{(0.5\frac{lb}{in.^2}+14.7\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{(1716\frac{ft\cdot lb}{slug\cdot{}^\circ R})(150+460)^\circ R}=0.00209\frac{slug}{ft^3}$

and $f_1=f_2=0.015$ Eq. (2) gives

$$(0.5\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})=\frac{1}{2}(0.00209\frac{slug}{ft^3})V_2^2\left[(0.015)\left(\left(\frac{20\text{ ft}}{\frac{1}{12}\text{ft}}\right)(0.25)^2+\left(\frac{20\text{ft}}{\frac{1}{24}\text{ft}}\right)\right)+1\right]$$

or

$V_2=90.4\ \frac{ft}{s}$ Thus, $Q=A_2V_2=\frac{\pi}{4}(\frac{1}{24}ft)^2(90.4\frac{ft}{s})=\underline{\underline{0.123\ \frac{ft^3}{s}}}$

If both pipes were 1 in. diameter, then $V_1=V_2$ and Eq. (1) becomes

$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}+f\frac{\ell_2}{D_2}+1\right]$ or with $f_1=f_2$, $\ell_1=\ell_2$, and $D_1=D_2$

$$p_0=\frac{1}{2}\rho V_2^2\left[f_2\frac{(2\ell_2)}{D_2}+1\right]$$

Hence,

$$(0.5\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})=\frac{1}{2}(0.00209\frac{slug}{ft^3})V_2^2\left[0.015\left(\frac{40\text{ ft}}{\frac{1}{12}\text{ft}}\right)+1\right]$$

or

$V_2=91.7\ \frac{ft}{s}$ Thus, $Q=A_2V_2=\frac{\pi}{4}(\frac{1}{12}ft)^2(91.7\frac{ft}{s})=\underline{\underline{0.500\frac{ft^3}{s}}}$

Since $p=\rho RT$ it follows that

$\frac{\rho_3}{\rho_0}=\frac{\left(\frac{p_3}{RT_3}\right)}{\left(\frac{p_0}{RT_0}\right)}=\frac{p_3}{p_0}\frac{T_0}{T_3}$ If we assume $T_3=T_0$ (it probably will not be, but it should be a reasonable approximation) then

$\frac{\rho_3}{\rho_0}\approx\frac{p_3}{p_0}=\frac{14.7\text{psi}}{(0.5+14.7)\text{psi}}=0.967$ The flow is nearly incompressible.

***8.108**

*8.108 Repeat Problem 8.107 if the pipes are galvanized iron and the friction factors are not known a priori.

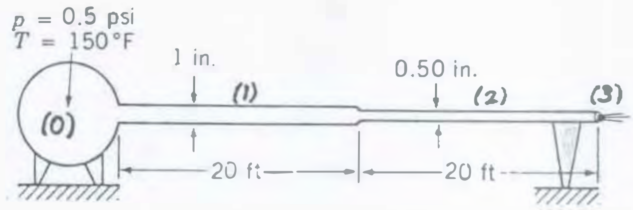


FIGURE P8.107

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=h_{L_1}+h_{L_2}+\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3\text{, where } V_0=0,\ z_0=z_2,\ p_3=0,\ V_2=V_3, \quad (1)$$

$$h_{L_1}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g},\ h_{L_2}=f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g},\text{ and } V_1=\frac{V_2A_2}{A_1}=V_2\left(\frac{D_2}{D_1}\right)^2=\left(\frac{0.5\text{ in.}}{1.0\text{ in.}}\right)^2V_2=0.25V_2$$

Thus, Eq. (1) becomes

$$\frac{p_0}{\gamma}=f_1\frac{\ell_1}{D_1}\frac{(0.25V_2)^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}+\frac{V_2^2}{2g}$$

or

$$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}(0.25)^2+f_2\frac{\ell_2}{D_2}+1\right] \quad (2)$$

With $p_0=\rho_0RT_0$ or $\rho_0=\frac{p_0}{RT_0}=\frac{(0.5\frac{lb}{in.^2}+14.7\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{(1716\frac{ft\cdot lb}{slug\cdot{}^\circ R})(150+460)^\circ R}=0.00209\ \frac{slug}{ft^3}$

Eq. (2) becomes

$$(0.5\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})=\frac{1}{2}(0.00209\frac{slug}{ft^3})V_2^2\left[(0.25)^2f_1\left(\frac{20\text{ ft}}{\frac{1}{12}\text{ ft}}\right)+f_2\left(\frac{20\text{ ft}}{\frac{1}{24}\text{ ft}}\right)+1\right]$$

or

$$6.89\times10^4=V_2^2(15f_1+480f_2+1) \quad (3)$$

Also from Table 8.1, $\frac{\varepsilon}{D_1}=\frac{0.0005\text{ ft}}{D_1}=\frac{0.0005\text{ ft}}{\frac{1}{12}\text{ ft}}=0.006$ (4)

and $\frac{\varepsilon}{D_2}=\frac{0.0005\text{ ft}}{\frac{1}{24}\text{ ft}}=0.012$ (5)

and

$$Re_1=\frac{V_1D_1}{\nu},\ Re_2=\frac{V_2D_2}{\nu}\text{, where from Table B.3}$$

$$\nu=\frac{\mu}{\rho_0}=\frac{4.18\times10^{-7}\frac{lb\cdot s}{ft^2}}{0.00209\frac{slug}{ft^3}}=2.00\times10^{-4}\frac{ft^2}{s}$$

Hence,

$$Re_1=\frac{(0.25V_2)(\frac{1}{12}\text{ ft})}{2.00\times10^{-4}\frac{ft^2}{s}}=104V_2 \quad (6)$$

and

$$Re_2=\frac{V_2(\frac{1}{24}\text{ ft})}{2.00\times10^{-4}\frac{ft^2}{s}}=208V_2 \quad (7)$$

For turbulent flow Eq. 8.35 gives $\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{\frac{\varepsilon}{D}}{3.7}+\frac{2.51}{Re\sqrt{f}}\right]$ (8)

By combining Eqs. (4) through (8) we obtain

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left[1.62\times10^{-3}+\frac{2.41\times10^{-2}}{V_2\sqrt{f_1}}\right] \quad (9)$$

and

$$\frac{1}{\sqrt{f_2}}=-2.0\log\left[3.24\times10^{-3}+\frac{1.21\times10^{-2}}{V_2\sqrt{f_2}}\right] \quad (10)$$

(con't)

*8.108 (con't)

A computer trial and error solution method gives the solution to Eqs. (3), (9), and (10) as:

$f_1 = 0.0425$, $f_2 = 0.0446$, and $V_2 = 5.47\ \frac{ft}{s}$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4}\left(\frac{0.50}{12}\ ft\right)^2\left(5.47\ \frac{ft}{s}\right) = \underline{\underline{7.46 \times 10^{-2}\ \frac{ft^3}{s}}}$$

If $D_1 = D_2$, then $V_1 = V_2$, $f_1 = f_2$ since $\frac{\varepsilon}{D_1} = \frac{\varepsilon}{D_2} = 0.006$, and

$$Re_1 = Re_2 = \frac{V_2 D_2}{\nu} = \frac{V_2\left(\frac{1}{12}\ ft\right)}{2.00 \times 10^{-4}\ \frac{ft^2}{s}} = 416\ V_2$$

Thus, Eq. (1) becomes

$$p_0 = \frac{1}{2}\rho V_2^2\left[f_2\left(\frac{\ell_1 + \ell_2}{D_2}\right) + 1\right]$$

or

$$\left(0.5\ \frac{lb}{in^2}\right)\left(144\ \frac{in^2}{ft^2}\right) = \frac{1}{2}\left(0.00209\ \frac{slug}{ft^3}\right) V_2^2\left[f_2\left(\frac{40 ft}{\frac{1}{12} ft}\right) + 1\right]$$

Hence,

$$6.89 \times 10^4 = V_2^2\left[480 f_2 + 1\right] \qquad (11)$$

Also, from Eq. (8)

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left[1.62 \times 10^{-3} + \frac{6.03 \times 10^{-3}}{V_2 \sqrt{f_2}}\right] \qquad (12)$$

A computer solution of Eqs. (11) and (12) gives

$f_2 = 0.0351$ and $V_2 = 62.2\ \frac{ft}{s}$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4}\left(\frac{1}{12}\ ft\right)^2\left(62.2\ \frac{ft}{s}\right) = \underline{\underline{0.339\ \frac{ft^3}{s}}}$$

Note: Since $p = \rho R T$ it follows that

$$\frac{\rho_3}{\rho_0} = \frac{\left(\frac{p_3}{RT_3}\right)}{\left(\frac{p_0}{RT_0}\right)} = \frac{p_3}{p_0}\frac{T_0}{T_3}$$ If we assume $T_3 = T_0$ (it probably will not be, but it should be a reasonable approximation) then

$$\frac{\rho_3}{\rho_0} = \frac{p_3}{p_0} = \frac{14.7\ psi}{(0.5 + 14.7)\ psi} = 0.967$$ The flow is nearly incompressible.

8.110

8.110 The flowrate between tank A and tank B shown in Fig. P8.110 is to be increased by 30% (i.e., from Q to $1.30Q$) by the addition of a second pipe (indicated by the dotted lines) running from node C to tank B. If the elevation of the free surface in tank A is 25 ft above that in tank B, determine the diameter, D, of this new pipe. Neglect minor losses and assume that the friction factor for each pipe is 0.02.

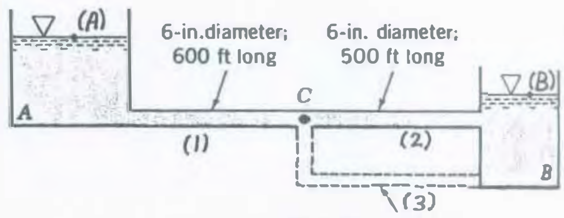


FIGURE P8.110

With the single pipe: $\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+z_A=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+z_B+f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}$ (1)

where $p_A=p_B=0$, $V_A=V_B=0$, $z_A=25\text{ ft}$, $z_B=0$,

and $V_1=V_2$ (since $D_1=D_2$).

Thus, $z_A=f_1\frac{(\ell_1+\ell_2)}{D_1}\frac{V_1^2}{2g}$, or $25\text{ ft}=(0.02)\frac{(600+500)\text{ft}}{(\frac{6}{12}\text{ft})}\frac{V_1^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$

or $V_1=6.05\frac{\text{ft}}{\text{s}}$ Hence, $Q=A_1V_1=\frac{\pi}{4}(\frac{6}{12}\text{ft})^2(6.05\frac{\text{ft}}{\text{s}})=1.188\frac{\text{ft}^3}{\text{s}}$

With the second pipe $Q=1.30(1.188\frac{\text{ft}^3}{\text{s}})=1.54\frac{\text{ft}^3}{\text{s}}$

Thus, $Q_1=1.54\frac{\text{ft}^3}{\text{s}}=Q_2+Q_3$ or $V_1=\frac{Q_1}{A_1}=\frac{1.54\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{6}{12}\text{ft})^2}=7.84\frac{\text{ft}}{\text{s}}$

For fluid flowing from A to B through pipes 1 and 2,

$z_A=h_{L_1}+h_{L_2}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}$ (see Eq. (1))

or

$$25\text{ ft}=(0.02)\frac{600\text{ ft}}{(\frac{6}{12}\text{ft})}\frac{(7.84\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}+(0.02)\frac{500\text{ ft}}{(\frac{6}{12}\text{ft})}\frac{V_2^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

Hence, $V_2=2.60\frac{\text{ft}}{\text{s}}$

and

$Q_2=A_2V_2=\frac{\pi}{4}(\frac{6}{12}\text{ft})^2(2.60\frac{\text{ft}}{\text{s}})=0.511\frac{\text{ft}^3}{\text{s}}$

Thus, $Q_3=Q_1-Q_2=1.54\frac{\text{ft}^3}{\text{s}}-0.511\frac{\text{ft}^3}{\text{s}}=1.03\frac{\text{ft}^3}{\text{s}}$

For fluid flowing from A to B through pipes 1 and 3,

$z_A=h_{L_1}+h_{L_3}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g}+f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$, where $V_3=\frac{Q_3}{A_3}=\frac{1.03\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}D_3^2}=\frac{1.31}{D_3^2}$

Thus,

$$25\text{ ft}=(0.02)\frac{600\text{ ft}}{(\frac{6}{12}\text{ft})}\frac{(7.84\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}+(0.02)\frac{500\text{ ft}}{D_3}\frac{(\frac{1.31}{D_3^2})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

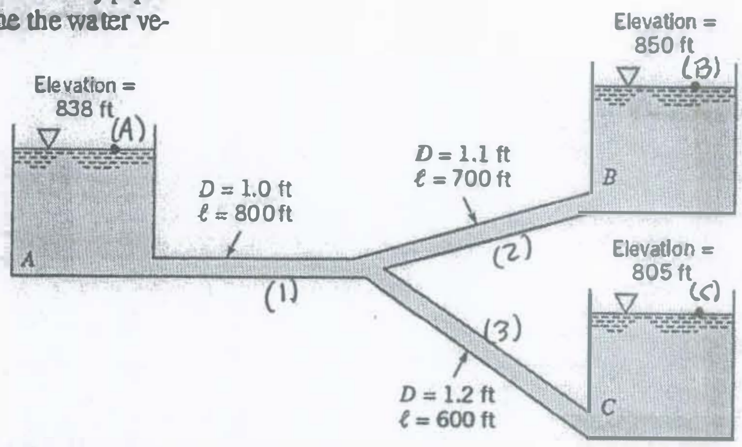
or

$D_3=0.662\text{ ft}$

Note: With the parameters given, the solution is quite sensitive to roundoff errors in the calculations

8.111

8.111 The three tanks shown in Fig. P8.111 are connected by pipes with friction factors of 0.03 for each pipe. Determine the water velocity in each pipe. Neglect minor losses.



■ FIGURE P8.111

Assume the flow from both tanks A and B is into tank C, or $Q_3 = Q_1 + Q_2$

Thus, $\frac{\pi}{4}D_3^2 V_3 = \frac{\pi}{4}D_1^2 V_1 + \frac{\pi}{4}D_2^2 V_2$, or $1.2^2 V_3 = 1.0^2 V_1 + 1.1^2 V_2$

Hence, $V_3 = 0.694\,V_1 + 0.840\,V_2$ (1)

For the flow from A to C, with $p_A = p_C = 0$, $V_A = V_C = 0$, we obtain

$z_A = z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$, or $838\,ft = 805\,ft + \frac{0.03}{2(32.2\frac{ft}{s^2})}\left[\frac{800\,ft}{1\,ft}V_1^2 + \frac{600\,ft}{1.2\,ft}V_3^2\right]$

or

$33 = 0.373\,V_1^2 + 0.233\,V_3^2$ (2)

Similarly for the flow from B to C, with $p_B = p_C = 0$, $V_B = V_C = 0$, we obtain

$z_B = z_C + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$, or $850\,ft = 805\,ft + \frac{0.03}{2(32.2\frac{ft}{s^2})}\left[\frac{700\,ft}{1.1\,ft}V_2^2 + \frac{600\,ft}{1.2\,ft}V_3^2\right]$

or

$45 = 0.296\,V_2^2 + 0.233\,V_3^2$ (3)

Thus, 3 equations ((1),(2), and (3)) for V_1, V_2, and V_3. Solve as follows:

Subtract (2) from (3) to obtain

$12 = 0.296\,V_2^2 - 0.373\,V_1^2$ (4)

From (2): $V_3 = \sqrt{141.6 - 1.6\,V_1^2}$, or when combined with (1):

$\sqrt{141.6 - 1.6V_1^2} = 0.694\,V_1 + 0.840\,V_2$, or $V_2 = \sqrt{200 - 2.27V_1^2} - 0.826\,V_1$ (5)

Combine Eqs. (4) and (5) to obtain:

$\frac{12}{0.296} = \left[\sqrt{200 - 2.27V_1^2} - 0.826\,V_1\right]^2 - \frac{0.373}{0.296}$, which can be simplified to

$V_1\sqrt{200 - 2.27\,V_1^2} = 96.5 - 1.725\,V_1^2$ By squaring this equation we (6)

obtain (after simplification):

$V_1^4 - 101.5\,V_1^2 + 1774 = 0$ Hence: $V_1^2 = \frac{101.5 \pm \sqrt{101.5^2 - 4(1774)}}{2} = \begin{matrix}79.1\\ \text{or}\\ 22.4\end{matrix}$

Thus, $V_1 = 8.89\,\frac{ft}{s}$ or $V_1 = 4.73\,\frac{ft}{s}$

Note: The $V_1 = 8.89$ solution is an extra root introduced by squaring Eq. (6). It is not a solution of the original Eqs. (1),(2),(3). For this value, Eq. (6) becomes $8.89\sqrt{200 - 2.27(8.89^2)} \overset{?}{=} 96.5 - 1.725\,(8.89)^2$, or "$40 = -40$"

Thus $V_1 = \underline{\underline{4.73\,\frac{ft}{s}}}$, from Eq. (2) $V_3 = \left[\frac{33 - 0.373(4.73)^2}{0.233}\right]^{1/2} = \underline{\underline{10.3\,\frac{ft}{s}}}$,

and from Eq. (1) $V_2 = \frac{10.3 - 0.694(4.73)}{0.840} = \underline{\underline{8.35\,\frac{ft}{s}}}$

8.112

8.112 The three water-filled tanks shown in Fig. P8.112 are connected by pipes as indicated. If minor losses are neglected, determine the flowrate in each pipe.

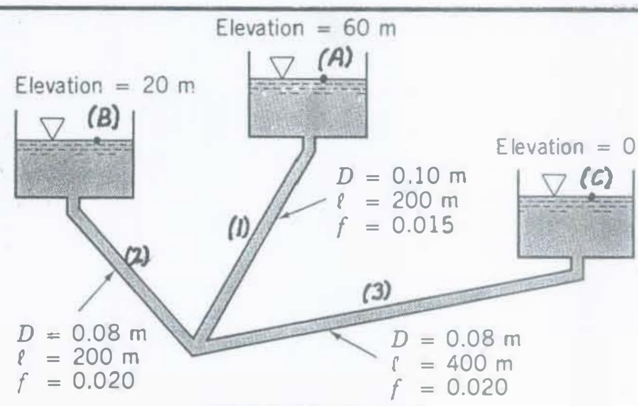


FIGURE P8.112

Assume the fluid flows from A to B and A to C. Thus, $Q_1 = Q_2 + Q_3$

or $\frac{\pi}{4}(0.1m)^2 V_1 = \frac{\pi}{4}(0.08m)^2 V_2 + \frac{\pi}{4}(0.08m)^2 V_3$

Thus, $V_1 = 0.64 V_2 + 0.64 V_3$ (1)

For fluid flowing from A to B with $p_A = p_B = 0$ and $V_A = V_B = 0$,

$$z_A = z_B + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g}$$

or

$$60m - 20m = (0.015)\left(\frac{200m}{0.1m}\right)\frac{V_1^2}{2(9.81\frac{m}{s^2})} + (0.020)\left(\frac{200m}{0.08m}\right)\frac{V_2^2}{2(9.81\frac{m}{s^2})}$$

Hence,

$$40 = 1.529 V_1^2 + 2.55 V_2^2 \quad (2)$$

Similarly, for fluid flowing from A to C with $p_A = p_C = 0$ and $V_A = V_C = 0$,

$$z_A = z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

or

$$60m = (0.015)\left(\frac{200m}{0.1m}\right)\frac{V_1^2}{2(9.81\frac{m}{s^2})} + (0.020)\left(\frac{400m}{0.08m}\right)\frac{V_3^2}{2(9.81\frac{m}{s^2})}$$

Hence,

$$60 = 1.529 V_1^2 + 5.10 V_3^2 \quad (3)$$

Solve Eqs. (1), (2), and (3) for V_1, V_2, and V_3. From Eqs. (1) and (3):

$$60 = 1.529(0.64)^2(V_2+V_3)^2 + 5.10 V_3^2, \text{ or } 95.8 = (V_2+V_3)^2 + 8.14 V_3^2 \quad (4)$$

Subtract Eq. (2) from Eq. (3):

$$60 - 40 = 5.10 V_3^2 + 2.55 V_2^2 \text{ or } V_2 = \sqrt{2V_3^2 - 7.84} \quad (5)$$

Thus, from Eqs. (4) and (5): $8.14 V_3^2 + \left(\sqrt{2V_3^2 - 7.84} + V_3\right)^2 - 95.8 = 0$

This can be simplified to

$2V_3\sqrt{2V_3^2 - 7.84} = 103.6 - 11.14 V_3^2$ (6) Square both sides and rearrange to give $V_3^4 - 19.63 V_3^2 + 92.5 = 0$ which can be solved by the quadratic formula to give

$$V_3^2 = \frac{19.63 \pm \sqrt{19.63^2 - 4(92.5)}}{2} = 11.77 \text{ or } 7.86$$ Thus $V_3 = 3.43\frac{m}{s}$ or $V_3 = 2.80\frac{m}{s}$

(con't)

8.112 (con't)

Note: The value $V_3 = 3.43 \frac{m}{s}$ is not a solution of the original equations, Eqs. (1), (2), and (3). With this value the right hand side of Eq.(6) is negative (i.e. $103.6 - 11.14V_3^2 = 103.6 - 11.14(3.43)^2 = -24.5$). As seen from the left hand side of Eq.(6), this cannot be. This extra root was introduced by squaring Eq.(6).

Thus, $Q_3 = A_3V_3 = \frac{\pi}{4}(0.08m)^2(2.80\frac{m}{s}) = \underline{\underline{0.0141 \frac{m^3}{s}}}$

Also, from Eq. (3):

$$60 = 1.529V_1^2 + 5.10(2.80)^2 \quad \text{or} \quad V_1 = 3.62\frac{m}{s}$$

or $Q_1 = A_1V_1 = \frac{\pi}{4}(0.10m)^2(3.62\frac{m}{s}) = \underline{\underline{0.0284\frac{m^3}{s}}}$

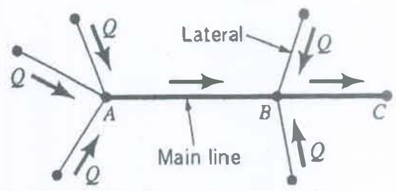
and from Eq. (1):

$$3.62 = 0.64V_2 + 0.64(2.80) \quad \text{or} \quad V_2 = 2.86\frac{m}{s}$$

or $Q_2 = A_2V_2 = \frac{\pi}{4}(0.08m)^2(2.86\frac{m}{s}) = \underline{\underline{0.0143\frac{m^3}{s}}}$

8.113

8.113 (See "Deepwater pipeline," Section 8.5.2.) Five oil fields, each producing an output of Q barrels per day, are connected to the 28-in.-diameter "main line pipe" (A–B–C) by 16-in.-diameter "lateral pipes" as shown in Fig. P8.113. The friction factor is the same for each of the pipes and elevation effects are negligible. (a) For section A–B determine the ratio of the pressure drop per mile in the main line pipe to that in the lateral pipes. (b) Repeat the calculations for section B–C.



■ FIGURE P8.113

For any of the pipe sections $\frac{\Delta p}{\gamma} = f\frac{\ell}{D}\frac{V^2}{2g}$, or $\Delta p = \frac{1}{2}\rho V^2 f\frac{\ell}{D}$

(a) Thus, $\Delta p_{AB} = \frac{1}{2}\rho V_{AB}^2 f_{AB}\frac{\ell_{AB}}{D_{AB}}$ and $\Delta p_{lat} = \frac{1}{2}\rho V_{lat}^2 f_{lat}\frac{\ell_{lat}}{D_{lat}}$, where $f_{AB} = f_{lat}$

Hence,

$$\frac{\Delta p_{AB}/\ell_{AB}}{\Delta p_{lat}/\ell_{lat}} = \frac{(V_{AB}^2/D_{AB})}{(V_{lat}^2/D_{lat})} \qquad (1)$$

Also,

$Q_{AB} = 3Q$ so that $\frac{\pi}{4}D_{AB}^2 V_{AB} = 3\frac{\pi}{4}D_{lat}^2 V_{lat}$ or $V_{AB}/V_{lat} = 3(D_{lat}/D_{AB})^2$

Thus, Eq. (1) becomes

$$\frac{\Delta p_{AB}/\ell_{AB}}{\Delta p_{lat}/\ell_{lat}} = \left[3\left(\frac{D_{lat}}{D_{AB}}\right)^2\right]^2\left(\frac{D_{lat}}{D_{AB}}\right) = 9\left(\frac{D_{lat}}{D_{AB}}\right)^5 = 9\left(\frac{16\text{ in.}}{28\text{ in.}}\right)^5 = 0.548$$

(b) Similarly, for section BC:

$\Delta p_{BC}/\ell_{BC} = \frac{1}{2}\rho V_{BC}^2 f/D_{BC}$ so that

$$\frac{\Delta p_{BC}/\ell_{BC}}{\Delta p_{lat}/\ell_{lat}} = \frac{(V_{BC}^2/D_{BC})}{(V_{lat}^2/D_{lat})} \qquad (2)$$

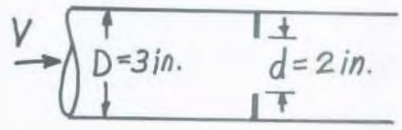
Also,

$Q_{BC} = 5Q_{lat}$, or $V_{BC}/V_{lat} = 5(D_{lat}/D_{BC})^2$ so that Eq. (2) becomes

$$\frac{\Delta p_{BC}/\ell_{BC}}{\Delta p_{lat}/\ell_{lat}} = \left[5\left(\frac{D_{lat}}{D_{BC}}\right)^2\right]^2\left(\frac{D_{lat}}{D_{BC}}\right) = 25\left(\frac{D_{lat}}{D_{BC}}\right)^5 = 25\left(\frac{16\text{ in.}}{28\text{ in.}}\right)^5 = \underline{\underline{1.52}}$$

8.116

8.116 A 2-in.-diameter orifice plate is inserted in a 3-in.-diameter pipe. If the water flowrate through the pipe is 0.90 cfs, determine the pressure difference indicated by a manometer attached to the flow meter.



$$Q = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}} \text{ , where } \beta = \frac{d}{D} = \frac{2\text{ in.}}{3\text{ in.}} = \frac{2}{3}\text{ , } Q = 0.70\,\frac{ft^3}{s}\text{ , and}$$

$$A_o = \frac{\pi}{4} d$$

Also,

$$Re = \frac{VD}{\nu}\text{ , where } V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.7\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{3}{12}ft\right)^2} = 14.26\,\frac{ft}{s}$$

Thus,

$$Re = \frac{\left(14.26\frac{ft}{s}\right)\left(\frac{3}{12}ft\right)}{1.21\times10^{-5}\frac{ft^2}{s}} = 2.95\times10^5 \quad \text{Hence, from Fig. 8.41 : } C_o = 0.608$$

so that,

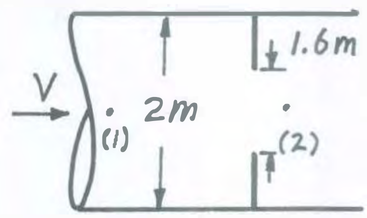
$$0.9\,\frac{ft^3}{s} = (0.608)\frac{\pi}{4}\left(\frac{2}{12}ft\right)^2\sqrt{\frac{2(p_1-p_2)}{\left(1.94\frac{slugs}{ft^3}\right)\left(1-\left(\frac{2}{3}\right)^4\right)}}$$

or

$$p_1 - p_2 = 3590\,\frac{lb}{ft^2} = \underline{\underline{24.9\,\frac{lb}{in.^2}}}$$

8.117

8.117 Air to ventilate an underground mine flows through a large 2-m-diameter pipe. A crude flowrate meter is constructed by placing a sheet metal "washer" between two sections of the pipe. Estimate the flowrate if the hole in the sheet metal has a diameter of 1.6 m and the pressure difference across the sheet metal is 8.0 mm of water.



$\Delta p = \gamma h$

$$Q = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}} = C_o \frac{\pi}{4}(1.6\,m)^2 \sqrt{\frac{2(0.008\,m)(9.80\times10^3\,\frac{N}{m^3})}{(1.23\,\frac{kg}{m^3})\left[1-\left(\frac{1.6\,m}{2.0\,m}\right)^4\right]}}$$

or

$$Q = 29.5\,C_o\ \frac{m^3}{s} \qquad (1)$$

Also, $Re = \frac{DV}{\nu} = \frac{(2\,m)\,V}{1.46\times10^{-5}\,\frac{m^2}{s}}$ or $Re = 1.37\times10^5\,V$ where $V \sim \frac{m}{s}$ (2)

and

$$\beta = \frac{d}{D} = \frac{1.6\,m}{2.0\,m} = 0.8$$

Trial and error solution:

Assume $C_o = 0.61$ so that from Eq. (1), $Q = 29.5\,(0.61) = 18.0\,\frac{m^3}{s}$

Hence, $V = \frac{Q}{A} = \frac{18.0\,\frac{m^3}{s}}{\frac{\pi}{4}(2.0\,m)^2} = 5.73\,\frac{m}{s}$

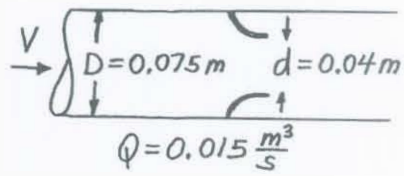
From Eq. (2), $Re = 1.37\times10^5\,(5.73) = 7.85\times10^5$

This Re and β give $C_o = 0.61$ (see Fig. 8.41) which agrees with the assumed value.

Thus, $\underline{\underline{Q = 18.0\,\frac{m^3}{s}}}$

8.118

8.118 Water flows through a 40-mm-diameter nozzle meter in a 75-mm-diameter pipe at a rate of 0.015 m³/s. Determine the pressure difference across the nozzle if the temperature is **(a)** 10 °C, or **(b)** 80 °C.



$Q = C_n A_n \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$, where $\beta = \frac{d}{D} = \frac{40\,mm}{75\,mm} = 0.533$

Thus,

$0.015 \frac{m^3}{s} = C_n \frac{\pi}{4}(0.04m)^2 \sqrt{\frac{2(p_1-p_2)}{\rho(1-0.533^4)}}$

or

$C_n\sqrt{p_1 - p_2} = 8.09\rho^{\frac{1}{2}}$, where $\rho \sim \frac{kg}{m^3}$, $p_1 - p_2 \sim \frac{N}{m^2}$ (1)

Also, $Re = \frac{VD}{\nu} = \frac{V(0.075\,m)}{\nu}$, with $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.015\frac{m^3}{s}}{\frac{\pi}{4}(0.075m)^2} = 3.40\frac{m}{s}$

a) Assume $T = 10°C$, or from Table B.2 : $\rho = 999.7\frac{kg}{m^3}$, $\nu = 1.307\times10^{-6}\frac{m^2}{s}$

Thus,

$Re = \frac{(3.40\frac{m}{s})(0.075m)}{1.307\times10^{-6}\frac{m^2}{s}} = 1.95\times10^5$ so that from Fig. 8.43:

$C_n = 0.986$

From Eq.(1): $0.986\sqrt{p_1-p_2} = 8.09(999.7)^{\frac{1}{2}}$ or $p_1 - p_2 = 6.73\times10^4\frac{N}{m^2}$

Thus,

$p_1 - p_2 = \underline{\underline{67.3\,kPa}}$

b) Assume $T = 80°C$, of from Table B.2 : $\rho = 971.8\frac{kg}{m^3}$, $\nu = 3.65\times10^{-7}\frac{m^2}{s}$

Thus,

$Re = \frac{(3.40\frac{m}{s})(0.075m)}{3.65\times10^{-7}\frac{m}{s}} = 6.99\times10^5$ so that from Fig. 8.43:

$C_n = 0.991$

From Eq.(1): $0.991\sqrt{p_1-p_2} = 8.09(971.8)^{\frac{1}{2}}$ or $p_1-p_2 = 6.48\times10^4\frac{N}{m^2}$

Thus,

$p_1 - p_2 = \underline{\underline{64.8\,kPa}}$

8.119.

8.119 Air at 200 °F and 60 psia flows in a 4-in.-diameter pipe at a rate of 0.52 lb/s. Determine the pressure at the 2-in-diameter throat of a Venturi meter placed in the pipe.

$$Q = C_v A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{2\text{ in.}}{4\text{ in.}} = 0.5 \text{ and } \gamma Q = 0.52\ \frac{lb}{s} \quad (1)$$

Also, $\rho = \frac{p}{RT} = \frac{(60\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{(1716\ \frac{ft \cdot lb}{slug \cdot °R})(200+460)°R} = 7.63 \times 10^{-3}\ \frac{slug}{ft^3}$

so that

$$\gamma = \rho g = (7.63 \times 10^{-3}\ \frac{slug}{ft^3})(32.2\ \frac{ft}{s}) = 0.246\ \frac{lb}{ft^3}$$

Thus, $Q = \frac{0.52\ \frac{lb}{s}}{0.246\ \frac{lb}{ft^3}} = 2.11\ \frac{ft^3}{s}$ and $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{2.11\ \frac{ft^3}{s}}{\frac{\pi}{4}(\frac{4}{12}ft)^2} = 24.2\ \frac{ft}{s}$

Also, from Table B.3, $\mu = 4.49 \times 10^{-7}\ \frac{lb \cdot s}{ft^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(7.63 \times 10^{-3}\ \frac{slug}{ft^3})(24.2\ \frac{ft}{s})(\frac{4}{12}ft)}{4.49 \times 10^{-7}\ \frac{lb \cdot s}{ft^2}} = 1.37 \times 10^5$$

Hence, from Fig. 8.45,

$C_v \approx 0.98$

From Eq. (1): $2.11\ \frac{ft^3}{s} = (0.98)\frac{\pi}{4}(\frac{2}{12}ft)^2 \sqrt{\frac{2(p_1 - p_2)}{(7.63 \times 10^{-3}\ \frac{slug}{ft^3})(1-0.5^4)}}$

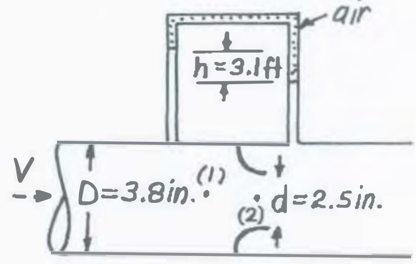
or

$p_1 - p_2 = 34.8\ \frac{lb}{ft^2}\left(\frac{1\ ft^2}{144\ in.^2}\right) = 0.242\ \frac{lb}{in.^2}$

Thus, $p_2 = (60 - 0.242)\,psia = \underline{\underline{59.76\ psia}}$

8.120

8.120 A 2.5-in.-diameter flow nozzle is installed in a 3.8-in.-diameter pipe that carries water at 160 °F. If the air–water manometer used to measure the pressure difference across the meter indicates a reading of 3.1 ft, determine the flowrate.



$$Q = C_n A_n \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{2.5\text{ in.}}{3.8\text{ in.}} = 0.658 \quad (1)$$

From Table B.1: $\rho = 1.896\ \frac{slugs}{ft^3}$, $\mu = 8.32\times10^{-6}\ \frac{lb\cdot s}{ft^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(1.896\ \frac{slugs}{ft^3})\,V\,(\frac{3.8}{12}\ ft)}{8.32\times10^{-6}\ \frac{lb\cdot s}{ft^2}}$$

or

$$Re = 7.22\times10^4\,V, \text{ where } V \sim \frac{ft}{s} \quad (2)$$

Also, with $Q = \frac{\pi}{4}D^2V$ Eq.(1) becomes (using $p_1 - p_2 = \gamma h$):

$$\frac{\pi}{4}\left(\frac{3.8}{12}\ ft\right)^2 V = C_n \frac{\pi}{4}\left(\frac{2.5}{12}\ ft\right)^2\left[\frac{2(32.2\frac{ft}{s^2})(1.896\ \frac{slugs}{ft^3})(3.1\ ft)}{(1.896\ \frac{slugs}{ft^3})(1-0.658^4)}\right]^{1/2}$$

or

$$V = 6.78\,C_n \quad (3)$$

Trial and error solution using Fig. 8.43 for $C_n = C_n(Re, \beta = 0.658)$:

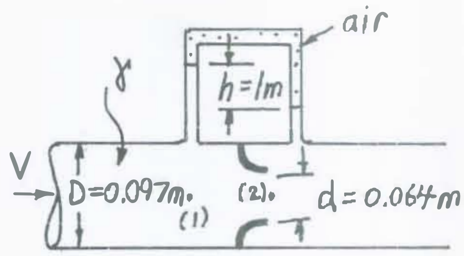
Assume $C_n = 0.99$ From Eq.(3) $V = 6.78\,(0.99) = 6.71\ \frac{ft}{s}$

From Eq. (2) $Re = 7.22\times10^4(6.71\frac{ft}{s}) = 4.84\times10^5$ which from Fig. 8.43 gives $C_n = 0.99$ (checks with assumed value)

Thus, $V = 6.71\ \frac{ft}{s}$ and $Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}\left(\frac{3.8}{12}ft\right)^2\left(6.71\ \frac{ft}{s}\right) = \underline{\underline{0.528\ \frac{ft^3}{s}}}$

8.121

8.121 A 0.064 m-diameter nozzle meter is installed in a 0.097 m-diameter pipe that carries water at 60 °C. If the inverted air–water U-tube manometer used to measure the pressure difference across the meter indicates a reading of 1 m, determine the flowrate.



(1) $Q = C_n A_n \sqrt{\dfrac{2(p_1-p_2)}{\rho(1-\beta^4)}}$, where $\beta = \dfrac{d}{D} = \dfrac{0.064\ m}{0.097\ m} = 0.660$

From Table B.2 : $\rho = 983.2\ \frac{kg}{m^3}$, $\mu = 4.665\times10^{-4}\ \frac{N\cdot s}{m^2}$ so that

$Re = \dfrac{\rho V D}{\mu} = \dfrac{(983.2\ \frac{kg}{m^3})\ V\ (0.097 m)}{4.665\times10^{-4}\ \frac{N\cdot s}{m^2}}$

or

(2) $Re = 2.04\times10^5\ V$, where $V \sim \frac{m}{s}$

Also, with $Q = \frac{\pi}{4}D^2V$ and $p_1-p_2 = \gamma h = \rho g h = 983.2\ \frac{kg}{m^3}(9.81\ \frac{m}{s^2})(1 m)$
$= 9.65\times10^3\ \frac{N}{m^2}$

equation (1) becomes

$$\frac{\pi}{4}(0.097\ m)^2 V = C_n \frac{\pi}{4}(0.064 m)^2\left[\frac{2\ (9.65\times10^3 \frac{N}{m^2})}{(983.2\ \frac{kg}{m^3})(1-0.660^4)}\right]^{1/2}$$

or

(3) $V = 2.14\ C_n$

Trial and error solution using Fig. 8.43 for $C_n = C_n(Re, \beta = 0.660)$

Assume $C_n = 0.99$ From Eq. (3), $V = 2.14(0.99) = 2.12\ \frac{m}{s}$

From Eq. (2), $Re = 2.04\times10^5(2.12) = 4.32\times10^5$ which from Fig. 8.43 gives $C_D = 0.99$ which checks with the assumed value.

Thus, $V = 2.12\ \frac{m}{s}$ and $Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}(0.097 m)^2(2.12\frac{m}{s}) = \underline{\underline{0.0157\ \frac{m^3}{s}}}$

8.122

8.122 Water flows through the Venturi meter shown in Fig. P8.122. The specific gravity of the manometer fluid is 1.52. Determine the flowrate.

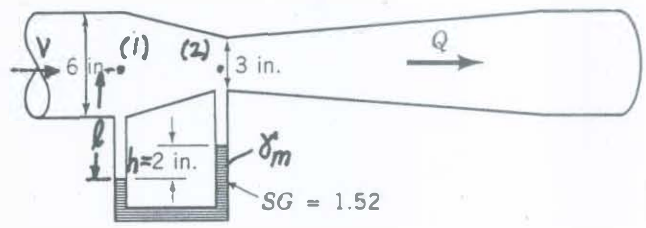


FIGURE P8.122

$$Q = C_v A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{3\text{ in.}}{6\text{ in.}} = 0.5$$

Also,

$$p_1 + \gamma\ell = p_2 + \gamma(\ell - h) + \gamma(SG)h \quad \text{or } p_1 - p_2 = \gamma(SG-1)h = \rho g(SG-1)h$$

Hence,

$$Q = C_v A_T \sqrt{\frac{2\rho g(SG-1)h}{\rho(1-\beta^4)}} \quad \text{or}$$

$$Q = C_v \frac{\pi}{4}\left(\frac{3}{12}\text{ft}\right)^2 \left[\frac{2(32.2\frac{\text{ft}}{\text{s}^2})(1.52-1)(\frac{2}{12}\text{ft})}{(1-0.5^4)}\right]^{1/2}$$

Thus,

$Q = 0.1198\, C_v$ Assume $C_v \approx 0.98$ so that $Q = 0.1198\,(0.98) = 0.117\,\frac{\text{ft}^3}{\text{s}}$

Hence,

$$V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.117\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{6}{12}\text{ft})^2} = 0.596\,\frac{\text{ft}}{\text{s}} \quad \text{so that}$$

$$Re = \frac{VD}{\nu} = \frac{(0.596\frac{\text{ft}}{\text{s}})(\frac{6}{12}\text{ft})}{1.21\times10^{-5}\frac{\text{ft}^2}{\text{s}}} = 2.46\times10^4$$

From Fig. 8.45 at this Re, $C_v \approx 0.96 \neq 0.98$, the assumed value.

Hence, assume $C_v = 0.96$, or

$$Q \approx 0.1198\,(0.96) = 0.115\,\frac{\text{ft}^3}{\text{s}} \text{ and } V = \frac{0.115}{\frac{\pi}{4}(\frac{6}{12})^2} = 0.586\,\frac{\text{ft}}{\text{s}}$$

Therefore, $Re = \dfrac{0.586\,(\frac{6}{12})}{1.21\times10^{-5}} = 2.42\times10^4$ so that from Fig. 8.45,

$C_v \approx 0.96$ Checks with assumed value.

Hence, $Q = \underline{\underline{0.115\,\frac{\text{ft}^3}{\text{s}}}}$

8.123

8.123 Water flows through the orifice meter shown in Fig. P8.123 at a rate of 0.10 cfs. If $d = 0.1$ ft, determine the value of h.

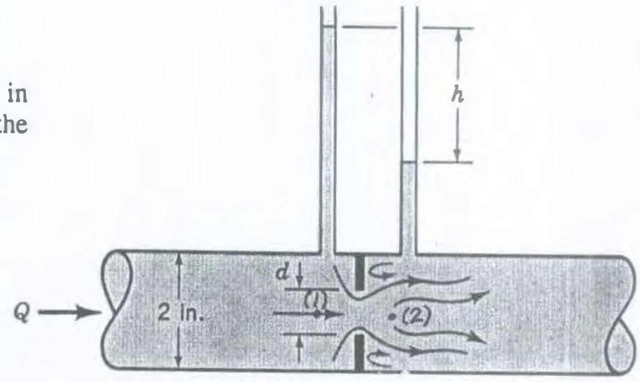


FIGURE P8.123

$$Q = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{0.1\,ft}{\frac{2}{12}\,ft} = 0.6,\ p_1 - p_2 = \gamma h = \rho g h \qquad (1)$$

Also,
$$V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.10\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2} = 4.58\,\frac{ft}{s} \text{ so that}$$

$$Re = \frac{VD}{\nu} = \frac{(4.58\,\frac{ft}{s})(\frac{2}{12}\,ft)}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 6.31\times10^4 \text{ Hence, from Fig. 8.41, } C_o = 0.616$$

Therefore, from Eq. (1):

$$0.10\,\frac{ft^3}{s} = (0.616)\frac{\pi}{4}(0.1\,ft)^2\sqrt{\frac{2\rho(32.2\,\frac{ft}{s^2})h}{\rho(1-0.6^4)}} \text{ or } h = \underline{\underline{5.77\,ft}}$$

8.124

8.124 Water flows through the orifice meter shown in Fig. P8.123 such that $h = 1.6$ ft with $d = 1.5$ in. Determine the flowrate.

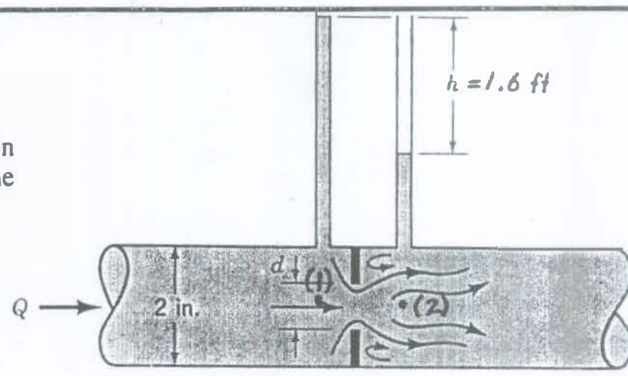


FIGURE P8.123

$Q = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}$, where $\beta = \frac{d}{D} = \frac{1.5\,in.}{2\,in.} = 0.75$ and $p_1 - p_2 = \gamma h = \rho g h$

Thus,

$$Q = C_o \frac{\pi}{4}\left(\frac{1.5}{12}ft\right)^2 \left[\frac{2\,\rho(32.2\frac{ft}{s^2})(1.6ft)}{\rho(1-0.75^4)}\right]^{\frac{1}{2}}$$

or

$Q = 0.151\, C_o$ (1)

Also, $Re = \frac{VD}{\nu} = \frac{V(\frac{2}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}} = 1.38\times10^4\, V$, where $V = \frac{Q}{\frac{\pi}{4}D^2} = 45.8Q$ (2)

Trial and error solution:

Assume $C_o = 0.6$; or from Eq.(1), $Q = 0.151(0.6) = 0.0906\ \frac{ft^3}{s}$

Hence, from Eq.(2), $V = 45.8(0.0906) = 4.15$ and $Re = 5.73\times10^4$

From Fig. 8.41 with this Re and β, $C_o = 0.62 \neq 0.6$ (the assumed value)

Assume $C_o = 0.62$ or $Q = 0.151(0.62) = 0.0936\ \frac{ft^3}{s}$. Thus $V = 45.8(0.0936)$ or $V = 4.29\frac{ft}{s}$ and $Re = 5.92\times10^4$, From Fig. 8.41, $C_o = 0.62$, the assumed value.

Hence, $Q = \underline{\underline{0.0936\ \frac{ft^3}{s}}}$

8.125

8.125 The scale reading on the rotameter shown in Fig. P8.125 and Video V8.14 (also see Fig. 8.46) is directly proportional to the volumetric flowrate. With a scale reading of 2.6 the water bubbles up approximately 3 in. How far will it bubble up if the scale reading is 5.0?

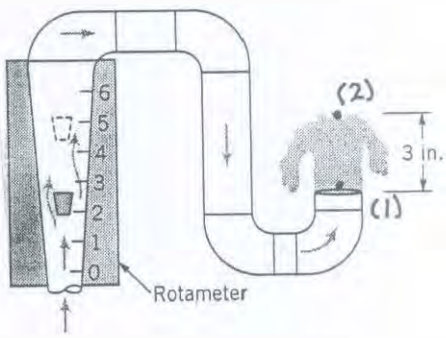


FIGURE P8.125

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0$, $z_1 = 0$, $V_2 = 0$, so that with no losses ($h_L = 0$),

(1) $$\frac{V_1^2}{2g} = z_2$$

For the rotameter $Q = K \times SR$ where SR = scale reading and K is a constant.

Thus,

$$V_1 = \frac{Q_1}{A_1} = \frac{K \times SR}{A_1}$$ so that when combined with Eq. (1),

$$\frac{K^2 (SR)^2}{A_1^2 (2g)} = z_2 \quad \text{or} \quad \frac{K^2 (2.6)^2}{A_1 (2g)} = \left(\tfrac{3}{12}\,\text{ft}\right) \quad \text{and} \quad \frac{K^2 (5.0)^2}{A_1 (2g)} = h$$

By dividing these two equations,

$$\frac{(5.0)^2}{(2.6)^2} = \frac{h}{\left(\frac{3}{12}\,\text{ft}\right)} \quad \text{or} \quad h = 0.925 \text{ ft} = \underline{\underline{11.1 \text{ in.}}}$$

8.126

8.126 Friction Factor for Laminar and Transitional Pipe Flow

Objective: Theoretically, the friction factor, f, for laminar pipe flow is given by $f = 64/\mathrm{Re}$, where the Reynolds number, $\mathrm{Re} = \rho VD/\mu$, is based on the average velocity, V, within the pipe and the pipe diameter, D. Also, the flow is normally laminar for $\mathrm{Re} < 2100$. The purpose of this experiment is to use the device shown in Fig. P8.126 to investigate these two properties.

Equipment: Small diameter metal tubes (pipes), air supply with flow regulator, rotameter flow meter, manometer.

Experimental Procedure: Attach a tube of length L and diameter D to the plenum. Adjust the flow regulator to obtain the desired flowrate as measured by the rotameter. Record the manometer reading, h, so that the pressure difference between the plenum (tank) and the free jet at the end of the tube can be determined. Repeat for several different flowrates and tube diameters. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For each of the data sets determine the pressure difference, $\Delta p = \gamma_m h$, between the plenum pressure and the free jet pressure. Here γ_m is the specific weight of the manometer fluid. Use the energy equation, Eq. 5.84, to determine the friction factor, f. Assume the loss coefficient for the pipe entrance is $K_L = 0.8$. Also calculate the Reynolds number, Re, for each data set.

Graph: On a log-log graph, plot the experimentally determined friction factor, f, as ordinates and the Reynolds number, Re, as abscissas.

Results: On the same graph, plot the theoretical friction factor for laminar flow, $f = 64/\mathrm{Re}$, as a function of the Reynolds number. Based on the experimental data, determine the maximum value of the Reynolds number for which the flow in these pipes is laminar.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

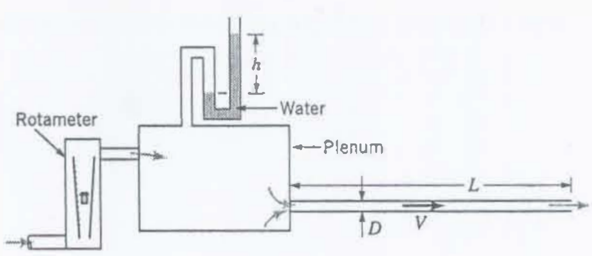


FIGURE P8.126

(con't)

8.126 (con't)

Solution for Problem 8.126: .Friction Factor for Laminar and Transitional Pipe Flow

L, in.	H_{atm}, in. Hg	T, deg F
24	28.9	73

h, in.	Q, ml/min	Q, cfs	V, fps	Re	f	Theoretical Re	Theoretical f
D = 0.108 in. Data						100	0.6400
7.5	6600	0.003887	61.11	3202	0.0341	2100	0.0305
6.75	6200	0.003652	57.40	3008	0.0349		
6.26	6000	0.003534	55.55	2911	0.0345		
5.54	5650	0.003328	52.31	2741	0.0344		
4.66	5150	0.003033	47.68	2499	0.0349		
4.29	5000	0.002945	46.29	2426	0.0339		
3.92	4860	0.002863	45.00	2358	0.0325		
3.48	4600	0.002709	42.59	2232	0.0322		
3.21	4500	0.002651	41.66	2183	0.0307		
2.34	3700	0.002179	34.26	1795	0.0338		
1.86	2900	0.001708	26.85	1407	0.0461		
1.11	1800	0.001060	16.67	873	0.0758		
0.63	1100	0.000648	10.18	534	0.1194		
D = 0.046 in. Data							
9.52	560	0.000330	28.58	638	0.1007		
7.68	475	0.000280	24.24	541	0.1134		
7.08	425	0.000250	21.69	484	0.1311		
5.26	315	0.000186	16.08	359	0.1785		
3.39	221	0.000130	11.28	252	0.2348		
2.61	165	0.000097	8.42	188	0.3256		
D = 0.063 in. Data							
4.58	925	0.000545	25.17	770	0.0838		
3.32	680	0.000401	18.50	566	0.1140		
2.51	530	0.000312	14.42	441	0.1431		
1.48	325	0.000191	8.84	270	0.2270		
0.86	190	0.000112	5.17	158	0.3893		

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{H2O} * H_{atm}$ = 847 lb/ft^3*(28.9/12 ft) = 2040 lb/ft^2

R = 1716 ft lb/slug deg R

T = 73 + 460 = 533 deg R

Thus, ρ = 0.00223 slug/ft^3 and $\gamma = \rho * g$ = 0.0718 lb/ft^3

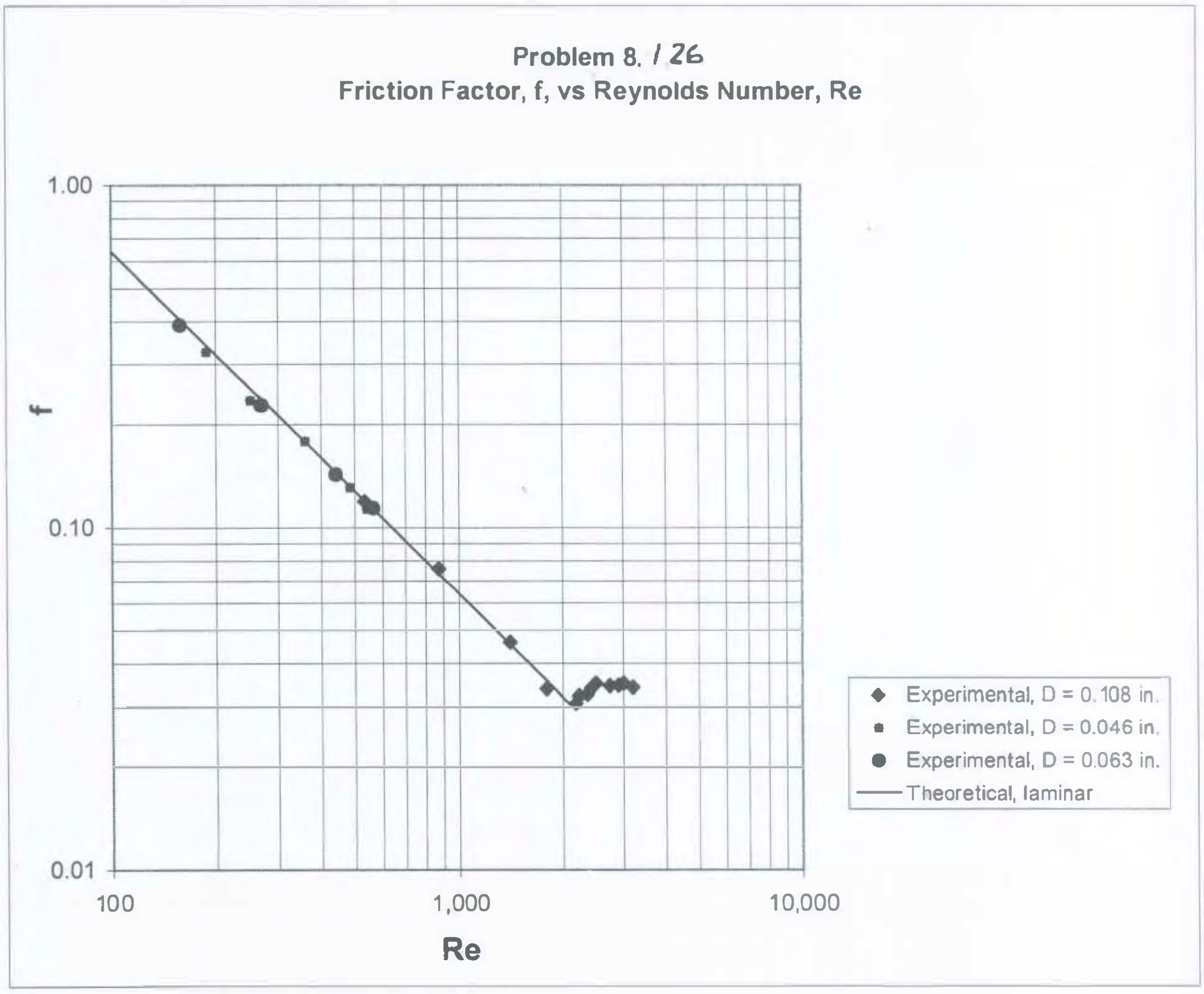
Also, μ = 3.83E-7 lb s/ft^2

Theoretical for laminar flow: f = 64/Re = 64/(ρDV/μ)

$\Delta p/\gamma$ = (fL/D + K_L + 1)(V^2/2g) where K_L = entrance loss coefficient = 0.8 and V = Q/(πD^2/4)

(con't)

8.126 (con't)



Problem 8.126
Friction Factor, f, vs Reynolds Number, Re
1.00
0.10
0.01
f
100
1,000
10,000
Re
Experimental, D = 0.108 in.
Experimental, D = 0.046 in.
Experimental, D = 0.063 in.
Theoretical, laminar

8.127

8.127 Calibration of an Orifice Meter and a Venturi Meter

Objective: Because of various real-world, nonideal conditions, neither orifice meters nor Venturi meters operate exactly as predicted by a simple theoretical analysis. The purpose of this experiment is to use the device shown in Fig. P8.127 to calibrate an orifice meter and a Venturi meter.

Equipment: Water tank with sight gage, pump, Venturi meter, orifice meter, manometers.

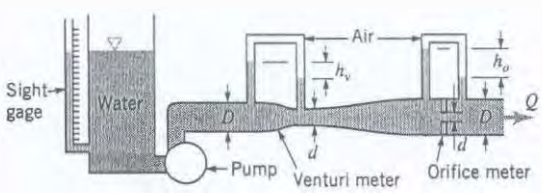
Experimental Procedure: Determine the pipe diameter, D, and the throat diameter, d, for the flow meters. Note that each meter has the same values of D and d. Make sure that the tubes connecting the manometers to the flow meters do not contain any unwanted air bubbles. This can be verified by noting that the manometer readings, h_v, and h_o, are zero when the system is full of water and the flowrate, Q, is zero. Turn on the pump and adjust the valve to give the desired flowrate. Record the time, t, it takes for a given volume, V, of water to be pumped from the tank. The volume can be determined from using the sight gage on the tank. At this flowrate record the manometer readings. Repeat for several different flowrates.

Calculations: For each data set determine the volumetric flowrate, $Q = V/t$, and the pressure differences across each meter, $\Delta p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. Use the flow meter equations (see Section 8.6.1) to determine the orifice discharge coefficient, C_o, and the Venturi discharge coefficient, C_v, for these meters.

Graph: On a log-log graph, plot flowrate, Q, as ordinates and pressure difference, Δp, as abscissas.

Result: On the same graph, plot the ideal flowrate, Q_{ideal} (see Eq. 8.37), as a function of pressure difference.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.127

(con't)

8.127 (con't)

Solution for Problem 8.127: Calibration of an Orifice Meter and a Venturi Meter

d, in.	D, in.	**V**, gallons							
0.625	1.025	2.00							Ideal C = 1
t, s	h_o, in.	h_v, in.	Δp_o, lb/ft^2	Δp_v, lb/ft^2	Q, ft^3/s	C_o	C_v		Δp, lb/ft^2
27.0	9.3	3.8	48.4	19.8	0.0099	0.611	0.956		18.0
13.2	37.1	14.5	192.9	75.4	0.0203	0.626	1.001		75.5
34.2	5.5	1.9	28.6	9.9	0.0078	0.627	1.067		11.2
16.6	23.9	10.1	124.3	52.5	0.0161	0.620	0.953		47.7
12.0	43.2	18.1	224.6	94.1	0.0223	0.638	0.985		91.4
11.7	51.3	21.7	266.8	112.8	0.0229	0.600	0.923		96.1
15.4	27.9	11.2	145.1	58.2	0.0174	0.618	0.976		55.5
25.1	10.1	4.2	52.5	21.8	0.0107	0.631	0.978		20.9
20.4	14.7	6.2	76.4	32.2	0.0131	0.643	0.990		31.6
17.3	21.4	8.7	111.3	45.2	0.0155	0.629	0.986		44.0
15.7	26.7	11.2	138.8	58.2	0.0170	0.620	0.957		53.4
			Average discharge coefficient:			0.624 orifice	0.979 venturi		

Q = **V** gal/t s x (231 in.^3/gal)x(1 ft^3/1728 in.^3)

$\Delta p = \gamma_{H2O}$*h = 62.4 lb/ft^3 *h ft

$Q_v = A_2/[1 - (A_2/A_1)^2]^{0.5} * C_v * (2*g*\Delta p_v/\gamma_{H2O})^{0.5}$

and

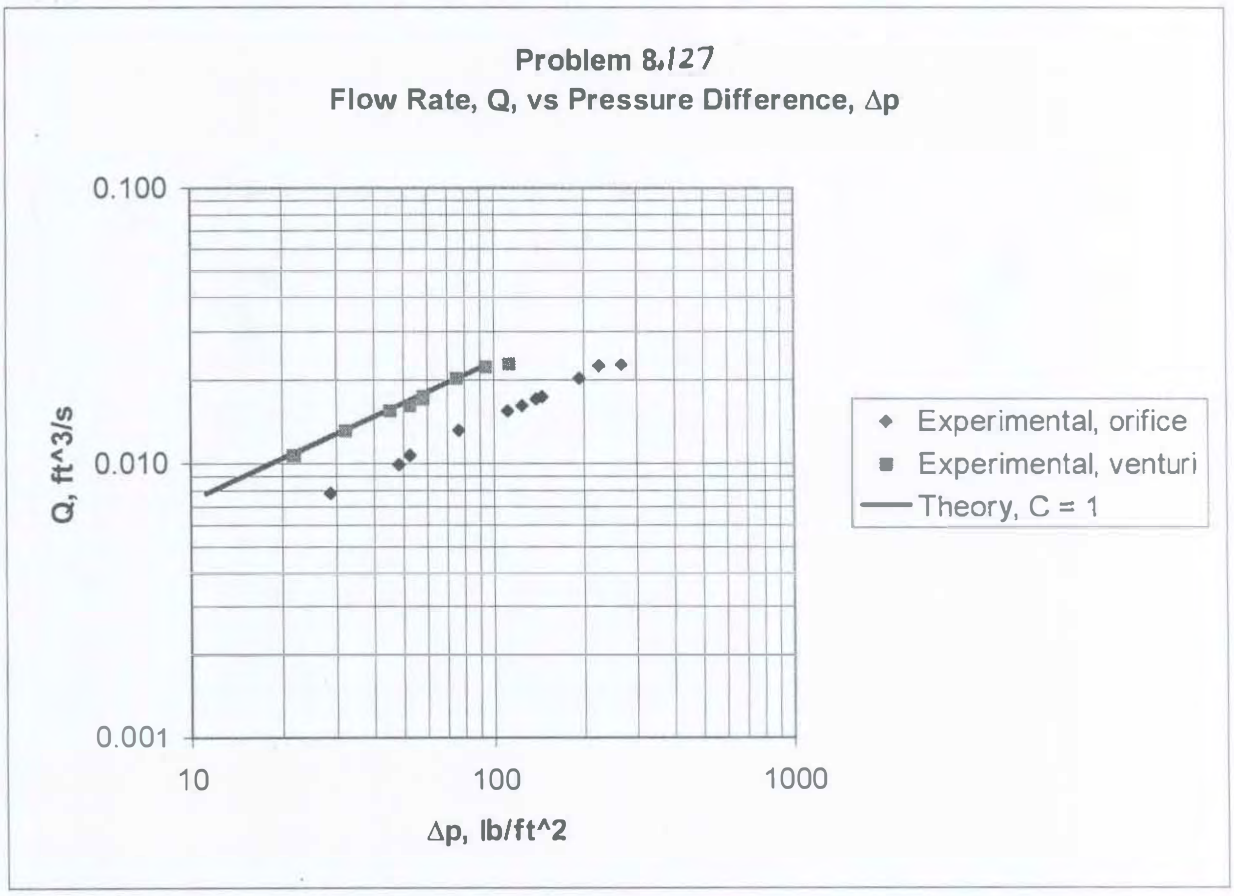
$Q_o = A_2/[1 - (A_2/A_1)^2]^{0.5} * C_o * (2*g*\Delta p_o/\gamma_{H2O})^{0.5}$

where

$A_1 = \pi$ D^2/4 = π (1.025/12 ft)^2/4 = 0.00573 ft^2

and

$A_2 = \pi$ d^2/4 = π (0.625/12 ft)^2/4 = 0.00213 ft^2



8.128

8.128 Flow from a Tank through a Pipe System

Objective: The rate of flow of water from a tank is a function of the pipe system used to drain the tank. The purpose of this experiment is to use a pipe system as shown in Fig. P8.128 to investigate the importance of major and minor head losses in a typical pipe flow situation.

Equipment: Water tank; various lengths of galvanized iron pipe; various threaded pipe fittings (valves, elbows, etc.); pipe wrenches; stop watch; thermometer.

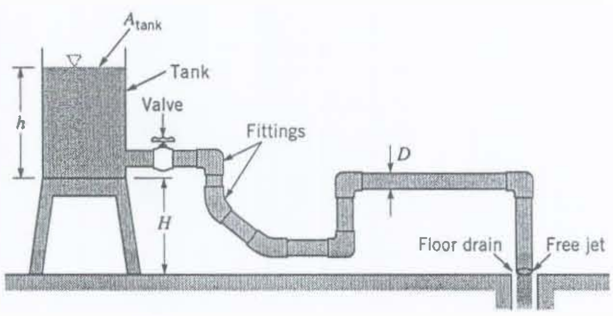
Experimental Procedure: Use the pipe segments and pipe fittings to construct a suitable pipeline through which the tank water may flow into a floor drain. Measure the pipe diameter, D, and the various pipe lengths and note the various valves and fittings used. Measure the elevation difference, H, between the bottom of the tank and the outlet of the pipe. Also determine the cross-sectional area of the tank, A_{tank}. Fill the tank with water and record the water temperature, T. With the pipeline valve wide open, measure the water depth, h, in the tank as a function of time, t, as the tank drains.

Calculations: Calculate the experimentally determined flowrate, Q_{ex}, from the tank as $Q_{ex} = -A_{tank}\, dh/dt$, where the time rate of change of water depth, dh/dt, is obtained from the slope of the h versus t graph. Select a typical water depth, h_1, for this calculation.

Graph: Plot the water depth, h, in the tank as ordinates and time, t, as abscissas.

Results: For the pipe system used in this experiment, use the energy equation to calculate the theoretical flowrate, Q_{th}, based on three different assumptions. Use the same typical water depth, h_1, for the theoretical calculations as was used in determining Q_{ex}. First, calculate Q_{th} under the assumption that all losses are negligible. Second, calculate Q_{th} if only major losses (pipe friction) are important. Third, calculate Q_{th} if both major and minor losses are important.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.128

(con't)

8.128 (con't)

Solution for Problem 8.128: Flow from a Tank Through a Pipe System

The pipe is galvanized iron with threaded fittings.
The system contains:

- one sharp edged entrance
- one fully open globe valve
- two 45-deg elbows
- four 90-deg elbows

D, in.	A_{tank}, ft^2	H, ft	Total pipe length, in.	T, deg F
0.595	0.654	1.00	135	71

h, ft	t, s
1.00	0
0.90	13
0.80	26
0.70	40
0.60	54
0.50	67
0.40	81

Experimental: Q_{ex} = -(dh/dt)*A_{tank} = -(0.0074 ft/s)*(0.654 ft^2) = 0.00484 ft^3/s

Theoretical with no losses: Q_{th} = V_2*A_2, where when h = 0.90 ft

V_2 = (2g*(h + H))^0.5 = (2*32.2*(0.9 +1.0))^0.5 = 11.06 ft/s

and with A_2 = πD^2/4 = π*(0.595/12 ft)^2/4 = 0.00193 ft^2

Q_{th} = 0.00193 ft^2*(11.06 ft/s) = 0.0213 ft^3/s

Theoretical with major losses: Q_{th} = V_2*A_2, where the energy equation gives

h + H = V_2^2/2g(1 + fL/D), where again use h = 0.90 ft and f is a function of Re and ε/D

Thus, with h = 0.90 ft,

1.9 = (V_2^2/64.4)*(1 + f*135/0.595), or

122.4 = V_2^2*(1 + 227f)

Re = V_2D/ν = V_2*(0.595/12 ft)/(1.04E-5 ft^2/s) = 4768*V_2

and

ε/D =0.0005 ft/(0.595/12 ft) = 0.0101

Trial and error solution*: Guess f, solve for V_2, calculate Re, obtain new f from Moody chart

The solution is: f = 0.041, V_2 = 3.44 ft/s, Re = 16,430

Q_{th} = 0.00193 ft^2*(3.44 ft/s) = 0.00664 ft^3/s

Theoretical with major and minor losses: The energy equation gives

h + H = (1 + fL/D +ΣK_L)V_2^2/2g

where ΣK_L = 0.5 + 10 + 2*0.4 + 4*1.5 = 17.3

Thus, with h = 0.9 ft

1.9 = (V_2^2/64.4)*(17.3 + f*135/0.595), or

122.4 = V_2^2*(17.3 + 227f)

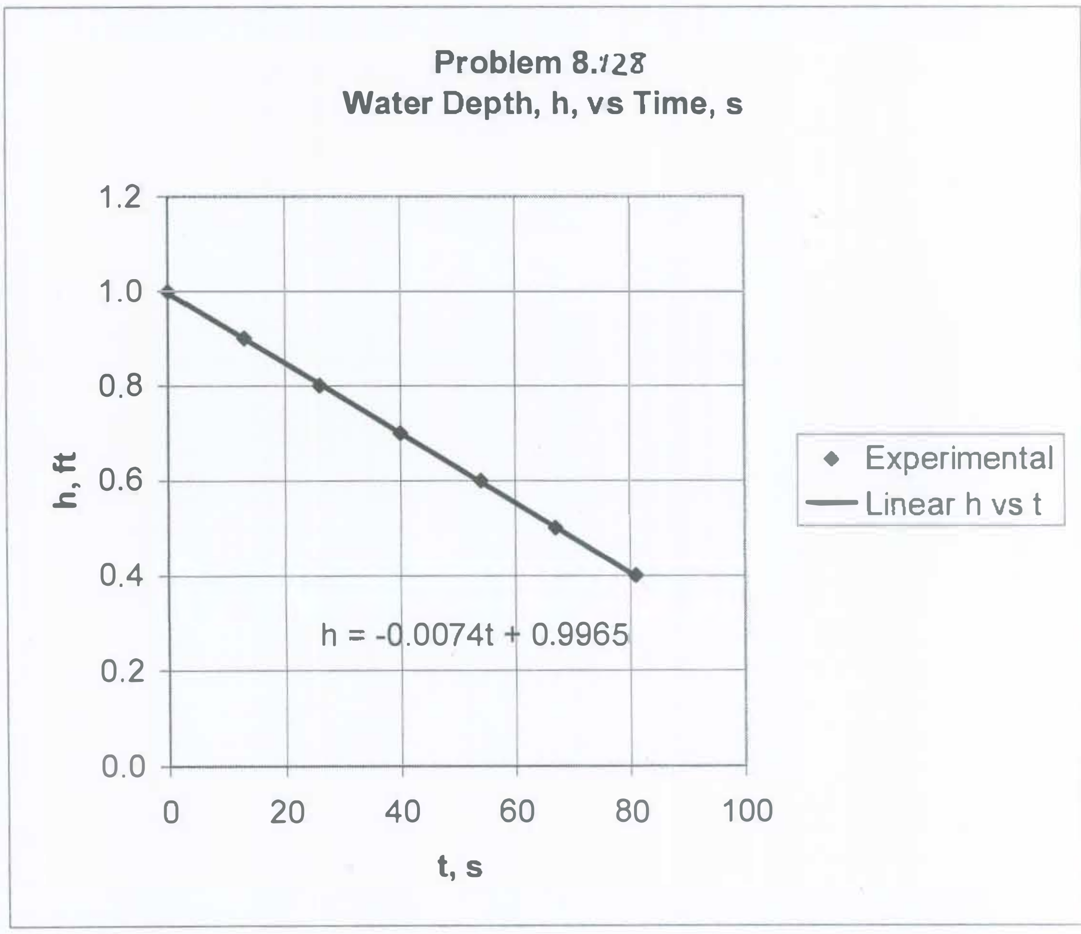
Trial and error solution gives*: f = 0.42, V_2 = 2.14 ft/s, Re = 10,200

Q_{th} = 0.00193 ft^2*(2.14 ft/s) = 0.00413 ft^3/s

* As an alternate solution method, use the Colebrook equation (Eq. 8.35) rather than the Moody chart (Fig. 8.20) and use a computer root-finding technique to solve the equation.

(con't)

8.128 (con't)



Problem 8.128
Water Depth, h, vs Time, s
1.2
1.0
0.8
0.6
0.4
0.2
0.0
h, ft
0
20
40
60
80
100
t, s
h = -0.0074t + 0.9965
Experimental
Linear h vs t

8.129

8.129 Flow of Water Pumped from a Tank and through a Pipe System

Objective: The rate of flow of water pumped from a tank is a function of the pump properties and of the pipe system used. The purpose of this experiment is to use a pump and pipe system as shown schematically in Fig. P8.129 to investigate the rate at which the water is pumped from the tank.

Equipment: Water tank; centrifugal pump; various lengths of galvanized iron pipe; various threaded pipe fittings (valves, elbows, unions, etc.); pipe wrenches; stop watch; thermometer.

Experimental Procedure: Use the pipe segments and pipe fittings to construct a suitable pipeline through which the tank water may be pumped into a sink. Measure the pipe diameter, D, and the various pipe lengths and note the various valves and fittings used. Measure the elevation difference, H, between the bottom of the tank and the outlet of the pipe. Also determine the cross-sectional area of the tank, A_{tank}. Fill the tank with water and record the water temperature, T. With the pipeline valves wide open, measure the water depth, h, in the tank as a function of time, t, as water is pumped from the tank.

Calculations: Calculate the experimentally determined flowrate, Q_{ex}, from the tank as $Q_{ex} = -A_{tank}\, dh/dt$, where the time rate of change of water depth, dh/dt, is obtained from the slope of the h versus t graph.

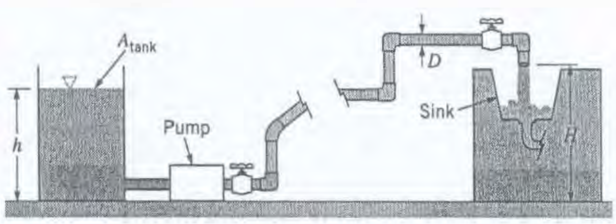
Graph: Plot the water depth, h, in the tank as ordinates and time, t, as abscissas.

Results: For the pipe system used in this experiment, use the energy equation to calculate the pump head, h_p, needed to in order to produce a given flowrate, Q. For these calculations include all major and minor losses in the pipe system. Plot the system curve (i.e., pump head as ordinates and flowrate as abscissas) based on the results of these calculations. On the same graph, plot the pump curve (i.e., h_p as a function of Q) as supplied by the pump manufacturer. For the pump used this curve is given by

$$h_p = -2.44 \times 10^5\, Q^2 + 51.0\, Q - 12.5$$

where Q is in ft^3/s and h_p is in ft. From the intersection of the system curve and the pump curve, determine the theoretical flowrate that the pump should provide for the pipe system used.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.129

(con't)

8.129 (con't)

Solution for Problem 8.129: Flowrate of Water Pumped from a Tank and Through a Pipe System

The pipe is galvanized iron with threaded fittings.
The system contains:

- one sharp entrance
- eight 90-deg elbows
- two 45-deg elbows
- two globe valves
- one union

D, in.	A_{tank}, ft^2	H, ft	Total pipe length, in.	T, deg F
0.625	0.647	3.50	242	62

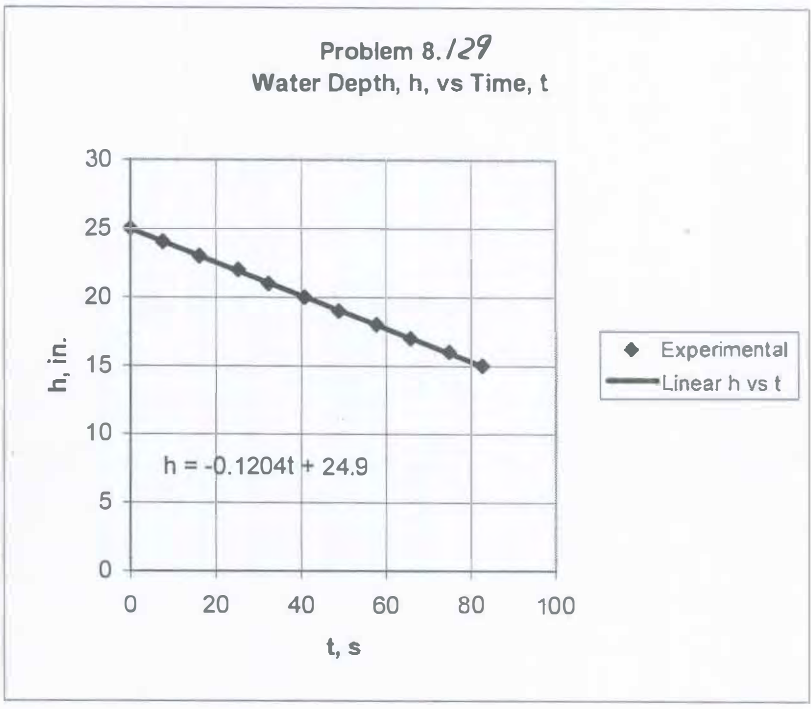
		Pump equation	System equation				
h, in.	t, s	h_p, ft	Q, ft^3/s	V, ft/s	Re	f	h_p, ft
25	0	12.50	0.000	0.00	0		2
24	7.6	12.31	0.001	0.47	2070	0.0309	2.16
23	16.1	11.63	0.002	0.94	4140	0.0490	2.73
22	25.2	10.46	0.003	1.41	6210	0.0470	3.62
21	32.3	8.80	0.004	1.88	8281	0.0450	4.84
20	40.8	6.66	0.005	2.35	10351	0.0430	6.37
19	48.9	4.02	0.006	2.81	12421	0.0425	8.27
18	57.7	0.90	0.007	3.28	14491	0.0420	10.50
17	65.7						
16	74.9						
15	82.7						

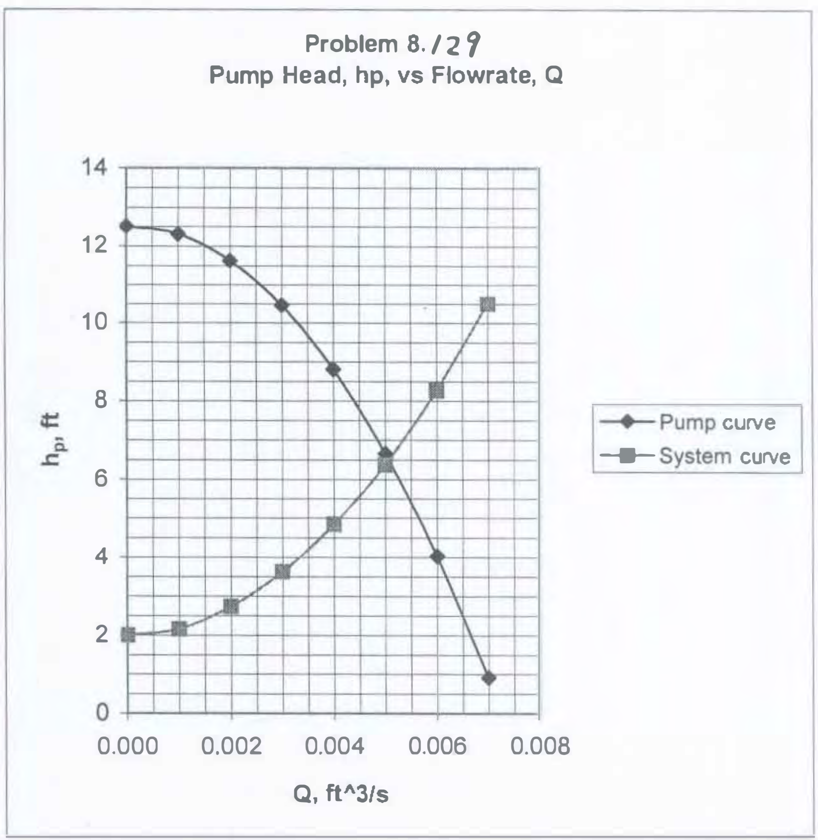
Experimental:
$Q_{ex} = -A_{tank}*(dh/dt)$ where from the graph, dh/dt = -0.1204 in./s
Thus,
Q_{ex} = -(0.647 ft^2)*(-0.1204/12 ft/s) = <u>0.00669 ft^3/s</u>

Theoretical:
The energy equation gives
$h + h_p - h_L = H + V^2/2g$, where
$h_L = (fL/D + \Sigma K_L)*V^2/2g = (f*(242 \text{ in.}/0.625 \text{ in.}) + 0.5 + 8*1.5 + 2*0.4 + 2*10 + 0.08)*V^2/2g$
$= (387*f + 33.4)*V^2/(2*32.2) = (6.01*f + 0.519)*V^2$
Thus, with h = 18 in. = 1.5 ft,
$h_p = H - h + h_L + V^2/2g = 3.5 - 1.5 + (6.01*f + 0.519)*V^2 + V^2/(64.4)$
or
$h_p = 2.0 + (6.01*f + 0.535)*V^2$
But $V = Q/A = Q/(\pi D^2/4) = Q/(\pi*(0.625/12 \text{ ft})^2/4) = 469*Q$
Thus, the system equation is
$h_p = 2.0 + (6.01*f + 0.535)*(469*Q)^2 = 2.0 + (1.32E{+}6*f + 1.18E{+}5)*Q^2$
Also, obtain f from the Moody chart with
$Re = VD/\nu = V*(0.625/12 \text{ ft})/(1.18E{-}5 \text{ ft}^2/s) = 4414*V$
$\varepsilon/D = 0.0005 \text{ ft}/(0.625/12 \text{ ft}) = 0.0096$
From the graph, the pump and system equations intersect at Q_{th} = <u>0.0051 ft^3/s</u>

(con't)

8.129 (con't)


Problem 8.129
Water Depth, h, vs Time, t
h, in.
t, s
h = -0.1204t + 24.9
Experimental
Linear h vs t
30
25
20
15
10
5
0
0
20
40
60
80
100


Problem 8.129
Pump Head, hp, vs Flowrate, Q
h_p, ft
Q, ft^3/s
Pump curve
System curve
14
12
10
8
6
4
2
0
0.000
0.002
0.004
0.006
0.008

8.130

8.130 Pressure Distribution in the Entrance Region of a Pipe

Objective: The pressure distribution in the entrance region of a pipe is different than that in the fully developed portion of the pipe. The purpose of this experiment is to use an apparatus, as shown in Fig. P8.130, to determine the pressure distribution and the head loss in the pipe entrance region.

Equipment: Air supply with flow meter, pipe with static pressure taps, manometer, ruler, barometer, thermometer.

Experimental Procedure: Measure the diameter, D, and length, L, of the pipe and the distance, x, from the pipe inlet to the various static pressure taps. Adjust the flowrate, Q, to the desired value. Record the manometer readings, h, at the various distances from the pipe entrance. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Determine the average velocity, $V = Q/A$, in the pipe and the pressure $p = \gamma_m h$ at the various locations, x, along the pipe. Here γ_m is the specific weight of the manometer fluid.

Graph: Plot the pressure, p, within the pipe as ordinates and the axial location, x, as abscissas.

RESULT: Use the graph to determine the entrance length, L_e, for the pipe. This can be done by noting the approximate location at which the pressure distribution becomes linear with distance along the pipe (i.e., where dp/dx becomes constant). Use the experimental data to determine the friction factor for fully developed flow in this pipe. Also determine the entrance loss coefficient, $K_{L ent}$.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

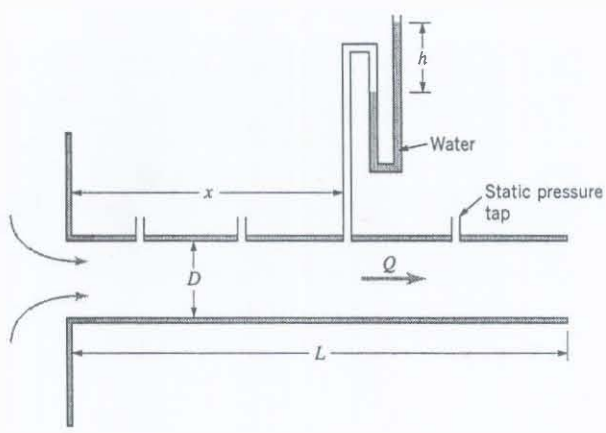


FIGURE P8.130

(con't)

8.130 (con't)

Solution for Problem 8.130: Pressure Distribution in the Entrance Region of a Pipe

D, in.	L, in.	Q, ft^3/s	H_{atm}, in. Hg	T, deg F
0.74	50	0.481	29.7	75

x, in.	h, in.	p, lb/ft^2
0	9.98	51.9
1	7.21	37.5
2	6.61	34.4
4	6.19	32.2
6	5.82	30.3
10	5.15	26.8
15	4.23	22.0
20	3.64	18.9
30	2.28	11.9
40	1.09	5.7
50	0	0.0

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.7/12 ft) = 2096 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

Thus, ρ = 0.00228 slug/ft^3

V = Q/A =(0.481 ft^3/s)/(π*(0.74/12 ft)^2/4) = 161 ft/s

$p = \gamma_{H2O} * h$

From the graph, the p vs x results are linear after (approximately) x = 15 in. Thus, L_e = 15 in.

For the fully developed flow portion, dp/dx = -fρV^2/2D and from the graph dp/dx = -0.635 (lb/ft^2)/in.
Thus,

f = 0.635 (lb/ft^2)/in.*2*0.74 in./(0.00228 slugs/ft^3*(161 ft/s)^2) = 0.0159

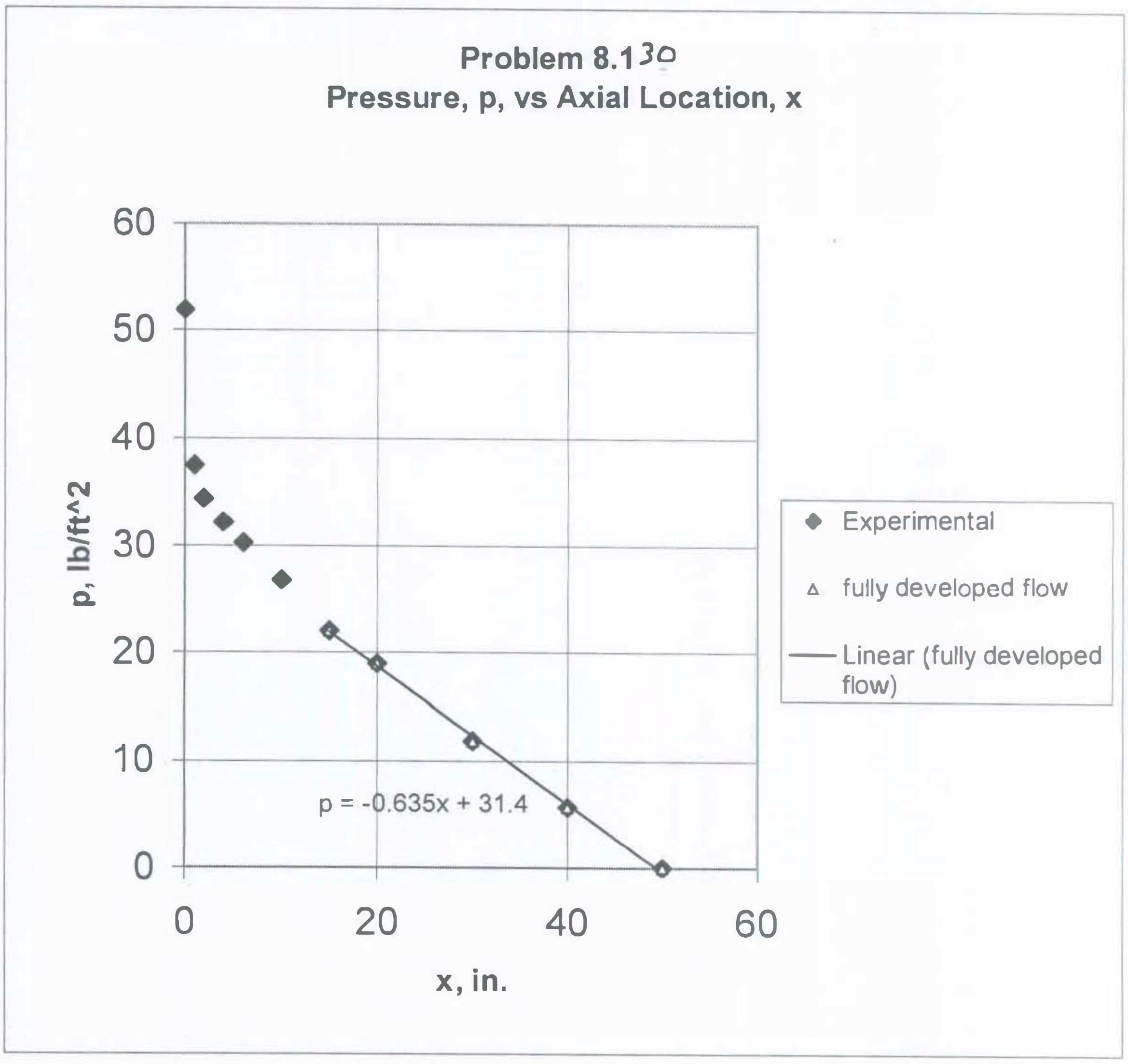
From the entrance to the exit of the pipe $p_{ent} = (K_L + fL/D)\rho V^2/2$
Thus,

$K_L = 2p_{ent}/(\rho V^2)$ -fL/D= 2*51.9 lb/ft^2/(0.00228 slugs/ft^3*(161 ft/s)^2) - 0.0159*50in./0.74 in.
=0.682

Results: L_e = 15 in.; f = 0.0159, and K_L = 0.682.

(con't)

8.130 (con't)



Problem 8.130
Pressure, p, vs Axial Location, x
p, lb/ft^2
x, in.
60
50
40
30
20
10
0
0
20
40
60
p = -0.635x + 31.4
Experimental
fully developed flow
Linear (fully developed flow)

8.131

8.131 Power Loss in a Coiled Pipe

Objective: The amount of power, P, dissipated in a pipe depends on the head loss, h_L, and the flowrate, Q. The purpose of this experiment is to use an apparatus as shown in Fig. P8.131 to determine the power loss in a coiled pipe and to determine how the coiling of the pipe affects the power loss.

Equipment: Air supply with a flow meter; flexible pipe that can be used either as a straight pipe or formed into a coil; manometer; barometer; thermometer.

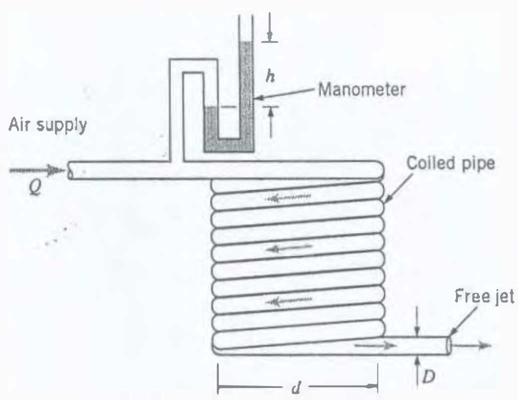
Experimental Procedure: Straighten the pipe and fasten it to the air supply exit. Measure the diameter, D, and length, L, of the pipe. Adjust the flowrate, Q, to the desired value and determine the manometer reading, h. Repeat the measurements for various flowrates. Form the pipe into a coil of diameter d and repeat the flowrate-pressure measurements. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer data to determine the pressure drop, $\Delta p = \gamma_m h$, and head loss, $h_L = \Delta p/\gamma$, as a function of flowrate, Q, for both the straight and coiled pipes. Here γ_m is the specific weight of the manometer fluid and γ is the specific weight of the flowing air. Also calculate the power loss, $P = \gamma Q h_L$, for both the straight and coiled pipes.

Graph: Plot head loss, h_L, as ordinates and flowrate, Q, as abscissas.

Results: On a log-log graph, plot the power loss, P, as a function of flowrate for both the straight and coiled pipes. Determine the best-fit straight lines through the data.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.131

(con't)

8.131 (con't)

Solution for Problem 8.131: Power Loss in a Coiled Pipe

D, in.	L, ft	H_{atm}, in. Hg	T, deg F
1.44	18	29.9	80

h, in.	Q, ft^3/s	Δp, lb/ft^2	h_L, ft	P, hp
Straight Pipe Data (d = infinity)				
10	1.19	52.0	709	0.1125
8	1.06	41.6	568	0.0802
6	0.913	31.2	426	0.0518
4	0.731	20.8	284	0.0276
2	0.505	10.4	142	0.0095
Coiled Pipe Data (d = 8 in.)				
10	0.835	52.0	709	0.0789
8	0.745	41.6	568	0.0563
6	0.641	31.2	426	0.0364
4	0.517	20.8	284	0.0196
2	0.357	10.4	142	0.0068

$\Delta p = \gamma_{H2O} h$ where γ_{H2O} = 62.4 lb/ft^3

$h_L = \Delta p/\gamma$ where $\gamma = g\rho$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} {}^* H_{atm}$ = 847 lb/ft^3*(29.9/12 ft) = 2110 lb/ft^2

R = 1716 ft lb/slug deg R

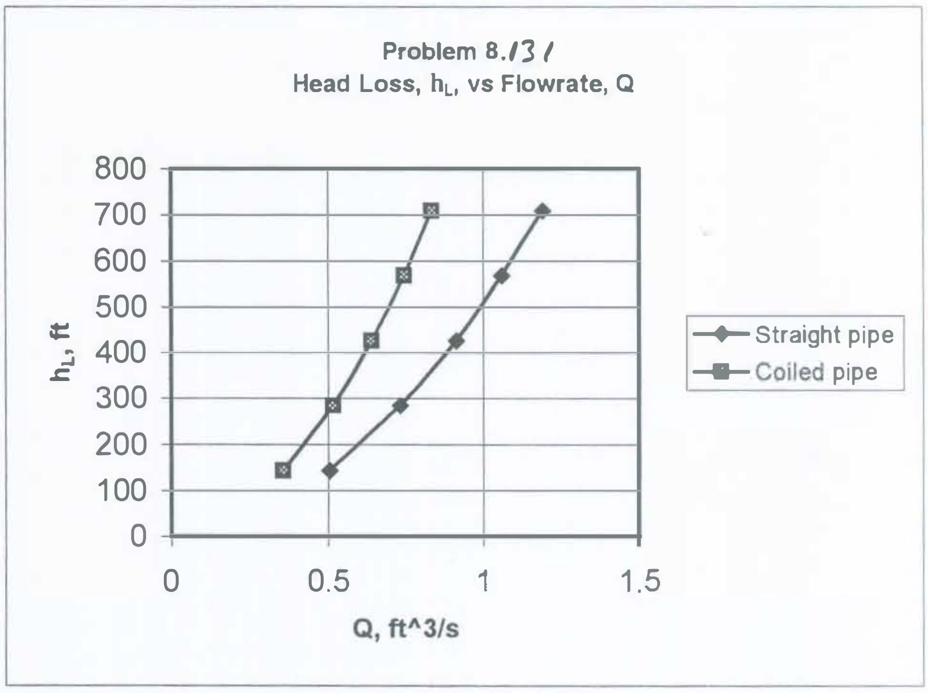
T = 80 +460 =540 deg R

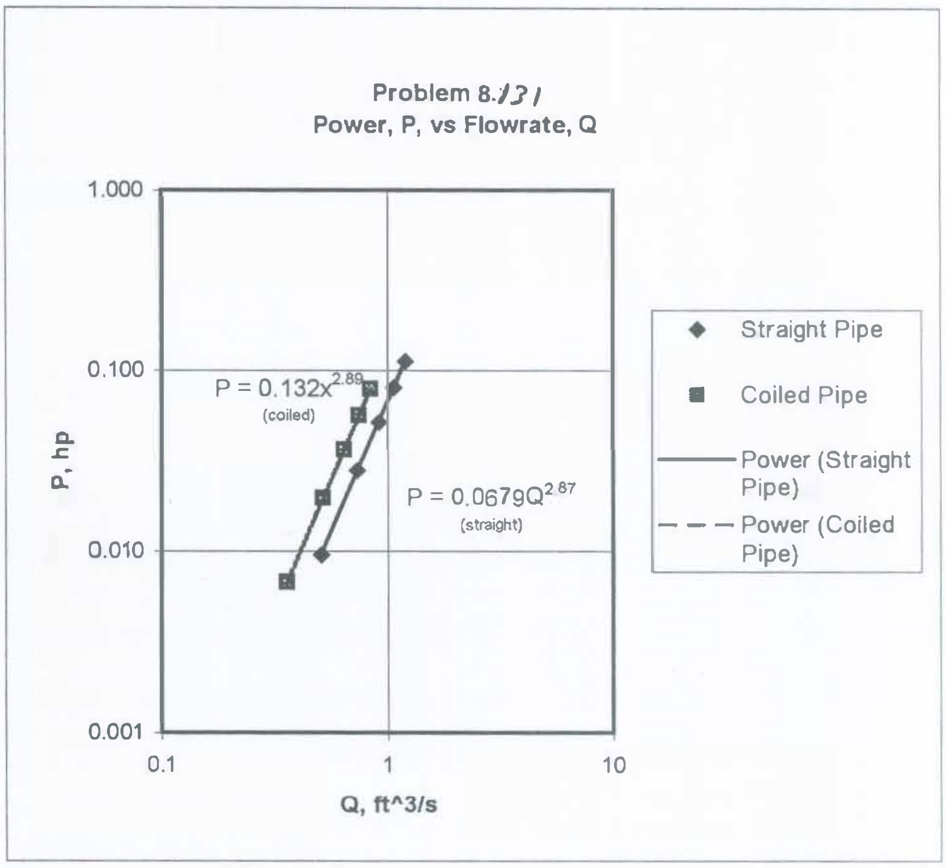
Thus, ρ = 0.00228 slug/ft^3 and γ = 0.0733 lb/ft^3

P = ($\gamma Q h_L$)ft lb/s*(1hp/550 ft lb/s)

(con't)

8.131 (con't)

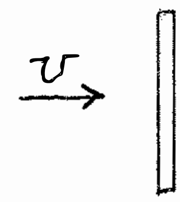

Problem 8.131
Head Loss, h_L, vs Flowrate, Q
800
700
600
500
400
300
200
100
0
h_L, ft
0
0.5
1
1.5
Q, ft^3/s
Straight pipe
Coiled pipe


Problem 8.131
Power, P, vs Flowrate, Q
1.000
0.100
0.010
0.001
P, hp
0.1
1
10
Q, ft^3/s
P = 0.132x^2.89
(coiled)
P = 0.0679Q^2.87
(straight)
Straight Pipe
Coiled Pipe
Power (Straight Pipe)
Power (Coiled Pipe)

9.2

9.2 A thin square is oriented perpendicular to the upstream velocity in a uniform flow. The average pressure on the front side of the square is 0.7 times the stagnation pressure and the average pressure on the back side is a vacuum (i.e., less than the free stream pressure) with a magnitude 0.4 times the stagnation pressure. Determine the drag coefficient for this square.

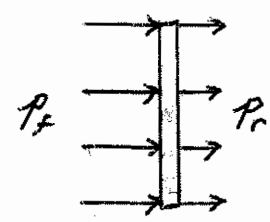
The drag can be determined by summing the pressure forces.



$$\mathcal{D} = p_f A - p_r A$$

$$= 0.7(\tfrac{1}{2}\rho V^2)A - (-0.4)(\tfrac{1}{2}\rho V^2)A$$

The pressure on the rear is in vacuum so is negative.



$$\mathcal{D} = 1.1(\tfrac{1}{2}\rho V^2)A$$

So,

$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho V^2 A} = \frac{1.1(\frac{1}{2}\rho V^2)A}{\frac{1}{2}\rho V^2 A}$$

$$C_D = 1.1$$

9.3

9.3 A small 15-mm-long fish swims with a speed of 20 mm/s. Would a boundary layer type flow be developed along the sides of the fish? Explain.

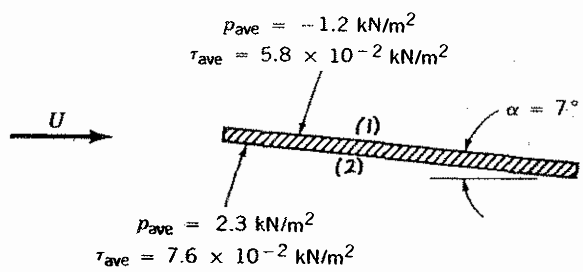
$Re = \frac{U\ell}{\nu}$, or with $\ell = 15 \times 10^{-3}\,m$, $U = 20 \times 10^{-3}\,\frac{m}{s}$ and
$\nu = 1.12 \times 10^{-6}\,\frac{m^2}{s}$ (i.e., $15.5\,°C$ water)

$$Re = \frac{(20 \times 10^{-3}\,\frac{m}{s})(15 \times 10^{-3}\,m)}{1.12 \times 10^{-6}\,\frac{m^2}{s}} = 268$$

This Reynolds number is not large enough to have true boundary layer type flow. ($Re \approx 1000$ is often assumed to be the lower limit.)

9.4

9.4 The average pressure and shear stress acting on the surface of the 1-m-square flat plate are as indicated in Fig. P9.4. Determine the lift and drag generated. Determine the lift and drag if the shear stress is neglected. Compare these two sets of results.



■ **FIGURE P9.4**

Since $\int p\, dA = p_{ave} A$ and $\int \tau_w\, dA = \tau_{ave} A$ it follows that

$$\mathcal{D} = -p_1 A_1 \sin\alpha + p_2 A_2 \sin\alpha + \tau_1 A_1 \cos\alpha + \tau_2 A_2 \cos\alpha$$

or with $A_1 = A_2 = 1\,m^2$ and $\alpha = 7°$,

$$\mathcal{D} = A_1 \sin\alpha (p_2 - p_1) + A_1 \cos\alpha (\tau_1 + \tau_2)$$
$$= (1\,m^2)\sin 7° (2.3 - (-1.2))\frac{kN}{m^2} + (1\,m^2)\cos 7° (5.8\times10^{-2} + 7.6\times10^{-2})\frac{kN}{m^2}$$
$$= 0.427\ kN + 0.133\ kN = \underline{0.560\ kN}$$

Note, if shear stress is neglected $\mathcal{D} = \underline{0.427\ kN}$ (i.e., $\tau_1 = \tau_2 = 0$)

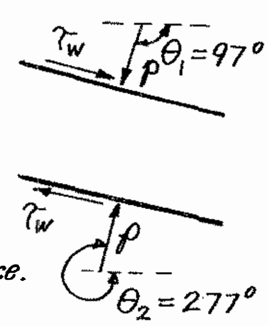
Also, $\mathcal{L} = -p_1 A_1 \cos\alpha + p_2 A_2 \cos\alpha - \tau_1 A_1 \sin\alpha - \tau_2 A_2 \sin\alpha$

or

$$\mathcal{L} = A_1 \cos\alpha (p_2 - p_1) - A_1 \sin\alpha (\tau_1 + \tau_2)$$
$$= (1\,m^2)\cos 7° (2.3 - (-1.2))\frac{kN}{m^2} - (1\,m^2)\sin 7° (5.8\times10^{-2} + 7.6\times10^{-2})\frac{kN}{m^2}$$
$$= 3.47\ kN - 0.0163\ kN = \underline{3.45\ kN}$$

Note, if shear stress is neglected $\mathcal{L} = \underline{3.47\ kN}$

Note: If the general expressions $\mathcal{D} = \int p \cos\theta\, dA + \int \tau_w \sin\theta\, dA$ and $\mathcal{L} = -\int p \sin\theta\, dA + \int \tau_w \cos\theta\, dA$ are use, be careful about the signs involved. On the upper surface $\theta_1 = 97°$ and p and τ_w are positive as indicated in the figure. On the lower surface $\theta_2 = 277°$ and p and τ_w are positive as indicated in the lower figure.



For example, with this notation $\tau_w < 0$ on the lower surface.

$$\mathcal{L} = -(-1.2\,\tfrac{kN}{m^2})\sin 97°(1\,m^2) - (2.3\,\tfrac{kN}{m^2})\sin 277°(1\,m^2)$$
$$+ (5.8\times10^{-2}\,\tfrac{kN}{m^2})\cos 97°(1\,m^2) + (-7.6\times10^{-2}\,\tfrac{kN}{m^2})\cos 277°(1\,m^2)$$
$$= 3.45\ kN, \text{ as obtained above.}$$

***9.5**

*9.5 The pressure distribution on the 1-m-diameter circular disk in Fig. P9.5 is given in the table. Determine the drag on the disk.

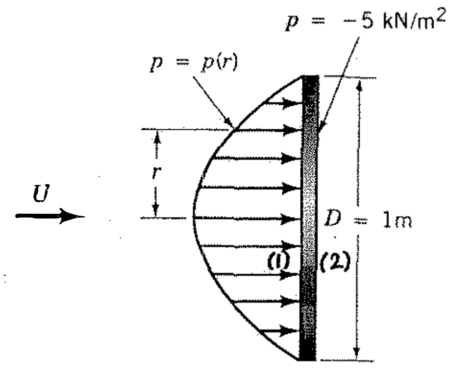


FIGURE P9.5

$$\mathcal{D} = \int_1 p\,dA - \int_2 p\,dA = \int_{r=0}^{r=\frac{D}{2}} p\,(2\pi r\,dr) - p_2 \frac{\pi}{4} D^2, \text{ since } dA = 2\pi r\,dr$$

Thus,

$$\mathcal{D} = 2\pi \int_0^{0.5\,m} p\,r\,dr - \left(-5\frac{kN}{m^2}\right)\frac{\pi}{4}(1\,m^2) = 2\pi \int_0^{0.5} p\,r\,dr + 3.93\,kN$$

where $p \sim \frac{kN}{m^2}$, $r \sim m$

Evaluate the integral numerically using the following integrand:

r, m	pr, kN/m
0	0
0.05	0.214
0.10	0.406
0.15	0.558
0.20	0.620
0.25	0.695
0.30	0.711
0.35	0.662
0.40	0.564
0.45	0.333
0.50	0.000

r (m)	p (kN/m²)
0	4.34
0.05	4.28
0.10	4.06
0.15	3.72
0.20	3.10
0.25	2.78
0.30	2.37
0.35	1.89
0.40	1.41
0.45	0.74
0.50	0.0

Using a standard numerical integration technique with the above integrand gives $\mathcal{D} = \underline{\underline{5.43\,kN}}$

9.6

9.6 When you walk through still air at a rate of 1 m/s, would you expect the character of the air flow around you to be most like that depicted in Fig. 9.6a, b, or c? Explain.

$Re = \frac{U\ell}{\nu}$, where $\nu = 1.46 \times 10^{-5} \frac{m^2}{s}$ and $U = 1 \frac{m}{s}$. Assume $\ell = 1\,m$.

Thus,

$$Re = \frac{(1 \frac{m}{s})(1m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 6.85 \times 10^4$$

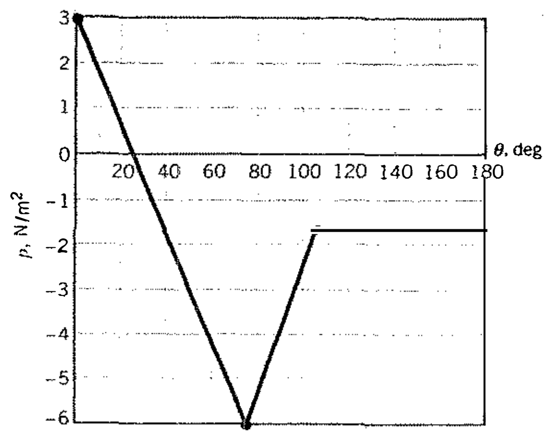
This flow has a large enough Reynolds number to develop a boundary layer. Thus, viscous effects would not be important far from your body, except in the wake region behind you.

Note: The above conclusion is true whether we assume $\ell = 1\,m$, $\ell = 2\,m$, $\ell = 0.1\,m$, or some other reasonable characteristic length of our body.

The flow would be most like that in <u>Fig. 9.6c.</u>

9.7

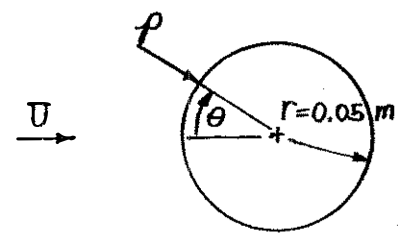
9.7 A 0.10 m-diameter circular cylinder moves through air with a speed U. The pressure distribution on the cylinder's surface is approximated by the three straight line segments shown in Fig. P9.7 Determine the drag coefficient on the cylinder. Neglect shear forces.



■ FIGURE P9.7

$\mathscr{D}_p = \int p\, b r \cos\theta\, d\theta = br \int p \cos\theta\, d\theta$

or $\mathscr{D}_p = 2br \int_{\theta=0}^{\pi} p \cos\theta\, d\theta$



$dA = b\, r d\theta$
b = length

Break up the integration into the following three segments:

1) $0 \le \theta \le 70^\circ = 1.222$ rad where

$p = -7.39\,\theta + 3 \ \frac{N}{m^2}$, where $\theta \sim$ rad.

i.e. $p\big|_{\theta=0} = 3$ and $p\big|_{\theta=1.222} = -6$

2) $70^\circ \le p \le 100^\circ$ or $1.222 \le \theta \le 1.745$ rad where

$p = 8.59\,\theta - 16.5 \ \frac{N}{m^2}$, where $\theta \sim$ rad

i.e. $p\big|_{\theta=1.222} = -6$ and $p\big|_{\theta=1.745} = -1.5$

and

3) $100^\circ \le p \le 180^\circ$ or $1.745 \le \theta \le 3.14$ rad where

$p = -1.5 \ \frac{N}{m^2}$

Thus,

$$\mathscr{D}_p = 2br\Big[\int_0^{70^\circ} p \cos\theta\, d\theta + \int_{70^\circ}^{100^\circ} p \cos\theta\, d\theta + \int_{100^\circ}^{180^\circ} p \cos\theta\, d\theta\Big] = 2br\big[I_1 + I_2 + I_3\big] \quad (1)$$

where

(con't)

9.7 (con't)

$$I_1 = \int_0^{1.222} (-7.39\theta + 3)\cos\theta \, d\theta = \left[-7.39(\cos\theta + \theta\sin\theta) + 3\sin\theta\right]_0^{1.222} = -0.791$$

$$I_2 = \int_{1.222}^{1.745} (8.59\theta - 16.5)\cos\theta \, d\theta = \left[8.59(\cos\theta + \theta\sin\theta) - 16.5\sin\theta\right]_{1.222}^{1.745} = -0.260$$

and

$$I_3 = \int_{1.745}^{3.14} (-1.5)\cos\theta \, d\theta = -1.5 \sin\theta \Big|_{1.745}^{3.14} = 1.477$$

Hence,

$$\mathcal{D}_p = 2br[0.791 - 0.260 + 1.477] = 0.852\, br$$

or with

$$C_D = \frac{\mathcal{D}_p}{\frac{1}{2}\rho U^2 A} = \frac{0.852\, br}{\frac{1}{2}\rho U^2 (2br)} = \frac{0.426}{\frac{1}{2}\rho U^2}$$

But the pressure at $\theta = 0$, the stagnation point, is $3 \frac{N}{m^2}$.
Thus, $\frac{1}{2}\rho U^2 = 3 \frac{N}{m^2}$ so that

$$C_D = \frac{0.426}{3} = \underline{\underline{0.142}}$$

9.8

9.8 Typical values of the Reynolds number for various animals moving through air or water are listed below. For which cases is inertia of the fluid important? For which cases do viscous effects dominate? For which cases would the flow be laminar; turbulent? Explain.

Animal	Speed	Re
(a) large whale	10 m/s	300,000,000
(b) flying duck	20 m/s	300,000
(c) large dragonfly	7 m/s	30,000
(d) invertebrate larva	1 mm/s	0.3
(e) bacterium	0.01 mm/s	0.00003

Inertia important if $Re \gtrsim 1$ (i.e whale, duck, dragonfly)

Viscous effects dominate if $Re \lesssim 1$ (i.e larva, bacterium)

Boundary layer flow becomes turbulent for Re on the order of 10^5 to 10^6. (i.e. whale and perhaps the duck)

The flow would be laminar for the dragonfly, larva, and bacterium and perhaps the duck.

9.12

9.12 Water flows past a flat plate that is oriented parallel to the flow with an upstream velocity of 0.5 m/s. Determine the approximate location downstream from the leading edge where the boundary layer becomes turbulent. What is the boundary layer thickness at this location?

$$Re_{cr} = 5\times10^5 = \frac{U x_{cr}}{\nu}$$

$$x_{cr} = \frac{5\times10^5 \nu}{U} = \frac{5\times10^5 (1.12\times10^{-6}\ m^2/s)}{0.5\ m/s} = \underline{\underline{1.12\ m}}$$

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{(1.12\times10^{-6}\ m^2/s)\ 1.12\ m}{0.5\ m/s}} = \underline{\underline{7.92\times10^{-3}\ m}}$$

9.13

9.13 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = C\sqrt{x}$, where C is a constant.

Thus,

$$C = \frac{\delta}{\sqrt{x}} = \frac{12 \times 10^{-3}\,m}{\sqrt{1.3\,m}} = 0.0105 \quad \text{or} \quad \delta = 0.0105\sqrt{x} \quad \text{where } x \sim m,\ \delta \sim m$$

x, m	δ, m	δ, mm
0.2	0.00470	4.70
2.0	0.0148	14.8
20.0	0.0470	47.0

9.14

9.14 If the upstream velocity of the flow in Problem 9.13 is $U = 1.5$ m/s, determine the kinematic viscosity of the fluid.

For laminar flow $\delta = 5\sqrt{\frac{\nu x}{U}}$, or $\nu = \frac{U\delta^2}{25\,x}$

Thus,

$$\nu = \frac{(1.5\,\frac{m}{s})(12 \times 10^{-3}\,m)^2}{25\,(1.3\,m)} = 6.65 \times 10^{-6}\,\frac{m^2}{s}$$

9.15

9.15 Water flows past a flat plate with an upstream velocity of $U = 0.02$ m/s. Determine the water velocity a distance of 10 mm from the plate at distances of $x = 1.5$ m and $x = 15$ m from the leading edge.

From the Blasius solution for boundary layer flow on a flat plate, $u = U f'(\eta)$, where η, the similarity variable, is $\eta = y\sqrt{\frac{U}{\nu x}}$. Values of $f'(\eta)$ are given in Table 9.1.

Since $Re_x = \frac{Ux}{\nu} = \frac{(0.02\frac{m}{s})(15m)}{1.12\times10^{-6}\frac{m^2}{s}} = 2.68\times10^5$ is less than the critical $Re_{x_{cr}} = 5\times10^5$, it follows that the boundary layer flow is laminar.

At $x_1 = 1.5\,m$ and $y = 10\times10^{-3}\,m$ we obtain:

$$\eta_1 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(1.5m)}} = 1.091$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.2647 + \frac{(0.3938 - 0.2647)}{(1.2 - 0.8)}(1.091 - 0.8)$$

Hence,

$$u_1 = U f'(\eta_1) = (0.02\frac{m}{s})(0.359) = \underline{\underline{0.00718\frac{m}{s}}}$$

Similarly, at $x_2 = 15m$ and $y = 10\times10^{-3}m$ we obtain:

$$\eta_2 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(15m)}} = 0.345$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.0 + \frac{(0.1328 - 0.0)}{(0.8 - 0.4)}(0.345 - 0.0) = 0.1145$$

Hence,

$$u_2 = U f'(\eta_2) = (0.02\frac{m}{s})(0.1145) = \underline{\underline{0.00229\frac{m}{s}}}$$

9.16

9.16 Approximately how fast can the wind blow past a 0.25-in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re < 1)? Explain. Repeat for a 0.004-in.-diameter hair and a 6-ft-diameter smokestack.

$Re = \frac{UD}{\nu} < 1$ or $U < \frac{\nu}{D}$ if viscous effects are to be important throughout the flow.

For standard air $\nu = 1.57 \times 10^{-4} \frac{ft^2}{s}$

Thus,

$U < \frac{1.57 \times 10^{-4}}{D}$, where D is the diameter in feet.

object	D, ft	U, $\frac{ft}{s}$
twig	2.08×10^{-2}	7.54×10^{-3}
hair	3.33×10^{-4}	0.471
smokestack	6	2.62×10^{-5}

9.17

9.17 As is indicated in Table 9.2, the laminar boundary layer results obtained from the momentum integral equation are relatively insensitive to the shape of the assumed velocity profile. Consider the profile given by $u = U$ for $y > \delta$, and $u = U\{1 - [(y - \delta)/\delta]^2\}^{1/2}$ for $y \le \delta$ as shown in Fig. P9.17. Note that this satisfies the conditions $u = 0$ at $y = 0$ and $u = U$ at $y = \delta$. However, show that such a profile produces meaningless results when used with the momentum integral equation. Explain.

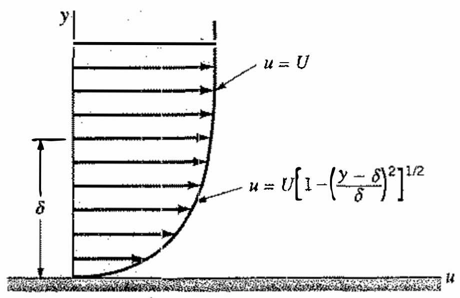


FIGURE P9.17

From the momentum integral equation

$$\delta = \sqrt{\frac{2C_2\nu x}{U C_1}}, \quad \text{where } \frac{u}{U} = g(Y) = \left[1-(Y-1)^2\right]^{\frac{1}{2}} \qquad (1)$$

Note: $\frac{u}{U} = 0$ at $Y=0$ and $\frac{u}{U} = 1$ and $Y=1$, as required.

Also, $C_1 = \int_0^1 g(1-g)\,dY$ which can be evaluated for the given $g(Y)$.

However,

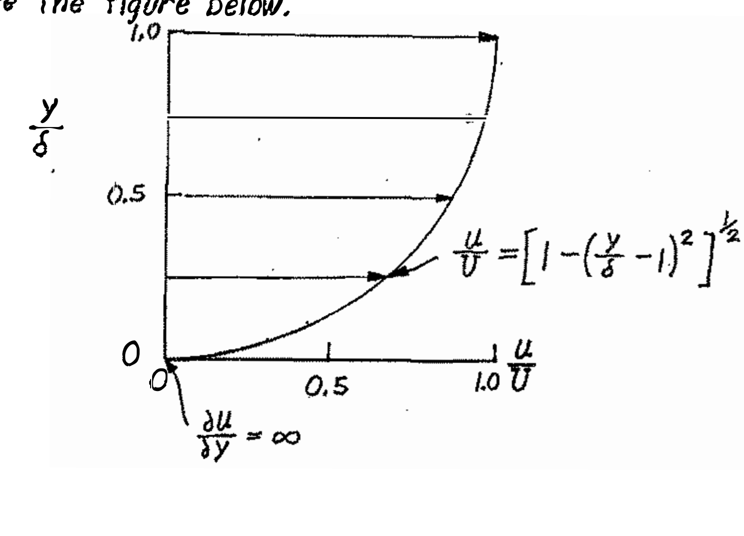
$$C_2 = \left.\frac{dg}{dY}\right|_{Y=0}, \quad \text{or since } \frac{dg}{dY} = \frac{1}{2}\left[1-(Y-1)^2\right]^{-\frac{1}{2}}(-2)(Y-1) = \frac{(1-Y)}{\left[1-(Y-1)^2\right]^{\frac{1}{2}}}$$

Thus,

$C_2 = \infty$, which from Eq.(1) gives $\delta = \infty$

This profile cannot be used since it gives $\delta = \infty$ due to the physically unrealistic $\frac{\partial u}{\partial y} = \infty$ at the surface ($y=0$).

See the figure below.



9.19

9.19 Because of the velocity deficit, $U - u$, in the boundary layer, the streamlines for flow past a flat plate are not exactly parallel to the plate. This deviation can be determined by use of the displacement thickness, δ^*. For air blowing past the flat plate shown in Fig. P9.19, plot the streamline A–B that passes through the edge of the boundary layer ($y = \delta_B$ at $x = \ell$) at point B. That is, plot $y = y(x)$ for streamline A–B. Assume laminar boundary layer flow.

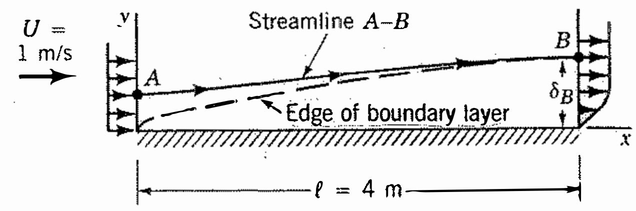
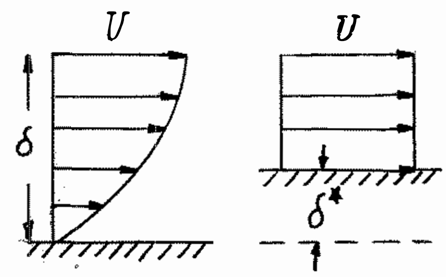


FIGURE P9.19

Since $Re_\ell = \frac{U\ell}{\nu} = \frac{(1\frac{m}{s})(4m)}{1.46\times10^{-5}\frac{m^2}{s}} = 2.74\times10^5 < 5\times10^5$, the boundary layer flow remains laminar along the entire plate. Hence,

$$\delta = 5\sqrt{\frac{\nu x}{U}} \quad \text{or} \quad \delta_B = 5\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.0382\,m$$

The flowrate carried by the actual boundary layer is by definition equal to that carried by a uniform velocity with by an amount δ^*. Since there is no flow through the plate or streamline A-B,



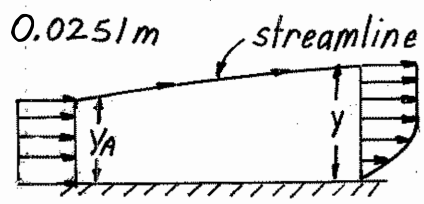
$$Q_A = Q_B, \text{ or } U y_A = (\delta_B - \delta_B^*)U \qquad (1)$$

where $\delta^* = 1.721\sqrt{\frac{\nu x}{U}}$

or $\delta_B^* = 1.721\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.01315\,m$

Thus,

$$y_A = \delta_B - \delta_B^* = 0.0382\,m - 0.01315\,m = 0.0251\,m$$



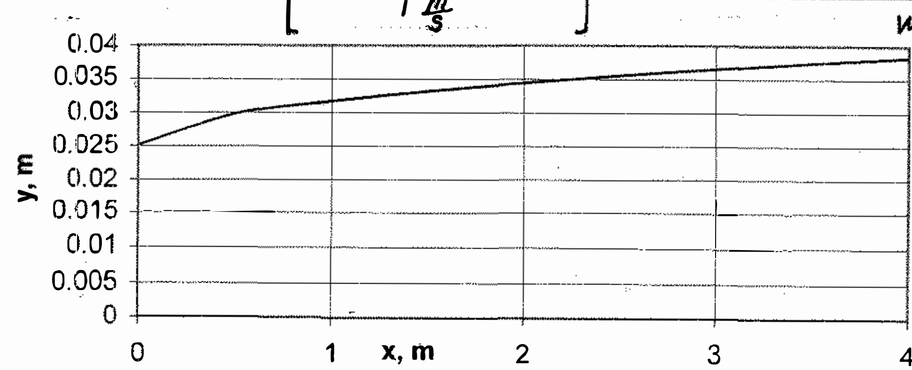
Hence, for any x-location

$Q_A = Q$ or $U y_A = U(y - \delta^*)$

or $y = y_A + \delta^* = y_A + 1.721\sqrt{\frac{\nu x}{U}}$

$$= 0.0251\,m + 1.721\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})\,x\,m}{1\frac{m}{s}}\right]^{\frac{1}{2}} = \underline{\underline{0.0251 + 6.58\times10^{-3}\sqrt{x}\ m,}}$$

where $x \sim m$



9.20

9.20 Air enters a square duct through a 1-ft opening as is shown in Fig. P9.20. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant $U = 2$ ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, d, as a function of x for $0 \le x \le 10$ ft if U is to remain constant. Assume laminar flow.

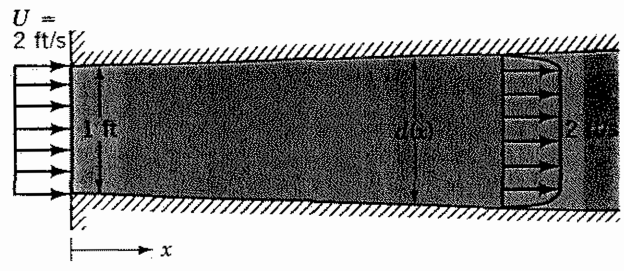


FIGURE P9.20

For incompressible flow $Q_0 = Q(x)$ where Q_0 = flowrate into the duct $= UA_0 = (2\,\frac{ft}{s})(1\,ft^2) = 2\,\frac{ft^3}{s}$

and

$Q(x) = UA$, where $A = (d - 2\delta^*)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_0 = U(d - 2\delta^*)^2 \quad \text{or} \quad d = 1\,ft + 2\delta^*, \qquad (1)$$

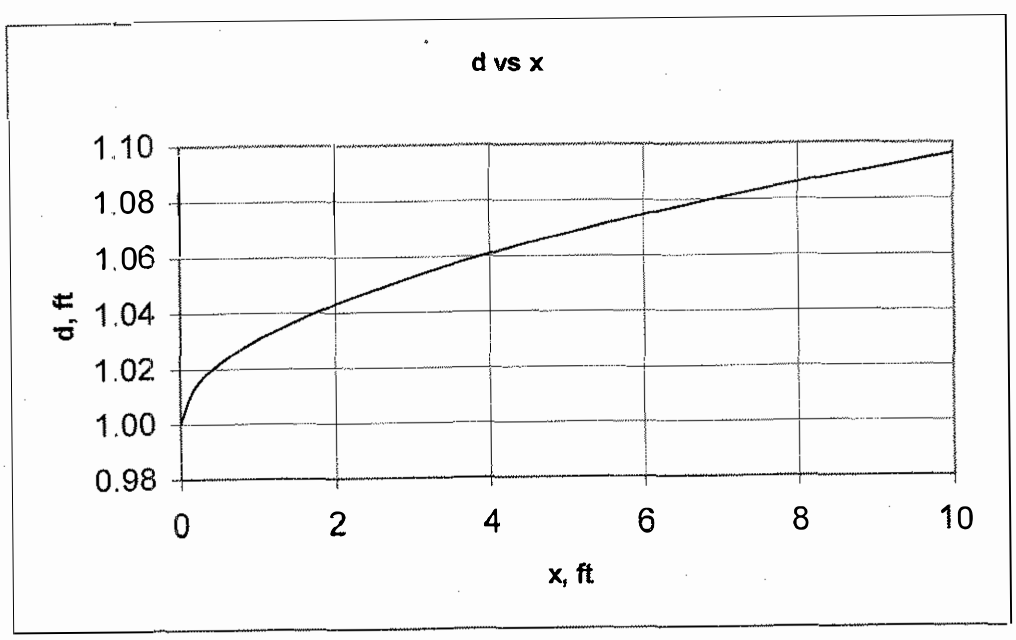
where

$$\delta^* = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\left[\frac{(1.57\times10^{-4}\,\frac{ft^2}{s})\,x}{2\,\frac{ft}{s}}\right]^{1/2} = 0.0152\sqrt{x}\ ft, \text{ where } x \sim ft$$

Hence, from Eq. (1)

$$\underline{\underline{d = 1 + 0.0304\sqrt{x}\ ft}}$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.



9.21

9.21 A smooth, flat plate of length $\ell = 6$ m and width $b = 4$ m is placed in water with an upstream velocity of $U = 0.5$ m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{(1.12\times10^{-6}\,\frac{m^2}{s})\,x}{0.5\,\frac{m}{s}}} = 7.48\times10^{-3}\sqrt{x}\ m,\ \text{where } x \sim m$$

and

$$\tau_w = 0.332\,U^{3/2}\sqrt{\frac{\rho\mu}{x}} = 0.332\,(0.5\,\tfrac{m}{s})^{3/2}\sqrt{\frac{(999\,\frac{kg}{m^3})(1.12\times10^{-3}\,\frac{N\cdot s}{m^2})}{x}}$$

$$= \frac{0.124}{\sqrt{x}}\ \frac{N}{m^2},\ \text{where } x \sim m$$

Thus, at $x = 3$ m $\quad \delta = 7.48\times10^{-3}\sqrt{3} = \underline{\underline{0.0130\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{3}} = \underline{\underline{0.0716\ \frac{N}{m^2}}}$$

while at $x = 6$ m $\quad \delta = 7.48\times10^{-3}\sqrt{6} = \underline{\underline{0.0183\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{6}} = \underline{\underline{0.0506\ \frac{N}{m^2}}}$$

9.22

9.22 An atmospheric boundary layer is formed when the wind blows over the earth's surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants a and n depend on the roughness of the terrain. As is indicated in Fig. P9. , typical values are $n = 0.40$ for urban areas, $n = 0.28$ for woodland or suburban areas, and $n = 0.16$ for flat open country (Ref. 23). **(a)** If the velocity is 20 ft/s at the bottom of the sail on your boat ($y = 4$ ft), what is the velocity at the top of the mast ($y = 30$ ft)? **(b)** If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

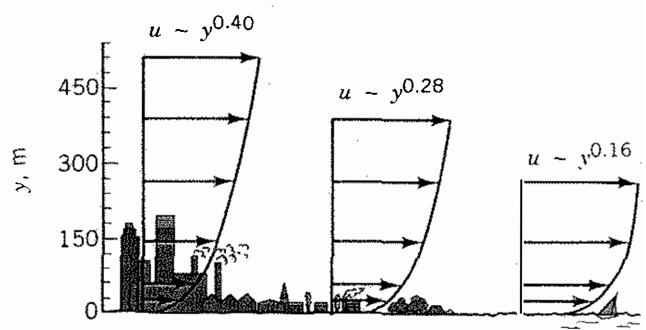


FIGURE P9.22

(a) $u = C\,y^{0.16}$, where C is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.16}$ or $u_2 = 20\,\dfrac{ft}{s}\left(\dfrac{30\,ft}{4\,ft}\right)^{0.16} = \underline{\underline{27.6\,\dfrac{ft}{s}}}$

(b) $u = \tilde{C}\,y^{0.4}$, where $\tilde{C}$ is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.40}$ or $u_2 = 10\,mph\left(\dfrac{60}{10}\right)^{0.4} = \underline{\underline{20.5\,mph}}$

9.24

9.24 A 30-story office building (each story is 12 ft tall) is built in a suburban industrial park. Plot the dynamic pressure, $\rho u^2/2$, as a function of elevation if the wind blows at hurricane strength (75 mph) at the top of the building. Use the atmospheric boundary layer information of Problem 9.22.

From Fig. P9.22 the boundary layer velocity profile is given by $u \sim y^{0.28}$, or $u = C\,y^{0.28}$, where C is a constant.

Thus, $\dfrac{u}{u_1} = \left(\dfrac{y}{y_1}\right)^{0.28}$

or

$u = 110\left(\dfrac{y}{360}\right)^{0.28} \dfrac{ft}{s}$ where $y \sim ft$

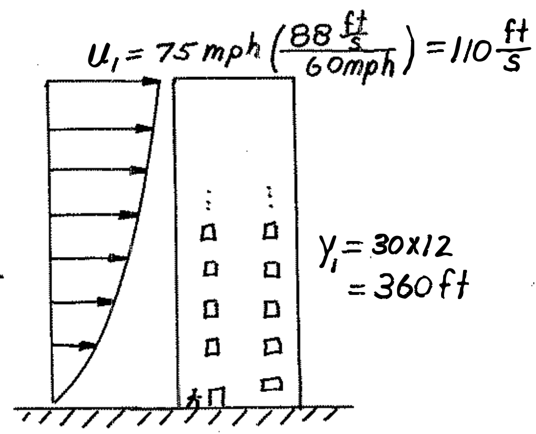
Hence,

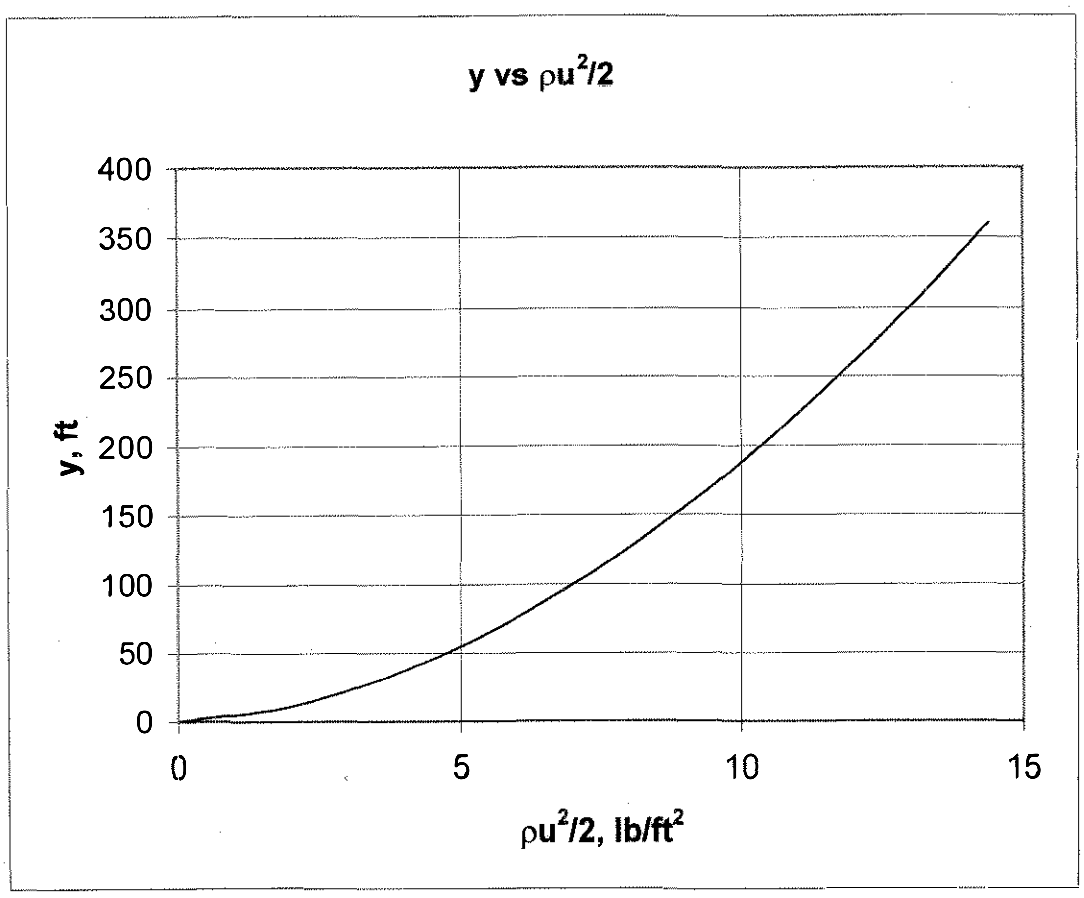
$\frac{1}{2}\rho u^2 = \frac{1}{2}(2.38 \times 10^{-3} \frac{slug}{ft^3})\left[110\left(\frac{y}{360}\right)^{0.28}\frac{ft}{s}\right]^2$

or

$\frac{1}{2}\rho u^2 = 14.4\left(\frac{y}{360}\right)^{0.56} \frac{lb}{ft^2}$, where $y \sim ft$

This is plotted in the figure below.





9.25

9.25 Show that for any function $f = f(\eta)$ the velocity components u and v determined by Eqs. 9.12 and 9.13 satisfy the incompressible continuity equation, Eq. 9.8.

Given $u = U f'(\eta)$, $v = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}(\eta f'(\eta) - f(\eta))$

where $\eta = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y$ and $(\ )' \equiv \frac{d}{d\eta}$

Show that $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ for any $f(\eta)$.

$\frac{\partial u}{\partial x} = U\frac{\partial f'}{\partial x} = U\frac{df'}{d\eta}\frac{\partial \eta}{\partial x}$, where $\frac{\partial \eta}{\partial x} = -\frac{U^{\frac{1}{2}} y}{2\nu^{\frac{1}{2}}} x^{-\frac{3}{2}}$

Thus,

$$\frac{\partial u}{\partial x} = -U f''\left[\frac{U^{\frac{1}{2}} y}{2\nu^{\frac{1}{2}} x^{\frac{3}{2}}}\right] = -\frac{U^{\frac{3}{2}} y f''}{2x^{\frac{3}{2}}\nu^{\frac{1}{2}}} \qquad (1)$$

and

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\frac{\partial \eta}{\partial y} f' + \eta\frac{\partial f'}{\partial \eta}\frac{\partial \eta}{\partial y} - \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y}\right]$$

$$= \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\frac{\partial \eta}{\partial y} f' + \eta f''\frac{\partial \eta}{\partial y} - \frac{\partial \eta}{\partial y} f'\right]$$

$$= \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\eta f''\frac{\partial \eta}{\partial y}\right] \text{ , where } \frac{\partial \eta}{\partial y} = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$$

Hence,

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y f''\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} = \frac{U^{\frac{3}{2}} y f''}{2x^{\frac{3}{2}}\nu^{\frac{1}{2}}} \qquad (2)$$

By combining Eqs. (1) and (2) we see that

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ for any function $f(\eta)$.

9.26

*9.26 Integrate the Blasius equation (Eq. 9.14) numerically to determine the boundary layer profile for laminar flow past a flat plate. Compare your results with those of Table 9.1.

9.26* Integrate the Blasius equation (Eq. 9.14) numerically to determine the boundary layer profile for laminar flow past a flat plate. Compare your results with those of Table 9.1.

Solve the following third order differential equation by a numerical integration technique:

$2f''' + ff'' = 0$ with boundary conditions

$f = f' = 0$ at $\eta = 0$ and $f' \rightarrow 1$ as $\eta \rightarrow \infty$ $\quad \left(( )' \equiv \frac{d}{d\eta} \right)$

Write this third order equation as 3 first order equations and use a Runga-Kutta numerical technique to integrate them. Thus, let

$y_1 \equiv f$, $y_1' = f' \equiv y_2$, $y_2' = f'' \equiv y_3$, and $y_3' = f''' = -\frac{1}{2}ff'' = -\frac{1}{2}y_1y_3$

That is:

$y_1' = y_2$

$y_2' = y_3$ and

$y_3' = -y_1y_3/2$

These can be approximated as

$\Delta y_1 = y_2\,\Delta\eta$, $\Delta y_2 = y_3\,\Delta\eta$, and $\Delta y_3 = (-y_1y_3/2)\Delta\eta$

Start with $y_1 = y_2 = 0$ at $\eta = 0$. Assume $y_3 = C$ at $\eta = 0$ (where C is some given constant) and "integrate to $\eta = \infty$" by $y_i = y_i(0) + \sum_j \Delta y_{ij}\Delta\eta$

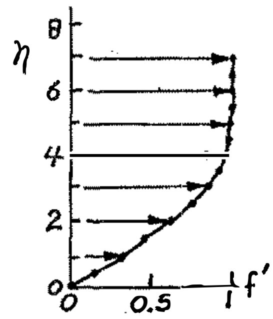
If $y_2(\infty) \neq 1$ (i.e., $f'(\infty) \neq 1$) adjust the value of C (i.e., $f''(0)$) and try again. The two-point boundary value problem (i.e., $f(0) = f'(0) = 0$ and $f'(\infty) = 1$) is solved by iteration as an initial value problem (i.e., $f(0) = f'(0) = 0$, $f''(0) = C$).

A step size of $\Delta\eta = 0.01$ was used, with $0 < \eta < 7$. That is, 700 steps were used. A value of $C = 0.332$ was found to give $f'(\infty) = 1$, or actually $f'(7) = 1$. This value of C and the corresponding velocity profile, $u = f'(\eta)$, shown on the next page agree very well with the standard values given in Table 9.1.

(con't)

9.26 (con't)

eta	f	f'	f''
0.5000	+4.07E-02	+1.66E-01	+3.31E-01
1.0000	+1.64E-01	+3.30E-01	+3.23E-01
1.5000	+3.68E-01	+4.87E-01	+3.03E-01
2.0000	+6.47E-01	+6.30E-01	+2.67E-01
2.5000	+9.93E-01	+7.52E-01	+2.17E-01
3.0000	+1.39E+00	+8.47E-01	+1.61E-01
3.5000	+1.83E+00	+9.14E-01	+1.07E-01
4.0000	+2.30E+00	+9.56E-01	+6.38E-02
4.5000	+2.79E+00	+9.80E-01	+3.36E-02
5.0000	+3.28E+00	+9.92E-01	+1.56E-02
5.5000	+3.78E+00	+9.97E-01	+6.41E-03
6.0000	+4.28E+00	+9.99E-01	+2.32E-03
6.5001	+4.78E+00	+1.00E+00	+7.36E-04
7.0001	+5.28E+00	+1.00E+00	+2.06E-04



9.27

9.27 An airplane flies at a speed of 400 mph at an altitude of 10,000 ft. If the boundary layers on the wing surfaces behave as those on a flat plate, estimate the extent of laminar boundary layer flow along the wing. Assume a transitional Reynolds number of $Re_{xcr} = 5 \times 10^5$. If the airplane maintains its 400-mph speed but descends to sea level elevation, will the portion of the wing covered by a laminar boundary layer increase or decrease compared with its value at 10,000 ft? Explain.

At 10,000 ft:

(a) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and from Table C.1, $\nu = \frac{\mu}{\rho} = \frac{3.534 \times 10^{-7}\,\frac{lb\cdot s}{ft^2}}{1.756 \times 10^{-3}\,\frac{slug}{ft^3}} = 2.01 \times 10^{-4}\,\frac{ft^2}{s}$

Hence, with $Re_{x_{cr}} = 5 \times 10^5$,

$$x_{cr} = \frac{\nu\, Re_{x_{cr}}}{U} = \frac{(2.01 \times 10^{-4}\,\frac{ft^2}{s})(5 \times 10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.171\,ft}}$$

At sea-level:

(b) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and $\nu = 1.57 \times 10^{-4}\,\frac{ft^2}{s}$

Hence,

$$x_{cr} = \frac{\nu\, Re_{cr}}{U} = \frac{(1.57 \times 10^{-4}\,\frac{ft^2}{s})(5 \times 10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.134\,ft}}$$

The laminar boundary layer occupies the first 0.134 ft of the wing at sea level and (from part (a) above) the first 0.171 ft at an altitude of 10,000 ft. This is due mainly to the lower density (larger kinematic viscosity). The dynamic viscosities are approximately the same.

9.29

9.29 A laminar boundary layer velocity profile is approximated by $u/U = [2 - (y/\delta)](y/\delta)$ for $y \le \delta$, and $u = U$ for $y > \delta$. (a) Show that this profile satisfies the appropriate boundary conditions. (b) Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$.

(a) $\frac{u}{U} = g(Y) = 2Y - Y^2$ where $Y = y/\delta$

Thus, $\left.\frac{u}{U}\right|_{y=0} = 0$ as it must, $\left.\frac{u}{U}\right|_{y=\delta} = 2 - 1 = 1$ or $u = U$ at $y = \delta$ as it must.

Also, $\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]$ so that $\left.\frac{du}{dy}\right|_{y=\delta} = U\left[\frac{2}{\delta} - \frac{2}{\delta}\right] = 0$

(b) From the momentum integral equation,

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0}$$

Thus,

$$C_1 = \int_0^1 (2Y - Y^2)(1 - 2Y + Y^2)\,dY = \int_0^1 (2Y - 5Y^2 + 4Y^3 - Y^4)\,dY$$

$$= 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{2}{15}$$

and

$$C_2 = (2 - 2Y)\Big|_{Y=0} = 2$$

so that

$$\delta = \sqrt{\frac{2(2)\nu x}{\frac{2}{15}U}} = \sqrt{\frac{30 \nu x}{U}}$$

Hence, with $Re_x = \frac{Ux}{\nu}$,

$$\frac{\delta}{x} = \frac{\sqrt{30}}{\sqrt{Re_x}} = \underline{\underline{\frac{5.48}{\sqrt{Re_x}}}}$$

9.30

9.30 A laminar boundary layer velocity profile is approximated by the two straight-line segments indicated in Fig. P9.30 Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$, and wall shear stress, $\tau_w = \tau_w(x)$. Compare these results with those in Table 9.2.

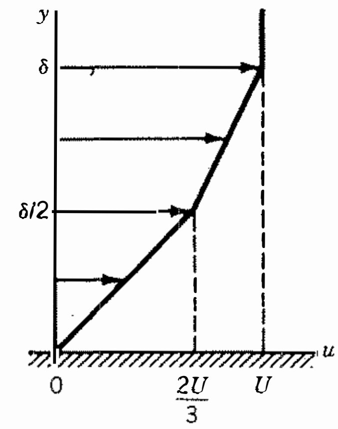


FIGURE P9.30

From the momentum integral equation

$$\delta = \sqrt{\frac{2 C_2 \nu x}{U C_1}}\ , \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0} \qquad (1)$$

and $\frac{u}{U} = g(Y)$ with $Y = \frac{y}{\delta}$,

For $0 \le Y < \frac{1}{2}$, $g = a_1 + b_1 Y$ with the constants a_1 and b_1 obtained from $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 0$ at $Y = 0$. Thus, $a_1 = 0$, $b_1 = \frac{4}{3}$

or $g = \frac{4}{3} Y$ for $0 \le Y < \frac{1}{2}$

Hence, $C_2 = \frac{4}{3}$ (2)

Similarly, for $\frac{1}{2} \le Y \le 1$, $g = a_2 + b_2 Y$ with $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 1$ at $Y = 1$

Thus,
$\frac{2}{3} = a_2 + \frac{1}{2} b_2$ and $1 = a_2 + b_2$ which give $a_2 = \frac{1}{3}$, $b_2 = \frac{2}{3}$

or
$g = \frac{1}{3} + \frac{2}{3} Y$, for $\frac{1}{2} \le Y < 1$

Hence, $C_1 = \int_0^1 g(1-g)\,dY = \int_0^{1/2} \frac{4}{3} Y \left(1 - \frac{4}{3} Y\right) dY + \int_{1/2}^1 \left(\frac{1}{3} + \frac{2}{3} Y\right)\left(1 - \frac{1}{3} - \frac{2}{3} Y\right) dY$

$= \frac{4}{9} \int_0^{1/2} (3Y - 4Y^2)\,dY + \frac{2}{9} \int_{1/2}^1 (1+2Y)(1-Y)\,dY$ which upon integration gives

$C_1 = 0.1574$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$$\delta = \left[\frac{2\left(\frac{4}{3}\right)\nu x}{0.1574\, U}\right]^{1/2} = \underline{\underline{4.12\sqrt{\frac{\nu x}{U}}}} \quad \text{or} \quad \frac{\delta\, Re_x^{1/2}}{x} = 4.12$$

Also,

$$\tau_w = \frac{\mu U}{\delta} C_2 = \underline{\underline{\frac{4 \mu U}{3 \delta}}} \quad \text{or} \quad C_f = \frac{\sqrt{2 C_1 C_2}}{\sqrt{Re_x}} = \frac{\sqrt{2(0.1574)\left(\frac{4}{3}\right)}}{\sqrt{Re_x}} = \frac{0.648}{\sqrt{Re_x}}$$

Compare these results to those in Table 9.2.

9.31*

9.31* For a fluid of specific gravity $SG = 0.86$ flowing past a flat plate with an upstream velocity of $U = 5$ m/s, the wall shear stress on a flat plate was determined to be as indicated in the table below. Use the momentum integral equation to determine the boundary layer momentum thickness, $\Theta = \Theta(x)$. Assume $\Theta = 0$ at the leading edge, $x = 0$.

Since $\tau_w = \rho U^2 \frac{d\Theta}{dx}$ it follows that $d\Theta = \frac{\tau_w}{\rho U^2} dx$

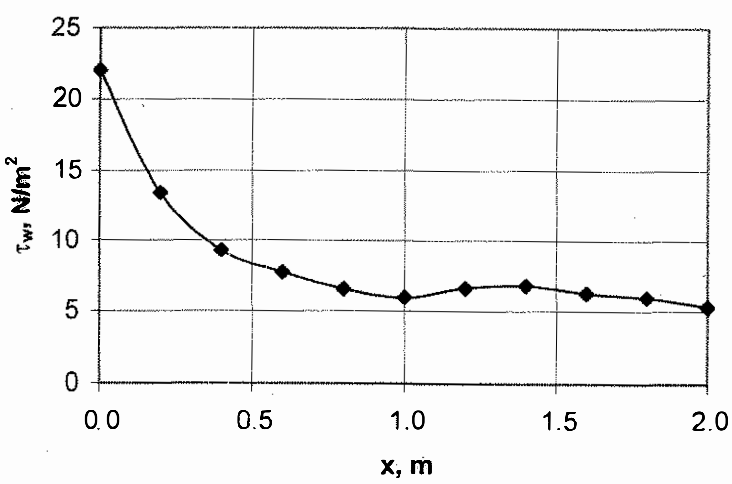
which can be integrated to give (using $\Theta = 0$ at $x = 0$)

$$\Theta = \frac{1}{\rho U^2} \int_0^x \tau_w \, dx = \frac{1}{(0.86)(1000 \frac{kg}{m^3})(5 \frac{m}{s})^2} \int_0^x \tau_w \, dx$$

or

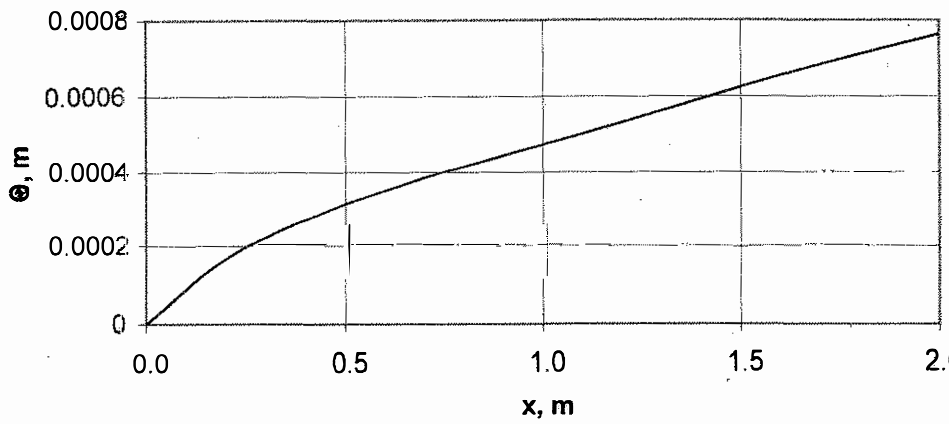
$$\Theta = 4.65 \times 10^{-5} \int_0^x \tau_w \, dx \,, \text{ where } \Theta \sim m,\ x \sim m, \text{ and } \tau_w \sim \frac{N}{m^2} \qquad (1)$$

For $0 \le x \le 2.0$ m, integrate Eq. (1) to determine Θ as a function of x. To do so, we need the value of τ_w at $x = 0$, which is not given in the table. Theoretically, $\tau_w = \infty$ at the leading. For our purposes, based on the extrapolated curve below, assume $\tau_w = 22 \frac{N}{m^2}$ at $x = 0$



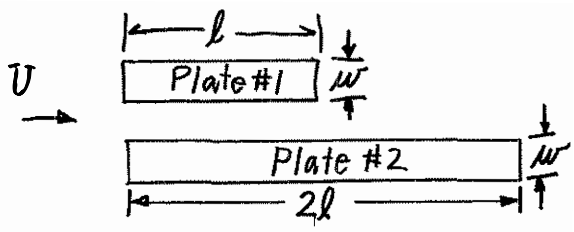
x (m)	τ_w (N/m^2)
0	—
0.2	13.4
0.4	9.25
0.6	7.68
0.8	6.51
1.0	5.89
1.2	6.57
1.4	6.75
1.6	6.23
1.8	5.92
2.0	5.26

A standard numerical integration technique gives the following results.



9.35

9.35 Water flows over two flat plates with the same laminar free-stream velocity. Both plates have the same width, but Plate #2 is twice as long as Plate #1. What is the relationship between the drag force for these two plates?



$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

Thus,

$$\mathcal{D}_1 = C_{D_1} \tfrac{1}{2}\rho U^2 \ell w$$

and

$$\mathcal{D}_2 = C_{D_2} \tfrac{1}{2}\rho U^2 (2\ell w) \quad \text{or}$$

$$\frac{\mathcal{D}_2}{\mathcal{D}_1} = \frac{C_{D_2}}{C_{D_1}} \frac{(2\ell w)}{\ell w} = 2\frac{C_{D_2}}{C_{D_1}} \qquad (1)$$

For laminar flow on a flat plate

$$C_D = \frac{1.328}{\sqrt{Re_\ell}}, \quad \text{where } Re_\ell = \frac{U\ell}{\nu}, \text{ so that } C_D = \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}$$

Thus,

$$\frac{C_{D_2}}{C_{D_1}} = \left(\frac{1.328\sqrt{\nu}}{\sqrt{U(2\ell)}}\right) \Big/ \left(\frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}\right) = \frac{1}{\sqrt{2}} \qquad (2)$$

Hence, from Eqs. (1) and (2),

$$\frac{\mathcal{D}_2}{\mathcal{D}_1} = 2/\sqrt{2} = \underline{\underline{1.414}}$$

9.36

9.36 Fluid flows past a flat plate with a drag force $\mathcal{D}_1$. If the freestream velocity is doubled, will the new drag force, $\mathcal{D}_2$, be larger or smaller than $\mathcal{D}_1$ and by what amount?

$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

If you assume that the doubling of U, which will change Re, does not significantly change C_D (see Fig. 9.22), then

$$\frac{\mathcal{D}_1}{\mathcal{D}_2} = \frac{C_D \frac{1}{2}\rho U_1^2 A}{C_D \frac{1}{2}\rho U_2^2 A} = \frac{U_1^2}{U_2^2} \quad \text{where } U_2 = 2U_1$$

$$= \frac{U_1^2}{(2U_1)^2} = \frac{1}{4}$$

So,

$$\underline{\underline{\mathcal{D}_2 = 4\mathcal{D}_1}}$$

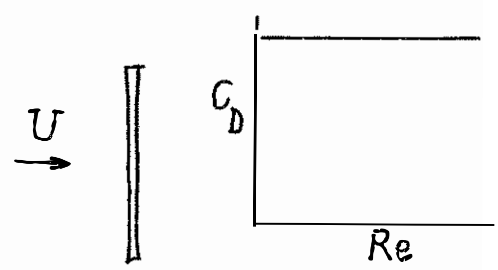


plate normal to flow

Note:

If the plate is parallel to the flow, then C_D changes with Re. See Fig. 9.22.

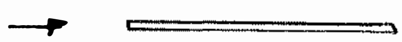


plate parallel to flow

Thus,

$$\frac{D_1}{D_2} = \frac{C_{D_1} U_1^2}{C_{D_2} U_2^2}$$

so that a numerical answer could not be obtained without additional data about the value of Re.

9.37

9.37 A model is placed in an air flow with a given velocity and then placed in water flow with the same velocity. If the drag coefficients are the same between these two cases, how do the drag forces compare between the two fluids?

$$\left(\frac{\mathcal{D}}{\frac{1}{2}\rho V^2 A}\right)_w = \left(\frac{\mathcal{D}}{\frac{1}{2}\rho V^2 A}\right)_a$$

$$\frac{\mathcal{D}_w}{\rho_w} = \frac{\mathcal{D}_a}{\rho_a}$$

$$\frac{\mathcal{D}_w}{\mathcal{D}_a} = \frac{\rho_w}{\rho_a} \quad \text{where } \rho_w \gg \rho_a$$

$$\underline{\underline{\mathcal{D}_w \gg \mathcal{D}_a}}$$

It should be noted that since $Re = \frac{V\ell}{\nu}$, matching ν_w and ν_a would be difficult. Therefore, depending on shape and velocity, the C_D values may not actually be the same. However, this difference would be small compared to the density difference.

Note: At standard conditions,

$$\frac{\mathcal{D}_w}{\mathcal{D}_a} = \frac{\rho_w}{\rho_a} = \frac{1.94\ slugs/ft^3}{2.38 \times 10^{-3}\ slugs/ft^3} = 815$$

9.38

9.38 The drag coefficient for a newly designed hybrid car is predicted to be 0.21. The cross-sectional area of the car is 30 ft^2. Determine the aerodynamic drag on the car when it is driven through still air at 55 mph.

$$\mathcal{D} = C_D \tfrac{1}{2}\rho V^2 A$$

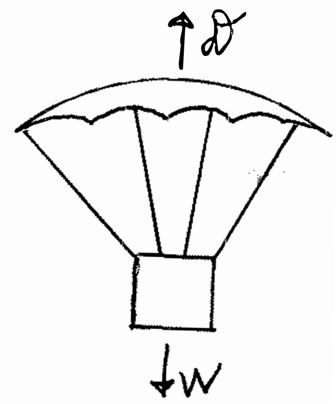
$$V = 55\,\text{mph} \times \frac{88\ \text{ft/s}}{60\,\text{mph}} = 80.7\ \text{ft/s}$$

$$\mathcal{D} = 0.21\left(\tfrac{1}{2}\right)(0.00238\ \text{slugs/ft}^3)(80.7\ \text{ft/s})^2(30\,\text{ft}^2)$$

$$\underline{\underline{\mathcal{D} = 48.8\ \text{lb}}}$$

9.39

9.39 A 5-m-diameter parachute of a new design is to be used to transport a load from flight altitude to the ground with an average vertical speed of 3 m/s. The total weight of the load and parachute is 200 N. Determine the approximate drag coefficient for the parachute.



$$C_D = \frac{\mathscr{D}}{\frac{1}{2}\rho V^2 A}$$

If in equilibrium, at constant velocity, then

$$W = \mathscr{D}$$

$$C_D = \frac{W}{\frac{1}{2}\rho V^2 A}$$

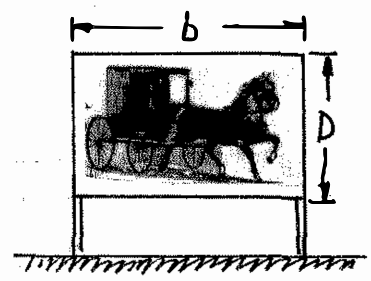
$$= \frac{200N}{\frac{1}{2}(1.23\,kg/m^3)(3\,m/s)^2 \frac{\pi}{4}(5m)^2}$$

$$\underline{\underline{C_D = 1.84}}$$

The sea-level density was used to solve this problem. Clearly during the drop, ρ will be changing, but the changes are relatively small.

9.40

9.40 A 50-mph wind blows against an outdoor movie screen that is 70 ft wide and 20 ft tall. Estimate the wind force on the screen.



$\mathscr{D} = C_D \frac{1}{2} \rho V^2 A$, where from Fig. 9.22

with $\frac{b}{D} = \frac{70\text{ ft}}{20\text{ ft}} = 3.5$ we obtain $C_D = 1.15$

Hence,

$$\mathscr{D} = 1.15\left(\frac{1}{2}\right)\left(0.00238\,\frac{\text{slugs}}{\text{ft}^3}\right)\left[\left(50\,\frac{\text{mi}}{\text{hr}}\right)\frac{\left(5280\,\frac{\text{ft}}{\text{mi}}\right)}{\left(3600\,\frac{\text{s}}{\text{hr}}\right)}\right]^2(70\text{ ft})(20\text{ ft})$$

or

$\mathscr{D} = \underline{10{,}300 \text{ lb}}$

9.41

9.41 The aerodynamic drag on a car depends on the "shape" of the car. For example, the car shown in Fig. P9.41 has a drag coefficient of 0.36 with the windows and roof closed. With the windows and roof open, the drag coefficient increases to 0.45. With the windows and roof open, at what speed is the amount of power needed to overcome aerodynamic drag the same as it is at 65 mph with the windows and roof closed? Assume the frontal area remains the same. Recall that power is force times velocity.

Windows and roof closed: $C_D = 0.35$

Windows open; roof open: $C_D = 0.45$

FIGURE P9.41

$\text{Power} = \mathcal{P} = F \cdot V$

The force is the drag force. Let $(\ )_c$ and $(\ )_o$ denote closed and open.

$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

We want to find U_o when $\mathcal{P}_o = \mathcal{P}_c$

$$\mathcal{P}_o = U_o \mathcal{D}_o = \tfrac{1}{2}\rho U_o^3 A_o C_{D_o} = \mathcal{P}_c = U_c \mathcal{D}_c = \tfrac{1}{2}\rho U_c^3 A_c C_{D_c}$$

The frontal areas are the same, so $A_o = A_c$

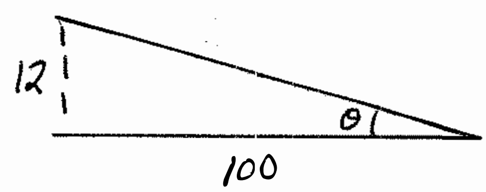
$$U_o^3 C_{D_o} = U_c^3 C_{D_c}$$

$$U_o = U_c\left(\frac{C_{D_c}}{C_{D_o}}\right)^{1/3} = (65\,\text{mph})\left(\frac{0.36}{0.45}\right)^{1/3}$$

$$\underline{\underline{U_o = 60.3\ \text{mph}}}$$

9.42

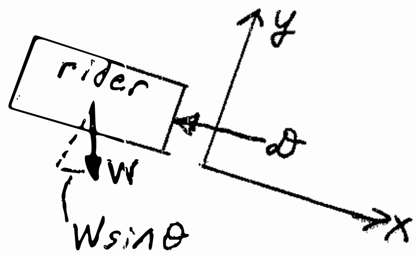
9.42 A rider on a bike with the combined mass of 100 kg attains a terminal speed of 15 m/s on a 12% slope. Assuming that the only forces affecting the speed are the weight and the drag, calculate the drag coefficient. The frontal area is 0.9 m². Speculate whether the rider is in the upright or racing position.



$\tan\theta = 12/100 = 0.12$

$\theta = 6.84°$

$\sin\theta = 0.119$



In equilibrium, $\Sigma F = 0$

$$\Sigma F_x = 0$$

$$W\sin\theta = \mathcal{D} = C_D \tfrac{1}{2}\rho V^2 A, \quad \text{where } W = mg = (100\,\text{kg})(9.81\,\text{m/s}^2) = 981\,\text{N}$$

$$C_D = \frac{W\sin\theta}{\frac{1}{2}\rho V^2 A}$$

$$= \frac{(981\,\text{N})(0.119)}{\frac{1}{2}(1.23\,\text{kg/m}^3)(15\,\text{m/s})^2(0.6\,\text{m}^2)}$$

$$C_D = 1.4$$

Looking at Fig. 9.30, given A and C_D, the rider is upright.

9.43

9.43 A baseball is thrown by a pitcher at 95 mph through standard air. The diameter of the baseball is 2.82 in. Estimate the drag force on the baseball.

$$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$$

$$U = 95\,mph \times \frac{88\,ft/s}{60\,mph} = 139.3\,ft/s$$

$$Re = \frac{UD}{\nu} = \frac{(139.3\,ft/s)\left(\frac{2.82}{12}\,ft\right)}{1.57\times10^{-4}\,ft^2/s} = 2.09\times10^5$$

From Fig. 9.25, and assuming a smooth sphere,

$$C_D \approx 0.5$$

$$\mathcal{D} = 0.5\left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)(139.3\,ft/s)^2\left(\frac{\pi}{4}\left(\frac{2.82}{12}\right)^2\right)$$

$$\underline{\underline{\mathcal{D} = 0.502\,lb}}$$

9.44

9.44 A logging boat is towing a log that is 2 m in diameter and 8 m long at 4 m/s through water. Estimate the power required if the axis of the log is parallel to the tow direction.

For power, $\mathcal{P} = F \cdot V$

$$F = \mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

For the aspect ratio, $D = 2m$ and $l = 8m$

From Fig. 9.

$$\frac{l}{D} = \frac{8}{2} = 4, \text{ so } C_D = 0.85$$

$$\mathcal{D} = 0.85\left(\tfrac{1}{2}\right)\left(999\,\tfrac{kg}{m^3}\right)\left(4\,\tfrac{m}{s}\right)^2\left(\tfrac{\pi}{4}(2m)^2\right)$$

$$\mathcal{D} = 21,341\,N$$

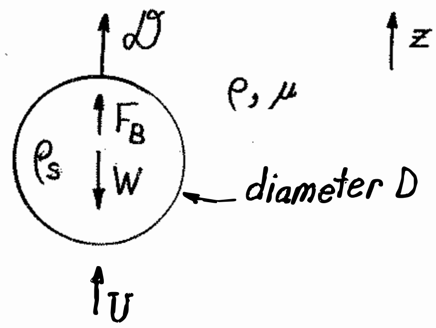
$$\mathcal{P} = \mathcal{D}U = (21,341\,N)(4\,\tfrac{m}{s}) = 85,400\,W$$

$$= \underline{\underline{85.4\,kW}}$$

Note: The above $C_D = 0.85$ assumes that the log is essentially submerged and wave making is not an important contribution to the drag.

9.45

9.45 A sphere of diameter D and density ρ_s falls at a steady rate through a liquid of density ρ and viscosity μ. If the Reynolds number, $\mathrm{Re} = \rho D U/\mu$, is less than 1, show that the viscosity can be determined from $\mu = gD^2(\rho_s - \rho)/18\,U$.



For steady flow $\sum F_z = 0$

or $\mathcal{D} + F_B = W$, where F_B = buoyant force $= \rho g \forall = \rho g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

W = weight $= \rho_s g \forall = \rho_s g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

and $\mathcal{D}$ = drag $= C_D \frac{1}{2}\rho \frac{\pi}{4} D^2$, or since $Re<1$

$\mathcal{D} = 3\pi D U \mu$

Thus,

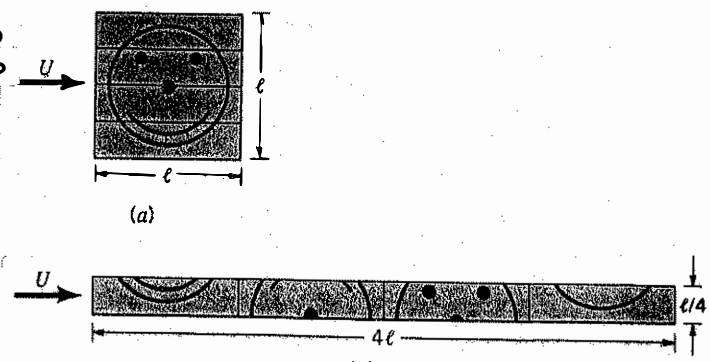
$3\pi D U\mu + \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3 = \rho_s g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

which can be rearranged to give

$$\mu = \frac{g D^2(\rho_s - \rho)}{18\,U}$$

9.46

9.46 The square flat plate shown in Fig. P9.46a is cut into four equal-sized prices and arranged as shown in Fig. P9.46b Determine the ratio of the drag on the original plate [case (a)] to the drag on the plates in the configuration shown in (b). Assume laminar boundary flow. Explain your answer physically.



■ FIGURE P9.46

For case (a):

$$\mathscr{D}_{f_a} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{U\ell}{\nu}}} \text{ and } A = \ell^2$$

Thus,

$$\mathscr{D}_{f_a} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}\ell^2 = 0.664\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2} \qquad (1)$$

For case (b):

$$\mathscr{D}_{f_b} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{\frac{U(4\ell)}{\nu}}} \text{ and } A = (4\ell)\left(\frac{\ell}{4}\right) = \ell^2$$

Thus,

$$\mathscr{D}_{f_b} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{4U\ell}}\ell^2 = \frac{1}{2}\left(0.664\rho U^{\frac{3}{2}}\sqrt{\nu}\,\ell^{\frac{3}{2}}\right) \qquad (2)$$

By comparing Eqs. (1) and (2) we see that

$$\underline{\underline{\mathscr{D}_{f_a} = 2.0\,\mathscr{D}_{f_b}}}$$

In case (b) the boundary layer on the rear plate is thicker than on the front plate. Hence the shear stress is less on the rear plate than it is on that plate in configuration (a), giving less drag for case (b) than for case (a), even though the total areas are the same.

9.47

9.47 If the drag on one side of a flat plate parallel to the upstream flow is $\mathcal{D}$ when the upstream velocity is U, what will the drag be when the upstream velocity is $2U$; or $U/2$? Assume laminar flow.

For laminar flow $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$, where $C_{Df} = \frac{1.328}{\sqrt{\frac{U\ell}{\nu}}}$

Thus,

$$\mathcal{D} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}} A = 0.664\,\rho A \frac{\sqrt{\nu}}{\sqrt{\ell}} U^{3/2} \sim U^{3/2}$$

Hence,

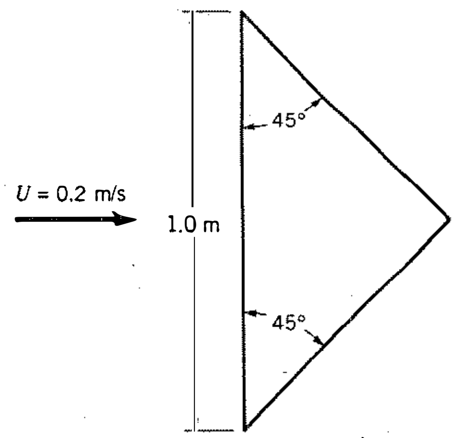
$$\frac{\mathcal{D}_U}{\mathcal{D}_{2U}} = \frac{U^{3/2}}{(2U)^{3/2}} \quad \text{or} \quad \underline{\underline{\mathcal{D}_{2U} = 2.83\,\mathcal{D}_U}}$$

and

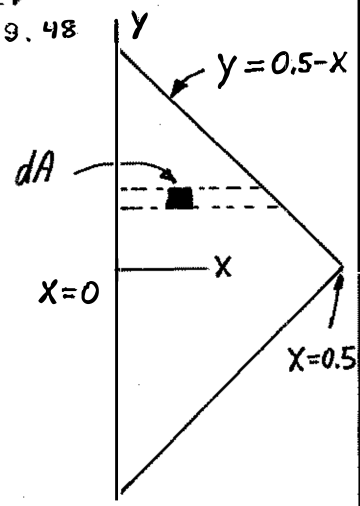
$$\frac{\mathcal{D}_U}{\mathcal{D}_{U/2}} \quad \frac{U^{3/2}}{\left(\frac{U}{2}\right)^{3/2}} \quad \text{or} \quad \underline{\underline{\mathcal{D}_{U/2} = 0.354\,\mathcal{D}_U}}$$

9.48

9.48 Water flows past a triangular flat plate oriented parallel to the free stream as shown in Fig. P9.48 Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow.



■ FIGURE P9.48



$$\mathscr{D} = \int \tau_w \, dA \quad \text{where} \quad \tau_w = 0.332\, U^{3/2} \sqrt{\frac{\rho \mu}{x}}$$

Thus,

$$\mathscr{D} = 0.332\, U^{3/2} \sqrt{\rho\mu} \int \frac{1}{\sqrt{x}}\, dA$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \int_{x=0}^{x=0.5} \int_{y=0}^{y=0.5-x} \frac{dy\, dx}{\sqrt{x}}$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \int_{x=0}^{0.5} \frac{0.5 - x}{\sqrt{x}}\, dx$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \left[0.5(2) x^{\frac{1}{2}} - \frac{2}{3} x^{\frac{3}{2}} \right]_0^{0.5}$$

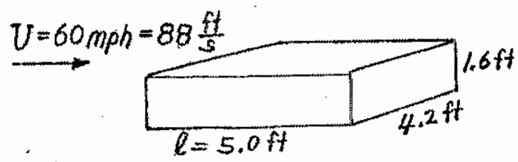
$$= 0.664 \left(0.2 \tfrac{m}{s}\right)^{3/2} \sqrt{999 \tfrac{kg}{m^3} \left(1.12 \times 10^{-3} \tfrac{N \cdot s}{m^2}\right)} \left[\sqrt{0.5} - \frac{2}{3}(0.5)^{3/2} \right]$$

or

$$\mathscr{D} = \underline{\underline{0.0296\, N}}$$

9.50

9.50 A rectangular car-top carrier of 1.6-ft height, 5.0-ft length (front to back), and 4.2-ft width is attached to the top of a car. Estimate the additional power required to drive the car with the carrier at 60 mph through still air compared with the power required to driving only the car at 60 mph.



$\mathcal{D} = C_D \frac{1}{2}\rho V^2 A$ and $\mathcal{P} = V\mathcal{D}$ = power (1)

From Fig. 9.31 with $\frac{\ell}{D} = \frac{5\,ft}{1.6\,ft} = 3.13$ we obtain $C_D = 1.3$

Hence,

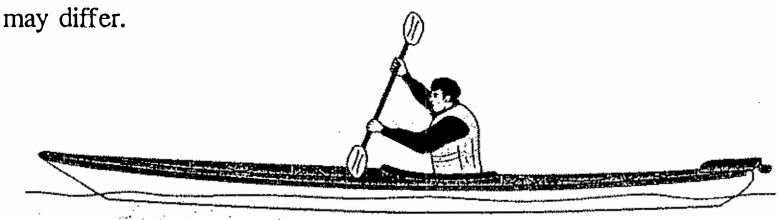
$$\mathcal{D} = 1.3\left(\tfrac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)(1.6\,ft)(4.2\,ft)\left(88\,\frac{ft}{s}\right)^2 = 80.5\,lb$$

Thus, from Eq. (1),

$$\mathcal{P} = \left(88\,\frac{ft}{s}\right)(80.5\,lb)\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}} = \underline{\underline{12.9\,hp}}$$

9.51

9.51 As shown in Video V9.2 and Fig. P9.51a a kayak is a relatively streamlined object. As a first approximation in calculating the drag on a kayak, assume that the kayak acts as if it were a smooth flat plate 17 ft long and 2 ft wide. Determine the drag as a function of speed and compare your results with the measured values given in Fig. P9.51b Comment on reasons why the two sets of values may differ.



(a)

For a flat plate $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$ where (1)

$A = 17\,ft(2\,ft) = 34\,ft^2$ and C_{Df} is a function of $Re_\ell = \frac{U\ell}{\nu}$

Thus,

$$Re_\ell = \frac{17\,ft\; U}{1.21\times10^{-5}\,ft^2/s} = 1.40\times10^6\, U \quad (2)$$

Consider $1 \le U \le 8\,\frac{ft}{s}$, or $1.40\times10^6 \le Re_\ell \le 1.12\times10^7$

From Fig. 9.15 we see that in this Re_ℓ range the boundary layer flow is in the transitional range. Thus, from Table 9.3

$$C_{Df} = 0.455/(\log Re_\ell)^{2.58} - 1700/Re_\ell \quad (3)$$

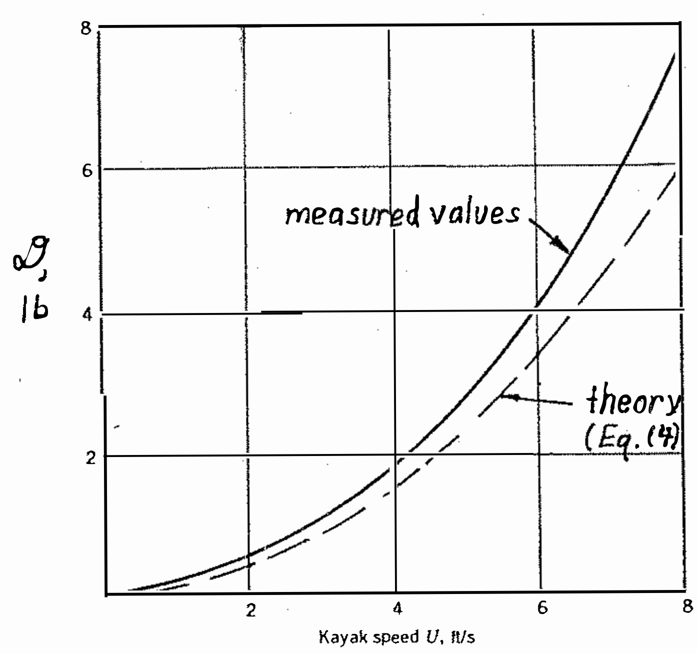
By combining Eqs. (1), (2), and (3):

$$\mathcal{D} = \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right) U^2 C_{Df}\,(34\,ft^2) \quad \text{or}$$

$$\mathcal{D} = 33.0\,U^2\left[0.455/(\log(1.40\times10^6 U))^{2.58} - 1700/(1.40\times10^6\, U)\right] \quad (4)$$

The results from this equation are plotted below.

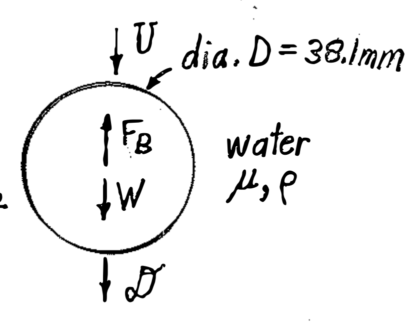
U, ft/s	$\mathcal{D}$, lb
1	0.0986
2	0.410
3	0.909
4	1.58
5	2.42
6	3.43
7	4.59
8	5.90



■ FIGURE P9.51(b)

9.52

9.52 A 38.1-mm-diameter, 0.0245-N table tennis ball is released from the bottom of a swimming pool. With what velocity does it rise to the surface? Assume it has reached its terminal velocity.



For steady rise $\Sigma F_z = 0$

or

$F_B = W + \mathcal{D}$, where $\mathcal{D} = \text{drag} = C_D \frac{1}{2} \rho U^2 \frac{\pi}{4} D^2$

$W = \text{weight} = 0.0245\ N$

$F_B = \text{buoyant force} = \gamma \forall = \gamma \frac{4\pi}{3} \left(\frac{D}{2}\right)^3$

Thus,

$$\gamma \frac{4\pi}{3} \left(\frac{D}{2}\right)^3 = W + C_D \frac{1}{2} \rho U^2 \frac{\pi}{4} D^2$$

or

$$\left(9.80 \times 10^3 \frac{N}{m^3}\right) \frac{4\pi}{3} \left(\frac{0.0381}{2} m\right)^3 = 0.0245\ N + \frac{1}{2} C_D \left(999 \frac{kg}{m^3}\right) U^2 \frac{\pi}{4} (0.0381 m)^3$$

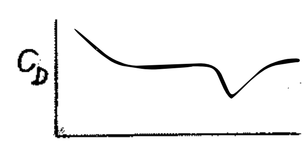
or

$$C_D U^2 = 0.455, \text{ where } U \sim \frac{m}{s} \qquad (1)$$

Also, $Re = \frac{UD}{\nu}$

or

$$Re = \frac{U (0.0381 m)}{1.12 \times 10^{-6} \frac{m^2}{s}} = 3.40 \times 10^4\ U, \text{ where } U \sim \frac{m}{s} \qquad (2)$$

Finally, from Fig. 9.21:  (3)

Trial and error solution: Assume C_D; obtain U from Eq. (1), Re from Eq. (2); check C_D from Eq. (3), the graph.

Assume $C_D = 0.5 \longrightarrow U = 0.954 \frac{m}{s} \longrightarrow Re = 3.24 \times 10^4 \longrightarrow C_D = 0.4 \neq 0.5$

Assume $C_D = 0.4 \longrightarrow U = 1.06 \frac{m}{s} \longrightarrow Re = 3.62 \times 10^4 \longrightarrow C_D = 0.4$ (checks)

Thus, $\underline{\underline{U = 1.06 \frac{m}{s}}}$

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.

9.53

9.53 To reduce aerodynamic drag on a bicycle, it is proposed that the cross-sectional shape of the handlebar tubes be made "tear-drop" shape rather than circular. Make a rough estimate of the reduction in aerodynamic drag for a bike with this type of handlebars compared with the standard handlebars. List all assumptions.

For a standard racing bike $\mathscr{D}_s = C_{D_s} \frac{1}{2}\rho U^2 A$, where from Fig. 9.33 $C_{D_s} = 0.88$, $A = 3.9\, ft^2$

Thus, $\mathscr{D}_s = 1.716\, \rho U^2$

For the modified bike assume $\mathscr{D}_m = \mathscr{D}_s - \mathscr{D}_c + \mathscr{D}_t$, where (1)

$\mathscr{D}_c$ = drag from standard circular cross section handle bars

and

$\mathscr{D}_t$ = drag from tear-drop shaped handle bars.

That is,

$\mathscr{D}_c = C_{D_c} \frac{1}{2}\rho U^2 A_H$ and $\mathscr{D}_t = C_{D_t} \frac{1}{2}\rho U^2 A_H$ where the handle bars are assumed to be 1 ft long and 1 in. in diameter. (i.e., $A_H = \frac{1}{12}\, ft^2$)

Typical C_D values are $C_{D_c} = 1$ (Fig. 9.23) and $C_{D_t} = 0.12$ (Fig. 9.21)

Thus, Eq. (1) gives $\mathscr{D}_m = 1.716\rho U^2 - 1(\frac{1}{2})\rho U^2(\frac{1}{12}) + 0.12(\frac{1}{2})\rho U^2(\frac{1}{12})$

$$= (1.716 - 0.0367)\rho U^2$$

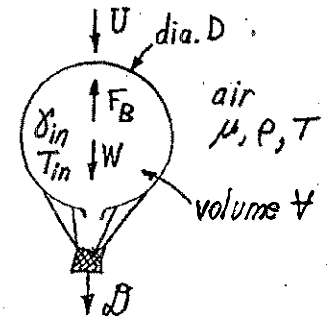
or

$$\frac{\mathscr{D}_s - \mathscr{D}_m}{\mathscr{D}_s} = \frac{1.716\,\rho U^2 - (1.716 - 0.0367)\rho U^2}{1.716\,\rho U^2} = 0.0214$$

i.e., a reduction in drag of approximately 2 percent

9.54

9.54 A hot air balloon roughly spherical in shape has a volume of 70,000 ft³ and a weight of 500 lb (including passengers, basket, balloon fabric, etc.). If the outside air temperature is 80 °F and the temperature within the balloon is 165 °F, estimate the rate at which it will rise under steady state conditions if the atmospheric pressure is 14.7 psi.



For steady rise $\sum F_z = 0$, or $F_B = W + \mathscr{D}$
where

$\mathscr{D} = \text{drag} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$F_B = \text{buoyant force} = \gamma \forall$

and

$W = \text{total weight} = 500 \text{ lb} + \gamma_{in} \forall$

Now $\rho = \dfrac{p}{RT} = \dfrac{(14.7 \frac{lb}{in.^2})(12 \frac{in.}{ft})^2}{(1715 \frac{ft \cdot lb}{slug \cdot {}^\circ R})(460+80)^\circ R} = 0.00229 \frac{slugs}{ft^3}$

$\gamma = \rho g = (0.00229 \frac{slugs}{ft^3})(32.2 \frac{ft}{s^2}) = 0.0736 \frac{lb}{ft^3}$

and

$\gamma_{in} = \dfrac{p g}{R\, T_{in}} = \dfrac{(14.7 \frac{lb}{in.^2})(12 \frac{in.}{ft})^2 (32.2 \frac{ft}{s^2})}{(1715 \frac{ft \cdot lb}{slug \cdot {}^\circ R})(460+165)^\circ R} = 0.0636 \frac{lb}{ft^3}$

Note: Since the balloon is open at the bottom, the pressure within the balloon is nearly the same as it is outside.

Thus, with $\forall = 7 \times 10^4 ft^3 = \frac{4\pi}{3}(\frac{D}{2})^3$,
or $D = 51.1$ ft we obtain

$\mathscr{D} = C_D \frac{1}{2}(0.00229) U^2 \frac{\pi}{4}(51.1)^2$

$= 2.36\, C_D U^2$ lb, where $U \sim \frac{ft}{s}$

Also,

$W = 500 \text{ lb} + (0.0636 \frac{lb}{ft^3})(70{,}000 \, ft^3) = 4952 \text{ lb}$

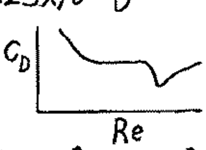
and

$F_B = (0.0736 \frac{lb}{ft^3})(70{,}000 \, ft^3) = 5152 \text{ lb}$ Thus, $F_B = W + \mathscr{D}$ gives

$5152 \text{ lb} = 4952 \text{ lb} + 2.36\, C_D U^2$ or $C_D U^2 = 84.7$ (1)

Also, $Re = \frac{UD}{\nu}$

or $Re = \dfrac{51.1 \text{ ft } U}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 3.25 \times 10^5\, U$ (2)

and from Fig. 9.23 

(3)

Trial and error solution: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 13.0 \frac{ft}{s} \rightarrow Re = 4.23 \times 10^6 \rightarrow C_D = 0.24 \neq 0.5$

Assume $C_D = 0.24 \rightarrow U = 18.8 \frac{ft}{s} \rightarrow Re = 6.11 \times 10^6 \rightarrow C_D = 0.30 \neq 0.24$

Assume $C_D = 0.30 \rightarrow \underline{\underline{U = 16.8 \frac{ft}{s}}} \rightarrow Re = 5.46 \times 10^6 \rightarrow C_D = 0.30$ (checks)

9.55

9.55 It is often assumed that "sharp objects can cut through the air better than blunt ones." Based on this assumption, the drag on the object shown in Fig. P9.55 should be less when the wind blows from right to left than when it blows from left to right. Experiments show that the opposite is true. Explain.

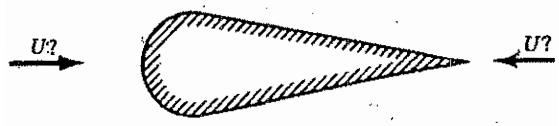
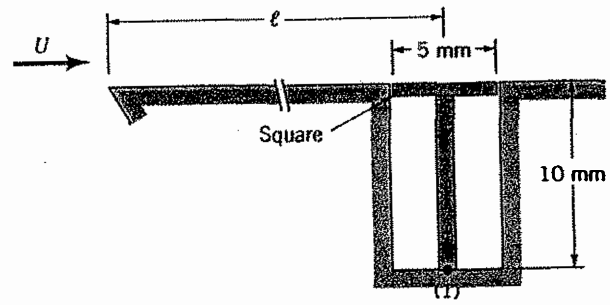


FIGURE P9.55

A significant portion of the drag on an object can be from the relatively low pressure developed in the wake region behind the object. By making the object streamlined (i.e., flow from left to right, not right to left in the above figure) boundary layer separation is avoided and a relatively thin wake with low drag is obtained. Whether the front of the object is "sharp" or "blunt" does not affect the contribution to the drag from the front part of the body—at least not as much a the width of the wake affects the drag.

9.56

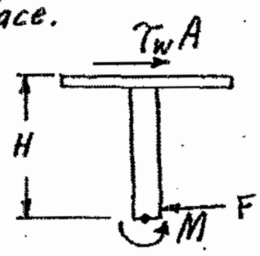
*9.56 The device shown in Fig. P9.56 is to be designed to measure the wall shear stress as air flows over the smooth surface with an upstream velocity U. It is proposed that τ_w can be obtained by measuring the bending moment, M, at the base [point (1)] of the support that holds the small surface element which is free from contact with the surrounding surface. Plot a graph of M as a function of U for $5 \le U \le 50$ m/s, with $\ell = 2, 3, 4$, and 5 m.



■ FIGURE P9.56

Since the length of the measuring surface is much less than its distance from the leading edge (i.e., $5mm \ll \ell$) we can assume that the shear stress is essentially constant on that surface.

Thus, $M = \tau_w A H$



or $M = (5\times10^{-3}m)^2(10\times10^{-3}m)\tau_w = 2.5\times10^{-7}\tau_w \ Nm$ (1)

where $\tau_w \sim \frac{N}{m^2}$

The flow will be laminar or turbulent depending whether $Re_\ell < 5\times10^5$ or $Re_\ell > 5\times10^5$, where $Re_\ell = \frac{U\ell}{\nu}$ and $\nu = 1.46\times10^{-5}\frac{m^2}{s}$.

Since $Re_{\ell_{min}} = \frac{(5\frac{m}{s})(2m)}{1.46\times10^{-5}\frac{m^2}{s}} = 6.84\times10^5$ the flow is always turbulent.

Also, since

$Re_{\ell_{max}} = \frac{(50\frac{m}{s})(5m)}{1.46\times10^{-5}\frac{m^2}{s}} = 1.71\times10^7$ it follows from Table 9.3 that*

$$\tau_w = 0.0225\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4} \quad \text{where } \delta = \frac{0.370x}{Re_x^{1/5}} = \frac{0.370\,\ell\,\nu^{1/5}}{U^{1/5}\,\ell^{1/5}}$$

That is,

$$\tau_w = 0.0225\,\rho U^2\left[\frac{\nu\,U^{\frac{1}{5}}\,\ell^{1/5}}{U\,(0.370)\,\ell\,\nu^{1/5}}\right]^{1/4} = 0.0225\,\rho U^{9/5}\nu^{1/5}\ell^{-\frac{1}{5}}(0.370)^{-\frac{1}{4}}$$

or

$$\tau_w = 0.0225\,(1.23\tfrac{kg}{m^3})U^{9/5}(1.46\times10^{-5}\tfrac{m^2}{s})^{\frac{1}{5}}\ell^{-\frac{1}{5}}(0.370)^{-\frac{1}{4}}$$

$$= 3.83\times10^{-3}\,U^{9/5}\,\ell^{-1/5}\ \frac{N}{m^2}, \quad \text{where } U\sim\tfrac{m}{s} \text{ and } \ell\sim m$$

Thus, from Eq. (1),

$$M = (2.5\times10^{-7})(4.34\times10^{-3})\,U^{9/5}\ell^{-1/5} = 9.57\times10^{-10}\,U^{9/5}\ell^{-1/5}\ N\cdot m$$

The values of M are calculated and plotted for $5 \le U \le 50\ \frac{m}{s}$, with $\ell = 2, 3, 4$, and 5 m.

(con't)

9.56 (con't)

For ℓ = 2.00 m

U, m/s	M, N.m
5.00	+1.510E-08
10.00	+5.257E-08
15.00	+1.091E-07
20.00	+1.830E-07
25.00	+2.735E-07
30.00	+3.798E-07
35.00	+5.012E-07
40.00	+6.374E-07
45.00	+7.879E-07
50.00	+9.525E-07

For ℓ = 4.00 m

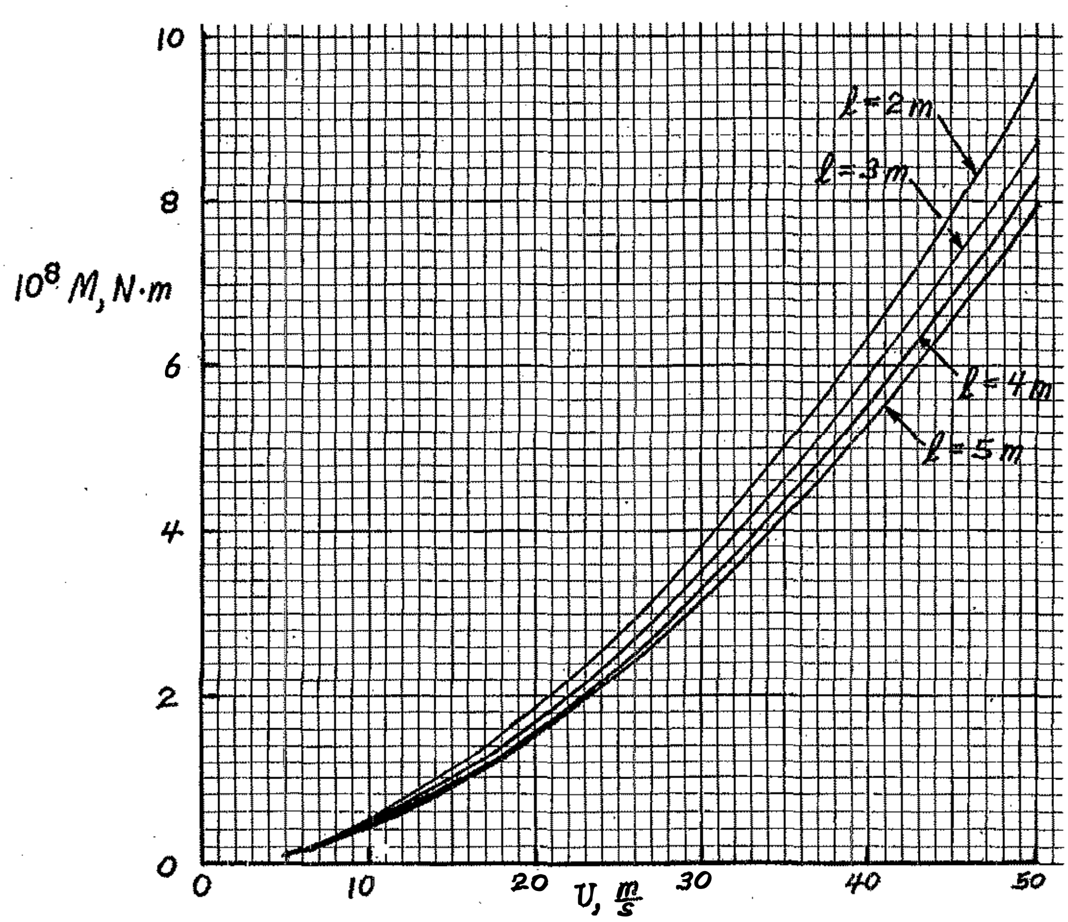
U, m/s	M, N.m
5.00	+1.314E-08
10.00	+4.576E-08
15.00	+9.494E-08
20.00	+1.594E-07
25.00	+2.381E-07
30.00	+3.306E-07
35.00	+4.363E-07
40.00	+5.549E-07
45.00	+6.859E-07
50.00	+8.292E-07

For ℓ = 3.00 m

U, m/s	M, N.m
5.00	+1.392E-08
10.00	+4.847E-08
15.00	+1.006E-07
20.00	+1.688E-07
25.00	+2.522E-07
30.00	+3.502E-07
35.00	+4.622E-07
40.00	+5.878E-07
45.00	+7.266E-07
50.00	+8.783E-07

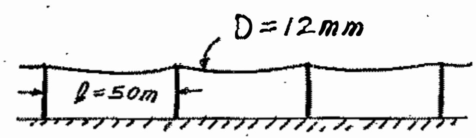
For ℓ = 5.00 m

U, m/s	M, N.m
5.00	+1.257E-08
10.00	+4.376E-08
15.00	+9.080E-08
20.00	+1.524E-07
25.00	+2.277E-07
30.00	+3.162E-07
35.00	+4.173E-07
40.00	+5.307E-07
45.00	+6.560E-07
50.00	+7.930E-07



9.57

9.57 A 12-mm-diameter cable is strung between a series of poles that are 50 m apart. Determine the horizontal force this cable puts on each pole if the wind velocity is 30 m/s.



F_P = force on one pole = $\mathcal{D}$

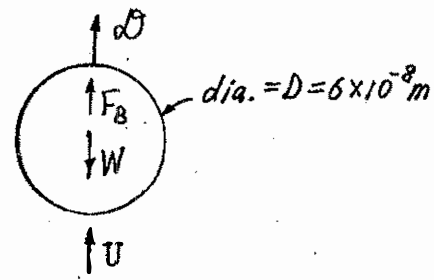
where $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$

Since $Re = \frac{UD}{\nu} = \frac{(30\frac{m}{s})(0.012m)}{1.46\times10^{-5}\frac{m^2}{s}} = 2.47\times10^4$ it follows from Fig. 9.23 that

$C_D = 0.4$. Hence, $F_P = 0.4\left(\frac{1}{2}\right)(1.23\frac{kg}{m^3})(30\frac{m}{s})^2(50m)(0.012m) = \underline{\underline{133\ N}}$

9.58

9.58 How fast do small water droplets of 0.06-μm (6×10^{-8} m) diameter fall through the air under standard sea-level conditions? Assume the drops do not evaporate. Repeat the problem for standard conditions at 5000-m altitude.



For steady conditions, $\mathcal{D} + F_B = W$,
where if $Re = \frac{UD}{\nu} < 1$

$\mathcal{D} = \text{drag} = 3\pi D U \mu$ Also, $W = \gamma_{H_2O} \forall = \gamma_{H_2O} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \text{weight}$

and $F_B = \gamma_{air} \forall = \gamma_{air} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \text{buoyant force}$

Since $\gamma_{air} \ll \gamma_{H_2O}$, we can neglect the buoyant force.
That is, $\mathcal{D} = W$, or

$$3\pi D U \mu = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \quad \text{or} \quad U = \frac{\gamma_{H_2O} D^2}{18\mu} \tag{1}$$

At sea level $\mu = 1.789 \times 10^{-5} \frac{N \cdot s}{m^2}$ so that

$$U = \frac{(9.80 \times 10^3 \frac{N}{m^3})(6 \times 10^{-8} m)^2}{18(1.789 \times 10^{-5} \frac{N \cdot s}{m^2})} = \underline{\underline{1.10 \times 10^{-7} \frac{m}{s}}}$$

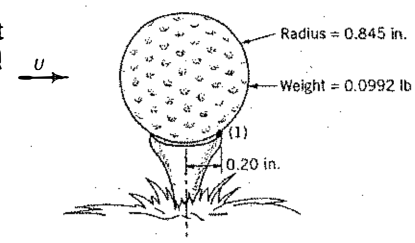
Note that $Re = \frac{(1.10 \times 10^{-7} \frac{m}{s})(6 \times 10^{-8} m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 4.52 \times 10^{-10} \ll 1$ so the use of the low Re drag equation is valid.

At an altitude of 5000 m, $\mu = 1.628 \times 10^{-5} \frac{N \cdot s}{m^2}$ and from Eq. (1)

$$U = \frac{(9.80 \times 10^3 \frac{N}{m^3})(6 \times 10^{-8} m)^2}{18(1.628 \times 10^{-5} \frac{N \cdot s}{m^2})} = \underline{\underline{1.20 \times 10^{-7} \frac{m}{s}}}$$

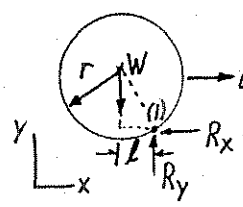
9.59

9.59 A strong wind can blow a golf ball off the tee by pivoting it about point 1 as shown in Fig. P9.59. Determine the wind speed necessary to do this.



■ FIGURE P9.59

When the ball is about to be blown from the tee the free body diagram is as shown. Hence, by summing moments about (1):

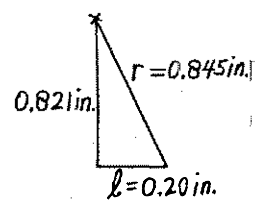


$\Sigma M_1 = 0$, or $W\ell = \mathcal{D} r$

Thus,

$(0.0992\,lb)(0.20\,in.) = \mathcal{D}(0.821\,in.)$

or



$\mathcal{D} = 0.0242\,lb$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 \pi r^2$

Thus,

$0.0242\,lb = C_D \frac{1}{2}(0.00238\,\frac{slugs}{ft^3})U^2 \pi \left(\frac{0.845\,in.}{12\,\frac{in.}{ft}}\right)^2$

or

$C_D U^2 = 1305$, where $U \sim \frac{ft}{s}$ (1)

For a sphere* $C_D = C_D(Re)$ (see Fig. 9.18) where (2)

$Re = \frac{\rho U D}{\mu} = \frac{(0.00238\,slugs/ft^3)\,U\,(2(0.845)/12\,ft)}{3.47\times10^{-7}(lb\cdot s/ft^2)}$

or

$Re = 966\,U$, where $U \sim \frac{ft}{s}$ (3)

Trial and error solution:

Assume $C_D = 0.4$ so that from Eq. (1), $U = 57.1\,\frac{ft}{s}$ and from Eq. (3), $Re = 966(57.1) = 5.52\times10^4$. Thus, from Fig. 9.18, $C_D \approx 0.25 \neq 0.40$ Try again.

Assume $C_D = 0.22$ so that $U = 77.0\,\frac{ft}{s}$ and $Re = 7.44\times10^4$

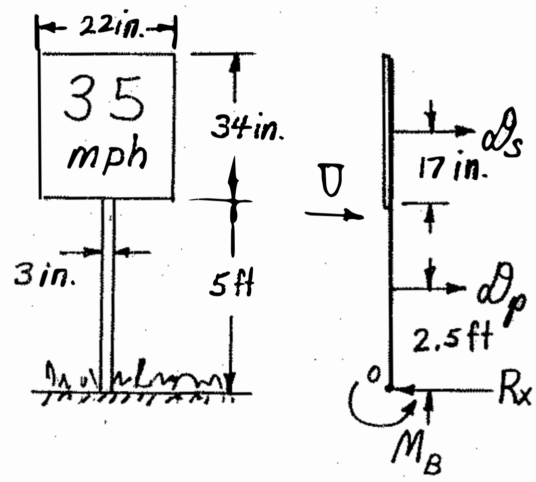
Thus, from Fig. 9.18, $C_D = 0.22$ Checks.

Hence, $\underline{\underline{U \approx 77.0\,\frac{ft}{s}}}$

* golf ball (i.e. with dimples)

9.60

9.60 A 22 in. by 34 in. speed limit sign is supported on a 3-in. wide, 5-ft-long pole. Estimate the bending moment in the pole at ground level when a 30-mph wind blows against the sign. (See Video V9.9.) List any assumptions used in your calculations.



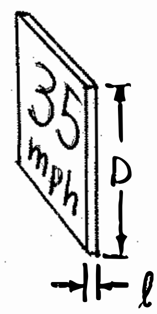
For equalibrium, $\Sigma M_0 = 0$ or

$M_B = 2.5\,ft\,\mathscr{D}_p + (5 + \frac{17}{12})ft\,\mathscr{D}_s$, where (1)

$\mathscr{D}_p$ = drag on the pole and $\mathscr{D}_s$ = drag on the sign

From Fig. 9.28 with $\ell/D < 0.1$ for the sign,

$C_{D_s} = 1.9$



From Fig. 9.19 if the post acts as a square rod with sharp corners $C_{Dp} = 2.2$ Thus, with $U = 30\,mph = 44\,\frac{ft}{s}$,

$\mathscr{D}_s = \frac{1}{2}\rho U^2 C_{D_s} A_s = \frac{1}{2}(0.00238\,\frac{slugs}{ft^3})(44\frac{ft}{s})^2(1.9)\left(\frac{22\,(34)}{144}ft^2\right) = 22.7\,lb$

and

$\mathscr{D}_p = \frac{1}{2}\rho U^2 C_{Dp} A_p = \frac{1}{2}(0.00238\,\frac{slugs}{ft^3})(44\frac{ft}{s})^2(2.2)\left(\frac{3}{12}(5)\,ft^2\right) = 6.34\,lb$

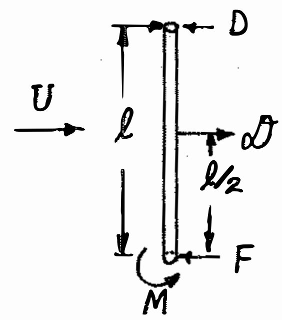
Thus, from Eq. (1):

$M_B = 2.5\,ft\,(6.34\,lb) + (5 + \frac{17}{12})ft\,(22.7\,lb) = \underline{\underline{162\;ft\cdot lb}}$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

9.61

9.61 Determine the moment needed at the base of 20-m-tall, 0.12-m-diameter flag pole to keep it in place in a 20 m/s wind.



For equilibrium, $M = \frac{\ell}{2}\mathscr{D}$ where (1)

$\mathscr{D} = C_D \frac{1}{2}\rho U^2 \ell D$

Since $Re = \frac{UD}{\nu} = \frac{(20\frac{m}{s})(0.12m)}{1.46\times10^{-5}\frac{m^2}{s}} = 1.64\times10^5$, it follows from Fig. 9.21

that $C_D = 1.2$

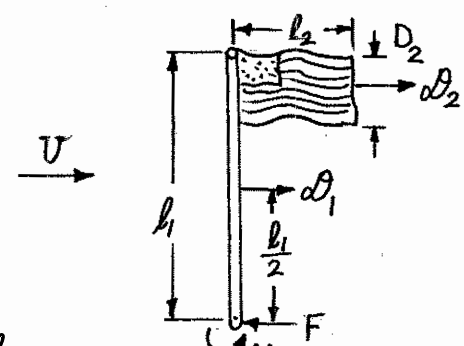
Thus, $\mathscr{D} = 1.2\left(\frac{1}{2}\right)(1.23\frac{kg}{m^3})(20\frac{m}{s})^2(20m)(0.12m) = 708N$

Hence, from Eq. (1)

$M = \frac{20m}{2}(708N) = \underline{\underline{7{,}080\ N\cdot m}}$

9.62

9.62 Repeat Problem 9.61 if a 2-m by 2.5-m flag is attached to the top of the pole. See Fig. 9.30 for drag coefficient data for flags.



For equilibrium, $M = \frac{\ell_1}{2}\mathcal{D}_1 + \left(\ell_1 - \frac{D_2}{2}\right)\mathcal{D}_2$ (1)

where $\ell_1 = 2.0\,m$, $\ell_2 = 2.5\,m$, and $D_2 = 2\,m$.

From the solution to Problem 9.48, $\frac{\ell_1}{2}\mathcal{D}_1 = 7{,}080\ N\cdot m$ (2)

Also,

$\mathcal{D}_2 = C_D \frac{1}{2}\rho U^2 \ell_2 D_2$, where from Fig. 9.30 with $\frac{\ell_2}{D_2} = \frac{2.5}{2} = 1.25$

we obtain $C_D = 0.08$.

Thus,

$\mathcal{D}_2 = 0.08\left(\frac{1}{2}\right)\left(1.23\frac{kg}{m^3}\right)\left(20\frac{m}{s}\right)^2(2.5\,m)(2\,m) = 98.4\,N$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$M = 7{,}080\ N\cdot m + (20\,m - 1\,m)(98.4\ N) = \underline{\underline{8{,}950\ N\cdot m}}$

9.64

9.64 How much more power is required to peddle a bicycle at 15 mph into a 20-mph headwind than at 15 mph through still air? Assume a frontal area of 3.9 ft² and a drag coefficient of $C_D = 0.88$.

$\mathcal{P}$ = power = $U_B \mathcal{D}$ and $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$, where U_B = speed of the bike $= 15\,\frac{mi}{hr}\left(\frac{88\,\frac{ft}{s}}{60\,\frac{mi}{hr}}\right) = 22\,\frac{ft}{s}$

and U = wind speed relative to bike.

Thus,

$$\mathcal{P} = \left(22\,\frac{ft}{s}\right)(0.88)\left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)U^2(3.9\,ft^2) = 0.0898\,U^2\ \frac{ft\cdot lb}{s} \quad (1)$$

with $U \sim \frac{ft}{s}$

a) With a 20 mph headwind, $U = (15+20)\frac{mi}{hr}\left(\frac{88\,\frac{ft}{s}}{60\,\frac{mi}{hr}}\right) = 51.3\,\frac{ft}{s}$

Thus,

$$\mathcal{P}_a = 0.0898\,(51.3)^2 = 236\ \frac{ft\cdot lb}{s}$$

b) With still air, $U = 15\,mph = 22\,\frac{ft}{s}$

Thus,

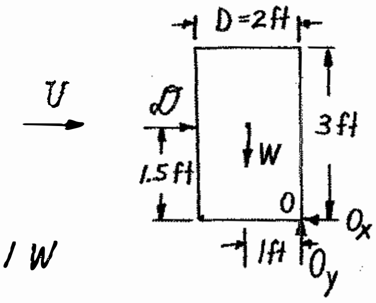
$$\mathcal{P}_b = 0.0898\,(22)^2 = 43.5\ \frac{ft\cdot lb}{s}$$

Hence, need an additional power of $\mathcal{P}_a - \mathcal{P}_b = (236 - 43.5)\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$

$= \underline{\underline{0.350\,hp}}$

9.65

9.65 Estimate the wind velocity necessary to knock over a 10-lb garbage can that is 3 ft tall and 2 ft in diameter. List your assumptions.



If the can is about to tip around corner O, then $\sum M_0 = 0$, or $1.5\mathscr{D} = 1\,W$

or $1.5\, C_D \frac{1}{2}\rho U^2 A = W$ A typical value of C_D for a cylinder is $C_D \approx 1$ (see Fig. 9.21)

Thus,

$(1.5\,ft)(1)(\frac{1}{2})(0.00238\,\frac{slugs}{ft^3})\,U^2(2\,ft)(3\,ft) = 10\,ft\cdot lb$, where $U \sim \frac{ft}{s}$

or $\underline{\underline{U = 30.6\,\frac{ft}{s}}}$

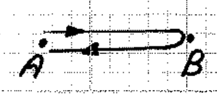
9.66

9.66 On a day without any wind, your car consumes x gallons of gasoline when you drive at a constant speed, U, from point A to point B and back to point A. Assume that you repeat the journey, driving at the same speed, on another day when there is a steady wind blowing from B to A. Would you expect your fuel consumption to be less than, equal to, or greater than x gallons for this windy round-trip? Support your answer with appropriate analysis.

Trip with the larger power lost due to aerodynamic drag will use the most gas. Let $(\ )_1$ mean "no wind" and $(\ )_2$ mean "wind".

(1) No wind:

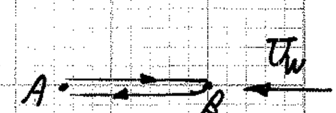
$$\mathcal{D}_1 = C_D \tfrac{1}{2}\rho U^2 A \quad \text{for both } A \to B \text{ and } B \to A$$



Thus,

$$\mathcal{P}_1 = \text{power} = U\mathcal{D}_1 = \tfrac{1}{2}\rho U^3 C_D A$$

(2) Wind (U_w = wind speed; assume $U_w < U$):



$$\mathcal{D}_2 = C_D \tfrac{1}{2}\rho (U+U_w)^2 A \quad \text{for } A \to B$$

$$\mathcal{D}_2 = C_D \tfrac{1}{2}\rho (U-U_w)^2 A \quad \text{for } B \to A$$

Thus,

$$\mathcal{P}_2 = \tfrac{1}{2}\rho (U+U_w)^2 U C_D A \quad \text{for } A \to B$$

$$\mathcal{P}_2 = \tfrac{1}{2}\rho (U-U_w)^2 U C_D A \quad \text{for } B \to A$$

Energy used $= \mathcal{P}t$, where t = time to go from $A \to B$ or $B \to A$

Thus,

$$E_1 = 2\left(\tfrac{1}{2}\rho U^3 C_D A\right)t \qquad \text{(Note: Factor of 2 for } A \to B + B \to A\text{)}$$

and

$$E_2 = \tfrac{1}{2}\rho (U+U_w)^2 U C_D A t + \tfrac{1}{2}\rho (U-U_w)^2 U C_D A t$$

Thus,

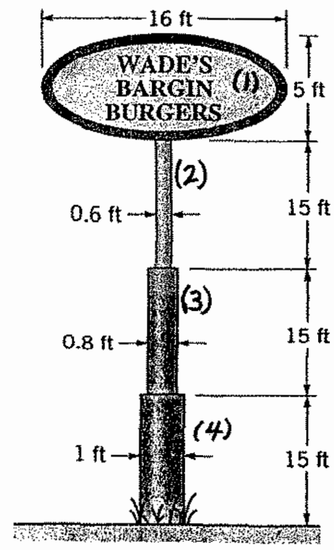
$$\frac{E_1}{E_2} = \frac{2U^3}{(U+U_w)^2 U + (U-U_w)^2 U} = \frac{2U^3}{2U^3 + 2U_w^2 U} = \frac{1}{1+(U_w/U)^2} < 1$$

Hence,

$$\frac{E_1}{E_2} < 1, \text{ i.e. } \underline{\underline{\text{more fuel needed when windy}}}$$

9.67

9.67 The structure shown in Fig. P9.67 consists of three cylindrical support posts to which an elliptical flat-plate sign is attached. Estimate the drag on the structure when a 50-mph wind blows against it.



■ FIGURE P9.67

For the composite body:

(1) $\mathscr{D} = \sum_{i=1}^{4} \mathscr{D}_i = \frac{1}{2}\rho U^2 \left[C_{D_1} A_1 + C_{D_2} A_2 + C_{D_3} A_3 + C_{D_4} A_4 \right]$

where if we assume the sign is an ellipse,

$A_1 = \frac{\pi}{4}(10\,ft)(5\,ft) = 39.3\,ft^2$, and the projected areas of the cylinders are

$A_2 = 0.6\,ft\,(15\,ft) = 9.00\,ft^2$

$A_3 = 0.8\,ft\,(15\,ft) = 12.0\,ft^2$ and

$A_4 = 1\,ft\,(15\,ft) = 15.0\,ft^2$

From Fig. 9.20, for a thin disc $C_{D_1} = 1.1$

For the cylindrical post, obtain C_D from Fig. 9.15 as: $(U = 50\,mph = 73.3\,\frac{ft}{s})$

$$Re_2 = \frac{U D_2}{\nu} = \frac{73.3\,\frac{ft}{s}(0.6\,ft)}{1.57\times10^{-4}\,\frac{ft^2}{s}} = 2.8\times10^5 \longrightarrow C_{D_2} = 0.6$$

Similarly,

$Re_3 = 3.7\times10^5 \longrightarrow C_{D_3} = 0.5$

$Re_4 = 4.7\times10^5 \longrightarrow C_{D_4} = 0.25$

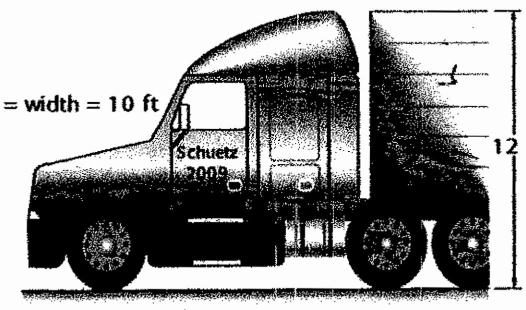
Thus, from Eq. (1):

$$\mathscr{D} = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left(73.3\,\frac{ft}{s}\right)^2\left[1.1(39.3\,ft^2) + 0.6(9.0\,ft^2) + 0.5(12\,ft^2) + 0.25(15\,ft^2)\right]$$

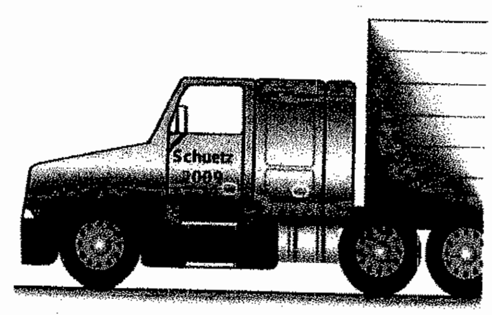
$= \underline{\underline{378\,lb}}$

9.68

9.68 As shown in Video V9.13 and Fig. P9.68, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from $C_D = 0.96$ to $C_D = 0.70$ corresponds to a reduction of how many horsepoer needed at a highway speed of 65 mph?



(a) $C_D = 0.70$



(b) $C_D = 0.96$

FIGURE P9.68

$\mathcal{P}$ = power = $\mathcal{D}U$ where

$\mathcal{D} = \frac{1}{2}\rho U^2 C_D A$

Thus, $\Delta\mathcal{P}$ = reduction in power

$= \mathcal{P}_b - \mathcal{P}_a$

$= \frac{1}{2}\rho U^3 A\left[C_{D_b} - C_{D_a}\right]$

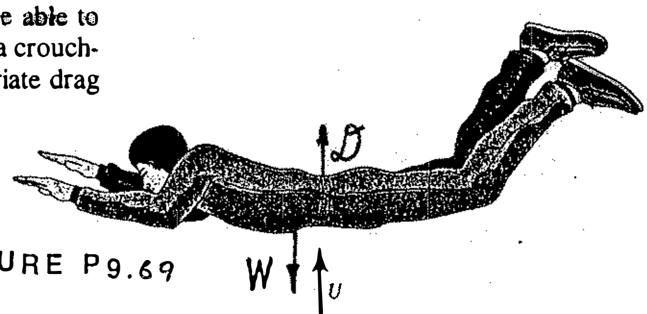
With $U = 65\ mph = 95.3\ fps$,

$$\Delta\mathcal{P} = \frac{1}{2}\left(0.00238\ \frac{slugs}{ft^3}\right)\left(95.3\ \frac{ft}{s}\right)^3(10\ ft)(12\ ft)\left[0.96 - 0.70\right]$$

$$= 32{,}100\ \frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}}\right) = \underline{\underline{58.4\ hp}}$$

9.69

9.69 As shown in Video V9.7 and Fig. P9.69 a vertical wind tunnel can be used for skydiving practice. Estimate the vertical wind speed needed if a 150-lb person is to be able to "float" motionless when the person (a) curls up as in a crouching position or (b) lies flat. See Fig. 9.30 for appropriate drag coefficient data.



■ FIGURE P9.69

For equilibrium conditions

$$W = \mathcal{D} = C_D \tfrac{1}{2} \rho U^2 A$$

Assume $W = 160\ lb$ and $C_D A = 9\ ft^2$ (see Fig. 9.30)

Thus,

$$160\ lb = \left(\tfrac{1}{2}\right)\left(0.00238 \tfrac{slugs}{ft^3}\right) U^2 (9\ ft^2) \quad \text{where } U \sim \tfrac{ft}{s}$$

or

$$U = \left(122 \tfrac{ft}{s}\right)\left(\frac{1\ mi}{5280\ ft}\right)\left(\frac{3600\ s}{1\ hr}\right) = \underline{\underline{83.2\ mph}}$$

Note: If the skydiver "curled up into a ball", then $C_D A \approx 2.5\ ft$ (see Fig. 9.30) and $U = 158\ mph$

9.70*

9.70 The helium-filled balloon shown in Fig. P9.70 is to be used as a wind speed indicator. The specific weight of the helium is $\gamma = 0.011 \text{ lb/ft}^3$, the weight of the balloon material is 0.20 lb, and the weight of the anchoring cable is negligible. Plot a graph of θ as a function of U for $1 \le U \le 50$ mph. Would this be an effective device over the range of U indicated? Explain.

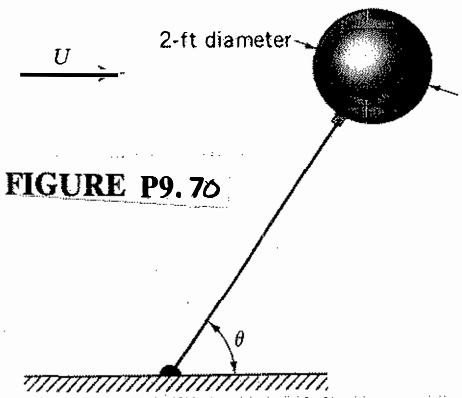


FIGURE P9.70

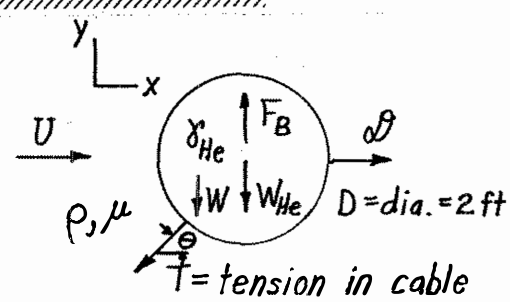
For the balloon to remain stationary

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Thus, $\mathcal{D} = T\cos\theta$ or $T = \dfrac{\mathcal{D}}{\cos\theta}$

and $F_B = W + T\sin\theta + W_{He}$

which combine to give



$$F_B = W + \mathcal{D}\tan\theta + W_{He} \tag{1}$$

But $W = 0.2 \text{ lb}$, $F_B = \rho g \forall = (7.65\times10^{-2}\,\frac{\text{lb}}{\text{ft}^3})\,\frac{4\pi}{3}(\frac{2}{2}\text{ft})^3 = 0.3204 \text{ lb}$

and $W_{He} = \gamma_{He}\forall = (0.011\,\frac{\text{lb}}{\text{ft}^3})\,\frac{4\pi}{3}(\frac{2}{2}\text{ft})^3 = 0.0461 \text{ lb}$

Thus, Eq. (1) becomes

$0.3204 \text{ lb} = 0.2 \text{ lb} + \mathcal{D}\tan\theta + 0.0461 \text{ lb}$

or

$\mathcal{D}\tan\theta = 0.0743 \text{ lb}$ Also, $\mathcal{D} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$= C_D U^2 (0.00238 \frac{\text{slugs}}{\text{ft}^3})\frac{\pi}{8}(2\text{ft})^2$

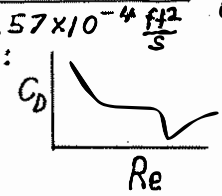
$= 0.00374\, C_D U^2 \text{ lb}$, where $U \sim \frac{\text{ft}}{\text{s}}$

Hence,

$$0.00374\, C_D U^2 \tan\theta = 0.0743 \quad \text{or} \quad \tan\theta = \frac{19.9}{C_D U^2} \tag{2}$$

$$\text{Also, } Re = \frac{UD}{\nu} = \frac{(2\text{ ft})\,U}{1.57\times10^{-4}\,\frac{\text{ft}^2}{\text{s}}} \quad \text{or} \quad Re = 1.27\times10^{4}\,U \tag{3}$$

and from Fig. 9.21:



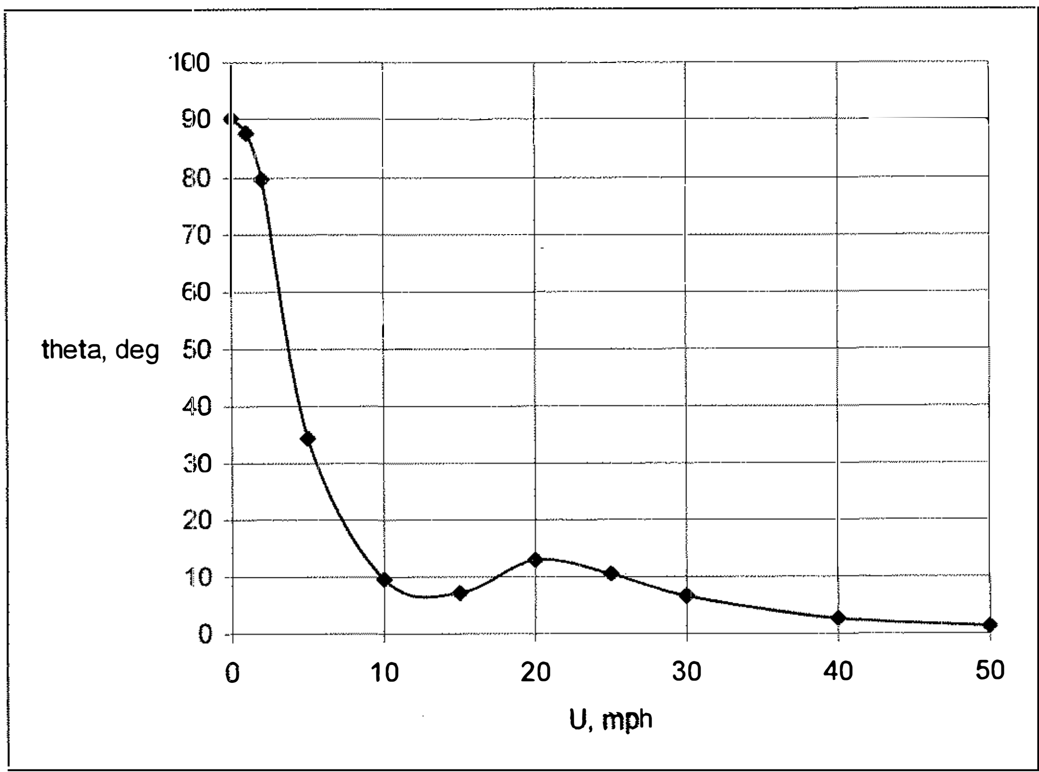
(4)

Thus, select various $1 \text{ mph} \le U \le 50 \text{ mph}$ (i.e., $1.47 \frac{\text{ft}}{\text{s}} \le U \le 73.3 \frac{\text{ft}}{\text{s}}$) and use Eqs. (2), (3), (4) to obtain θ. Plotted results are shown below.

(con't)

9.70* (con't)

U, mph	Re	CD	Θ, deg
0	0	---	90
1	12700	0.40	87.52
2	25400	0.42	79.71
5	63500	0.54	34.42
10	127000	0.55	9.55
15	190500	0.33	7.10
20	254000	0.10	13.02
25	317500	0.08	10.48
30	381000	0.09	6.52
40	508000	0.12	2.76
50	635000	0.16	1.32



Note: Because of the sudden change in C_D when the boundary layer becomes turbulent (at about 15 mph), the θ vs U curve is highly non-linear. In fact, for some values of θ there is more than one possible value of U. It would not work well as a wind speed indicator in this range.

9.71

9.71 A 0.30-m-diameter cork ball ($SG = 0.21$) is tied to an object on the bottom of a river as is shown in Fig. P9.61. Estimate the speed of the river current. Neglect the weight of the cable and the drag on it.

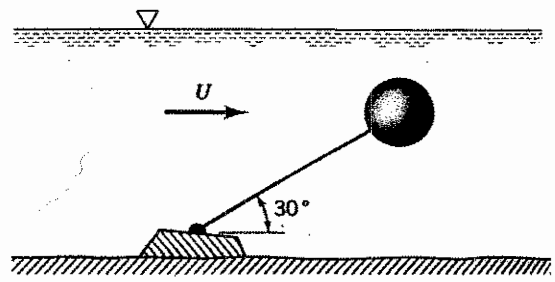
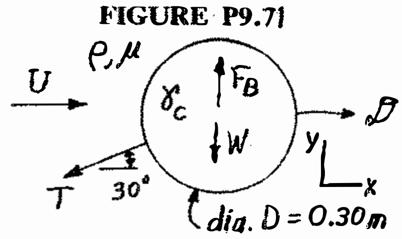


FIGURE P9.71



For the ball to remain stationary

$\sum F_x = 0$ and $\sum F_y = 0$

Thus, $\mathcal{D} = T\cos 30°$ or $T = \dfrac{\mathcal{D}}{\cos 30°}$

and

$$F_B = W + T\sin 30°$$

Hence, $F_B = W + \mathcal{D}\tan 30°$, where $F_B = \rho g \forall = (9.80\,\frac{kN}{m^3})\left(\frac{4\pi}{3}\left(\frac{0.30}{2}m\right)^3\right)$

$= 0.1385\ kN$

and

$W = \gamma_c \forall = \left(\frac{\gamma_c}{\gamma}\right)\gamma\forall = (SG)F_B$

$= 0.21\,(0.1385\,kN)$

$= 0.0291\,kN$

Thus,

$0.1385\,kN = 0.0291\,kN + \mathcal{D}\tan 30°$

or

$\mathcal{D} = 0.189\ kN$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A = C_D U^2 (\frac{1}{2})(999\,\frac{kg}{m^3})(\frac{\pi}{4}(0.3m)^2)$

$= 35.3\, C_D U^2\ N$, where $U \sim \frac{m}{s}$

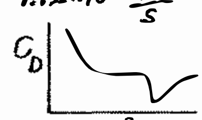
Hence

$35.3 C_D U^2 = 189$ or $C_D U^2 = 5.35$ (1)

Also, $Re = \dfrac{UD}{\nu} = \dfrac{(0.3m)\,U}{1.12\times10^{-6}\frac{m^2}{s}} = 2.68\times10^5\,U$ (2)

and

from Fig. 9.21 (3)



Trial and error solution for U: Assume C_D; calculate U from Eq.(1) and Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 3.27\,\frac{m}{s} \rightarrow Re = 8.76\times10^5 \rightarrow C_D = 0.15 \neq 0.5$

Assume $C_D = 0.15 \rightarrow U = 5.97\,\frac{m}{s} \rightarrow Re = 1.60\times10^6 \rightarrow C_D = 0.20 \neq 0.15$

Assume $C_D = 0.19 \rightarrow U = 5.31\,\frac{m}{s} \rightarrow Re = 1.42\times10^6 \rightarrow C_D = 0.19$ (checks)

Thus, $U = \underline{\underline{5.31\,\frac{m}{s}}}$

9.72

9.72 A shortwave radio antenna is constructed from circular tubing, as is illustrated in Fig. P9.72. Estimate the wind force on the antenna in a 100 km/hr wind.

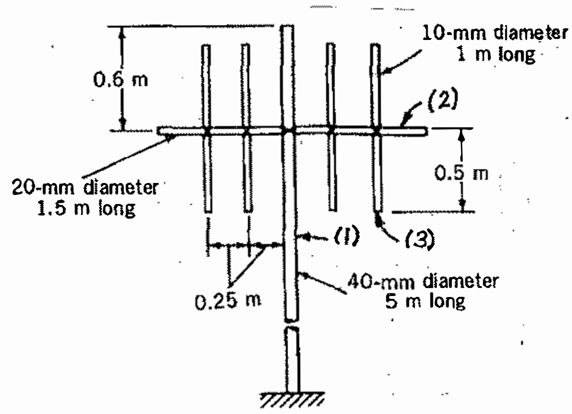


FIGURE P9.72

$$\mathscr{D} = \mathscr{D}_1 + \mathscr{D}_2 + 4\mathscr{D}_3$$

$$= \frac{1}{2}\rho U^2 \left[C_{D_1} A_1 + C_{D_2} A_2 + C_{D_3} A_3 \right]$$

where $U = 100 \frac{km}{hr} \left(\frac{10^3 m}{km}\right)\left(\frac{1h}{3600s}\right) = 27.8 \frac{m}{s}$

Obtain C_{D_i} from Fig. 9.23 for the given $Re_i = \frac{UD_i}{\nu}$.

Thus, $Re_1 = \frac{(27.8 \frac{m}{s})(0.04m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 7.62 \times 10^4 \longrightarrow C_{D_1} = 1.4$

$Re_2 = \frac{(27.8 \frac{m}{s})(0.02m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 3.81 \times 10^4 \longrightarrow C_{D_2} = 1.4$

and

$Re_3 = \frac{(27.8 \frac{m}{s})(0.01m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 1.90 \times 10^4 \longrightarrow C_{D_3} = 1.4 = C_{D_2} = C_{D_1}$

so that

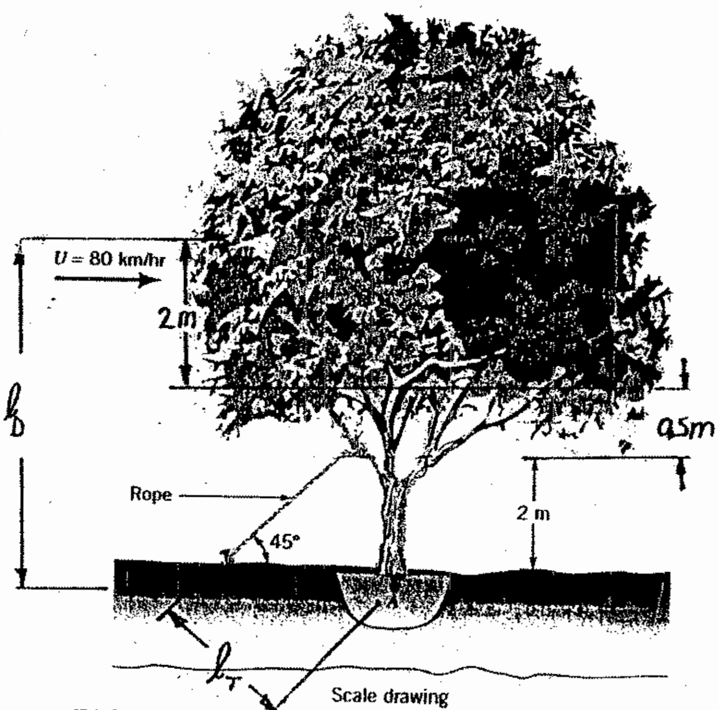
$$\mathscr{D} = \frac{1}{2}(1.23 \frac{kg}{m^3})(27.8 \frac{m}{s})^2 (1.4) \left[(5m)(0.04m) + (1.5m)(0.02m) + 4(1m)(0.01m) \right]$$

or

$$\mathscr{D} = \underline{\underline{180\ N}}$$

9.73

9.73 The large, newly planted tree shown in Fig. P9.73 is kept from tipping over in a wind by use of a rope as shown. It is assumed that the sandy soil cannot support any moment about the center of the soil ball, point A. Estimate the tension in the rope if the wind is 80 km/hr. See Fig. 9.30 for drag coefficient data.



■ FIGURE P9.73

$\Sigma M_A = 0$ where the moments are due to the drag, $\mathcal{D}$, and the tension in the rope, T.

Thus,

$\ell_D \mathcal{D} = \ell_T T$, where from the figure $\ell_D \approx (2+2.5+0.5)m = 5.0m$

and $\ell_T = \frac{3\,m}{\sqrt{2}} = 2.12\,m$

Hence,

$T = \frac{\ell_D \mathcal{D}}{\ell_T} = \frac{\ell_D \frac{1}{2}\rho U^2 A C_D}{\ell_T}$ where from the figure $A \approx \frac{\pi}{4}(5m)^2$

Thus, with $U = (80\frac{km}{hr})(\frac{1hr}{3600s})(1000\frac{m}{km}) = 22.2\frac{m}{s}$

and $C_D = 0.26$ (see Fig. 9.21) we obtain

$T = \frac{5.0m}{2.12\,m}(\frac{1}{2})(1.23\frac{kg}{m^3})(22.2\frac{m}{s})^2\frac{\pi}{4}(5m)^2(0.26) = 3650\ N = \underline{3.65kN}$

9.74

9.74 Estimate the wind force on your hand when you hold it out of your car window while driving 55 mph. Repeat your calculations if you were to hold your hand out of the window of an airplane flying 550 mph.

$\mathscr{D} = C_D \frac{1}{2} \rho U^2 A$, where $U = (55\,mph)\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 80.7\,\frac{ft}{s}$

Assume your hand is 4 in. by 6 in. in size and acts like a thin disc with $C_D \approx 1.1$ (see Fig. 9.29).

Thus,

$$\mathscr{D} = (1.1)\left(\tfrac{1}{2}\right)(0.00238)\left(80.7\,\tfrac{ft}{s}\right)^2\left(\tfrac{4}{12}\,ft\right)\left(\tfrac{6}{12}\,ft\right) = \underline{\underline{1.42\ lb}}$$

If your hand is normal to the the lift force is zero.

For $U = 550\,mph = 807\,\frac{ft}{s}$ (i.e., a 10 fold increase in U) the drag will increase by a factor of 100 (i.e., $\mathscr{D} \sim U^2$), or $\underline{\underline{\mathscr{D} = 142\ lb}}$

Note: We have assumed that C_D is not a function of U. That is, it is not a function of either $Re = \frac{UD}{\nu}$ or $Ma = \frac{U}{c}$.

9.76

9.76 A 2-mm-diameter meteor of specific gravity 2.9 has a speed of 6 km/s at an altitude of 50,000 m where the air density is 1.03×10^{-3} kg/m^3. If the drag coefficient at this large Mach number condition is 1.5, determine the deceleration of the meteor.

$$\mathcal{D} = ma \quad \text{where } m = \rho \forall = \rho \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = (2.9)\left(999 \frac{kg}{m^3}\right)\frac{4\pi}{3}\left(\frac{2 \times 10^{-3}\ m}{2}\right)^3$$

$$= 1.21 \times 10^{-5}\ kg$$

Also, $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$

$$= 1.5\left(\frac{1}{2}\right)\left(1.03 \times 10^{-3} \frac{kg}{m^3}\right)\left(6 \times 10^3 \frac{m}{s}\right)^2 \frac{\pi}{4}\left(2 \times 10^{-3} m\right)^2 = 8.74 \times 10^{-2} N$$

Thus,

$$a = \frac{\mathcal{D}}{m} = \frac{8.74 \times 10^{-2} N}{1.21 \times 10^{-5} kg} = 7220 \frac{m}{s^2}$$

9.77

9.77 Air flows past two equal sized spheres (one rough, one smooth) that are attached to the arm of a balance as is indicated in Fig. P9.77. With $U = 0$ the beam is balanced. What is the minimum air velocity for which the balance arm will rotate clockwise?

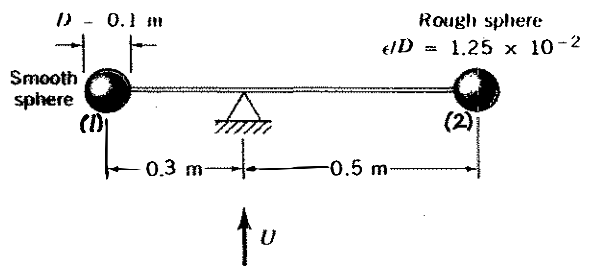


FIGURE P9.77

For clockwise rotation to start, $\Sigma M_o < 0$

That is $0.3\,\mathcal{D}_1 \geq 0.5\,\mathcal{D}_2$, where $\mathcal{D}_1 = C_{D_1}\frac{1}{2}\rho U_1^2 A_1$ and

$$\mathcal{D}_2 = C_{D_2}\frac{1}{2}\rho U_2^2 A_2$$

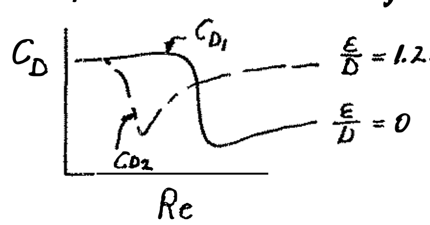
Thus,

$$0.3\,C_{D_1}\frac{1}{2}\rho U_1^2 A_1 = 0.5\,C_{D_2}\frac{1}{2}\rho U_2^2 A_2$$, or since $U_1 = U_2$ and $A_1 = A_2$

this gives

$$C_{D_2} = 0.6\,C_{D_1} \qquad (1)$$

Consider the curves in Fig. 9.25 with $\frac{\varepsilon}{D} = 0$ and $\frac{\varepsilon}{D} = 1.25 \times 10^{-2}$



Trial and error solution to find Re so that Eq. (1) is satisfied.

Assume $Re = 6\times10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.46$ or $\frac{C_{D2}}{C_{D_1}} = 0.92 \neq 0.6$

Assume $Re = 8\times10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.21$ or $\frac{C_{D2}}{C_{D1}} = 0.42 \neq 0.6$

Assume $Re = 7\times10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.33$ or $\frac{C_{D2}}{C_{D_1}} = 0.66 \approx 0.6$

Thus, $Re \approx 7.1\times10^4 = \frac{UD}{\nu} = \frac{(0.1\,m)\,U}{1.46\times10^{-5}\,\frac{m^2}{s}}$ or $U \approx \underline{\underline{10.4\,\frac{m}{s}}}$

9.78

9.78 A 2-in.-diameter sphere weighing 0.14 lb is suspended by the jet of air shown in Fig. P9.78 and Video V3.2. The drag coefficient for the sphere is 0.5. Determine the reading on the pressure gage if friction and gravity effects can be neglected for the flow between the pressure gage and the nozzle exit.

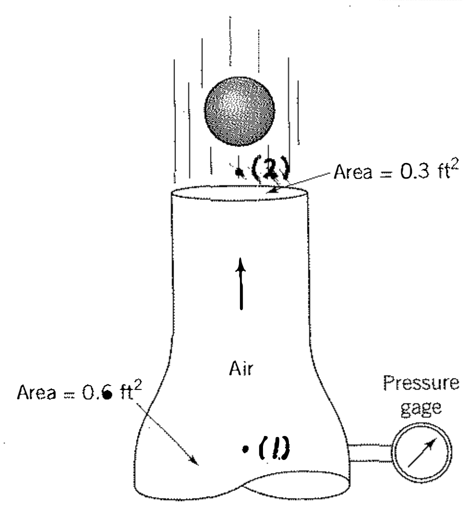
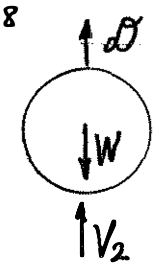


FIGURE P9.78



For equilibrium, $\mathscr{D} = W$ or

$$C_D \tfrac{1}{2}\rho V_2^2 A = W, \text{ where } A = \tfrac{\pi}{4}D^2$$

Thus,

$$V_2 = \left[\frac{2W}{C_D \rho \pi D^2/4}\right]^{1/2}$$

$$= \left[\frac{8\,(0.14\,lb)}{0.5(0.00238\,\frac{slugs}{ft^3})\pi(\frac{2}{12}\,ft)^2}\right]^{1/2} = 104\,\frac{ft}{s}$$

Also,

$$V_1 A_1 = V_2 A_2 \text{ or } V_1 = V_2 \frac{A_2}{A_1} = \left(104\,\frac{ft}{s}\right)\frac{0.3\,ft^2}{0.6\,ft^2} = 52.0\,\frac{ft}{s}$$

and

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2 \text{ where } p_2 = 0$$

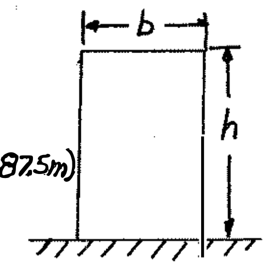
Thus,

$$p_1 = \tfrac{1}{2}\rho\left[V_2^2 - V_1^2\right] = \tfrac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left[\left(104\,\frac{ft}{s}\right)^2 - \left(52.0\,\frac{ft}{s}\right)^2\right]$$

$$= \underline{\underline{9.65\,\frac{lb}{ft^2}}}$$

9.79

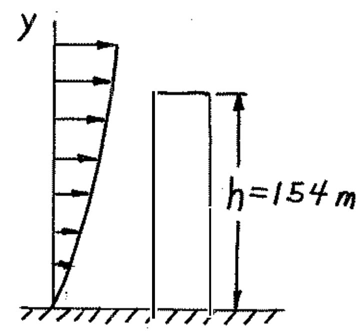
9.79 The United Nations Building in New York is approximately 87.5-m wide and 154-m tall. **(a)** Determine the drag on this building if the drag coefficient is 1.3 and the wind speed is a uniform 20 m/s. **(b)** Repeat your calculations if the velocity profile against the building is a typical profile for an urban area (see Problem 9.22) and the wind speed halfway up the building is 20 m/s.



(a) $\mathscr{D} = C_D \frac{1}{2}\rho U^2 A = 1.3\left(\frac{1}{2}\right)(1.23\ \frac{kg}{m^3})(20\frac{m}{s})^2(154m)(87.5m)$

or

$\mathscr{D} = 4.31 \times 10^6\ N = \underline{\underline{4.31\ MN}}$



(b) For an urban area, $u = C\,y^{0.4}$

Thus, with $u = 20\frac{m}{s}$ at $y = \frac{h}{2} = 77m$

we obtain

$C = \frac{20}{77^{0.4}} = 3.52$, or $u = 3.52\ y^{0.4}$ with $u \sim \frac{m}{s}$, $y \sim m$

The total drag is

$$\mathscr{D} = \int d\mathscr{D} = \int C_D \frac{1}{2}\rho u^2\, dA = \frac{1}{2}\rho C_D \int_{y=0}^{y=154} (3.52\ y^{0.4})^2 (87.5)\, dy$$

or

$$\mathscr{D} = \frac{1}{2}(1.23)(1.3)(3.52)^2(87.5)\int_0^{154} y^{0.8}\,dy = 867\left(\frac{1}{1.8}\right)(154)^{1.8} = 4.17 \times 10^6\ N$$

Thus,

$\mathscr{D} = \underline{\underline{4.17\ MN}}$

9.80

9.80 A regulation football is 6.78 in. in diameter and weighs 0.91 lb. If its drag coefficient is $C_D = 0.2$, determine its deceleration if it has a speed of 20 ft/s at the top of its trajectory.

$$\mathcal{D} = ma \quad , \quad \text{where } m = \frac{W}{g} = \frac{0.91 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}} = 0.0283 \text{ slugs}$$

and

$$\mathcal{D} = C_D \frac{1}{2} \rho U^2 A = 0.2\left(\frac{1}{2}\right)\left(0.00238 \frac{\text{slugs}}{\text{ft}^3}\right)\left(20 \frac{\text{ft}}{\text{s}}\right)^2\left(\frac{\pi}{4}\left(\frac{6.78}{12}\text{ft}^2\right)\right) = 0.0239 \text{ lb}$$

Thus,

$$a = \frac{\mathcal{D}}{m} = \frac{0.0238 \text{ lb}}{0.0283 \text{ slugs}} = \underline{\underline{0.841 \frac{\text{ft}}{\text{s}^2}}}$$

9.81

9.81 An airplane tows a banner that is b = 0.8 m tall and ℓ = 25 m long at a speed of 150 km/hr. If the drag coefficient based on the area $b\ell$ is $C_D = 0.06$, estimate the power required to tow the banner. Compare the drag force on the banner with that on a rigid flat plate of the same size. Which has the larger drag force and why?

$\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ with $A = b\ell$.

Thus, with $C_D = 0.06$ and $U = (150\,\frac{km}{hr})(\frac{1\,hr}{3600\,s})(\frac{1000\,m}{1\,km}) = 41.7\,\frac{m}{s}$

this gives

$$\mathcal{P} = (0.06)(\tfrac{1}{2})(1.23\,\tfrac{kg}{m^3})(41.7\,\tfrac{m}{s})^3(0.8\,m)(25\,m) = 53.5\times10^3\,W = \underline{\underline{53.5\ kW}}$$

For a rigid flat plate

$\mathcal{P} = \mathcal{D}U = 2C_D\frac{1}{2}\rho U^3 b\ell$ (the factor of two is needed because the drag coefficient is based on the drag on one side of the plate)

With $Re_\ell = \frac{U\ell}{\nu} = \frac{(41.7\frac{m}{s})(25\,m)}{1.46\times10^{-5}\,\frac{m^2}{s}} = 7.14\times10^7$ we obtain from Fig. 9.15 a value of $C_D \approx 0.0025$ for a smooth plate.

Thus,

$$\mathcal{P} = 2(0.0025)(\tfrac{1}{2})(1.23\,\tfrac{kg}{m^3})(41.7\,\tfrac{m}{s})^3(0.8\,m)(25\,m) = 4.46\times10^3\,W = \underline{\underline{4.46\ kW}}$$

For the flat plate case the drag is relatively small because it is due entirely to shear (viscous) forces. Due to the "fluttering" of the banner, a good portion of its drag (and hence power) is a result of pressure forces. It is not as streamlined as a rigid flat plate.

9.83

9.83 The paint stirrer shown in Fig. P9.83 consists of two circular disks attached to the end of a thin rod that rotates at 80 rpm. The specific gravity of the paint is $SG = 1.1$ and its viscosity is $\mu = 2 \times 10^{-2}\ \text{lb}\cdot\text{s/ft}^2$. Estimate the power required to drive the mixer if the induced motion of the liquid is neglected.

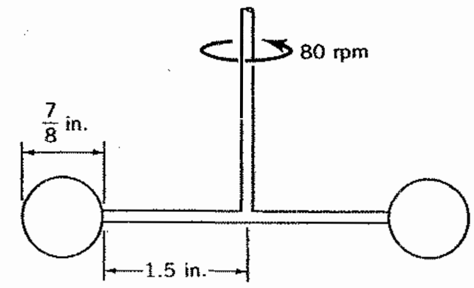


FIGURE P9.83

If we neglect the effects of the shaft and rod and consider the paint to be stationary, then

$\mathcal{M} = 2\mathcal{D}R$, where $\mathcal{M}$ = torque to rotate shaft
and $\mathcal{D}$ = drag on one disk $= C_D \frac{1}{2}\rho U^2 A$

Also, $U = \omega R$ and $\mathcal{P}$ = power to rotate shaft $= \mathcal{M}\omega$

Thus,

$$\mathcal{P} = 2\mathcal{D}R\,\omega = 2C_D\tfrac{1}{2}\rho(\omega R)^2\tfrac{\pi}{4}D^2 R\,\omega$$

or

$$\mathcal{P} = \tfrac{\pi}{4}C_D\rho\omega^3R^3D^2 = \tfrac{\pi}{4}C_D\rho U^3D^2 \quad \text{where } \rho = SG\,\rho_{H_2O} \qquad (1)$$

With $Re = \dfrac{\rho U D}{\mu} = \dfrac{SG\rho_{H_2O}UD}{\mu}$

where

$$U = \omega R = \left(80\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1.5+\frac{7}{16}}{12}\ \text{ft}\right) = 1.353\ \frac{\text{ft}}{\text{s}}$$

we have

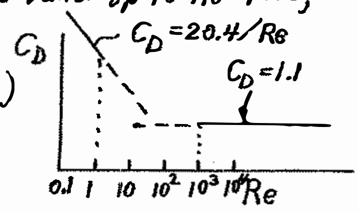
$$Re = \frac{(1.1)(1.94\frac{\text{slugs}}{\text{ft}^3})(1.353\frac{\text{ft}}{\text{s}})\left(\frac{7}{8(12)}\text{ft}\right)}{2\times10^{-2}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}} = 10.5$$

For a circular disk, $C_D = 1.1$ if $Re > 10^3$ (see Fig. 9.29)

while $\qquad C_D = \dfrac{20.4}{Re}$ if $Re < 1$ (see Table 9.4) $\qquad$ (2)

For this particular problem $1 < Re = 10.5 < 10^3$

Note: If the low Reynolds number result (Eq. (2)) is valid up to $Re = 10.5$,
then $C_D = \dfrac{20.4}{10.5} = 1.94$



To be on the conservative side (i.e., maximum power)
use the larger C_D — $C_D = 1.94$. From Eq. (1)

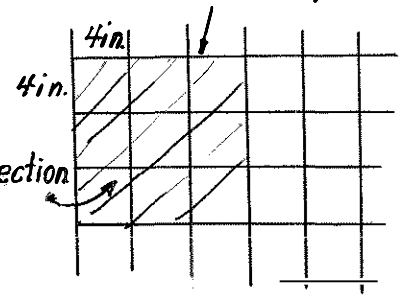
$$\mathcal{P} = \tfrac{\pi}{4}(1.94)(1.1)(1.94\tfrac{\text{slugs}}{\text{ft}^3})(1.353\tfrac{\text{ft}}{\text{s}})^3\left(\tfrac{7}{8(12)}\text{ft}\right)^2$$
$$= 0.0428\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

or

$$\mathcal{P} = \left(0.0428\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}\right)\left(\frac{1\ \text{hp}}{550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right) = \underline{\underline{7.78\times10^{-5}\ \text{hp}}}$$

9.85

9.85 A fishnet consists of 0.10-in.-diameter strings tied into squares 4 in. per side. Estimate the force needed to tow a 15 ft by 30 ft section of this net through seawater at 5 ft/s.



The net can be treated as one long 0.10-in.-diameter circular cylinder with

$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$, where $U = 5\frac{ft}{s}$. Each 1 ft^2 section of the net contains 6 feet of string (do not count the edges twice). Thus, the total string length is approximately $\ell = (6\frac{ft}{ft^2})(15\,ft)(30\,ft) = 2700\,ft$

Also, since $\rho = 1.99\,\frac{slugs}{ft^3}$ and $\nu = 1.26\times10^{-5}\,\frac{ft^2}{s}$ (see Table 1.5)

$$Re = \frac{UD}{\nu} = \frac{(5\frac{ft}{s})(\frac{0.10}{12}ft)}{1.26\times10^{-5}\frac{ft^2}{s}} = 3310$$

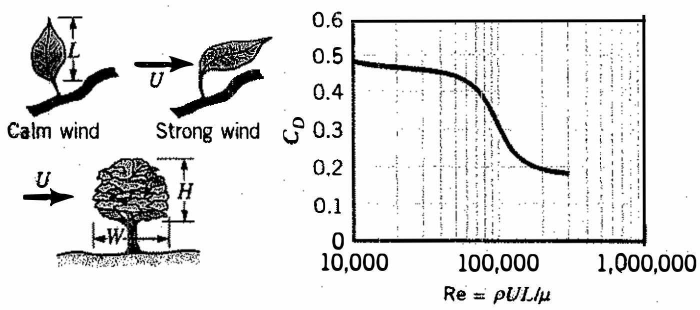
Hence, from Fig. 9.21 that $C_D = 1.1$

Thus,

$$\mathcal{D} = (1.1)(\tfrac{1}{2})(1.99\,\tfrac{slugs}{ft^3})(5\,\tfrac{ft}{s})^2(\tfrac{0.1}{12}\,ft)(2700\,ft) = \underline{\underline{616\ lb}}$$

9.86

9.86 As indicated in Fig. P9.86, the orientation of leaves on a tree is a function of the wind speed, with the tree becoming "more streamlined" as the wind increases. The resulting drag coefficient for the tree (based on the frontal area of the tree, HW) as a function of Reynolds number (based on the leaf length, L) is approximated as shown. Consider a tree with leaves of length $L = 0.3$ ft. What wind speed will produce a drag on the tree that is 6 times greater than the drag on the tree in a 15 ft/s wind?



■ FIGURE P9.86

$$\mathscr{D} = C_D \tfrac{1}{2}\rho U^2 A \quad \text{and} \quad Re = \frac{\rho U L}{\mu}$$

or

$$\mathscr{D} = C_D \tfrac{1}{2}(0.00238)U^2 HW = 0.00119\, HW C_D U^2 \qquad (1)$$

and

$$Re = \frac{0.00238 \frac{slugs}{ft^3}\, U\,(0.3\ ft)}{3.74 \times 10^{-7}\ lb\cdot s/ft^2} = 1909\, U, \quad \text{where } U \sim ft/s \qquad (2)$$

Thus, with $U = 15$ ft/s, $Re = 1909(15) = 28,600$ so that from Fig. P9.84,

$C_D = 0.46$ so

$$\mathscr{D}_{15} = 0.00119\, HW\,(0.46)(15)^2 = 0.123\, HW$$

For the drag 6 time as great, $\mathscr{D} = 6\mathscr{D}_{15} = 6(0.123\, HW) = 0.738\, HW$ (3)

Thus, from Eqs. (1) and (3):

$$0.738\, HW = 0.00119\, HW\, C_D U^2$$

or

$$C_D U^2 = 621 \qquad (4)$$

Trial and error solution:

Assume $C_D = 0.3$ so that from Eq. (4), $U = \sqrt{\frac{621}{0.3}} = 45.5$ ft/s and from Eq. (2)

$Re = 1909(45.5) = 86,900$. Thus, from Fig. P9.84, $C_D = 0.33 \neq 0.3$, the assumed value.

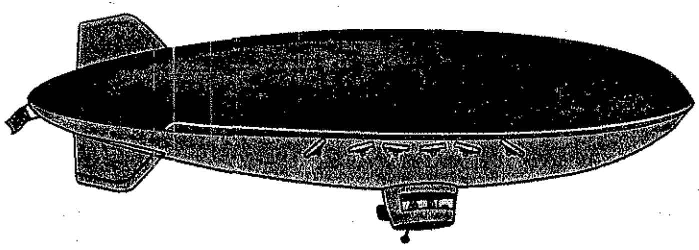
Try again. Assume $C_D = 0.33 \rightarrow U = 43.4$ ft/s $\rightarrow Re = 82,900 \rightarrow C_D = 0.36 \neq 0.33$

Try $C_D = 0.36 \rightarrow U = 41.5$ ft/s $\rightarrow Re = 79,300 \rightarrow C_D = 0.36$

Thus, $\underline{\underline{U = 41.5 \text{ ft/s}}}$

9.87

9.87 The blimp shown in Fig. P9.87 is used at various athletic events. It is 128 ft long and has a maximum diameter of 33 ft. If its drag coefficient (based on the frontal area) is 0.060, estimate the power required to propel it (a) at its 35-mph cruising speed, or (b) at its maximum 55-mph speed.



■ FIGURE P9.87

$\mathcal{P} = \mathcal{D}U$ where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$

Thus, with

$\mathcal{D} = 0.060\left(\frac{1}{2}\right)\left(0.00238 \frac{slugs}{ft^3}\right) U^2 \frac{\pi}{4}(33\,ft)^2$

$= 0.0611\, U^2\ lb$, where $U \sim ft/s$

(a) Thus, with $U = 35 \frac{mi}{hr}\left(\frac{5280\,ft/mi}{3600\,s/hr}\right) = 51.3\,ft/s$,

$\mathcal{D} = 0.0611(51.3)^2 = 161\,lb$

so that

$\mathcal{P} = \mathcal{D}U = 161\,lb\,(51.3\,ft/s)\left(\frac{1\,hp}{550\,ft\cdot lb/s}\right) = \underline{\underline{15.0\,hp}}$

(b) Similarly, with $U = 55\,mph = 80.7\,ft/s$,

$\mathcal{D} = 0.0611\,(80.7)^2 = 398\,lb$

so that

$\mathcal{P} = \mathcal{D}U = 398\,lb\,(80.7\,\frac{ft}{s})\left(\frac{1\,hp}{550\,ft\cdot lb/s}\right) = \underline{\underline{58.4\,hp}}$

9.88

9.88 Show that for level flight at a given speed, the power required to overcome aerodynamic drag decreases as the altitude increases. Assume that the drag coefficient remains constant. This is one reason why airlines fly at high altitudes.

For level flight $\mathcal{L} = W$, where W = airplane weight = constant
and $\mathcal{L} = C_L \frac{1}{2}\rho V^2 A$

If V is to remain constant, then C_L must increase as ρ decreases (i.e., altitude increases).

Also, $\mathcal{P} = \mathcal{D} V$, where $\mathcal{D} = C_D \frac{1}{2}\rho V^2 A$
or

$\mathcal{P} = C_D \frac{1}{2}\rho V^3 A$. For constant V, C_D, and A, the power decreases as altitude increases (ρ decreases).

9.89

9.89 (See Fluids in the News article "Dimpled baseball bats," Section 9.3.3.) How fast must a 3.5-in.-diameter, dimpled baseball bat move through the air in order to take advantage of drag reduction produced by the dimples on the bat. Although there are differences, assume the bat (a cylinder) acts the same as a golf ball in terms of how the dimples affect the transition from a laminar to a turbulent boundary layer.

From Fig. 9.25, for a golf ball the dimples reduce drag for $Re = \frac{\rho U D}{\mu} \gtrsim 4 \times 10^4$
Thus, assume $Re = 4 \times 10^4$ for the bat so that

$$\frac{\rho U D}{\mu} = 4 \times 10^4$$

or

$$\frac{\left(0.00238 \frac{slugs}{ft^3}\right) U \left(\frac{3.5}{12} ft\right)}{\left(3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)} = 4 \times 10^4$$

Thus,

$$U = 21.6 \frac{ft}{s}$$

9.90

9.90 (See Fluids in the News article "At 10,240 mpg it doesn't cost much to 'fill 'er up,'" Section 9.3.3.) (a) Determine the power it takes to overcome aerodynamic drag on a small (6 ft^2 cross section), streamlined ($C_D = 0.12$) vehicle traveling 15 mph. (b) Compare the power calculated in part (a) with that for a large (36 ft^2 cross-sectional area), nonstreamlined ($C_D = 0.48$) SUV traveling 65 mph on the interstate.

$\mathcal{P}$ = power $= U\mathcal{D}$, where $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$

so that

$$\mathcal{P} = C_D \frac{1}{2} \rho U^3 A$$

$$(a)\ \mathcal{P} = 0.12\left(\frac{1}{2}\right)\left(0.00238\ \frac{slugs}{ft^3}\right)\left[\left(15\ \frac{mi}{hr}\right)\left(\frac{5280\ ft/mi}{3600\ s/hr}\right)\right]^3 (6\ ft^2)$$

$$= 9.12\ \frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ ft\cdot lb/s}\right) = \underline{\underline{0.0166\ hp}}$$

$$(b)\ \mathcal{P} = 0.48\left(\frac{1}{2}\right)\left(0.00238\ \frac{slugs}{ft^3}\right)\left[\left(65\ \frac{mi}{hr}\right)\left(\frac{5280\ ft/mi}{3600\ s/hr}\right)\right]^3 (36\ ft^2)$$

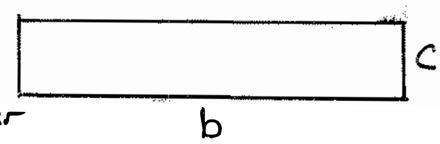
$$= 17{,}800\ \frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ ft\cdot lb/s}\right) = \underline{\underline{32.4\ hp}}$$

9.92

9.92 A rectangular wing with an aspect ratio of 6 is to generate 1000 lb of lift when it flies at a speed of 200 ft/s. Determine the length of the wing if its lift coefficient is 1.0.

Aspect ratio, $\mathcal{A} = b^2/A = 6$

$= b/c$ for rectangular wing



The lift coefficient is given by,

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho V^2 A}$$

$$\mathcal{L} = C_L \frac{1}{2}\rho V^2 A \quad \text{where } A = bc = 6c^2$$

$$\mathcal{L} = C_L \frac{1}{2}\rho V^2 (6c^2)$$

$$1000\ lb = 1.0\left(\tfrac{1}{2}\right)(0.00238\ slugs/ft^3)(200\ ft/s)^2(6c^2)$$

$$6c^2 = 21.0$$

$$c = 1.87\ ft$$

$$b = 6(c) = 6(1.87\ ft)$$

$$\underline{\underline{b = 11.2\ ft}}$$

9.94

9.94 A Piper Cub airplane has a gross weight of 1750 lb, a cruising speed of 115 mph, and a wing area of 179 ft^2. Determine the lift coefficient of this airplane for these conditions.

For equilibrium $\mathcal{L} = W = 1750\ lb$, where $\mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, with $U = (115\ mph)\dfrac{(88\ \frac{ft}{s})}{(60\ mph)} = 169\ \frac{ft}{s}$

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{1750\ lb}{\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(169\ \frac{ft}{s})^2(179\ ft^2)} = \underline{\underline{0.288}}$$

9.95

9.95 A light aircraft with a wing area of 200 ft^2 and a weight of 2000 lb has a lift coefficient of 0.40 and a drag coefficient of 0.05. Determine the power required to maintain level flight.

For equilibrium $\mathcal{L} = W = 2000 lb = C_L \frac{1}{2}\rho U^2 A$

or

$$2000\ lb = (0.40)\tfrac{1}{2}(0.00238\ \tfrac{slugs}{ft^3})\,U^2(200\ ft^2)$$

Hence,

$$U = 145\ \tfrac{ft}{s}$$

Also, $\mathcal{P}$ = power $= \mathcal{D}U$, where

$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A = (0.05)\tfrac{1}{2}(0.00238\ \tfrac{slugs}{ft^3})(145\ \tfrac{ft}{s})^2(200 ft^2) = 250\ lb$$

Note: This value of $\mathcal{D}$ could be obtained from

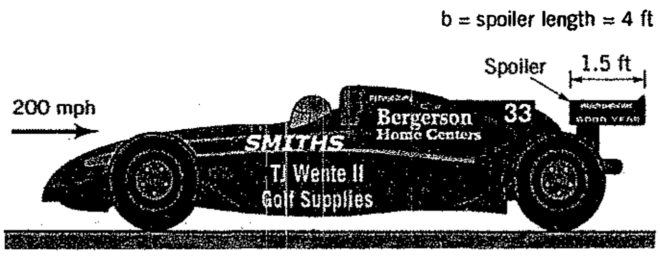
$$\frac{W}{\mathcal{D}} = \frac{\mathcal{L}}{\mathcal{D}} = \frac{C_L}{C_D} = \frac{0.40}{0.05} = 8, \text{ or } \mathcal{D} = \frac{W}{8} = \frac{2000 lb}{8} = 250\ lb$$

Thus,

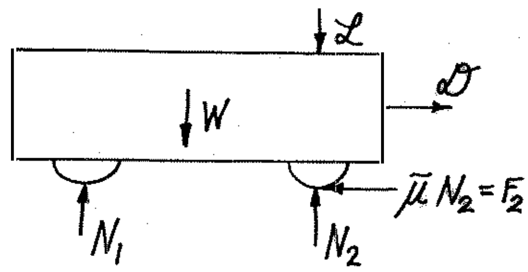
$$\mathcal{P} = 250 lb(145\ \tfrac{ft}{s}) = 3.63\times10^4\ \tfrac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}}\right) = \underline{\underline{65.9\ hp}}$$

9.96

9.96 As shown in Video V9.19 and Fig. P9.96, a spoiler is used on race cars to produce a negative lift, thereby giving a better tractive force. The lift coefficient for the airfoil shown is $C_L = 1.1$, and the coefficient of friction between the wheels and the pavement is 0.6. At a speed of 200 mph, by how much would use of the spoiler increase the maximum tractive force that could be generated between the wheels and ground? Assume the air speed past the spoiler equals the car speed and that the airfoil acts directly over the drive wheels.



■ FIGURE P9.96



Tractive force $= F_2 = \tilde{\mu} N_2$

where $\tilde{\mu}$ = coefficient of friction = 0.6

Thus,

$\Delta F_2 = \tilde{\mu}\, \Delta N_2 = \tilde{\mu}\, \mathscr{L}$, where ΔF_2 is the increase in tractive force due to the (downward) lift.

Hence, with $U = 200\ mph = 293\ ft/s$,

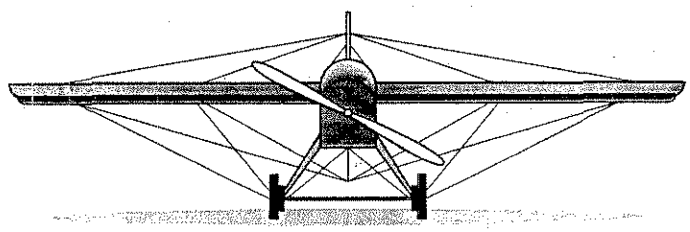
$\mathscr{L} = \frac{1}{2}\rho U^2 C_L A = \frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(293\ \frac{ft}{s})^2(1.1)(1.5ft)(4ft) = 674\ lb$,

and

$\Delta F_2 = 0.6\,(674\ lb) = \underline{\underline{405\ lb}}$

9.97

9.97 The wings of old airplanes are often strengthened by the use of wires that provided cross-bracing as shown in Fig. P9.97. If the drag coefficient for the wings was 0.020 (based on the planform area), determine the ratio of the drag from the wire bracing to that from the wings.



Speed: 70 mph
Wing area: 148 ft^2
Wire: length = 160 ft
diameter = 0.05 in.

FIGURE P9.97

$\mathscr{D}_{wing} = \frac{1}{2}\rho U^2 C_{D_{wing}} A_{wing}$

and

$\mathscr{D}_{wire} = \frac{1}{2}\rho U^2 C_{D_{wire}} A_{wire}$ so that

$$\frac{\mathscr{D}_{wire}}{\mathscr{D}_{wing}} = \frac{C_{D_{wire}} A_{wire}}{C_{D_{wing}} A_{wing}}, \text{ where } A_{wing} = 148\text{ ft}^2,\ C_{D_{wing}} = 0.02$$

Also, $A_{wire} = \ell D = (160\text{ ft})\left(\frac{0.05}{12}\text{ ft}\right) = 0.667\text{ ft}^2$

and since

$$Re = \frac{UD}{\nu} = \frac{(70\text{ mph})\left(\frac{88\frac{\text{ft}}{\text{s}}}{60\text{ mph}}\right)\left(\frac{0.05}{12}\text{ ft}\right)}{1.57\times10^{-4}\frac{\text{ft}^2}{\text{s}}} = 2720.$$

From Fig. 9.21, with $Re = 2720$ we obtain $C_D = 1.0$

Hence,

$$\frac{\mathscr{D}_{wire}}{\mathscr{D}_{wing}} = \frac{(1.0)(0.667\text{ ft}^2)}{(0.02)(148\text{ ft}^2)} = 0.225, \text{ or } \underline{\underline{22.5\%}}$$

9.98

9.98 A wing generates a lift $\mathscr{L}$ when moving through sea-level air with a velocity U. How fast must the wing move through the air at an altitude of 10,000 m with the same lift coefficient if it is to generate the same lift?

$\mathscr{L} = C_L \frac{1}{2} \rho U^2 A$ so with $\mathscr{L}$, C_L, and A constant

$$(\rho U^2)_{sea\ level} = (\rho U^2)_{10,000\,m}$$

Hence,

$$U_{10,000\,m} = \left(\frac{\rho_{sea\ level}}{\rho_{10,000\,m}}\right)^{1/2} U_{sea\ level} = \left(\frac{1.23\ \frac{kg}{m^3}}{0.414\ \frac{kg}{m^3}}\right)^{1/2} U_{sea\ level}$$

or

$$\underline{\underline{U_{10,000\,m} = 1.72\ U_{sea\ level}}}$$

*9.99

9.99 Air blows over the flat-bottomed, two-dimensional object shown in Fig. P9.91. The shape of the object, $y = y(x)$, and the fluid speed along the surface, $u = u(x)$, are given in the table. Determine the lift coefficient for this object.

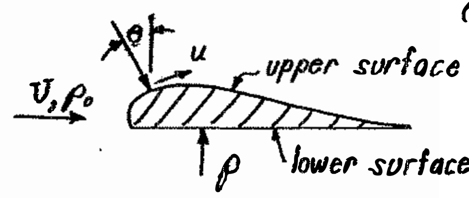
x (% c)	y (% c)	u/U
0	0	0
2.5	3.72	0.971
5.0	5.30	1.232
7.5	6.48	1.273
10	7.43	1.271
20	9.92	1.276
30	11.14	1.295
40	11.49	1.307
50	10.45	1.308
60	9.11	1.195
70	6.46	1.065
80	3.62	0.945
90	1.26	0.856
100	0	0.807

If viscous effects are negligible, then

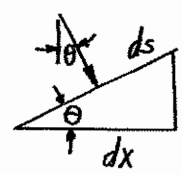
$$\mathcal{L} = \int_{lower} p \cos\theta \, dA - \int_{upper} p \cos\theta \, dA \qquad (1)$$

where from the Bernoulli equation

$$p + \tfrac{1}{2}\rho u^2 = p_0 + \tfrac{1}{2}\rho U^2 \qquad (2)$$



The effect of atmospheric pressure, p_0, drops out when the integration over the entire surface is performed.
With $\theta = 0$ on the lower surface and with $\cos\theta \, dA = \cos\theta \, (\ell \, ds) = \ell \, dx$, where ℓ = wing span, Eqs.(1) and (2) give



$$\mathcal{L} = \int_{lower} \left[p_0 + \tfrac{1}{2}\rho(U^2 - u^2)\right]\ell \, dx - \int_{upper}\left[p_0 + \tfrac{1}{2}\rho(U^2 - u^2)\right]\ell \, dx$$

or, since $u = U$ on the lower surface

$$\mathcal{L} = -\tfrac{1}{2}\rho\ell\int_{x=0}^{x=c}(U^2 - u^2)\,dx = \tfrac{1}{2}\rho U^2 \ell c \int_{x'=0}^{x'=1}\left[\left(\frac{u}{U}\right)^2 - 1\right]dx' \text{, where } x' = \frac{x}{c} \qquad (3)$$

Thus, since

$$C_L = \frac{\mathcal{L}}{\tfrac{1}{2}\rho U^2 A} = \frac{\mathcal{L}}{\tfrac{1}{2}\rho U^2 \ell c}$$

it follows from Eq.(3) that

$$C_L = \int_{x'=0}^{x'=1}\left[\left(\frac{u}{U}\right)^2 - 1\right]dx'$$

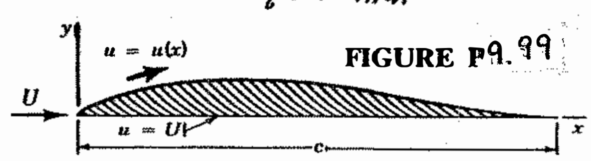


FIGURE P9.99

By using a standard numerical integration routine with the data given we obtain

$$\underline{\underline{C_L = 0.327}}$$

x'	$\left(\frac{u}{U}\right)^2 - 1$
0	-1.00
0.025	-0.0572
0.050	0.518
0.075	0.621
0.100	0.615
0.200	0.628
0.300	0.677
0.400	0.708
0.500	0.711
0.600	0.428
0.700	0.134
0.800	-0.107
0.900	-0.267
1.000	-0.349

9.101

9.101 A Boeing 747 aircraft weighing 580,000 lb when loaded with fuel and 100 passengers takes off with an airspeed of 140 mph. With the same configuration (i.e., angle of attack, flap settings, etc.) what is its takeoff speed if it is loaded with 372 passengers. Assume each passenger with luggage weighs 200 lb.

For steady flight $\mathcal{L} = C_L \frac{1}{2}\rho U^2 A = W$ (1)

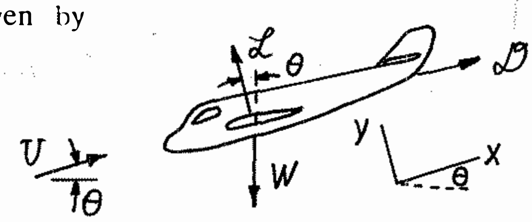
Let $(\ )_{100}$ denote conditions with 100 passengers and $(\ )_{372}$ with 372 passengers. Thus, with $C_{L100} = C_{L372}$, $A_{100} = A_{372}$, and $\rho_{100} = \rho_{372}$ Eq. (1) gives

$$\frac{\mathcal{L}_{100}}{\mathcal{L}_{372}} = \frac{U_{100}^2}{U_{372}^2} \quad \text{or} \quad U_{372} = U_{100}\left\{\frac{[580{,}000 + (372-100)(200)]\,lb}{580{,}000\ lb}\right\}^{1/2}, \text{ with } U_{100} = 140\,mph$$

Thus, $U_{372} = \underline{\underline{146\ mph}}$

9.102

9.102 Show that for unpowered flight (for which the lift, drag, and weight forces are in equilibrium) the glide slope angle, θ, is given by $\tan\theta = C_D/C_L$.



For steady unpowered flight

$\Sigma F_x = 0$ gives $\mathcal{D} = W\sin\theta$

and

$\Sigma F_y = 0$ gives $\mathcal{L} = W\cos\theta$

Thus,

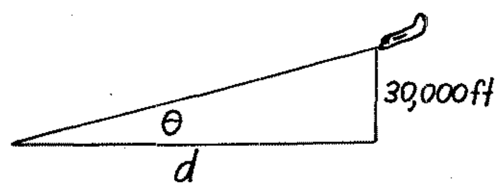
$$\frac{\mathcal{D}}{\mathcal{L}} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta, \text{ where } \frac{\mathcal{D}}{\mathcal{L}} = \frac{\frac{1}{2}\rho U^2 A C_D}{\frac{1}{2}\rho U^2 A C_L} = \frac{C_D}{C_L}$$

Hence, $\underline{\underline{\tan\theta = \frac{C_D}{C_L}}}$

9.103

9.103 If the lift coefficient for a Boeing 777 aircraft is 15 times greater than its drag coefficient, can it glide from an altitude of 30,000 ft to an airport 80 mi away if it loses power from its engines? Explain. (See Problem 9.102)

From Problem 9.102, $\tan\theta = \frac{C_D}{C_L} = \frac{1}{15}$



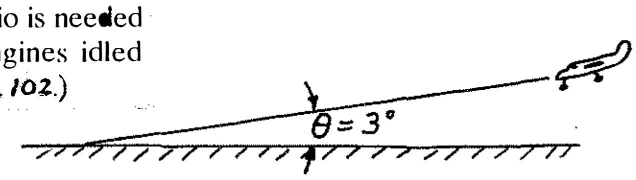
Hence,

$$\frac{30{,}000}{d} = \frac{1}{15}, \text{ or } d = 4.5\times10^5\,ft = 85.2\,mi$$

Hence, the plane can glide 80mi.

9.104

9.104 On its final approach to the airport an airplane flies on a flight path that is 3.0° relative to the horizontal. What lift-to-drag ratio is needed if the airplane is to land with its engines idled back to zero power? (See Problem 9.102.)



From Problem 9.102,

$\tan\theta = \frac{C_D}{C_L}$

or

$$\frac{C_D}{C_L} = \tan 3° = 0.0524 \qquad \frac{C_L}{C_D} = 19.1$$

9.105

9.105 Over the years there has been a dramatic increase in the flight speed (U) and altitude (h), weight ($\mathcal{W}$), and wing loading ($\mathcal{W}/A$ = weight divided by wing area) of aircraft. Use the data given in the table below to determine the lift coefficient for each of the aircraft listed.

Aircraft	Year	$\mathcal{W}$, lb	U, mph	$\mathcal{W}/A$, lb/ft^2	h, ft
Wright Flyer	1903	750	35	1.5	0
Douglas DC-3	1935	25,000	180	25.0	10,000
Douglas DC-6	1947	105,000	315	72.0	15,000
Boeing 747	1970	800,000	570	150.0	30,000

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{W}{\frac{1}{2}\rho U^2 A} = \frac{2}{\rho U^2}\left(\frac{W}{A}\right)$$

Thus,

	ρ, $slugs/ft^3$	U, ft/s	W/A, lb/ft^2	C_L
Wright Flyer	2.38×10^{-3}	51.3	1.5	0.480
DC-3	1.76×10^{-3}	264	25.0	0.409
DC-6	1.50×10^{-3}	462	72.0	0.451
747	8.91×10^{-4}	836	150	0.482

9.106

9.106 The landing speed of an airplane such as the Space Shuttle is dependent on the air density. (See **Video V9.1**.) By what percent must the landing speed be increased on a day when the temperature is 110 deg F compared to a day when it is 50 deg F? Assume the atmospheric pressure remains constant.

For equalibrium, lift = weight, or

$$\frac{1}{2}\rho U^2 C_L A = W$$

Thus, with constant W, C_L, and A,

$$(\rho U^2)_{T=110°} = (\rho U^2)_{T=50°} \quad \text{or}$$

$$U_{110°} = \left(\frac{\rho_{50}}{\rho_{110}}\right)^{\frac{1}{2}} U_{50°}$$

But $p = \rho R T$ so that $\dfrac{\rho_{50}}{\rho_{110}} = \dfrac{(p_{50}/RT_{50})}{(p_{110}/RT_{110})} = \dfrac{(460+110)}{(460+50)} = 1.1176$

Thus,

$$U_{110°} = \sqrt{1.1176}\, U_{50°} = 1.0572\, U_{50°} \quad \text{or a } \underline{5.72\% \text{ increase}}$$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

9.107

9.107 Commercial airliners normally cruise at relatively high altitudes (30,000 to 35,000 ft). Discuss how flying at this high altitude (rather than 10,000 ft, for example) can save fuel costs.

For level flight W = aircraft weight $= \mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, for given W, C_L, and A the dynamic pressure is constant, independent of altitude. That is

$$\left.\tfrac{1}{2}\rho U^2\right)_{10,000\,ft} = \left.\tfrac{1}{2}\rho U^2\right)_{30,000\,ft}, \text{ or } U_{30,000} = \left(\frac{\rho_{10,000}}{\rho_{30,000}}\right)^{\frac{1}{2}} U_{10,000}$$

Hence, $U_{30,000} > U_{10,000}$

Also, since the drag is $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ it follows that

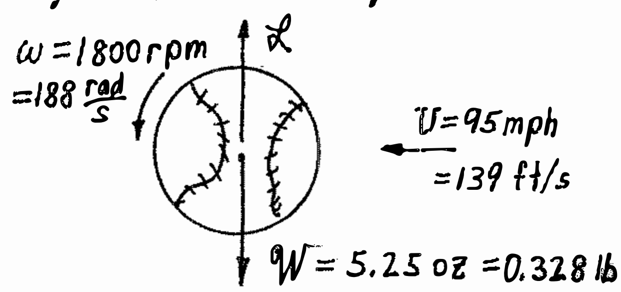
$$\mathcal{D}_{30,000} = \left.C_D \tfrac{1}{2}\rho U^2 A\right|_{30,000} = \left.C_D \tfrac{1}{2}\rho U^2 A\right)_{10,000} \text{ since } \tfrac{1}{2}\rho U^2_{30,000} = \tfrac{1}{2}\rho U^2_{10,000}$$

Hence, the aircrart can fly faster at high altituded with the same amount of drag ($\mathcal{D}_{30,000} = \mathcal{D}_{10,000}$)

9.109

9.109 For many years, hitters have claimed that some baseball pitchers have the ability to actually throw a rising fastball. Assuming that a top major leaguer pitcher can throw a 95-mph pitch and impart a 1800-rpm spin to the ball, is it possible for the ball to actually rise? Assume the baseball diameter is 2.9 in. and its weight is 5.25 oz.

If the lift produced on the spinning ball is greater than its weight the ball will rise.



$$\mathscr{L} = C_L \tfrac{1}{2}\rho U^2 A$$

where C_L is a function of $\frac{\omega D}{2U}$ as shown in Fig. 9.39.

Thus, with

$$\frac{\omega D}{2U} = \frac{(188\,\frac{rad}{s})(\frac{2.9}{12}\,ft)}{2\,(139\,ft/s)} = 0.163$$

$C_L \approx 0.04$

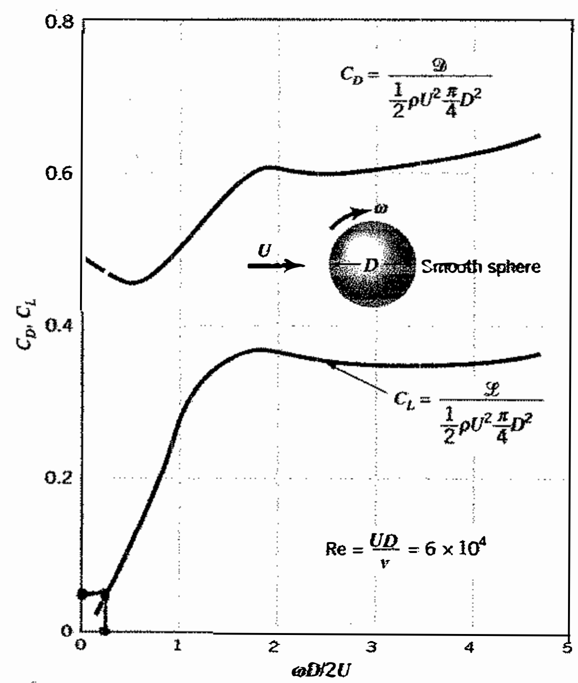
Hence, for the given conditions

$$\mathscr{L} = 0.04\left(\tfrac{1}{2}\right)\left(0.00238\,\tfrac{slugs}{ft^3}\right)\left(139\,\tfrac{ft}{s}\right)^2 \times \tfrac{\pi}{4}\left(\tfrac{2.9}{12}\,ft\right)^2 = 0.0422\,lb$$

so that

$$\mathscr{L} = 0.0422\,lb < \mathscr{W} = 0.328\,lb$$

The ball will not rise.

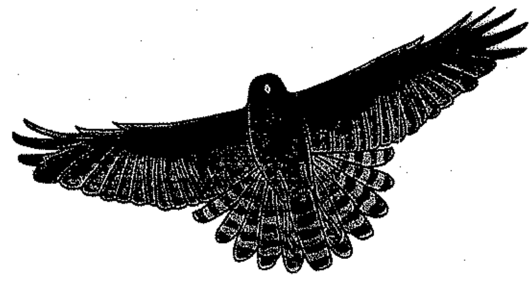


■ FIGURE 9.39 Lift and drag coefficients for a spinning smooth sphere (Ref. 23).

Note: The above result is based on smooth-sphere data. The results for a baseball (with its rough surface containing seams) will probably give a somewhat larger lift because for a given angular velocity it can "drag" more air along as it spins.

9.110

9.110 (See "**Learning from nature,**" Section 9.4.1.) As indicated in Fig. P9.110, birds can significantly alter their body shape and increase their planform area, A, by spreading their wing and tail feathers, thereby reducing their flight speed. If during landing the planform area is increased by 50% and the lift coefficient increased by 30% while all other parameters are held constant, by what percent is the flight speed reduced?



■ FIGURE P9.110

$$\mathcal{L} = C_L \tfrac{1}{2}\rho U^2 A$$

Let $(\ )_2$ denote landing conditions and $(\ )_1$ denote normal flight conditions.

Thus, with $\mathcal{L}_1 = \mathcal{L}_2$,

$$C_{L_1} \tfrac{1}{2}\rho U_1^2 A_1 = C_{L_2} \tfrac{1}{2}\rho U_2^2 A_2$$

or

$$U_2 = U_1 \sqrt{\frac{A_1}{A_2}} \sqrt{\frac{C_{L1}}{C_{L2}}} = U_1 \sqrt{\frac{A_1}{1.5A_1}} \sqrt{\frac{C_{L1}}{1.3C_{L_1}}}$$

or

$$U_2 = 0.716\, U_1$$

Hence,

$$\frac{U_2 - U_1}{U_1} = 0.716 - 1 = -0.284$$

i.e., a $\underline{\underline{28.4\,\%\text{ reduction}}}$ in flight speed

9.111

9.111 (See Fluids in the News article "Why winglets?," Section 9.4.2.) It is estimated that by installing appropriately designed winglets on a certain airplane the drag coefficient will be reduced by 5%. For the same engine thrust, by what percent will the aircraft speed be increased by use of the winglets?

Let $(\ )_1$ denote without winglets and $(\ )_2$ with winglets.
Thus, since drag equals thrust and $\text{thrust}_1 = \text{thrust}_2$, it follows that

$$\mathcal{D}_1 = \mathcal{D}_2$$

or

$$C_{D_1} \tfrac{1}{2} \rho U_1^2 A_1 = C_{D_2} \tfrac{1}{2} \rho U_2^2 A_2$$

so that with $A_1 = A_2$,

$$U_2 = U_1 \sqrt{\frac{C_{D1}}{C_{D2}}} = U_1 \sqrt{\frac{C_{D1}}{0.95\, C_{D_1}}} = 1.0260\, U_1$$

Thus, a 2.60% increase in speed is realized.

9.112

9.112 Boundary Layer on a Flat Plate

Objective: A boundary layer is formed on a flat plate when air blows past the plate. The thickness, δ, of the boundary layer increases with distance, x, from the leading edge of the plate. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.112, to measure the boundary layer thickness.

Equipment: Wind tunnel; flat plate; boundary layer mouse consisting of ten Pitot tubes positioned at various heights, y, above the flat plate; inclined multiple manometer; measuring calipers; barometer, thermometer.

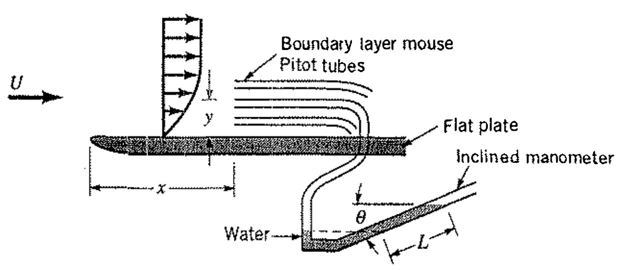
Experimental Procedure: Position the tips of the Pitot tubes of the boundary layer mouse a known distance, x, downstream from the leading edge of the plate. Use calipers to determine the distance, y, between each Pitot tube and the plate. Fasten the tubing from each Pitot tube to the inclined multiple manometer and determine the angle of inclination, θ, of the manometer board. Adjust the wind tunnel speed, U, to the desired value and record the manometer readings, L. Move the boundary layer mouse to a new distance, x, downstream from the leading edge of the plate and repeat the measurements. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For each distance, x, from the leading edge, use the manometer data to determine the air speed, u, as a function of distance, y, above the plate (see Eq. 3.13). That is, obtain $u = u(y)$ at various x locations. Note that both the wind tunnel test section and the open end of the manometer tubes are at atmospheric pressure.

Graph: Plot speed, u, as ordinates and distance from the plate, y, as abscissas for each location, x, tested.

Results: Use the $u = u(y)$ results to determine the approximate boundary layer thickness as a function of distance, $\delta = \delta(x)$. Plot a graph of boundary layer thickness as a function of distance from the leading edge. Note that the air flow within the wind tunnel is quite turbulent so that the measured boundary layer thickness is not expected to match the theoretical laminar boundary layer thickness given by the Blassius solution (see Eq. 9.15).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P9.112

(con't)

9.112 (con't)

Solution for Problem 9.112: Boundary Layer on a Flat Plate

θ, deg	H_{atm}, in. Hg	T, deg F	γ_{H2O}, lb/ft^3
25	29.09	80	62.4

y, in.	L, in.	u, ft/s	y, in.	L, in.	u, ft/s
Data for x = 7.75 in.			Data for x = 3.75 in.		
0.020	0.20	19.9	0.020	0.15	17.2
0.035	0.35	26.3	0.035	0.35	26.3
0.044	0.48	30.8	0.044	0.45	29.8
0.060	0.70	37.2	0.060	0.71	37.5
0.096	0.95	43.4	0.096	1.20	48.7
0.110	1.06	45.8	0.110	1.30	50.7
0.138	1.21	48.9	0.138	1.56	55.6
0.178	1.44	53.4	0.178	1.77	59.2
0.230	1.70	58.0	0.230	1.95	62.1
0.270	1.85	60.5	0.270	2.00	62.9
Data for x = 5.75 in.			Data for x = 1.75 in.		
0.020	0.20	19.9	0.020	0.20	19.9
0.035	0.42	28.8	0.035	0.50	31.5
0.044	0.50	31.5	0.044	0.68	36.7
0.060	0.71	37.5	0.060	0.90	42.2
0.096	0.98	44.0	0.096	1.51	54.7
0.110	1.06	45.8	0.110	1.70	58.0
0.138	1.30	50.7	0.138	1.90	61.3
0.178	1.54	55.2	0.178	1.95	62.1
0.230	1.76	59.0	0.230	2.00	62.9
0.270	1.88	61.0	0.270	2.00	62.9

$\rho u^2/2 = \gamma_{H2O}$*L sinθ

where

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{H2O}$*H_{atm} = 847 lb/ft^3*(29.09/12 ft) = 2053 lb/ft^2

R = 1716 ft lb/slug deg R

T = 80 + 460 = 540 deg R

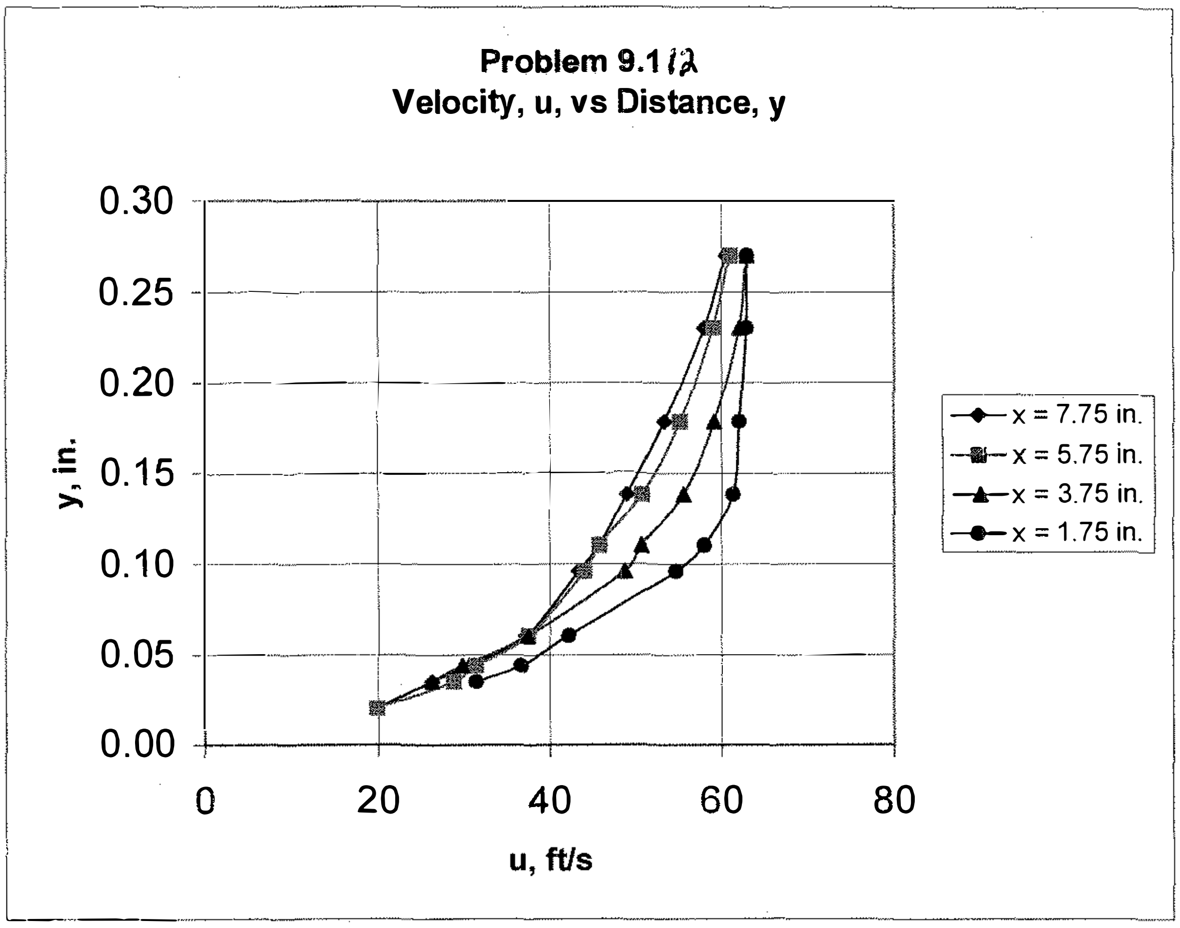
Thus, ρ = 0.00222 slug/ft^3

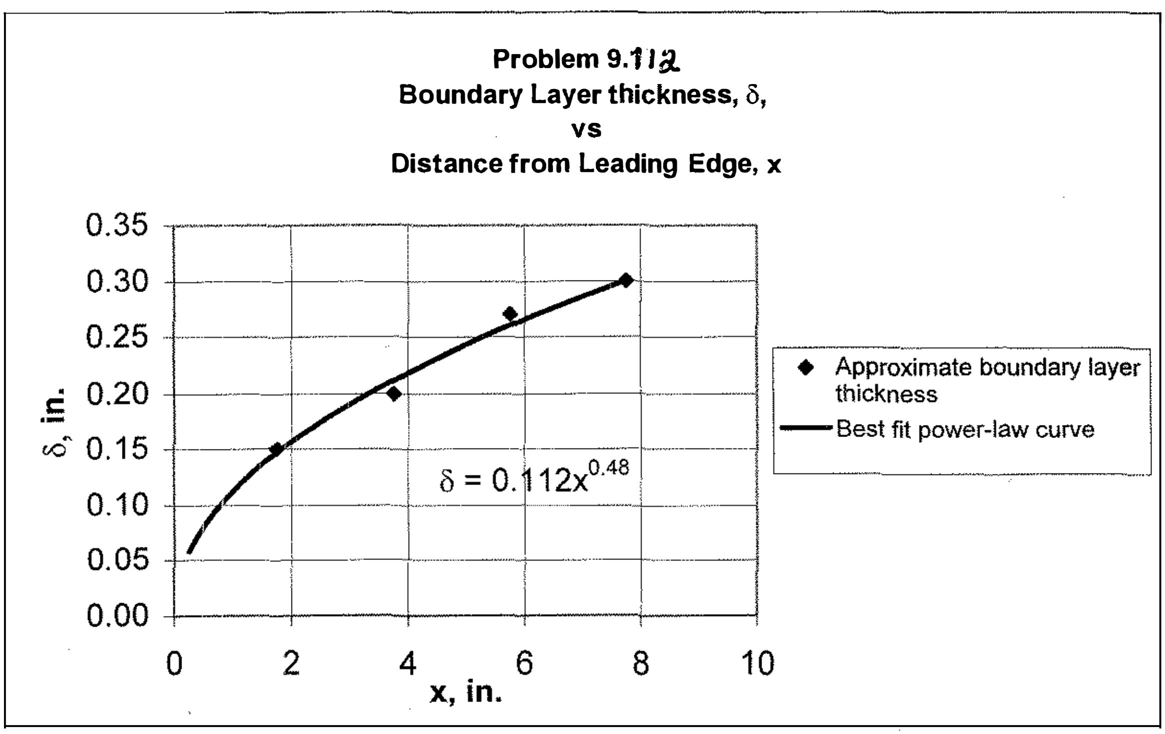
Approximate boundary layer thickness as obtained from the graph:

x, in.	δ, in.
1.75	0.15
3.75	0.20
5.75	0.27
7.75	0.30

(con't)

9.112 (con't)


Problem 9.112
Velocity, u, vs Distance, y
0.30
0.25
0.20
0.15
0.10
0.05
0.00
y, in.
0
20
40
60
80
u, ft/s
x = 7.75 in.
x = 5.75 in.
x = 3.75 in.
x = 1.75 in.


Problem 9.112
Boundary Layer thickness, δ,
vs
Distance from Leading Edge, x
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
δ, in.
0
2
4
6
8
10
x, in.
δ = 0.112x^0.48
Approximate boundary layer thickness
Best fit power-law curve

9.113

9.113 Pressure Distribution on a Circular Cylinder

Objective: Viscous effect within the boundary layer on a circular cylinder cause boundary layer separation, thereby causing the pressure distribution on the rear half of the cylinder to be different than that on the front half. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.113, to determine the pressure distribution on a circular cylinder.

Equipment: Wind tunnel; circular cylinder with 18 static pressure taps arranged equally from the front to the back of the cylinder; inclined multiple manometer; barometer; thermometer.

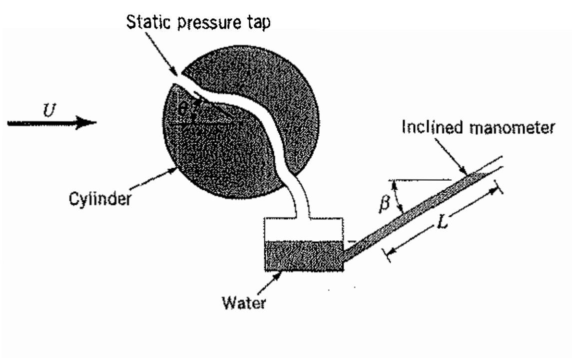
Experimental Procedure: Mount the circular cylinder in the wind tunnel so that a static pressure tap points directly upstream. Measure the angle, β, of the inclined manometer. Adjust the wind tunnel fan speed to give the desired free stream speed, U, in the test section. Attach the tubes from the static pressure taps to the multiple manometer and record the manometer readings, L, as a function of angular position, θ. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the data to determine the pressure coefficient, $C_p = (p - p_0)/(\rho U^2/2)$, as a function of position, θ. Here $p_0 = 0$ is the static pressure upstream of the cylinder in the free stream of the wind tunnel, and $p = \gamma_m L \sin\beta$ is the pressure on the surface of the cylinder.

Graph: Plot the pressure coefficient, C_p, as ordinates and the angular location, θ, as abscissas.

Results: On the same graph, plot the theoretical pressure coefficient, $C_p = 1 - 4 \sin^2\theta$, obtained from ideal (inviscid) theory (see Section 6.6.3).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P9.113

(con't)

9.113 (con't)

Solution for Problem 9.113: Pressure Distribution on a Circular Cylinder

β, deg	H_{atm}, in. Hg	T, deg F	U, ft/s
25	29.97	75	47.9

		Experiment		Theory
θ, deg	L, in.	p, lb/ft^2	C_p	C_p
0	1.2	2.64	1.00	1.00
10	1.1	2.42	0.92	0.88
20	0.7	1.54	0.58	0.53
30	0.1	0.22	0.08	0.00
40	-0.6	-1.32	-0.50	-0.65
50	-1.6	-3.52	-1.33	-1.35
60	-2.4	-5.27	-2.00	-2.00
70	-3.1	-6.81	-2.58	-2.53
80	-3.0	-6.59	-2.50	-2.88
90	-2.7	-5.93	-2.25	-3.00
100	-2.7	-5.93	-2.25	-2.88
110	-2.6	-5.71	-2.17	-2.53
120	-2.6	-5.71	-2.17	-2.00
130	-2.6	-5.71	-2.17	-1.35
140	-2.6	-5.71	-2.17	-0.65
150	-2.6	-5.71	-2.17	0.00
160	-2.7	-5.93	-2.25	0.53
170	-2.7	-5.93	-2.25	0.88
180	-2.8	-6.15	-2.33	1.00

$p = \gamma_{H20}{*}L \sin\beta$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}{*}H_{atm}$ = 847 lb/ft^3*(29.97/12 ft) = 2115 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

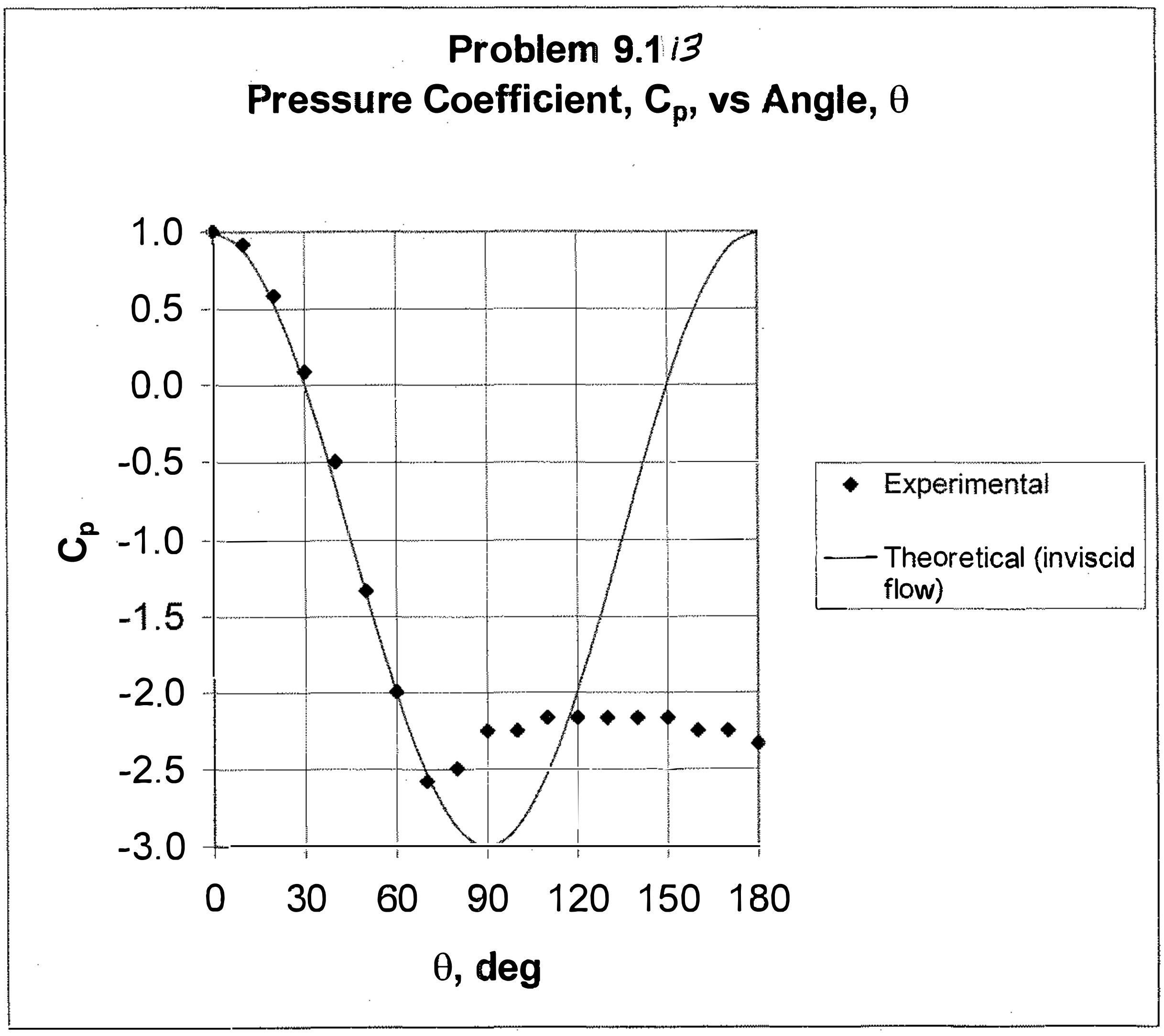
Thus, ρ = 0.00230 slug/ft^3

$C_p = p/(\rho U^2/2)$

Theory: $C_p = 1 - 4 \sin^2\theta$

(con't)

9.113 (con't)



Problem 9.113
Pressure Coefficient, C_p, vs Angle, θ
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
C_p
0
30
60
90
120
150
180
θ, deg
Experimental
Theoretical (inviscid flow)

10.2

10.2 On a distant planet small amplitude waves travel across a 1-m-deep pond with a speed of 5 m/s. Determine the acceleration of gravity on the surface of that planet.

$c = \sqrt{gy}$, where $c = 5\,\frac{m}{s}$ and $y = 1\,m$

Thus,

$$g = \frac{c^2}{y} = \frac{(5\,\frac{m}{s})^2}{1\,m} = \underline{\underline{25\,\frac{m}{s^2}}}$$

10.3

10.3 The flowrate in a 50-ft-wide, 2-ft-deep river is $Q = 190$ cfs. Is the flow subcritical or supercritical?

$Fr = \frac{V}{\sqrt{gy}}$, where $V = \frac{Q}{A} = \frac{190\,\frac{ft^3}{s}}{(2ft)(50ft)} = 1.90\,\frac{ft}{s}$

Thus,

$$Fr = \frac{1.90\,\frac{ft}{s}}{\sqrt{(32.2\,\frac{ft}{s^2})(2ft)}} = 0.237 < 1$$ $\underline{\underline{\text{The flow is subcritical.}}}$

10.4

10.4 The flowrate per unit width in a wide channel is $q = 2.3\ m^2/s$. Is the flow subcritical or supercritical if the depth is **(a)** 0.2 m, **(b)** 0.8m, or **(c)** 2.5 m?

$$V = \frac{Q}{A} = \frac{qb}{yb} = \frac{q}{y} \quad \text{so that} \quad Fr = \frac{V}{\sqrt{gy}} = \frac{q}{y\sqrt{gy}} = \frac{q}{\sqrt{g}\, y^{3/2}}$$

or

$$Fr = \frac{2.3\frac{m^2}{s}}{\sqrt{9.81\frac{m}{s^2}}\, y^{3/2}} = \frac{0.734}{y^{3/2}}, \quad \text{where } y \sim m$$

	y, m	Fr	flow type
a)	0.2	8.21	supercritical
b)	0.8	1.03	supercritical
c)	2.5	0.186	subcritical

10.5

10.5 A rectangular channel 3 m wide carries 10 m^3/s at a depth of 2 m. Is the flow subcritical or supercritical? For the same flowrate, what depth will give critical flow?

$Q = AV$ or $V = \frac{Q}{by} = \frac{10\,\frac{m^3}{s}}{(3m)(2m)} = 1.667\,\frac{m}{s}$

Thus,

$Fr = \frac{V}{\sqrt{gy}} = \frac{1.667\,\frac{m}{s}}{\left[(9.81\,\frac{m}{s^2})(2m)\right]^{\frac{1}{2}}} = 0.376 < 1$ The flow is <u>subcritical.</u>

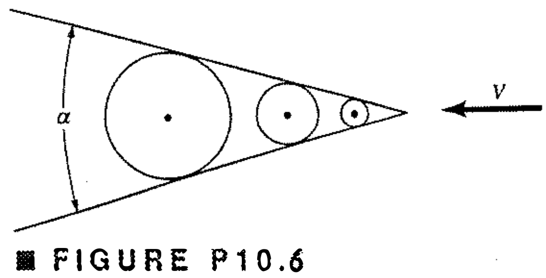
Also,

$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$, where $q = \frac{Q}{b} = \frac{10\,\frac{m^3}{s}}{3m} = 3.33\,\frac{m^2}{s}$ so that

$y_c = \left(\frac{(3.33\,\frac{m^2}{s})^2}{9.81\,\frac{m}{s^2}}\right)^{\frac{1}{3}} = \underline{1.04\,m}$

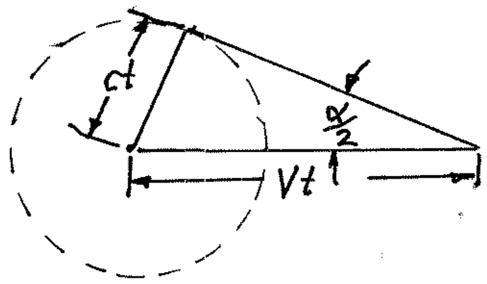
10.6

10.6 Consider waves made by dropping objects (one after another from a fixed location) into a stream of depth y that is moving with speed V as shown in Fig. P10.6 (see Video V10.5). The circular wave crests that are produced travel with speed $c = (gy)^{1/2}$ relative to the moving water. Thus, as the circular waves are washed downstream, their diameters increase and the center of each circle is fixed relative to the moving water. **(a)** Show that if the flow is supercritical, lines tangent to the waves generate a wedge of half-angle $\alpha/2 = \arcsin(1/Fr)$, where $Fr = V/(gy)^{1/2}$ is the Froude number. **(b)** Discuss what happens to the wave pattern when the flow is subcritical, $Fr < 1$.



■ FIGURE P10.6

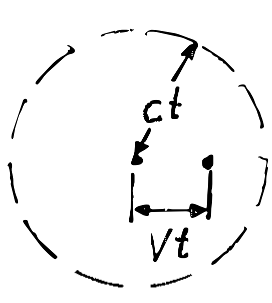
(a) In a time interval of t since the object hit the water (and iniatiated the wave), the center of the wave has been swept downstream a distance Vt and the wave has expanded to be a distance ct from its center. This is shown in the figure below. Note that $Vt > ct$ if $V > c$ (i.e. $Fr > 1$).



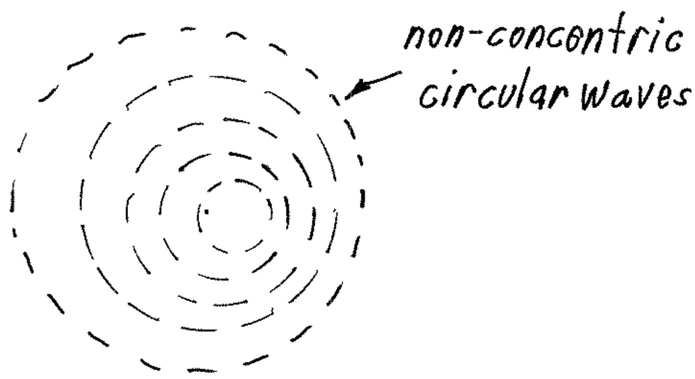
Thus, from the figure, $\sin\frac{\alpha}{2} = \dfrac{ct}{Vt} = \dfrac{c}{V} = \dfrac{\sqrt{gy}}{V} = \dfrac{1}{Fr}$

or $\frac{\alpha}{2} = \arcsin(1/Fr)$

(b) If $Fr < 1$ the above result gives $\sin\frac{\alpha}{2} > 1$, which is impossible. For $Fr < 1$ the following wave pattern would result. There is no "wedge" produced.



$Vt < ct$ if $Fr < 1$



10.7

10.7 Waves on the surface of a tank are observed to travel at a speed of 2 m/s. How fast would these waves travel if (a) the tank were in an elevator accelerating downward at a rate of 4 m/s^2, (b) the tank accelerates horizontally at a rate of 9.81 m/s^2, (c) the tank were aboard the orbiting Space Shuttle? Explain.

Since $c = \sqrt{gy}$ it follows that the tank depth is

$$y = \frac{c^2}{g} = \frac{(2\,\frac{m}{s})^2}{9.81\,\frac{m}{s^2}} = 0.408\,m$$

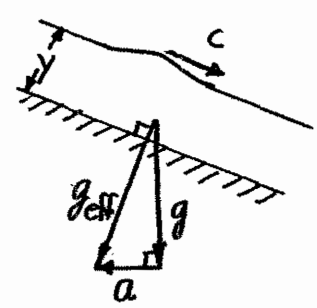
(a) If the tank accelerates down with acceleration a, the effective acceleration of gravity is $g_{eff} = g - a = (9.81-4)\frac{m}{s^2} = 5.81\,\frac{m}{s^2}$

Thus,

$$c = \sqrt{g_{eff}\,y} = \sqrt{(5.81\,\tfrac{m}{s^2})(0.408m)} = \underline{\underline{1.54\,\tfrac{m}{s}}}$$

(b) If the tank accelerates horizontally with acceleration a, the effective acceleration is

$$g_{eff} = \sqrt{g^2 + a^2} = \sqrt{9.81^2 + 9.81^2} = 13.87\,\frac{m}{s^2}$$



Thus,

$$c = \sqrt{(13.87\tfrac{m}{s^2})(0.408m)} = \underline{\underline{2.38\,\tfrac{m}{s}}}$$

(c) In orbit $g_{eff} = 0$ (weightless) so $c = \underline{\underline{0}}$

10.8

10.8 In flowing from section (1) to section (2) along an open channel, the water depth decreases by a factor of two and the Froude number changes from a subcritical value of 0.5 to a supercritical value of 3.0. Determine the channel width at (2) if it is 12 ft wide at (1).

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = 0.5, \text{ or } \sqrt{g y_1} = 2.0\, V_1 \qquad (1)$$

and

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = 3.0 \quad \text{where } y_2 = 0.5\, y_1$$

Thus, $\frac{V_2}{\sqrt{0.5 g y_1}} = 3.0$, or $\sqrt{g y_1} = V_2/(3\sqrt{0.5})$ (2)

By equating Eq. (1) and (2): $2.0 V_1 = V_2/(3\sqrt{0.5})$

or

$V_2 = 4.24\, V_1$

However, $Q_1 = Q_2$ or $b_1 y_1 V_1 = b_2 y_2 V_2$, where b = channel width.

Thus, with $b_1 = 12$ ft:

$(12\text{ ft})\, y_1 (V_1) = b_2 (0.5 y_1)(4.24 V_1)$, or $b_2 = \frac{12 \text{ ft}}{0.5(4.24)} = \underline{\underline{5.66 \text{ ft}}}$

10.9

10.9 Observations at a shallow sandy beach show that even though the waves several hundred yards out from the shore are not parallel to the beach, the waves often "break" on the beach nearly parallel to the shore as is indicated in Fig. P10.9. Explain this behavior based on the wave speed $c = (gy)^{1/2}$.

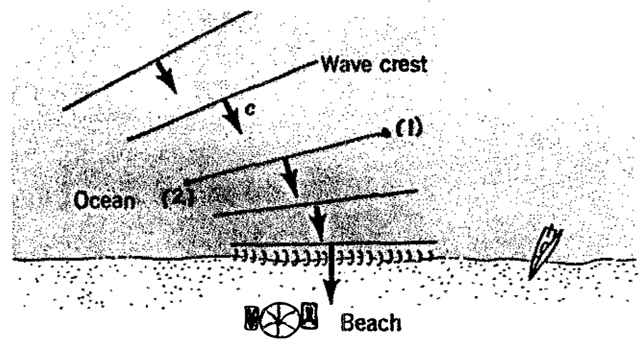


FIGURE P10.9

Since $c = \sqrt{gy}$ it follows that $c_1 > c_2$ because of the fact that $y_1 > y_2$. Therefore, as the waves move, that portion in the deeper water tends to "catch up" with that portion closer to shore in the shallower water. The wave crest tends to become more nearly parallel to the shore line. The waves "break" on the shore as if the wind were blowing normal to the shore.

10.11

10.11 Often when an earthquake shifts a segment of the ocean floor, a relatively small amplitude wave of very long wavelength is produced. Such waves go unnoticed as they move across the open ocean; only when they approach the shore do they become dangerous (a tsunami or "tidal wave").
length, λ, is 6000 ft and the ocean depth is 15,000 ft.

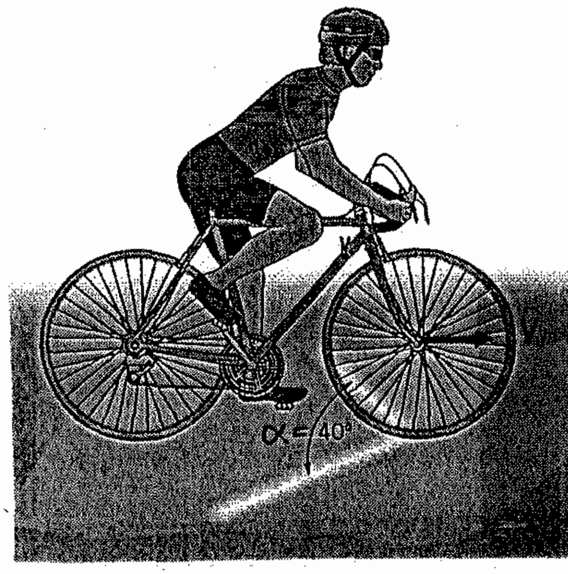
From Eq. 10.4: $c = \left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{\frac{1}{2}}$

or

$$c = \left[\frac{(32.2\frac{ft}{s^2})(6000\,ft)}{2\pi}\tanh\left(\frac{2\pi(15{,}000\,ft)}{6000\,ft}\right)\right]^{\frac{1}{2}} = \underline{\underline{175\,\frac{ft}{s}}}$$

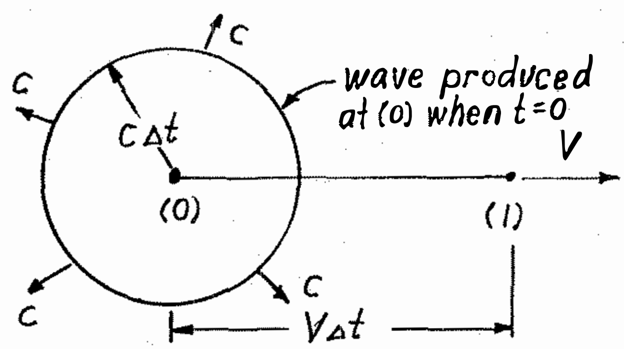
10.12

10.12 A bicyclist rides through a 3-in. deep puddle of water as shown in **Video V10.5** and Fig. P10.12. If the angle made by the V-shaped wave pattern produced by the front wheel is observed to be 40 deg, estimate the speed of the bike through the puddle. *Hint:* Make a sketch of the current location of the bike wheel relative to where it was Δt seconds ago. Also indicate on this sketch the current location of the wave that the wheel made Δt seconds ago. Recall that the wave moves radially outward in all directions with speed c relative to the stationary water.

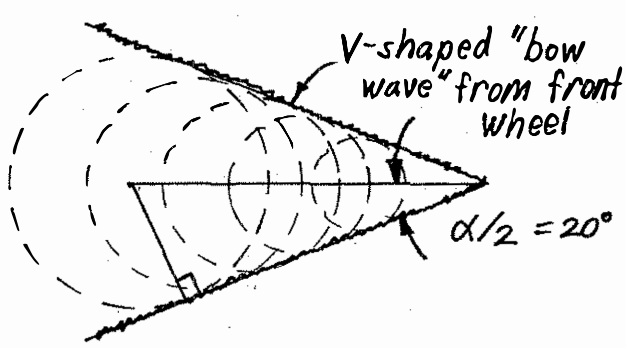


■ FIGURE P10.12

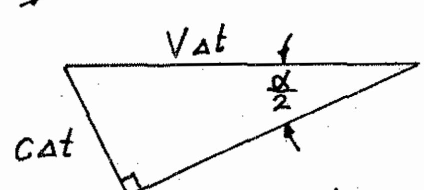
At time $t=0$ the front wheel was at point (0). At the current time, $t=\Delta t$, the wheel has traveled a distance $d = V\Delta t$ and is at point (1). At time $t=\Delta t$, a wave produced by the wheel when it was at (0) will be a distance $c\Delta t$ from (0) as indicated in the figure.



Waves produced at various times (from $t=0$ to $t=\Delta t$) by the front wheel will form a V-shaped wave as shown in the second figure (provided $V > c$; supercritical bike speed).



From the geometry of the figure



$$\sin\frac{\alpha}{2} = \frac{c\Delta t}{V\Delta t}$$

or

$$V = \frac{c}{\sin\frac{\alpha}{2}} \quad \text{where } c = \sqrt{gy} = \left[32.2\,\frac{ft}{s^2}\left(\frac{3}{12}\,ft\right)\right]^{\frac{1}{2}} = 2.84\,\frac{ft}{s}$$

Thus,

$$V = \frac{2.84\,\frac{ft}{s}}{\sin 20^\circ} = \underline{\underline{8.30\,\frac{ft}{s}}}$$

10.13

10.13 Determine the minimum depth in a 3-m-wide rectangular channel if the flow is to be subcritical with a flowrate of $Q = 60\ \text{m}^3/\text{s}$.

$$V = \frac{Q}{A} = \frac{60\ \frac{m^3}{s}}{(3m)\ y} = \frac{20}{y}, \text{ where } V \sim \frac{m}{s} \text{ when } y = \text{depth} \sim m$$

Also,

$$Fr = \frac{V}{\sqrt{gy}} = \frac{\left(\frac{20}{y}\ \frac{m}{s}\right)}{\left[\left(9.81\ \frac{m}{s^2}\right)y\right]^{1/2}} = \frac{6.39}{y^{3/2}}$$

Note: As y decreases, Fr increases

Thus, to have $Fr < 1$ we must have $\frac{6.39}{y^{3/2}} < 1$, or

$$y > (6.39)^{2/3} = \underline{\underline{3.44m}}$$

10.14

10.14 (See Fluids in the News article titled "Tsunami, the nonstorm wave," Section 10.2.1.) An earthquake causes a shift in the ocean floor that produces a tsunami with a wavelength of 100 km. How fast will this wave travel across the ocean surface if the ocean depth is 3000 m?

$$c = \left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{1/2}, \text{ where } \lambda = 100\,km = 10^5\,m \text{ and } y = 3000\,m.$$

Thus,

$$c = \left[\frac{9.81\,\frac{m}{s^2}\,(10^5\,m)}{2\pi}\tanh\left(\frac{2\pi(3000\,m)}{10^5\,m}\right)\right]^{1/2} = 171\,\frac{m}{s}$$

or

$$c = 171\,\frac{m}{s}\left(\frac{3600\,s}{hr}\right)\left(\frac{1\,km}{10^3\,m}\right) = \underline{\underline{616\,\frac{km}{hr}}}$$

10.15

10.15 Water flows in a 10-m-wide open channel with a flowrate of 5 m^3/s. Determine the two possible depths if the specific energy of the flow is $E = 0.6$ m.

(1) $$E = y + \frac{q^2}{2gy^2},$$ where $E = 0.6\,m$ and

$$q = \frac{Q}{b} = \frac{5\,\frac{m^3}{s}}{10\,m} = 0.5\,\frac{m^2}{s}$$

Thus, Eq. (1) becomes

$$0.6\,m = y + \frac{(0.5\,\frac{m^2}{s})^2}{(2)(9.80\,\frac{m}{s^2})\,y^2 m^2}$$

or

$$0.6 = y + \frac{0.0128}{y^2},$$ where $y \sim m$

Solution to this equation give

$y = 0.560\,m$ and $y = 0.173\,m$

10.16

10.16 Water flows in a rectangular channel with a flowrate per unit width of $q = 2.5 \text{ m}^2/\text{s}$. Plot the specific energy diagram for this flow. Determine the two possible depths of flow if $E = 2.5$ m.

$$E = y + \frac{q^2}{2g\,y^2} = y + \frac{(2.5\frac{m^2}{s})^2}{2(9.81\frac{m}{s^2})y^2} = y + \frac{0.319}{y^2}$$

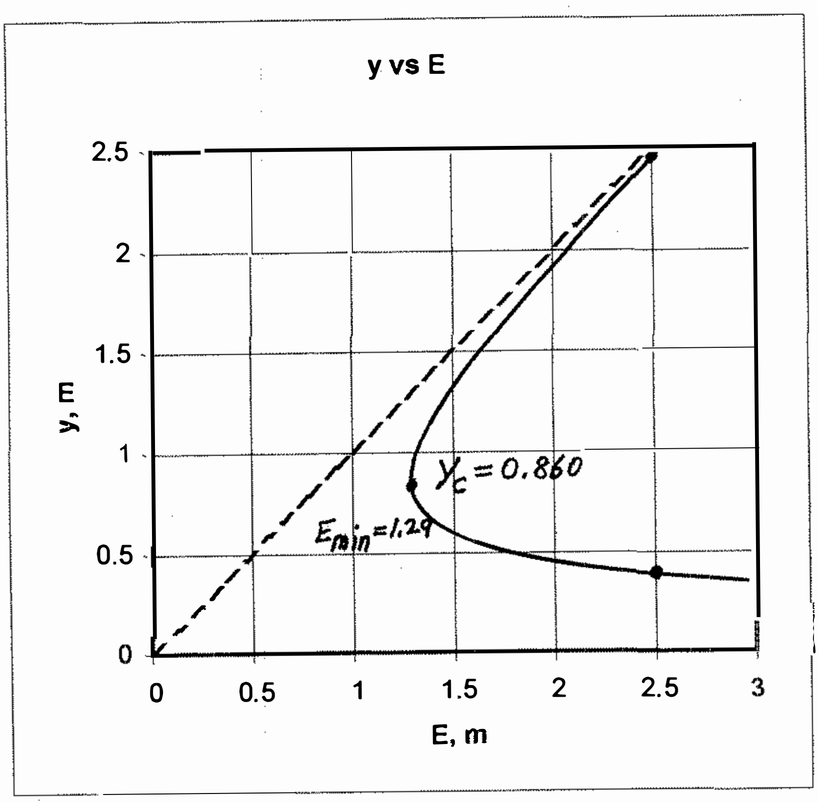
Thus, plot

$$E = y + \frac{0.319}{y^2}, \text{ where } E \sim m,\ y \sim m$$

Note: $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(2.5\frac{m^2}{s})^2}{9.81\frac{m}{s^2}}\right)^{1/3} = 0.860\,m$

and

$$E_{min} = \frac{3}{2}y_c = \frac{3}{2}(0.860m) = 1.29\,m$$



For $E = 2.5\,m$, Eq. (1) is $2.5 = y + \frac{0.319}{y^2}$

or $y^3 - 2.5y^2 + 0.319 = 0$

The roots to this equation are $y = 2.45$, 0.338, and -0.335

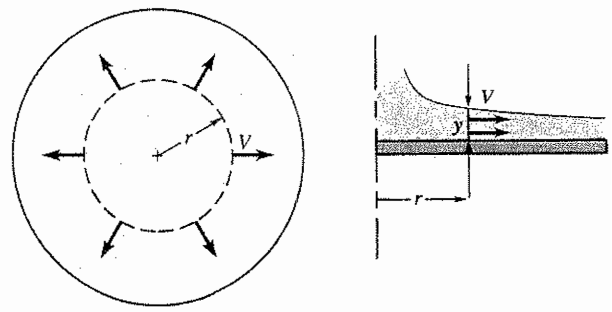
Thus, $\underline{\underline{y = 2.45\ m \text{ or } y = 0.388\ m}}$

10.17

10.17 Water flows radially outward on a horizontal round disk as is shown in **Video V10.12** and Fig. P10.17. **(a)** Show that the specific energy can be written in terms of the flowrate, Q, the radial distance from the axis of symmetry, r, and the fluid depth, y, as

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) For a constant flowrate, sketch the specific energy diagram. Recall Fig. 10.7, but note that for the present case r is a variable. Explain the important characteristics of your sketch. **(c)** Based on the results of Part (b), show that the water depth increases in the flow direction if the flow is subcritical, but that it decreases in the flow direction if the flow is supercritical.



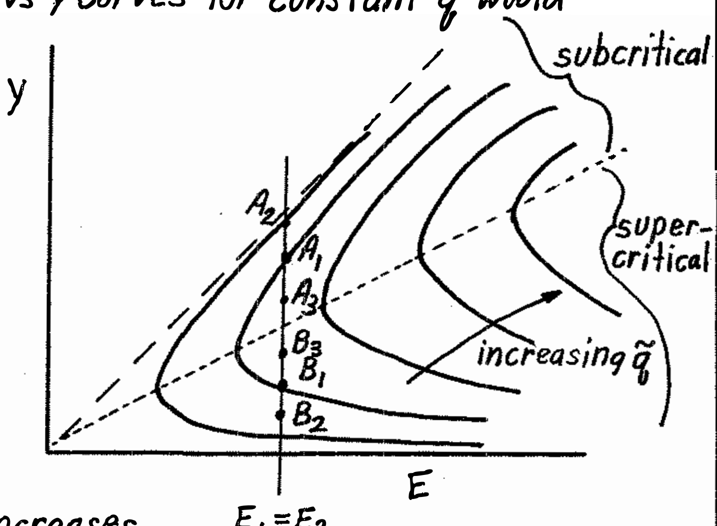
■ **FIGURE P10.17**

(a) The specific energy is $E = y + \frac{V^2}{2g}$, where $V = \frac{Q}{A} = \frac{Q}{2\pi r y}$

Thus,

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) Let $\tilde{q} \equiv \frac{Q}{2\pi r}$ so that $E = y + \frac{\tilde{q}^2}{2gy^2}$ which is the same as for two dimensional flow with $q = \frac{Q}{b}$ being replaced by $\tilde{q}$. However, for two dimensional flow q is constant; for radial flow $\tilde{q}$ is a variable since r varies. But E vs y curves for constant $\tilde{q}$ would look as shown below (Fig. 10.7).



(c) From the Bernoulli equation $E_1 = E_2$ or $E =$ constant for this flow. Consider subcritical flow — point A. For outflow r increases so that $\tilde{q}$ decreases. Thus since $E = $ const., the flow goes from state A_1 to A_2; the depth increases. For subcritical inflow r decreases, $\tilde{q}$ increases, the flow goes from A_1 to A_3 and the depth decreases. For supercritical flow ↑ is true. Thus, outflow increases r, decreases $\tilde{q}$; or from B_1 to B_2 — decreasing depth. Supercritical inflow from B_1 to B_3 — increasing depth.

	subcritical	supercritical
inflow	depth decreases	depth increases
outflow	depth increases	depth decreases

10.18

10.18 Water flows in a 10-ft-wide rectangular channel with a flowrate of 200 ft³/s. Plot the specific energy diagram for this flow. Determine the two possible flowrates when the specific energy is 6 ft.

$$E = y + \frac{q^2}{2gy^2}\ , \quad \text{where } q = \frac{Q}{b} = \frac{200\ \frac{ft^3}{s}}{10\ ft} = 20\ \frac{ft^2}{s}$$

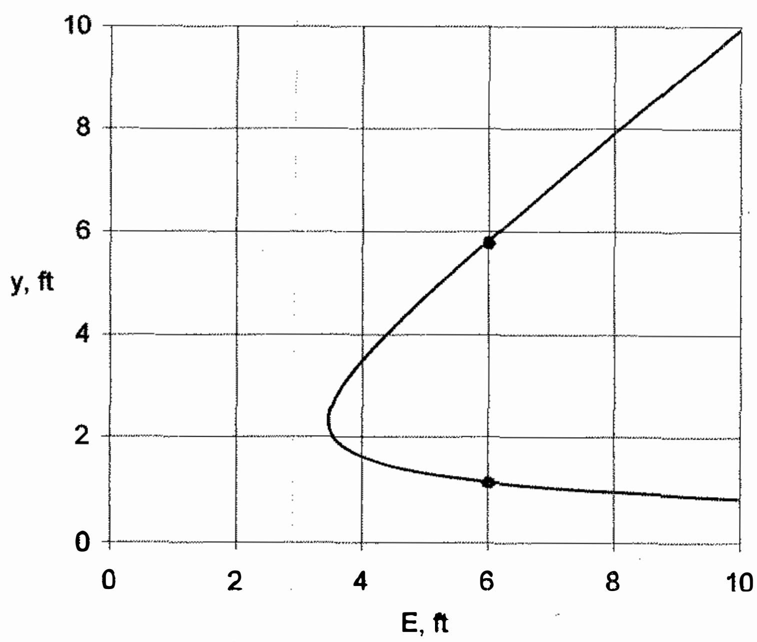
Thus,

$$E = y + \frac{(20\ ft^2/s)^2}{2(32.2\ ft/s^2)\,y^2}$$

or

(1) $$E = y + \frac{6.21}{y^2}\ , \quad \text{where } E \text{ and } y \sim ft$$

Eq. (1) is plotted below.



From Eq. (1), when $E = 6\ ft$,

$$6 = y + \frac{6.21}{y^2}$$

or

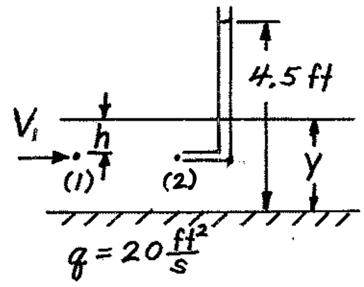
$y^3 - 6y^2 + 6.21 = 0$ which has solutions

$y = 5.82\ ft$ or $y = 1.129\ ft$

These values are shown in the above figure.

10.19

10.19 Water flows in a rectangular channel at a rate of $q = 20$ cfs/ft. When a Pitot tube is placed in the stream, water in the tube rises to a level of 4.5 ft above the channel bottom. Determine the two possible flow depths in the channel. Illustrate this flow on a specific energy diagram.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{, where } z_1 = z_2,$$
$$V_2 = 0,\ \frac{p_1}{\gamma} = h \text{, and } \frac{p_2}{\gamma} = 4.5 - (y-h)$$

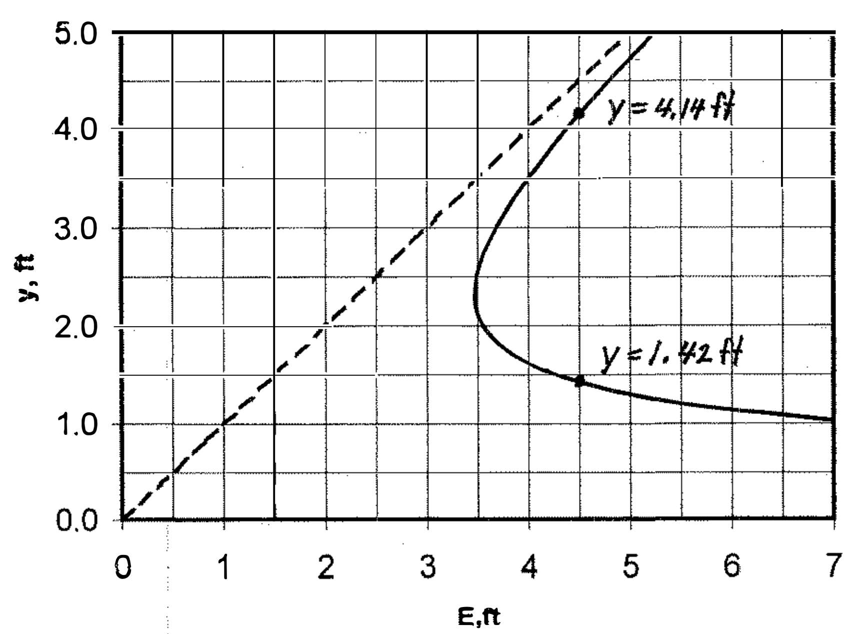
Thus,
$$h + \frac{V_1^2}{2g} = 4.5\,\text{ft} - y + h \text{, or } \frac{V_1^2}{2g} = 4.5 - y$$
but,
$$V_1 = \frac{q}{y} = \frac{20\frac{ft^2}{s}}{y}$$
Hence,
$$\frac{\left(\frac{20}{y}\right)^2}{2(32.2\frac{ft}{s^2})} = 4.5 - y \quad \text{or} \quad y^3 - 4.5y^2 + 6.21 = 0 \text{, where } y \sim ft$$

The roots of this equation are $y = 4.14, 1.42$, and -1.06

Thus,
$$\underline{\underline{y = 4.14\,\text{ft or } y = 1.42\,\text{ft}}}$$

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(20\frac{ft^2}{s})^2}{2(32.2\frac{ft}{s^2})y^2} \quad \text{or } E = y + \frac{6.21}{y^2} \qquad (1)$$

The specific energy diagram (plot of Eq. (1)) is shown below.



10.20

10.20 Water flows in a 5-ft-wide rectangular channel with a flowrate of $Q = 30\ \text{ft}^3/\text{s}$ and an upstream depth of $y_1 = 2.5$ ft as is shown in Fig. P10.20. Determine the flow depth and the surface elevation at section (2).

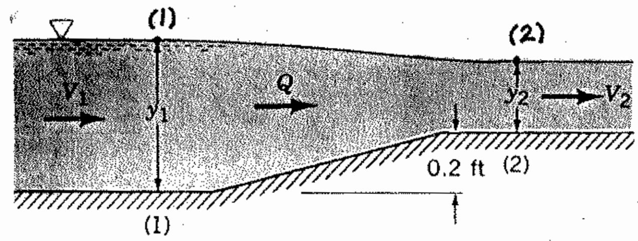


FIGURE P10.20

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\text{, where } p_1 = p_2 = 0,\ z_1 = y_1 = 2\text{ ft},\ z_2 = 0.2\text{ ft} + y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{(30\frac{ft^3}{s})}{(2\,ft)(5\,ft)} = 3\frac{ft}{s}\text{, and } V_2 = \frac{Q}{A_2} = \frac{30\frac{ft^3}{s}}{(5\,ft)\,y_2} = \frac{6}{y_2}$$

Thus,

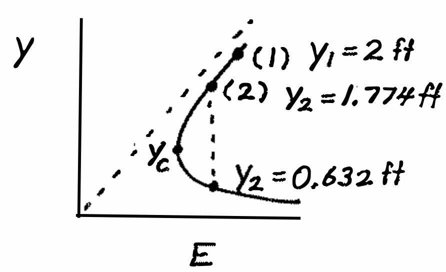
$$\frac{(3\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 2\,ft = \frac{(\frac{6}{y_2}\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 0.2\,ft + y_2$$

or

$$y_2^3 - 1.94y_2^2 + 0.559 = 0 \quad \text{which has roots } y_2 = 1.774,\ 0.632,\text{ and } -0.632$$

$$\text{Note: } Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{3\frac{ft}{s}}{[(32.2\frac{ft}{s^2})(2\,ft)]^{1/2}} = 0.374 < 1$$

If $y_2 = 0.632$, then $Fr_2 > 1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $\underline{\underline{y_2 = 1.774\text{ ft and } z_2 = 1.974\text{ ft}}}$

10.21

10.21 Repeat Problem 10.20 if the upstream depth is $y_1 = 0.5$ ft.

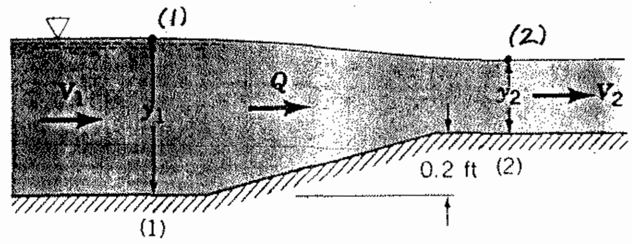


FIGURE P10.20

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 0.5\,ft,\ z_2 = 0.2\,ft + y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{30\,\frac{ft^3}{s}}{(0.5\,ft)(5\,ft)} = 12\,\frac{ft}{s}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{30\,\frac{ft^3}{s}}{(5\,ft)y_2} = \frac{6}{y_2}$$

Thus,

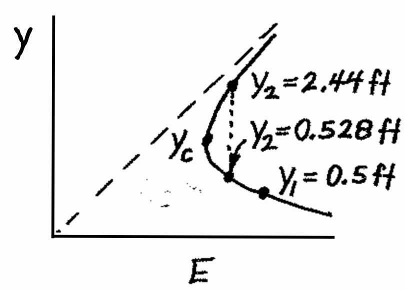
$$\frac{(12\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} + 0.5\,ft = \frac{(\frac{6}{y_2})^2}{2(32.2\,\frac{ft}{s^2})} + 0.2\,ft + y_2$$

or

$$y_2^3 - 2.53\,y_2^2 + 0.559 = 0 \text{ which has roots } y_2 = 2.44,\ 0.528, \text{ and } -0.434$$

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12\,\frac{ft}{s}}{[(32.2\,\frac{ft}{s^2})(0.5\,ft)]^{1/2}} = 2.99 > 1$

If $y_2 = 2.44$ ft, then $Fr_2 < 1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $\underline{\underline{y_2 = 0.528 \text{ ft and } z_2 = 0.728 \text{ ft}}}$

*10.22

10.22* Water flows over the bump in the bottom of the rectangular channel shown in Fig. P10.22 with a flowrate per unit width of $q = 4\ m^2/s$. The channel bottom contour is given by $z_B = 0.2e^{-x^2}$, where z_B and x are in meters. The water depth far upstream of the bump is $y_1 = 2$ m. Plot a graph of the water depth, $y = y(x)$, and the surface elevation, $z = z(x)$, for $-4\ m \le x \le 4\ m$. Assume one-dimensional flow.

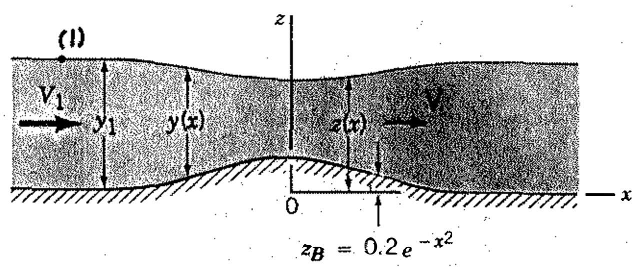


FIGURE P10.22

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p}{\gamma} + \frac{V^2}{2g} + z, \text{ where } p_1 = p = 0,\ z_1 = y_1 = 2m,\ z_2 = y + z_B$$

or $z = y + 0.2\,e^{-x^2}$, $V_1 = \frac{q}{y_1} = \frac{4\frac{m^2}{s}}{2m} = 2\frac{m}{s}$, and $V = \frac{q}{y} = \frac{4}{y}$

Thus, $$\frac{(2\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} + 2m = \frac{(\frac{4}{y}\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} + y + 0.2\,e^{-x^2}$$

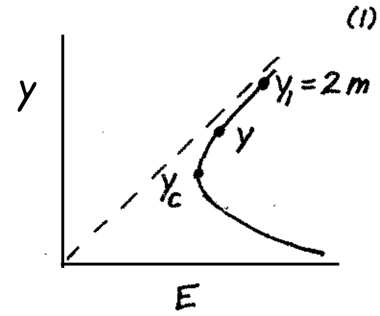
or

$$y^3 - (2.20 - 0.2e^{-x^2})y^2 + 0.815 = 0 \text{ where } y \sim m \qquad (1)$$

Solve for y with $-4 \le x \le 4m$

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2\frac{m}{s}}{[(9.81\frac{m}{s^2})(2m)]^{\frac{1}{2}}} = 0.452 < 1$

Thus, the flow will remain subcritical throughout — the largest root of Eq. (1) will be the correct one.



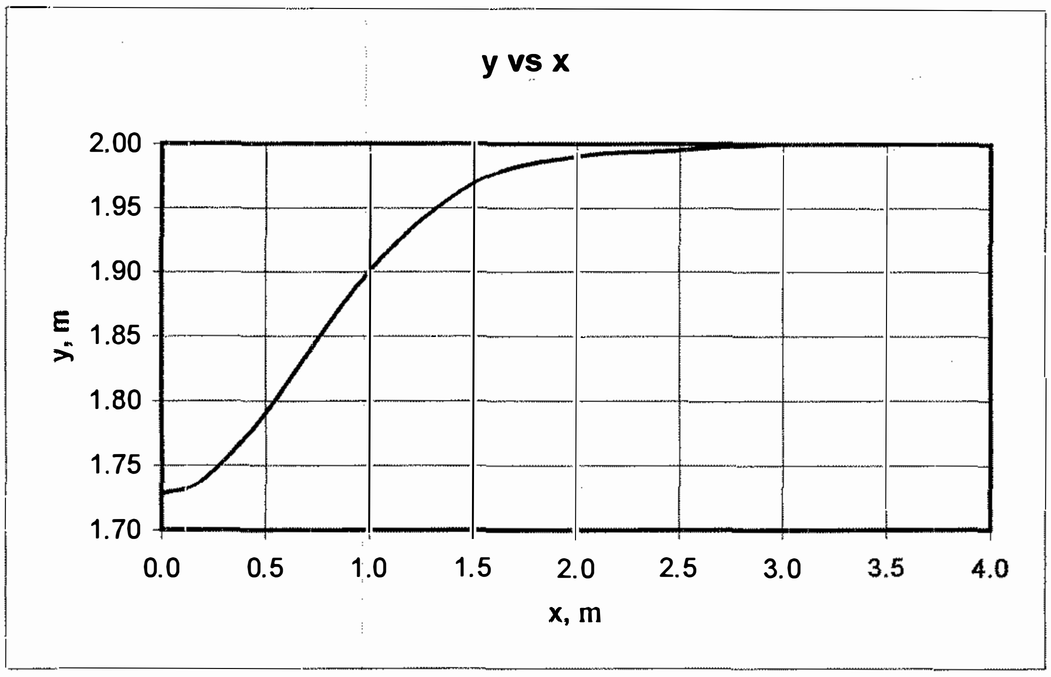
The following results are obtained by solving Eq. (1) for y and then $z = y + 0.2e^{-x^2}$ for $-4m \le x \le 4m$.

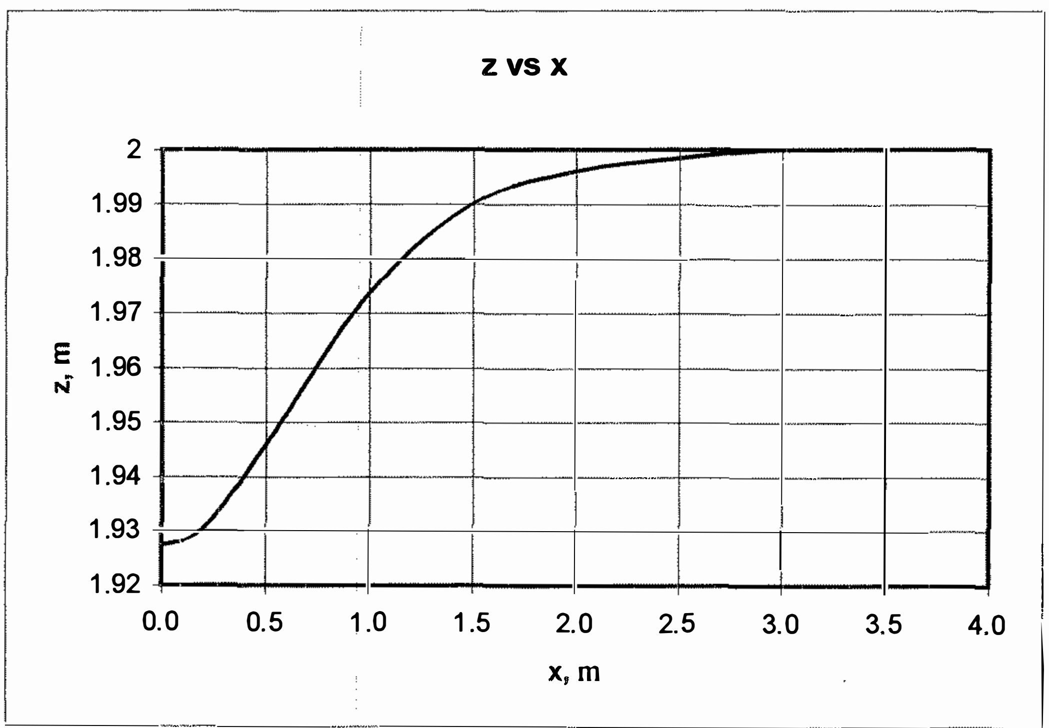
±x, m	y, m	z, m
0.0	1.727	1.927
0.5	1.790	1.946
1.0	1.901	1.974
1.5	1.969	1.990
2.0	1.991	1.994
2.5	1.995	1.995
3.0	1.995	1.995
3.5	1.995	1.995
4.0	1.995	1.995

(con't)

10.22* (con't)

The above results are plotted in the graph below.





*10.23

*10.23 Repeat Problem 10.22 if the upstream depth is 0.4 m.

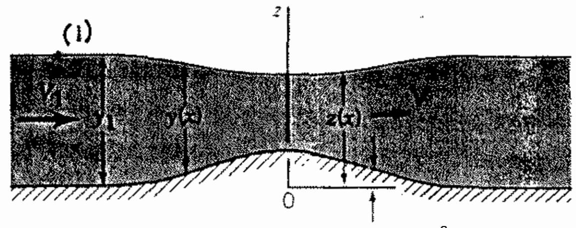


FIGURE P10.22

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p}{\gamma}+\frac{V^2}{2g}+z, \text{ where } p_1=p=0,\ z_1=y_1=0.4\,m,\ z_2=y+z_B$$

or $z_2 = y + 0.2e^{-x^2}$, $V_1=\frac{q}{y_1}=\frac{4\frac{m^2}{s}}{0.4m}=10\frac{m}{s}$, and $V=\frac{q}{y}=\frac{4}{y}$

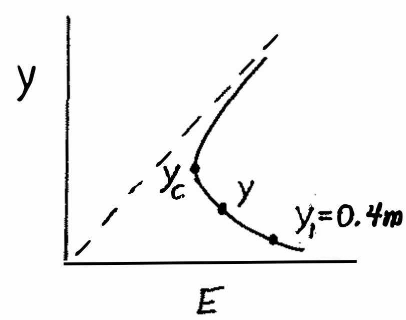
Thus, $$\frac{(10\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}+0.4m=\frac{(\frac{4}{y}\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}+y+0.2e^{-x^2}$$

or
$$y^3-(5.50-0.2e^{-x^2})y^2+0.815=0 \quad \text{where } y\sim m \tag{1}$$

Solve for y with $-4\le x\le 4\,m$

Note: $Fr_1=\frac{V_1}{\sqrt{gy_1}}=\frac{10\frac{m}{s}}{[(9.81\frac{m}{s^2})(0.4m)]^{1/2}}=5.05>1$

Thus, the flow will remain supercritical throughout — the smallest positive root of Eq.(1) will be the correct one.



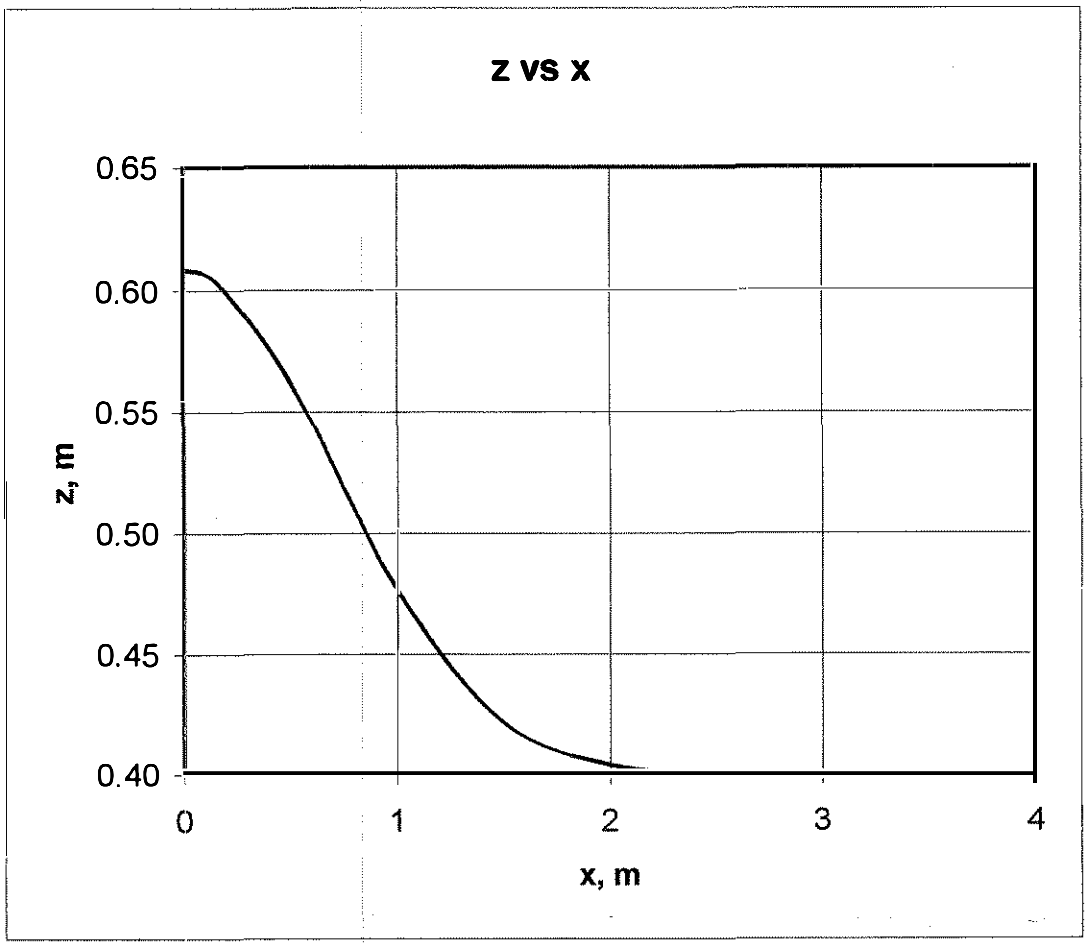
The following results are obtained by solving Eq. (1) for y and then $z=y+0.2e^{-x^2}$ for $-4m\le x\le 4m$.

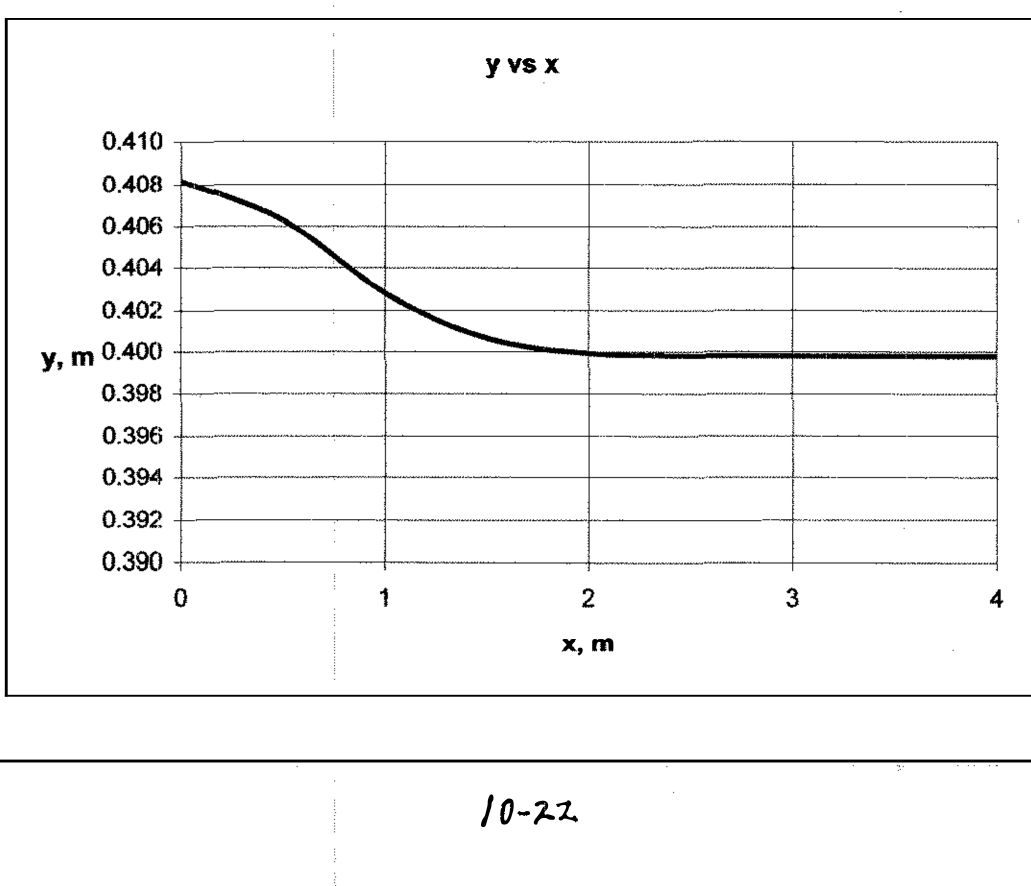
± x, m	y, m	z, m
0.0	0.408	0.608
0.5	0.406	0.562
1.0	0.403	0.476
1.5	0.401	0.422
2.0	0.400	0.404
2.5	0.400	0.400
3.0	0.400	0.400
3.5	0.400	0.400
4.0	0.400	0.400

(con't)

10.23* (con't)

The above results are plotted on the graph below.





10.24

10.24 Water in a rectangular channel flows into a gradual contraction section as is indicated in Fig. P10.24. If the flowrate is $Q = 25\ ft^3/s$ and the upstream depth is $y_1 = 2$ ft, determine the downstream depth, y_2.

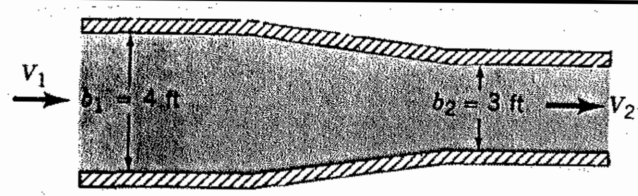
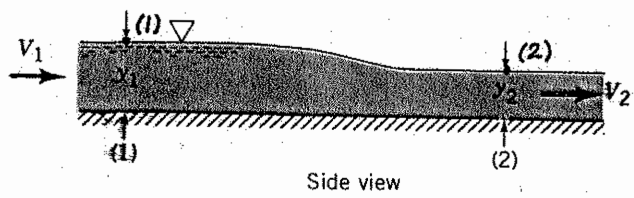


FIGURE P10.24



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\,, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 2\text{ ft},\ z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25\frac{ft^3}{s}}{(4ft)(2ft)} = 3.13\frac{ft}{s}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{25\frac{ft^3}{s}}{(3ft)\,y_2} = \frac{8.33}{y_2}$$

Thus,

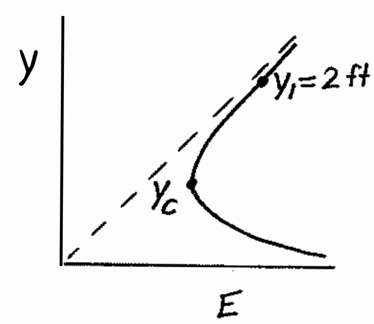
$$\frac{(3.13\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 2ft = \frac{\left(\frac{8.33}{y_2}\right)^2}{2(32.2\frac{ft}{s^2})} + y_2$$

or

$$y_2^3 - 2.15y_2^2 + 1.077 = 0 \text{ which has roots } y_2 = 1.828,\ 0.946, \text{ and } -0.623 \qquad (1)$$

$$\text{Note: } Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{3.13\frac{ft}{s}}{\left[(32.2\frac{ft}{s})(2ft)\right]^{\frac{1}{2}}} = 0.390 < 1$$

Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 < 1$ also. Thus, it is not possible to have $y_2 = 0.946$



Thus, $\underline{\underline{y_2 = 1.828\text{ ft}}}$

10.25

10.25 Sketch the specific energy diagram for the flow of Problem 10.24 and indicate its important characteristics. Note that $q_1 \neq q_2$.

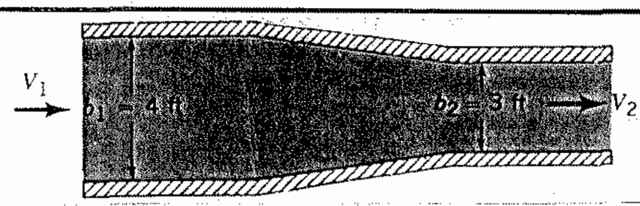
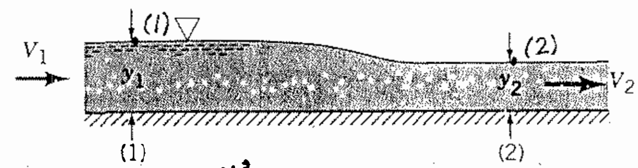


FIGURE P10.24



$$E = y + \frac{q^2}{2gy^2}$$

Thus, for the $b_1 = 4\,ft$ channel, $q_1 = \frac{Q}{b_1} = \frac{25\,\frac{ft^3}{s}}{4\,ft} = 6.25\,\frac{ft^2}{s}$

or

$$E = y + \frac{(6.25\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2} \quad \text{or} \quad E = y + \frac{0.607}{y^2} \qquad (1)$$

For the $b_2 = 3\,ft$ channel, $q_2 = \frac{Q}{b_2} = \frac{25\,\frac{ft^3}{s}}{3\,ft} = 8.33\,\frac{ft^2}{s}$

or

$$E = y + \frac{(8.33\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2} \quad \text{or} \quad E = y + \frac{1.077}{y^2} \qquad (2)$$

Note: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ so that $y_{c_1} = \left(\frac{q_1^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(6.25\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right)^{\frac{1}{3}} = 1.067\,ft$

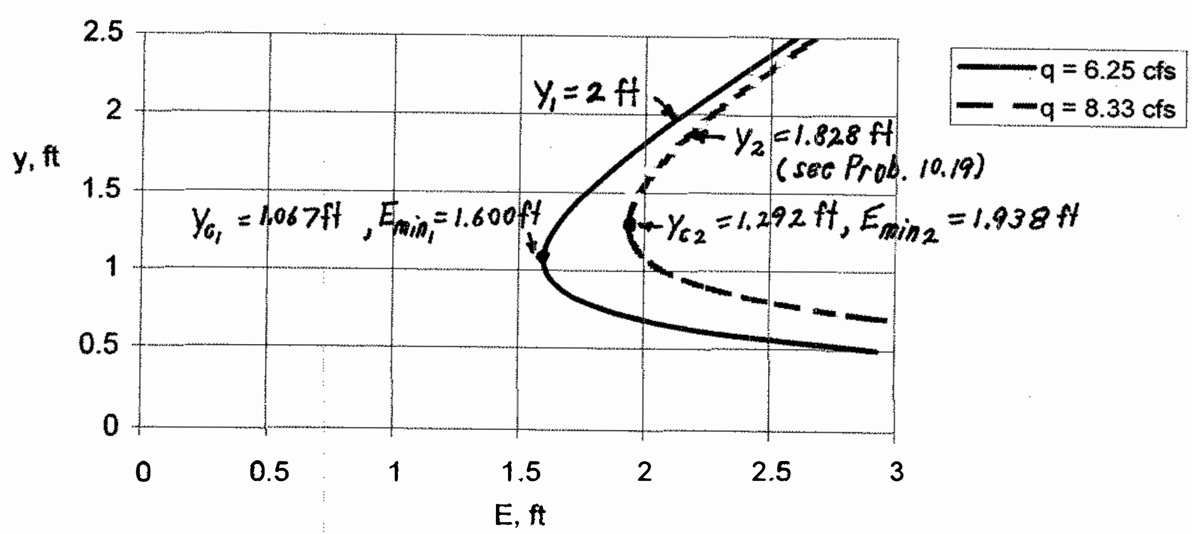
and

$$y_{c_2} = \left(\frac{q_2^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(8.33\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right)^{\frac{1}{3}} = 1.292\,ft$$

Also, $E_{min} = \frac{3}{2}\,y_c$, or $E_{min_1} = \frac{3}{2}(1.067\,ft) = 1.600\,ft$

$$E_{min_2} = \frac{3}{2}(1.292\,ft) = 1.938\,ft$$

The specific energy diagrams (Eqs. (1) and (2) are plotted below:



Note: $E_1 = y_1 + \frac{V_1^2}{2g} = E_2 = y_2 + \frac{V_2^2}{2g} = 2\,ft + \frac{(3.13\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = 2.15\,ft$

10.26

10.26 Repeat Problem 10.24 if the upstream depth is $y_1 = 0.5$ ft. Assume that there are no losses between sections (1) and (2).

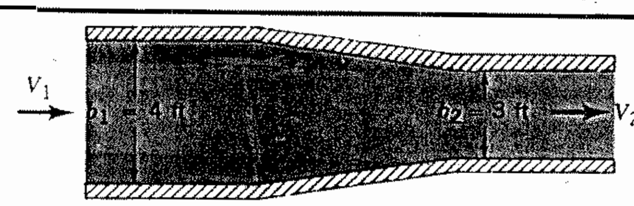
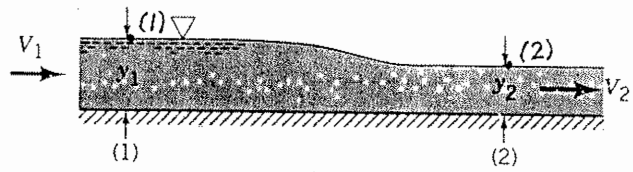


FIGURE P10.24



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\,, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 0.5\text{ ft},\ z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25\,\frac{ft^3}{s}}{(4\,ft)(0.5\,ft)} = 12.5\,\frac{ft}{s}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{25\,\frac{ft^3}{s}}{(3\,ft)\,y_2} = \frac{8.33}{y_2}$$

Thus,

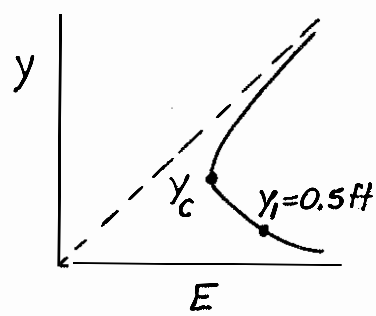
$$\frac{(12.5\,\frac{ft}{s})^2}{2\,(32.2\,\frac{ft}{s^2})} + 0.5\,ft = \frac{\left(\frac{8.33}{y_2}\right)^2}{2\,(32.2\,\frac{ft}{s^2})} + y_2$$

or

$$y_2^3 - 2.93\,y_2^2 + 1.077 = 0 \text{ which has roots } y_2 = 2.79,\ 0.694, \text{ and } -0.555$$

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12.5\,\frac{ft}{s}}{[(32.2\,\frac{ft}{s^2})(0.5\,ft)]^{1/2}} = 3.12 > 1$

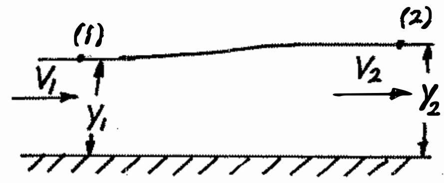
Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 > 1$ also. Thus, it is not possible to have $y_2 = 2.79$ (the subcritical root).



Thus, $y_2 = \underline{\underline{0.694\text{ ft}}}$

10.27

10.27 Water flows in a rectangular channel with a flowrate per unit width of $q = 1.5\ m^2/s$ and a depth of 0.5 m at section (1). The head loss between sections (1) and (2) is 0.03 m. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram. Is it possible to have a head loss of 0.06 m? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L, \text{ where } p_1 = p_2 = 0,\ z_1 = 0.5\,m,\ z_2 = y_2, \qquad (1)$$

$$V_1 = \frac{q}{y_1} = \frac{1.5\frac{m^2}{s}}{0.5m} = 3\frac{m}{s}, \text{ and } V_2 = \frac{q}{y_2} = \frac{1.5\frac{m^2}{s}}{y_2}$$

Thus, with $E = y + \frac{V^2}{2g}$ and $h_L = 0.03\,m$ Eq.(1) is

$$E_1 = E_2 + 0.03$$

Also, $E = y + \frac{q^2}{2g\,y^2} = y + \frac{(1.5\frac{m^2}{s})^2}{2(9.81\frac{m}{s^2})y^2}$

or

$$E = y + \frac{0.1146}{y^2} \qquad (2)$$

Eq.(2) is plotted below.

Note: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(1.5\frac{m^2}{s})^2}{9.81\frac{m}{s^2}}\right)^{\frac{1}{3}} = 0.612\ m$

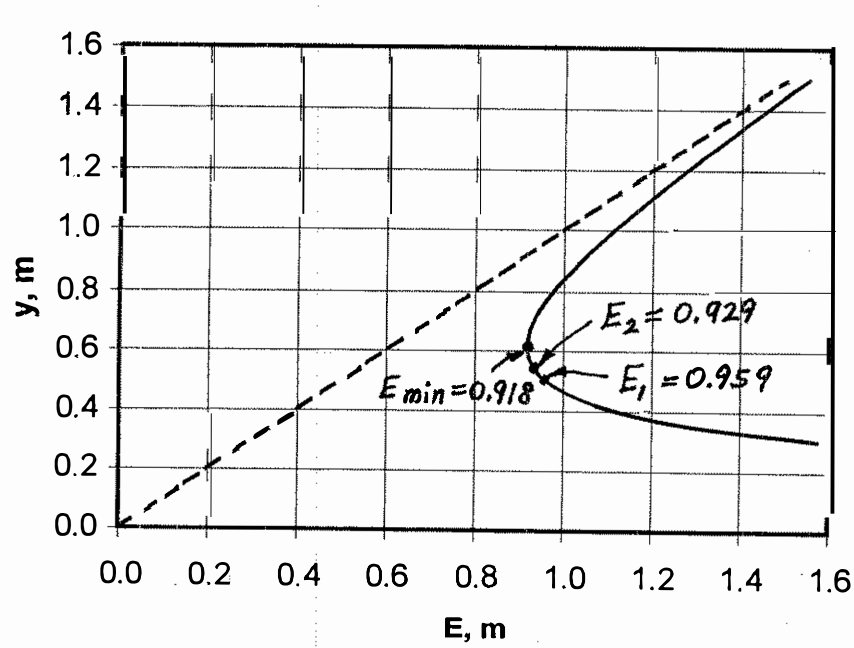
and

$$E_{min} = \frac{3}{2}y_c = \frac{3}{2}(0.612m) = 0.918m$$

Also,

$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 0.5 + \frac{(1.5\frac{m^2}{s})}{2(9.81\frac{m}{s^2})(0.5m)^2} = 0.959m$$

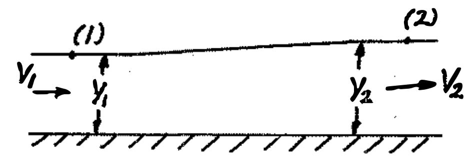
and $E_2 = E_1 - 0.03 = 0.929\,m$



Note: If $h_L = 0.06m$ with $E_1 = 0.959m$ so that $E_2 = E_1 - 0.06$, then $E_2 = 0.899\ m < E_{min}$. Thus, it is not possible to have $h_L = 0.06$ with the given q and y_1.

10.28

10.28 Water flows in a horizontal rectangular channel with a flowrate per unit width of $q = 10$ ft²/s and a depth of 1.0 ft at the downstream section (2). The head loss between section (1) upstream and section (2) is 0.2 ft. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L, \text{ where } p_1 = p_2 = 0,\ V_2 = \frac{q}{y_2} = \frac{10\,\frac{ft^2}{s}}{1\,ft} = 10\,\frac{ft}{s}, \quad (1)$$

and $y_2 = 1\,ft$

Thus, with $E = y + \frac{V^2}{2g}$

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(10\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2}$$

or

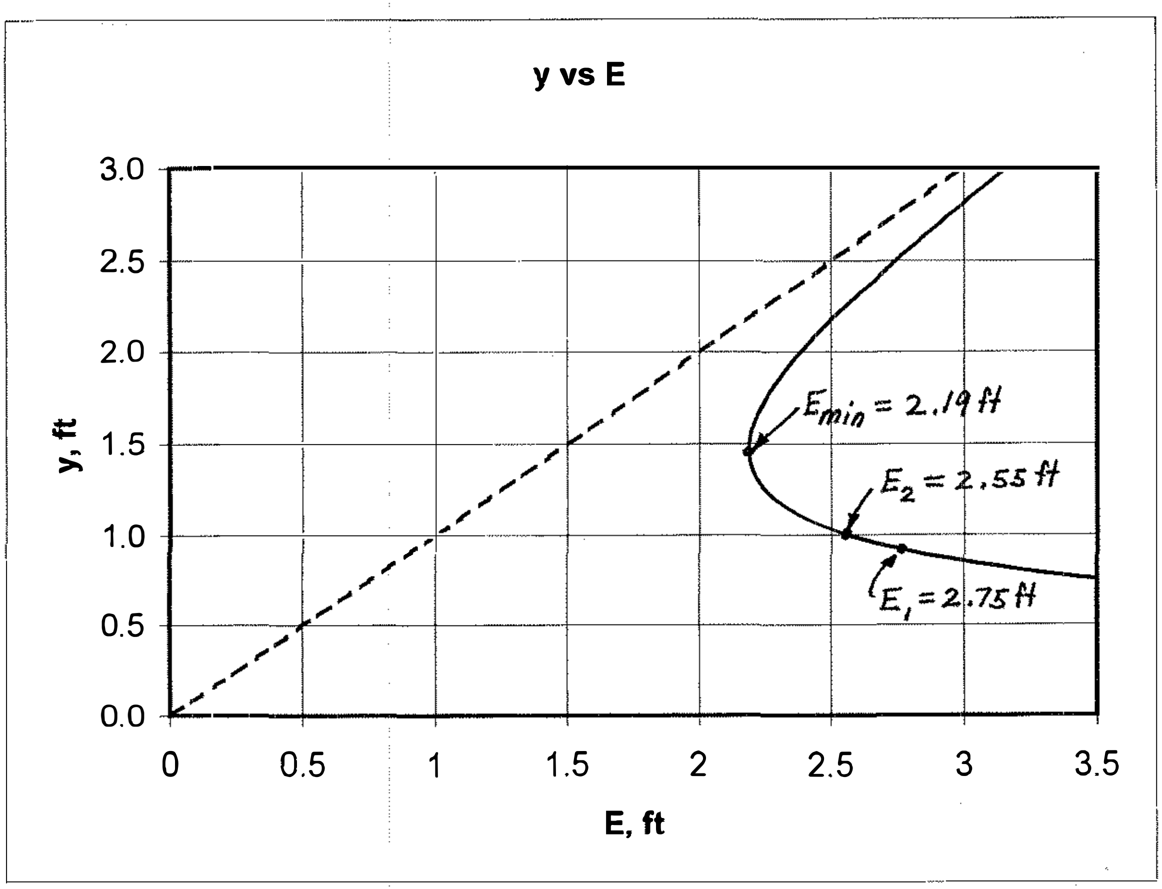
$$E = y + \frac{1.553}{y^2} \text{ where } E \sim ft,\ y \sim ft. \quad (2)$$

and Eq.(1) gives $E_1 = E_2 + h_L = E_2 + 0.2\,ft$

Eq.(2) is plotted below.

Note: $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(10\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right)^{1/3} = 1.459\,ft,\ E_{min} = \frac{3}{2}y_c = \frac{3}{2}(1.459\,ft) = 2.19\,ft,$

$$E_2 = y_2 + \frac{1.553}{y_2^2} = 1 + \frac{1.553}{1^2} = 2.55\,ft, \text{ and } E_1 = E_2 + h_L = 2.75\,ft$$



10.29

10.29 Water flows in a horizontal, rectangular channel with an initial depth of 1 m and an initial velocity of 4 m/s. Determine the depth downstream if losses are negligible. Note that there may be more than one solution.

$E = y + \frac{q^2}{2gy^2}$, where from the initial conditions,

$y_1 = 1m$ and $q_1 = \frac{Q}{b} = \frac{V_1 y_1 b}{b} = (4 m/s)(1m) = 4 \frac{m^2}{s}$

Thus,

$E_1 = y_1 + \frac{q_1^2}{2gy_1} = 1m + \frac{(4\frac{m^2}{s})^2}{2(9.81\frac{m}{s^2})(1m)^2} = 1.815\ m$

(Note: Since $q = Vy$, $E = y + \frac{(Vy)^2}{2gy^2} = y + \frac{V^2}{2g}$)

With no losses, $E_2 = E_1$, so that $q_1 = q_2$ (i.e., $Q_1 = q_1 b_1 = Q_2 = q_2 b_2$, with $b_1 = b_2$)

Hence,

$E_2 = 1.815 m = y_2 + \frac{(4\frac{m^2}{s})^2}{2(9.81\frac{m}{s^2})y_2^2}$

or

$1.815 = y_2 + \frac{0.815}{y_2^2}$

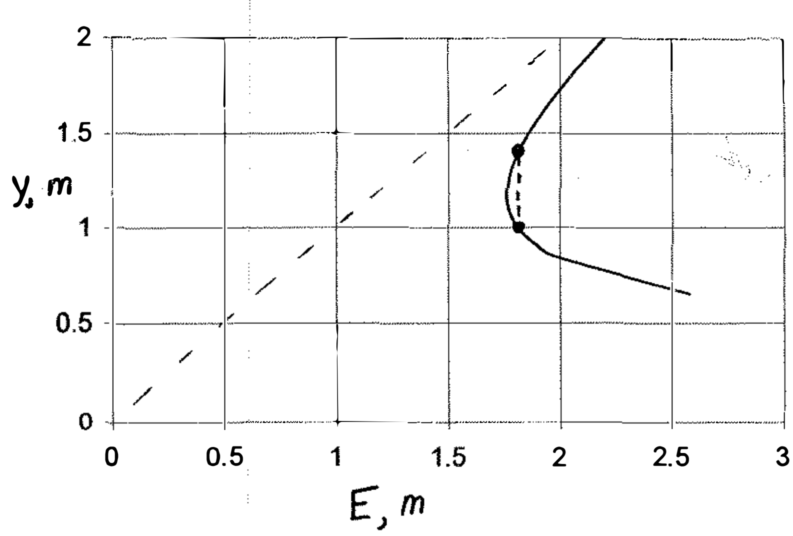
which has solutions

$y_2 = \underline{\underline{1\ m}}$ (same as the initial depth)

or

$y_2 = \underline{\underline{1.40\ m}}$

The energy diagram and these two depths are shown below.



10.30

10.30 A smooth transition section connects two rectangular channels as shown in Fig. P10.30. The channel width increases from 6.0 to 7.0 ft and the water surface elevation is the same in each channel. If the upstream depth of flow is 3.0 ft, determine h, the amount the channel bed needs to be raised across the transition section to maintain the same surface elevation.

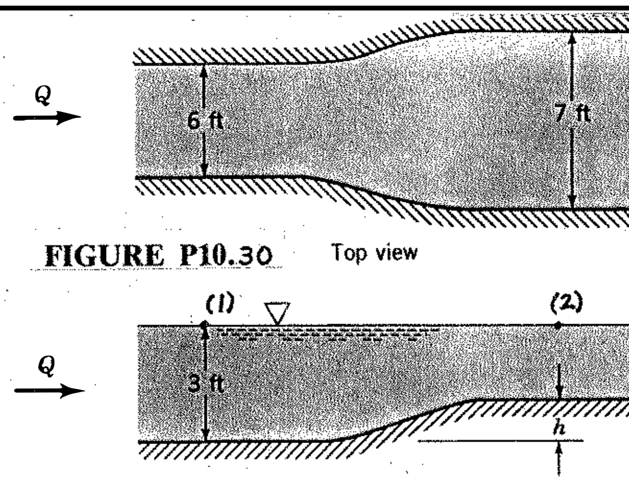


FIGURE P10.30

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0 \text{ and } z_1 = z_2$$

Thus, $V_1 = V_2$ or

$$\frac{Q}{A_1} = \frac{Q}{A_2} \quad \text{Hence, } A_1 = A_2 \text{ or } (6\,ft)(3\,ft) = (7\,ft)(3\,ft - h)$$

or

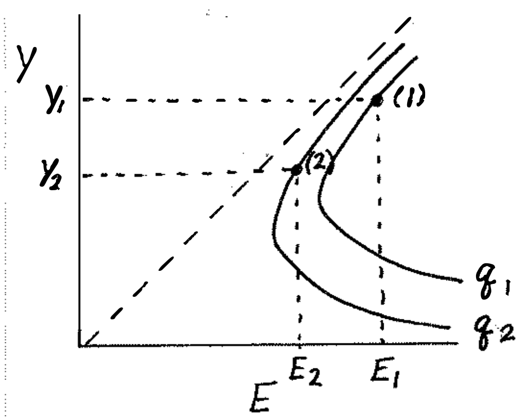
$$h = \underline{\underline{0.429\ ft}}$$

Note: $q_1 = \frac{Q}{b_1} = \frac{Q}{6}$ and $q_2 = \frac{Q}{b_2} = \frac{Q}{7} < q_1$

and

$E_1 = y_1 + \frac{V_1^2}{2g}$ and $E_2 = y_2 + \frac{V_2^2}{2g}$ Thus, since $V_1 = V_2$ it follows that $E_1 - E_2 = y_1 - y_2$

The corresponding specific energy diagram is as indicated below:



10.31

10.31 Water flows over a bump of height $h = h(x)$ on the bottom of a wide rectangular channel as is indicated in Fig. P10.31 If energy losses are negligible, show that the slope of the water surface is given by $dy/dx = -(dh/dx)/[1 - (V^2/gy)]$, where $V = V(x)$ and $y = y(x)$ are the local velocity and depth of flow. Comment on the sign (i.e., <0, =0, or >0) of dy/dx relative to the sign of dh/dx.

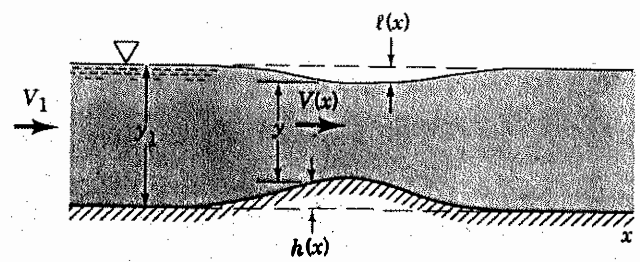


FIGURE P10.31

For any two points on the free surface:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{, where } p_1 = p_2 = 0,\ z_1 = y_1 \text{, and } z_2 = h + y_2$$

Thus, $\frac{V^2}{2g} + h + y = \text{constant}$ so that by differentiating

$$\frac{2V\frac{dV}{dx}}{2g} + \frac{dh}{dx} + \frac{dy}{dx} = 0 \qquad (1)$$

Also, for conservation of mass

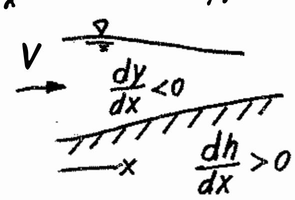
$$V_1 y_1 = Vy \text{ or } V\frac{dy}{dx} + y\frac{dV}{dx} = 0 \text{ or } \frac{dV}{dx} = -\frac{V}{y}\frac{dy}{dx} \qquad (2)$$

Combine Eqs. (1) and (2):

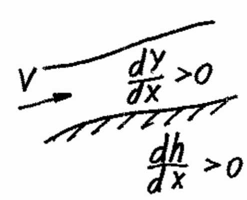
$$\frac{V}{g}\left(-\frac{V}{y}\frac{dy}{dx}\right) + \frac{dh}{dx} + \frac{dy}{dx} = 0 \text{, or } \underline{\underline{\frac{dy}{dx} = \frac{-\left(\frac{dh}{dx}\right)}{\left(1 - \left(\frac{V^2}{gy}\right)\right)}}}$$

Note: If $Fr = \frac{V}{\sqrt{gy}} < 1$, then $\frac{dh}{dx}$ and $\frac{dy}{dx}$ have the opposite sign

If $Fr > 1$, then $\frac{dh}{dx}$ and $\frac{dy}{dx}$ have the same sign.



$Fr < 1$



$Fr > 1$

10.32

10.32 Integrate the differential equation obtained in Problem 10.31 to determine the "draw-down" distance, $\ell = \ell(x)$, indicated in Fig. P10.31. Comment on your results.

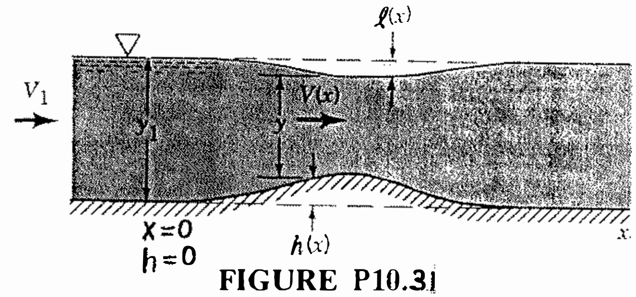


FIGURE P10.31

From Problem 10.31:

$$\frac{dy}{dx} = \frac{-\left(\frac{dh}{dx}\right)}{\left(1-\left(\frac{V^2}{gy}\right)\right)}, \text{ where } V_1 y_1 = Vy, \text{ or } V = \frac{V_1 y_1}{y}$$

Thus, $\frac{V^2}{gy} = \frac{\left(\frac{V_1 y_1}{y}\right)^2}{gy} = \frac{V_1^2 y_1^2}{gy^3}$ so that

$$\frac{dy}{dx} = \frac{-\left(\frac{dh}{dx}\right)}{\left[1-\frac{V_1^2 y_1^2}{gy^3}\right]}, \text{ or } \left[1-\left(\frac{V_1^2 y_1^2}{g}\right)y^{-3}\right]dy = -\left(\frac{dh}{dx}\right)dx$$

Integrate from $y=y_1$ and $x=0$, with $\frac{dh}{dx}$ a given function of x.

$$\int_{y=y_1}^{y}\left[1-\left(\frac{V_1^2 y_1^2}{g}\right)y^{-3}\right]dy = -\int_{x=0}^{x}\left(\frac{dh}{dx}\right)dx = -\int_{h=0}^{h} dh = -h$$

or

$$\left[y-\left(\frac{V_1^2 y_1^2}{g}\right)\left(-\tfrac{1}{2}\right)y^{-2}\right]_{y_1}^{y} = -h \quad \text{Thus, } y+\left(\frac{V_1^2 y_1^2}{2g}\right)y^{-2} - y_1 - \frac{V_1^2}{2g} = -h$$

or

$$y^3 - \left(y_1 + \frac{V_1^2}{2g} - h\right)y^2 + \left(\frac{V_1^2 y_1^2}{2g}\right) = 0 \qquad (1)$$

Obtain $y = y(x)$ from Eq. (1) and then $\ell = \ell(x)$ from $y_1 = h + y + \ell$ or $\ell = y_1 - h - y$

Note: Eq. (1) is nothing more than the Bernoulli equation:

$$\frac{V_1^2}{2g} + y_1 = \frac{V^2}{2g} + y + h \quad \text{with } V = \frac{V_1 y_1}{y} \text{ so that}$$

$$\frac{V_1^2}{2g} + y_1 = \frac{\left(\frac{V_1 y_1}{y}\right)^2}{2g} + y + h \quad \text{which simplifies to Eq. (1).}$$

10.33

10.33 Water flows in the river shown in Fig. P10.33 with a uniform bottom slope. The total head at each section is measured by using Pitot tubes as indicated. Determine the value of dy/dx at a location where the Froude number is 0.357.

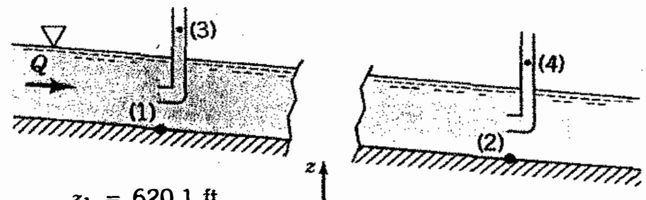


FIGURE P10.33

$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}$, where from the figure $S_f = \frac{h_L}{\ell} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0)ft}{4100 ft}$

or $S_f = 8.05 \times 10^{-4}$ and $S_o = \frac{z_1 - z_2}{\ell} = \frac{(620.1 - 618.7)ft}{4100 ft} = 3.41 \times 10^{-4}$

Thus,

$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (0.357)^2} = \underline{\underline{0.000532}}$ (i.e., $2.81 \frac{ft}{mi}$)

10.34

10.34 Repeat Problem 10.33 if the Froude number is 2.75.

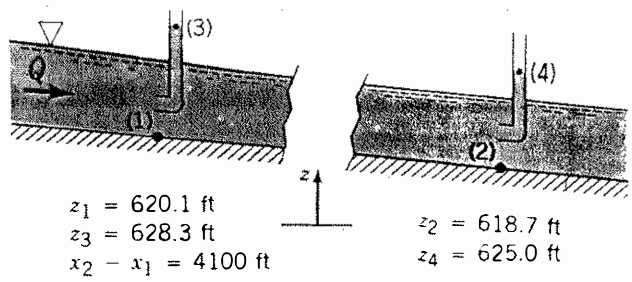


FIGURE P10.33

$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}$, where from the figure $S_f = \frac{h_L}{\ell} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0)\,ft}{4100\,ft}$

or $S_f = 8.05 \times 10^{-4}$ and $S_o = \frac{z_1 - z_2}{\ell} = \frac{(620.1 - 618.7)\,ft}{4100\,ft} = 3.41 \times 10^{-4}$

Thus,

$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (2.75)^2} = \underline{\underline{-7.07 \times 10^{-5}}}$ (i.e., $-0.373 \frac{ft}{mi}$)

10.35

10.35 Water flows in a horizontal rectangular channel at a depth of 0.5 ft and a velocity of 8 ft/s. Determine the two possible depths at a location slightly downstream. Viscous effects between the water and the channel surface are negligible.

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{8\ ft/s}{\sqrt{32.2\ ft/s^2 (0.5\ ft)}} = 1.99$$

Thus, with $Fr_1 > 1$ there could be a hydraulic jump with $y_2 > y_1 = 0.5\ ft$. If so, then

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 Fr_1^2}\right] = \frac{1}{2}\left[-1 + \sqrt{1 + 8(1.99)^2}\right] = 2.36$$

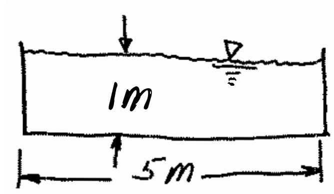
So that

$$y_2 = 2.36\, y_1 = 2.36(0.5\ ft) = 1.18\ ft$$

Hence, either $y_2 = 0.5\ ft$ (no jump) or $y_2 = 1.18\ ft$ (with jump)

10.36

10.36 Water flows in a 5-m-wide channel with a speed of 2 m/s and a depth of 1 m. The channel bottom slopes at a rate of 1 m per 1000 m. Determine the Manning coefficient for this channel.



(1) $V = \frac{K}{n} R_h^{2/3} \sqrt{S_o}$, where

$S_o = \frac{1m}{1000m} = 0.001$, $K = 1$, and

$R_h = \frac{A}{P} = \frac{1m(5m)}{(5m+1m+1m)} = \frac{5}{7}m$

Thus, with $V = 2\frac{m}{s}$, from Eq. (1)

$2 = \frac{1}{n}\left(\frac{5}{7}\right)^{2/3}\sqrt{0.001}$

or

$n = \underline{\underline{0.0126}}$

10.37

10.37 Fluid properties such as viscosity or density do not appear in the Manning equation (Eq. 10.20). Does this mean that this equation is valid for any open-channel flow such as that involving mercury, water, oil, or molasses? Explain.

The Manning equation, $Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2}$, was "derived" specifically for water. It is not in dimensionless form and cannot be use without alteration (i.e. different n values; different dependance on R_h; etc) for other fluids.

10.38

10.38 The following data are taken from measurements on Indian Fork Creek: $A = 26\text{ m}^2$, $P = 16$ m, and $S_0 = 0.02$ m/62 m. Determine the average shear stress on the wetted perimeter of this channel.

$\tau_w = \gamma R_h S_o$, where $R_h = \frac{A}{P}$

Thus, $\tau_w = \left(9800\,\frac{N}{m^3}\right)\left(\frac{26 m^2}{16 m}\right)\left(\frac{0.02 m}{62 m}\right) = \underline{\underline{5.14\,\frac{N}{m^2}}}$

10.39

10.39 The following data are obtained for a particular reach of the Provo River in Utah: A = 183 ft^2, free-surface width = 55 ft, average depth = 3.3 ft, R_h = 3.22 ft, V = 6.56 ft/s, length of reach = 116 ft, and elevation drop of reach = 1.04 ft. Determine the **(a)** average shear stress on the wetted perimeter, **(b)** the Manning coefficient, n, and **(c)** the Froude number of the flow.

a) $\tau_w = \gamma R_h S_o$, where $S_o = \frac{1.04\,ft}{116\,ft} = 0.00897$

Thus, $\tau_w = (62.4\,\frac{lb}{ft^3})(3.22ft)(0.00897) = \underline{\underline{1.80\,\frac{lb}{ft^2}}}$

b) $Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2} = AV$, where $K = 1.49$

Thus, $n = \frac{1.49 R_h^{2/3} S_o^{1/2}}{V} = \frac{(1.49)(3.22)^{2/3}(0.00897)^{1/2}}{6.56} = \underline{\underline{0.0469}}$

c) $Fr = \frac{V}{\sqrt{gy}} = \frac{6.56\,\frac{ft}{s}}{[(32.2\,\frac{ft}{s^2})(3.3ft)]^{1/2}} = \underline{\underline{0.636}}$ <1 (subcritical)

10.40

10.40 At a particular location the cross section of the Columbia River is as indicated in Fig. P10.40. If on a day without wind it takes 5 min to float 0.5 mi along the river, which drops 0.46 ft in that distance, determine the value of the Manning coefficient, n.

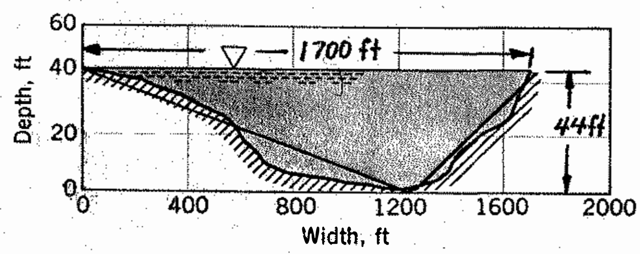


FIGURE P10.40

From the given data, $V = \dfrac{(0.5\,mi)(5280\,\frac{ft}{mi})}{(5\,min)(60\,\frac{s}{min})} = 8.8\,\frac{ft}{s}$.

From the Manning equation,

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49,\ S_o = \frac{0.46\,ft}{(0.5\,mi)(5280\,\frac{ft}{mi})} = 0.000174, \quad (1)$$

and $R_h = \frac{A}{P}$.

Approximate A and P from the figure as

$$A \approx \tfrac{1}{2} b y = \tfrac{1}{2}(1700\,ft)(44\,ft) = 37{,}400\,ft^2$$

and

$P \approx 1800$ ft Thus, $R_h \approx \dfrac{37{,}400\,ft^2}{1800\,ft} = 20.8$ ft

Hence, from Eq. (1):

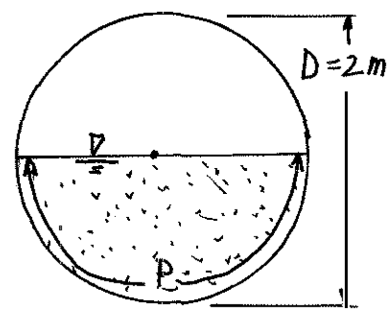
$$8.8 = \frac{1.49}{n}(20.8)^{2/3}(0.000174)^{1/2}$$

or

$$n = \underline{\underline{0.0169}}$$

10.41

10.41 A 2-m-diameter pipe made of finished concrete lies on a slope of 1 m elevation change per 1000 m horizontal distance. Determine the flowrate when the pipe is half full.



(1) $Q = AV = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}$, where $K = 1$ and $S_o = \frac{1m}{1000m} = 0.001$

Also, $A = \frac{1}{2}\left(\frac{\pi}{4}D\right)^2 = \frac{\pi}{8}(2m)^2 = 1.57\ m^2$

and $R_h = \frac{A}{P} = \frac{\frac{1}{2}\left(\frac{\pi}{4}D^2\right)}{\frac{\pi}{2}D} = \frac{D}{4} = \frac{2m}{4} = 0.5\ m$

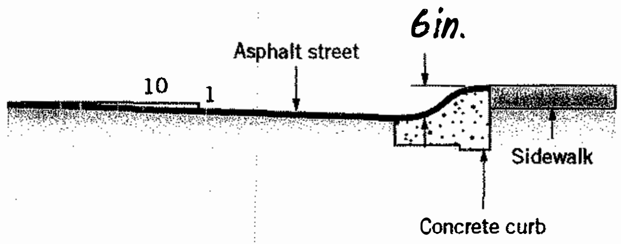
From Table 10.1, for finished concrete, $n = 0.012$

Thus, from Eq. (1),

$Q = \frac{1}{0.012}(1.57)(0.5)^{2/3}\sqrt{0.001} = \underline{\underline{2.61\ \frac{m^3}{s}}}$

10.42

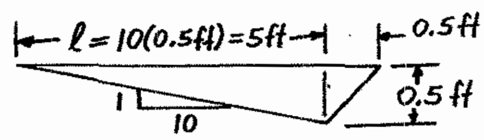
10.42 Rainwater flows down a street whose cross section is shown in Fig. P10.42. The street is on a hill at an angle of 2°. Determine the maximum flowrate possible if the water is not to overflow onto the sidewalk.



■ FIGURE P10.42

$$Q = \frac{\kappa}{n} A R_h^{\frac{2}{3}} \sqrt{S_0} \quad , \text{ where } \kappa = 1.49 \text{ and } S_0 = \tan 2^\circ = 0.0349 \qquad (1)$$

Although part of the channel is asphalt and part is concrete, since the value of n is similar for each, use $n = 0.016$, the value for asphalt. Also, approximate the cross section as a triangle as indicated.



Thus,

$$A \approx \tfrac{1}{2}(5 + 0.5)(0.5)\,ft^2 = 1.375\,ft^2$$

and

$$P \approx \sqrt{2}\,(0.5ft) + \sqrt{(5ft)^2 + (0.5ft)^2} = 5.73\,ft$$

so that

$$R_h = \frac{A}{P} = \frac{1.375\,ft^2}{5.73\,ft} = 0.240\,ft$$

Hence, from Eq. (1):

$$Q = \frac{1.49}{0.016}(1.375)(0.240)^{\frac{2}{3}}\sqrt{0.0349} = \underline{\underline{9.25\,\frac{ft^3}{s}}}$$

10.43

10.43 By what percent is the flowrate reduced in the rectangular channel shown in Fig. P10.43 because of the addition of the thin center board? All surfaces are of the same material.

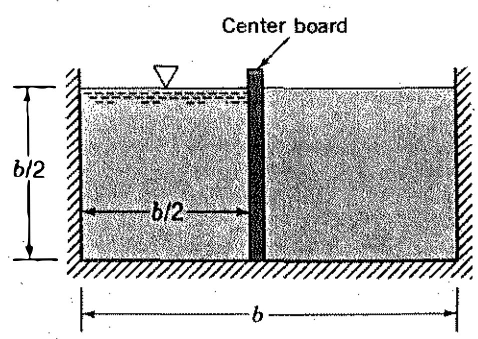


FIGURE P10.43

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$$

Without the centerboard $A = b\left(\frac{b}{2}\right) = \frac{b^2}{2}$, $R_h = \frac{A}{P} = \frac{\frac{b^2}{2}}{2b} = \frac{b}{4}$

or

$$Q_{without} = \frac{K}{n}\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3} S_o^{1/2} \qquad (1)$$

With the centerboard $Q_{with} = 2Q_2$, where $A_2 = \left(\frac{b}{2}\right)^2$,

$$R_{h_2} = \frac{A_2}{P_2} = \frac{\left(\frac{b}{2}\right)^2}{3\left(\frac{b}{2}\right)} = \frac{b}{6}$$

or

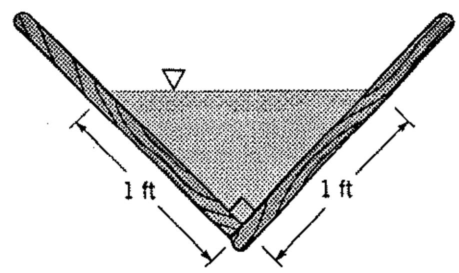
$$Q_{with} = 2\frac{K}{n}\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3} S_o^{1/2} \qquad (2)$$

Divide Eq. (2) by Eq. (1) to obtain $\frac{Q_{with}}{Q_{without}} = \frac{2\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3}}{\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3}} = 0.763$

a $100 - 76.3 = \underline{\underline{23.7\ \%\ \text{reduction}}}$

10.44

10.44 The great Kings River flume in Fresno County, California, was used from 1890 to 1923 to carry logs from an elevation of 4500 ft where trees were cut to an elevation of 300 ft at the railhead. The flume was 54 miles long, constructed of wood, and had a V-cross section as indicated in Fig. P10.44. It is claimed that logs would travel the length of the flume in 15 hours. Do you agree with this claim? Provide appropriate calculations to support your answer.



■ FIGURE P10.44

ℓ = distance traveled = $V_{log}\, t$. Thus,

$$V_{log} = \frac{\ell}{t} = \frac{(54\,mi)(5280\,ft/mi)}{(15\,hr)(3600\,s/hr)} = 5.28\,\frac{ft}{s}$$

Determine the average water velocity, V, and compare it with this V_{log}.

$$V = \frac{K}{n} R_h^{2/3} \sqrt{S_o}\,,\quad \text{where } K = 1.49,\ A = \tfrac{1}{2}(1\,ft^2) = 0.5\,ft^2,\ P = 2\,ft$$

so that $R_h = \frac{A}{P} = \frac{0.5\,ft^2}{2\,ft} = 0.25\,ft$

Also,

$$S_o = \frac{\Delta z}{\ell} = \frac{(4500-300)\,ft}{(54\,mi)(5280\,ft/mi)} = 0.0147$$

Thus, with $n = 0.012$ (see Table 10.1, planed wood),

$$V = \frac{1.49}{0.012}(0.25)^{2/3}\sqrt{0.0147} = 5.97\,\frac{ft}{s}$$

Note: V is slightly larger than V_{log}. Thus, the claim appears to be correct. <u>Yes.</u>

10.45

10.45 Water flows in a channel as shown in Fig. P10.45. The velocity is 4.0 ft/s when the channel is half full with depth d. Determine the velocity when the channel is completely full, depth $2d$.

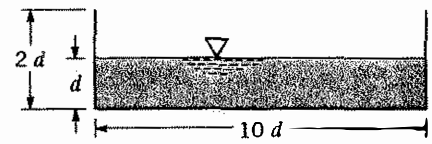


FIGURE P10.45

$$V = \frac{K}{n} R_h^{2/3} \sqrt{S_o} \quad \text{so that}$$

$$(1) \quad \frac{V_{full}}{V_{\frac{1}{2}full}} = \frac{\left(\frac{K}{n} R_h^{2/3} \sqrt{S_o}\right)_{full}}{\left(\frac{K}{n} R_h^{2/3} \sqrt{S_o}\right)_{\frac{1}{2}full}} = \left(\frac{R_{h\,full}}{R_{h\,\frac{1}{2}full}}\right)^{2/3}$$

since K, n, and S_o are the same for both the full and half-full conditions.

Also,

$$R_{h\,full} = \left(\frac{A}{P}\right)_{full} = \frac{(2d)(10d)}{(10d+2d+2d)} = \frac{10}{7} d$$

and

$$R_{h\,\frac{1}{2}full} = \left(\frac{A}{P}\right)_{\frac{1}{2}full} = \frac{(d)(10d)}{(10d+d+d)} = \frac{5}{6} d$$

Thus, from Eq. (1)

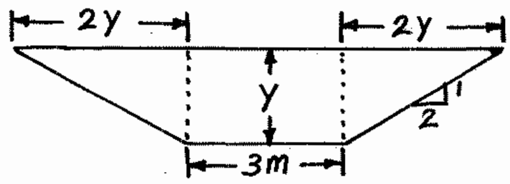
$$\frac{V_{full}}{V_{\frac{1}{2}full}} = \left(\frac{\frac{10}{7}d}{\frac{5}{6}d}\right)^{2/3} = 1.432$$

or

$$V_{full} = 1.432\, V_{\frac{1}{2}full} = 1.432\left(4\,\frac{ft}{s}\right) = \underline{\underline{5.73\,\frac{ft}{s}}}$$

10.46

10.46 A trapezoidal channel with a bottom width of 3.0 m and sides with a slope of 2:1 (horizontal:vertical) is lined with fine gravel (n = 0.020) and is to carry 10 m³/s. Can this channel be built with a slope of S_0 = 0.00010 if it is necessary to keep the velocity below 0.75 m/s to prevent scouring of the bottom? Explain.



Determine V with $Q = 10\,\frac{m^3}{s}$ and $S_0 = 0.00010$.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}\,, \text{ where } A = \tfrac{1}{2} y[3 + (3+4y)] = 2y^2 + 3y \qquad (0)$$

and $R_h = \frac{A}{P}$, with $P = 3 + 2(\sqrt{5}\,y)$

Thus,

$$10 = \frac{1}{0.02}(2y^2+3y)\left[\frac{2y^2+3y}{3+2\sqrt{5}\,y}\right]^{2/3}(0.0001)^{1/2}$$

or

$$20 = \frac{(2y^2+3y)^{5/3}}{(3+2\sqrt{5}\,y)^{2/3}} \quad \text{which can be written as}$$

$$2y^2 + 3y - 6.03\,(3 + 2\sqrt{5}\,y)^{0.4} = 0 \qquad (1)$$

A standard root-finding computer program gives the solution to Eq. (1) as
$y = 2.25\,m$

Hence, from Eq. (0) $A = 2(2.25)^2 + 3(2.25) = 16.9\,m^2$
so that

$$V = \frac{Q}{A} = \frac{10\,\frac{m^3}{s}}{16.9\,m^2} = 0.592\,\frac{m}{s}$$

Thus, $V < 0.75\,\frac{m}{s}$ so that scouring will not occur.

10.47

10.47 Water flows in a 2-m-diameter finished concrete pipe so that it is completely full and the pressure is constant all along the pipe. If the slope is $S_0 = 0.005$, determine the flowrate by using open-channel flow methods. Compare this result with that obtained by using pipe flow methods of Chapter 8.

For open channel flow $Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2}$, where $\kappa = 1$

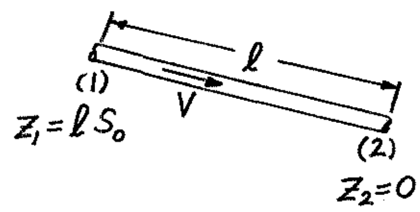
Also, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(2m)^2 = 3.14 m^2$ and $P = \pi D = 6.28 m$ so that

$$R_h = \frac{A}{P} = \frac{3.14 m^2}{6.28 m} = 0.5 m$$

Hence, with $n = 0.012$ for finished concrete (see Table 10.1)

$$Q = \frac{1}{0.012}(3.14)(0.5)^{2/3}(0.005)^{1/2} = \underline{\underline{11.7 \frac{m^3}{s}}} \text{ (open channel)}$$

For pipe flow with constant pressure:



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f \frac{\ell}{D} \frac{V^2}{2g}$$

where $p_1 = p_2$ and $V_1 = V_2$

Thus, with $z_1 - z_2 = \ell S_o$,

$$\ell S_o = f \frac{\ell}{D} \frac{V^2}{2g}$$

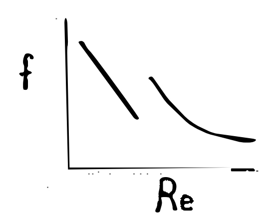
or

$$fV^2 = 2gDS_o = 2(9.81 \tfrac{m}{s^2})(2m)(0.005) \quad \text{Thus, } fV^2 = 0.196 \qquad (1)$$

From Table 8.1, for smooth concrete $\frac{\varepsilon}{D} = 0.3 \times 10^{-3} m / 2m = 1.5 \times 10^{-4}$

$$\text{Also, } Re = \frac{VD}{\nu} = \frac{V(2m)}{1.12 \times 10^{-6} \frac{m^2}{s}} = 1.79 \times 10^6 V \qquad (2)$$

and from the Moody chart (Fig. 8.20):



(3)

Solve Eqs. (1), (2), and (3) for f, V, Re:

Assume $f = 0.015$ so that from Eq.(1)

$$V = \left[\frac{0.196}{0.015}\right]^{1/2} = 3.61 \frac{m}{s}$$

or

$Re = 1.79 \times 10^6 (3.61) = 6.46 \times 10^6$ Thus, from Eq.(3) (Moody chart)

$f = 0.013 \neq 0.015$. Assume $f = 0.013$, or $V = \left[\frac{0.196}{0.013}\right]^{1/2} = 3.88 \frac{m}{s}$

so that $Re = 1.79 \times 10^6 (3.88) = 6.95 \times 10^6$ Thus, from Eq.(3) $f = 0.013$ (checks with the assumed value) Hence, $V = 3.88 \frac{m}{s}$ or

$$Q = AV = \frac{\pi}{4}(2m)^2 (3.88 \tfrac{m}{s}) = \underline{\underline{12.2 \frac{m^3}{s}}} \text{ (pipe flow)} \approx 11.7 \frac{m^3}{s} \text{ (open channel flow)}$$

10.48

10.48 Water flows in a weedy earthen channel at a rate of 30 m^3/s. What flowrate can be expected if the weeds are removed and the depth remains constant?

$Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2}$ Let $(\ )_{nw}$ denote no weeds; $(\ )_w$ denote with weeds.

Thus, since $A_w = A_{nw}$, $R_{hw} = R_{hnw}$ and $S_{ow} = S_{onw}$ it follows that

$$\frac{Q_w}{Q_{nw}} = \frac{\frac{\kappa}{n_w} A_w R_{hw}^{2/3} S_{ow}^{1/2}}{\frac{\kappa}{n_{nw}} A_{nw} R_{h_{nw}}^{2/3} S_{o_{nw}}^{1/2}} = \frac{n_{nw}}{n_w}$$

From Table 10.1 $n_w = 0.030$, $n_{nw} = 0.022$

or

$$Q_{nw} = \frac{n_w}{n_{nw}} Q_w = \frac{0.030}{0.022}\left(30\,\frac{m^3}{s}\right) = \underline{\underline{40.9\,\frac{m^3}{s}}}$$

10.49

10.49 A round concrete storm sewer pipe used to carry rainfall runoff from a parking lot is designed to be half full when the rainfall rate is a steady 1 in./hr. Will this pipe be able to handle the flow from a 2-in./hr rainfall without water backing up into the parking lot? Support your answer with appropriate calculations.

$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_o}$

Let $(\ )_1$ denote conditions when the pipe is half full and $(\ )_2$ when the pipe is full.

Thus, $A_1 = \frac{\pi}{8} D^2$, $R_{h_1} = A_1/P_1 = (\frac{\pi}{8} D^2)/(\frac{\pi}{2} D) = D/4$

and $A_2 = \frac{\pi}{4} D^2$, $R_{h_2} = A_2/P_2 = (\frac{\pi}{4} D^2)/(\pi D) = D/4$

Also, $S_{o_1} = S_{o_2}$ and $n_1 = n_2$

Therefore,

$$\frac{Q_1}{Q_2} = \frac{\frac{\kappa}{n_1} A_1 R_{h_1}^{2/3} \sqrt{S_{o_1}}}{\frac{\kappa}{n_2} A_2 R_{h_2}^{2/3} \sqrt{S_{o_2}}} = \frac{A_1 R_{h_1}^{2/3}}{A_2 R_{h_2}^{2/3}} = \frac{(\frac{\pi}{8} D^2)(\frac{D}{4})^{2/3}}{(\frac{\pi}{4} D^2)(\frac{D}{4})^{2/3}} = \frac{1}{2}$$

That is, $Q_2 = 2 Q_1$. The full pipe can carry twice that of the half-full pipe. It can carry the 2 in./hr rainfall.

10.50

10.50 A 10-ft-wide rectangular channel is built to bypass a dam so that fish can swim upstream during their migration. During normal conditions when the water depth is 4 ft, the water velocity is 5 ft/s. Determine the velocity during a flood when the water depth is 8 ft.

Let $(\ )_n$ and $(\ )_f$ denote normal and flood conditions, respectively. Thus,

(1) $V_n = \frac{k}{n_n} R_{h_n}^{2/3} \sqrt{S_{on}}$ and

(2) $V_f = \frac{k}{n_f} R_{h_f}^{2/3} \sqrt{S_{of}}$

where $n_n = n_f$, $S_{on} = S_{of}$ and

$A_n = 10\,ft(4\,ft) = 40\,ft^2$, $A_f = 10\,ft(8\,ft) = 80\,ft^2$

$P_n = 10\,ft + 2(4\,ft) = 18\,ft$, $P_f = 10\,ft + 2(8\,ft) = 26\,ft$

Thus, $R_{h_n} = \frac{A_n}{P_n} = \frac{40\,ft^2}{18\,ft} = 2.22\,ft$

and

$R_{h_f} = \frac{A_f}{P_f} = \frac{80\,ft^2}{26\,ft} = 3.08\,ft$

Hence, divide Eq(2) by Eq(1) to obtain:

$$\frac{V_f}{V_n} = \left(\frac{R_{hf}}{R_{hn}}\right)^{2/3} = \left(\frac{3.08\,ft}{2.22\,ft}\right)^{2/3} = 1.24$$

so that

$$V_f = 1.24\,V_n = 1.24\left(5\,\frac{ft}{s}\right) = \underline{\underline{6.22\,\frac{ft}{s}}}$$

10.52*

*10.52 Water flows in the painted steel rectangular channel with rounded corners shown in Fig. P10.52. The bottom slope is 1 ft/200 ft. Plot a graph of flowrate as a function of water depth for $0 \le y \le 1$ ft with corner radii of $r = 0, 0.2, 0.4, 0.6, 0.8$, and 1.0 ft.

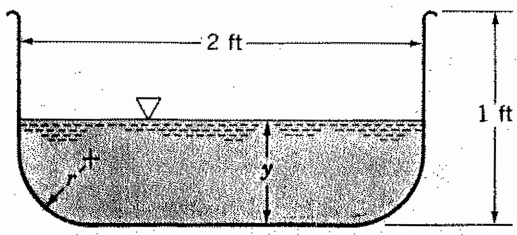
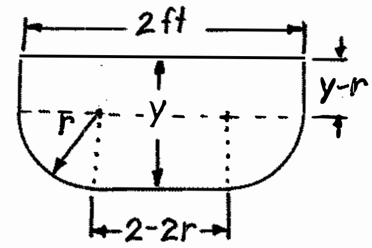


FIGURE P10.52

$Q = \frac{K}{n} A R_h S_o^{1/2}$, where $K = 1.49$, from Table 10.1 $n = 0.014$, and (1)

$S_o = \frac{1\,ft}{200\,ft} = 0.005$



(a) Assume $y \ge r$:

Thus, $A = 2(y-r) + r(2-2r) + \frac{1}{2}\pi r^2$

or $A = 2y - (2 - \frac{\pi}{2})r^2$ (2)

and $P = 2(y-r) + (2-2r) + \pi r$

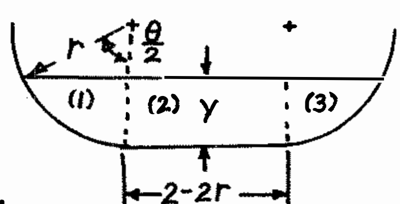
or $P = 2y - (4-\pi)r + 2$ (3)

Hence, with $R_h = \frac{A}{P}$ Eqs. (1), (2), and (3) give

$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$

or

$Q = 7.53 \frac{\left[2y - (2-\frac{\pi}{2})r^2\right]^{5/3}}{\left[2y - (4-\pi)r + 2\right]^{2/3}}$ for $r \le y \le 1$, where $r \sim ft$, $y \sim ft$, $Q \sim \frac{ft^3}{s}$ (4)



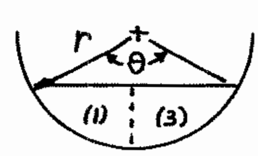
(b) Assume $y \le r$:

Thus, $A = A_1 + A_2 + A_3$

From Example 10.5, with $D = 2r$

$A_1 + A_3 = \frac{(2r)^2}{8}(\theta - \sin\theta)$ where $\theta \sim rad$ and

$\cos\frac{\theta}{2} = \frac{r-y}{r}$



Hence, $A = \frac{r^2}{2}(\theta - \sin\theta) + (2-2r)y$ (5)

Also, $P = 2 - 2r + P_1 + P_3$, where

from Example 10.5, $P_1 + P_3 = \frac{(2r)\theta}{2} = r\theta$

Thus, $P = 2 - 2r + r\theta = 2 + (\theta - 2)r$ (6)

By combing Eqs. (1, (5), and (6) we obtain:

$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$

or

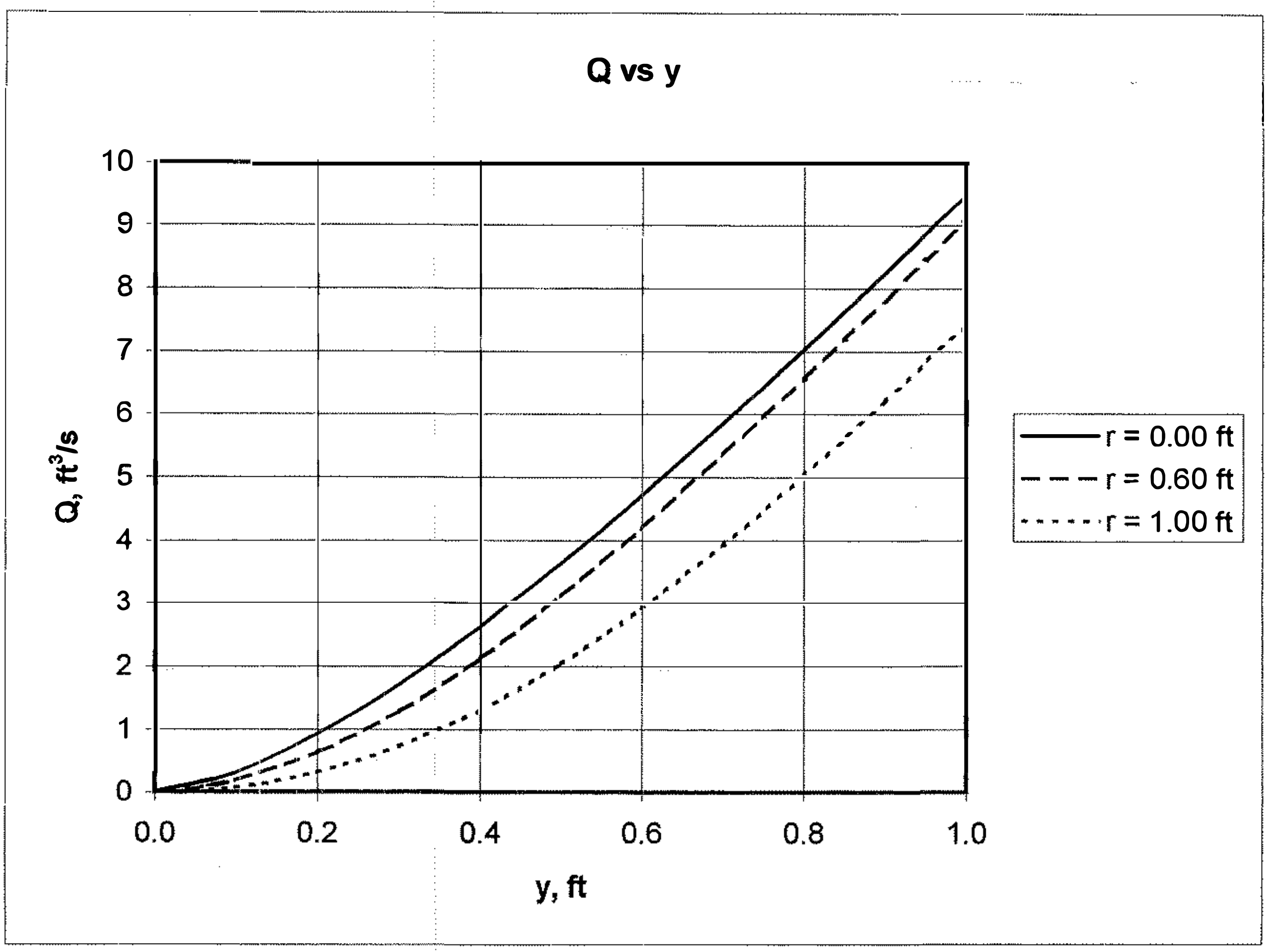
$Q = 7.53 \frac{\left[\frac{r^2}{2}(\theta - \sin\theta) + (2-2r)y\right]^{5/3}}{\left[2 + (\theta-2)r\right]^{2/3}}$ for $0 \le y \le r$, where $r \sim ft$, $y \sim ft$, (7)

$Q \sim \frac{ft^3}{s}$, and $\theta = 2\cos^{-1}\left(\frac{r-y}{r}\right) \sim rad$

(con't)

10.52* (con't)

The results, $Q=Q(y)$, are plotted below for $r = 0$, 0.6, and 1 ft.



10.53*

10.53* The cross section of a long tunnel carrying water through a mountain is as indicated in Fig. P10.53. Plot a graph of flowrate as a function of water depth, y, for $0 \le y \le 18$ ft. The slope is 2 ft/mi and the surface of the tunnel is rough rock (equivalent to rubble masonry). At what depth is the flowrate maximum? Explain.

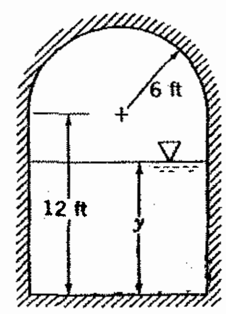


FIGURE P10.53

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49,\ S_o = \frac{2\text{ ft}}{5280\text{ ft}} = 0.000379, \quad (1)$$

and from Table 10.5 $n = 0.025$

(a) Assume $y \le 12$ ft : Thus, $A = 12y$ and $P = 2y + 12$

so that $R_h = \frac{A}{P} = \frac{12y}{2y+12} = \frac{6y}{y+6}$

Hence,

$$Q = \frac{1.49}{0.025}(12y)\left[\frac{6y}{y+6}\right]^{2/3}(0.000379)^{1/2}$$

or

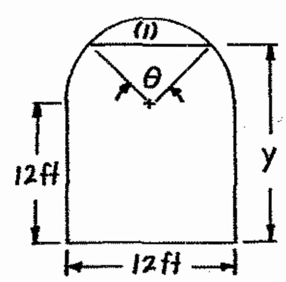
$$Q = 46.0\frac{y^{5/3}}{(y+6)^{2/3}}, \text{ for } y \le 12 \text{ where } y \sim \text{ft},\ Q \sim \frac{\text{ft}^3}{\text{s}} \quad (2)$$

(b) Assume $12 \le y \le 18$ ft :

Thus, $A = (12\text{ ft})^2 + \frac{\pi}{2}(6\text{ ft})^2 - A_1$, $\quad (3)$

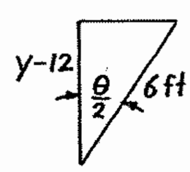
where from Example 10.5

$A_1 = \frac{D^2}{8}(\theta - \sin\theta)$, with $\cos\frac{\theta}{2} = \frac{y-12}{6}$



Hence, from Eq. (3)

$$A = 201\text{ft}^2 - \frac{(12\text{ ft})^2}{8}(\theta - \sin\theta)$$



or

$$A = 201 - 18(\theta - \sin\theta)\ \text{ft, where } \theta \sim \text{rad} \quad (4)$$

Also,

$$P = 3(12\text{ft}) + (\pi - \theta)(6\text{ft}) = 36 + 6(\pi - \theta)\ \text{ft} \quad (5)$$

Thus, with $R_h = \frac{A}{P}$ Eqs. (1), (4), and (5) give

$$Q = \frac{1.49}{0.025}\frac{[201 - 18(\theta - \sin\theta)]^{5/3}}{[36 + 6(\pi - \theta)]^{2/3}}(0.000379)^{1/2}$$

or

$$Q = 43.4\frac{(11.15 - \theta + \sin\theta)^{5/3}}{(9.14 - \theta)^{2/3}}, \text{ for } 12 \le y \le 18\text{ft, where } \theta \sim \text{rad}, \quad (6)$$

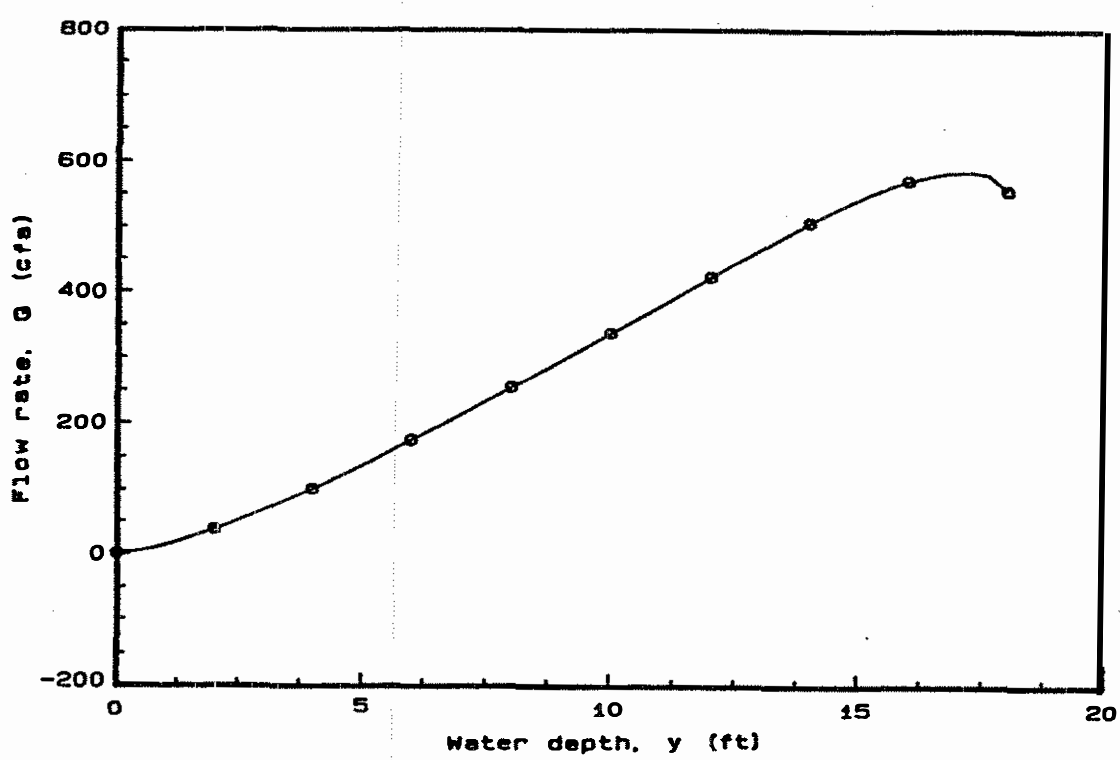
$Q \sim \frac{\text{ft}^3}{\text{s}}$, and $\theta = 2\cos^{-1}\left(\frac{y-12}{6}\right)$

For $0 \le y \le 18$ ft calculate $Q = Q(y)$ from either Eq. (2) and Eq. (6),

(con't)

10.53* (con't)

depending on the value of y. The results from this calculation are given below. The maximum flowrate, $Q_{max} = 583 \frac{ft^3}{s}$, occurs at $y = \underline{17.1 \; ft}$. For $17.1 ft \le y \le 18 ft$, an increase in depth adds only little to the flow area, A, but greatly increases the wetted perimeter, P. Thus, the retarding force is increased considerably.



10.54

10.54 The smooth concrete-lined channel shown in Fig. P10.54 is built on a slope of 2 m/km. Determine the flowrate if the depth is $y = 1.5$ m.

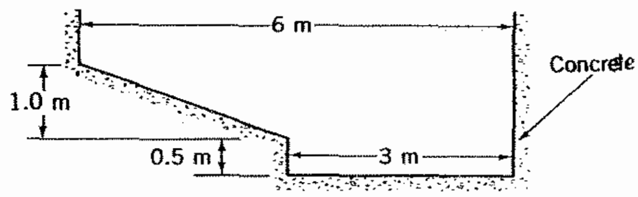


FIGURE P10.54

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1,\ S_o = \frac{2\,m}{1000\,m} = 0.002, \text{ and from Table 10.1} \qquad (1)$$

$n = 0.012$

With $y = 1.5\,m$, $A = (3m)(0.5m) + \frac{1}{2}(3m + 6m)(1.0m) = 6\,m^2$

and

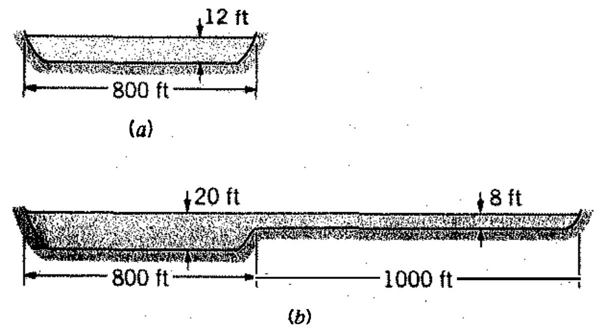
$$P = 1.5\,m + 3\,m + 0.5\,m + (1^2 + 3^2)^{\frac{1}{2}}\,m = 8.16\,m$$

Thus, $R_h = \frac{A}{P} = \frac{6\,m^2}{8.16\,m} = 0.735\,m$, and Eq. (1) gives

$$Q = \frac{1}{0.012}(6)(0.735)^{2/3}(0.002)^{1/2} = \underline{\underline{18.2\,\frac{m^3}{s}}}$$

10.55

*10.55 At a given location, under normal conditions a river flows with a Manning coefficient of 0.030 and a cross section as indicated in Fig. P10.55*a*. During flood conditions at this location, the river has a Manning coefficient of 0.040 (because of trees and brush in the floodplain) and a cross section as shown in Fig. P10.55*b*. Determine the ratio of the flowrate during flood conditions to that during normal conditions.



■ FIGURE P10.55

(1) $Q_a = \frac{K}{n_a} A_a R_{h_a}^{2/3} \sqrt{S_{oa}}$, where $A_a = 12\,ft(800\,ft) = 9600\,ft^2$, $P_a = 2(12\,ft) + 800\,ft = 824\,ft$,

so that $R_{h_a} = A_a / P_a = 9600\,ft^2/(824\,ft) = 11.65\,ft$

Similarly,

(2) $Q_b = \frac{K}{n_b} A_b R_{h_b}^{2/3} \sqrt{S_{ob}}$, where $A_b = 20\,ft(800\,ft) + 8\,ft(1000\,ft) = 24{,}000\,ft^2$,

$P_b = 800\,ft + 1000\,ft + 2(20\,ft) = 1840\,ft$ so that

$R_{h_b} = A_b / P_b = 24{,}000\,ft^2/(1840\,ft) = 13.04\,ft$

Thus, from Eqs. (1) and (2), with $S_{oa} = S_{ob}$,

$$(3)\quad \frac{Q_b}{Q_a} = \frac{\frac{K}{n_b} A_b R_{h_b}^{2/3} \sqrt{S_{ob}}}{\frac{K}{n_a} A_a R_{h_a}^{2/3} \sqrt{S_{oa}}} = \frac{n_a A_b R_{h_b}^{2/3}}{n_b A_a R_{h_a}^{2/3}}$$

By using the given and calculated data,

$$\frac{Q_b}{Q_a} = \left(\frac{0.03}{0.04}\right)\left(\frac{24{,}000\,ft^2}{9{,}600\,ft^2}\right)\left(\frac{13.04\,ft}{11.65\,ft}\right)^{2/3} = \underline{\underline{2.02}}$$

10.56

10.56 Repeat Problem 10.54 if the surfaces are smooth concrete as is indicated, except for the diagonal surface, which is gravelly with $n = 0.025$.

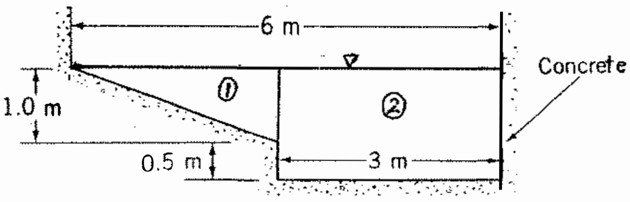


FIGURE P10.54

$$Q = Q_1 + Q_2 = \frac{K}{n_1} A_1 R_{h_1}^{2/3} S_o^{1/2} + \frac{K}{n_2} A_2 R_{h_2}^{2/3} S_o^{1/2}, \text{ where } K=1,\ S_o = 0.002, \tag{1}$$

$n_1 = 0.025$, and from Table 10.1 $n_2 = 0.012$

Also, $A_1 = \frac{1}{2}(1.0\,m)(3m) = 1.50\,m^2$, $P_1 = (1.0^2 + 3.0^2)^{\frac{1}{2}} m = 3.16\,m$

or $R_{h_1} = \frac{A_1}{P_1} = \frac{1.50\,m^2}{3.16\,m} = 0.475\,m$

and

$A_2 = (3m)(1.5m) = 4.5\,m^2$, $P_2 = 0.5m + 3m + 1.5m = 5m$

or $R_{h_2} = \frac{A_2}{P_2} = \frac{4.5\,m^2}{5m} = 0.90\,m$

Hence, from Eq.(1):

$$Q = \frac{1}{0.025}(1.50)(0.475)^{2/3}(0.002)^{1/2} + \frac{1}{0.012}(4.5)(0.90)^{2/3}(0.002)^{1/2}$$

or

$$Q = \underline{\underline{17.3\ \frac{m^3}{s}}}$$

Note: With all surfaces concrete, $Q = 18.2\ \frac{m^3}{s}$ (see Problem 10.64).

10.57*

10.57* Water flows through the storm sewer shown in Fig. P10.57. The slope of the bottom is 2 m/400 m. Plot a graph of the flowrate as a function of depth for $0 \le y \le 1.7$ m. On the same graph plot the flowrate expected if the entire surface were lined with material similar to that of a clay tile.

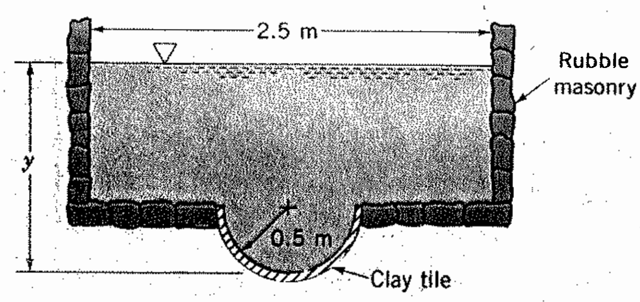


FIGURE P10.57

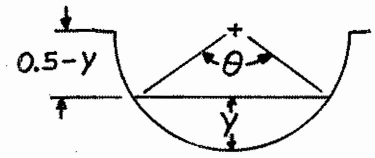
(a) For $0 \le y = 0.5\,m$: The flow is the same as that in a circular pipe. Thus, from Example 10.5 with $D = 1\,m$, $K = 1$, and $n = 0.014$ (Table 10.1):

$$Q = \frac{K}{n} S_o^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} = \frac{1}{0.014}\left(\frac{2\,m}{400\,m}\right)^{1/2} \frac{(1)^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}$$

or

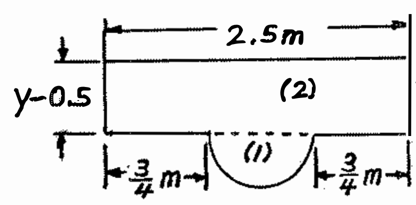
$$Q = 0.251 \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} \frac{m^3}{s}, \text{ where } \theta \sim rad \qquad (1)$$

and $\theta = 2\cos^{-1}\left(\frac{0.5 - y}{0.5}\right)$



(b) For $y \ge 0.5\,m$:

$Q = Q_1 + Q_2$, where



$Q_1 = \frac{K}{n_1} A_1 R_{h_1}^{2/3} S_o^{1/2}$, with $n_1 = 0.014$,

$A_1 = \frac{\pi}{2}(0.5m)^2 = 0.393\,m^2$, $P_1 = \pi(0.5) = 1.57\,m$ so that

$R_{h_1} = \frac{A_1}{P_1} = \frac{0.393\,m^2}{1.57\,m} = 0.250\,m$

Thus,

$$Q_1 = \frac{1}{0.014}(0.393)(0.250)^{2/3}\left(\frac{2}{400}\right)^{1/2} = 0.787\,\frac{m^3}{s}$$

Also,

$$Q_2 = \frac{K}{n_2} A_2 R_{h_2}^{2/3} S_o^{1/2}, \text{ with } n_2 = 0.025 \text{ (see Table 10.1)} \qquad (2)$$

$A_2 = (2.5m)(y - 0.5) = 2.5y - 1.25$ and $P_2 = 2(y-0.5) + 2\left(\frac{3}{4}\right) = 2y + 0.5$

Hence, with $R_{h_2} = \frac{A_2}{P_2}$, Eq. (2) becomes

$$Q_2 = \frac{1}{0.025}(2.5y - 1.25)^{5/3} \frac{1}{(2y+0.5)^{2/3}} \left(\frac{2}{400}\right)^{1/2} = 13.0 \frac{(y-0.5)^{5/3}}{(2y+0.5)^{2/3}}$$

Therefore,

$$Q = 0.787 + 13.0 \frac{(y-0.5)^{5/3}}{(2y+0.5)^{2/3}} \; \frac{m^3}{s} \text{ for } y \ge 0.5\,m \qquad (3)$$

Plot $Q = Q(y)$ for $0 \le y \le 1.7\,m$ using Eqs. (1) and (3).

(con't)

10.57* (con't)

If the entire surface were lined with material with $n_1 = n_2 = 0.014$, Eqn. (1) would remain valid. The coefficient "13.0" in Eq. (3) would become $13.0\left(\frac{0.025}{0.014}\right) = 23.2$. For this case,

$$Q = 0.787 + 23.2\,\frac{(y-0.5)^{5/3}}{(2y+0.5)^{2/3}}\ \frac{m^3}{s} \text{ for } y \geq 0.5m \qquad (4)$$

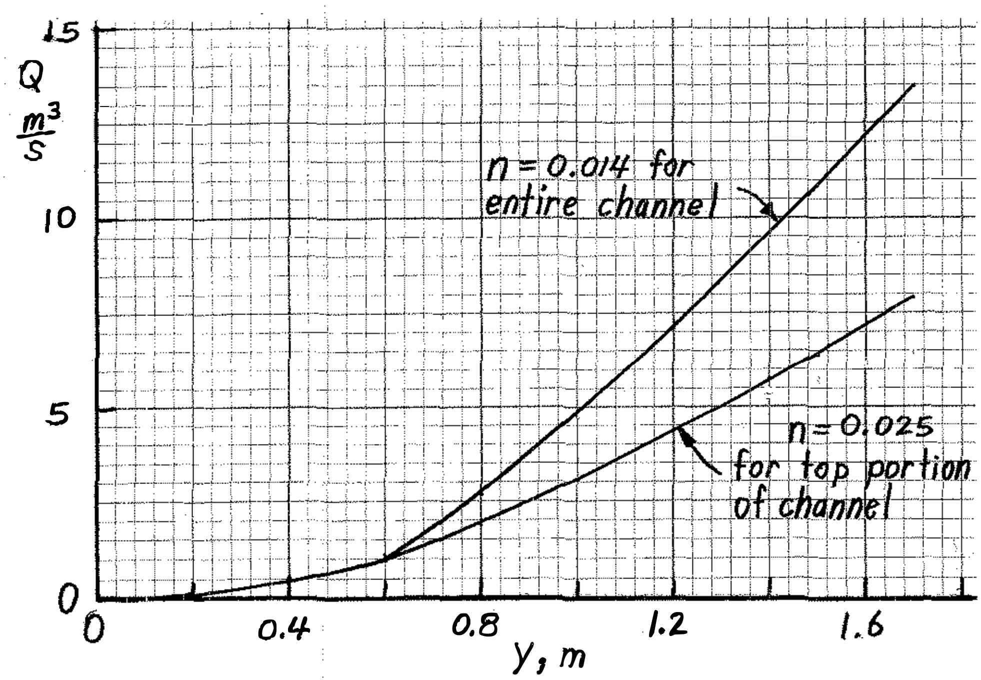
This result is also plotted (i.e. Q from Eq. (1) for $0 \leq y \leq 0.5$, and Q from Eq. (4) for $0.5 < y \leq 1.7m$).

With n = 0.025 for part of the channel

y, m	Q, m3/s		
0.0	7.552E-11	0.9	2.407E+00
0.1	3.293E-02	1.0	3.010E+00
0.2	1.381E-01	1.1	3.649E+00
0.3	3.089E-01	1.2	4.315E+00
0.4	5.315E-01	1.3	5.003E+00
0.5	7.870E-01	1.4	5.708E+00
0.6	9.837E-01	1.5	6.426E+00
0.7	1.367E+00	1.6	7.157E+00
0.8	1.853E+00	1.7	7.897E+00

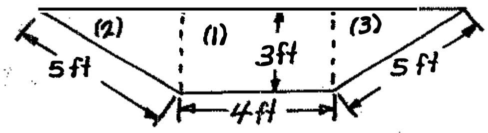
With n = 0.014 for the entire channel

y, m	Q, m3/s		
0.0	7.552E-11	0.9	3.678E+00
0.1	3.293E-02	1.0	4.754E+00
0.2	1.381E-01	1.1	5.894E+00
0.3	3.089E-01	1.2	7.083E+00
0.4	5.315E-01	1.3	8.310E+00
0.5	7.870E-01	1.4	9.568E+00
0.6	1.138E+00	1.5	1.085E+01
0.7	1.822E+00	1.6	1.215E+01
0.8	2.689E+00	1.7	1.348E+01



10.58

10.58 Determine the flowrate for the symmetrical channel shown in Fig. P10.80 if the bottom is smooth concrete and the sides are weedy. The bottom slope is $S_0 = 0.001$.



$Q = Q_1 + Q_2 + Q_3 = Q_1 + 2Q_2$, where $Q_i = \frac{K}{n_i} A_i R_{h_i}^{2/3} S_0^{1/2}$ with $K = 1.49$

Also, $A_1 = (3\,ft)(4\,ft) = 12\,ft^2$, $A_2 = \frac{1}{2}(3\,ft)(4\,ft) = 6\,ft^2$, $P_1 = 4\,ft$, and $P_2 = 5\,ft$,

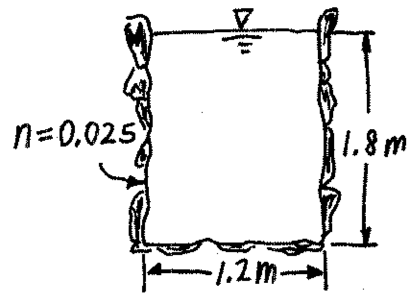
so that $R_{h_1} = \frac{A_1}{P_1} = \frac{12\,ft^2}{4\,ft} = 3\,ft$ and $R_{h_2} = \frac{A_2}{P_2} = \frac{6\,ft^2}{5\,ft} = 1.2\,ft$

Hence, with $n_1 = 0.012$ and $n_2 = 0.030$ (see Table 10.1) we obtain:

$$Q = \frac{1.49}{0.012}(12)(3)^{2/3}(0.001)^{1/2} + (2)\frac{1.49}{0.030}(6)(1.2)^{2/3}(0.001)^{1/2} = \underline{\underline{119\ \frac{ft^3}{s}}}$$

10.59

10.59 (See Fluids in the News article titled "Done without a GPS or lasers," Section 10.4.3.) Determine the number of gallons of water delivered per day by a rubble masonary, 1.2-m-wide aqueduct laid on an average slope of 14.6 m per 50 km if the water depth is 1.8 m.



$$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_o},$$

where $A = 1.2\,m\,(1.8\,m) = 2.16\,m^2$ and

$P = 1.2\,m + 2(1.8\,m) = 4.8\,m$ so that

$R_h = A/P = (2.16\,m^2)/(4.8\,m) = 0.450\,m$

Thus, with $\kappa = 1$,

$$Q = \frac{1}{0.025}(2.16\,m^2)(0.450\,m)^{2/3}\left(\frac{14.6\,m}{50\times10^3\,m}\right)^{1/2} = 0.867\,m^3/s$$

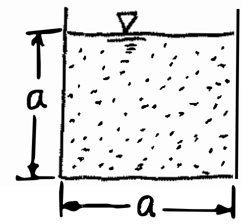
or

$$Q = 0.867\frac{m^3}{s}\left(\frac{3600\,s}{1\,hr}\right)\left(\frac{24\,hr}{1\,day}\right)\left(\frac{1\,ft^3}{0.0283\,m^3}\right)\left(\frac{1728\,in.^3}{1\,ft^3}\right)\left(\frac{1\,gal}{231\,in.^3}\right)$$

$$= 19.8\times10^6\,gal/day$$

10.60

10.60 Water flows in a rectangular, finished concrete channel at a rate of 2 m³/s. The bottom slope is 0.001. Determine the channel width if the water depth is to be equal to its width.



(1) $Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}$, where $S_o = 0.001$ and $Q = 2\,\frac{m^3}{s}$

Also, $K = 1$ and from Table 10.1, $n = 0.012$

For the square channel

$A = a^2$ and $R_h = \frac{A}{P} = \frac{a^2}{3a} = \frac{a}{3}$

Thus, from Eq. (1)

$$2\,\frac{m^3}{s} = \frac{1}{0.012}\, a^2 \left(\frac{a}{3}\right)^{2/3} \sqrt{0.001}$$

or

$$a^{8/3} = 1.58$$

Hence,

$$a = \underline{\underline{1.19\,m}}$$

10.61

10.61 An old, rough-surfaced, 2-m-diameter concrete pipe with a Manning coefficient of 0.025 carries water at a rate of 5.0 m^3/s when it is half full. It is to be replaced by a new pipe with a Manning coefficient of 0.012 that is also to flow half full at the same flowrate. Determine the diameter of the new pipe.

$$Q_{old} = \frac{\kappa}{n_{old}} A_{old} R_{h_{old}}^{2/3} \sqrt{S_{o_{old}}} \quad (1)$$

and

$$Q_{new} = \frac{\kappa}{n_{new}} A_{new} R_{h_{new}}^{2/3} \sqrt{S_{o_{new}}} \quad (2)$$

where $Q_{old} = Q_{new}$ and $S_{o_{old}} = S_{o_{new}}$

Thus, by equating Eqs. (1) and (2),

$$\frac{A_{old} R_{h_{old}}^{2/3}}{n_{old}} = \frac{A_{new} R_{h_{new}}^{2/3}}{n_{new}} \quad (3)$$

But for a half full pipe, $A = \frac{\pi}{8} D^2$ and $R_h = \frac{A}{P} = \frac{\frac{\pi}{8} D^2}{\frac{\pi}{2} D} = \frac{D}{4}$

Thus,

$A R_h^{2/3} = \frac{\pi}{8} D^2 \left(\frac{D}{4}\right)^{2/3}$ so that Eq. (3) becomes

$$\frac{\frac{\pi}{8} D_{old}^2 \left(\frac{D_{old}}{4}\right)^{2/3}}{n_{old}} = \frac{\frac{\pi}{8} D_{new}^2 \left(\frac{D_{new}}{4}\right)^{2/3}}{n_{new}}$$

or

$$\frac{D_{old}^{8/3}}{n_{old}} = \frac{D_{new}^{8/3}}{n_{new}}$$

Thus,

$$D_{new} = \left(\frac{n_{new}}{n_{old}}\right)^{3/8} D_{old} = \left(\frac{0.012}{0.025}\right)^{3/8} (2\ m) = \underline{\underline{1.52\ m}}$$

10.62

10.62 Four sewer pipes of 0.5-m diameter join to form one pipe of diameter D. If the Manning coefficient, n, and the slope are the same for all of the pipes, and if each pipe flows half-full, determine D.

$Q_1 = 4Q_o$, where $Q_1 = \frac{K}{n_1} A_1 R_{h_1}^{2/3} S_{o_1}^{1/2}$ and $Q_o = \frac{K}{n_o} A_o R_{h_o}^{2/3} S_{oo}^{1/2}$, with (1)

$n_1 = n_o$, $S_{o1} = S_{oo}$, $A_1 = \frac{\pi}{8}D^2$, $R_{h1} = \frac{A_1}{P_1} = \frac{\frac{\pi}{8}D^2}{\frac{\pi}{2}D} = \frac{D}{4}$,

$A_o = \frac{\pi}{8}(0.5)^2$, and $R_{h_o} = \frac{A_o}{P_o} = \frac{0.5}{4}$

Thus, from Eq. (1)

$A_1 R_{h_1}^{2/3} = 4 A_o R_{h_o}^{2/3}$, or

$\frac{\pi}{8}D^2\left(\frac{D}{4}\right)^{2/3} = 4\,\frac{\pi}{8}(0.5)^2\left(\frac{0.5}{4}\right)^{2/3}$

Hence,

$D^{8/3} = 4\left(\frac{1}{2}\right)^{8/3}$, or $D = \underline{\underline{0.841\ m}}$

10.63

10.63 The flowrate in the clay-lined channel (n = 0.025) shown in Fig. P10.63 is to be 300 ft³/s. To prevent erosion of the sides, the velocity must not exceed 5 ft/s. For this maximum velocity, determine the width of the bottom, b, and the slope, S_0.

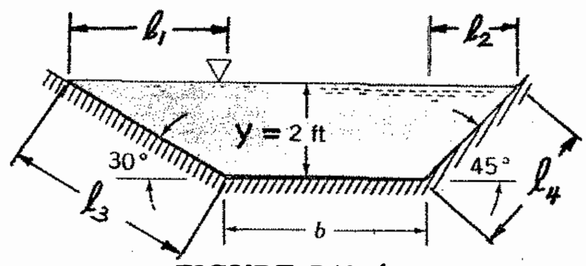


FIGURE P10.63

$V = \frac{Q}{A}$, where $A = \frac{1}{2}[b + (b + \ell_1 + \ell_2)]y$ with $\ell_1 = \frac{2\,ft}{\tan 30°} = 3.46\,ft$ (1)

and $\ell_2 = \frac{2\,ft}{\tan 45°} = 2\,ft$

Thus, $5\,\frac{ft}{s} = \frac{300\,\frac{ft^3}{s}}{\frac{1}{2}[b + (b + 3.46\,ft + 2\,ft)](2\,ft)}$, or $b = \underline{\underline{27.3\,ft}}$

Also, $V = \frac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1, $n = 0.025$ (2)

From Eq. (1), $A = \frac{1}{2}[2(27.3\,ft) + 3.46\,ft + 2\,ft](2\,ft) = 60.0\,ft^2$

Also, $P = b + \ell_3 + \ell_4 = 27.3\,ft + \frac{2\,ft}{\sin 30°} + \frac{2\,ft}{\sin 45°} = 34.1\,ft$

Thus, $R_h = \frac{A}{P} = \frac{60.0\,ft^2}{34.1\,ft} = 1.76\,ft$ so that Eq. (2) becomes

$$5 = \frac{1.49}{0.025}(1.76)^{2/3} S_o^{1/2}, \text{ or } S_o = \underline{\underline{0.00331}}$$

10.64

10.64 Overnight a thin layer of ice forms on the surface of a 40-ft-wide river that is essentially of rectangular cross-sectional shape. Under these conditions the flow depth is 3 ft. During the following day the sun melts the ice cover. Determine the new depth if the flowrate remains the same and the surface roughness of the ice is essentially the same as that for the bottom and sides of the river.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}$$

Let $(\ )_i$ denote conditions with the ice cover and $(\ )_n$ with no ice cover. Thus,

$A_i = (40\,ft)(3\,ft) = 120\,ft^2$, $P_i = 2(40\,ft) + 2(3\,ft) = 86\,ft$,

and $R_{h_i} = A_i / P_i = 120\,ft^2 / 86\,ft = 1.395\,ft$

Also,

$A_n = 40y$, $P_n = 40 + 2y$, and $R_{h_n} = A_n / P_n = 40y/(40+2y)$

Hence, since $Q_i = Q_n$ it follows that

$$\frac{K}{n_i} A_i R_{h_i}^{2/3} \sqrt{S_{o_i}} = \frac{K}{n_n} A_n R_{h_n}^{2/3} \sqrt{S_{o_n}}$$

so that with $n_i = n_n$ and $S_{o_i} = S_{o_n}$ this becomes

$$A_i R_{h_i}^{2/3} = A_n R_{h_n}^{2/3}$$

Hence,

$$120(1.395)^{2/3} = 40y\left(\frac{40y}{40+2y}\right)^{2/3}$$

or

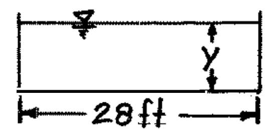
$$3.75 = y\left(\frac{40y}{40+2y}\right)^{2/3} \qquad (1)$$

A standard root-finding computer program gives the solution to Eq. (1) as

$y = \underline{\underline{2.31\,ft}}$

10.65

10.65 A rectangular unfinished concrete channel of 28-ft-width is laid on a slope of 8 ft/mi. Determine the flow depth and Froude number of the flow if the flowrate is 400 ft³/s.



$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49,\ S_o = \frac{8\text{ ft}}{5280\text{ ft}} = 0.001515, \text{ and}$$

from Table 10.1 $n = 0.014$

Also, $A = 28y$ and $P = 2y + 28$ so that $R_h = \frac{A}{P} = \frac{28y}{2y+28}$

Thus,
$$400 = \frac{1.49}{0.014}\left(\frac{28y}{2y+28}\right)^{2/3}(28y)(0.001515)^{1/2}$$

or
$$0.594 = \frac{y^{5/3}}{(y+14)^{2/3}}$$

Hence,
$$0.458(y+14) - y^{5/2} = 0 \qquad (1)$$

The solution to Eq. (1) is $y = \underline{\underline{2.23\text{ ft}}}$

Thus,
$$V = \frac{Q}{A} = \frac{400\frac{\text{ft}^3}{\text{s}}}{(28\text{ ft})(2.23\text{ ft})} = 6.41\frac{\text{ft}}{\text{s}}$$

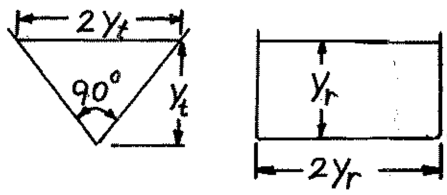
so that
$$Fr = \frac{V}{\sqrt{gy}} = \frac{6.41\frac{\text{ft}}{\text{s}}}{\left[(32.2\frac{\text{ft}}{\text{s}^2})(2.23\text{ ft})\right]^{1/2}} = \underline{\underline{0.756}}$$

10.66

10.66 An engineer is to design a channel lined with planed wood to carry water at a flowrate of 2 m³/s on a slope of 10 m/800 m. The channel cross section can be either a 90° triangle or a rectangle with a cross section twice as wide as its depth. Which would require less wood and by what percent?

$$Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2} \qquad (1)$$

Let $(\ )_t$ denote the triangular cross-section and $(\ )_r$ denote the rectangular cross-section



Thus, $Q_r = Q_t = 2\,\frac{m^3}{s}$, $S_{or} = S_{ot} = \frac{10}{800}$

and $n_r = n_t$ so that Eq. (1) gives

$$A_r R_{h_r}^{2/3} = A_t R_{h_t}^{2/3}, \quad \text{where } R_h = \frac{A}{P} \qquad (2)$$

Hence,

$A_r = 2y_r^2$, $P_r = 4y_r$ so that $R_{h_r} = \frac{2y_r^2}{4y_r} = \frac{1}{2}y_r$

Also,

$A_t = \frac{1}{2}(2y_t)y_t = y_t^2$, $P_t = 2(\sqrt{2}\,y_t)$ so that $R_{h_t} = \frac{y_t}{2\sqrt{2}}$

Thus, from Eq. (2):

$2y_r^2\left(\frac{1}{2}y_r\right)^{2/3} = y_t^2\left(\frac{1}{2\sqrt{2}}y_t\right)^{2/3}$, or $y_r = 0.707\,y_t$

The amount of wood is proportional to the wetted perimeter, P.

Since

$$\frac{P_t}{P_r} = \frac{2\sqrt{2}\,y_t}{4y_r} = \frac{2\sqrt{2}\,y_t}{4(0.707)y_t} = 1.00$$

the triangle requires the same amount of wood as the rectangle

10.67

10.67 A circular finished concrete culvert is to carry a discharge of 50 ft³/s on a slope of 0.0010. It is to flow not more than half-full. The culvert pipes are available from the manufacture with diameters that are multiples of 1 ft. Determine the smallest suitable culvert diameter.

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = 0.001$, and (from Table 10.1) $n = 0.012$

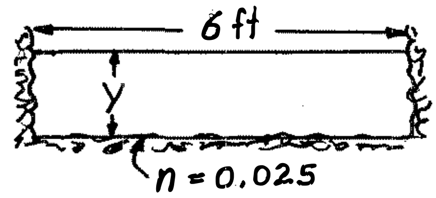
For a circular pipe half full $A = \frac{\pi}{8} D^2$, $P = \frac{\pi}{2} D$ so that $R_h = \frac{A}{P} = \frac{D}{4}$

Thus, $50.0 = \frac{1.49}{0.012}\left(\frac{\pi}{8} D^2\right)\left(\frac{D}{4}\right)^{2/3}(0.001)^{1/2}$, or $D = 5.12 \text{ ft}$

To make sure it is not more than half full use the <u>6 ft diameter</u> pipe.

10.68

10.68 At what depth will 50 ft^3/s of water flow in a 6-ft-wide rectangular channel lined with rubble masonry set on a slope of 1 ft in 500 ft? Is a hydraulic jump possible under these conditions? Explain.



$$Q = \frac{1.49}{n} A R_h^{2/3} \sqrt{S_0} \quad \text{where}$$

$$A = 6y\,, \quad R_h = \frac{A}{P} = \frac{6y}{2y+6}\,, \quad S_0 = \frac{1\ ft}{500\ ft}$$

and

$n = 0.025$ (see Table 10.1)

Thus,

$$50 = \frac{1.49}{0.025}(6y)\left[\frac{6y}{2y+6}\right]^{2/3}(0.002)^{1/2}$$

which becomes

$$y^{5/3} = (2y+6)^{2/3}(0.948)$$

By use of a root-finding computer program, the solution is

$$\underline{\underline{y = 2.53\ ft}}$$

Thus, $V = \frac{Q}{A} = \frac{50\ ft^3/s}{6(2.53)\ ft^2} = 3.29\ ft/s$

so that

$$Fr = \frac{V}{\sqrt{gy}} = \frac{3.29\ ft/s}{\left[(32.2\ ft/s^2)(2.53\ ft)\right]^{1/2}} = 0.365$$

Since $Fr < 1$ it is $\underline{\underline{\text{not possible}}}$ to have a hydraulic jump.

10.69

10.69 The rectangular canal shown in Fig. P10.69 changes to a round pipe of diameter D as it passes through a tunnel in a mountain. Determine D if the surface material and slope remain the same and the round pipe is to flow completely full.

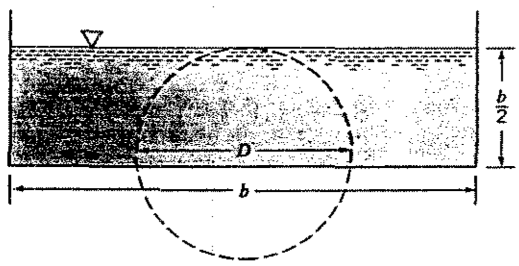


FIGURE P10.69

$Q = \frac{\kappa}{n} A R_h^{\frac{2}{3}} \sqrt{S_o}$ (1)

Let $(\ )_r$ denote the rectangular channel and $(\ )_c$ denote the circular pipe

Thus, since $Q_r = Q_c$, $n_r = n_c$, $\kappa_r = \kappa_c$, $S_{or} = S_{oc}$ it follows from Eq. (1) that

$A_r R_{h_r}^{\frac{2}{3}} = A_c R_{h_c}^{\frac{2}{3}}$ where $A_r = \frac{b^2}{2}$ and $P_r = b + 2\left(\frac{b}{2}\right) = 2b$

so that $R_{h_r} = \frac{A_r}{P_r} = \frac{b}{4}$

and

$A_c = \frac{\pi}{4} D^2$ and $P_c = \pi D$

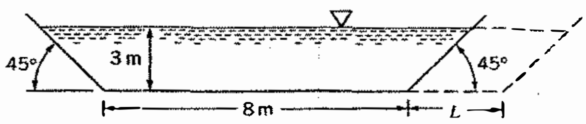
so that $R_{h_c} = \frac{A_c}{P_c} = \frac{D}{4}$

Thus,

$\left(\frac{b}{2}\right)\left(\frac{b}{4}\right)^{\frac{2}{3}} = \left(\frac{\pi}{4} D^2\right)\left(\frac{D}{4}\right)^{\frac{2}{3}}$ or $D = \left(\frac{2}{\pi}\right)^{\frac{3}{8}} b = \underline{\underline{0.844 b}}$

10.70

10.70 The flowrate through the trapezoidal canal shown in Fig. P10.70 is Q. If it is desired to double the flowrate to $2Q$ without changing the depth, determine the additional width, L, needed. The bottom slope, surface material, and the slope of the walls are to remain the same.



■ FIGURE P10.70

Initially: $Q_i = \frac{\kappa}{n} A_i R_{h_i}^{\frac{2}{3}} \sqrt{S_o}$

where $A_i = (3m)(8m) + 2\left[\frac{1}{2}(3m)(3m)\right] = 33\ m^2$

and $P_i = 8m + 2\left[\sqrt{2}\,(3m)\right] = 16.49\,m$

Thus, $R_{h_i} = \frac{A_i}{P_i} = \frac{33\ m^2}{16.49\ m} = 2.00\,m$

so that

$$Q_i = \frac{\kappa\sqrt{S_o}}{n}(33)(2.00)^{2/3} = 52.4\,\frac{\kappa\sqrt{S_o}}{n} \qquad (1)$$

Finally: $Q_f = \frac{\kappa}{n} A_f R_{h_f}^{\frac{2}{3}} \sqrt{S_o}$ $\quad$ (κ, n, S_o are constant)

where $A_f = (3m)(8+L)m + 2\left[\frac{1}{2}(3m)(3m)\right] = 33+3L\ m^2$, where $L \sim m$

and $P_f = P_i + L = 16.49 + L\ m$

Thus, $R_{h_f} = \frac{A_f}{P_f} = \frac{33+3L}{16.49+L}\ m$

so that

$$Q_f = \frac{\kappa\sqrt{S_o}}{n}(33+3L)\left(\frac{33+3L}{16.49+L}\right)^{\frac{2}{3}} \qquad (2)$$

But $Q_f = 2Q_i$, so that from Eqs. (1) and (2):

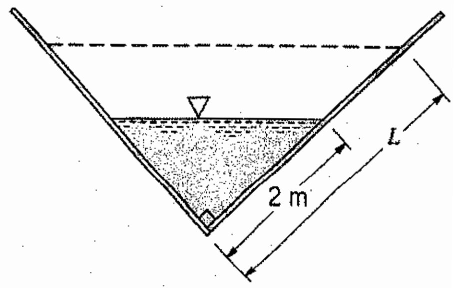
$$(33+3L)\left(\frac{33+3L}{16.49+L}\right)^{\frac{2}{3}} = 2(52.4) = 104.8$$

A standard computer root-finding program gives the solution to this equation as

$L = 8.77\,m$

10.71

10.71 When the channel of triangular cross section shown in Fig. P10.71 was new, a flowrate of Q caused the water to reach $L = 2$ m up the side as indicated. After considerable use, the walls of the channel became rougher and the Manning coefficient, n, doubled. Determine the new value of L if the flowrate stayed the same.



■ FIGURE P10.71

$$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_o} \qquad (1)$$

Let $(\ )_o$ and $(\ )_n$ represent the old and new conditions.

Thus, $n_o = 2n_n$, $A_n = \frac{1}{2}(2m)^2 = 2m^2$, $P_n = 4m$, so that

$$R_{h_n} = A_n/P_n = (2m^2)/(4m) = \frac{1}{2}m$$

Also, $A_o = \frac{1}{2}L^2$, $P_o = 2L$, so that $R_{h_o} = A_o/P_o = (\frac{1}{2}L^2)/(2L) = L/4$

Therefore, using Eq.(1) with $Q_o = Q_n$ gives

$$\frac{\kappa}{n_o} A_o R_{h_o}^{2/3} \sqrt{S_{oo}} = \frac{\kappa}{n_n} A_n R_{h_n}^{2/3} \sqrt{S_{on}}$$

or since $S_{on} = S_{oo}$,

$$\frac{1}{n_o} A_o R_{h_o}^{2/3} = \frac{1}{n_n} A_n R_{h_n}^{2/3}$$

By using the above data this becomes

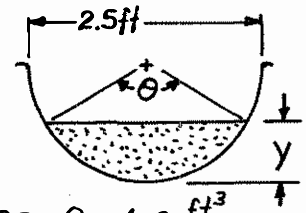
$$\frac{1}{2n_n}\left(\frac{1}{2}L^2\right)\left(\frac{L}{4}\right)^{2/3} = \frac{1}{n_n}(2m^2)\left(\frac{1}{2}m\right)^{2/3} \text{ or } L^{8/3} = 8(2)^{2/3}$$

or

$L = \underline{\underline{2.59\ m}}$

10.72

10.72 A smooth steel water slide at an amusement park is of semicircular cross section with a diameter of 2.5 ft. The slide descends a vertical distance of 35 ft in its 420 ft length. If pumps supply water to the slide at a rate of 6 cfs, determine the depth of flow. Neglect the effects of the curves and bends of the slide.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \frac{35\,ft}{420\,ft} = 0.0833$, $Q = 6.0\,\frac{ft^3}{s}$

and from Table 10.1 $n = 0.012$

Also (see Example 10.5), $A = \frac{D^2}{8}(\theta - \sin\theta)$ and

$R_h = \frac{D(\theta - \sin\theta)}{4\theta}$, where $D = 2.5\,ft$

Thus,

$$Q = \frac{K}{n} S_o^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}\right], \text{ where } \theta \sim rad,$$

or

$$6.0 = \frac{1.49}{0.012}(0.0833)^{1/2} \frac{(2.5)^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}\right]$$

Hence,

$0.293\,\theta^{2/3} = (\theta - \sin\theta)^{5/3}$ $\quad 0.0252\,\theta^2 - (\theta - \sin\theta)^5 = 0$ (1)

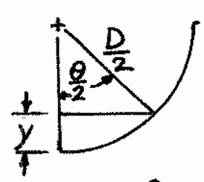
Using a standard root-finding technique gives the solution to Eq. (1) as $\theta = 1.574$ rad.

Thus, $\theta = (1.574\,rad)\left(\frac{180\,deg}{\pi\,rad}\right) = 90.2°$

or since

$y = \frac{D}{2}\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$ it follows that

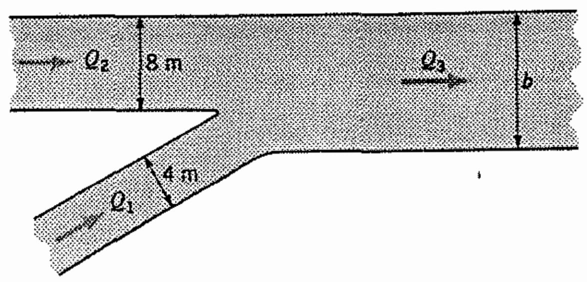
$y = \left(\frac{2.5}{2}\,ft\right)\left(1 - \cos\left(\frac{90.2}{2}\right)\right) = \underline{\underline{0.368\,ft}}$



$y + \frac{D}{2}\cos\frac{\theta}{2} = \frac{D}{2}$

10.73

10.73 Two canals join to form a larger canal as shown in Video V10.6 and Fig. P10.73. Each of the three rectangular canals is lined with the same material and has the same bottom slope. The water depth in each is to be 2 m. Determine the width of the merged canal, b. Explain physically (i.e., without using any equations) why it is expected that the width of the merged canal is less than the combined widths of the two original canals (i.e., $b < 4 \text{ m} + 8 \text{ m} = 12 \text{ m}$).



■ FIGURE P10.73

$$Q_3 = Q_1 + Q_2 \quad \text{where for } i=1,2,3$$

$$Q_i = \frac{\kappa}{n_i} A_i R_{h_i}^{2/3} \sqrt{S_{0i}}$$

Thus,

$$\frac{\kappa}{n_3} A_3 R_{h_3}^{2/3} \sqrt{S_{0_3}} = \frac{\kappa}{n_2} A_2 R_{h_2}^{2/3} \sqrt{S_{0_2}} + \frac{\kappa}{n_1} A_1 R_{h_1}^{2/3} \sqrt{S_{0_1}} \tag{1}$$

But $n_1 = n_2 = n_3$ and $S_{0_1} = S_{0_2} = S_{0_3}$ so that Eq. (1) becomes

$$A_3 R_{h_3}^{2/3} = A_2 R_{h_2}^{2/3} + A_1 R_{h_1}^{2/3} \tag{2}$$

where

$$A_1 = 2m(4m) = 8m^2, \; P_1 = (2+2+4) = 8m \text{ so that } R_{h_1} = \frac{A_1}{P_1} = \frac{8m^2}{8m} = 1\,m$$

$$A_2 = 2m(8m) = 16m^2, \; P_2 = (2+2+8) = 12m \text{ so that } R_{h_2} = \frac{A_2}{P_2} = \frac{16m^2}{12m} = 1.333m$$

and

$$A_3 = 2b\,m^2, \; P_3 = (2+2+b) = (b+4)m \text{ so that } R_{h_3} = \frac{A_3}{P_3} = \frac{2b}{(b+4)}$$

Thus, Eq. (2) becomes

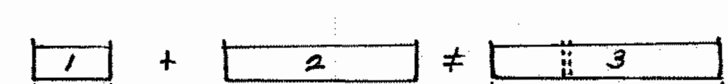
$$(2b)\left[\frac{2b}{(b+4)}\right]^{2/3} = 16\,(1.333)^{2/3} + 8\,(1)^{2/3} = 27.4$$

or

$$b^{5/3} = 8.63\,(b+4)^{2/3} \tag{3}$$

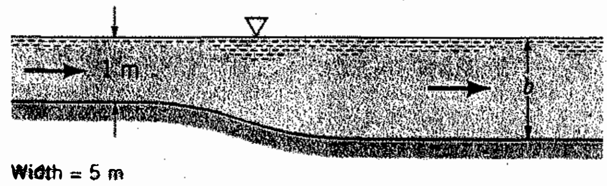
Using a standard root-finding technique gives the solution to Eq. (3):
$b = 10.66 \text{ m}$

If the two original canals merged to form a 12m wide canal, the water depth would be less than 2m because without the two walls there would be less friction force hold the water back. Thus, to maintain the 2m depth we must have $b < 12m$.



10.74

10.74 Water flows uniformly at a depth of 1 m in a channel that is 5 m wide as shown in Fig. P10.74. Further downstream the channel cross section changes to that of a square of width and height b. Determine the value of b if the two portions of this channel are made of the same material and are constructed with the same bottom slope.



■ FIGURE P10.74

$Q_U = Q_D$, where $(\ )_U$ and $(\ )_D$ denote upstream and downstream conditions.
Thus, since $Q = \frac{\kappa}{n} A\, R_h^{2/3} \sqrt{S_o}$ it follows that

$$\frac{\kappa}{n_U} A_U R_{h_U}^{2/3} \sqrt{S_{oU}} = \frac{\kappa}{n_D} A_D R_{h_D}^{2/3} \sqrt{S_{oD}}$$

Also, $S_{oU} = S_{oD}$ and $n_U = n_D$

Hence,

(1) $A_U R_{h_U}^{2/3} = A_D R_{h_D}^{2/3}$, where $A_U = (1m)(5m) = 5m^2$, $P_U = 2(1m) + 5m = 7m$,
so that $R_{h_U} = A_U / P_U = 5m^2/7m = 0.714m$.

Also, $A_D = b^2$, $P_D = 3b$, so that $R_{h_D} = A_D/P_D = b^2/(3b) = \frac{1}{3} b$

Thus, from Eq. (1):

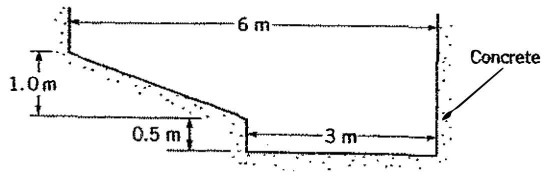
$$(5m^2)(0.714m)^{2/3} = b^2 \left(\tfrac{1}{3} b\right)^{2/3}$$

or

$b = \underline{\underline{2.21\,m}}$

10.75

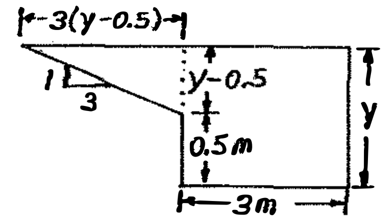
10.75 Determine the flow depth for the channel shown in Fig. P10.54 if the flowrate is 15 m^3/s.



■ FIGURE P10.54

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1$, $S_o = \frac{3m}{3000m} = 0.003$, and from Table 10.1 $n = 0.012$

Also, $A = 3y + \frac{1}{2}[3(y-0.5)](y-0.5) = \frac{3}{2}y^2 + \frac{3}{2}y + \frac{3}{8}$



and

$$P = y + 3 + 0.5 + \left[(y-\tfrac{1}{2})^2 + 9(y-\tfrac{1}{2})^2\right]^{\frac{1}{2}}$$

$$= y + 3.5 + \sqrt{10}\,(y-0.5) = 4.16y + 1.92$$

Hence, with $R_h = \frac{A}{P}$ and $Q = 15\frac{m^3}{s}$ we obtain

$$15 = \frac{1}{0.012}(1.5y^2 + 1.5y + 0.375)^{5/3} \frac{1}{(4.16y + 1.92)^{2/3}} (0.003)^{\frac{1}{2}}$$

or

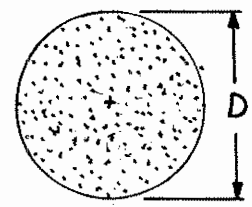
$$2.04(4.16y + 1.92)^{0.4} - 1.5y^2 - 1.5y - 0.375 = 0 \qquad (1)$$

Using a standard root-finding technique, the solution to Eq. (1) is found to be $y = \underline{\underline{1.22 m}}$

Note: Since $y < 1.5m$ the water does not contact the left vertical wall.

10.76

10.76 Rainwater runoff from a 200-ft by 500-ft parking lot is to drain through a circular concrete pipe that is laid on a slope of 3 ft/mi. Determine the pipe diameter if it is to be full with a steady rainfall of 1.5 in./hr.



$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49, \quad S_o = \frac{3 \text{ ft}}{5280 \text{ ft}} = 0.000568, \qquad (1)$$

$$A = \frac{\pi}{4} D^2 \quad \text{and} \quad R_h = \frac{A}{P} = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$$

From Table 10.1, $n = 0.012$

Also, $Q = A_{lot}\, r$, where r = rainfall rate $= 1.5 \frac{\text{in.}}{\text{hr}}$

Thus,

$$Q = (200\text{ft})(500\text{ft})\left(1.5 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12} \frac{\text{ft}}{\text{in.}}\right)\left(\frac{1 \text{ hr}}{3600 \text{s}}\right) = 3.47 \frac{\text{ft}^3}{\text{s}}$$

Hence, from Eq. (1):

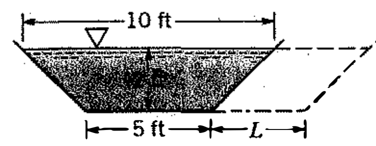
$$3.47 = \frac{1.49}{0.012}\left(\frac{\pi}{4} D^2\right)\left(\frac{D}{4}\right)^{2/3} (0.000568)^{1/2}$$

or

$$D = \underline{\underline{1.64 \text{ft}}}$$

10.77

10.77 (See Fluids in the News article titled "Plumbing the Everglades," Section 10.4.1.) The canal shown in Fig. P10.77 is to be widened so that it can carry twice the amount of water. Determine the additional width, L, required if all other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same.



■ FIGURE P10.77

Let $(\ )_o$ denote the original canal and $(\ )_w$ the widened canal.

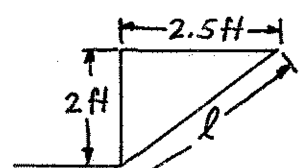
Thus,

(1) $Q_o = \frac{\kappa}{n_o} A_o R_{h_o}^{2/3} \sqrt{S_{o_o}}$ and

(2) $Q_w = \frac{\kappa}{n_w} A_w R_{h_w}^{2/3} \sqrt{S_{ow}}$, where $n_o = n_w$ and $S_{o_o} = S_{ow}$

Hence, from Eqs. (1) and (2)

(3) $\dfrac{Q_w}{Q_o} = \dfrac{\frac{\kappa}{n_w} A_w R_{hw}^{2/3} \sqrt{S_{ow}}}{\frac{\kappa}{n_o} A_o R_{h_o}^{2/3} \sqrt{S_{o_o}}} = \dfrac{A_w}{A_o}\left(\dfrac{R_{hw}}{R_{ho}}\right)^{2/3}$, where $Q_w = 2Q_o$



$\ell = [2.5^2 + 2^2]^{1/2}\ ft = 3.20\ ft$

Also, $A_o = \frac{1}{2}(5\,ft + 10\,ft)(2\,ft) = 15\ ft^2$

$P_o = 5\ ft + 2(3.20\,ft) = 11.4\,ft$, so that

$R_{h_o} = A_o / P_o = 15\ ft^2 / 11.4\ ft = 1.316\ ft$

and

$A_w = \frac{1}{2}\left[(5\,ft + L) + (10\,ft + L)\right](2\,ft) = (15 + 2L)\ ft^2$

$P_w = 5\,ft + L + 2(3.20\,ft) = (11.4 + L)\,ft$, so that

$R_{h_w} = A_w / P_w = (15 + 2L)/(11.4 + L)$

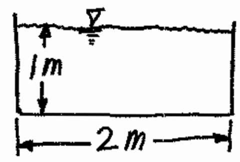
Hence, from Eq. (3) with $\frac{Q_w}{Q_o} = 2$,

(4) $2 = \dfrac{(15 + 2L)}{15}\left[\dfrac{(15+2L)/(11.4+L)}{1.316}\right]^{2/3}$, where $L \sim ft$

By using a standard root-finding program, the solution to Eq. (4) is determined to be $L = \underline{\underline{5.94\ ft}}$

10.78

10.78 Water flows 1 m deep in a 2-m-wide finished concrete channel. Determine the slope if the flowrate is 3 m^3/s.



(1) $Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}$, where $K = 1$, and from Table 10.1, $n = 0.012$

Also,

$A = (1\,m)(2\,m) = 2\,m^2$

and

$R_h = \frac{A}{P} = \frac{2\,m^2}{(2\,m + 1\,m + 1\,m)} = 0.5\,m$

Hence, with $Q = 3\,\frac{m^3}{s}$ Eq.(1) becomes

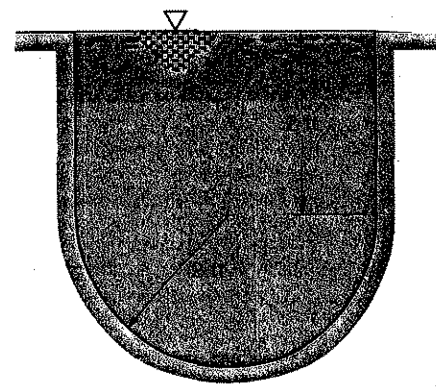
$3\,\frac{m^3}{s} = \frac{1}{0.012}(2\,m^2)(0.5\,m)^{2/3}\sqrt{S_o}$

or

$S_o = \underline{\underline{0.000816}}$

10.79

10.79 Water flows in the channel shown in Fig. P10.79 at a rate of 90 ft^3/s. Determine the minimum slope that this channel can have so that the water does not overflow the sides. The Manning coefficient for this channel is $n = 0.014$.



■ FIGURE P10.79

(1) $$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}, \text{ where } K = 1.49,\ n = 0.014, \text{ and } Q = 90\,\frac{ft^3}{s}$$

Also,

$$A = (2\,ft)(4\,ft) + \frac{1}{2}\pi(2\,ft)^2 = 14.28\,ft^2$$

and

$$R_h = \frac{A}{P} = \frac{14.28\,ft^2}{(2\,ft + 2\,ft + \pi(2\,ft))} = 1.389\,ft$$

Thus, from Eq.(1)

$$90\,\frac{ft^3}{s} = \frac{1.49}{0.014}(14.28\,ft^2)(1.389\,ft)^{2/3}\sqrt{S_o}$$

or

$$S_o = \underline{\underline{0.00226}}$$

10.80

10.80 To prevent weeds from growing in a clean earthen-lined canal, it is recommended that the velocity be no less than 2.5 ft/s. For the symmetrical canal shown in Fig. P10.80, determine the minimum slope needed.

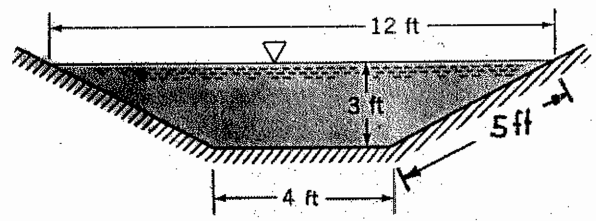


FIGURE P10.80

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \tag{1}$$

$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2$ and $P = 4\,ft + 2(5\,ft) = 14\,ft$

Thus, $R_h = \frac{24\,ft^2}{14\,ft} = 1.714\,ft$

From Table 10.1, $n = 0.022$ so that Eq.(1) gives (with $V = 2.5\,\frac{ft}{s}$)

$2.5 = \frac{1.49}{0.022}(1.714)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.000664}}$

10.81

10.81 The smooth, concrete-lined, symmetrical channel shown in Video V10.7 and Fig. P10.80 carries water from the silt-laden Colorado River. If the velocity must be 4.0 ft/s to prevent the silt from settling out (and eventually clogging the channel), determine the minimum slope needed.

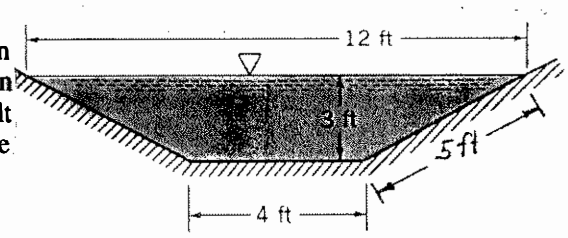


FIGURE P10.80

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \tag{1}$$

$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2$ and $P = 4\,ft + 2(5\,ft) = 14\,ft$

Thus, $R_h = \frac{24\,ft^2}{14\,ft} = 1.714\,ft$

From Table 10.1, $n = 0.012$ so that Eq.(1) gives (with $V = 4\,\frac{ft}{s}$)

$4.0 = \frac{1.49}{0.012}(1.714)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.000505}}$

10.82

10.82 The symmetrical channel shown in Fig. P10.80 is dug in sandy loam soil with $n = 0.020$. For such surface material it is recommended that to prevent scouring of the surface the average velocity be no more than 1.75 ft/s. Determine the maximum slope allowed.

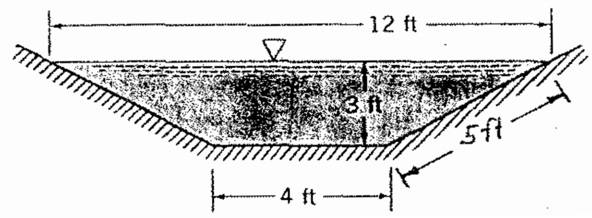


FIGURE P10.80

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \tag{1}$$

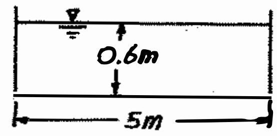
$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2$ and $P = 4\,ft + 2(5\,ft) = 14\,ft$

Thus, $R_h = \frac{24\,ft^2}{14\,ft} = 1.714\,ft$

With $n = 0.020$ and $V = 1.75\,\frac{ft}{s}$ Eq.(1) gives

$1.75 = \frac{1.49}{0.020}(1.714)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.000269}}$

10.83 The depth downstream of a sluice gate in a rectangular wooden channel of width 5 m is 0.60 m. If the flowrate is 18 m³/s, determine the channel slope needed to maintain this depth. Will the depth increase or decrease in the flow direction if the slope is (a) 0.02; (b) 0.01?



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1$ and from Table 10.1, $n = 0.012$ (1)

Also $A = (5m)(0.6m) = 3\,m^2$, $P = 5m + 2(0.6m) = 6.2\,m$

so that $R_h = \frac{A}{P} = \frac{3\,m^2}{6.2\,m} = 0.484\,m$

Hence, from Eq. (1):

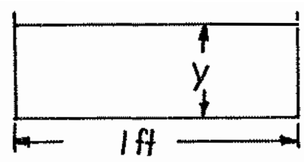
$18 = \frac{1}{0.012}(3)(0.484)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.0136}}$

With $S_o = 0.02 > 0.0136$ the velocity will increase and the water will become less than 0.6 m deep.

With $S_o = 0.01 < 0.0136$ the velocity will decrease and the water will become greater than 0.6 m deep.

10.84

10.84 Water in a rectangular painted steel channel of width $b = 1$ ft and depth y is to flow at critical conditions, Fr = 1. Plot a graph of the critical slope, S_{0c}, as a function of y for 0.05 ft $\le y \le$ 5 ft. What is the maximum slope allowed if critical flow is not to occur regardless of the depth?



$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1 $n = 0.014$

Also, $R_h = \frac{A}{P} = \frac{y}{2y+1}$ and with $Fr = \frac{V}{\sqrt{gy}} = 1$, $V = \sqrt{gy}$

Thus,

$$\sqrt{32.2\,y} = \frac{1.49}{0.014}\left(\frac{y}{2y+1}\right)^{2/3} S_{oc}^{1/2} \quad \text{or} \quad S_{oc} = 0.00284\left[\frac{(2y+1)^4}{y}\right]^{\frac{1}{3}} \qquad (1)$$

Equation (1) is plotted below. To determine the minimum critical slope set $\frac{dS_{oc}}{dy} = 0$. That is:

$$\frac{dS_{oc}}{dy} = \left(\tfrac{1}{3}\right)(0.00284)\left[\frac{(2y+1)^4}{y}\right]^{-2/3}\left[\frac{4(2y+1)^3(2)y - (2y+1)^4}{y^2}\right] = 0$$

Thus, $y = \frac{1}{6}$ so that from Eq. (1)

$$S_{oc_{min}} = 0.00284\left[\frac{\left(\frac{2}{6}+1\right)^4}{\frac{1}{6}}\right]^{1/3} = \underline{\underline{0.00757}}$$

If $S_o < 0.00757$ critical flow cannot occur at any depth.

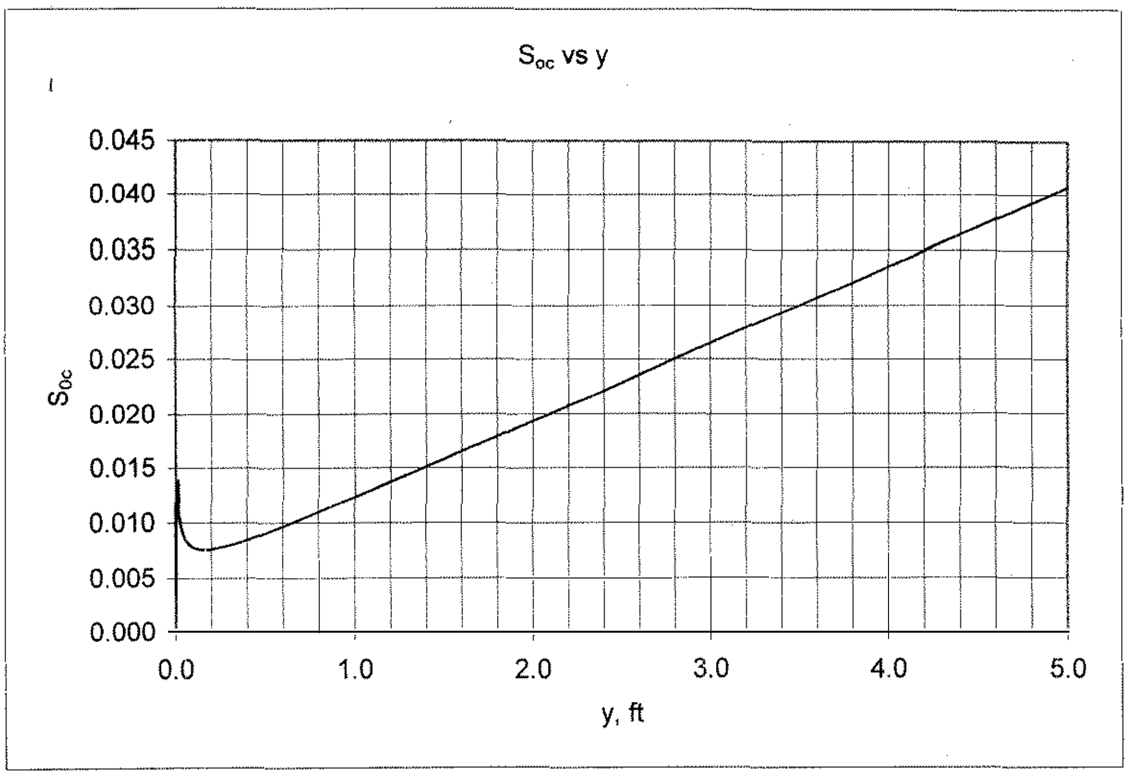
The following values are obtained from Eq. (1). Note that

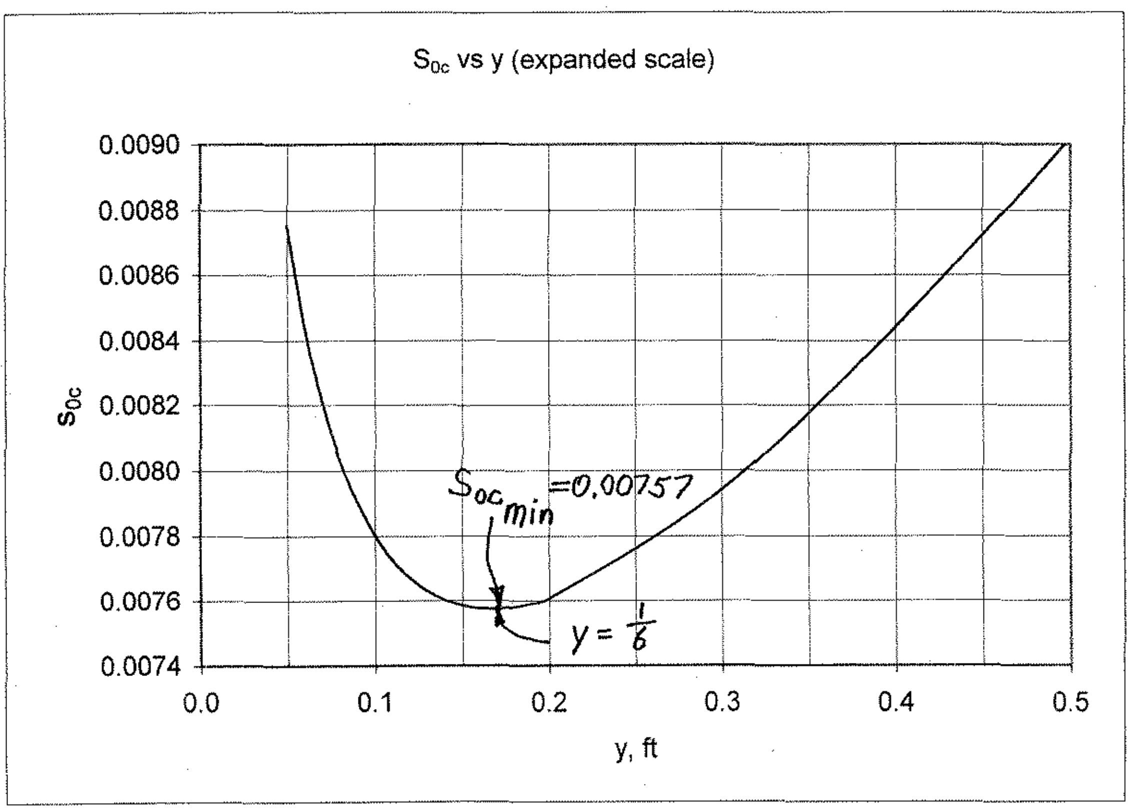
$$\lim_{y\to 0} S_{oc} = 0.00284 \lim_{y\to 0}\left[\frac{(2y+1)^4}{y}\right]^{1/3} = \infty \quad \text{and} \quad \lim_{y\to\infty} S_{oc} = \infty$$

See next page for graphs.

(con't)

10.84 (con't)


S_{oc} vs y
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
S_{0c}
0.0
1.0
2.0
3.0
4.0
5.0
y, ft


S_{0c} vs y (expanded scale)
0.0090
0.0088
0.0086
0.0084
0.0082
0.0080
0.0078
0.0076
0.0074
S_{0c}
$S_{oc_{min}} = 0.00757$
$y = \frac{1}{6}$
0.0
0.1
0.2
0.3
0.4
0.5
y, ft

10.85

10.85 A 50-ft-long aluminum gutter (Manning coefficient $n = 0.011$) on a section of a roof is to handle a flowrate of 0.15 ft^3/s during a heavy rain storm. The cross section of the gutter is shown in Fig. P10.85. Determine the vertical distance that this gutter must be pitched (i.e., the difference in elevation between the two ends of the gutter) so that the water does not overflow the gutter. Assume uniform depth channel flow.

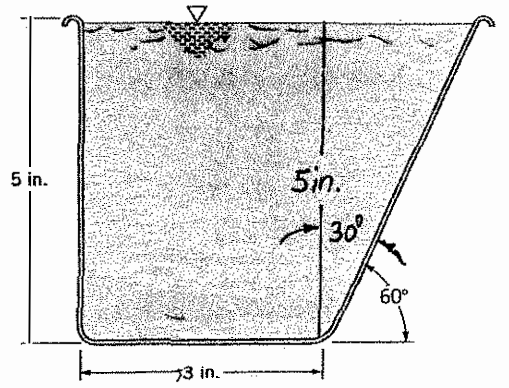


FIGURE P10.85

(1) $Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}$, where $K = 1.49$, $n = 0.011$, and

$A = (5\,in.)(3\,in.) + \frac{1}{2}(5\,in.)(5\,in.\ \tan 30°) = 22.2\,in.^2 \left(\frac{1\,ft^2}{144\,in.^2}\right) = 0.154\,ft^2$

Also,

$R_h = \frac{A}{P}$, where $P = (5\,in.) + (3\,in.) + \left(\frac{5\,in.}{\cos 30°}\right) = 13.77\,in. \left(\frac{1\,ft}{12\,in.}\right) = 1.148\,ft$

Hence,

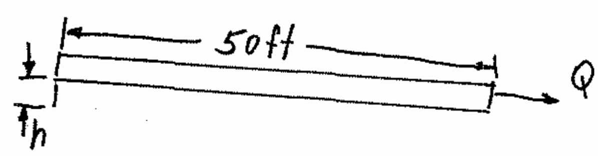
$R_h = \frac{0.154\,ft^2}{1.148\,ft} = 0.134\,ft$

Thus, from Eq. (1)

$0.15\,\frac{ft^3}{s} = \frac{1.49}{0.011}(0.154\,ft^2)(0.134\,ft)^{2/3}\sqrt{S_o}$

so that

$S_o = 7.54 \times 10^{-4}$



But

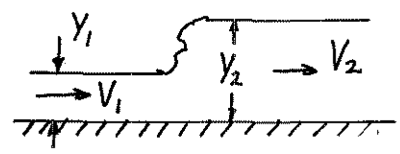
$S_o = \frac{h}{50\,ft}$

so that

$h = (50\,ft)S_o = (50\,ft)(7.54 \times 10^{-4}) = 0.0377\,ft = \underline{\underline{0.452\,in.}}$

10.87

10.87 Water flows upstream of a hydraulic jump with a depth of 0.5 m and a velocity of 6 m/s. Determine the depth of the water downstream of the jump.



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\,\right], \text{ where}$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{6\ m/s}{\sqrt{(9.81\ m/s^2)(0.5\ m)}} = 2.71$$

Thus,

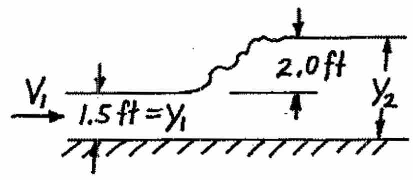
$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8(2.71)^2}\right] = 3.36$$

so that

$$y_2 = 3.36(0.5\ m) = \underline{\underline{1.68\ m}}$$

10.88

10.88 A 2.0-ft standing wave is produced at the bottom of the rectangular channel in an amusement park water ride. If the water depth upstream of the wave is estimated to be 1.5 ft, determine how fast the boat is traveling when it passes through this standing wave (hydraulic jump) for its final "splash."



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\,\right]$$

or $$\left(\frac{2.0\,ft + 1.5\,ft}{1.5\,ft}\right) = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\,\right]$$

Thus, $Fr_1 = 1.97$, or since $Fr_1 = \dfrac{V_1}{\sqrt{g\,y_1}}$

$$V_1 = Fr_1\sqrt{g y_1} = 1.97\sqrt{\left(32.2\,\tfrac{ft}{s^2}\right)(1.5\,ft)} = \underline{\underline{13.7\,\tfrac{ft}{s}}}$$

10.89

10.89 The water depths upstream and downstream of a hydraulic jump are 0.3 and 1.2 m, respectively. Determine the upstream velocity and the power dissipated if the channel is 50 m wide.

$$\frac{y_2}{y_1} = \frac{1.2\,m}{0.3\,m} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\,\right] \text{ or } Fr_1 = 3.16 \quad \text{Thus, since } Fr_1 = \frac{V_1}{(g\,y_1)^{1/2}}$$

it follows that $V_1 = (3.16)\left[\left(9.81\,\frac{m}{s^2}\right)(0.3\,m)\right]^{1/2} = \underline{\underline{5.42\,\frac{m}{s}}}$

The power dissipated is given by

$$\mathcal{P} = \gamma Q h_L, \text{ where } \frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left(1-\left(\frac{y_1}{y_2}\right)^2\right)$$

or

$$h_L = (0.3\,m)\left[1 - \frac{1.2\,m}{0.3\,m} + \frac{(3.16)^2}{2}\left(1-\left(\frac{0.3\,m}{1.2\,m}\right)^2\right)\right] = 0.504\,m$$

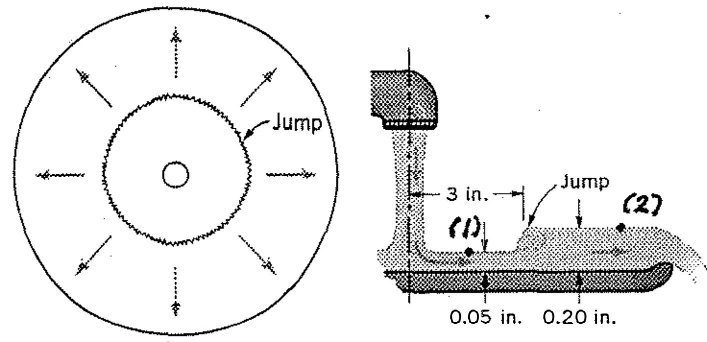
Also, $Q = A_1V_1 = y_1 b V_1 = (0.3\,m)(50\,m)\left(5.42\,\frac{m}{s}\right) = 81.3\,\frac{m^3}{s}$

Thus,

$$\mathcal{P} = \left(9.8\,\frac{kN}{m^3}\right)\left(81.3\,\frac{m^3}{s}\right)(0.504\,m) = 401\,\frac{kN\cdot m}{s} = \underline{\underline{401\,kW}}$$

10.90

10.90 Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in Fig. P10.90 and Video V10.12. Consider a situation where a jump forms 3.0 in. from the center of the plate with depths upstream and downstream of the jump of 0.05 in. and 0.20 in., respectively. Determine the flowrate from the faucet.



■ FIGURE P10.90

For a hydraulic jump:

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \quad \text{or}$$

$$\frac{0.20\text{ in.}}{0.05\text{ in.}} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \quad \text{so that} \quad Fr_1 = 3.16 = \frac{V_1}{\sqrt{g y_1}}$$

Thus,

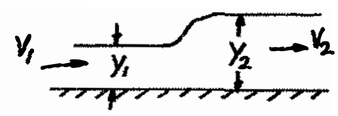
$$V_1 = 3.16\sqrt{32.2\frac{ft}{s^2}(0.05/12)ft} = 1.16\frac{ft}{s}$$

and

$$Q = A_1 V_1 = 2\pi R_1 y_1 V_1 = 2\pi\left(\frac{3}{12}ft\right)\left(\frac{0.05}{12}ft\right)\left(1.16\frac{ft}{s}\right) = \underline{\underline{0.00759\frac{ft^3}{s}}}$$

10.91

10.91 Show that the Froude number downstream of a hydraulic jump in a rectangular channel is $(y_1/y_2)^{3/2}$ times the Froude number upstream of the jump, where (1) and (2) denote the upstream and downstream conditions, respectively.



$$Fr_2 = \frac{V_2}{(g\,y_2)^{1/2}}\,, \text{ where } V_1A_1 = V_2A_2\,, \text{ or } V_2 = \frac{A_1}{A_2}V_1 = \frac{y_1}{y_2}V_1$$

Thus,

$$Fr_2 = \frac{\left(\frac{y_1}{y_2}\right)V_1}{(g\,y_2)^{1/2}} = \left(\frac{y_1}{y_2}\right)^{3/2}\frac{V_1}{(g\,y_1)^{1/2}}$$

Hence, $\underline{\underline{Fr_2 = \left(\frac{y_1}{y_2}\right)^{3/2} Fr_1}}$

10.92

10.92 Water flows in a 2-ft-wide rectangular channel at a rate of 10 ft³/s. If the water depth downstream of a hydraulic jump is 2.5 ft, determine **(a)** the water depth upstream of the jump, **(b)** the upstream and downstream Froude numbers, and **(c)** the head loss across the jump.

$Q = 10\,\frac{ft^3}{s}$, $b = \text{width} = 2\,ft$

(a) Use $\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8\,Fr_1^2}\right]$ with $y_2 = 2.5\,ft$ so that

$5 + y_1 = y_1\sqrt{1 + 8\,Fr_1^2}$ Now, with $Fr_1^2 = \frac{V_1^2}{g y_1} = \frac{(Q/(b y_1))^2}{g\,y_1} = \frac{(10/(2 y_1))^2}{32.2\,y_1}$,

or $Fr_1^2 = \frac{0.776}{y_1^3}$, we obtain

$5 + y_1 = y_1\left[1 + 8\left(\frac{0.776}{y_1^3}\right)\right]^{\frac{1}{2}}$. By squaring both sides and simplifying we obtain $y_1^2 + 2.5 y_1 - 0.621 = 0$ which gives $y_1 = \underline{\underline{0.228\,ft}}$

(b) From the above results

$$Fr_1^2 = \frac{0.776}{(0.228)^3} \quad \text{or} \quad Fr_1 = \underline{\underline{8.09}}$$

Also,

$$V_2 = \frac{Q}{A_2} = \frac{10\,\frac{ft^3}{s}}{(2.5ft)(2ft)} = 2.0\,\frac{ft}{s} \text{ so that } Fr_2 = \frac{V_2}{(g y_2)^{1/2}} = \frac{2\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(2.5ft)\right]^{1/2}}$$

or

$Fr_2 = \underline{\underline{0.223}}$

(c) Also,

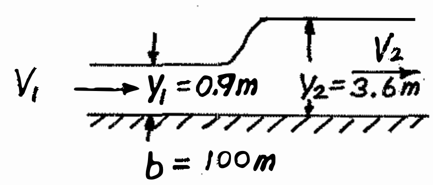
$$h_L = y_1\left[1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left(1 - \left(\frac{y_1}{y_2}\right)^2\right)\right] = 0.228ft\left[1 - \frac{2.5}{0.228} + \frac{(8.10)^2}{2}\left(1 - \left(\frac{0.228}{2.5}\right)^2\right)\right]$$

or

$h_L = \underline{\underline{5.15\,ft}}$

10.93

10.93 A hydraulic jump at the base of a spillway of a dam is such that the depths upstream and downstream of the jump are 0.90 and 3.6 m, respectively (see Video V10.11). If the spillway is 10 m wide, what is the flowrate over the spillway?



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right], \text{ or } \frac{3.6m}{0.9m} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

Hence, $Fr_1 = 3.16$, but $Fr_1 = \frac{V_1}{(g\,y_1)^{1/2}}$ so that

$$V_1 = 3.16\left[(9.81\tfrac{m}{s^2})(0.9m)\right]^{1/2} = 9.39\ \tfrac{m}{s}$$

Thus,

$$Q = A_1V_1 = by_1V_1 = (10.0m)(0.9m)(9.39\tfrac{m}{s}) = \underline{\underline{84.5\ \tfrac{m^3}{s}}}$$

10.94

10.94 Determine the head loss and power dissipated by the hydraulic jump of Problem 10.93.

$$h_L = y_1\left[1-\frac{y_2}{y_1}+\frac{Fr_1^2}{2}\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right], \text{ where from } \frac{y_2}{y_1} = \frac{3.6m}{0.9m} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

Hence, $Fr_1 = 3.16$ so that

$$h_L = (0.9m)\left[1-\frac{3.6m}{0.9m}+\frac{(3.16)^2}{2}\left(1-\left(\frac{0.9m}{3.6m}\right)^2\right)\right] = \underline{\underline{1.51\ m}}$$

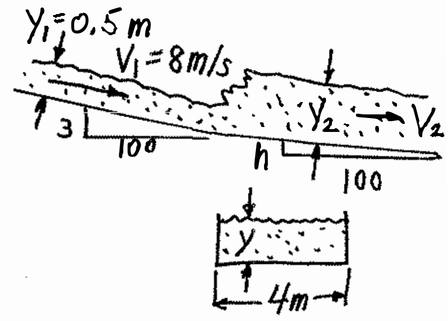
Also, $\mathcal{P} = \gamma Q h_L$, where $V_1 = (gy_1)^{\frac{1}{2}}Fr_1 = \left[(9.81\tfrac{m}{s^2})(0.9m)\right]^{1/2}(3.16) = 9.39\tfrac{m}{s}$

Hence,

$$\mathcal{P} = (9.80\tfrac{kN}{m^3})\left[(0.9m)(100m)(9.39\tfrac{m}{s})\right](1.51\ m) = 12{,}500\tfrac{kN\cdot m}{s} = \underline{\underline{12{,}500kW}}$$

10.95

10.95 A hydraulic jump occurs in a 4-m-wide rectangular channel at a point where the slope changes from 3 m per 100 m upstream of the jump to h m per 100 m downstream of the jump. The depth and velocity of the uniform flow upstream of the jump are 0.5 m and 8 m/s, respectively. Determine the value of h if the flow downstream of the jump is to be uniform flow.



Upstream of the jump

$V_1 = \frac{\kappa}{n} R_{h_1}^{2/3} \sqrt{S_{o_1}}$, where $S_{o_1} = \frac{3\,m}{100\,m} = 0.03$

and $R_{h_1} = \frac{A_1}{P_1} = \frac{(4m)(0.5m)}{(4m+0.5m+0.5m)} = 0.4m$

Thus,

$Q_1 = A_1 V_1 = (4m)(0.5m)\frac{\kappa}{n}(0.4m)^{2/3}\sqrt{0.03}$

or

(1) $Q_1 = 0.188 \frac{\kappa}{n} \frac{m^3}{s}$

Also,

$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+(1+8Fr_1^2)^{1/2}\right]$, where $Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{8\frac{m}{s}}{\sqrt{(9.81\frac{m}{s^2})(0.5m)}} = 3.61$

Thus,

$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+(1+8(3.61)^2)^{1/2}\right] = 4.63$

so that

$y_2 = 4.63\,y_1 = 4.63(0.5m) = 2.31m$

Therefore,

$A_2 = (4m)(2.31m) = 9.24m^2$ and $R_{h_2} = \frac{A_2}{P_2} = \frac{9.24m^2}{(4m+2.31m+2.31m)} = 1.07m$

so that

$Q_2 = \frac{\kappa}{n} A_2 R_{h_2}^{2/3}\sqrt{S_{o_2}} = \frac{\kappa}{n}(9.24m^2)(1.07m)^{2/3}\sqrt{S_{o_2}}$

or

(2) $Q_2 = 9.67\frac{\kappa}{n}\sqrt{S_{o_2}}\ \frac{m^3}{s}$

But, $Q_1 = Q_2$, so that from Eqs. (1) and (2)

$0.188\frac{\kappa}{n} = 9.67\frac{\kappa}{n}\sqrt{S_{o_2}}$

or

$S_{o_2} = 0.000378 = \frac{h}{100m}$ Hence, $h = \underline{\underline{0.0378\,m}}$

10.96

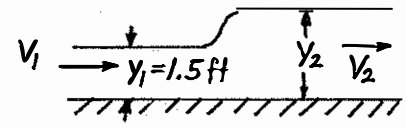
10.96 At a given location in a 12-ft-wide rectangular channel the flowrate is 900 ft³/s and the depth is 4 ft. Is this location upstream or downstream of the hydraulic jump that occurs in this channel? Explain.

$$V = \frac{Q}{A} = \frac{900\,\frac{ft^3}{s}}{(12\,ft)(4\,ft)} = 18.75\,\frac{ft}{s} \quad \text{so that } Fr = \frac{V}{(gy)^{1/2}} = \frac{18.75\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(4\,ft)\right]^{1/2}} = 1.65$$

Since $Fr > 1$, the location is upstream of the jump.

*10.97

*10.97 A rectangular channel of width b is to carry water at flowrates from $30 \le Q \le 600$ cfs. The water depth upstream of the hydraulic jump that occurs (if one does occur) is to remain 1.5 ft for all cases. Plot the power dissipated in the jump as a function of flowrate for channels of width $b = 10, 20, 30,$ and 40 ft.



$$\mathcal{P} = \gamma Q h_L \, , \text{ where } h_L = y_1\left[1-\left(\frac{y_2}{y_1}\right) + \frac{Fr_1^2}{2}\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right] \quad (1)$$

and

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \text{, provided } Fr_1 \ge 0 \quad (2)$$

Also,

$$Fr_1 = \frac{V_1}{(g y_1)^{1/2}} \text{, where } V_1 = \frac{Q}{A_1} = \frac{Q}{1.5\,b} \text{ so that}$$

$$Fr_1 = \frac{\left(\frac{Q}{1.5b}\right)}{\left[(32.2\frac{ft}{s^2})(1.5ft)\right]^{1/2}} = 0.0959\,\frac{Q}{b} \quad \text{Hence, from Eq.(1)}$$

$$h_L = (1.5)\left[1-\left(\frac{y_2}{y_1}\right)+(0.00460)\left(\frac{Q}{b}\right)^2\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right] \text{ ft, where } b\sim ft,\ Q\sim\frac{ft^3}{s} \quad (3)$$

and from Eq.(2)

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\left(1+0.0736\left(\frac{Q}{b}\right)^2\right)^{1/2}\right] \quad (4)$$

For the given values of plot $\mathcal{P}$ from

$$\mathcal{P} = 62.4 Q h_L \ \frac{ft\cdot lb}{s} \quad \text{for } 30 \le Q \le 600 \ \frac{ft\cdot lb}{s} \quad (5)$$

Note: If $Fr_1 < 1$ there is no jump and $\mathcal{P} = 0$. From above, $Fr_1 = 1$ when

$$Q = \frac{b}{0.0959} = 10.4\,b \quad (6)$$

Let Q_1 = flowrate when $Fr_1 = 1$. From Eq.(6) we obtain

b, ft	Q_1, $\frac{ft^3}{s}$
10	104
20	208
30	312
40	416

With $b = 10, 20, 30,$ or 40 ft calculate and plot $\mathcal{P}$ from:

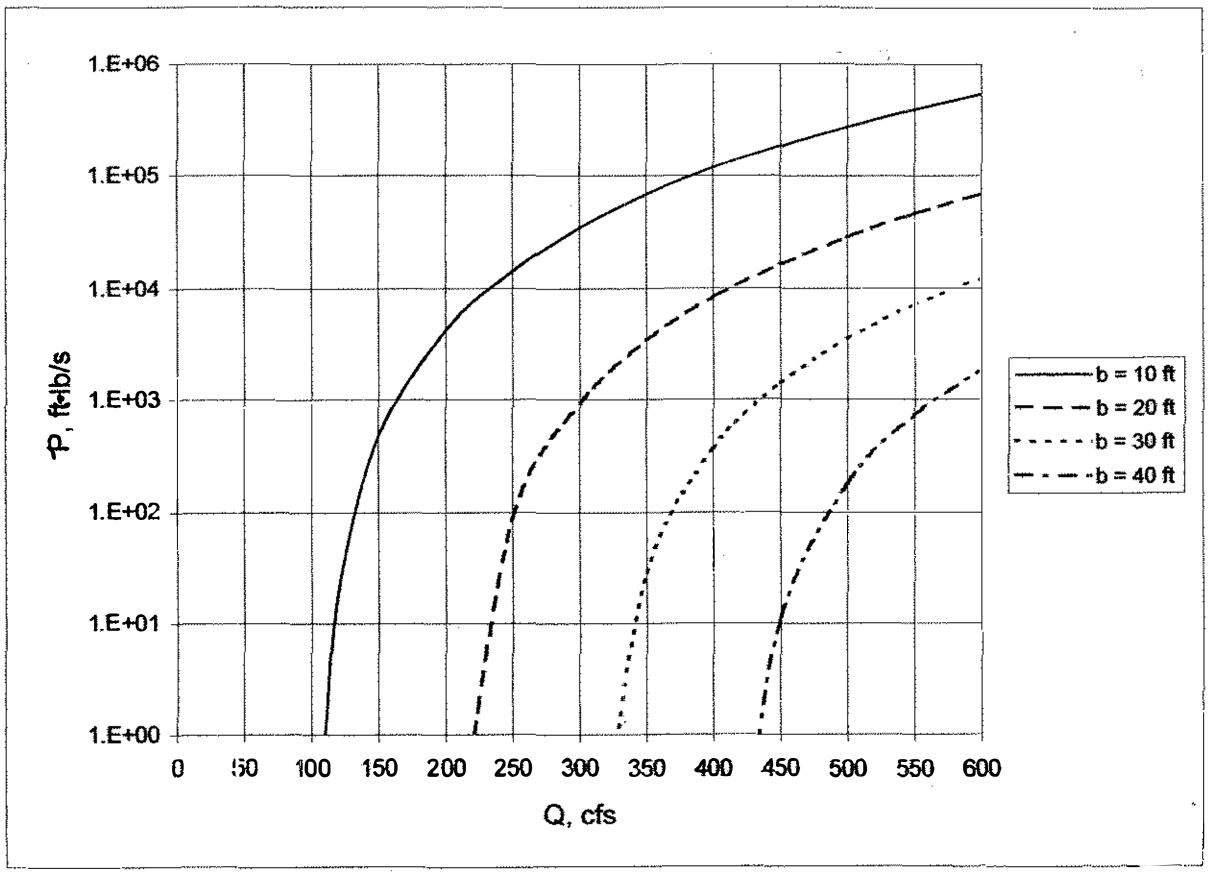
a) $\mathcal{P} = 0$ if $Q < Q_1$

b) $\mathcal{P} = 62.4\,Q h_L \ \frac{ft\cdot lb}{s}$, where obtain h_L from Eq. (3) with $\frac{y_2}{y_1}$ from Eq. (4) if $Q_1 \le Q \le 600 \ \frac{ft^3}{s}$

(con't)

10.97* (con't)

The results of the above calculations are plotted below.



10.98

10.98 Water flows in a rectangular channel at a depth of $y = 1$ ft and a velocity of $V = 20$ ft/s. When a gate is suddenly placed across the end of the channel, a wave (a moving hydraulic jump) travels upstream with velocity V_w as is indicated in Fig. P10.98. Determine V_w. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w, the flow appears as a steady hydraulic jump.

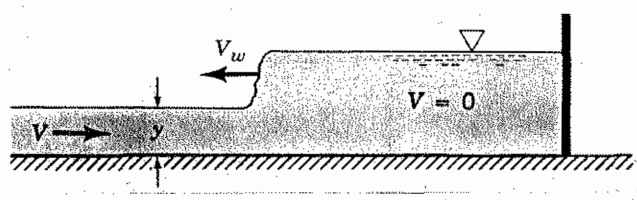
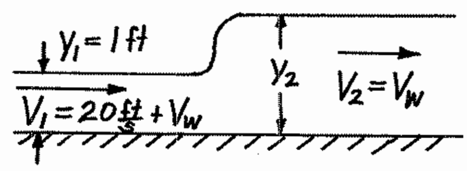


FIGURE P10.98

For an observer moving to the left with speed V_w the flow appears as shown below.

Thus, treat the flow as a jump with



$$Fr_1 = \frac{V_1}{(g y_1)^{1/2}} = \frac{(20 + V_w)}{\left[(32.2\frac{ft}{s})(1\,ft)\right]^{1/2}}$$

or

$$Fr_1 = 0.176\,(20 + V_w) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$, or $\dfrac{y_2}{y_1} = \dfrac{V_1}{V_2} = \dfrac{20 + V_w}{V_w}$ (2)

and

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 Fr_1^2}\right]$$ which when combined with Eqs.(1) and (2) becomes

$$\frac{20 + V_w}{V_w} = \frac{1}{2}\left[-1 + \sqrt{1 + 8(0.176)^2(20 + V_w)^2}\right]$$

or

$$2(20 + V_w) + V_w = V_w\left(1 + (0.248)(20 + V_w)^2\right)^{\frac{1}{2}}$$

or

$$(40 + 3V_w)^2 = V_w^2\left[1 + (0.248)(20 + V_w)^2\right]$$, which can be written as

$$0.248\, V_w^4 + 9.92\, V_w^3 + 91.2\, V_w^2 - 240\, V_w - 1600 = 0 \qquad (3)$$

By using a standard root-finding program, the solution to Eq. (3) is determined to be $V_w = 4.36$ ft/s.

10.99

10.99 Water flows in a rectangular channel with velocity $V = 6$ m/s. A gate at the end of the channel is suddenly closed so that a wave (a moving hydraulic jump) travels upstream with velocity $V_w = 2$ m/s as is indicated in Fig. P10.98 Determine the depths ahead of and behind the wave. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w, the flow appears as a steady hydraulic jump.

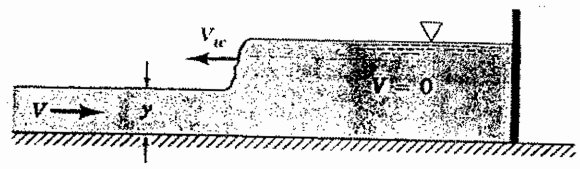
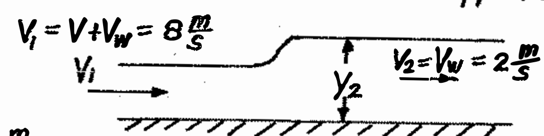


FIGURE P10.98

For an observer moving to the left with speed $V_w = 2\,\frac{m}{s}$ the flow appears as shown below.



Thus, treat as a jump with

$V_1 = 8\,\frac{m}{s}$, $V_2 = 2\,\frac{m}{s}$

Since $A_1 V_1 = A_2 V_2$ or $\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{8\,\frac{m}{s}}{2\,\frac{m}{s}} = 4$ it follows that

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^{\,2}}\right] = 4 \qquad \text{Hence, } Fr_1 = 3.16$$

However, $Fr_1 = \frac{V_1}{(g y_1)^{1/2}}$ so that

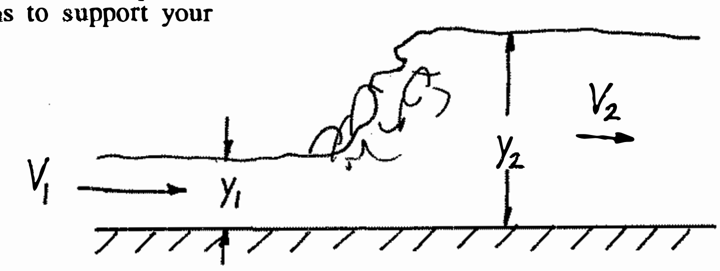
$$y_1 = \frac{V_1^{\,2}}{g\,Fr_1^{\,2}} = \frac{(8\,\frac{m}{s})^2}{(9.81\,\frac{m}{s^2})(3.16)^2} = \underline{\underline{0.652\,m}}$$

and

$$y_2 = 4y_1 = 4(0.652\,m) = \underline{\underline{2.61\,m}}$$

10.100

10.100 (See Fluids in the News article titled "Grand Canyon rapids building," Section 10.6.1.) During the flood of 1983, a large hydraulic jump formed at "Crystal Hole" rapid on the Colorado River. People rafting the river at that time report "entering the rapid at almost 30 mph, hitting a 20-ft-tall wall of water, and exiting at about 10 mph." Is this information (i.e., upstream and downstream velocities and change in depth) consistent with the principles of a hydraulic jump? Show calculations to support your answer.



Is the given data consistent with a hydraulic jump?

$V_1 = 30\ mph = 44\ ft/s$

$V_2 = 10\ mph = 14.7\ ft/s$

$y_2 - y_1 = 20\ ft$

From conservation of mass: $A_1V_1 = A_2V_2$

or $y_1 V_1 = y_2 V_2$ since $b_1 = \text{width} = b_2$

Thus,

$$\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{44\ ft/s}{14.7\ ft/s} = 2.99 \qquad (1)$$

Also, for a hydraulic jump

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right) \text{ so that } 2.99 = \frac{1}{2}\left(-1 + \sqrt{1+8Fr_1^2}\right)$$

or

$Fr_1 = 2.44$

Thus, since $Fr_1 = \frac{V_1}{\sqrt{gy_1}}$ it follows that

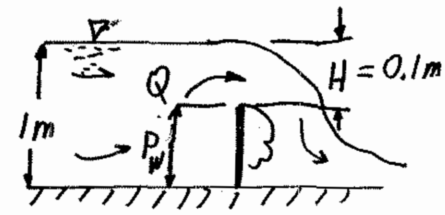
$$2.44 = \frac{44\ ft/s}{(32.2\ ft/s^2\ y_1)^{1/2}} \text{ or } y_1 = 10.1\ ft \text{, so that from Eq. (1),}$$

$y_2 = 2.99 y_1 = 2.99(10.1\ ft) = 30.2\ ft$

Hence, the given data gives $y_2 - y_1 = 30.2\ ft - 10.1\ ft = 20.1\ ft$, which is surprisingly close to the reported depth. Yes, the data is consistent with the principles of a hydraulic jump.

10.102

10.102 Water flows over a 2-m-wide rectangular sharp-crested weir. Determine the flowrate if the weir head is 0.1 m and the channel depth is 1 m.



(1) $Q = C_{wr}\frac{2}{3}\sqrt{2g}\, b H^{3/2}$, where $b = 2\,m$, $H = 0.1\,m$

Also,

$C_{wr} = 0.611 + 0.075\left(\frac{H}{P_w}\right)$, where $P_w = 1\,m - H = 1\,m - 0.1\,m = 0.9\,m$

Thus,

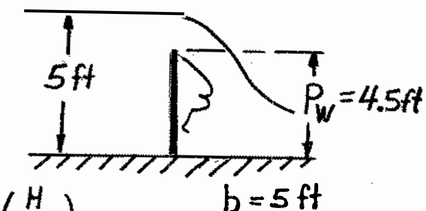
$C_{wr} = 0.611 + 0.075\left(\frac{0.1\,m}{0.9\,m}\right) = 0.619$

so that from Eq. (1),

$Q = 0.619\left(\frac{2}{3}\right)\sqrt{2\left(9.81\frac{m}{s^2}\right)}\,(2m)(0.1m)^{3/2} = \underline{\underline{0.116\frac{m^3}{s}}}$

10.103

10.103 Water flows over a 5-ft-wide, rectangular sharp-crested weir that is $P_w = 4.5$ ft tall. If the depth upstream is 5 ft, determine the flowrate.



$$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\;b\,H^{3/2}, \quad \text{where } C_{wr} = 0.611 + 0.075\left(\frac{H}{P_w}\right)$$

with

$$H = 5\text{ ft} - 4.5\text{ ft} = 0.5\text{ ft}$$

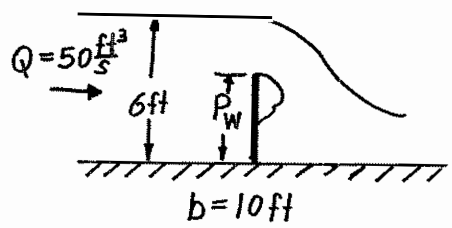
Hence, $C_{wr} = 0.611 + 0.075\left(\frac{0.5\text{ ft}}{4.5\text{ ft}}\right) = 0.619$

and

$$Q = (0.619)\left(\frac{2}{3}\right)\left(2\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)\right)^{1/2}(5\text{ ft})(0.5)^{3/2} = \underline{\underline{5.85\,\frac{\text{ft}^3}{\text{s}}}}$$

10.104

10.104 A rectangular sharp crested weir is used to measure the flowrate in a channel of width 10 ft. It is desired to have the channel flow depth be 6 ft when the flowrate is 50 cfs. Determine the height, P_w, of the weir plate.



$$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\;b\,H^{3/2}, \quad \text{where } H = 6\text{ ft} - P_w \text{ and}$$

$$C_{wr} = 0.611 + 0.075\,\frac{H}{P_w}$$

Thus,

$$Q = \left(0.611 + 0.075\left(\frac{6 - P_w}{P_w}\right)\right)\left(\frac{2}{3}\right)(2g)^{1/2}\,b\,(6 - P_w)$$

or

$$50\,\frac{\text{ft}^3}{\text{s}} = \left(0.611 + 0.075\left(\frac{6 - P_w}{P_w}\right)\right)\left(\frac{2}{3}\right)\left(64.4\,\frac{\text{ft}}{\text{s}^2}\right)^{1/2}(10\text{ ft})(6 - P_w), \quad \text{where } P_w \sim \text{ft}$$

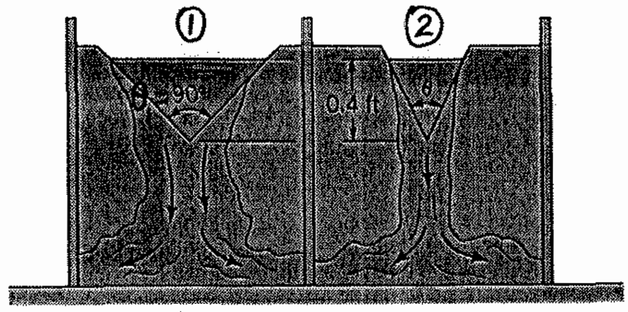
Hence,

$$\left[8.15 + \frac{(6 - P_w)}{P_w}\right](6 - P_w)^{3/2} - 12.5 = 0 \qquad (1)$$

By using a standard root-finding program, the solution to Eq. (1) is found to be $P_w = \underline{\underline{4.70\text{ ft}}}$.

10.105

10.105 Water flows from a storage tank, over two triangular weirs, and into two irrigation channels as shown in **Video V10.13** and Fig. P10.105. The head for each weir is 0.4 ft, and the flowrate in the channel fed by the 90°-V-notch weir is to be twice the flowrate in the other channel. Determine the angle θ for the second weir.



■ FIGURE P10.105

$$Q = C_{wt}\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2} \qquad (1)$$

where

$$\theta_1 = 90^\circ,\ H_1 = H_2 = 0.4\,ft,\ \text{and}\ Q_1 = 2Q_2 \qquad (2)$$

Thus, from Fig. 10.21,

$$C_{wt_1} = 0.590$$

From Eqs. (1) and (2),

$$C_{wt_1}\frac{8}{15}\tan\left(\frac{\theta_1}{2}\right)\sqrt{2g}\,H_1^{5/2} = C_{wt_2}\frac{8}{15}\tan\left(\frac{\theta_2}{2}\right)\sqrt{2g}\,H_2^{5/2} \times 2$$

or

$$0.590\tan 45^\circ = C_{wt_2}\tan\left(\frac{\theta_2}{2}\right)\times 2$$

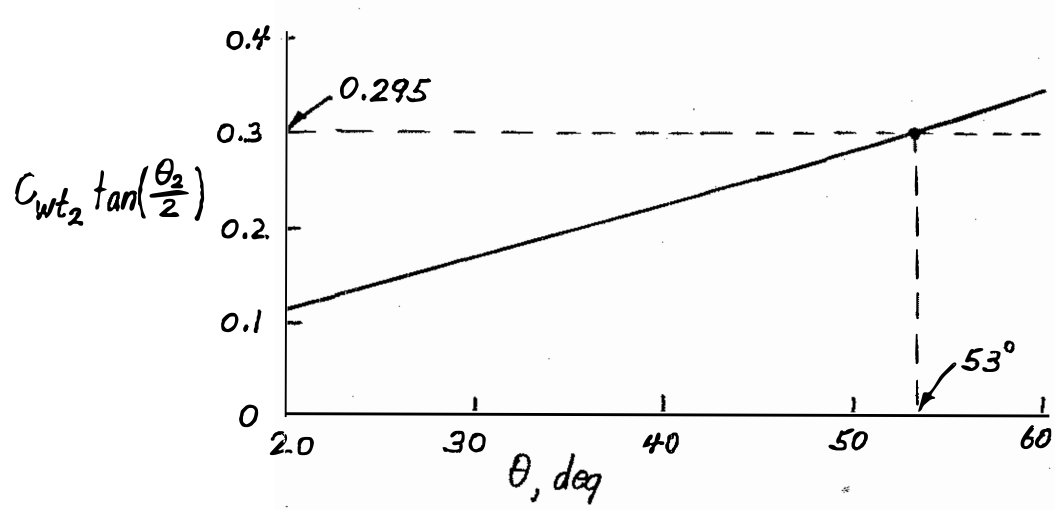
or

$$C_{wt_2}\tan\left(\frac{\theta_2}{2}\right) = 0.295 \qquad (3)$$

Trial and error solution: Assume $\theta_2 = 20^\circ$. From Fig. 10.21, $C_{wt_2} = 0.626$

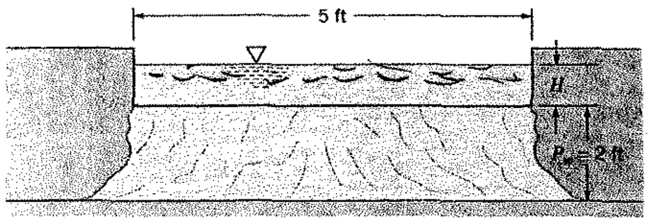
Thus, $C_{wt_2}\tan\left(\frac{\theta_2}{2}\right) = 0.626\tan(10^\circ) = 0.110 \neq 0.295$. Thus, $\theta_2 \neq 20^\circ$

Repeated tries result in the graph below from which we conclude that $\theta_2 = \underline{\underline{53^\circ}}$



10.106

10.106 Rain water from a parking lot flows into a 2-acre (8.71×10^4 ft²) retention pond. After a heavy rain when there is no more inflow into the pond, the rectangular weir shown in Fig. P10.106 at the outlet of the pond has a head of $H = 0.6$ ft. **(a)** Determine the rate at which the level of the water in the pond decreases, dH/dt, at this condition. **(b)** Determine how long it will take to reduce the pond level by half a foot; that is, to $H = 0.1$ ft.



■ FIGURE P10.106

For a rectangular weir,

$$Q = C_{wr}\frac{2}{3}\sqrt{2g}\, b H^{3/2}, \text{ where } C_{wr} = 0.611 + 0.075\frac{H}{P_w}$$

Thus, with $b = 5$ ft, and $P_w = 2$ ft

$$Q = \left(0.611 + 0.075\frac{H}{(2)}\right)\frac{2}{3}\sqrt{2(32.2\text{ ft/s}^2)}\,(5\text{ ft})\,H^{3/2}$$

or

(1) $$Q = 26.7(0.611 + 0.0375H)H^{3/2}\ \frac{ft^3}{s}, \text{ where } H \sim ft$$

(2) $$\text{Also, } Q = -A_{pond}\frac{dH}{dt} = -8.71\times10^4\text{ft}^2\frac{dH}{dt}$$

Thus, from Eqs. (1) and (2),

$$-8.71\times10^4\frac{dH}{dt} = 26.7(0.611 + 0.0375H)H^{3/2}$$

or

(3) $$\frac{dH}{dt} = -3.07\times10^{-4}(0.611 + 0.0375H)H^{3/2} = -1.87\times10^{-4}H^{3/2} - 1.15\times10^{-5}H^{5/2}$$

(a) When $H = 0.6$ ft,

$$\frac{dH}{dt} = -1.87\times10^{-4}(0.6)^{3/2} - 1.15\times10^{-5}(0.6)^{5/2} = -9.01\times10^{-5}\frac{ft}{s}\left(\frac{3600\ s}{hr}\right)$$

$$= \underline{\underline{-0.324\frac{ft}{hr}}}$$

(b) Integrate Eq(3) from $H = 0.6$ ft when $t = 0$ to $H = 0.1$ ft when $t = T$

Thus, from Eq(3)

$$\int_{t=0}^{t=T} dt = \int_{H=0.6}^{H=0.1}\frac{-dH}{(1.87\times10^{-4}H^{3/2} + 1.15\times10^{-5}H^{5/2})}$$

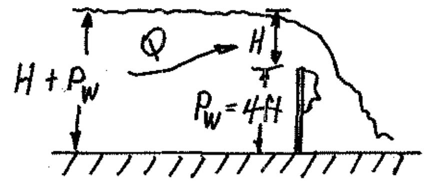
or

(4) $$T = \int_{0.1}^{0.6}\frac{dH}{(1.87\times10^{-4}H^{3/2} + 1.15\times10^{-5}H^{5/2})}$$

Numerical integration of Eq(4) gives $T = 19{,}900\,s\left(\frac{1\,hr}{3600\,s}\right) = \underline{\underline{5.53\,hr}}$

10.107

10.107 A basin at a water treatment plant is 60 ft long, 10 ft wide, and 5 ft deep. Water flows from the basin over a 3-ft-long, rectangular weir whose crest is 4 ft above the bottom of the basin. Estimate how long it will take for the depth of the water in the basin to change from 4.5 ft to 4.4 ft if there is no flow into the basin.



$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\;b\,H^{3/2}$, where $b = 3\text{ ft}$ and $C_{wr} = (0.611 + 0.075(\frac{H}{P_W}))$

Thus,

$Q = (0.611 + 0.075(\frac{H}{4}))\frac{2}{3}\sqrt{2(32.2\text{ ft/s})}\,(3\text{ft})H^{3/2}$

or

(1) $Q = 9.81\,H^{3/2} + 0.301\,H^{5/2}\ \text{ft}^3/\text{s}$, where $H \sim \text{ft}$

Also,

$Q = -A_{tank}\frac{dH}{dt} = -(60\text{ft})(10\text{ft})\frac{dH}{dt}$

or

(2) $Q = -600\,\frac{dH}{dt}$

Thus, from Eqs (1) and (2),

$-600\frac{dH}{dt} = 9.81\,H^{3/2} + 0.301\,H^{5/2}$

(3) $\frac{dH}{dt} = -0.0164\,H^{3/2} - 0.000502\,H^{5/2}$, where H = water depth in channel $- 4$ft.

Thus, with $H = 0.5$ ft at $t = 0$ and $H = 0.4$ ft at $t = T$ it follows that

$$\int_{t=0}^{t=T} dt = -\int_{H=0.5}^{H=0.4}\frac{dH}{(0.0164\,H^{3/2} + 0.000502\,H^{5/2})}$$

or

(4) $$T = \int_{0.4}^{0.5}\frac{dH}{(0.0164\,H^{3/2} + 0.000502\,H^{5/2})}$$

A standard numerical integration of Eq. (4) gives

$T = \underline{20.1\text{ s}}$

Note: From Eq. (3), $\frac{dH}{dt} = -0.00589\,\frac{\text{ft}}{\text{s}}$ when $H = 0.5$ft and $-0.00420\,\frac{\text{ft}}{\text{s}}$ when $H = 0.4$ft. At these rates with $\frac{dH}{dt} = \frac{\Delta H}{T} = \frac{-0.1\text{ ft}}{T}$ we would obtain $T = -0.1\text{ft}/(\frac{dH}{dt}) = -0.1\text{ ft}/(-0.00589\,\frac{\text{ft}}{\text{s}}) = 17.0\text{ s}$ or $T = -0.1\text{ft}/(-0.00420\,\frac{\text{ft}}{\text{s}}) = 23.8\text{ s}$, which brackets the actual $T = 20.1$ s.

10.108

10.108 Water flows over a sharp crested triangular weir with $\theta = 90°$. The head range covered is $0.2 \le H \le 1.0$ ft and the accuracy in the measurement of the head, H, is $\delta H = \pm 0.01$ ft. Plot a graph of the percent error expected in Q as a function of Q.

$$Q = C_{wt}\,\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\;H^{5/2}, \text{ where } \theta = 90°$$

Thus,

$$\frac{dQ}{dH} = \frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\left[C_{wt}\left(\frac{5}{2}\right)H^{3/2} + H^{5/2}\frac{dC_{wt}}{dH}\right]$$

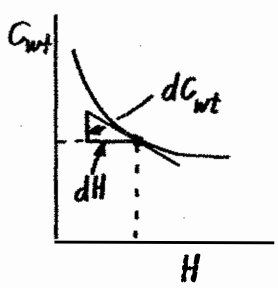
or

$$\frac{dQ}{Q} = \frac{\left[C_{wt}\left(\frac{5}{2}\right)H^{3/2} + H^{5/2}\frac{dC_{wt}}{dH}\right]dH}{C_{wt}\,H^{5/2}}$$

Hence,

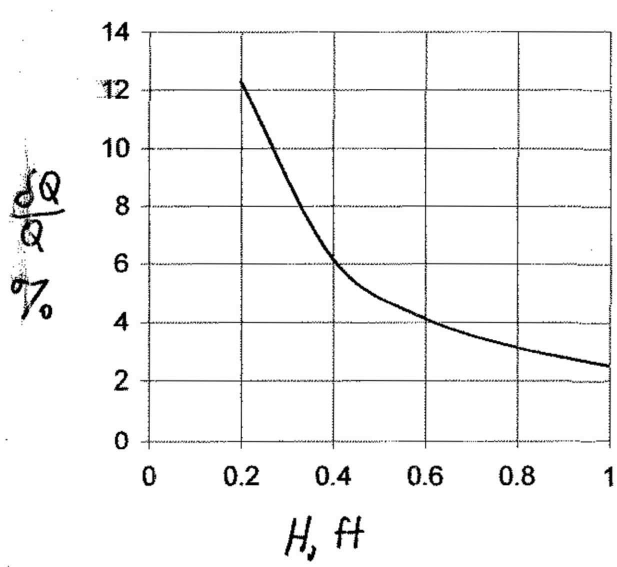
$$\frac{\delta Q}{Q} = \frac{5}{2}\,\frac{\delta H}{H} + \frac{1}{C_{wt}}\left(\frac{dC_{wt}}{dH}\right)\delta H \qquad (1)$$

With $0.2 \le H \le 1.0$ ft and $\delta H = 0.01$ ft calculate $\frac{\delta Q}{Q}$ after obtaining C_{wt} and $\frac{dC_{wt}}{dH}$ from Fig. 10.21.



H, ft	C_{wt}	$\frac{dC_{wt}}{dH}$	$\frac{\delta Q}{Q}$
0.2	0.60	−0.100	0.123
0.4	0.588	−0.042	0.0618
0.6	0.582	−0.018	0.0414
0.8	0.581	−0.005	0.0312
1.0	0.581	0	0.0250

The above results are plotted below:



10.109

10.109 (a) The rectangular sharp-crested weir shown in Fig. P10.109*a* is used to maintain a relatively constant depth in the channel upstream of the weir. How much deeper will the water be upstream of the weir during a flood when the flowrate is 45 ft^3/s, compared to normal conditions when the flowrate is 30 ft^3/s? Assume the weir coefficient remains constant at $C_{wr} = 0.62$. (b) Repeat the calculations if the weir of part (a) is replaced by a rectangular sharp-crested "duck bill" weir which is oriented at an angle of 30° relative to the channel centerline as shown in Fig. P10.109*b*. The weir coefficient remains the same.

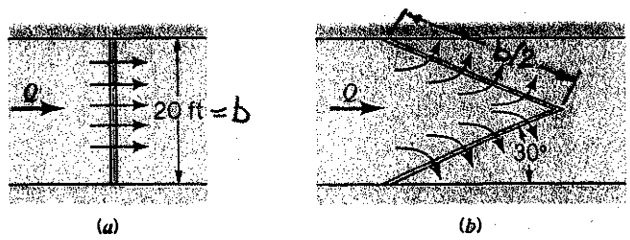


FIGURE P10.109

In either case

$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\;bH^{3/2} = 0.62\left(\frac{2}{3}\right)\sqrt{2(32.2\ ft/s^2)}\;bH^{3/2}$,

or

$Q = 3.32\,bH^{3/2}$, where $Q \sim ft^3/s$ when $b \sim ft$ and $H \sim ft$ (1)

(a) From Eq. (1) with $b = 20\ ft$, if $Q = 30\ ft^3/s$ then

$30 = 3.32\,(20)\,H_{30}^{3/2}$, or $H_{30} = 0.589\ ft$

If $Q = 45\ ft^3/s$, then

$45 = 3.32\,(20)\,H_{45}^{3/2}$, or $H_{45} = 0.772\ ft$

Thus, $\Delta H_a = H_{45} - H_{30} = 0.772\ ft - 0.589\ ft = \underline{0.183\ ft}$

(b) From Eq. (1) with $b = 2\,(10\ ft)/\sin 30^\circ = 40\ ft$, if $Q = 30\ ft^3/s$ then

$30 = 3.32\,(40)\,H_{30}^{3/2}$, or $H_{30} = 0.371\ ft$

If $Q = 45\ ft^3/s$, then

$45 = 3.32\,(40)\,H_{45}^{3/2}$, or $H_{45} = \underline{0.486\ ft}$

Thus, $\Delta H_b = H_{45} - H_{30} = 0.486\ ft - 0.371\ ft = \underline{0.115\ ft}$

Note that the "duck bill" weir gives a smaller change in the head than does the "regular" weir.

10.110

10.110 Water flows in a rectangular channel of width $b = 20$ ft at a rate of 100 ft³/s. The flowrate is to be measured by using either a rectangular weir of height $P_w = 4$ ft or a triangular ($\theta = 90°$) sharp crested weir. Determine the head, H, necessary. If measurement of the head is accurate to only ±0.04 ft, determine the accuracy of the measured flowrate expected for each of the weirs. Which weir would be the most accurate? Explain.

(a) Rectangular weir:

$Q = (0.611 + 0.075(\frac{H}{P_w}))(\frac{2}{3})\sqrt{2g}\, b\, H^{3/2}$, where $P_w = 4$ ft

Thus,

$Q = [0.611 + 0.075(\frac{H}{4})](\frac{2}{3})[2(32.2\frac{ft}{s^2})]^{1/2}(20ft)\, H^{3/2}$

or

$Q = 107(0.611 + 0.0188H)H^{3/2}$, where $Q \sim \frac{ft^3}{s}$ and $H \sim ft$ (1)

With $Q = 100\frac{ft^3}{s}$ this gives $0.935 = (0.611 + 0.0188H)H^{3/2}$

or $(32.5 + H)H^{3/2} - 49.7 = 0$ (2)

By using a standard root-finding program, the solution to Eq. (2) is determined to be

$H = 1.294$ ft

(b) Triangular weir:

$Q = C_{wt}\frac{8}{15}\tan(\frac{\theta}{2})\sqrt{2g}\, H^{5/2} = C_{wt}(\frac{8}{15})(\tan 45°)[2(32.2\frac{ft}{s^2})]^{1/2} H^{5/2}$

or

$Q = 4.28\, C_{wt} H^{5/2}\ \frac{ft^3}{s}$, where $H \sim ft$ and C_{wt} is from Fig. 10.21. (2)

For $Q = 100\frac{ft^3}{s}$, assume $C_{wt} = 0.58$ so that

$4.28(0.58)H^{5/2}$, or $H = 4.39$ ft Note: The assumed $C_{wt} = 0.58$ checks (see Fig. 10.21)

Calculate Q for $H = H_{100}$, $H_{100}+0.04$, and $H_{100}-0.04$ from Eqs. (1) and (2):

(Rectangular) H, ft	Q, cfs
1.254	95.3
H_{100} = 1.294	100
1.334	104.9

(Triangular) H, ft	Q, cfs
4.35	98.0
H_{100} = 4.39	100
4.43	102.5

With $H \pm 0.04$ft it is seen that triangular weir is more accurate (i.e. smaller variation in Q).

10.111

10.111 Water flows under a sluice gate in a 60-ft-wide finished concrete channel as is shown in Fig. P10.111. Determine the flowrate. If the slope of the channel is 2.5 ft/200 ft, will the water depth increase or decrease downstream of the gate? Assume $C_c = y_2/a = 0.65$. Explain.

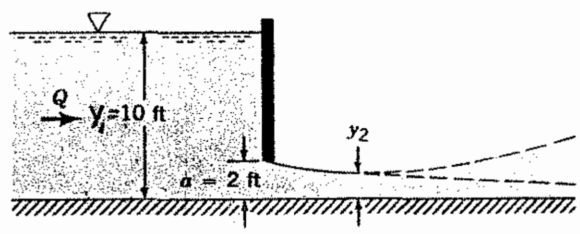


FIGURE P10.111

$Q = bq = bC_d\, a\sqrt{2gy_1}$, where $b = 60\text{ ft}$, $a = 2\text{ ft}$, and from Fig. 10.24

since $\frac{y_1}{a} = \frac{10\text{ ft}}{2\text{ ft}} = 5$ it follows that $C_d = 0.55$

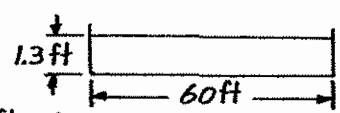
Hence,

$Q = (60\text{ft})(0.55)(2\text{ft})\left[2(32.2\frac{\text{ft}}{\text{s}^2})(10\text{ft})\right]^{1/2} = \underline{\underline{1670\frac{\text{ft}^3}{\text{s}}}}$

Determine the slope needed to maintain uniform flow downstream of the gate:

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1 $n = 0.012$ (1)

Also, $y_2 = C_c\, a = 0.65\,(2\text{ ft}) = 1.3\text{ ft}$



so that

and $A = (1.3\text{ft})(60\text{ft}) = 78\text{ ft}^2$, $P = (60 + 2(1.3))\text{ft} = 62.6\text{ ft}$

$R_h = \frac{A}{P} = \frac{78\text{ ft}^2}{62.6\text{ ft}} = 1.245\text{ ft}$

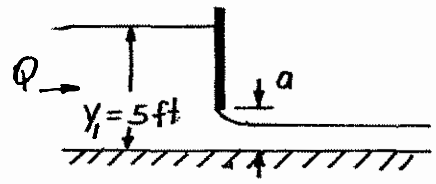
Thus, from Eq. (1):

$1670 = \frac{1.49}{0.012}(78)(1.245)^{2/3} S_o^{1/2}$, or $S_o = 0.0222$

Hence, the required slope for uniform flow is $S_o = 0.0222$ but the actual slope is $S_o = \frac{2.5\text{ ft}}{200\text{ ft}} = 0.0125$, less than required. The fluid will slow down and the depth increase.

10.112

10.112 Water flows under a sluice gate in a channel of 10-ft width. If the upstream depth remains constant at 5 ft, plot a graph of flowrate as a function of the distance between the gate and channel bottom as the gate is slowly opened. Assume free outflow.

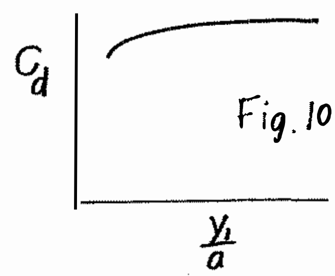


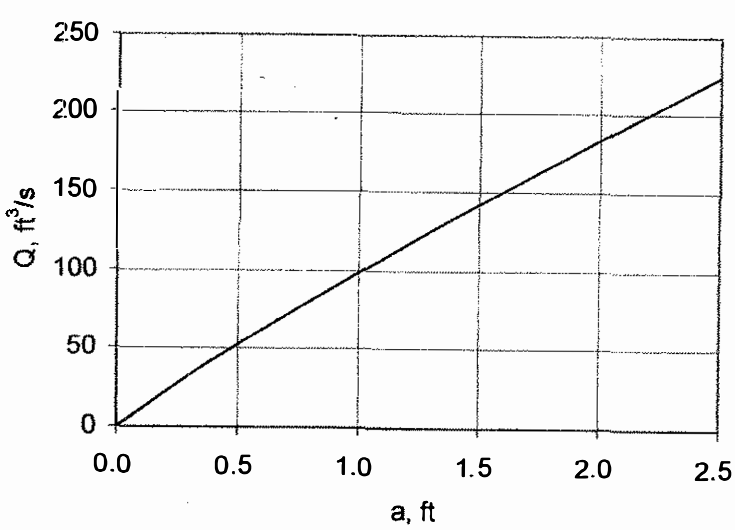
$Q = qb = b\, C_d\, a\sqrt{2gy_1}$, where $y_1 = 5$ ft, $b = 10$ ft, and C_d is from Fig. 10.25.

Thus,

$Q = C_d\,(10\text{ft})\,a\left[2(32.2\tfrac{\text{ft}}{\text{s}^2})(5\text{ ft})\right]^{\frac{1}{2}} = 179\, C_d\, a\ \tfrac{\text{ft}^3}{\text{s}}$, where $a \sim$ ft

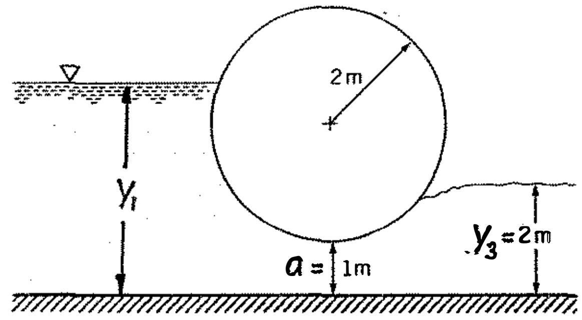
a, ft	$\frac{y_1}{a}$	C_d	Q, $\frac{\text{ft}^3}{\text{s}}$
0	∞	0.6	0
0.5	10	0.58	51.9
1.0	5	0.55	98.5
1.5	3.33	0.53	142
2.0	2.5	0.51	183
2.5	2	0.50	224





10.113

10.113 A water-level regulator (not shown) maintains a depth of 2.0 m downstream from a 10-m-wide drum gate as shown in Fig. P10.113. Plot a graph of flowrate, Q, as a function of water depth upstream of the gate, y_1, for $2.0 \leq y_1 \leq 5.0$ m.



■ FIGURE P10.113

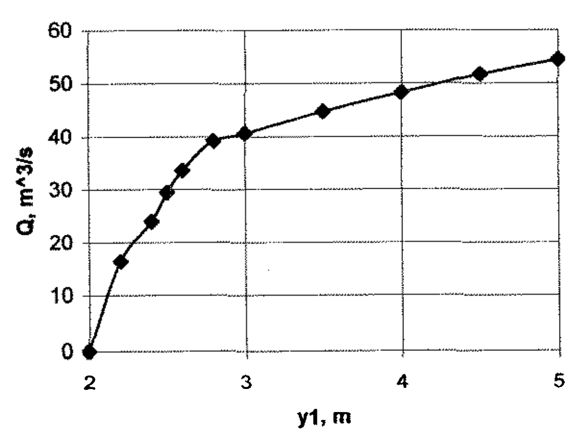
$Q = bq = b\,C_d\,a\sqrt{2gy_1}$, where $a = 1\,m$ and $b = 10\,m$.

Thus,

$Q = (10\,m)\,C_d\,(1\,m)\sqrt{2(9.81\frac{m}{s^2})(y_1\,m)} = 44.3\,C_d\sqrt{y_1}$, where $Q \sim \frac{m^3}{s}$ when $y_1 \sim m$

Obtain C_d from Fig. 10.25 with $\frac{y_3}{a} = 2$.

y1, m	y1/a	Cd	Q, m^3/s
2.00	2.00	0.00	0.00
2.20	2.20	0.25	16.43
2.40	2.40	0.35	24.02
2.50	2.50	0.42	29.42
2.60	2.60	0.47	33.57
2.80	2.80	0.53	39.29
3.00	3.00	0.53	40.67
3.50	3.50	0.54	44.75
4.00	4.00	0.55	48.29
4.50	4.50	0.55	51.69
5.00	5.00	0.55	54.48



10.114

10.114 Calibration of a Triangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.114 to calibrate a triangular weir and determine the relationship between flowrate, Q, and weir head, H.

Equipment: Water channel (flume) with a pump and a flow control valve; triangular weir; float; point gage; stop watch.

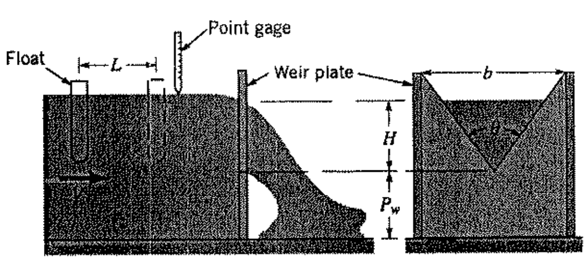
Experimental Procedure: Measure the width, b, of the channel, the distance, P_w, between the channel bottom and the bottom of the V-notch in the weir plate, and the angle, θ, of the V-notch. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q, over the weir. Use the point gage to measure the weir head, H. Insert the float into the water well upstream from the weir and measure the time, t, it takes for the float to travel a known distance, L. Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q, as ordinates and weir head, H, as abscissas. Draw the best-fit line with a slope of 5/2 through the data.

Results: Use the flowrate-weir head data to determine the triangular weir coefficient, C_{wt}, for this weir (see Eq. 10.32). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.114

(con't.)

10.114 (con't)

Solution for Problem 10.114 Calibration of a Triangluar Weir

θ, deg	b, in.	P_w, in.	L, ft
90	6.00	6.55	1.50

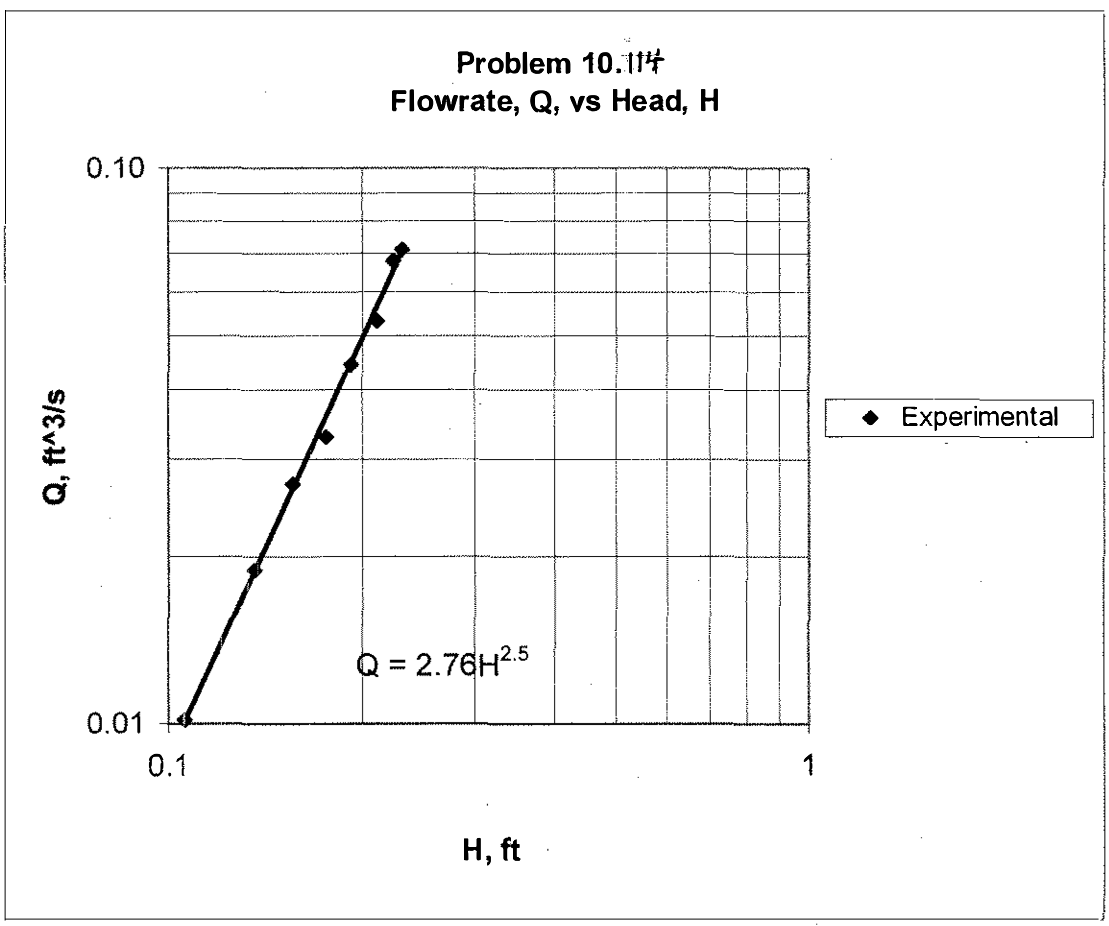
H, ft	t, s	V, ft/s	Q, ft^3/s
0.231	8.2	0.183	0.0711
0.224	8.5	0.176	0.0679
0.211	10.7	0.140	0.0530
0.192	12.5	0.120	0.0443
0.176	16.5	0.091	0.0328
0.156	19.5	0.077	0.0270
0.136	27.1	0.055	0.0189
0.106	48.2	0.031	0.0101
0.091	62.9	0.024	0.0076
0.088	68.1	0.022	0.0070

$Q = VA = V*b(P_w + H)$ where $V = L/t$

$Q = C_{wt}\ (8/15)\ \tan(\theta/2)\ (2g)^{1/2}\ H^{5/2}$ where from the graph

$Q = 2.76\ H^{2.5}$

Thus, $C_{wt} = (15/8)*2.76/(2*32.2)^{1/2} = \underline{0.645}$



10.115 Calibration of a Rectangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.115 to calibrate a rectangular weir and determine the relationship between flowrate, Q, and weir head, H.

Equipment: Water channel (flume) with a pump and a flow control valve; rectangular weir; float; point gage; stop watch.

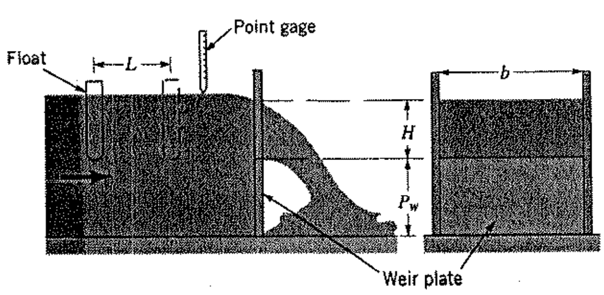
Experimental Procedure: Measure the width, b, of the channel and the distance, P_w, between the channel bottom and the top of the weir plate. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q, over the weir. Use the point gage to measure the weir head, H. Insert the float into the water well upstream from the weir and measure the time, t, it takes for the float to travel a known distance, L. Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q, as ordinates and weir head, H, as abscissas. Draw the best-fit line with a slope of 3/2 through the data.

Results: Use the flowrate-weir head data to determine the rectangular weir coefficient, C_{wr}, for this weir (see Eq. 10.30). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.115

(con't)

10.115 (con't)

Solution for Problem 10.115 Calibration of a Rectangular Weir

b, in.	P_w, in.	L, ft
6.00	6.00	1.40

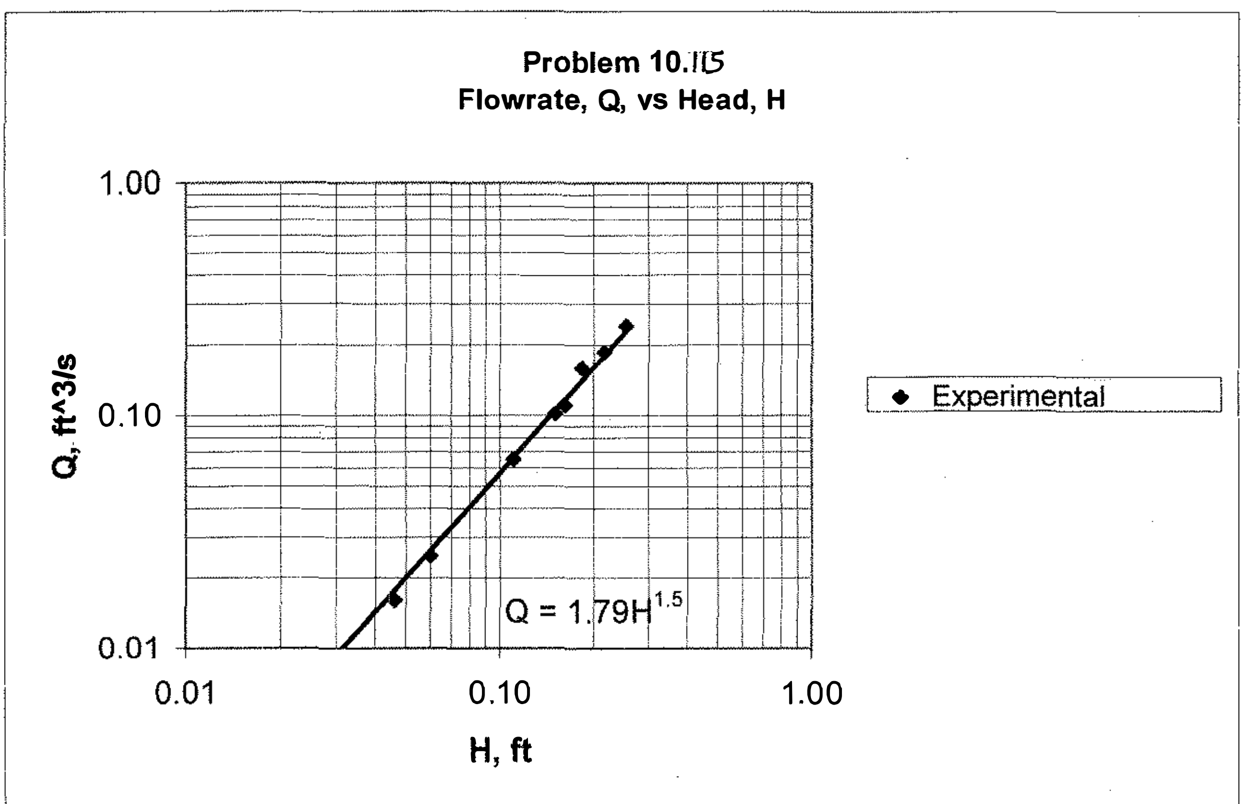
H, ft	t, s	V, ft/s	Q, ft^3/s
0.254	2.2	0.636	0.240
0.216	2.7	0.519	0.186
0.184	3.0	0.467	0.160
0.162	4.2	0.333	0.110
0.151	4.5	0.311	0.101
0.111	6.6	0.212	0.065
0.060	15.8	0.089	0.025
0.046	23.8	0.059	0.016
0.031	38.4	0.036	0.010

$Q = VA = V{*}b(P_w + H)$ where $V = L/t$

$Q = C_{wr}\,(2/3)\,(2g)^{1/2}\,H^{3/2}\,b$ where from the graph

$Q = 1.79\,H^{1.5}$

Thus, $C_{wr} = (3/2){*}1.79/(0.5{*}(2{*}32.2)^{1/2}) = \underline{0.669}$



10.116

10.116 Hydraulic Jump Depth Ratio

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.116 to determine the depth ratio, y_2/y_1, across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1.

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; adjustable tail gate.

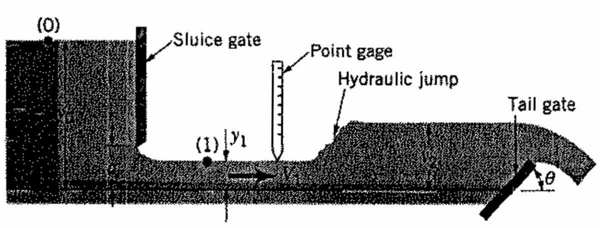
Experimental Procedure: Position the sluice gate so that the distance, a, between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0, upstream of the sluice gate. Carefully adjust the angle, θ, of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0, the depth just upstream from the jump, y_1, and the depth downstream from the jump, y_2. Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1, and Froude number, $Fr_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump (see Eq. 3.21). Also use the measured depths to determine the depth ratio, y_2/y_1, across the jump.

Graph: Plot the depth ratio, y_2/y_1, as ordinates and Froude number, Fr_1, as abscissas.

Results: On the same graph, plot the theoretical depth ratio as a function of Froude number (see Eq. 10.24).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.116

(con't)

10.116 (con't)

Solution for Problem 10.116 Hydraulic Jump Depth Ratio

			Experimental			Theoretical	
y_0, ft	y_1, ft	y_2, ft.	V_1, ft/s	Fr_1	y_2/y_1	Fr_1	y_2/y_1
0.855	0.055	0.404	7.19	5.40	7.35	1	1.00
0.759	0.055	0.386	6.75	5.07	7.02	2	2.37
0.691	0.055	0.367	6.42	4.82	6.67	3	3.77
0.578	0.055	0.337	5.83	4.38	6.13	4	5.18
0.492	0.055	0.308	5.34	4.01	5.60	5	6.59
0.414	0.055	0.280	4.85	3.65	5.09	6	8.00
0.289	0.055	0.233	3.95	2.97	4.24		
0.248	0.055	0.211	3.62	2.72	3.84		

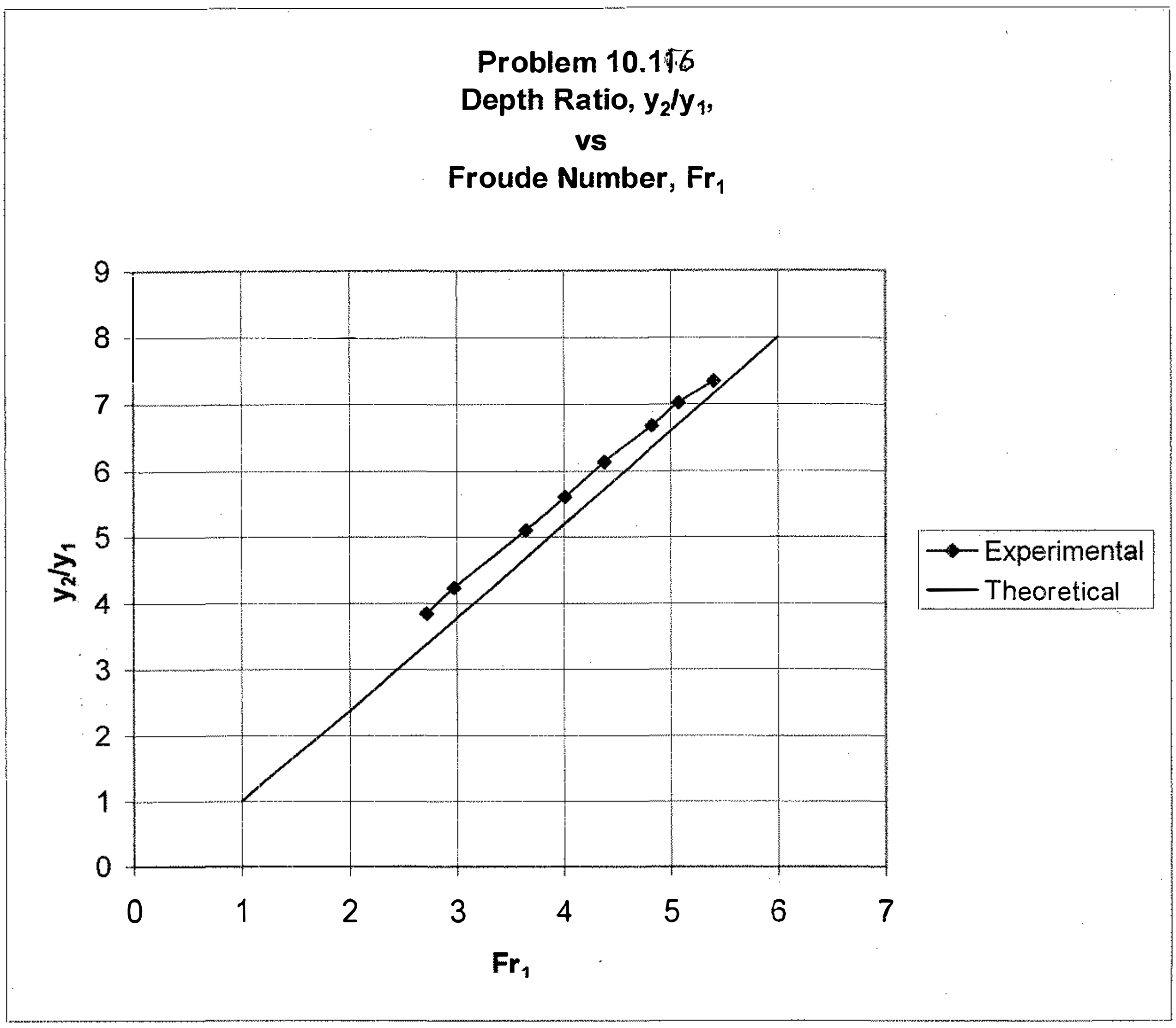
For flow under a sluice gate:

$$V_1 = [2g^*(y_0 - y_1)/(1 - (y_1/y_0)^2)]^{1/2}$$

Theory:

$$y_2/y_1 = [-1 +(1 +8Fr_1^2)^{1/2}]/2$$

$$Fr_1 = V_1/(gy_1)^{1/2}$$



10.117

10.117 Hydraulic Jump Head Loss

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.117 to determine the head loss ratio, h_L/y_1, across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1.

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; Pitot tubes; adjustable tail gate.

Experimental Procedure: Position the sluice gate so that the distance, a, between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0, upstream of the sluice gate. Carefully adjust the angle, θ, of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0, and the depth just upstream from the jump, y_1. Also measure the head loss, h_L, as the difference in the water elevations in the piezometer tubes connected to the two Pitot tubes located upstream and downstream of the jump. Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1, and the Froude number, $Fr_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump. Also calculate the dimensionless head loss, h_L/y_1, for each data set.

Graph: Plot the dimensionless head loss across the jump, h_L/y_1, as ordinates and the Froude number, Fr_1, as abscissas.

Results: On the same graph, plot the theoretical dimensionless head loss as a function of Froude number (see Eqs. 10.24 and 10.25).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

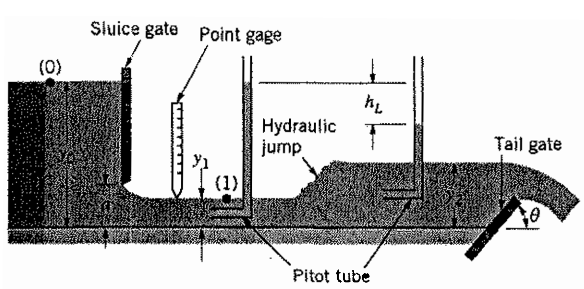


FIGURE P10.117

(con't)

10.117 (con't)

Solution for Problem 10.117 Hydraulic Jump Head Loss

				Experimental			Theoretical		
y_0, ft	y_1, ft	y_2, ft.	h_L, ft	V_1, ft/s	Fr_1	h_L/y_1	Fr_1	y_2/y_1	h_L/y_1
0.855	0.055	0.404	0.364	7.19	5.40	6.62	1	1.00	0.00
0.759	0.055	0.386	0.313	6.75	5.07	5.69	2	2.37	0.27
0.691	0.055	0.367	0.271	6.42	4.82	4.93	3	3.77	1.41
0.578	0.055	0.337	0.201	5.83	4.38	3.65	4	5.18	3.52
0.492	0.055	0.308	0.152	5.34	4.01	2.76	5	6.59	6.62
0.414	0.055	0.280	0.117	4.85	3.65	2.13	6	8.00	10.72
0.289	0.055	0.233	0.058	3.95	2.97	1.05			
0.248	0.055	0.211	0.042	3.62	2.72	0.76			

For flow under a sluice gate:

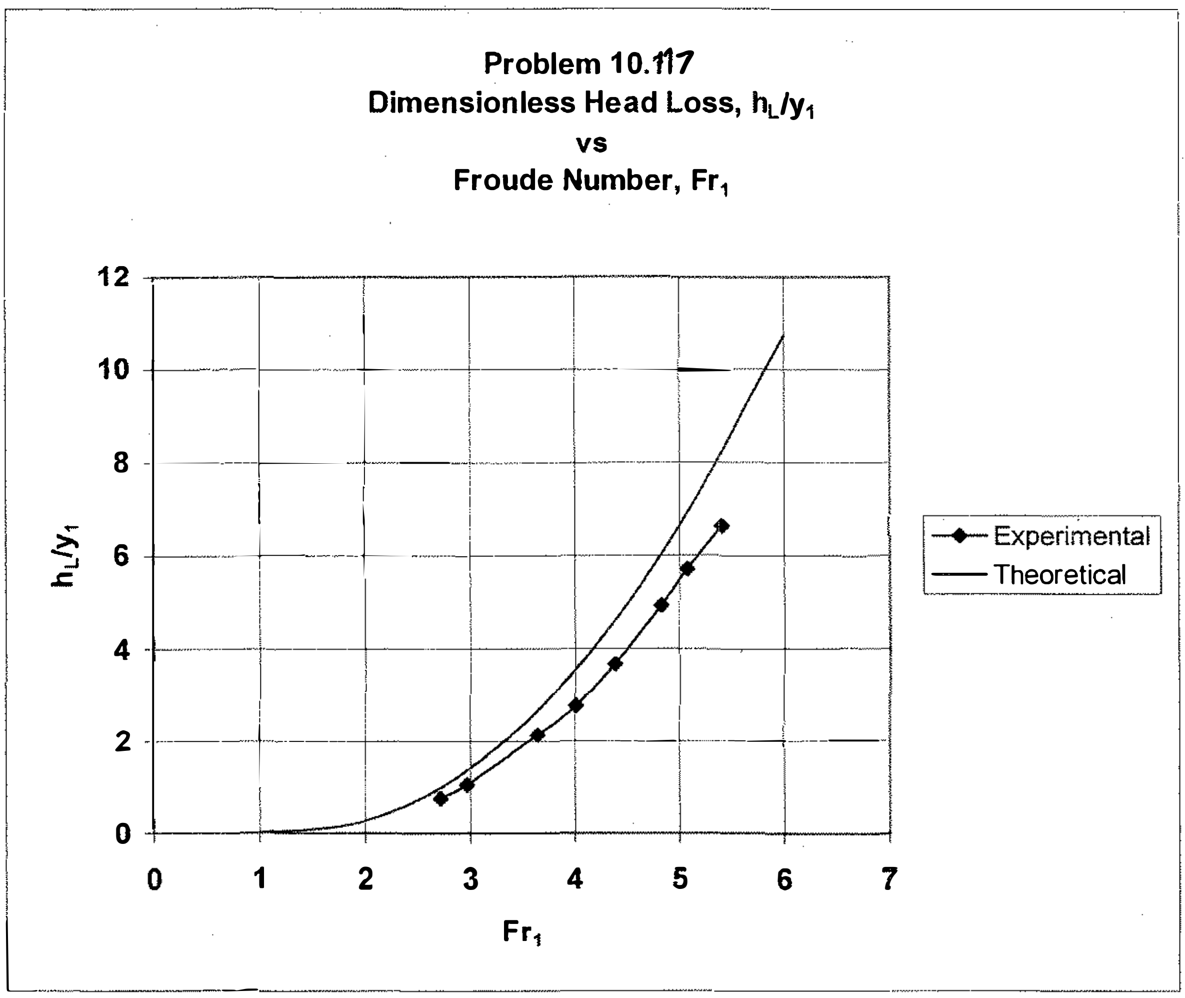
$$V_1 = [2g^*(y_0 - y_1)/(1 - (y_1/y_0)^2)]^{1/2}$$

Theory:

$$h_L/y_1 = 1 - (y_2/y_1) + Fr_1^2[1 - (y_1/y_2)^2]/2$$

where

$$y_2/y_1 = [-1 + (1 + 8Fr_1^2)^{1/2}]/2$$



11.1

11.1 Distinguish between flow of an ideal gas and inviscid flow of a fluid.

The flow of an ideal gas involves a gas that obeys the equation of state, Eq. 11.1

$$\rho = \frac{p}{RT}$$

and for which internal energy, $\check{u}$, is a function of temperature only. An ideal gas may have non-zero viscosity.

The inviscid flow of a fluid involves a fluid that has zero viscosity. That fluid may or may not be an ideal gas.

11.3

11.3 Five pounds mass of air are heated in a closed, rigid container from 80 °F, 15 psia to 500 °F. Estimate the final pressure of the air and the entropy rise involved.

To determine the final pressure, p_{final}, we can use the ideal gas equation (Eq. 11.1). Thus, for constant $\frac{mass}{volume}$ = density,

$$p_{final} = \frac{p_{initial}\, T_{final}}{T_{initial}} = \frac{(15\ psia)(960^\circ R)}{540^\circ R} = \underline{\underline{26.7}}\ psia$$

Eq. 11.22 may be used to determine the entropy rise, $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_{final}}{T_{initial}} - R \ln \frac{p_{final}}{p_{initial}}$$

and

$$s_2 - s_1 = \left(6006\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right) \ln\left(\frac{960^\circ R}{540^\circ R}\right) - \left(\underset{\text{From Table 1.7}}{1716}\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right) \ln\left(\frac{26.7\ psia}{15\ psia}\right) = \underline{\underline{2466}}\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}$$

11.4

11.4 Air flows steadily between two sections in a duct. At section (1), the temperature and pressure are $T_1 = 80\,°C$, $p_1 = 301$ kPa(abs), and at section (2), the temperature and pressure are $T_2 = 180\,°C$, $p_2 = 181$ kPa(abs). Calculate the (a) change in internal energy between sections (1) and (2), (b) change in enthalpy between sections (1) and (2), (c) change in density between sections (1) and (2), (d) change in entropy between sections (1) and (2). How would you estimate the loss of available energy between the two sections of this flow?

(a) Eq. 11.5 may be used to evaluate the change in internal energy, $\check{u}_2 - \check{u}_1$. Thus,

$$\check{u}_2 - \check{u}_1 = c_v (T_2 - T_1) = \left(717.2 \frac{J}{kg \cdot K}\right)(453\,K - 353\,K) = \underline{\underline{71{,}720 \frac{J}{kg}}}$$

(b) Eq. 11.9 may be used to evaluate the change in enthalpy, $\check{h}_2 - \check{h}_1$. Thus,

$$\check{h}_2 - \check{h}_1 = c_p (T_2 - T_1) = \left(1004 \frac{J}{kg \cdot K}\right)(453\,K - 353\,K) = \underline{\underline{100{,}400 \frac{J}{kg}}}$$

(c) The ideal gas equation (Eq. 11.1) may be used to evaluate the density at each section. Thus,

$$\rho_2 - \rho_1 = \frac{p_2}{RT_2} - \frac{p_1}{RT_1} = \frac{1}{R}\left(\frac{p_2}{T_2} - \frac{p_1}{T_1}\right)$$

or

$$\rho_2 - \rho_1 = \frac{1}{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)} \left[\frac{\left(181{,}000 \frac{N}{m^2}\right)}{(453\,K)} - \frac{\left(301{,}000 \frac{N}{m^2}\right)}{(353\,K)} \right] = \underline{\underline{-1.58 \frac{kg}{m^3}}}$$

From Table 1.8 ↑

(d) Eq. 11.22 may be used to evaluate the change in entropy, $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = \left(1004 \frac{J}{kg \cdot K}\right) \ln\left[\frac{(453\,K)}{(353\,K)}\right] - \left(286.9 \frac{N \cdot m}{kg \cdot K}\right) \ln\left[\frac{(181\,kPa)}{(301\,kPa)}\right]$$

or

$$s_2 - s_1 = \underline{\underline{396}} \frac{J}{kg \cdot K}$$

(con't)

11.4 (con't)

Since the flow involves a significant change in density, see solution to part (c) above, it is compressible and Eq. 5.108 must be used to evaluation the loss in available energy between sections (1) and (2). So from Eq. 5.108 we get

$$\text{loss} = \check{u}_2 - \check{u}_1 + \int_1^2 p\,d\left(\frac{1}{\rho}\right) - q_{\substack{net\\in}}$$

and to complete this solution we need more information so we can evaluate the integral and $q_{\substack{net\\in}}$.

11.5

11.5 Does the entropy change during the process of Example 11.2 indicate a loss of available energy by the flowing fluid?

We combine Eq. 5.106

$$d\check{u} + p\,d\left(\frac{1}{\rho}\right) - \delta q_{\substack{net\\in}} = \delta(\text{loss})$$

with Eq. 5.92

$$T\,ds = d\check{u} + p\,d\left(\frac{1}{\rho}\right)$$

to get

$$T\,ds - \delta q_{\substack{net\\in}} = \delta(\text{loss})$$

and conclude that if this flow is adiabatic ($\delta q_{\substack{net\\in}} = 0$) then entropy change is related to loss.

11.6

11.6 As demonstrated in Video V11.1, fluid density differences in a flow may be seen with the help of a schlieren optical system. Discuss what variables affect fluid density and the different ways in which a variable density flow can be achieved.

For an ideal gas:

$$\rho = \frac{p}{RT}$$

so changes in density, ρ, will accompany changes in pressure, p, gas composition, R, and/or temperature, T. Variations in fluid velocity and/or heating and cooling may result in pressure and temperature changes. Changes in gas composition that affect the value of the gas constant, R, will result in changes of density, ρ.

11.7

11.7 Describe briefly how a schlieren optical visualization system (**Videos V11.1** and **V11.4**, also Fig. 11.4) works. How else might density changes in a fluid flow be made visible to the eye?

Density variations in a transparent flowing fluid result in variations

These light speed variations result in changes in light ray direction and phase. Changes in light

local variations in perceived light brightness. The shadowgraph and schlieren methods make visible these variations in light brightness. An interferometer makes visible the local variations in light ray phase. A good description of these three flow vizualization methods may be found in The Handbook of Fluid Dynamics edited by Richard W. Johnson and published by the CRC Press (1998).

11.8

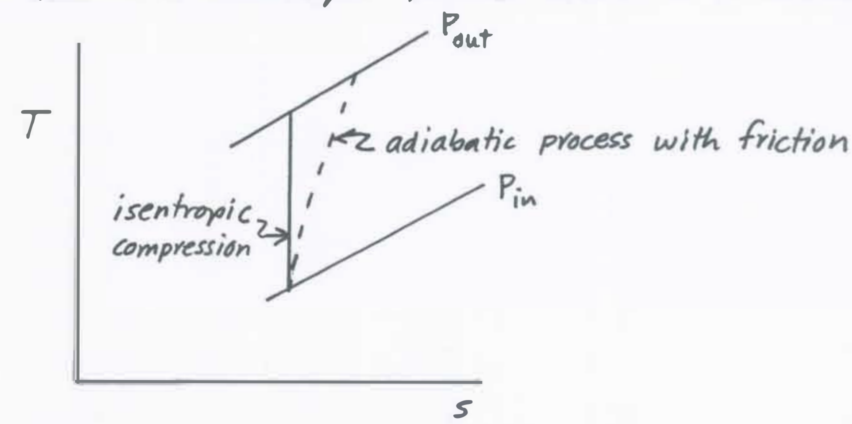
11.8 Explain why the Bernoulli equation (Eq. 3.7) cannot be accurately used for compressible flows.

Refer to Section 3.8.1 Compressibility Effects

11.9

11.9 Air at 14.7 psia and 70 °F is compressed adiabatically by a centrifugal compressor to a pressure of 100 psia. What is the minimum temperature rise possible? Explain.

The minimum temperature rise would occur with an adiabatic and frictionless process which involves a constant entropy or isentropic flow. According to the second law of thermodynamics, Eq. 5.101, the entropy must increase or remain constant during an adiabatic process, it cannot decrease. The T-s diagram sketched below illustrates how the isentropic process results in a minimum temperature rise.



For the isentropic process, Eq. 11.24 is valid. Thus,

$$T_{\substack{out\\minimum}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} = (530°R)\left(\frac{100\,psia}{14.7\,psia}\right)^{\frac{1.4-1}{1.4}} = 917°R$$

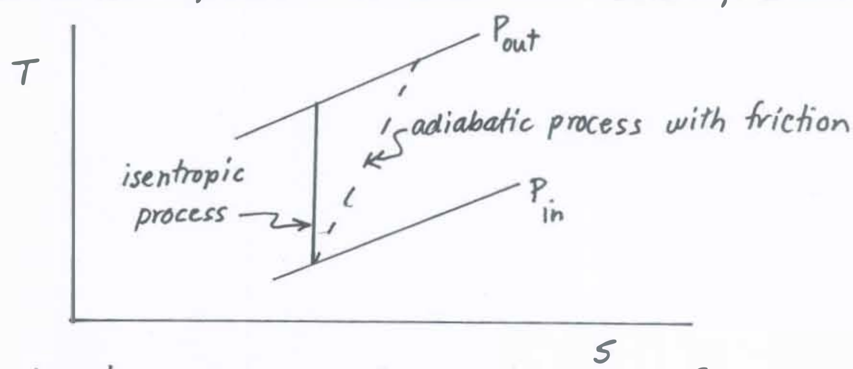
and

$$T_{\substack{out\\minimum}} - T_{in} = 917°R - 530°R = \underline{\underline{387\,°R}}$$

11.10

11.10 Methane is compressed adiabatically from 100 kPa(abs) and 25 °C to 200 kPa(abs). What is the minimum compressor exit temperature possible? Explain.

The minimum compressor exit temperature would occur with an adiabatic and frictionless process which involves a constant entropy or isentropic flow. According to the second law of thermodynamics, Eq. 5.101, the entropy must increase or remain constant during an adiabatic process, it cannot decrease. The T-s diagram sketched below illustrates how the isentropic process results in a lower exit temperature than any actual adiabatic process between the same pressures.



For the isentropic compression, we conclude from Eq. 11.24 that

$$T_{\substack{out\\minimum}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}}$$

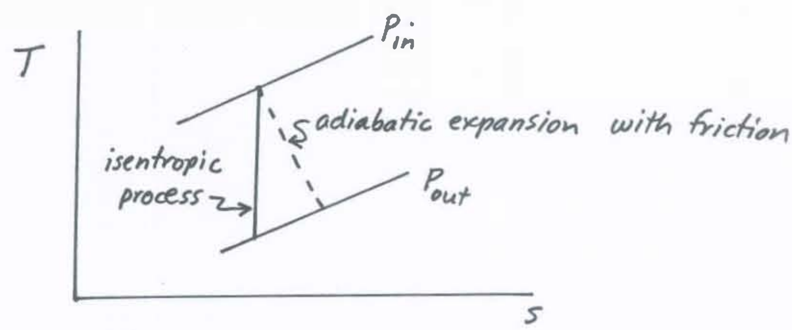
or

$$T_{\substack{out\\minimum}} = (298\text{ K})\left(\frac{200\text{ kPa}}{100\text{ kPa}}\right)^{\frac{1.31-1}{1.31}} = \underline{\underline{351\text{ K}}}$$

11.11

11.11 Air expands adiabatically through a turbine from a pressure and temperature of 180 psia, 1600 °R to a pressure of 14.7 psia. If the actual temperature change is 85% of the ideal temperature change, determine the actual temperature of the expanded air and the actual enthalpy and entropy differences across the turbine.

To determine the actual temperature of the expanded air and the actual enthalpy and entropy differences across the turbine we need first to determine the ideal temperature change across the turbine. The ideal temperature change across the turbine is associated with an adiabatic and frictionless and thus isentropic turbine expansion. The actual process involves a smaller temperature change as illustrated with the T-s diagram sketch below.



Eq. 11.24 is valid for the isentropic expansion. Thus,

$$T_{\substack{out\\ideal}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} = (1600\,°R)\left(\frac{14.7\,psia}{180\,psia}\right)^{\frac{1.4-1}{1.4}} = 782\,°R$$

Since

$$\left(T_{\substack{out\\actual}} - T_{in}\right) = 0.85\left(T_{\substack{out\\ideal}} - T_{in}\right)$$

then

$$T_{\substack{out\\actual}} = 0.85\left(782\,°R - 1600\,°R\right) + 1600\,°R = \underline{\underline{905}}\,°R$$

The actual enthalpy difference, $\check{h}_{\substack{out\\actual}} - \check{h}_{in}$, may be obtained with Eq. 11.9. Thus,

$$\check{h}_{\substack{out\\actual}} - \check{h}_{in} = c_p\left(T_{\substack{out\\actual}} - T_{in}\right) = \left(6006\,\frac{ft\cdot lb}{slug\cdot °R}\right)\left(905\,°R - 1600\,°R\right) = \underline{\underline{-4.17\times10^{6}\,\frac{ft\cdot lb}{slug}}}$$

The actual entropy difference, $s_{\substack{out\\actual}} - s_{in}$, may be calculated with Eq. 11.22. Thus,

$$s_{\substack{out\\actual}} - s_{in} = c_p \ln\left(\frac{T_{\substack{out\\actual}}}{T_{in}}\right) - R\ln\left(\frac{P_{out}}{P_{in}}\right) = \left(6006\,\frac{ft\cdot lb}{slug\cdot °R}\right)\ln\left(\frac{905\,°R}{1600\,°R}\right) - \left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)\ln\left(\frac{14.7\,psia}{180\,psia}\right)$$

↑ From Table 1.7

or

$$s_{\substack{out\\actual}} - s_{in} = \underline{\underline{877}}\,\frac{ft\cdot lb}{slug\cdot °R}$$

11.12

11.12 An expression for the value of c_p for carbon dioxide as a function of temperature is

$$c_p = 286 - \frac{1.15 \times 10^5}{T} + \frac{2.49 \times 10^6}{T^2}$$

where c_p is in (ft · lb)/(lbm · °R) and T is in °R. Compare the change in enthalpy of carbon dioxide using the constant value of c_p (see Table 1.7) with the change in enthalpy of carbon dioxide using the expression above, for $T_2 - T_1$ equal to **(a)** 10 °R, **(b)** 1000 °R, **(c)** 3000 °R. Set $T_1 = 540$ °R.

For constant c_p, the change in enthalpy, $\check{h}_2 - \check{h}_1$, may be evaluated with Eq. 11.9. Thus,

$$\left(\check{h}_2 - \check{h}_1\right)_{\substack{constant \\ c_p}} = c_p\,(T_2 - T_1)$$

For varying c_p, the change in enthalpy, $\check{h}_2 - \check{h}_1$, may be evaluated with Eq. 11.8. Thus,

$$\check{h}_2 - \check{h}_1 = \int_{T_1}^{T_2} c_p\, dT = \int_{T_1}^{T_2}\left(286 - \frac{1.15 \times 10^5}{T} + \frac{2.49 \times 10^6}{T^2}\right) dT$$

or

$$\left(\check{h}_2 - \check{h}_1\right)_{\substack{varying \\ c_p}} = 286\,\frac{\text{ft.lb}}{\text{lbm.°R}}(T_2 - T_1) - 1.15\times10^5\,\frac{\text{ft.lb}}{\text{lbm}}\ln\left(\frac{T_2}{T_1}\right) - 2.49\times10^6\,\frac{\text{ft.lb.°R}}{\text{lbm}}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

(a) For $T_1 = 540$°R and $T_2 = 550$°R

$$\left(\check{h}_2 - \check{h}_1\right)_{\substack{constant \\ c_p}} = \left(152\,\frac{\text{ft.lb}}{\text{lbm.°R}}\right)(550\text{°R} - 540\text{°R}) = \underline{\underline{1520}}\ \frac{\text{ft.lb}}{\text{lbm}}$$

and

$$\left(\check{h}_2 - \check{h}_1\right)_{\substack{varying \\ c_p}} = \left(286\,\frac{\text{ft.lb}}{\text{lbm.°R}}\right)(550\text{°R} - 540\text{°R}) - \left(1.15\times10^5\,\frac{\text{ft.lb}}{\text{lbm}}\right)\ln\left(\frac{550\text{°R}}{540\text{°R}}\right) - \left(2.49\times10^6\,\frac{\text{ft.lb.°R}}{\text{lbm}}\right)\left(\frac{1}{550\text{°R}} - \frac{1}{540\text{°R}}\right)$$

$$\left(\check{h}_2 - \check{h}_1\right)_{\substack{varying \\ c_p}} = \underline{\underline{1580}}\ \frac{\text{ft.lb}}{\text{lbm}}$$

(con't)

11.12 (con't)

(b) For $T_1 = 540°R$ and $T_2 = 1540°R$

$$(\check{h}_2 - \check{h}_1)_{\substack{constant \\ c_p}} = \left(152 \frac{ft.lb}{lbm.°R}\right)(1540°R - 540°R) = \underline{\underline{1.52 \times 10^5}} \frac{ft.lb}{lbm}$$

and

$$(\check{h}_2 - \check{h}_1)_{\substack{varying \\ c_p}} = \left(286 \frac{ft.lb}{lbm.°R}\right)(1540°R - 540°R)$$

$$-\left(1.15 \times 10^5 \frac{ft.lb}{lbm}\right) \ln\left(\frac{1540°R}{540°R}\right)$$

$$-\left(2.49 \times 10^6 \frac{ft.lb.°R}{lbm}\right)\left(\frac{1}{1540°R} - \frac{1}{540°R}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\substack{varying \\ c_p}} = \underline{\underline{1.95 \times 10^5}} \frac{ft.lb}{lbm}$$

(c) For $T_1 = 540°R$ and $T_2 = 3540°R$

$$(\check{h}_2 - \check{h}_1)_{\substack{constant \\ c_p}} = \left(152 \frac{ft.lb}{lbm.°R}\right)(3540°R - 540°R) = \underline{\underline{4.56 \times 10^5}} \frac{ft.lb}{lbm}$$

$$(\check{h}_2 - \check{h}_1)_{\substack{varying \\ c_p}} = \left(286 \frac{ft.lb}{lbm.°R}\right)(3540°R - 540°R) - \left(1.15 \times 10^5 \frac{ft.lb}{lbm}\right) \ln\left(\frac{3540°R}{540°R}\right)$$

$$-\left(2.49 \times 10^6 \frac{ft.lb.°R}{lbm}\right)\left(\frac{1}{3540°R} - \frac{1}{540°R}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\substack{varying \\ c_p}} = \underline{\underline{6.80 \times 10^5}} \frac{ft.lb}{lbm}$$

11.14

11.14 Confirm the speed of sound for air at 70 °F listed in Table B.3.

Eq. 11.36 is suitable for calculating the speed of sound in air. Thus,

$$c = \sqrt{RTk} = \sqrt{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^{\circ}R}\right) \frac{(530\,{}^{\circ}R)(1.401)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1129 \frac{ft}{s}$$

From Table B.3

$c = 1128 \frac{ft}{s}$ for air at $70^{\circ}F$. The values of c are comparable.

11.15

11.15 From Table B.1 we can conclude that the speed of sound in water at 60 °F is 4814 ft/s. Is this value of c consistent with the value of bulk modulus, E_v, listed in Table 1.5?

The speed of sound in water may be approximated from a nominal value of the bulk modulus, E_v, and density, ρ, with Eq. 11.38. Thus

$$c = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{\left(3.12 \times 10^5 \frac{lb}{in.^2}\right)\left(144 \frac{in^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{4812 \frac{ft}{s}}}$$

From Table B.1

$$c = 4814 \frac{ft}{s}$$

11.16

11.16 If the observed speed of sound in steel is 5300 m/s, determine the bulk modulus of elasticity of steel in N/m^3. The density of steel is nominally 7790 kg/m^3. How does your value of E_v for steel compare with E_v for water at 15.6 °C? Compare the speeds of sound in steel, water, and air at standard atmospheric pressure and 15 °C and comment on what you observe.

The speed of sound, c, is related to the bulk modulus of elasticity, E_v, and density, ρ, by Eq. 11.38 as follows

$$c = \sqrt{\frac{E_v}{\rho}}$$

Thus

$$E_v = \rho c^2$$

Table 1.7).

and for steel

$$E_{v_{steel}} = \left(7790 \frac{kg}{m^3}\right)\left(5300 \frac{m}{s}\right)^2 \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

or

$$E_{v_{steel}} = \underline{\underline{2.19 \times 10^{11}}} \frac{N}{m^2}$$

For water at 15.6 °C we get from Table 1.6 m Table 1.7.

$$E_{v_{water}} = 2.15 \times 10^9 \frac{N}{m^2}$$

For water at 15.6°C

$$c_{water} = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{\left(2.15 \times 10^9 \frac{N}{m^2}\right)}{\left(999 \frac{kg}{m^3}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = \underline{\underline{1470}} \frac{m}{s}$$

For steel

$$c_{steel} = \underline{\underline{5300}} \frac{m}{s}$$ which is much higher than the speed of sound in water

For air at 15°C we get from Table B.4

$$c_{air} = 340.4 \ m/s$$

The least compressible material, steel, involves the largest speed of sound. The most compressible material, air, involves the smallest speed of sound. This matches our intuition.

11.17

11.17 Using information provided in Table C.1, develop a table of speed of sound in ft/s as a function of elevation for U.S. standard atmosphere.

We can use Eq. 11.36 to determine the speed of sound in U.S. standard atmosphere at the elevations listed in Table C.1. Thus,

$$c = \sqrt{RTk}$$

We use $R = 1716 \frac{ft \cdot lb}{slug \cdot {}^{\circ}R}$ and $k = 1.40$ from Table 1.7. For absolute temperature we add $460\,{}^{\circ}R$ to ${}^{\circ}F$. For altitude = −5000 ft

$$c = \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^{\circ}R}\right)(538.84\,{}^{\circ}R)(1.40)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1136 \frac{ft}{s}$$

For all elevations, the same procedure shown above was used. The results are:

altitude ft	c ft/s
−5000	1136
0	1117
5000	1097
10,000	1078
15,000	1058
20,000	1037
25,000	1016
30,000	995
35,000	973
40,000	968
45,000	968
50,000	968
60,000	968
70,000	971
80,000	978
90,000	984
100,000	991
150,000	1073
200,000	1028
250,000	944

11.18

11.18 Using information provided in Table C.2, develop a table of speed of sound in m/s as a function of elevation for U.S. standard atmosphere.

We can use Eq. 11.36 to determine the speed of sound in U.S. standard atmosphere at the elevations listed in Table C.2. Thus,

$$c = \sqrt{RTk}$$

We use $R = 286.9 \frac{J}{kg \cdot K}$ and $k = 1.40$ from Table 1.8. For absolute temperature we add 273 K to °C. For altitude = -1000 m,

$$c = \sqrt{\frac{\left(286.9 \frac{J}{kg \cdot K}\right)(294.5\,K)(1.40)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 344 \frac{m}{s}$$

For all elevations, the above procedure was used. The results are:

altitude m	c m/s
-1000	344
0	340
1000	336
2000	332
3000	328
4000	324
5000	320
6000	316
7000	312
8000	308
9000	304
10,000	299
15,000	295
20,000	295
25,000	298
30,000	302
40,000	317
50,000	330
60,000	315
70,000	297
80,000	282

11.19

11.19 Determine the Mach number of a car moving in standard air at a speed of **(a)** 25 mph, **(b)** 55 mph, and **(c)** 100 mph.

The Mach number is the ratio of local velocity to speed of sound. Thus

$$Ma = \frac{V}{c}$$

For standard air

$$c = \sqrt{RTk} = \sqrt{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(519\ {}^\circ R)(1.4)} = 1117 \frac{ft}{s}$$

or

$$c = \left(1117 \frac{ft}{s}\right)\frac{\left(3600 \frac{s}{hr}\right)}{\left(5280 \frac{ft}{mi}\right)} = 761.6 \text{ mph}$$

(a) For $V = 25$ mph

$$Ma = \frac{25 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.0328}}$$

(b) For $V = 55$ mph

$$Ma = \frac{55 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.0722}}$$

(c) For $V = 100$ mph

$$Ma = \frac{100 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.131}}$$

11.23

11.23 At a given instant of time, two of the pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest are shown in Fig. P11.23 Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.

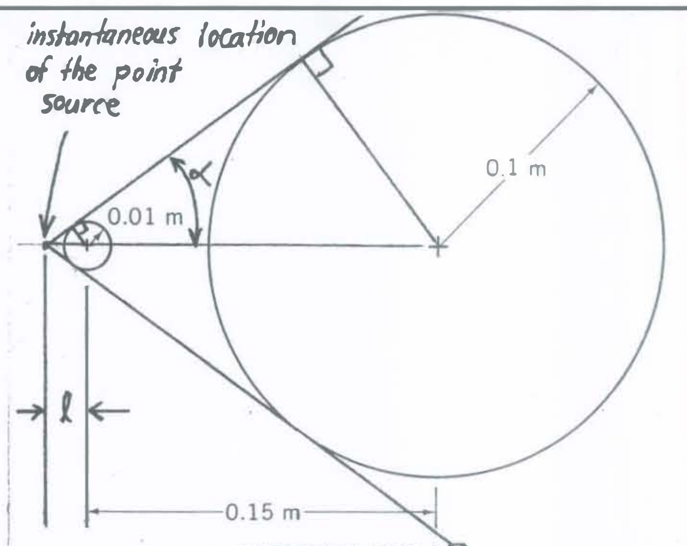


FIGURE P11.23

The Mach number associated with the motion of the point source involved in the sketch above is easily obtained with Eq. 11.39 as shown below.

$$Ma = \frac{1}{\sin\alpha}$$

From the sketch above we note that

$$\sin\alpha = \frac{0.01\ m}{\ell} = \frac{0.1\ m}{0.15\ m + \ell}$$

Thus

$$(0.01m)(0.15m + \ell) = (0.1\ m)\ \ell$$

or

$$\ell = \frac{(0.01m)(0.15m)}{(0.09m)} = \underline{\underline{0.0167\ m}}$$

and

$$\sin\alpha = \frac{0.01\ m}{0.0167\ m} = 0.599$$

Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{1}{0.599} = \underline{\underline{1.67}}$$

11.24

11.24 At a given instant of time, two of the pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest are shown in Fig. P11.24. Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.

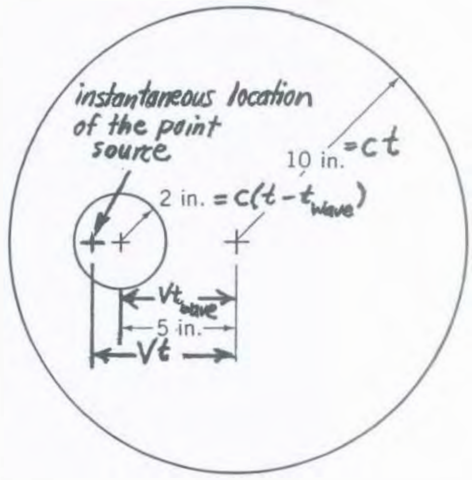


FIGURE P11.24

To determine the Mach number, Ma, we use

$$Ma = \frac{V t_{wave}}{c\, t_{wave}} \qquad (1)$$

However, from the sketch above we have

$$c(t - t_{wave}) = 2 \text{ in.} = ct - ct_{wave} = 10 \text{ in.} - ct_{wave}$$

Thus,

$$ct_{wave} = 10 \text{ in.} - 2 \text{ in.} = 8 \text{ in.}$$

and with Eq. 1

$$Ma = \frac{5 \text{ in.}}{8 \text{ in.}} = \underline{\underline{0.625}}$$

Also

$$Ma = \frac{Vt}{ct} = \frac{Vt}{10 \text{ in.}} = 0.625$$

Thus,

$$Vt = (0.625)(10 \text{ in.}) = \underline{\underline{6.25 \text{ in.}}}$$

11.25

11.25 Sound waves are very small amplitude pressure pulses that travel at the "speed of sound." Do very large amplitude waves such as a blast wave caused by an explosion (see **Video V11.7**) travel less than, equal to, or greater than the speed of sound? Explain.

The speed of sound is the speed at which an infinitesimal pressure disturbance travels through a fluid and it represents the minimum speed of this disturbance. Finite pressure disturbances travel faster than sound waves because the larger pressure difference acts as a driver of faster movement.

11.26

11.26 How would you estimate the distance between you and an approaching storm front involving lightning and thunder?

One way to estimate the distance between you and approaching storm clouds, x, is to count the number of seconds, t, between seeing the lightning and hearing thunder. Using an approximate value of the speed of sound, $1145\ \frac{ft}{s}$ (see Table B.3) we can approximate distance, x from

$$x = \left(1145\ \frac{ft}{s}\right)(t)$$

11.27

11.27 If a person inhales helium and then talks, his or her voice sounds like "Donald Duck." Explain why this happens.

The speed of sound in helium is nearly three times the speed of sound in air.

11.28

11.28 If a high-performance aircraft is able to cruise at a Mach number of 3.0 at an altitude of 80,000 ft, how fast is this in **(a)** mph, **(b)** ft/s, **(c)** m/s?

(b) With Eq. 11.46

$$V = (Ma)\,c$$

and at 80,000 ft in U.S. standard atmosphere, we have from the solution of problem 11.16

$$c = 978\ \frac{ft}{s}$$

Thus

$$V = (3.0)\left(978\ \frac{ft}{s}\right) = \underline{\underline{2930}}\ \frac{ft}{s}$$

(a) Then

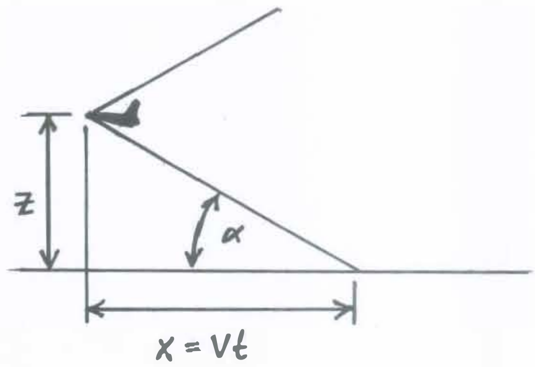
$$V = \left(2930\ \frac{ft}{s}\right)\frac{\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = \underline{\underline{2000}}\ mph$$

(c) Also

$$V = \left(2930\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = \underline{\underline{893}}\ \frac{m}{s}$$

11.29

11.29 At the seashore, you observe a high-speed aircraft moving overhead at an elevation of 10,000 ft. You hear the plane 8 s after it passes directly overhead. Using a nominal air temperature of 40 °F, estimate the Mach number and speed of the aircraft.



The Mach number is related to the angle α by Eq. 11.39. Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{V}{c} \qquad (1)$$

Also

$$\tan\alpha = \frac{z}{Vt} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{\sin\alpha}{\cos\alpha} = \frac{z \sin\alpha}{c\ t}$$

or

$$\alpha = \cos^{-1}\left(\frac{c\,t}{z}\right)$$

Now

$$c = \sqrt{RTk} = \sqrt{\frac{\left(1716\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(500\,^\circ R)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1096\ \frac{ft}{s}$$

Then

$$\alpha = \cos^{-1}\left[\frac{\left(1096\ \frac{ft}{s}\right)(8\ s)}{(10000\ ft)}\right] = 28.7^\circ$$

and

$$Ma = \frac{1}{\sin 28.7^\circ} = \underline{\underline{2.08}}$$

Further

$$V = (Ma)\ c = (2.08)\left(1096\ \frac{ft}{s}\right) = \underline{\underline{2280}}\ \frac{ft}{s}$$

11-30

11.30 Explain how you could vary the Mach number but not the Reynolds number in air flow past a sphere. For a constant Reynolds number of 300,000, estimate how much the drag coefficient will increase as the Mach number is increased from 0.3 to 1.0.

Considering air as an ideal gas, we can express the Mach number Ma, as

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (1)$$

The Reynolds number, Re, is

$$Re = \frac{\rho V d}{\mu} = \frac{pVd}{RT\mu} \qquad (2)$$

Looking at equations 1 and 2 we reason that we can vary Ma while holding Re constant by varying V and p only with pV held constant.

From the graph below we conclude that at $Re = 3\times10^5$, the drag coefficient increases from 0.47 to 0.75 at Ma increases from 0.3 to 1.0.

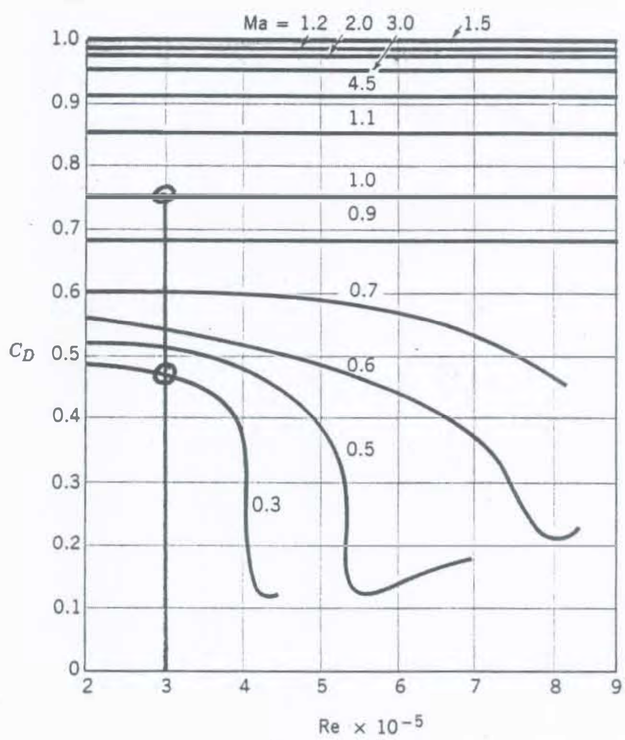


FIGURE 11.2 The variation of the drag coefficient of a sphere with Reynolds number and Mach number. (Adapted from Fig. 1.8 in Ref. 1 of Chapter 9)

11.33

11.33 Starting with the enthalpy form of the energy equation (Eq. 5.69), show that for isentropic flows, the stagnation temperature remains constant. Why is this important?

Starting with Eq. 5.69 we have

$$\dot{m}\left[\check{h}_{out} - \check{h}_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\substack{net \\ in}} + \dot{W}_{\substack{shaft \\ net\ in}}$$

For isentropic flow the entropy remains constant and $\dot{Q}_{net} = 0$. Stagnation enthalpy is defined as

$$\check{h}_0 = \check{h} + \frac{V^2}{2}$$

So, for negligible change in elevation (okay for gases) and no shaft work, $\dot{W}_{shaft}$, then

$\check{h}_0$ remains constant.

and since for an ideal gas enthalpy is a function of temperature only, we conclude that constant $\check{h}_0$ means constant stagnation temperature T_0.

This constant stagnation temperature provides us with a convenient reference property at every location in a specific isentropic flow.

11.34

11.34 Explain how fluid pressure varies with cross section area change for the isentropic flow of an ideal gas when the flow is **(a)** subsonic; **(b)** supersonic.

With the help of Eq. 11.47 we can comment on how pressure varies with area change in an isentropic flow. From Eq. 11.47 we obtain

$$dp = \frac{\rho V^2}{(1-Ma^2)} \frac{dA}{A} \qquad (1)$$

(a) For subsonic flow, Eq. 1 suggests that changes of p follow changes of A. If A increases, p increases and vice versa.

(b) For supersonic flow, Eq. 1 suggests that changes of p are opposite to changes of A. If A increases, p decreases and vice versa.

11.35

11.35 For any ideal gas, prove that the slope of constant pressure lines on a temperature–entropy diagram is positive and that higher pressure lines are above lower pressure lines. Why is this important?

From the second Tds equation (Eq. 11.18) we note that for a constant pressure line

$$\frac{d\check{h}}{ds} = T$$

and since for an ideal gas Eq. 11.7 is valid, we have

$$d\check{h} = c_p\, dT$$

and thus

$$\frac{dT}{ds} = \frac{T}{c_p} \qquad (1)$$

With Eq. 1 we conclude that the slope of a constant pressure line on a temperature - entropy diagram is positive.

Further, from Eq. 11.24 we conclude that

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{k}{k-1}\right)}$$

for any isentropic process and thus higher pressure lines are above lower pressure lines in temperature - entropy diagrams. This information is important because it enables us to sketch T-s diagrams correctly.

11.36

11.36 Air flows steadily and isentropically from standard atmospheric conditions to a receiver pipe through a converging duct. The cross-sectional area of the throat of the converging duct is 0.05 ft^2. Determine the mass flowrate through the duct if the receiver pressure is **(a)** 10 psia, **(b)** 5 psia. Sketch temperature–entropy diagrams for situations **(a)** and **(b)**. Verify results obtained with values from the appropriate graph in Appendix D with calculations involving ideal gas equations. Is condensation of water vapour a concern? Explain.

This problem is similar to Example 11.5

The mass flowrate is obtained at the throat with Eq. 11.40. Thus,

$$\dot{m} = \rho_{th} A_{th} V_{th} \qquad (1)$$

The throat density can be obtained with Eq. 11.60. Thus,

$$\rho_{th} = \rho_0 \left[\frac{1}{1 + \left(\frac{k-1}{2}\right) Ma_{th}^2} \right]^{\frac{1}{k-1}} \qquad (2)$$

To determine the throat Mach number we use Eq. 11.59. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{k-1}\right)\left[\left(\frac{p_0}{p_{th}}\right)^{\frac{k-1}{k}} - 1\right]} \qquad (3)$$

The critical throat pressure is obtained with Eq. 11.61. Thus,

$$p_{th}^* = p_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = (14.7\ psia)\left(\frac{2}{1.40+1}\right)^{\frac{1.4}{1.4-1}} = 7.76\ psia$$

If the receiver pressure, p_{re}, is greater than or equal to p_{th}^*, then $p_{th} = p_{re}$ and the flow is not choked. If $p_{re} < p_{th}^*$, then $p_{th} = p_{th}^*$ and the flow is choked.

The velocity at the throat is obtained with Eqs. 11.36 and 11.46 combined to yield

$$V_{th} = Ma_{th} \sqrt{R T_{th} k} \qquad (4)$$

where T_{th} is obtained with Eq. 11.56. Thus,

$$T_{th} = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_{th}^2} \qquad (5)$$

(con't)

11.36	(con't)

(a) For $p_{re} = 10$ psia $> p_{th}^* = 7.76$ psia, $p_{th} = 10$ psia and we use Eq. 3 to calculate the throat Mach number. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{1.40-1}\right)\left[\left(\frac{14.7\text{ psia}}{10\text{ psia}}\right)^{\frac{1.40-1}{1.40}} - 1\right]} = 0.7628$$

From Eq. 2 we obtain

$$\rho_{th} = \left(2.38\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}\right)\left[\frac{1}{1+\left(\frac{1.40-1}{2}\right)(0.7628)^2}\right]^{\frac{1}{1.40-1}} = 1.807\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}$$

From Eq. 5 we get

$$T_{th} = \frac{519°R}{1+\left(\frac{1.40-1}{2}\right)(0.7628)^2} = 464.9°R$$

and with Eq. 4

$$V_{th} = (0.7628)\sqrt{\frac{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(1.40)(464.9°R)}{\left(1\,\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 806.2\,\frac{\text{ft}}{\text{s}}$$

With Eq. 1 we obtain

$$\dot m = \left(1.807\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}\right)\left(0.05\text{ ft}^2\right)\left(806.2\,\frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.0728\,\frac{\text{slug}}{\text{s}}}}$$

$$\dot m = \left(0.0728\,\frac{\text{slug}}{\text{s}}\right)\left(32.174\,\frac{\text{lbm}}{\text{s}}\right)$$

$$\dot m = \underline{\underline{2.34\,\frac{\text{lbm}}{\text{s}}}}$$

Alternatively, using Fig. D.1 with

$$\frac{p_{th}}{p_0} = \frac{10\text{ psia}}{14.7\text{ psia}} = 0.68$$

The value of Ma_{th} is

$$Ma_{th} = 0.76$$

For $Ma_{th} = 0.76$, we get from Fig. D.1

$$T_{th} = (0.9\quad)T_0 = (0.9\quad)(519°R) = 467°R$$

Then with Eq. 4

$$V_{th} = 0.76\sqrt{\frac{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(1.40)(467°R)}{\left(1\,\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 805\,\frac{\text{ft}}{\text{s}}$$

(con't)

11.36 (con't)

For $Ma_{th} = 0.76$ we get from Fig. D.1

$$\rho_{th} = 0.76086\,\rho_o = (0.76)\left(2.38\times10^{-3}\,\frac{slug}{ft^3}\right) = 1.8\times10^{-3}\,\frac{slug}{ft^3}$$

Now, with Eq. 1 we obtain

$$\dot{m} = \left(1.8\times10^{-3}\,\frac{slug}{ft^3}\right)(0.05\,ft^2)\left(805\,\frac{ft}{s}\right) = \underline{\underline{0.076}}\,\frac{slug}{ft^3} = \underline{\underline{2.44}}\,\frac{lbm}{s}$$

(b) For $p_{re} = 5\,psia < p^* = 7.76\,psia$, $p_{th} = 7.76\,psia$ and $Ma_{th} = 1.0$. From Eq. 2,

$$\rho_{th} = \left(2.38\times10^{-3}\,\frac{slug}{ft^3}\right)\left[\frac{1}{1+\left(\frac{1.40-1}{2}\right)}\right]^{\frac{1}{1.40-1}} = 1.509\times10^{-3}\,\frac{slug}{ft^3}$$

From Eq. 5 we obtain

$$T_{th} = \frac{519\,°R}{1+\left(\frac{1.40-1}{2}\right)} = 432.5\,°R$$

and with Eq. 4

$$V_{th} = \sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(1.40)(432.5\,°R)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1019\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$\dot{m} = \left(1.509\times10^{-3}\,\frac{slug}{ft^3}\right)(0.05\,ft^2)\left(1019\,\frac{ft}{s}\right) = \underline{\underline{0.0769}}\,\frac{slug}{s} = \underline{\underline{2.47}}\,\frac{lbm}{s}$$

Alternatively, from Fig. D.1 for Ma = 1.0

$$T_{th} = (0.83)(519\,°R) = 431\,°R$$

and

$$\rho_{th} = (0.64)\left(2.38\times10^{-3}\,\frac{slug}{ft^3}\right) = 1.52\times10^{-3}\,\frac{slug}{ft^3}$$

Then with Eq. 4

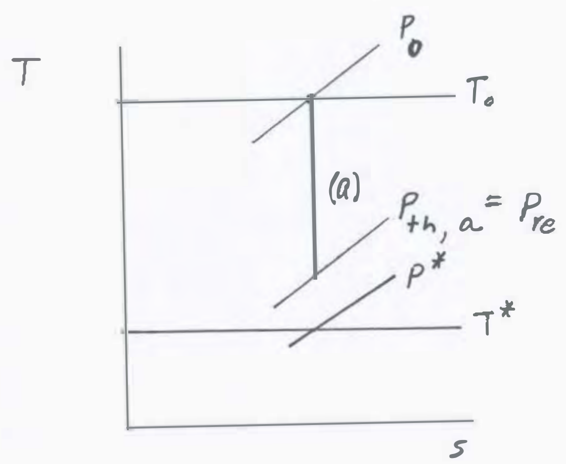
$$V_{th} = \sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(1.40)(431\,°R)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1020\,\frac{ft}{s}$$

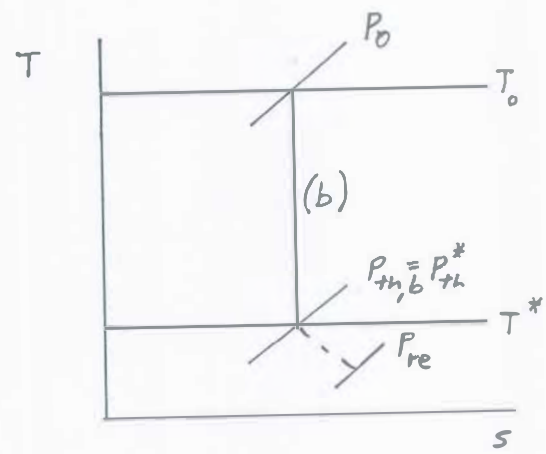
(con't)

11.36 (con't)

and with Eq. 1 we obtain

$$\dot{m} = \left(1.52 \times 10^{-3} \frac{slug}{ft^3}\right)\left(0.05\ ft^2\right)\left(1020 \frac{ft}{s}\right) = \underline{\underline{0.078}}\ \frac{slug}{s} = \underline{\underline{2.51}}\ \frac{lbm}{s}$$





Condensation of water vapour is a topic that deserves further study and discussion.

11.37

11.37 Determine the static pressure to stagnation pressure ratio associated with the following motion in standard air: **(a)** a runner moving at the rate of 10 mph, **(b)** a cyclist moving at the rate of 40 mph, **(c)** a car moving at the rate of 65 mph, **(d)** an airplane moving at the rate of 500 mph.

With a value of Mach number calculated with

$$Ma = \frac{V}{c} \qquad (1)$$

we can calculate $\frac{p}{p_0}$ with

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]^{\frac{k}{k-1}} \qquad (11.59)$$

For c we use for parts a, b and c

$$c = \sqrt{RTk} = \sqrt{\frac{\left(1716\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(519\ {}^\circ R)(1.40)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1117\ \frac{ft}{s}$$

or

$$c = \frac{\left(1117\ \frac{ft}{s}\right)\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = 761.6\ mph$$

(a) For $V = 10\ mph$

$$Ma = \frac{10\ mph}{761.6\ mph} = 0.0131$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{1.4-1}{2}\right)(0.0131)^2}\right]^{\frac{1.4}{1.4-1}} = \left[\frac{1}{1+(0.2)(0.0131)^2}\right]^{3.5} = 0.99988$$

(b) For $V = 40\ mph$

$$Ma = \frac{40\ mph}{761.6\ mph} = 0.0525$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0525)^2}\right]^{3.5} = 0.998$$

(c) For $V = 65\ mph$

$$Ma = \frac{65\ mph}{761.6\ mph} = 0.0854$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0854)^2}\right]^{3.5} = 0.9949$$

(con't)

11.37 (con't)

(d) For airplane we assume a nominal altitude of 30,000 ft. From Table C.1 we note a corresponding temperature of $-47.83^\circ F$. Then

$$c = \sqrt{\left(1716\ \frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)\frac{\left[(-47.83+460)^\circ R\right](1.4)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}}$$

$$c = 995\ \frac{ft}{s}$$

or

$$c = \left(995\ \frac{ft}{s}\right)\frac{\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = 678\ mph$$

Then for

$$Ma = \frac{500\ mph}{678\ mph} = 0.738$$

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.738)^2}\right]^{3.5} = 0.696$$

11.38

11.38 The static pressure to stagnation pressure ratio at a point in a gas flow field is measured with a Pitot-static probe as being equal to 0.6. The stagnation temperature of the gas is 20 °C. Determine the flow speed in m/s and the Mach number if the gas is air. What error would be associated with assuming that the flow is incompressible?

To determine the flow speed and Mach number having been given the static pressure to stagnation pressure ratio, $\frac{p}{p_0}$, and stagnation temperature, T_0, for air we enter Fig. D.1 with the given value of $\frac{p}{p_0}$ and read the corresponding value of Ma. Thus with $\frac{p}{p_0} = 0.6$, the corresponding value in Fig. D.1 is

$$Ma = \underline{\underline{0.89}}$$

For $Ma = 0.89$, Fig. D.1 gives

$$\frac{T}{T_0} = 0.86$$

and thus

$$T = \left(\frac{T}{T_0}\right) T_0 = (0.86)(293\,K) = 252\ K$$

Then

$$V = (Ma)c = Ma\sqrt{RTk} = 0.89\sqrt{\frac{\left(286.9\,\frac{N \cdot m}{kg \cdot K}\right)(252\ K)(1.4)}{\left(1\,\frac{N}{kg \cdot \frac{m}{s^2}}\right)}}$$

or

$$V = \underline{\underline{283}}\ \frac{m}{s}$$

Inspection of Fig. 3.24 suggests that for this Mach number level, the error associated with assuming that the flow is incompressible would be unacceptably large.

11.39

11.39 The stagnation pressure and temperature of air flowing past a probe are 120 kPa (abs) and 100 °C, respectively. The air pressure is 80 kPa (abs). Determine the air speed and Mach number considering the flow to be **(a)** incompressible; **(b)** compressible.

(a) Assuming incompressible flow we use Bernoulli's equation (Eq. 3.7) to connect the static and stagnation states and get

$$V = \sqrt{\frac{2(p_0 - p)}{\rho_0}} \qquad (1)$$

With the ideal gas equation of state (Eq. 1) we obtain

$$\rho_0 = \frac{p_0}{RT_0} \qquad (2)$$

and combining Eqs. 1 and 2 we obtain

$$V = \sqrt{\frac{2(p_0 - p)RT_0}{p_0}}$$

or

$$V = \sqrt{\frac{2\,[120\text{ kPa(abs)} - 80\text{ kPa(abs)}]\left(286.9\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(373\text{ K})}{[120\text{ kPa(abs)}]\left(1\,\frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)}} = \underline{\underline{267\,\frac{\text{m}}{\text{s}}}}$$

For Mach number we need

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (3)$$

To determine T we use the equation of motion (Eq. 11.54) to obtain

$$T = T_0 - \frac{V^2(k-1)}{2kR} = 373\text{K} - \frac{\left(267\,\frac{\text{m}}{\text{s}}\right)^2(1.4-1)}{2(1.4)\left(286.9\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)}\left(1\,\frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)$$

or $T = 337.5\text{ K}$

(con't)

11.39 (con't)

With Eq. 3 we obtain

$$Ma = \frac{267 \frac{m}{s}}{\sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)\frac{(337.5 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = \underline{\underline{0.725}}$$

(b) For compressible flow

$$\frac{P}{P_0} = \frac{80 \text{ kPa}(abs)}{120 \text{ kPa}(abs)} = 0.67$$

and from Fig. D.1 we read

$$Ma = \underline{\underline{0.78}}$$

Also from Fig. D.1 we read

$$\frac{T}{T_0} = 0.89$$

and thus

$$T = (0.89)(373 K) = 332 \ K$$

Thus,

$$V = Ma\sqrt{RTk} = (0.78)\sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)\frac{(332 \ K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}$$

and

$$V = \underline{\underline{285}} \ \frac{m}{s}$$

11.40

11.40 The stagnation pressure indicated by a Pitot tube mounted on an airplane in flight is 45 kPa (abs). If the aircraft is cruising in standard atmosphere at an altitude of 10,000 m, determine the speed and Mach number involved.

For 10,000 m standard atmosphere we get from Table C.2

$$p = 26.50 \text{ kPa (abs)}$$

and

$$T = 223.1\,K$$

Thus

$$\frac{p}{p_0} = \frac{26.50 \text{ kPa (abs)}}{45 \text{ kPa (abs)}} = 0.59$$

and from Fig. D.1 we read

$$Ma = \underline{\underline{0.90}}$$

Thus

$$V = (Ma)c = Ma\sqrt{RTk} = (0.9)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(223.1\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}}$$

or

$$V = \underline{\underline{269}}\ \frac{m}{s}$$

11.42

*11.42 An ideal gas enters subsonically and flows isentropically through a choked converging–diverging duct having a circular cross-sectional area A that varies with axial distance from the throat, x, according to the formula

$$A = 0.1 + x^2$$

where A is in square feet and x is in feet. For this flow situation, sketch the side view of the duct and graph the variation of Mach number, static temperature to stagnation temperature ratio, T/T_0, and static pressure to stagnation pressure ratio, p/p_0, through the duct from $x = -0.6$ ft to $x = +0.6$ ft. Also show the possible fluid states at $x = -0.6$ ft, 0 ft, and $+0.6$ ft using temperature–entropy coordinates. Consider the gas as being helium (use $0.051 \le \text{Ma} \le 5.193$). Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and under-expanded duct exit flows (see **Video V11.6**) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

This is like Example 11.8.
Since

$$A = \pi r^2$$

and

$$A = 0.1 + x^2$$

then

$$r = \frac{0.1 + x^2}{\pi} \qquad (1)$$

With Eq. 1 we can determine r values corresponding to values of x. The are summarized in the graph and tables
duct is choked,

$$A^* = 0.1 \text{ ft}^2$$

and

$$\frac{A}{A^*} = 1 + \frac{x^2}{0.1} \qquad (2)$$

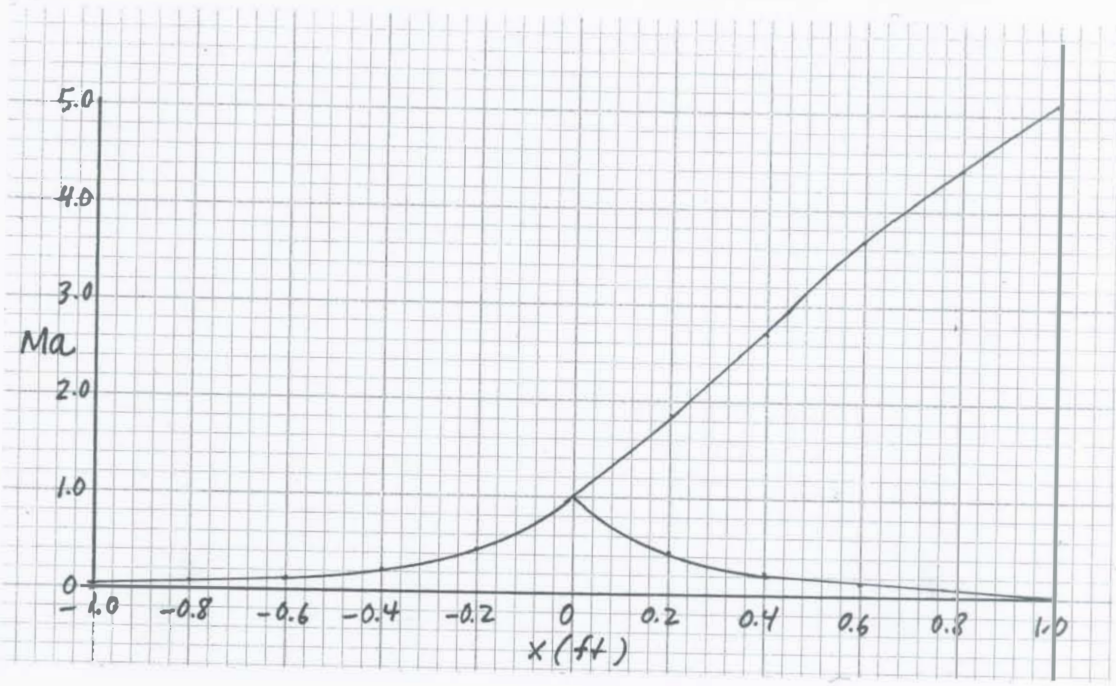
With Eq. 2 we can determine $\frac{A}{A^*}$ values corresponding to values of x. These $\frac{A}{A^*}$ values are tabulated

For helium we enter program ISENTROP with $k = 1.66$ and with Ma values within the range specified in the problem statement and obtain values of $\frac{A}{A^*}$ (Eq. 11.71), x (Eq. 2), $\frac{T}{T_0}$ (Eq. 11.56) and $\frac{p}{p_0}$ (Eq. 11.59). These values are tabulated and graphed on pages that follow.

(con't)

11.42 (con't)

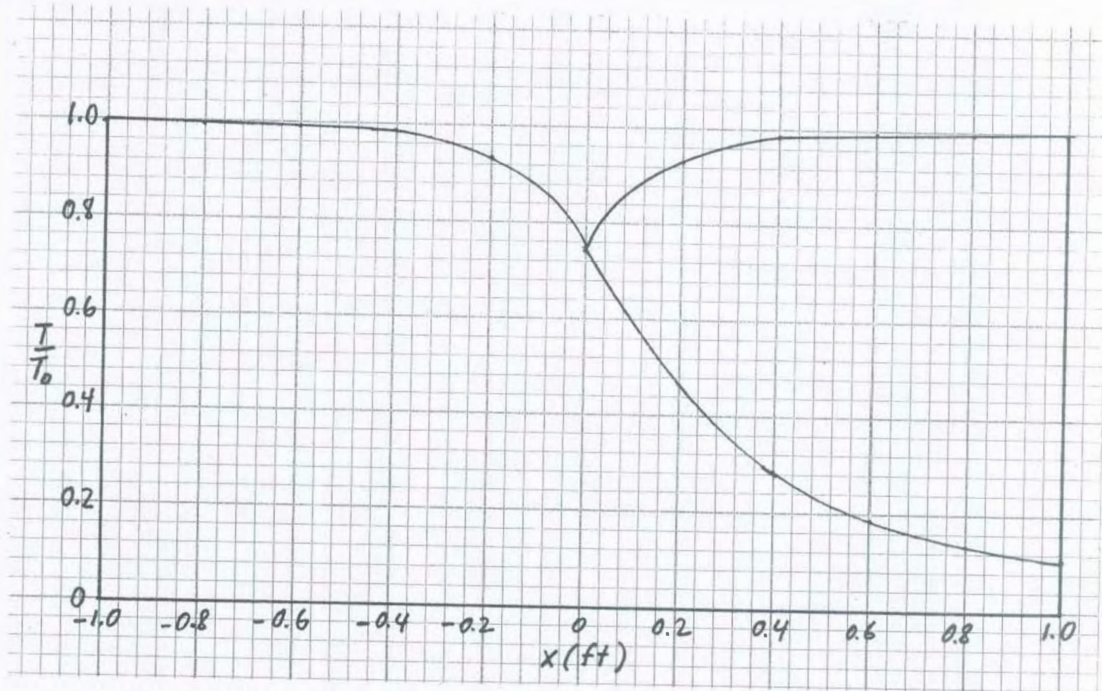
Ma	$\frac{A}{A^*}$	Eq. 2 x(ft)	$\frac{T}{T_0}$	$\frac{P}{P_0}$	state
	From program ISENTROP with $k = 1.66$				
		subsonic solution			
0.051	11.06	±1.00	0.99914	0.99784	a, c
0.076	7.43	±0.80	0.99809	0.99522	
0.123	4.62	±0.60	0.99503	0.98755	
0.223	2.61	±0.40	0.98385	0.95989	
0.460	1.40	±0.20	0.93473	0.84386	
1.00	1.00	0	0.75188	0.48808	b
		supersonic solution			
1.855	1.40	0.20	0.46827	0.14833	
2.778	2.60	0.40	0.28195	0.04141	
3.647	4.60	0.60	0.18556	0.01446	
4.448	7.40	0.80	0.13282	0.00624	
5.193	11.0	1.00	0.10102	0.00313	d



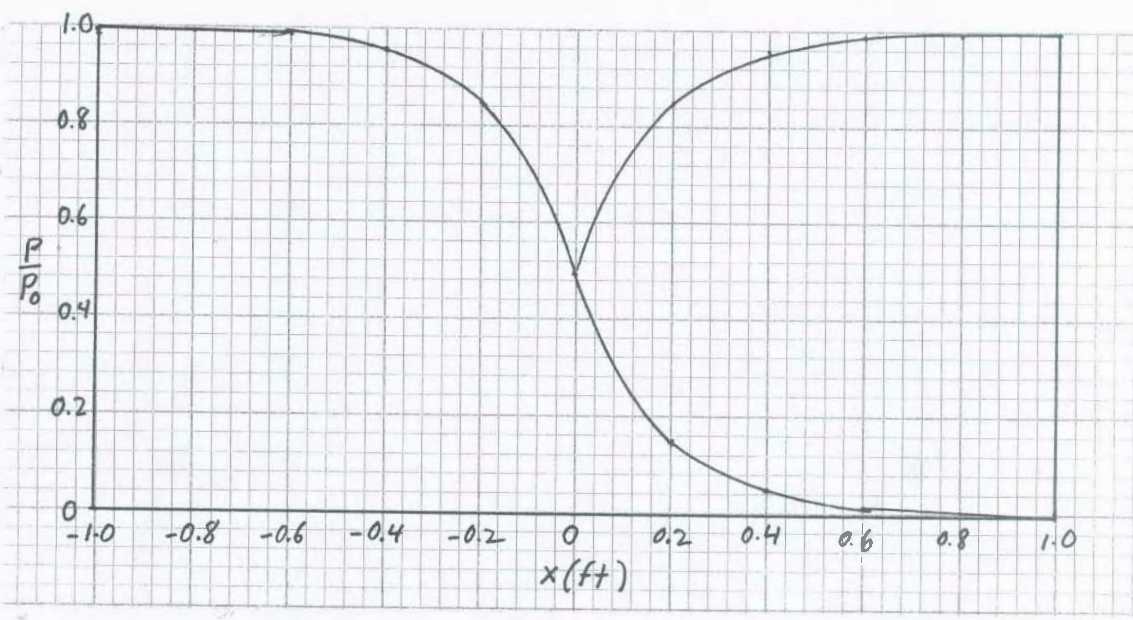
Variation of Mach number for helium

(con't)

11.42 (con't)



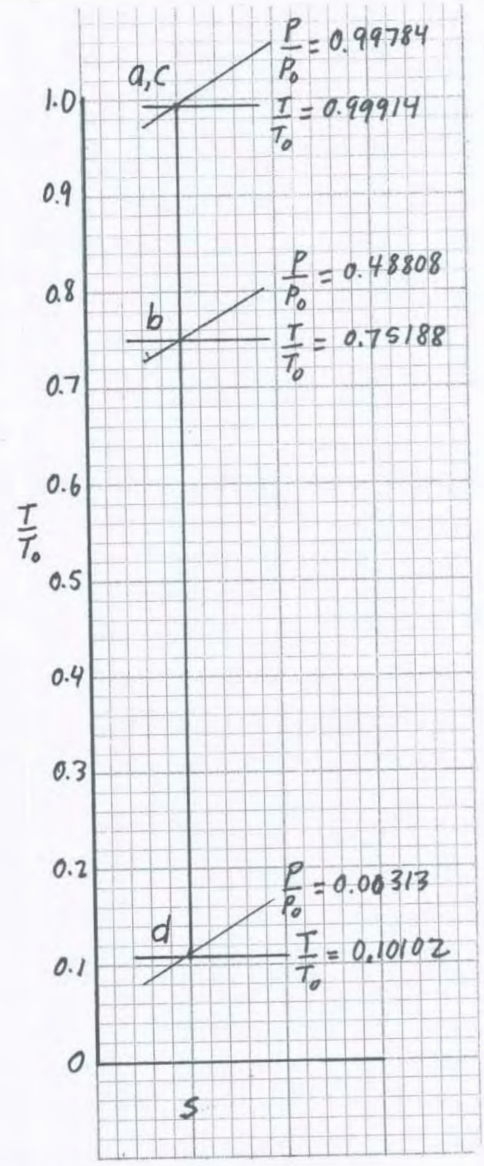
Variation of static temperature to stagnation temperature ratio for helium



Variation of static pressure to stagnation pressure ratio for helium

(con't)

11.42 (con't)

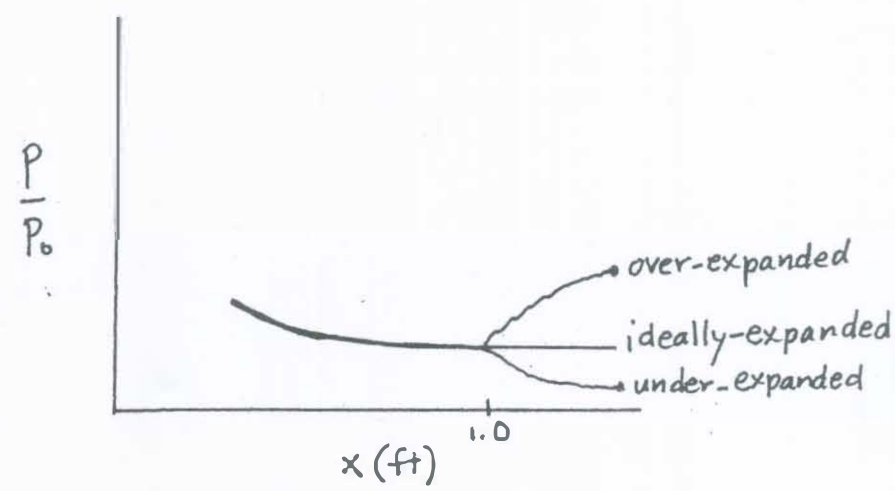


Temperature entropy diagram for helium

(con't)

11.42 (con't)

Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit.This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.43

*11.43 An ideal gas enters supersonically and flows isentropically through the choked converging–diverging duct described in Problem 11.42. Graph the variation of Ma, T/T_0, and p/p_0 from the entrance to the exit sections of the duct for helium (use $0.051 \le \text{Ma} \le 5.193$). Show the possible fluid states at $x = -0.6$ ft, 0 ft, and $+0.6$ ft using temperature–entropy coordinates. Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and underexpanded duct exit flows (see **Video V11.6**) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

This is similar to Example 11.9.
This problem involves the duct of Problem 11· . However the flow enters supersonically. We can use values from the tables of problem 11· with a little rearrangement to account for the supersonic entering flow.

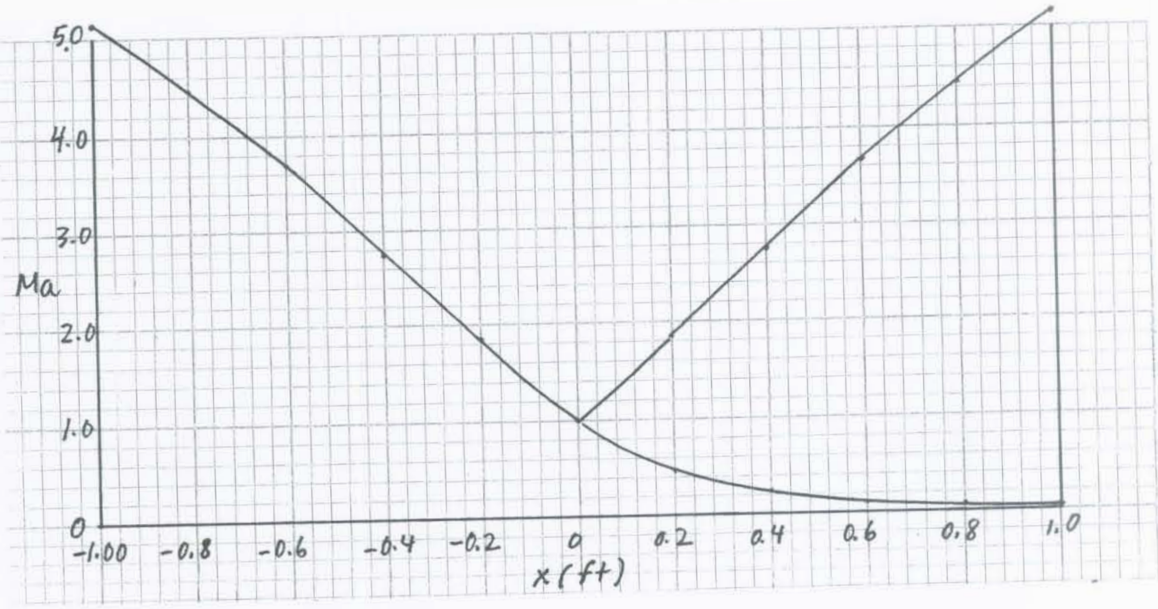
For helium we have

From program ISENTROP with $k = 1.66$

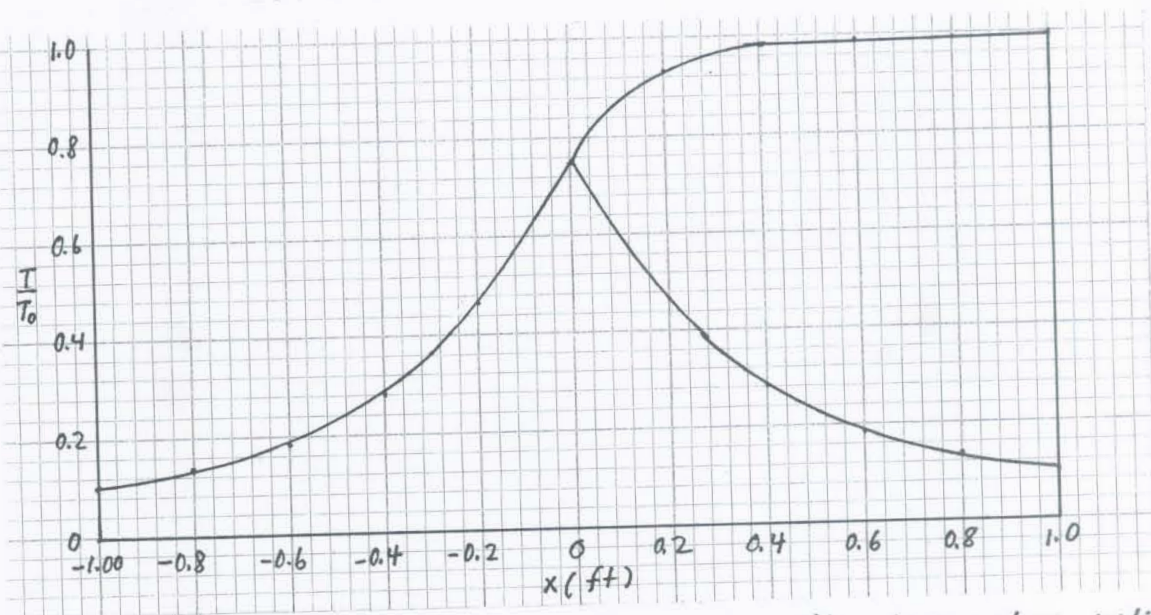
Ma	$\frac{A}{A^*}$	Eq. 2 of 11.39 x (ft)	$\frac{T}{T_0}$	$\frac{p}{p_0}$	state
		supersonic solution			
5.193	11.0	-1.00	0.10102	0.00313	a
4.448	7.4	-0.80	0.13282	0.00624	
3.647	4.6	-0.60	0.18556	0.01446	
2.778	2.6	-0.40	0.28195	0.04141	
1.855	1.4	-0.20	0.46827	0.14833	
1.0	1.0	0	0.75188	0.48808	b
1.855	1.4	0.20	0.46827	0.14833	
2.778	2.6	0.40	0.28195	0.04141	
3.647	4.6	0.60	0.18556	0.01446	
4.448	7.4	0.80	0.13282	0.00624	
5.193	11.0	1.00	0.10102	0.00313	c
		subsonic solution			
0.460	1.40	0.20	0.93473	0.84386	
0.223	2.61	0.40	0.98385	0.95989	
0.123	4.62	0.60	0.99503	0.98755	
0.076	7.43	0.80	0.99809	0.99522	
0.051	11.06	1.00	0.99914	0.99784	d

(con't)

11.43 (con't)



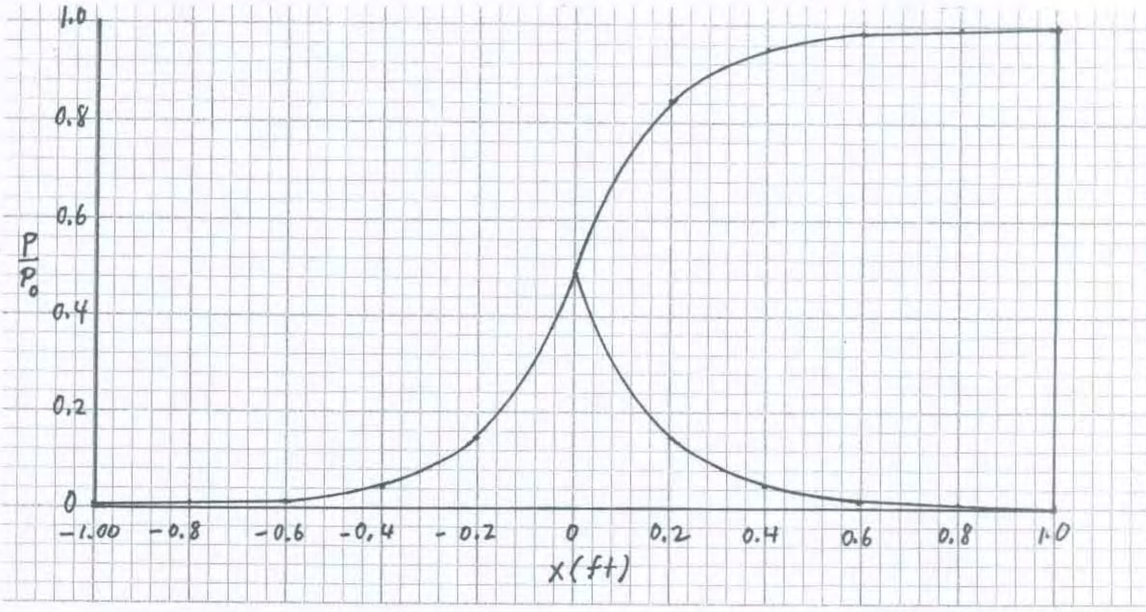
Variation of Mach number for helium



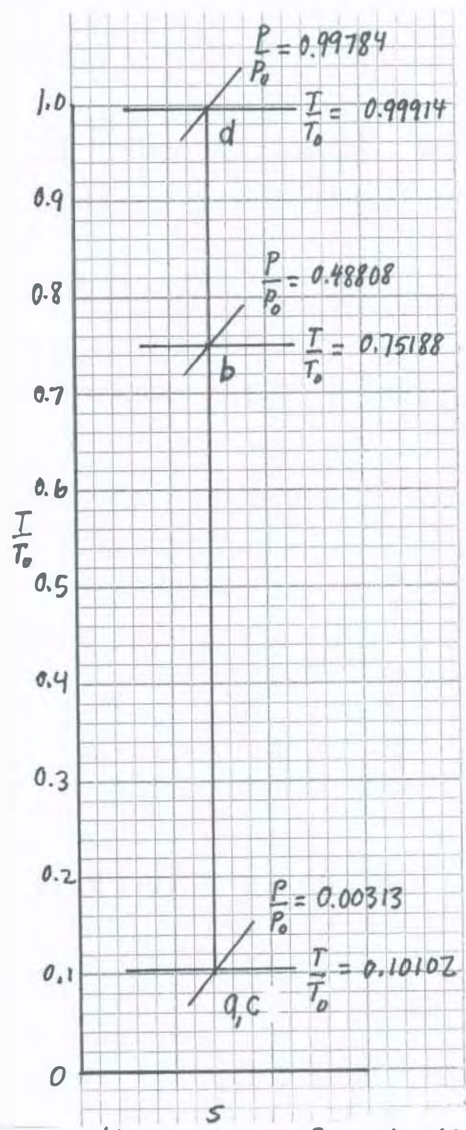
Variation of static temperature to stagnation temperature ratio for helium

(con't)

11.43 (con't)



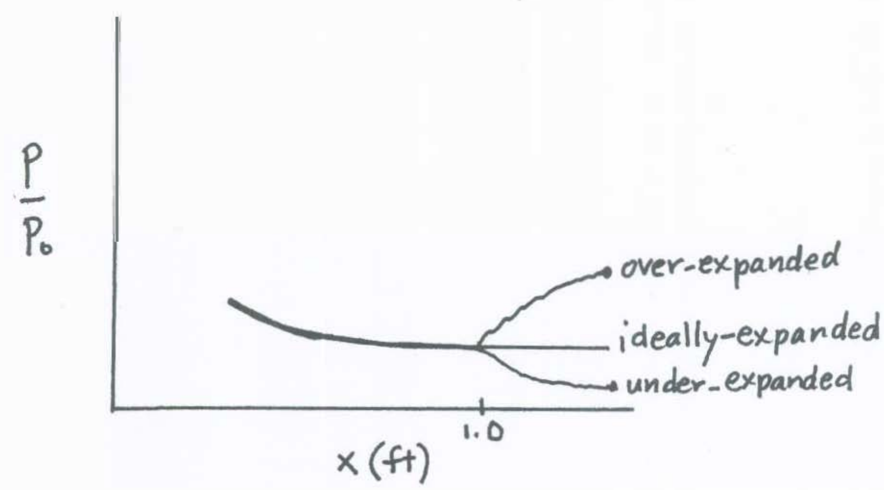
Variation of static pressure to stagnation pressure ratio for helium



(con't) T-s diagram for helium

11.43 (con't)

Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit.This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.44

11.44 An ideal gas flows subsonically and isentropically through the converging–diverging duct described in Problem 11.42. Graph the variation of Ma, T/T_0, and p/p_0 from the entrance to the exit sections of the duct for air. The value of p/p_0 is 0.6708 at $x = 0$ ft. Sketch important states on a $T-s$ diagram.

This is like Example 11.10.

Since $\frac{p}{p_0} = 0.6708$ at $x = 0$ is greater than $\frac{p^*}{p_0} = 0.5283$ for air the air flow through the converging-diverging duct is not choked. For values of $\frac{A}{A^*}$ at different values of x we obtain corresponding values of Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$.

(a) For air we enter Fig. D.1 with values of $\frac{A}{A^*}$ to get Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$. For A^* we use

$$A^* = \frac{A}{\left(\frac{A}{A^*}\right)}$$

evaluated at $x = 0$ where $A = 0.1\ ft^2$. We determine $\frac{A}{A^*}$ at $x = 0$ from Fig. D.1 for the subsonic flow value of $\frac{p}{p_0} = 0.6708$, we get

$$\frac{A}{A^*} = 1.05 \quad \text{and thus}$$

$$A^* = \frac{0.1\ ft^2}{1.05} = 0.095\ ft^2$$

We then determine the $\frac{A}{A^*}$ variation through the duct with

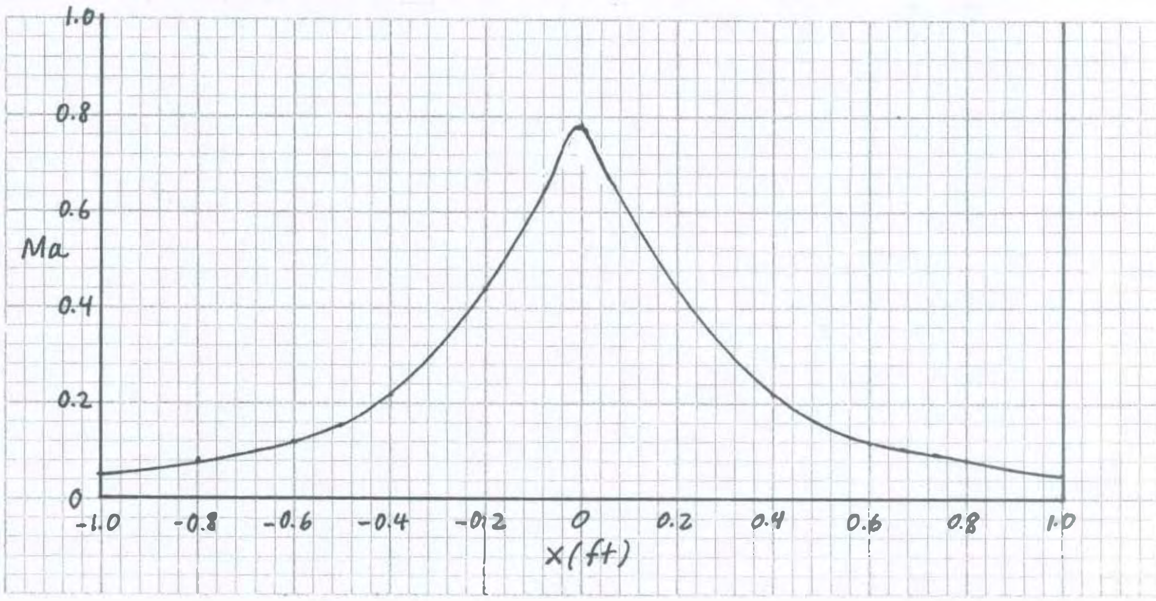
$$\frac{A}{A^*} = \frac{x^2 + 0.1}{A^*} = \frac{x^2 + 0.1}{0.095} \qquad (1)$$

The corresponding values of $\frac{A}{A^*}$, Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$ from Fig. D.1 are also tabulated on the next page.

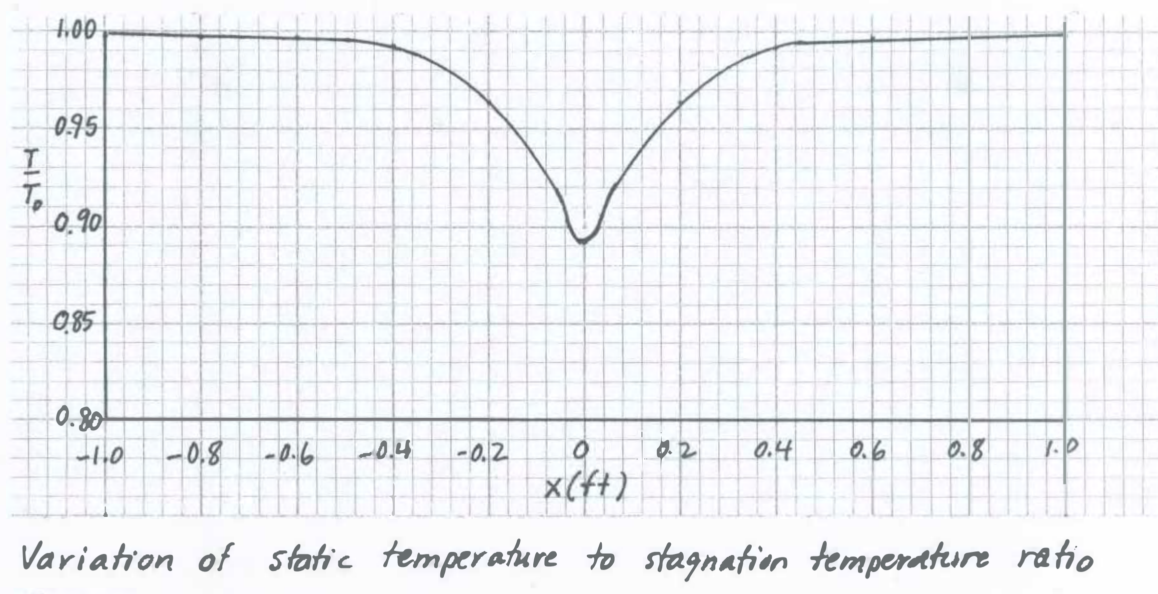
(con't)

11.44 (con't)

x (ft)	$\frac{A}{A^*}$ (With Eq. 1)	Ma (From Fig. D.1)	$\frac{T}{T_o}$ (From Fig. D.1)	$\frac{P}{P_o}$ (From Fig. D.1)	state
-1.0	11.6	0.05	0.99	0.99	a
-0.8	7.8	0.08	0.99	0.99	
-0.6	4.8	0.12	0.99	0.98	
-0.4	2.7	0.22	0.99	0.966	
-0.2	1.5	0.44	0.96	0.87	
0	1.0	0.78	0.89	0.66	b
0.2	1.5	0.44	0.96	0.87	
0.4	2.7	0.22	0.99	0.96	
0.6	4.8	0.12	0.99	0.98	
0.8	7.8	0.08	0.99	0.99	
1.0	11.6	0.05	0.99	0.99	c



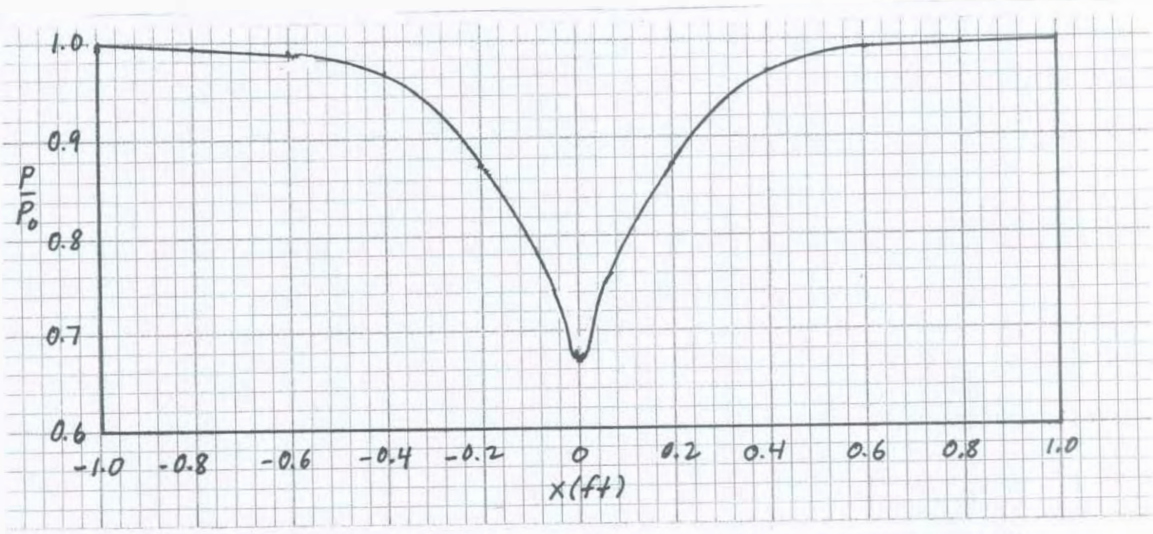
Variation of Mach number for air



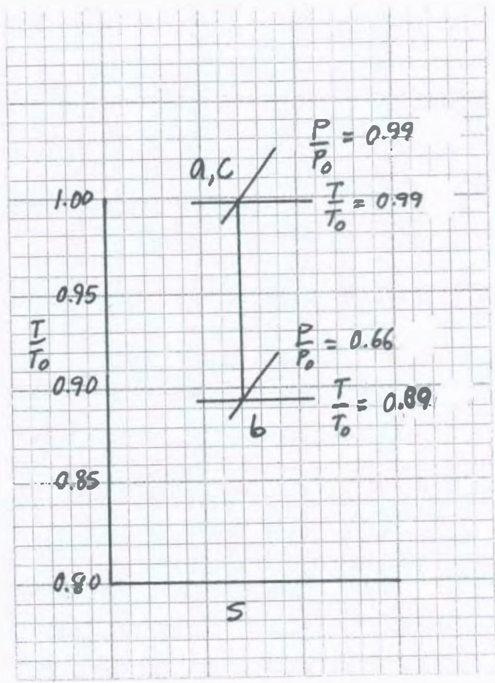
Variation of static temperature to stagnation temperature ratio for air

(con't)

11.44 (con't)



Variation of static pressure to stagnation pressure ratio for air



T-s diagram for air

11.45

11.45 An ideal gas is to flow isentropically from a large tank where the air is maintained at a temperature and pressure of 59 °F and 80 psia to standard atmospheric discharge conditions. Describe in general terms the kind of duct involved and determine the duct exit Mach number and velocity in ft/s if the gas is air.

To determine the duct exit Mach number, Ma_{exit}, we use Eq. 11.59 or for air, Fig. D.1. Thus,

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{p_{exit}}{p_0}\right)^{\frac{k-1}{k}}} - 1\right]\left(\frac{2}{k-1}\right)} \qquad (1)$$

or for air

$$Ma_{exit} = \text{Fig. D.1 value as a function of } \frac{p_{exit}}{p_0} \qquad (2)$$

To determine exit velocity, V_{exit}, we use

$$V_{exit} = (Ma_{exit})\,c_{exit} = Ma_{exit}\sqrt{R\,T_{exit}\,k} \qquad (3)$$

where

$$T_{exit} = \frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma_{exit}^2} \qquad (4)$$

or for air

$$T_{exit} = T_0\left(\frac{T_{exit}}{T_0} \text{ value from Fig. D.1 for } Ma_{exit}\right) \qquad (5)$$

$$\frac{p_{exit}}{p_0} = \frac{14.7 \text{ psia}}{80 \text{ psia}} = 0.1838$$

and thus from Fig. D.1, the corresponding values are

$$Ma_{exit} = \underline{\underline{1.8}}$$

and

$$\frac{T_{exit}}{T_0} = 0.62$$

(con't)

11.45 (con't)

Then with Eq. 5 we obtain

$$T_{exit} = (519\,^\circ R)(0.62\quad) = 322\,^\circ R$$

and with Eq. 3 we conclude that

$$V_{exit} = (1.8\quad)\sqrt{\frac{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(322\ {}^\circ R)(1.4)}{\left(1\ \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{1580}}\ \frac{ft}{s}$$

A converging-diverging nozzle is required because the exit flow is supersonic.

11.46

11.46 An ideal gas flows isentropically through a converging–diverging nozzle. At a section in the converging portion of the nozzle, $A_1 = 0.1\ \text{m}^2$, $p_1 = 600\ \text{kPa(abs)}$, $T_1 = 20\ °\text{C}$, and $\text{Ma}_1 = 0.6$. For section (2) in the diverging part of the nozzle, determine A_2, p_2, and T_2 if $\text{Ma}_2 = 3.0$ and the gas is air.

To determine A_2 we use Eq. 11.71 or for air, Fig. D.1 Thus,

$$A_2 = A_1 \frac{\left(\frac{A_2}{A^*}\right)}{\left(\frac{A_1}{A^*}\right)} = A_1 \left\{ \frac{\frac{1}{Ma_2}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_2^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}}{\frac{1}{Ma_1}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_1^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}} \right\} \quad (1)$$

or for air

$$A_2 = A_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{A_2}{A^*} \text{ for } Ma_2\right)}{\left(\text{Fig. D.1 value of } \frac{A_1}{A^*} \text{ for } Ma_1\right)}\right] \quad (2)$$

To determine P_2 we use Eq. 11.59 or for air, Fig. D.1. Thus,

$$P_2 = P_1 \frac{\left(\frac{P_2}{P_0}\right)}{\left(\frac{P_1}{P_0}\right)} = P_1 \left\{ \frac{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_2^2}\right]^{\frac{k}{k-1}}}{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{k}{k-1}}} \right\} \quad (3)$$

or for air,

$$P_2 = P_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{P_2}{P_0} \text{ for } Ma_2\right)}{\left(\text{Fig D.1 value of } \frac{P_1}{P_0} \text{ for } Ma_1\right)}\right] \quad (4)$$

To determine T_2 we use Eq. 11.56 or for air, Fig. D.1. Thus,

$$T_2 = T_1 \frac{\left(\frac{T_2}{T_0}\right)}{\left(\frac{T_1}{T_0}\right)} = T_1 \left\{ \frac{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_2^2}\right]}{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]} \right\} \quad (5)$$

or for air

$$T_2 = T_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{T_2}{T_0} \text{ for } Ma_2\right)}{\left(\text{Fig D.1 value of } \frac{T_1}{T_0} \text{ for } Ma_1\right)}\right] \quad (6)$$

Eq. 2 leads to

$$A_2 = (0.1\ m^2)\frac{(4.3)}{(1.2)} = \underline{\underline{0.36\ m^2}}$$

Eq. 4 leads to

$$P_2 = [600\ kPa(abs)]\frac{(0.03)}{(0.78)} = \underline{\underline{23\ kPa(abs)}}$$

and Eq. 6 gives

$$T_2 = (293\ K)\frac{(0.36)}{(0.93)} = \underline{\underline{113\ K}}$$

11.47

11.47 Upstream of the throat of an isentropic converging–diverging nozzle at section (1), $V_1 = 150$ m/s, $p_1 = 100$ kPa(abs), and $T_1 = 20$ °C. If the discharge flow is supersonic and the throat area is 0.1 m^2, determine the mass flowrate in kg/s for the flow of air.

We determine the Mach number at section (1) with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{R T_1 k}} \qquad (1)$$

For the gas involved it is likely that Ma_1 is less than 1.0 because V_1 is low. Thus, the flow at the throat is choked since the entering flow is subsonic and the leaving flow is supersonic. For mass flowrate we use Eq. 11.40 to obtain

$$\dot{m} = \rho^* A^* V^* \qquad (2)$$

For throat velocity, V^*, we use

$$V^* = \sqrt{R T^* k} \qquad (3)$$

To obtain T^* we use Eq. 11.63. Thus,

$$T^* = T_0 \left(\frac{2}{k+1}\right) \qquad (4)$$

or for air,

$$T^* = T_0 \left(\text{value of } \frac{T}{T_0} \text{ from Fig. D.1 for } Ma = 1.0\right) \qquad (5)$$

To determine T_0 we use Eq. 11.56. Thus,

$$T_0 = T_1 \left[1 + \left(\frac{k-1}{2}\right) Ma_1^2\right] \qquad (6)$$

or for air,

$$T_0 = \frac{T_1}{\left(\text{value of } \frac{T_1}{T_0} \text{ from Fig. D.1 for } Ma_1\right)} \qquad (7)$$

(con't)

11.47	(con't)

To determine ρ^* we use the ideal gas equation of state (Eq. 11.1). Thus,

$$\rho^* = \frac{p^*}{RT^*} \qquad (8)$$

For p^* we use Eq. 11.61. Thus

$$p^* = p_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad (9)$$

or for air,

$$p^* = p_o \left(\text{value of } \frac{p^*}{p_o} \text{ from Fig. D.1 for } Ma = 1.0\right) \qquad (10)$$

For p_o we use Eq. 11.59. Thus,

$$p_o = p_1 \left[1 + \left(\frac{k-1}{2}\right) Ma_1^2\right]^{\frac{k}{k-1}} \qquad (11)$$

or for air

$$p_o = \frac{p_1}{\left(\text{value of } \frac{p_1}{p_o} \text{ from Fig. D.1 for } Ma_1\right)} \qquad (12)$$

(a) For air we use Eq. 1 to obtain

$$Ma_1 = \frac{\left(150 \frac{m}{s}\right)}{\sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)\frac{(293 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = 0.4372$$

Thus, the flow is choked at the throat. From Eq. 7 we obtain for corresponding value in Fig. D.1 for $Ma_1 = 0.44$

$$T_o = \frac{293 K}{(0.96)} = 305 \text{ K}$$

With Eq. 5 we obtain

$$T^* = (305 \text{ K})(0.83333) = 254 \text{ K}$$

Thus

$$V^* = \sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)\frac{(254 \text{ K})(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 319 \frac{m}{s}$$

(con't)

11.47 (con't)

From Eq. 12 we obtain with the help of Fig. D.1

$$P_0 = \frac{100 \text{ kPa(abs)}}{0.87} = 115 \text{ kPa(abs)}$$

and with Eq. 10

$$p^* = [115 \text{ kPa(abs)}](0.52828) = 60.8 \text{ kPa(abs)}$$

Then with Eq. 8

$$\rho^* = \frac{\left(60.8 \times 10^3 \frac{N}{m^2}\right)}{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(254 \text{ K})} = 0.83 \frac{kg}{m^3}$$

Finally, with Eq. 2 we obtain

$$\dot{m} = \left(0.83 \frac{kg}{m^3}\right)(0.1 \, m^2)\left(319 \frac{m}{s}\right) = \underline{\underline{26.5}} \frac{kg}{s}$$

11.48

11.48 The flow blockage associated with the use of an intrusive probe can be important. Determine the percentage increase in section velocity corresponding to a 0.5% reduction in flow area due to probe blockage for air flow if the section area is 1.0 m², T_0 = 20 °C, and the unblocked flow Mach numbers are **(a)** Ma = 0.2; **(b)** Ma = 0.8; **(c)** Ma = 1.5; **(d)** Ma = 3.0.

We want to ascertain

$$\frac{V_{blocked} - V_{unblocked}}{V_{unblocked}} \times 100$$

To determine the unblocked area velocity, $V_{unblocked}$, we use

$$V_{unblocked} = Ma_{unblocked} \sqrt{R T_{unblocked} k} \qquad (1)$$

For $T_{unblocked}$ we use

$$T_{unblocked} = T_0 \left(\frac{T}{T_0} \text{ for } Ma_{unblocked} \text{ from Eq. 11.56 for } Ma_{unblocked} \right) \qquad (2)$$

To determine the blocked area velocity, $V_{blocked}$, we use

$$V_{blocked} = Ma_{blocked} \sqrt{R T_{blocked} k} \qquad (3)$$

For $Ma_{blocked}$ we use $\frac{A_{blocked}}{A^*}$ and determine $Ma_{blocked}$ from Eq. 11.71.

Solution of Eq. 11.71 for $Ma_{blocked}$ from $\frac{A_{blocked}}{A^*}$ requires trial and error.

To determine $\frac{A_{blocked}}{A^*}$ we set

$$\frac{A_{blocked}}{A^*} = 0.995 \frac{A_{unblocked}}{A^*} \qquad (4)$$

We obtain $\frac{A_{unblocked}}{A^*}$ from Eq. 11.71 with the given value of $Ma_{unblocked}$.

To determine $T_{blocked}$ we use Eq. 11.56 to obtain

$$T_{blocked} = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_{blocked}^2} \qquad (5)$$

(con't)

11.48 (con't)

(a) For $Ma_{unblocked} = 0.2$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.99206) = 290.7\,K$$

Then with Eq. 1 we have

$$V_{unblocked} = (0.2)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(290.7\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 68.34\,\frac{ft}{s}$$

we use Eqs. 4 and 11.71 to get

$$\frac{A_{blocked}}{A^*} = (0.995)(2.9635) = 2.949$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 0.201$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\,K}{1+\left(\frac{1.4-1}{2}\right)(0.201)^2} = 290.6\,K$$

With Eq. 3 we have

$$V_{blocked} = (0.201)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(290.6\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 68.67\,\frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})\times 100}{V_{unblocked}} = \frac{\left(68.67\,\frac{m}{s} - 68.34\,\frac{m}{s}\right)(100)}{68.34\,\frac{m}{s}} = \underline{\underline{0.483\,\%}}$$

(b) For $Ma = 0.8$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.88652) = 259.8\,K$$

Then with Eq. 1 we get

$$V_{unblocked} = 0.8\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(259.8\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 258.4\,\frac{m}{s}$$

we use Eqs. 4 and 11.71

$$\frac{A_{blocked}}{A^*} = (0.995)(1.03823) = 1.033$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 0.813$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\,K}{1+\left(\frac{1.4-1}{2}\right)(0.813)^2} = 258.8\,K$$

(con't)

11.48 (con't)

With Eq. 3 we have

$$V_{blocked} = (0.813)\sqrt{\frac{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)(258.8\ K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 262.1\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(262.1\ \frac{m}{s} - 258.4\ \frac{m}{s}\right)(100)}{\left(258.4\ \frac{m}{s}\right)} = \underline{\underline{1.43\ \%}}$$

(c) For $Ma = 1.5$, we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\ K)(0.68965) = 202.1\ K$$

Then with Eq. 1 we get

$$V_{unblocked} = (1.5)\sqrt{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)\frac{(202.1\ )(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 427.4\ \frac{m}{s}$$

We use Eqs. 4 and 11.71 to get

$$\frac{A_{blocked}}{A^*} = (0.995)(1.1762) = 1.17$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 1.491$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\ K}{1 + \left(\frac{1.4 - 1}{2}\right)(1.491)^2} = 202.8\ K$$

With Eq. 3 we have

$$V_{blocked} = (1.491)\sqrt{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)\frac{(202.8\ K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 425.5\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(425.5\ \frac{m}{s} - 427.4\ \frac{m}{s}\right)(100)}{427.4\ \frac{m}{s}} = \underline{\underline{-0.445\%}}$$

(d) For $Ma = 3.0$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\ K)(0.35714) = 104.6\ K$$

Then with Eq. 1 we get

$$V_{unblocked} = (3.0)\sqrt{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)\frac{(104.6\ K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 614.9\ \frac{m}{s}$$

(con't)

11.48 (con't)

We use Eq. 4 and 11.71

$$\frac{A_{blocked}}{A^*} = (0.995)(4.2346) = 4.213$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 2.995$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\ K}{1+\left(\frac{1.4-1}{2}\right)(2.995)^2} = 104.9\,K$$

With Eq. 3 we have

$$V_{blocked} = (2.995)\sqrt{\frac{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(104.9\,K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 614.8\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(614.8\ \frac{m}{s} - 614.9\ \frac{m}{s}\right)(100)}{\left(614.9\ \frac{m}{s}\right)} = \underline{\underline{-0.0163\%}}$$

11.49

11.49 (See Fluids in the News article titled "Rocket nozzles," Section 11.4.2.) Comment on the practical limits of area ratio for the diverging portion of a converging–diverging nozzle designed to achieve supersonic exit flow.

From Fig. D.1 we see that the A/A^* vs. Ma curve becomes very steep with increasing values of Ma (very large increase in A/A^* needed to achieve even small gains in Ma level) suggesting practical limits to area divergence ratio is actual devices. For example, using Eq. 11.71, the A/A^* divergence ratio needed for $Ma = 5$ is 3450!

11.51

11.51 An ideal gas enters [section (1)] an insulated, constant cross-sectional area duct with the following properties:

$$T_0 = 293 \text{ K}$$
$$p_0 = 101 \text{ kPa(abs)}$$
$$\text{Ma}_1 = 0.2$$

For Fanno flow, determine corresponding values of fluid temperature and entropy change for various levels of pressure and plot the Fanno line if the gas is helium.

This is similar to Example 11.11. For Fanno flow of an ideal gas we use Eqs. 11.75 and 11.76 to establish the Fanno line states. Thus,

$$T + \frac{(\rho V)^2 T^2}{2c_p\left(\frac{p^2}{R^2}\right)} = T_0 \qquad (1)$$

and

$$s - s_1 = c_p \ln\left(\frac{T}{T_1}\right) - R\ln\left(\frac{p}{p_1}\right) \qquad (2)$$

For helium, $k = 1.66$ and $R = 2077 \frac{N \cdot m}{kg \cdot K}$ (Table 1.8) and $c_p = 5224 \frac{N \cdot m}{kg \cdot K}$ from Problem 11.1 (c). We determine the constant value of ρV by calculating ρ_1 with the ideal gas equation of state (Eq. 3) and V_1 with Eq. 4. For T_1 we use Eq. 11.56 to obtain

$$T_1 = \frac{T_0}{1 + \left(\frac{k-1}{2}\right)Ma_1^2} = \frac{(293 K)}{1 + \left(\frac{1.66-1}{2}\right)(0.2)^2} = 289.2 K$$

Then, with Eq. 4 we obtain

$$V_1 = 0.2 \sqrt{\frac{\left(2077 \frac{N \cdot m}{kg \cdot K}\right)(289.2 K)(1.66)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 199.7 \frac{m}{s}$$

For p_1 we use Eq. 11.59 to get

$$p_1 = p_0\left[\frac{1}{1 + \left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{k}{k-1}} = [101 \text{ kPa(abs)}]\left[\frac{1}{1 + \left(\frac{1.66-1}{2}\right)(0.2)^2}\right]^{\frac{1.66}{1.66-1}} = 97.72 \text{ kPa(abs)}$$

and with Eq. 3 we obtain

$$\rho_1 = \frac{\left(97.72 \times 10^3 \frac{N}{m^2}\right)}{\left(2077 \frac{N \cdot m}{kg \cdot K}\right)(289.2 K)} = 0.1627 \frac{kg}{m^3}$$

Thus, the value of ρV is

$$\rho_1 V_1 = \left(0.1627 \frac{kg}{m^3}\right)\left(199.7 \frac{m}{s}\right) = 32.49 \frac{kg}{m^2 \cdot s}$$

(con't)

11.51 (con't)

Eq. 1 becomes for helium

$$T + \frac{\left(32.49 \frac{kg}{m^2 \cdot s}\right)^2 T^2 \left(\frac{1\,N}{kg \cdot \frac{m}{s^2}}\right)}{2\left(5224 \frac{N \cdot m}{kg \cdot K}\right) \frac{p^2}{\left(2077 \frac{N \cdot m}{kg \cdot K}\right)^2}} = 293\,K$$

or

$$T + 4.358 \times 10^5 \frac{T^2}{p^2} = 293 \qquad (6)$$

where T is in K and p is in $\frac{N}{m^2}$.

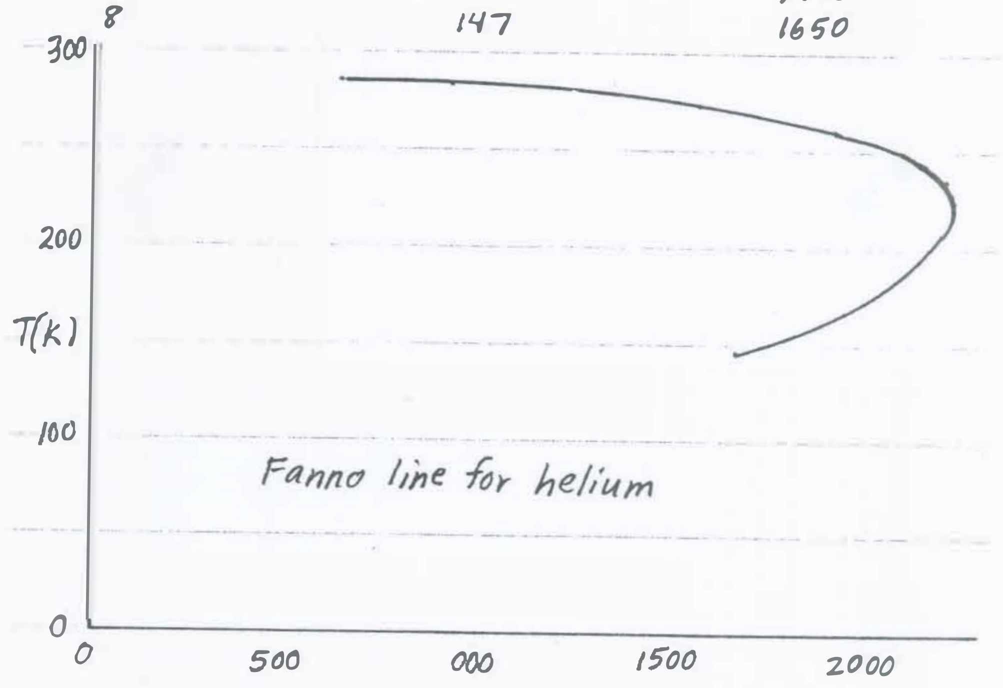
Eq. 2 becomes for helium

$$s - s_1 = \left(5224 \frac{N \cdot m}{kg \cdot K}\right) \ln\left(\frac{T}{289.2\,K}\right) - \left(2077 \frac{N \cdot m}{kg \cdot K}\right) \ln\left[\frac{p}{97.72\,kPa(abs)}\right] \qquad (7)$$

where T is in K and p is in kPa(abs).

With Eqs. 6 and 7 we construct the table of values shown below.

p [kPa(abs)]	T (K)	$s - s_1$ $\left(\frac{N \cdot m}{kg \cdot K}\right)$
70	286	630
60	283	905
50	279	1210
40	273	1550
30	260	1900
25	250	2060
20	234	2179
18	225	2200
15	209	2188
10	169	1923
8	147	1650



$s - s_1 \left(\frac{N \cdot m}{kg \cdot K}\right)$

11.52

11.52 For Fanno flow, prove that

$$\frac{dV}{V} = \frac{fk(\mathrm{Ma}^2/2)(dx/D)}{1 - \mathrm{Ma}^2}$$

and in so doing show that when the flow is subsonic, friction accelerates the fluid, and when the flow is supersonic, friction decelerates the fluid.

Starting with Eq. 11.95 we have

$$\frac{1}{2}\left(1 + k\,Ma^2\right)\frac{d(V^2)}{V^2} - \frac{d(Ma^2)}{Ma^2} + \frac{fk}{2}\,Ma^2\,\frac{dx}{D} = 0 \qquad (1)$$

From Eq. 11.93 we have

$$\frac{d(Ma^2)}{Ma^2} = \frac{d(V^2)}{V^2}\left[1 + \left(\frac{k-1}{2}\right)Ma^2\right] \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{1}{2}\left(1 + k\,Ma^2\right)\frac{d(V^2)}{V^2} - \left[1 + \left(\frac{k-1}{2}\right)Ma^2\right]\frac{d(V^2)}{V^2} + \frac{fk}{2}\,Ma^2\frac{dx}{D} = 0 \qquad (3)$$

or

$$\frac{1}{2}\left(Ma^2 - 1\right)\frac{d(V^2)}{V^2} = -\frac{fk}{2}\,Ma^2\,\frac{dx}{D}$$

and

$$\frac{d(V^2)}{V^2} = \frac{Ma^2}{(Ma^2-1)}\,fk\,\frac{dx}{D} \qquad (4)$$

However

$$d(V^2) = 2V\,dV \qquad (5)$$

Thus combining Eqs. 4 and 5 we get

$$\frac{dV}{V} = \frac{fk\left(\frac{Ma^2}{2}\right)\left(\frac{dx}{D}\right)}{1 - Ma^2} \qquad (6)$$

When the flow is subsonic ($Ma < 1.0$), Eq. 6 leads to $\frac{dV}{V} = +$ and thus friction accelerates the fluid. On the other hand when the flow is supersonic ($Ma > 1.0$), Eq. 6 leads to $\frac{dV}{V} = -$ and in this case friction decelerates the fluid.

11-53

11.53 Standard atmospheric air (T_0 = 59 °F, p_0 = 14.7 psia) is drawn steadily through a frictionless and adiabatic converging nozzle into an adiabatic, constant cross section area duct. The duct is 10 ft long and has an inside diameter of 0.5 ft. The average friction factor for the duct may be estimated as being equal to 0.03. What is the maximum mass flowrate in slugs/s through the duct? For this maximum flowrate determine the values of static temperature, static pressure, stagnation temperature, stagnation pressure, and velocity at the inlet [section (1)] and exit [section (2)] of the constant area duct. Sketch a temperature–entropy diagram for this flow.

This is similar to Example 11.12. As explained in Example 11.12, the maximum flowrate through the duct will occur when the constant area duct chokes and the Mach number at the duct exit [section (2)] is 1.0. The maximum flowrate can be obtained with

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad (1)$$

We note that T_0 is constant throughout the entire flow since the flow is adiabatic. Thus, $T_{0,1} = T_{0,2} = 519\,°R$. Also, p_0 is constant in the converging nozzle but decreases through the constant area duct because of friction. Thus, $p_{0,1} = 14.7$ psia.

For choked flow

$$\frac{f(\ell_2 - \ell_1)}{D} = \frac{(0.03)(10\,ft)}{0.5\,ft} = 0.6 = \frac{f(\ell^* - \ell_1)}{D}$$

and from Fig. D.2 we can read values of Ma_1, $\frac{T_1}{T^*}$, $\frac{V_1}{V^*}$, $\frac{p_1}{p^*}$ and $\frac{p_{0,1}}{p^*}$. Then $T^* = T_2$ can be obtained with Eq. 11.63 since T_0 is constant. Thus,

$$T^* = \left(\frac{2}{k+1}\right)T_0 = \left(\frac{2}{1.4+1}\right)(519\,°R) = \underline{\underline{432\,°R}} = T_2$$

and $V^* = V_2$ can be determined with

$$V^* = \sqrt{RT^*k} = \sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(432\,°R)(1.4)}{\left(1\,\frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{1020\,\frac{ft}{s}}} = V_2$$

(con't)

11.53 (con't)

For $\frac{f(\ell^* - \ell)}{D} = 0.6$, from Fig. D-2 we read

$Ma_1 = 0.57$

$$\frac{T_1}{T^*} = 1.13 \qquad (2)$$

$$\frac{V_1}{V^*} = 0.6 \qquad (3)$$

$$\frac{p_1}{p^*} = 1.86 \qquad (4)$$

$$\frac{p_{0,1}}{p_0^*} = 1.22 \qquad (5)$$

From Eq. 2 we get

$$T_1 = (1.13)(432\,^\circ R) = \underline{\underline{488\,^\circ R}}$$

With Eq. 3 we obtain

$$V_1 = (0.6)\left(1020\ \frac{ft}{s}\right) = \underline{\underline{612}}\ \frac{ft}{s}$$

With Eq. 5 we have

$$p_0^* = p_{0,2} = \frac{p_{0,1}}{1.22} = \frac{14.7\ psia}{1.22} = \underline{\underline{12}}\ psia$$

To determine p_1 we enter Fig. D.1 with $Ma_1 = 0.57$ and read

$$\frac{p_1}{p_{0,1}} = 0.8$$

and

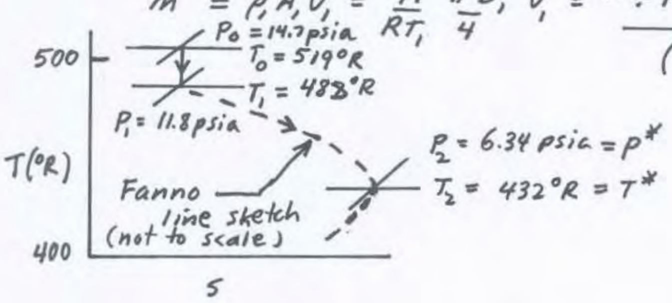
$$p_1 = (0.8)(14.7\ psia) = 11.8\ psia$$

With Eq. 4 we obtain

$$p^* = p_2 = \frac{p_1}{1.86} = \frac{11.8\ psia}{1.86} = 6.34\ psia$$

With Eq. 1 we have

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(11.8\ psia)\left(144\ \frac{in.^2}{ft^2}\right)\pi(0.5\ ft)^2\left(612\ \frac{ft}{s}\right)}{\left(1716\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(488\,^\circ R)(4)} = \underline{\underline{0.244\ \frac{slug}{s}}}$$



11.54

11.54 The upstream pressure of a Fanno flow venting to the atmosphere is increased until the flow chokes. What will happen to the flowrate when the upstream pressure is further increased?

For a Fanno flow

$$\rho V = \frac{p}{RT} Ma \sqrt{RTk} = \text{constant}$$

Also at any one axial location in the flow, from Eq. 11.56

$$\frac{T}{T_0} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma^2}$$

Combining we get

$$\rho V = \frac{p}{R\left[\frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]} Ma \sqrt{R\left[\frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]k} = \text{constant}$$

So for any one axial location of the flow where the Ma level is the same, T_0 is also the same but p is higher. Thus ρV is also higher and we conclude that increasing the inlet pressure of a choked Fanno flow into the atmosphere results in an <u>increase of flowrate</u> also.

Following the procedure of Example 11.11 one could plot a series of Fanno lines for different values of increased inlet pressure.

11.55

11.55 The duct in Problem 11.53 is shortened by 50%. The duct discharge pressure is maintained at the choked flow value determined in Problem 11.53. Determine the change in mass flowrate through the duct associated with the 50% reduction in length. The average friction factor remains constant at a value of 0.03.

This is like Example 11.13. We guess that the shortened duct will still choke and check our assumption by comparing p_d with p^*. If $p_d < p^*$, the flow is choked. If not, another assumption must be made. For choked flow we calculate the mass flowrate as we did in Example 11.12 or in the solution of problem 11.51. For unchoked flow, we must devise another strategy.

For choked flow

$$\frac{f\,(\ell_2-\ell_1)}{D} = \frac{(0.03)(5\,ft)}{(0.5\,ft)} = 0.3 = \frac{f\,(\ell^*-\ell)}{D}$$

From Fig. D.2 we read

$$Ma_1 = 0.66$$

$$\frac{T_1}{T^*} = 1.1 \qquad (1)$$

$$\frac{V_1}{V^*} = 0.7 \qquad (2)$$

$$\frac{p_1}{p^*} = 1.6 \qquad (3)$$

With $Ma_1 = 0.66$, we enter Fig. D.1 and read

$$\frac{p_1}{p_{0,1}} = 0.75$$

Thus

$$p_1 = (0.75)(14.7\,psia) = 11 \quad psia$$

(con't)

11.55 (con't)

and with Eq. 3 we obtain

$$p^* = p_2 = \frac{p_1}{1.6} = \frac{11 \text{ psia}}{1.6} = 6.88 \text{ psia}$$

Since

$$p_2 = 6.88 \text{ psia} > p_d = 6.34 \text{ psia}$$

the flow is choked as assumed.

$T^* = T_2$ can be obtained with Eq. 11.63 since T_0 is constant. Thus,

$$T^* = \left(\frac{2}{k+1}\right) T_0 = \left(\frac{2}{1.4+1}\right)(519\,°R) = 432\,°R = T_2$$

and $V_2 = V^*$ can be determined with

$$V_2 = V^* = \sqrt{RT^* k} = \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(432\,°R)(1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1020 \frac{ft}{s}$$

With Eq. 1 we have

$$\dot{m} = \rho_2 A_2 V_2 = \frac{p_2}{RT_2} \frac{\pi D_2^2}{4} V_2 = \frac{(6.88 \text{ psia})\left(144 \frac{in.^2}{ft^2}\right)}{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(432\,°R)} \frac{\pi (0.5\,ft)^2}{(4)} \left(1020 \frac{ft}{s}\right)$$

or

$$\dot{m} = 0.268 \frac{slug}{s}$$

The change in mass flowrate is

$$\left(\frac{\dot{m}_{5\,ft} - \dot{m}_{10\,ft}}{\dot{m}_{10\,ft}}\right) \times 100 = \left(\frac{0.268 \frac{slug}{s} - 0.244 \frac{slug}{s}}{0.244 \frac{slug}{s}}\right)(100) = \underline{\underline{9.8\,\%}}$$

The mass flowrate increased by 9.8% when the tube was shortened by 50%.

11.56

11.56 If the same mass flowrate of air obtained in Problem 11.53 is desired through the shortened duct of Problem 11.55, determine the back pressure, p_2, required. Assume f remains constant at a value of 0.03.

This is similar to Example 11.14. Since the same mass flowrate achieved in Problem 11.51 is desired with the shortened duct of Problem 11.53, we need to achieve the value of Ma_1 obtained in Problem 11.51. Thus, for the same value of Ma_1 as in Problem 11.51 we have

$$\frac{f(\ell^* - \ell_1)}{D} = 0.6$$

However,

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_1)}{D} - \frac{f(\ell_2 - \ell_1)}{D}$$

or

$$\frac{f(\ell^* - \ell_2)}{D} = 0.6 - \frac{(0.03)(5\ ft)}{0.5\ ft} = 0.3$$

With $\frac{f(\ell^* - \ell_2)}{D} = 0.3$ we enter Fig. D.2 and read

$$\frac{p_2}{p^*} = 1.6 \qquad (1)$$

The value of p^* obtained in Problem 11.53 is still valid, so

$$p^* = 6.88\ psia$$

and with Eq. 1 we get

$$p_2 = (1.6\ \ )(6.88\ psia) = \underline{\underline{11\ psia}}$$

11.57

11.57 If the average friction factor of the duct of Example 11.12 is changed to **(a)** 0.01 or **(b)** 0.03, determine the maximum mass flowrate of air through the duct associated with each new friction factor and compare with the maximum mass flowrate value of Example 11.12.

(a) For $f = 0.01$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{(0.01)(2\,m)}{(0.1\,m)} = 0.2$$

and on Fig. D.2 we read

$$Ma_1 = 0.7 \qquad (1)$$

$$\frac{T_1}{T^*} = 1.1 \qquad (2)$$

$$\frac{V_1}{V^*} = 0.73$$

From Example 11.12

$$T^* = 240\,K$$

and

$$V^* = 310\,\frac{m}{s}$$

Thus, with Eq. 1 we get

$$T_1 = (1.1)(240K) = 264\,K$$

and with Eq. 2 we obtain

$$V_1 = (0.73)(310\,\frac{m}{s}) = 226\,\frac{m}{s}$$

To determine p_1 we enter Fig. D.1 with $Ma_1 = 0.7$ and read

$$\frac{p_1}{p_{0,1}} = 0.72$$

Thus,

$$p_1 = (0.72)[101\,kPa\,(abs)] = 72.7\,kPa\,(abs)$$

To determine the mass flowrate we use

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(72.7\,10^3\,\frac{N}{m^2})\,\pi\,(0.1m)^2(226\,\frac{m}{s})}{(286.9\,\frac{N \cdot m}{kg \cdot K})(264K)(4)} = \underline{\underline{1.7}}\,\frac{kg}{s}$$

(Con't)

11.57 (con't)

For $f = 0.03$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{(0.03)(2m)}{(0.1m)} = 0.6$$

and on Fig. D.2 we read

$$Ma_1 = 0.57$$

$$\frac{T_1}{T^*} = 1.13$$

$$\frac{V_1}{V^*} = 0.6$$

Thus,

$$T_1 = (1.13)(240K) = 271K$$

$$V_1 = (0.6)(310\ \frac{m}{s}) = 186\ \frac{m}{s}$$

From Fig. D.1 we read for $Ma_1 = 0.57$

$$\frac{P_1}{P_{0,1}} = 0.8$$

Thus,

$$P_1 = (0.8)[101\ kPa(abs)] = 81\ kPa(abs)$$

To determine $\dot{m}$ we use

$$\dot{m} = \frac{P_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(81\times10^3\ \frac{N}{m^2})\pi(0.1m)^2(186\ \frac{m}{s})}{(286.9\ \frac{N.m}{kg.K})(271K)(4)} = \underline{\underline{1.52\ \frac{kg}{s}}}$$

The maximum (choked duct) flowrates for different values of f are

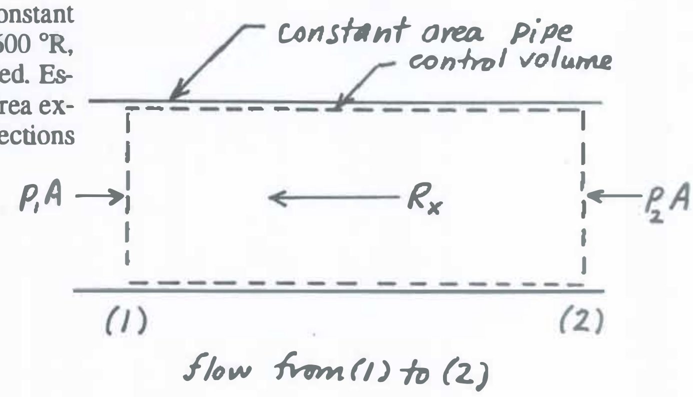
$$\dot{m}_{f=0.01} = 1.70\ \frac{kg}{s}$$

$$\dot{m}_{f=0.02} = 1.65\ \frac{kg}{s}$$

$$\dot{m}_{f=0.03} = 1.52\ \frac{kg}{s}$$

11.58

11.58 Air flows adiabatically between two sections in a constant area pipe. At upstream section (1), $p_{0,1} = 100$ psia, $T_{0,1} = 600$ °R, and $\mathrm{Ma}_1 = 0.5$. At downstream section (2), the flow is choked. Estimate the magnitude of the force per unit cross-sectional area exerted by the inside wall of the pipe on the fluid between sections (1) and (2).



The control volume sketched above is used. Applying the axial component of the linear momentum equation (Eq. 5.22) to the contents of this control volume we get for the force exerted by the pipe wall on the fluid, R_x,

$$R_x = p_1 A - p_2 A + \dot{m}(V_1 - V_2)$$

or

$$\frac{R_x}{A} = p_1 - p_2 + \rho_1 V_1 (V_1 - V_2) \qquad (1)$$

Thus we need p_1, p_2, ρ_1, V_1 and V_2.

(a) For air we enter Fig. D.1 with $\mathrm{Ma}_1 = 0.5$ and get

$$\frac{T_1}{T_{0,1}} = 0.95$$

and

$$\frac{p_1}{p_{0,1}} = 0.84$$

Thus

$$T_1 = (0.95)(600\,°R) = 570\,°R$$

and

$$p_1 = (0.84)(100\text{ psia}) = 84\text{ psia}$$

Then

$$V_1 = \mathrm{Ma}_1\sqrt{RT_1k} = (0.5)\sqrt{\frac{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(570\,°R)(1.4)}{\left(1\,\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 585\,\frac{\text{ft}}{\text{s}}$$

and

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(84\text{ psia})\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(570\,°R)} = 0.0124\,\frac{\text{slug}}{\text{ft}^3}$$

(con't)

11.58

At section (2) the flow is choked. Thus we use the $*$ state of the Fanno flow, Fig. D.2 for section (2). Entering Fig. D.2 with $Ma_1 = 0.5$ we read

$$\frac{p_1}{p^*} = 2.14 = \frac{p_1}{p_2}$$

and

$$\frac{V_1}{V^*} = 0.54 = \frac{V_1}{V_2}$$

Thus

$$p_2 = \frac{p_1}{2.14} = \frac{(84.3\ psia)}{(2.14\ )} = 39.4\ psia$$

and

$$V_2 = \frac{V_1}{0.54} = \frac{\left(586\ \frac{ft}{s}\right)}{(0.54\ \ )} = 1080\ \frac{ft}{s}$$

Now with Eq. 1 we have

$$\frac{R_x}{A} = (84\ psia)\left(144\ \frac{in.^2}{ft^2}\right) - (39.4\ psia)\left(144\ \frac{in.^2}{ft^2}\right)$$
$$+\left(0.0124\ \frac{slug}{ft^3}\right)\left(585\ \frac{ft}{s}\right)\left(585\ \frac{ft}{s} - 1080\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$\frac{R_x}{A} = \underline{\underline{2830}}\ \frac{lb}{ft^2}$$

11.59

11.59 Cite an example of an actual subsonic flow of practical importance that may be approximated with a Rayleigh flow.

The flow through the combustor of a gas turbine engine is sometimes approximated with a Rayleigh flow.

11.60

11.60 Standard atmospheric air [$T_0 = 288$ K, $p_0 = 101$ kPa (abs)] is drawn steadily through an isentropic converging nozzle into a frictionless and diabatic ($q = 500$ kJ/kg) constant cross section area duct. For maximum flow determine the values of static temperature, static pressure, stagnation temperature, stagnation pressure, and flow velocity at the inlet [section (1)] and exit [section (2)] of the constant area duct. Sketch a temperature–entropy diagram for this flow.

For maximum flow, the Rayleigh flow is choked. For the isentropic nozzle

$$T_{0,1} = T_0 = \underline{\underline{288\ K}}$$

$$p_{0,1} = p_0 = \underline{\underline{101\ kPa\ (abs)}}$$

To determine the static state at the nozzle exit, Rayleigh flow inlet, we need the value of Ma_1. To determine Ma_1 we use

$$h_{0,2} - h_{0,1} = q = c_p\,(T_{0,2} - T_{0,1})$$

or

$$T_{0,2} = \frac{q}{c_p} + T_{0,1} = \frac{\left(500{,}000\ \frac{N \cdot m}{kg}\right)}{\left(1004\ \frac{N \cdot m}{kg \cdot K}\right)} + 288K = \underline{\underline{786\ K}}$$

and noting that for choked flow, $T_{0,2} = T_{0,a}$ we get

$$\frac{T_{0,1}}{T_{0,2}} = \frac{T_{0,1}}{T_{0,a}} = \frac{288\ K}{786\ K} = 0.37$$

With $\frac{T_{0,1}}{T_{0,a}} = 0.37$ we enter Fig. D.3 and read

$$Ma_1 = 0.31$$

$$\frac{p_1}{p_a} = 2.1 \qquad (1)$$

$$\frac{T_1}{T_a} = 0.42 \qquad (2)$$

(con't)

11.60 (con't)

$$\frac{V_1}{V_a} = 0.2 \qquad (3)$$

$$\frac{p_{0,1}}{p_{0,a}} = 1.19 \qquad (4)$$

With Eq. 4 we obtain

$$p_{0,a} = \frac{p_{0,1}}{1.19} = \frac{101\ kPa(abs)}{1.19} = \underline{\underline{84.9\ kPa(abs)}} = p_{0,2}$$

With $Ma_1 = 0.31$ we read from Fig. D.1

$$\frac{p_1}{p_{0,1}} = 0.94 \qquad (5)$$

and

$$\frac{T_1}{T_{0,1}} = 0.98 \qquad (6)$$

With Eqs. 5 and 6 we get

$$p_1 = (0.94)[101\ kPa(abs)] = \underline{\underline{95\ kPa(abs)}} \qquad (7)$$

and

$$T_1 = (0.98114)(288K) = \underline{\underline{282K}}$$

Thus

$$V_1 = Ma_1\sqrt{RT_1k} = (0.31)\sqrt{\frac{\left(286.9\ \frac{N.m}{kg.K}\right)(282K)(1.4)}{\left(1\ \frac{N}{kg.\frac{m}{s^2}}\right)}} = \underline{\underline{104\ \frac{m}{s}}} \qquad (9)$$

Combining Eqs. 1 and 7 we obtain

$$p_a = \frac{p_1}{2.1} = \frac{[95\ kPa(abs)]}{(2.1)} = \underline{\underline{45\ kPa(abs)}} = p_2$$

Combining Eqs. 2 and 8 we have

$$T_a = \frac{T_1}{0.42} = \frac{(283\ K)}{(0.42)} = \underline{\underline{674K}} = T_2$$

Combining Eqs. 3 and 9 we have

$$V_a = \frac{V_1}{0.2} = \frac{104\ \frac{m}{s}}{0.2} = \underline{\underline{520\ \frac{m}{s}}} = V_2$$

(con't)

11.60 (con't)

To sketch a T-s diagram we obtain $s_2 - s_1$ from

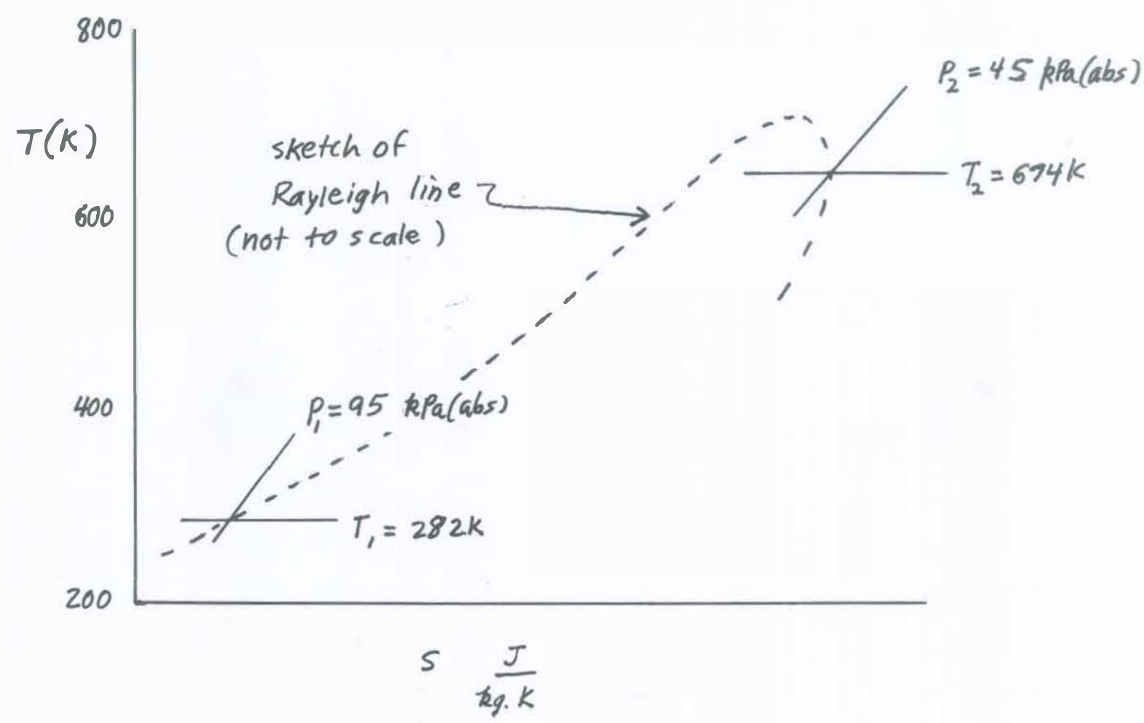
$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

or

$$s_2 - s_1 = \left(1004 \frac{N \cdot m}{kg \cdot K}\right) \ln\left(\frac{674}{282}\right) - \left(286.9 \frac{N \cdot m}{kg \cdot K}\right) \ln\left[\frac{45 \text{ kPa(abs)}}{95 \text{ kPa(abs)}}\right]$$

and

$$s_2 - s_1 = 1090 \frac{N \cdot m}{kg \cdot K}$$



11.61

11.61 Air enters a 0.5-ft inside diameter duct with $p_1 = 20$ psia, $T_1 = 80$ °F, and $V_1 = 200$ ft/s. What frictionless heat addition rate in Btu/s is necessary for an exit gas temperature $T_2 = 1500$ °F? Determine p_2, V_2, and Ma_2 also.

To determine the heat transfer rate we use the energy equation (Eq. 5.69) to get

$$\dot{Q}_{\substack{net\\in}} = \dot{m}\left(h_{0,2} - h_{0,1}\right) = \dot{m}\, c_p \left(T_{0,2} - T_{0,1}\right) \qquad (1)$$

For mass flowrate we use

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\,\frac{\pi D_1^2}{4}\, V_1 \qquad (2)$$

To determine $T_{0,2}$ and $T_{0,1}$ we use Eq. 11.56. Thus,

$$\frac{T}{T_0} = \frac{1}{1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2} \qquad (3)$$

or for air

$$\frac{T}{T_0} = f(\mathrm{Ma}) \text{ in Fig. D.1} \qquad (4)$$

To determine p_2 we use

$$p_2 = p_1 \left(\frac{p_a}{p_1}\right)\left(\frac{p_2}{p_a}\right) \qquad (5)$$

where with Eq. 11.123 for Rayleigh flow

$$\frac{p}{p_a} = \frac{1+k}{1+k\,\mathrm{Ma}^2} \qquad (6)$$

or for air

$$\frac{p}{p_a} = f(\mathrm{Ma}) \text{ in Fig. D.3} \qquad (7)$$

For exit velocity, V_2, we use

$$V_2 = \mathrm{Ma}_2 \sqrt{RT_2 k} \qquad (8)$$

We determine Ma_1 with

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{RT_1 k}} \qquad (9)$$

(con't)

11.61	(con't)

and we determine Ma_2 with

$$\frac{T_2}{T_a} = \left(\frac{T_2}{T_1}\right)\left(\frac{T_1}{T_a}\right) \tag{10}$$

and Eq. 11.128 for Rayleigh flow, namely

$$\frac{T}{T_a} = \left[\frac{(1+k)\,Ma}{1+k\,Ma^2}\right]^2 \tag{11}$$

or for air with

$$\frac{T}{T_a} = f(Ma) \text{ on Fig. D.3} \tag{12}$$

For air we determine Ma_1 with Eq. 9. Thus,

$$Ma_1 = \frac{\left(200\ \frac{ft}{s}\right)}{\sqrt{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)\dfrac{(540\,{}^\circ R)(1.4)}{\left(1\ \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}}} = 0.18$$

For $Ma_1 = 0.18$ we read on Fig. D.1

$$\frac{T_1}{T_{0,1}} = 0.99$$

Thus

$$T_{0,1} = \frac{540\,{}^\circ R}{0.99} = 545\,{}^\circ R$$

With $Ma_1 = 0.18$ we read on Fig. D.3 the values

$$\frac{T_1}{T_a} = 0.17$$

and

$$\frac{P_1}{P_a} = 2.3$$

Thus with Eq. 10 we obtain

$$\frac{T_2}{T_a} = \left(\frac{1960\,{}^\circ R}{540\,{}^\circ R}\right)(0.17) = 0.62$$

(con't)

11.61 (con't)

For $\frac{T_2}{T_a} = 0.62$ we get from Fig. D.3

$$Ma_2 = \underline{\underline{0.40}}$$

and

$$\frac{p_2}{p_a} = 1.96$$

With $Ma_2 = 0.40$ we read on Fig. D.1

$$\frac{T_2}{T_{0,2}} = 0.97$$

Thus,

$$T_{0,2} = \frac{1960}{0.97} = 2020\,°R$$

Then with Eq. 5 we have

$$p_2 = (20\,psia)\left(\frac{1}{2.3}\right)(1.96) = \underline{\underline{17}}\ psia$$

With Eq. 8 we have

$$V_2 = (0.40)\sqrt{\frac{\left(1716\ \frac{ft.lb}{slug.°R}\right)(1960\,°R)(1.4)}{\left(1\ \frac{lb}{slug.\frac{ft}{s^2}}\right)}} = \underline{\underline{868}}\ \frac{ft}{s}$$

With Eq. 2 we get

$$\dot{m} = \frac{(20\,psia)\left(144\ \frac{in.^2}{ft^2}\right)\ \pi\ (0.5\,ft)^2\left(200\ \frac{ft}{s}\right)}{\left(1716\ \frac{ft.lb}{slug.°R}\right)(540\,°R)\quad(4)} = 0.122\ \frac{slug}{s}$$

and with Eq. 1 we obtain

$$\dot{Q}_{net\ in} = \left(0.122\ \frac{slug}{s}\right)\left(6006\ \frac{ft.lb}{slug.°R}\right)\frac{(2020°R - 545°R)}{\left(778\ \frac{ft.lb}{Btu}\right)} = \underline{\underline{1390}}\ \frac{Btu}{s}$$

11.62

11.6 Air enters a length of constant cross section area pipe with $p_1 = 200$ kPa (abs), $T_1 = 500$ K, and $V_1 = 400$ m/s. If 500 kJ/kg of energy is removed from the air by frictionless heat transfer between sections (1) and (2), determine p_2, T_2, and V_2. Sketch a temperature–entropy diagram for the flow between sections (1) and (2).

To determine the state of the air at section (2) we use the energy equation (Eq. 5.69) to calculate the value of $T_{0,2}$. Thus,

$$q_{\substack{net\\in}} = h_{0,2} - h_{0,1} = c_p\left(T_{0,2} - T_{0,1}\right)$$

or

$$T_{0,2} = \frac{q_{\substack{net\\in}}}{c_p} + T_{0,1} = -\frac{q_{\substack{net\\out}}}{c_p} + T_{0,1} \qquad (1)$$

We obtain $T_{0,1}$ from $\frac{T_1}{T_{0,1}}$ which we read from Fig. D.1 with a value of Ma_1. We determine Ma_1 with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{RT_1 k}} \qquad (2)$$

With Ma_1 we also enter Fig. D.3 and read values of $\frac{p_1}{p_a}$, $\frac{T_1}{T_a}$, $\frac{V_1}{V_a}$, and $\frac{T_{0,1}}{T_{0,a}}$. Then we determine $\frac{T_{0,2}}{T_{0,a}}$ with

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{T_{0,2}}{T_{0,1}}\right)\left(\frac{T_{0,1}}{T_{0,a}}\right) \qquad (3)$$

With this value of $\frac{T_{0,2}}{T_{0,a}}$ we enter Fig. D.3 and read corresponding values of $\frac{p_2}{p_a}$, $\frac{T_2}{T_a}$, and $\frac{V_2}{V_a}$. Then we determine p_2, T_2 and V_2 with

$$p_2 = \left(\frac{p_2}{p_a}\right)\left(\frac{p_a}{p_1}\right) p_1 \qquad (4)$$

$$T_2 = \left(\frac{T_2}{T_a}\right)\left(\frac{T_a}{T_1}\right) T_1 \qquad (5)$$

and

$$V_2 = \left(\frac{V_2}{V_a}\right)\left(\frac{V_a}{V_1}\right) V_1 \qquad (6)$$

(con't)

11.62 (con't)

We use Eq. 2 to get

$$Ma_1 = \frac{\left(400\ \frac{m}{s}\right)}{\sqrt{\frac{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(500K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}}} = 0.89$$

For $Ma_1 = 0.89$ we get from Fig. D.1

$$\frac{T_1}{T_{0,1}} = 0.86$$

Thus,

$$T_{0,1} = \frac{(500\ K)}{(0.86\ )} = 580\ K$$

and with Eq. 1 we have

$$T_{0,2} = -\frac{\left(500{,}000\ \frac{J}{kg}\right)}{\left(1004\ \frac{J}{kg\cdot K}\right)} + 580K = 82\ K$$

With $Ma_1 = 0.893$ we enter Fig. D.3 and read

$$\frac{P_1}{P_a} = 1.14$$

$$\frac{T_1}{T_a} = 1.02$$

$$\frac{V_1}{V_a} = 0.9$$

and

$$\frac{T_{0,1}}{T_{0,a}} = 0.99$$

Now with $\frac{T_{0,1}}{T_{0,a}} = 0.99$ and Eq. 3 we obtain

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{82K}{579K}\right)(0.99\ ) = 0.14$$

(con't)

11.62 (con't)

which has as corresponding values in Fig. D.3 of

$Ma_2 = 0.18$

$\frac{p_2}{p_a} = 2.3$

$\frac{T_2}{T_a} = 0.17$

and

$\frac{V_2}{V_a} = 0.07$

With these ratios and those ratios corresponding to $Ma_1 = 0.89$ we use Eqs. 4, 5 and 6 to obtain

$$p_2 = (2.3)\left(\frac{1}{1.14}\right)\left[200\ kPa(abs)\right] = \underline{\underline{404}}\ kPa\,(abs)$$

$$T_2 = (0.17)\left(\frac{1}{1.02}\right)(500K) = \underline{\underline{83}}\ K$$

and

$$V_2 = (0.07)\left(\frac{1}{0.9}\right)\left(400\ \frac{m}{s}\right) = \underline{\underline{31}}\ \frac{m}{s}$$

Note that according to our calculations, $T_2 = 83.2K$ is slightly larger than $T_{0,2} = 82K$. This is not correct and is a result of the inaccuracy associated with using the graphs.

For more precision we ascertain the value of Ma_2 knowing $\frac{T_{0,2}}{T_{0,a}}$ using Eq. 11.131. First however, we determine $\frac{T_{0,1}}{T_{0,a}}$ knowing Ma_1 with Eq. 11.131. Thus,

$$\frac{T_{0,1}}{T_{0,a}} = \frac{2(k+1)\,Ma_1^2\left(1+\frac{k-1}{2}Ma_1^2\right)}{\left(1+kMa_1^2\right)^2} = \frac{2(1.4+1)(0.893)^2\left[1+\left(\frac{1.4-1}{2}\right)(0.893)^2\right]}{\left[1+(1.4)(0.893)^2\right]^2}$$

or

$$\frac{T_{0,1}}{T_{0,a}} = 0.9908$$

(con't)

11.62 (con't)

Now we use Eq. 11.56 to determine $\frac{T_1}{T_{0,1}}$. Thus,

$$\frac{T_1}{T_{0,1}} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2} = \frac{1}{1+\left(\frac{1.4-1}{2}\right)(0.893)^2} = 0.8624$$

and

$$T_{0,1} = \frac{T_1}{0.8624} = \frac{(500\,K)}{0.8624} = 579.8\,K$$

Now with Eq. 1 we have

$$T_{0,2} = \frac{-\left(500{,}000\,\frac{J}{kg}\right)}{\left(1004\,\frac{J}{kg\cdot K}\right)} + 579.8\,K = 81.79\,K$$

With Eq. 3 we obtain

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{81.79\,K}{579.8\,K}\right)(0.9908) = 0.1398$$

With Eq. 11.131 and $\frac{T_{0,2}}{T_{0,a}} = 0.1398$ we get

$$Ma_2 = 0.1776$$

Then with Eq. 11.128 and $Ma_1 = 0.893$ and $Ma_2 = 0.1776$ we get

$$\frac{T_1}{T_a} = \left[\frac{(1+k)Ma_1}{1+kMa_1^2}\right]^2 = \left[\frac{(1+1.4)(0.893)}{1+(1.4)(0.893)^2}\right]^2 = 1.026$$

and

$$\frac{T_2}{T_a} = \left[\frac{(1+1.4)(0.1776)}{1+(1.4)(0.1776)^2}\right]^2 = 0.1666$$

(con't)

11.62 (con't)

Now with Eq. 5 we have

$$T_2 = (0.1666)\left(\frac{1}{1.026}\right)(500K) = 81.19\ K$$

and

$$T_2 = 81.19K < T_{0,2} = 81.79\ K$$

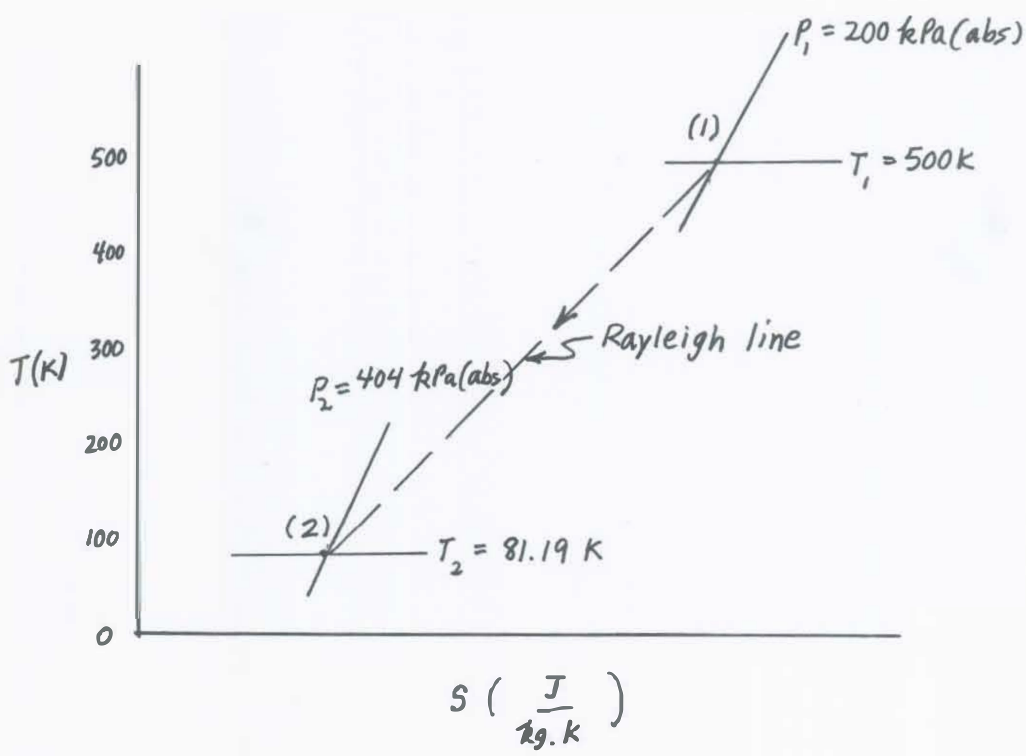
as it should be.

For our T-s sketch we use Eq. 11.76 to calculate $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 1004 \frac{J}{kg \cdot K} \ln\left(\frac{81.19\ K}{500\ K}\right) - 286.9 \frac{J}{kg \cdot K} \ln\left[\frac{404\ kPa(abs)}{200\ kPa(abs)}\right]$$

and

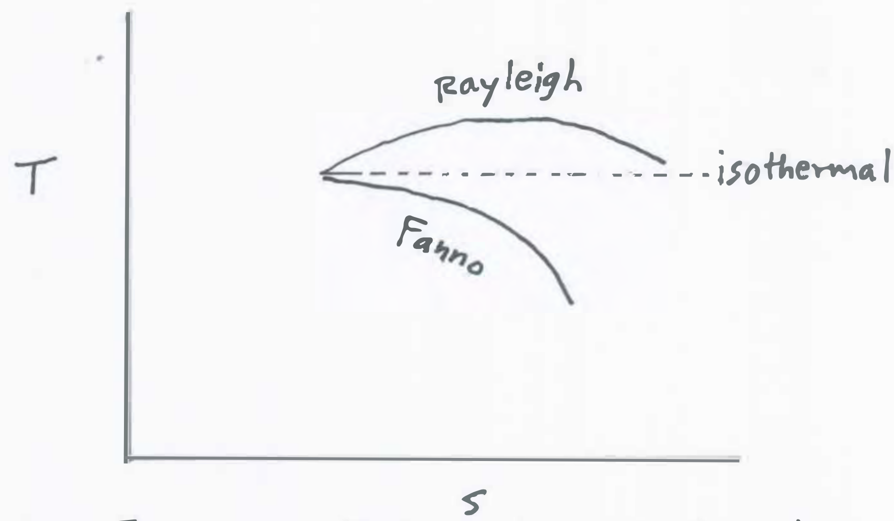
$$s_2 - s_1 = -2030 \frac{J}{kg \cdot K}$$



11.63

11.63 Describe what happens to a Fanno flow when heat transfer is allowed to occur. Is this the same as a Rayleigh flow with friction considered?

One way to respond to this problem statement is to consider what the path of these flows would look like on temperature-entropy (T-s) coordinates. Starting with the subsonic portions of Fig. 11.25



we can show Fanno and Rayleigh flows. Another classical case described in a number of fluid mechanics texts is isothermal pipe flow (constant temperature pipe flow with friction and heat transfer). This kind of flow approximates what occurs in long underground pipelines. As shown in the sketch above by the broken line the isothermal flow path is generally above the Fanno flow path and below the Rayleigh flow path. We conclude that the path for pipe flow with friction and heating would be above the Fanno flow path and the path for pipe flow with friction and cooling would be below the Fanno flow path with friction and flow rates constant. A Rayleigh flow with friction would track below the Rayleigh flow path shown other things equal. Rayleigh flows approximate flows with heat transfer over short path lengths over which friction can be ignored as an approximation of reality.

11.65

11.65 The Mach number and stagnation pressure of air are 2.0 and 200 kPa(abs) just upstream of a normal shock. Estimate the stagnation pressure loss across the shock.

We want to determine the stagnation pressure loss across a normal shock, or

$$p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) \qquad (1)$$

To determine the stagnation pressure ratio we use Eq. 11.156. Thus,

$$\frac{p_{0,y}}{p_{0,x}} = \frac{\left[\left(\frac{k+1}{2}\right)Ma_x^2\right]^{\frac{k}{k-1}}\left[1+\left(\frac{k-1}{2}\right)Ma_x^2\right]^{\frac{k}{1-k}}}{\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right]^{\frac{1}{k-1}}} \qquad (2)$$

or for air

$$\frac{p_{0,y}}{p_{0,x}} = f(Ma_x) \quad \text{in Fig. D.4.}$$

For air ($k=1.4$) we have from Fig. D.4 for $Ma_x = 2.0$,

$$\frac{p_{0,y}}{p_{0,x}} = 0.72$$

Thus, with Eq. 1 we obtain

$$p_{0,x} - p_{0,y} = [200 \text{ kPa(abs)}](1 - 0.72) = \underline{\underline{56}} \text{ kPa}$$

21

11.66

11.66 The stagnation pressure ratio across a normal shock in an air flow is 0.6. Estimate the Mach number of the flow entering the shock.

To determine the Mach number of the air flow entering a normal shock, Ma_x, given the stagnation pressure ratio, $\frac{p_{o,x}}{p_{o,y}}$, we enter Fig. D.4 with

$$\frac{p_{o,x}}{p_{o,y}} = 0.6$$

and read on Fig. D.4

$$Ma_x = \underline{\underline{2.29}}$$

11.67

11.67 Just upstream of a normal shock in an air flow, Ma = 3.0, $T = 600\,°R$, and $p = 30$ psia. Estimate values of Ma, T_0, T, p_0, p, and V downstream of the shock.

To determine Ma_y knowing Ma_x we use Eq. 11.149. Thus,

$$Ma_y = \sqrt{\frac{Ma_x^2 + \left(\frac{2}{k-1}\right)}{\left(\frac{2k}{k-1}\right)Ma_x^2 - 1}} \qquad (1)$$

or for air we use Fig. D.4 for Ma_y as a function of Ma_x.

To determine $T_{0,y}$ we use Eq. 11.56. Thus,

$$T_{0,y} = T_y\left[1 + \left(\frac{k-1}{2}\right)Ma_y^2\right] \qquad (2)$$

or for air we use Fig. D.1 for $\frac{T_y}{T_{0,y}}$ as a function of Ma_y.

To obtain T_y we use Eq. 11.151. Thus,

$$T_y = T_x\left\{\frac{\left[1+\left(\frac{k-1}{2}\right)Ma_x^2\right]\left[2\left(\frac{k}{k-1}\right)Ma_x^2 - 1\right]}{\left[\frac{(k+1)^2}{2(k-1)}\right]Ma_x^2}\right\} \qquad (3)$$

or for air we use Fig. D.4 for $\frac{T_y}{T_x}$ as a function of Ma_x.

For $p_{0,y}$ we use Eq. 2 of Example 11.19 to get

$$p_{0,y} = p_x\left\{\frac{\left[\left(\frac{k+1}{2}\right)Ma_x^2\right]^{\frac{k}{k-1}}}{\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right]^{\frac{1}{k-1}}}\right\} \qquad (4)$$

or for air we use Fig. D.4 for $\frac{p_{0,y}}{p_x}$ as a function of Ma_x.

For p_y we use Eq. 11.150 to obtain

$$p_y = p_x\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right] \qquad (5)$$

or for air we use Fig. D.4 for $\frac{p_y}{p_x}$ as a function of Ma_x.

For V_y we use

$$V_y = Ma_y\sqrt{RT_y k} \qquad (6)$$

(con't)

11.67 (con't)

For air we read from Fig. D.4 for $Ma_x = 3.0$

$$Ma_y = \underline{\underline{0.475}}$$

$$\frac{P_y}{P_x} = 10.3 \qquad (7)$$

$$\frac{T_y}{T_x} = 2.7 \qquad (8)$$

$$\frac{P_{0,y}}{P_x} = 12 \qquad (9)$$

and we obtain from Fig. D.1 for $Ma_y = 0.475$

res

$$\frac{T_y}{T_{0,y}} = 0.96 \qquad (10)$$

From Eq. 8 we get

$$T_y = (2.7)(600°R) = \underline{\underline{1620°R}}$$

and thus with Eq. 10

$$T_{0,y} = \frac{T_y}{0.96} = \frac{1620°R}{0.96} = \underline{\underline{1690°R}}$$

With Eq. 7 we obtain

$$P_y = (10.3)(30\,psia) = \underline{\underline{309}}\,psia$$

and Eq. 9 yields

$$P_{0,y} = (12)(30\,psia) = \underline{\underline{360}}\,psia$$

Then with Eq. 6 we obtain

$$V_y = (0.475)\sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(1620°R)(1.4)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = \underline{\underline{937}}\,\frac{ft}{s}$$

11.68

11.68 A total pressure probe like the one shown in **Video V3.8** is inserted into a supersonic air flow. A shock wave forms just upstream of the impact hole. The probe measures a total pressure of 500 kPa(abs). The stagnation temperature at the probe head is 500 K. The static pressure upstream of the shock is measured with a wall tap to be 100 kPa(abs). From these data, estimate the Mach number and velocity of the flow.

This is like Example 11.19.
We enter Fig. D.4 with

$$\frac{p_{0,y}}{p_x} = \frac{500\ \text{kPa(abs)}}{100\ \text{kPa(abs)}} = 5$$

and read

$$Ma_x = \underline{\underline{1.9}}$$

We determine the value of V_x with

$$V_x = Ma_x \sqrt{R T_x k} \qquad (1)$$

For T_x we read from Fig. D.1 for $Ma_x = \underline{\underline{1.9}}$

$$\frac{T_x}{T_{0,x}} = 0.58$$

and since

$$T_{0,x} = T_{0,y} = 500\ \text{K}$$

we have

$$T_x = (0.58)\,500\,\text{K} = 290\ \text{K}$$

and with Eq. 1 we obtain

$$V_x = 1.9 \sqrt{\frac{\left(286.9\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290\,\text{K})(1.4)}{\left(1\ \frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)}} = \underline{\underline{648}}\ \frac{\text{m}}{\text{s}}$$

11.69

11.69 The Pitot tube on a supersonic aircraft (see **Video V3.8**) cruising at an altitude of 30,000 ft senses a stagnation pressure of 12 psia. If the atmosphere is considered standard, determine the airspeed and Mach number of the aircraft. A shock wave is present just upstream of the probe impact hole.

At 30,000 ft, we read from Table C.1 for standard atmosphere

$$T = -47.83\,°F = 412.2\,°R$$

and

$$p = 4.373\ psia$$

Thus,

$$\frac{p_{0,y}}{p_x} = \frac{12\ psia}{4.373\ psia} = 2.74$$

and with this value of $\frac{p_{0,y}}{p_x}$ we read from Fig. D.4

$$Ma_y = \underline{\underline{1.25}}$$

Thus,

$$V_x = Ma_x\sqrt{RT_x k} = 1.25\sqrt{\frac{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)(412.2\,°R)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}}$$

and

$$V_x = \underline{\underline{1240}}\ \frac{ft}{s}$$

11.7D

11.7D An aircraft cruises at a Mach number of 2.0 at an altitude of 15 km. Inlet air is decelerated to a Mach number of 0.4 at the engine compressor inlet. A normal shock occurs in the inlet diffuser upstream of the compressor inlet at a section where the Mach number is 1.2. For isentropic diffusion, except across the shock, and for standard atmosphere determine the stagnation temperature and pressure of the air entering the engine compressor.

The deceleration process in the inlet diffuser is assumed to be adiabatic since we are considering isentropic diffusion except across the shock. Thus,

$$T_0 = \text{constant}$$

and

$$T_{0,\text{comp inlet}} = T_{0,\text{diffuser inlet}} \qquad (1)$$

To determine the diffuser inlet stagnation temperature we enter Fig. D.1 with $Ma = 2.0$ and read

$$\frac{T}{T_0} = 0.55 \qquad (2)$$

At 15 km elevation in standard atmosphere we read from Table C.2

$$T = -56.5\,^\circ C = 216.5\text{ K}$$

Thus, with Eqs. 1 and 2 we obtain

$$T_{0,\text{comp inlet}} = T_{0,\text{diffuser inlet}} = \frac{(216.5\text{ K})}{(0.55\ \ )} = \underline{\underline{394\text{ K}}}$$

To determine the stagnation pressure at the compressor inlet we use

$$p_{0,\text{comp inlet}} = p_{0,\text{diffuser inlet}}\left(\frac{p_{0,x}}{p_{0,\text{diffuser inlet}}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right)\left(\frac{p_{0,\text{comp inlet}}}{p_{0,y}}\right) \qquad (3)$$

For $p_{0,\text{diffuser inlet}}$ we use

$$p_{0,\text{diffuser inlet}} = \left(\frac{p_{0,\text{diffuser inlet}}}{p_{\text{diffuser inlet}}}\right) p_{\text{diffuser inlet}} \qquad (4)$$

where $p_{\text{diffuser inlet}} = p_{\text{atm}}$ at 15 km or $p_{\text{diffuser inlet}} = 1.211 \times 10^4 \frac{N}{m^2}$ (abs) from Table C.2.

(con't)

11.70 (con't)

We obtain $\frac{p_{\text{diffuser inlet}}}{p_{0,\text{diffuser inlet}}}$ from Fig. D.1 for $Ma_{\text{diffuser inlet}} = 2.0$.

Thus from Fig. D.1 we have

$$\frac{p_{\text{diffuser inlet}}}{p_{0,\text{diffuser inlet}}} = 0.13 \qquad (5)$$

Combining Eqs. 4 and 5 we obtain

$$p_{0,\text{diffuser inlet}} = \frac{1.211 \times 10^4 \frac{N}{m^2}(abs)}{(0.13)} = 93{,}000 \frac{N}{m^2}(abs)$$

For $Ma_x = 1.2$, we read from Fig. D.4

$$\frac{p_{0,y}}{p_{0,x}} = 0.99$$

Also, since the flow is isentropic except across the shock,

$$\frac{p_{0,x}}{p_{0,\text{diffuser inlet}}} = 1.0$$

and

$$\frac{p_{0,\text{comp inlet}}}{p_{0,y}} = 1.0$$

Thus, with Eq 3 we obtain

$$p_{0,\text{comp inlet}} = \left[93{,}000 \frac{N}{m^2}(abs)\right](1.0)(0.9928)(1.0) = \underline{\underline{92{,}000 \frac{N}{m^2}(abs)}} = \underline{\underline{92 \text{ kPa}(abs)}}$$

To determine the static pressure at the compressor inlet we enter Fig. D.1 with $Ma_{\text{comp inlet}} = 0.4$ and read

$$\frac{p_{\text{comp inlet}}}{p_{0,\text{comp inlet}}} = 0.89$$

Thus,

$$p_{\text{comp inlet}} = (0.89)\left[92 \text{ kPa}(abs)\right] = \underline{\underline{82}} \text{ kPa}(abs)$$

11.71

11.71 Determine, for the air flow through the frictionless and adiabatic converging–diverging duct of Example 11.8, the ratio of duct exit pressure to duct inlet stagnation pressure that will result in a standing normal shock at: **(a)** $x = +0.1$ m; **(b)** $x = +0.2$ m; **(c)** $x = +0.4$ m. How large is the stagnation pressure loss in each case?

This is similar to Example 11.20.

(a) For a standing normal shock at $x = +0.1\,m$ we note from the table of Example 11.8 that

$$Ma_x = 1.37$$

and

$$\frac{p_x}{p_{0,x}} = 0.33 \qquad (1)$$

From Fig. D.4, for $Ma_x = 1.37$ we obtain

$$Ma_y = 0.75$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.96 \qquad (2)$$

From Fig. D.1 we find for

$$Ma_y = 0.75$$

$$\frac{A_y}{A^*} = 1.1 \qquad (3)$$

For $x = +0.1\,m$, the ratio of duct exit area to local area (A_2/A_y) is

$$\frac{A_2}{A_y} = \frac{0.1\,m^2 + (0.5\,m)^2}{0.1\,m^2 + (0.1\,m)^2} = 3.18 \qquad (4)$$

and using Eqs. 3 and 4 we get

$$\frac{A_2}{A^*} = \left(\frac{A_y}{A^*}\right)\left(\frac{A_2}{A_y}\right) = (1.1)(3.18) = 3.5$$

(con't)

11.71 (con't)

With $\frac{A_2}{A^*} = 3.5$ we get from Fig. D.1

$$Ma_2 = 0.17$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.98$$

Thus

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.98)(0.96) = \underline{\underline{0.94}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\ kPa(abs)](1 - 0.96) = \underline{\underline{4\ kPa}}$$

(b) For a standing normal shock at $x = +0.2\ m$ we note from the table of Example 11.8 that

$$Ma_x = 1.76$$

and

$$\frac{p_x}{p_{0,x}} = 0.18$$

From Fig. D.4, for $Ma_x = 1.76$ we obtain

$$Ma_y = 0.62$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.83$$

From Fig. D.1 we find for

$$Ma_y = 0.63$$

$$\frac{A_y}{A^*} = 1.16$$

For $x = +0.2\ m$, the ratio of duct exit area to local area, $\frac{A_2}{A_y}$, is

$$\frac{A_2}{A_y} = \frac{0.1\ m^2 + (0.5\ m)^2}{0.1\ m^2 + (0.2\ m)^2} = 2.5$$

(con't)

11.71 (con't)

and thus

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) = (2.5)(1.16) = 2.9$$

With $\frac{A_2}{A^*} = 2.9$ we get from Fig. D.1

$$Ma_2 = 0.20$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.97$$

Thus

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.97)(0.83) = \underline{\underline{0.8}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\,kPa(abs)](1-0.83) = \underline{\underline{17}}\ kPa$$

(c) For a standing normal shock at $x = +0.4\,m$ we note from the table of Example 11.8 that

$$Ma_x = 2.48$$

and

$$\frac{p_x}{p_{0,x}} = 0.06$$

From Fig. D.4, for $Ma_x = 2.48$ we obtain

$$Ma_y = 0.515$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.51$$

From Fig. D.1 we find

$$Ma_y = 0.51$$

$$\frac{A_y}{A^*} = 1.3$$

(con't)

11.71 (con't)

For $x = +0.4\,m$, the ratio of duct exit area to local area, $\frac{A_2}{A_y}$, is

$$\frac{A_2}{A_y} = \frac{0.1\,m^2 + (0.5\,m)^2}{0.1\,m^2 + (0.4\,m)^2} = 1.35$$

and thus

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) = (1.35)(1.3\quad) = 1.8$$

With $\frac{A_2}{A^*} = 1.8$ we get from Fig. D.1

$$Ma_2 = 0.34$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.92$$

Thus,

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.92\quad)(0.51) = \underline{\underline{0.47}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\,kPa(abs)](1-0.51\quad) = \underline{\underline{50}}\,kPa$$

11.72

11.72 A normal shock is positioned in the diverging portion of a frictionless, adiabatic, converging–diverging air flow duct where the cross section area is 0.1 ft² and the local Mach number is 2.0. Upstream of the shock, $p_0 = 200$ psia and $T_0 = 1200$ °R. If the duct exit area is 0.15 ft², determine the exit area temperature and pressure and the duct mass flowrate.

To determine the duct exit temperature, T_2, and pressure, P_2, we need $\frac{T_2}{T_{0,2}}$ and $\frac{P_2}{P_{0,2}}$. We can obtain these ratios from Fig. D.1 knowing the value of Ma_2. The value of Ma_2 we obtain from Fig. D.1 with a known value of $\frac{A_2}{A^*}$ which we get from

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) \qquad (1)$$

The value of $\left(\frac{A_y}{A^*}\right)$ is obtained from Fig. D.1 with the value of Ma_y obtained from Fig. D.4 with a known value of $Ma_x = 2.0$. Thus from Fig. D.4 for $Ma_x = 2.0$

$$Ma_y = 0.58$$

and from Fig. D.1 we read for $Ma_y = 0.58$

$$\frac{A_y}{A^*} = 1.2$$

From the problem statement

$$\frac{A_2}{A_y} = \frac{0.15\ ft^2}{0.1\ ft^2} = 1.5$$

and thus with Eq. 1 we have

$$\frac{A_2}{A^*} = (1.5)(1.2) = 1.8$$

(con't)

11.72 (con't)

With $\frac{A_2}{A^*} = 1.8$ we get from Fig. D.1

$$Ma_2 = 0.34 \quad (2)$$

$$\frac{T_2}{T_{0,2}} = 0.97 \quad (3)$$

and

$$\frac{p_2}{p_{0,2}} = 0.92 \quad (4)$$

The value of $T_{0,2}$ is obtained from

$$T_{0,2} = T_{0,x} = T_{0,y} = T_0 = 1200\,^\circ R \quad (5)$$

The value of $p_{0,2}$ is obtained from

$$p_{0,2} = p_{0,y} = p_{0,x}\left(\frac{p_{0,y}}{p_{0,x}}\right)$$

where

$$\frac{p_{0,y}}{p_{0,x}} = 0.72$$

from Fig. D.4 for $Ma_x = 2.0$.

Thus

$$p_{0,2} = (200\ psia)(0.72) = 144\ psia \quad (6)$$

With Eqs 3 and 5 we obtain

$$T_2 = T_{0,2}\left(\frac{T_2}{T_{0,2}}\right) = (1200\,^\circ R)(0.97) = \underline{\underline{1160}}\,^\circ R$$

With Eqs. 4 and 6 we have

$$p_2 = p_{0,2}\left(\frac{p_2}{p_{0,2}}\right) = (144\ psia)(0.92) = \underline{\underline{132}}\ psia$$

For mass flowrate we use

$$\dot{m} = \rho_2 A_2 V_2 = \frac{p_2}{RT_2} A_2 Ma_2 c_2 = \frac{p_2}{RT_2} A_2 Ma_2 \sqrt{RT_2 k}$$

and

$$\dot{m} = \frac{(132 psia)\left(144 \frac{in.^2}{ft^2}\right)(0.15\,ft^2)(0.34)}{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(1160\,^\circ R)} \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(1160\,^\circ R)(1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}}$$

$$\dot{m} = \underline{\underline{0.81}}\ \frac{slug}{s} = \underline{\underline{26.1}}\ \frac{lbm}{s}$$

11.73

11.73 Supersonic air flow enters an adiabatic, constant cross section area (inside diameter = 1 ft) pipe 30 ft long with $Ma_1 = 3.0$. The pipe friction factor is estimated to be 0.02. What ratio of pipe exit pressure to pipe inlet stagnation pressure would result in a normal shock wave standing at **(a)** $x = 5$ ft, or **(b)** $x = 10$ ft, where x is the distance downstream from the pipe entrance? Determine also the duct exit Mach number and sketch the temperature–entropy diagram for each situation.

This is similar to Example 11.21.

With $Ma_1 = 3.0$ we enter Fig. D.2 and get

$$\frac{f(\ell^* - \ell_1)}{D} = 0.52$$

We note that

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{f(\ell^* - \ell_x)}{D} + \frac{f(\ell_x - \ell_1)}{D} \qquad (1)$$

(a) With Eq. 1 we get for $\ell_x - \ell_1 = 5$ ft

$$\frac{f(\ell^* - \ell_x)}{D} = \frac{f(\ell^* - \ell_1)}{D} - \frac{f(\ell_x - \ell_1)}{D} = 0.52 - \frac{(0.02)(5\,ft)}{(1\,ft)}$$

or

$$\frac{f(\ell^* - \ell_x)}{D} = 0.42$$

With $\frac{f(\ell^* - \ell_x)}{D} = 0.42$ we enter Fig. D.2 and find

$$Ma_x = 2.5$$

With $Ma_x = 2.5$ we enter Fig. D.4 and read

$$Ma_y = 0.52$$

Now with $Ma_y = 0.52$ we obtain from Fig. D.2

$$\frac{f(\ell^* - \ell_y)}{D} = 0.9$$

Since

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_y)}{D} - \frac{f(\ell_2 - \ell_y)}{D}$$ (con't)

11.73 (con't)

we get

$$\frac{f(\ell^* - \ell_2)}{D} = 0.9 \qquad \frac{(0.02)(25\ ft)}{(1\ ft)} = 0.4$$

and entering Fig. D.2 with $\frac{f(\ell^* - \ell_2)}{D} = 0.4$ we obtain

$$Ma_2 = \underline{\underline{0.62}} \quad (\text{subsonic flow})$$

Now we note that

$$\frac{p_2}{p_{0,1}} = \left(\frac{p_2}{p^*}\right)\left(\frac{p^*}{p_y}\right)\left(\frac{p_y}{p_x}\right)\left(\frac{p_x}{p^*}\right)\left(\frac{p^*}{p_1}\right)\left(\frac{p_1}{p_{0,1}}\right) \qquad (2)$$

With $Ma_2 = 0.62$ we obtain from Fig. D.2

$$\frac{p_2}{p^*} = 1.7 \qquad (3)$$

With $Ma_y = 0.52$ we obtain from Fig. D.2

$$\frac{p_y}{p^*} = 2.05 \qquad (4)$$

With $Ma_x = 2.5$ we get from Fig. D.4

$$\frac{p_y}{p_x} = 7 \qquad (5)$$

and we obtain from Fig. D.2

$$\frac{p_x}{p^*} = 0.3 \qquad (6)$$

For $Ma_1 = 3.0$ we get from Fig. D.2.

$$\frac{p_1}{p^*} = 0.22 \qquad (7)$$

and from Fig. D.1

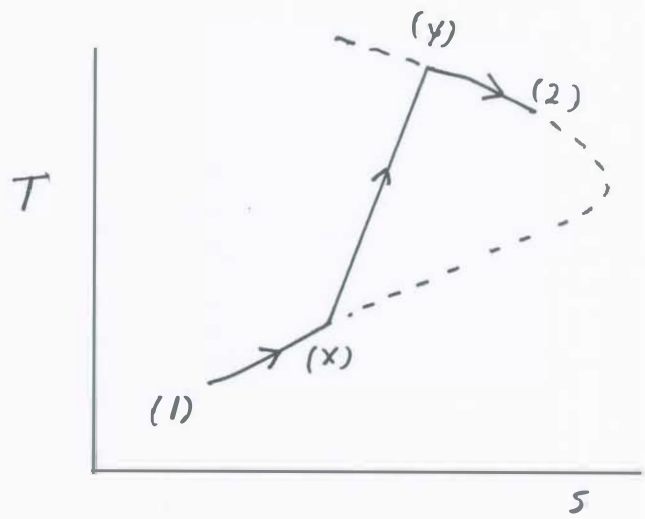
$$\frac{p_1}{p_{0,1}} = 0.03 \qquad (8)$$

(con't)

11.73 (con't)

Combining Eqs. 2 through 8 we obtain

$$\frac{p_2}{p_{0,1}} = (1.7\ \ )\left(\frac{1}{2.05}\ \ \right)(7\ \ )(0.3\ \ )\left(\frac{1}{0.22}\ \ \right)(0.03\ \ ) = \underline{\underline{0.213}}$$



Since we do not have values of temperature or pressure anywhere in the flow, we can only sketch qualitatively what happens on T-s coordinates. The T-s diagram will be similar to the one of Fig. E11.21(b) as indicated above.

(b) With Eq. 1 we get for $\ell_x - \ell_1 = 10\,ft$

$$\frac{f(\ell^* - \ell_x)}{D} = 0.52 - \frac{(0.02)(10\,ft)}{(1\,ft)} = 0.32$$

With $\frac{f(\ell^* - \ell_x)}{D} = 0.32$ we enter Fig. D.2 and find

$$Ma_x = 2$$

With $Ma_x = 2$ we enter Fig. D.4 and read

$$Ma_y = 0.58$$

Now with $Ma_y = 0.58$ we obtain from Fig. D.2

(con't)

11.73 (con't)

$$\frac{f(\ell^* - \ell_y)}{D} = 0.62$$

Since

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_y)}{D} - \frac{f(\ell_2 - \ell_y)}{D}$$

we get

$$\frac{f(\ell^* - \ell_2)}{D} = 0.62 \quad - \frac{(0.02)(20\,ft)}{(1\,ft)} = 0.22$$

and entering Fig. D.2 with $\frac{f(\ell^* - \ell_2)}{D} = 0.22$ we obtain

$$Ma_2 = \underline{\underline{0.89}}$$

With $Ma_2 = 0.89$ we obtain from Fig. D.2

$$\frac{p_2}{p^*} = 1.14 \tag{9}$$

With $Ma_y = 0.57$ we obtain from Fig. D.2

$$\frac{p_y}{p^*} = 1.86 \tag{10}$$

With $Ma_x = 2$ we get from Fig. D.4

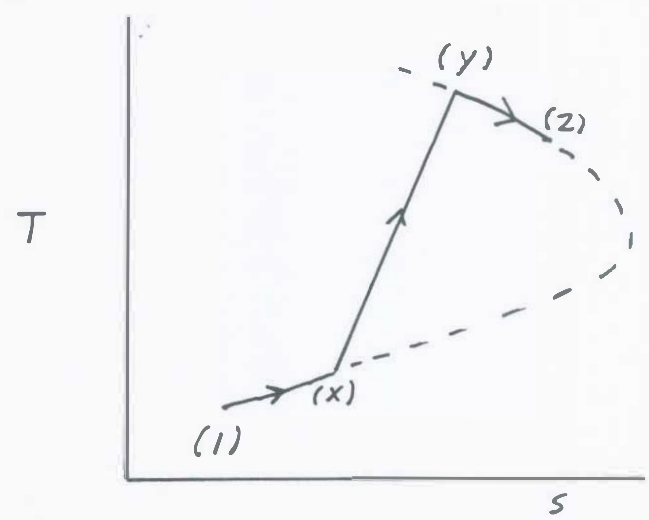
$$\frac{p_y}{p_x} = 4.8 \tag{11}$$

and we obtain from Fig. D.2

$$\frac{p_x}{p^*} = 0.4 \tag{12}$$

Combining Eqs. 2, 7, 8, 9, 10, 11 and 12 we obtain

$$\frac{p_2}{p_{0,1}} = (1.14)\left(\frac{1}{1.86}\right)(4.8\ )(0.4\ )\left(\frac{1}{0.22}\right)(0.03\ ) = \underline{\underline{0.16}}$$



11.74

11.74 Supersonic air flow enters an adiabatic, constant area pipe (inside diameter = 0.1 m) with $Ma_1 = 2.0$. The pipe friction factor is 0.02. If a standing normal shock is located right at the pipe exit, and the Mach number just upstream of the shock is 1.2, determine the length of the pipe.

We note that

$$\frac{f(\ell_2-\ell_1)}{D} = \frac{f(\ell^*-\ell_1)}{D} - \frac{f(\ell^*-\ell_2)}{D} \qquad (1)$$

where according to Eq. 11.98

$$\frac{f(\ell^*-\ell)}{D} = \frac{1}{k}\frac{(1-Ma^2)}{(Ma^2)} + \left(\frac{k+1}{2k}\right)\ln\left[\frac{\left(\frac{k+1}{2}\right)Ma^2}{1+\left(\frac{k-1}{2}\right)Ma^2}\right] \qquad (2)$$

or for air, $\frac{f(\ell^*-\ell)}{D}$ is graphed as a function of Ma in Fig. D.2.

Thus, knowing Ma_1 and Ma_2 we can determine $\frac{f(\ell^*-\ell_1)}{D}$ and $\frac{f(\ell^*-\ell_2)}{D}$ and with Eq. 1 we obtain $\frac{f(\ell_2-\ell_1)}{D}$. With f and D also known we can determine $\ell_2-\ell_1$.

For air, we find in Fig. D.2 corresponding to $Ma_1 = 2.0$ and $Ma_2 = 1.2$,

$$\frac{f(\ell^*-\ell_1)}{D} = 0.3$$

and

$$\frac{f(\ell^*-\ell_2)}{D} = 0.03$$

Thus, with Eq. 1 we have

$$\frac{f(\ell_2-\ell_1)}{D} = 0.3 - 0.03 = 0.27$$

and

$$\ell_2-\ell_1 = \frac{(0.27)(0.1\,m)}{0.02} = \underline{\underline{1.35\ m}}$$

11.75

11.75 Air enters a frictionless, constant cross section area duct with $Ma_1 = 2.0$, $T_{0,1} = 59$ °F, and $p_{0,1} = 14.7$ psia. The air is decelerated by heating until a normal shock wave occurs where the local Mach number is 1.5. Downstream of the normal shock, the subsonic flow is accelerated with heating until it chokes at the duct exit. Determine the static temperature and pressure, the stagnation temperature and pressure, and the fluid velocity at the duct entrance, just upstream and downstream of the normal shock and at the duct exit. Sketch the temperature–entropy diagram for this flow.

At the duct entrance, section (1), we have

$$T_{0,1} = \underline{\underline{59°F}} = \underline{\underline{519°R}}$$

and

$$p_{0,1} = \underline{\underline{14.7 \text{ psia}}}$$

With $Ma_1 = 2.0$ we enter Fig. D.1 and read

$$\frac{T_1}{T_{0,1}} = 0.56 \qquad (1)$$

and

$$\frac{p_1}{p_{0,1}} = 0.13 \qquad (2)$$

Thus with Eqs. 1 and 2 we obtain

$$T_1 = (0.56)(519°R) = \underline{\underline{291°R}}$$

and

$$p_1 = (0.13)(14.7 \text{ psia}) = \underline{\underline{1.91 \text{ psia}}}$$

Then

$$V_1 = Ma_1\sqrt{RT_1k} = (2.0)\sqrt{\frac{\left(1716 \frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(288°R)(1.4)}{\left(1 \frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = \underline{\underline{1660 \frac{\text{ft}}{\text{s}}}}$$

At section (x) just upstream of the shock

$$T_{0,x} = T_{0,1}\left(\frac{T_{0,a}}{T_{0,1}}\right)\left(\frac{T_{0,x}}{T_{0,a}}\right) \qquad (3)$$

(con't)

11.75 (con't)

and

$$p_{0,x} = p_{0,1}\left(\frac{p_{0,a}}{p_{0,1}}\right)\left(\frac{p_{0,x}}{p_{0,a}}\right) \qquad (4)$$

For $Ma_1 = 2.0$ and $Ma_x = 1.5$ we read from Fig. D.3

$$\frac{T_{0,1}}{T_{0,a}} = 0.79 \qquad (5)$$

$$\frac{p_{0,1}}{p_{0,a}} = 1.5 \qquad (6)$$

$$\frac{T_{0,x}}{T_{0,a}} = 0.91$$

$$\frac{p_{0,x}}{p_{0,a}} = 1.12$$

With these ratios and Eqs. 3 and 4 we obtain

$$T_{0,x} = (519\,°R)\left(\frac{1}{0.79}\right)(0.91) = \underline{\underline{598\,°R}}$$

$$p_{0,x} = (14.7\,psia)\left(\frac{1}{1.5}\right)(1.12) = \underline{\underline{11}}\ psia$$

With $Ma_x = 1.5$ we enter Fig. D.1 and read

$$\frac{T_x}{T_{0,x}} = 0.69$$

and

$$\frac{p_x}{p_{0,x}} = 0.27$$

Thus,

$$T_x = (0.69)(595\,°R) = \underline{\underline{411}}\,°R$$

and

$$p_x = (0.27)(11\,psia) = \underline{\underline{3}}\ psia$$

Then

$$V_x = Ma_x\sqrt{RT_x k} = (1.5)\sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot°R}\right)(410\,°R)(1.4)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = \underline{\underline{1490\,\frac{ft}{s}}}$$

(con't)

11.75 (Con't)

At section (y) just downstream of the shock we obtain from Fig. D.4 for $Ma_x = 1.5$

$$Ma_y = 0.7$$

$$\frac{p_y}{p_x} = 2.5$$

$$\frac{T_y}{T_x} = 1.3$$

$$\frac{V_x}{V_y} = 1.9$$

$$\frac{p_{0,y}}{p_{0,x}} = 0.93$$

With these ratios and values of properties at section (x) previously determined we have

$$p_y = (2.5)(3.00\ psia) = \underline{\underline{7.5\ psia}}$$

$$T_y = (1.3)(410°R) = \underline{\underline{533°R}}$$

$$V_y = \frac{(1490\ \frac{ft}{s})}{1.9} = \underline{\underline{784\ \frac{ft}{s}}}$$

$$p_{0,y} = (0.93)(11.0\ psia) = \underline{\underline{10.2\ psia}}$$

Also, since the flow across the normal shock is adiabatic,

$$T_{0,y} = T_{0,x} = \underline{\underline{598°R}}$$

At the duct exit, section (2) we have the subscript "a" state in Fig. D.3 since the flow is choked there. Thus from Eqs. 5 and 6 we conclude that

$$T_{0,a} = \frac{T_{0,1}}{0.79} = \frac{(519°R)}{(0.79)} = \underline{\underline{657°R}} = T_{0,2}$$

and

$$p_{0,a} = \frac{p_{0,1}}{1.5} = \frac{(14.7\ psia)}{(1.5)} = \underline{\underline{9.8}}\ psia = p_{0,2}$$

(con't)

11.75 (con't)

With $Ma_1 = 2.0$ we read further from Fig. D.3

$$\frac{p_1}{p_a} = 0.36$$

$$\frac{T_1}{T_a} = 0.53$$

$$\frac{V_1}{V_a} = 1.45$$

Thus,

$$p_a = \frac{(1.91\ psia)}{(0.36)} = \underline{\underline{5.31\ psia}} = p_2$$

$$T_a = \frac{(291\ °R)}{(0.53)} = \underline{\underline{549\ °R}} = T_2$$

and

$$V_a = \frac{\left(1660\ \frac{ft}{s}\right)}{(1.45)} = \underline{\underline{1140\ \frac{ft}{s}}} = V_2$$

To sketch a T-s diagram we need values of $s - s_1$ and we calculate these values with

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p_x}{p_1}$$

So, for example,

$$s_x - s_1 = \left(6006\ \frac{ft \cdot lb}{slug \cdot °R}\right) \ln\left(\frac{411\ °R}{519\ °R}\right) - \left(1716\ \frac{ft\ lb}{slug \cdot °R}\right) \ln\left(\frac{3\ psia}{14.7\ psia}\right) = 1310\ \frac{ft \cdot lb}{slug\ °R}$$

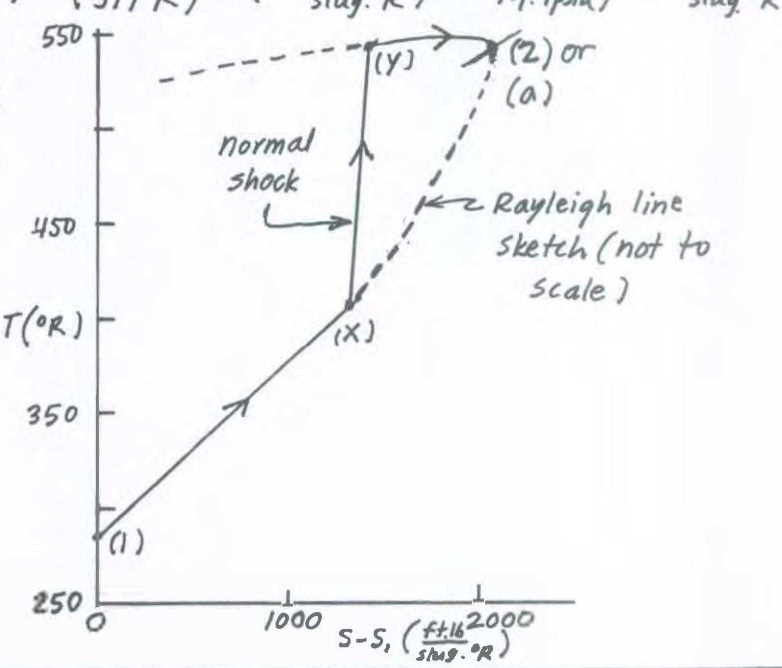
Similarly

$$s_y - s_1 = 6006 \ln \frac{533}{519} - 1716 \ln \frac{7.5}{14.7}$$

$$s_y - s_1 = 1320\ \frac{ft \cdot lb}{slug \cdot °R}$$

$$s_2 - s_1 = 6006 \ln \frac{549}{519} - 1716 \ln \frac{5.31}{14.7}$$

$$s_2 - s_1 = 2080\ \frac{ft \cdot lb}{slug \cdot °R}$$



11.76

11.76 Air enters a frictionless, constant area duct with Ma = 2.5, $T_0 = 20\ ^\circ\mathrm{C}$, and $p_0 = 101$ kPa(abs). The gas is decelerated by heating until a normal shock occurs where the local Mach number is 1.3. Downstream of the shock, the subsonic flow is accelerated with heating until it exits with a Mach number of 0.9. Determine the static temperature and pressure, the stagnation temperature and pressure, and the fluid velocity at the duct entrance, just upstream and downstream of the normal shock, and at the duct exit. Sketch the temperature–entropy diagram for this flow.

(a) For air we have at the duct entrance, section (1)

$$Ma_1 = 2.5$$

$$T_{0,1} = 20^\circ C = \underline{\underline{293\ K}}$$

$$p_{0,1} = \underline{\underline{101\ kPa\,(abs)}}$$

With $Ma_1 = 2.5$, we enter Fig. D.1 and read

$$\frac{T_1}{T_{0,1}} = 0.44 \qquad (1)$$

and

$$\frac{p_1}{p_{0,1}} = 0.06 \qquad (2)$$

Thus we have with Eqs. 1 and 2

$$T_1 = (0.44)(293\,K) = \underline{\underline{130\,K}}$$

and

$$p_1 = (0.06)[101\ kPa\,(abs)] = \underline{\underline{6.0}}\ kPa\,(abs)$$

Then,

$$V_1 = Ma_1\sqrt{RT_1 k} = (2.5)\sqrt{\frac{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(130\,K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = \underline{\underline{571}}\ \frac{m}{s}$$

At section (x) just upstream of the shock,

$$T_{0,x} = T_{0,1}\left(\frac{T_{0,a}}{T_{0,1}}\right)\left(\frac{T_{0,x}}{T_{0,a}}\right) \qquad (3)$$

and

$$p_{0,x} = p_{0,1}\left(\frac{p_{0,a}}{p_{0,1}}\right)\left(\frac{p_{0,x}}{p_{0,a}}\right) \qquad (4)$$

(con't)

11.76 (con't)

For $Ma_1 = 2.5$ and $Ma_x = 1.3$ we read from Fig. D.3

$$\frac{T_{0,1}}{T_{0,a}} = 0.71$$

$$\frac{P_{0,1}}{P_{0,a}} = 2.2$$

$$\frac{T_{0,x}}{T_{0,a}} = 0.95$$

$$\frac{P_{0,x}}{P_{0,a}} = 1.04$$

With these values and Eqs. 3 and 4 we obtain

$$T_{0,x} = (293\,K)\left(\frac{1}{0.71}\right)(0.95) = \underline{\underline{396\,K}}$$

$$P_{0,x} = [101\,kPa(abs)]\left(\frac{1}{2.2}\right)(1.04) = \underline{\underline{47.7\,kPa(abs)}}$$

With $Ma_x = 1.3$ we enter Fig. D.1 and read

$$\frac{T_x}{T_{0,x}} = 0.75$$

and

$$\frac{P_x}{P_{0,x}} = 0.36$$

Thus,

$$T_x = (0.75)(395\,K) = \underline{\underline{296\,K}}$$

and

$$P_x = (0.36)[47.4\,kPa(abs)] = \underline{\underline{17\,kPa(abs)}}$$

Then

$$V_x = Ma_x\sqrt{RT_x k} = (1.3)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(296\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{448\,\frac{m}{s}}}$$

(con't)

11.76 (con't)

At section (y) just downstream of the shock we obtain from Fig. D.4 for $Ma_x = 1.3$

$$Ma_y = 0.79$$

$$\frac{p_y}{p_x} = 1.8$$

$$\frac{T_y}{T_x} = 1.2$$

$$\frac{V_x}{V_y} = 1.5$$

$$\frac{p_{0,y}}{p_{0,x}} = 0.98$$

With these ratios and values of properties at section (x) previously determined we have

$$p_y = (1.8)\left[17.1\ kPa(abs)\right] = \underline{\underline{30.8}}\ kPa(abs)$$

$$T_y = (1.2)(295\ K) = \underline{\underline{354}}\ K$$

$$V_y = \frac{\left(448\ \frac{m}{s}\right)}{(1.5)} = \underline{\underline{299}}\ \frac{m}{s}$$

$$p_{0,y} = (0.98)\left[47.4\ kPa(abs)\right] = \underline{\underline{46.4}}\ kPa(abs)$$

Also, since the flow across the normal shock is adiabatic,

$$T_{0,y} = T_{0,x} = \underline{\underline{396}}\ K$$

At the duct exit, section (2), we have

$$p_2 = p_y\left(\frac{p_a}{p_y}\right)\left(\frac{p_2}{p_a}\right) \qquad (5)$$

$$T_2 = T_y\left(\frac{T_a}{T_y}\right)\left(\frac{T_2}{T_a}\right) \qquad (6)$$

$$T_{0,2} = T_{0,y}\left(\frac{T_{0,a}}{T_{0,y}}\right)\left(\frac{T_{0,2}}{T_{0,a}}\right) \qquad (7)$$

$$p_{0,2} = p_{0,y}\left(\frac{p_{0,a}}{p_{0,y}}\right)\left(\frac{p_{0,2}}{p_{0,a}}\right) \qquad (8)$$

$$V_2 = V_y\left(\frac{V_a}{V_y}\right)\left(\frac{V_2}{V_y}\right) \qquad (9)$$

(con't)

11.76 (con't)

Appropriate ratios to use in Eqs. 5 through 9 are obtained from Fig. D.3 for $Ma_y = 0.79$ and $Ma_2 = 0.9$.

Thus,

$$\frac{p_y}{p_a} = 1.3$$

$$\frac{p_2}{p_a} = 1.12$$

$$\frac{T_y}{T_a} = 1.02$$

$$\frac{T_2}{T_a} = 1.02$$

$$\frac{T_{0,y}}{T_{0,a}} = 0.96$$

$$\frac{T_{0,2}}{T_{0,a}} = 0.99$$

$$\frac{p_{0,y}}{p_{0,a}} = 1.02$$

$$\frac{p_{0,2}}{p_{0,a}} = 1.01$$

$$\frac{V_y}{V_a} = 0.8$$

$$\frac{V_2}{V_a} = 0.91$$

With these ratios and Eqs. 5 through 9 we obtain

$$p_2 = [30.9\ kPa\,(abs)]\left(\frac{1}{1.3}\right)(1.12) = \underline{\underline{26.6\ kPa\,(abs)}}$$

$$T_2 = (351\,K)\left(\frac{1}{1.02}\right)(1.02) = \underline{\underline{351\ K}}$$

$$T_{0,2} = (395\,K)\left(\frac{1}{0.96}\right)(0.99) = \underline{\underline{407\,K}}$$

(con't)

11.76 (con't)

$$P_{0,2} = [46.4 \text{ kPa(abs)}]\left(\frac{1}{1.02}\right)(1.01) = \underline{\underline{45.9 \text{ kPa(abs)}}}$$

$$V_2 = \left(296 \frac{m}{s}\right)\left(\frac{1}{0.8}\right)(0.91) = \underline{\underline{337 \frac{m}{s}}}$$

For sketching a T-s diagram we need values of $s - s_1$. We use,

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p}{p_1}$$

Thus, for example,

$$s_x - s_1 = \left(1004 \frac{J}{kg \cdot K}\right) \ln\left(\frac{296 \text{ K}}{130 \text{ K}}\right) - \left(286.9 \frac{J}{kg \cdot K}\right) \ln\left[\frac{17 \text{ kPa(abs)}}{6.0 \text{ kPa(abs)}}\right]$$

or

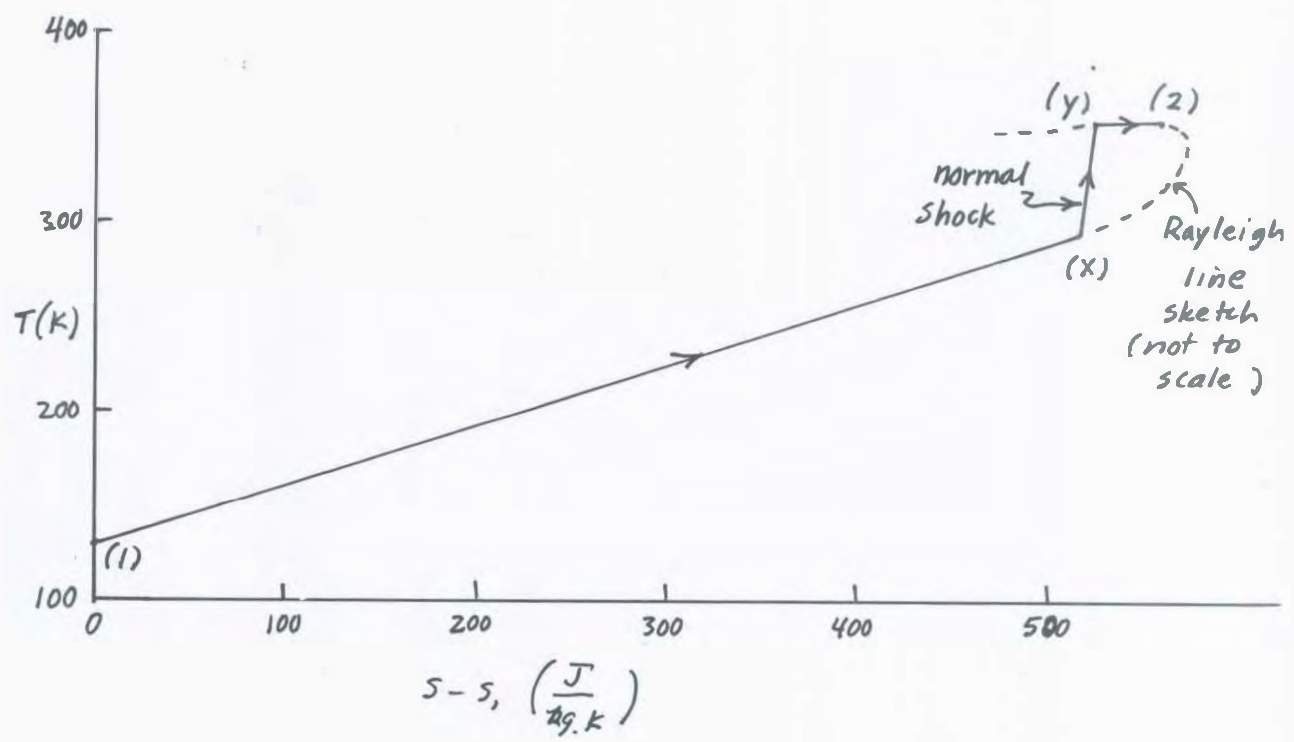
$$s_x - s_1 = 527 \frac{J}{kg \cdot K}$$

Similarly

$$s_y - s_1 = 536 \frac{J}{kg \cdot K}$$

and

$$s_2 - s_1 = 570 \frac{J}{kg \cdot K}$$



11.80

11.80 [See Fluids in the News article titled "Hilsch tube (Ranque vortex tube)," Section 11.1.] Explain why a Hilsch tube works and cite some high and low gas temperatures actually achieved. What is the most important limitation of a Hilsch tube and how can it be overcome?

A Hilsch tube works because the core flow of the associated compressible swirling flow is in solid body rotation (forced vortex). As shown by Eckert and Drake (Eckert, E.R.G. and Drake, Jr., R.M., Analysis of Heat and Mass Transfer, McGraw-Hill, New York, 1972), the difference is total temperature across the radius of this forced vortex can be appreciable, especially when the flow is turbulent. Kurosaka (Kurosaka, M., Acoustic Streaming in Swirling Flow and the Ranque-Hilsch (Vortex Tube) Effect, Journal of Fluid Mechanics 124: 139-172, 1982) concluded that periodic unsteadiness of the swirling flow is the primary cause of the formation of this forced vortex.

According to measurements (Ahlborn, B., Keller, J.U., Staudt, R., Trietz, G. and Rebhan, E., Limits of Temperature Separation in a Vortex Tube, J. Phys. D: Appl. Phys. 27: 480-488, 1994) typical hot and cold stream temperatures are 57°C and -13°C.

The most important limitation of the Hilsch tube is the inefficiency of the process, a challenge that remains to be resolved.

11.81

11.81 [See Fluids in the News article titled "Supersonic and compressible flows in gas turbines," Section 11.3.] Using typical physical dimensions and rotation speeds of manufactured gas turbine rotors, consider the possibility that supersonic Fluid velocities relative to blade surfaces are possible. How do designers use this knowledge?

For the fan of a regional turbofan gas turbine engine

tip radius = 19 in.

rotation speed = 8300 rpm

So blade tip speed, U_t, is

$$U_t = r_t \omega = \frac{(19\ \text{in.})\left(8300\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)}{\left(12\ \frac{\text{in.}}{\text{ft}}\right)\left(60\ \frac{\text{s}}{\text{min}}\right)} = 1376\ \frac{\text{ft}}{\text{s}}$$

For a typical fan inflow velocity triangle (see Fig. 12.3) the velocity relative to the fan blade, W, is much larger than the blade velocity, U. At takeoff and nominal ambient temperatures we see from Table B.3 that the relative velocity of the air flow over the fan blade near its leading edge is most likely supersonic.

For the core compressor of this same engine

tip radius = 10 in.

rotation speed = 16250 rpm

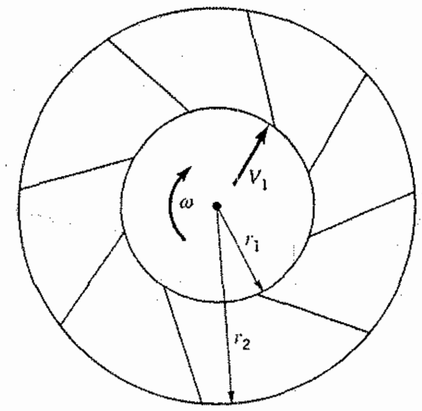
the resultant blade tip speed is $1417\ \frac{\text{ft}}{\text{s}}$

Even at higher temperatures within the core compressor the relative velocity, W, is quite likely to be supersonic.

Designers continue to improve the fan, compressor and turbine components of gas turbines.

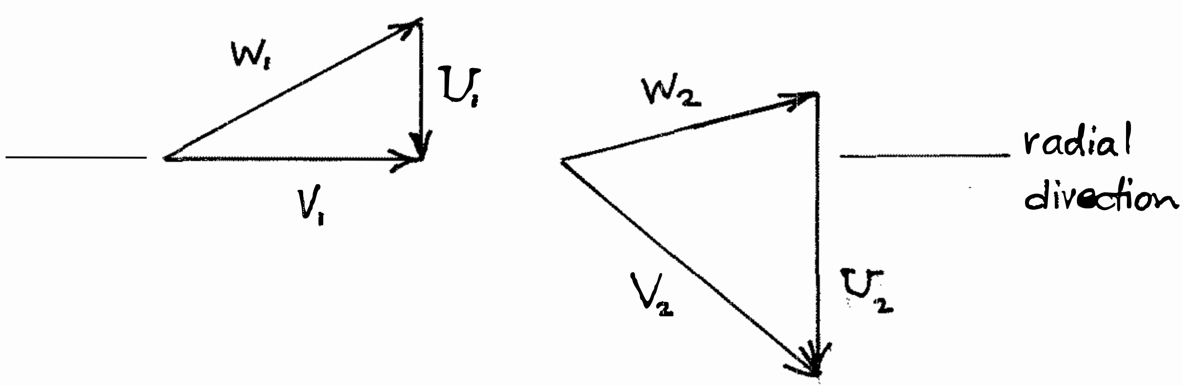
12.4

12.4 The rotor shown in Fig. P12.4 rotates clockwise. Assume that the fluid enters in the radial direction and the relative velocity is tangent to the blades and remains constant across the entire rotor. Is the device a pump or a turbine? Explain.



■ FIGURE P12.4

$W_1 = W_2$ according to the problem statement and $U_2 > U_1$ since $r_2 > r_1$. Thus a reasonable set of velocity triangles for this situation looks like



By comparing the velocity triangles at the rotor inlet (1) and exit (2) we see that the absolute velocity vector, V, has been turned in the direction of blade motion and work has been done on the fluid. This is a pump.

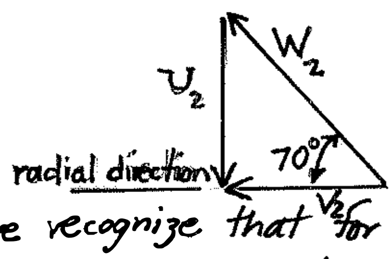
12.10

12.10 Water flows through a rotating sprinkler arm as shown in Fig. P12.10 and **Video V12.2**. Estimate the minimum water pressure necessary for an angular velocity of 150 rpm. Is this a turbine or a pump?

To estimate the minimum water pressure we consider the flow through the sprinkler and into the atmosphere to be without any loss of available energy.

So, using Bernoulli's equation we get

$$P_{min} = P_2 + \rho \frac{V_2^2}{2}$$



where $P_2 = P_{atm}$

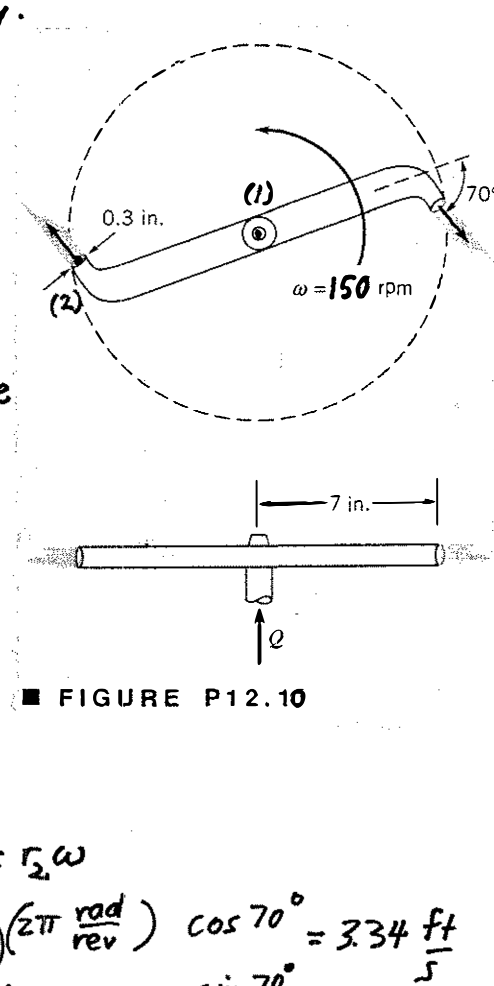
To determine V_2 we recognize that for the minimum pressure condition, the torque resisting sprinkler rotation is zero.

So

$$T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = 0$$

since $V_{\theta 1} = 0$

then $V_{\theta 2} = 0$



■ FIGURE P12.10

From the exit (2) velocity triangle,

$$V_2 = W_2 \cos 70° \quad \text{and} \quad W_2 \sin 70° = U_2 = r_2 \omega$$

So

$$V_2 = r_2 \omega \frac{\cos 70°}{\sin 70°} = \frac{(7 \text{ in.})\left(150 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)}{\left(12 \frac{in.}{ft}\right)\left(60 \frac{s}{min}\right)} \frac{\cos 70°}{\sin 70°} = 3.34 \frac{ft}{s}$$

Then

$$P_{min} = P_{atm} + \frac{1}{2} \frac{\left(62.4 \frac{lbm}{ft^3}\right)\left(3.34 \frac{ft}{s}\right)^2}{\left(32.2 \frac{lbm \cdot ft}{lb \cdot s^2}\right)} = P_{atm} + 10.8 \frac{lb}{ft^2}$$

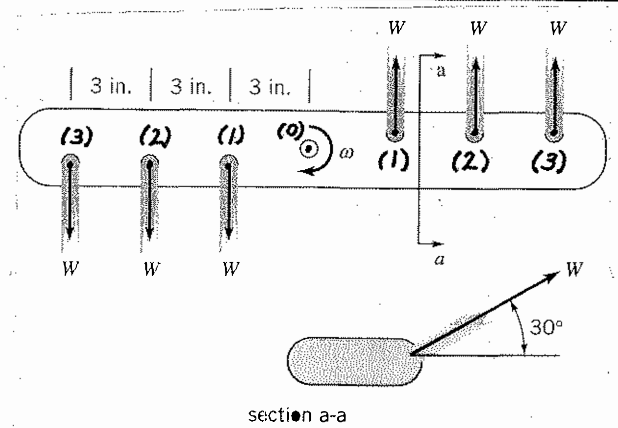
So $P_{min} = 10.8 \frac{lb}{ft^2}$ above P_{atm}

The actual pressure needed for sprinkler rotation is larger because of fluid flow losses and finite torque resisting rotation.

This is a turbine, the sprinker moves in the same direction as the fluid force on it.

12.11

12.11 Water is supplied to a dishwasher through the manifold shown in Fig. P12.11. Determine the rotational speed of the manifold if bearing friction and air resistance are neglected. The total flowrate of 2.3 gpm is divided evenly among the six outlets, each of which produces a 5/16-in.-diameter stream.



■ FIGURE P12.11

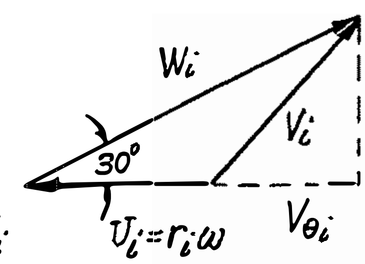
With points (0), (1), (2), and (3) located as in the diagram above,

$T = \text{"}\dot{m}(r_{out}V_{\theta out} - r_{in}V_{\theta in})\text{"}$ where $V_{\theta in} = 0$. Thus,

$T = 2\dot{m}_1 r_1 V_{\theta 1} + 2\dot{m}_2 r_2 V_{\theta 2} + 2\dot{m}_3 r_3 V_{\theta 3} = 0$ since there is no friction.

But $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$ so that the above becomes

$r_1 V_{\theta 1} + r_2 V_{\theta 2} + r_3 V_{\theta 3} = 0$ (1)



But $U_i + V_{\theta i} = W_i \cos 30^\circ$, $i = 1,2,3$ (2)

where

$Q = 6A_i W_i = 6\left[\frac{\pi}{4}\left(\frac{(\frac{5}{16})}{12}\,ft\right)^2\right]W_i = 0.00320\,W_i$

with

$Q = \left(2.3\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(231\frac{in.^3}{gal}\right)\left(\frac{1\,ft^3}{1728\,in.^3}\right) = 0.00512\,\frac{ft^3}{s}$

Thus, $W_i = 1.60\,\frac{ft}{s}$ so that from Eq. (2) $V_{\theta i} = W_i \cos 30^\circ - U_i$

or $V_{\theta i} = (1.60\,\frac{ft}{s})\cos 30^\circ - r_i\omega$. With $r_1 = \frac{3}{12}\,ft$, $r_2 = \frac{6}{12}\,ft$, and $r_3 = \frac{9}{12}\,ft$

Eq. (1) becomes

$\frac{3}{12}(1.386 - \frac{3}{12}\omega) + \frac{6}{12}(1.386 - \frac{6}{12}\omega) + \frac{9}{12}(1.386 - \frac{9}{12}\omega) = 0$

or $24.9 = 10.5\,\omega$

Thus, $\omega = 2.37\,\frac{rad}{s} \times \frac{1\,rev}{2\pi\,rad} = \underline{\underline{0.378\,\frac{rev}{s}}}$

12.12

12.12 Water flows axially up the shaft and out through the two sprinkler arms as sketched in Fig. P12.10 and as shown in **Video V12.2**. With the help of the moment-of-momentum equation explain why only at a threshold amount of water flow, the sprinkler arms begin to rotate. What happens when the flowrate increases above this threshold amount? If the exit nozzle could be varied, what would happen for a set flowrate above the threshold amount, when the angle is increased to 90°? Decreased to 0°?

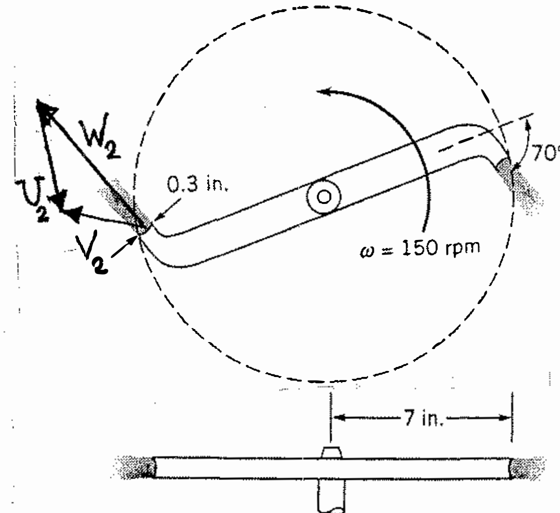


FIGURE P12.10

This sprinkler is similar to the one of Example 5.18.

Thus,

$$T_{shaft} = -r_2 V_{\theta 2}\, \dot{m}$$

From the velocity triangle shown in the sketch above, we conclude that

$$V_{\theta 2} = (W_2 \sin 70° - U_2)$$

where

$$U_2 = r_2 \omega$$

Combining, we get

$$T_{shaft} = -r_2 \left(W_2 \sin 70° - r_2 \omega\right) \dot{m} \qquad (1)$$

So, when W_2 and $\dot{m}$ combined is large enough with $\omega = 0$ to overcome T_{shaft}, the sprinkler rotor begins to rotate.

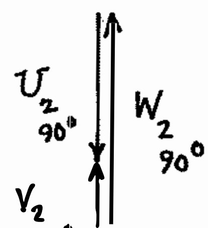
When flowrate increases further, ω is no longer zero but set at a value that satisfies Eq. 1

(con't)

12.12 (con't)

When the nozzle angle is increased from 70° to 90°

$$T_{shaft_{90°}} = (W_{2_{90°}} \sin 90° - r_2 \omega)\, \dot{m}_{90°}$$



For

$$T_{shaft_{90°}} \approx T_{shaft_{70°}}$$

$$W_{2_{90°}} = W_{2_{70°}}$$

$$\dot{m}_{90°} = \dot{m}_{70°}$$

We get

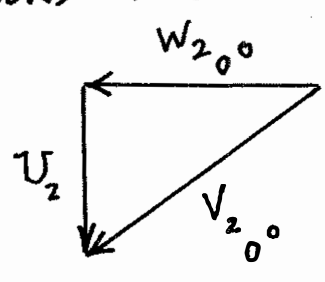
$$W_{2_{90°}} - r_2 \omega_{90°} = W_{2_{70°}} \sin 70° - r_2 \omega_{70°}$$

For this to be true

$$\omega_{90°} > \omega_{70°}$$

or the sprinkler speeds up when the nozzle angle is increased from 70° to 90°.

When the nozzle angle is decreased to 0°, the exit velocity triangle now looks like



and the shaft torque associated with this flow opposes and eventually stops sprinkler rotation.

12.13

12.13 At a given radial location, a 15-mph wind against a windmill (see **Video V12.1**) results in the upstream (1) and downstream (2) velocity triangles shown in Fig. P12.13. Sketch an appropriate blade section at that radial location and determine the energy transferred per unit mass of fluid.

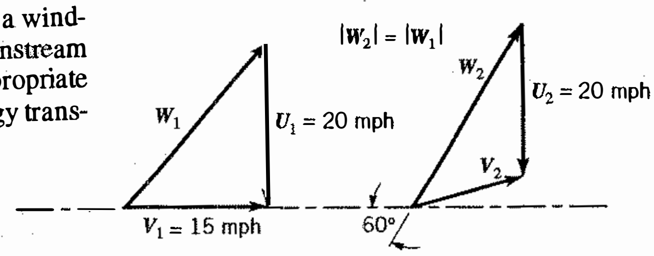


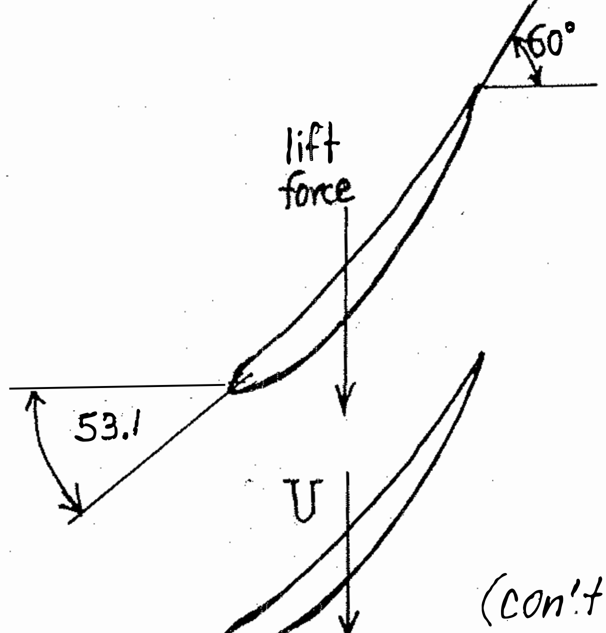
FIGURE P12.13

We can determine whether the axial flow turbomachine involved is a turbine or a fan by comparing the direction of the lift force on the rotor blade section with the direction of the blade velocity, U. If the lift force and the blade velocity are in the same direction a turbine is involved. If the lift force and blade velocity are in opposite directions, a fan is involved. The direction of the lift force can be inferred from the shape of the rotor blade section sketched to be tangent to the relative flows entering and leaving the rotor row.

The entering

$$\beta_1 = \tan^{-1}\frac{U_1}{V_1} = \tan^{-1}\frac{(20\,mph)}{(15\,mph)} = 53.1^\circ$$

Thus, the rotor blade sections sketched below are appropriate



(con't)

12.13 (con't)

Since the lift force acting on each rotor blade section is in the same direction as the blade velocity we conclude that this turbomachine is a turbine. The energy transferred per unit mass is the shaft work per unit mass, w_{shaft}, which we can determine with Eq. 11.5. Thus

$$w_{shaft} = -U_2 V_{\theta,2} \qquad (1)$$

From the velocity triangles we obtain

$$V_{\theta,2} = W_2 \sin 60° - U_2$$

and

$$W_2 = W_1 = \sqrt{V_1^2 + U_1^2}$$

Thus

$$w_{shaft} = -U_2\left(\sqrt{V_1^2 + U_1^2}\,\sin 60° - U_2\right)$$

$$w_{shaft} = -(20\,mph)\left[\sqrt{(15\,mph)^2 + (20\,mph)^2}\,\sin 60° - 20\,mph\right]\left(\frac{5280\,\frac{ft}{mi}}{3600\,\frac{s}{hr}}\right)^2\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

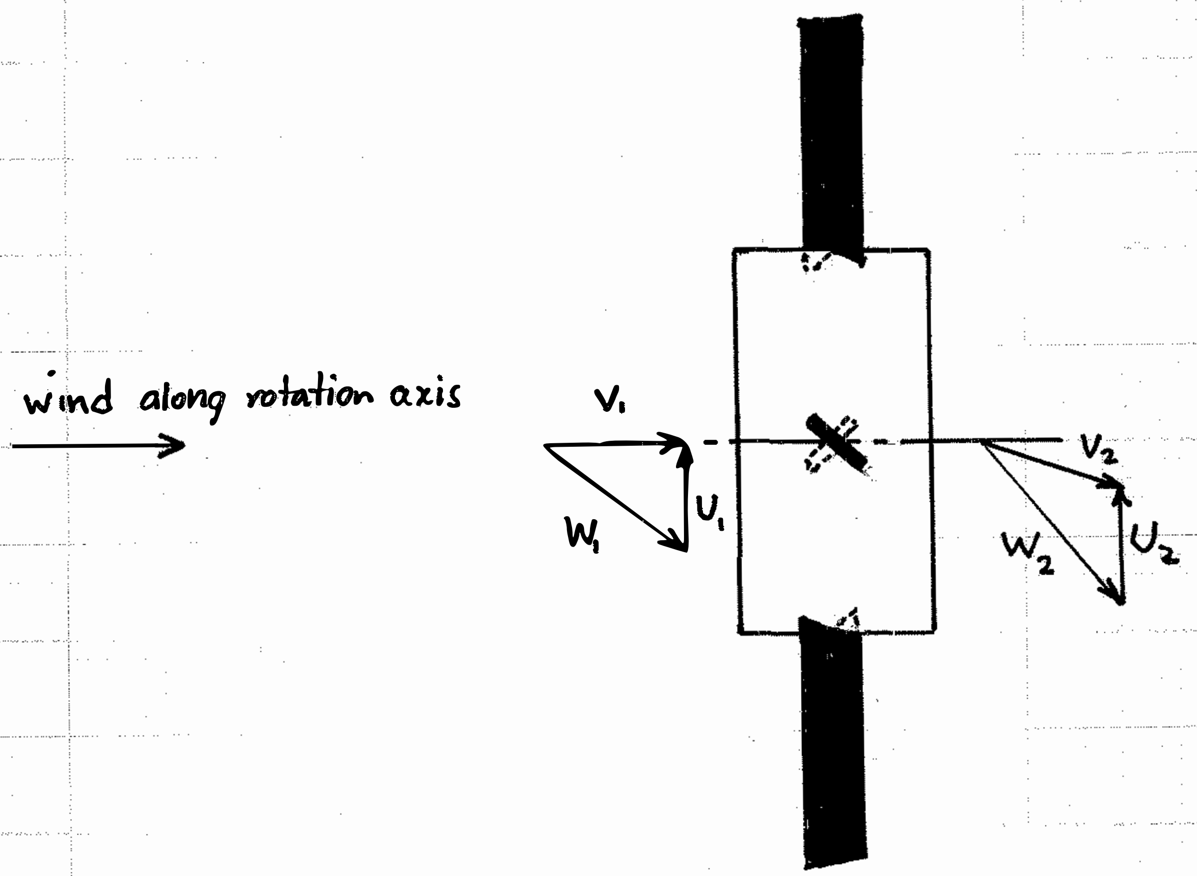
$$w_{shaft} = \underline{\underline{-71}}\ \frac{ft\cdot lb}{slug}$$

or

$$w_{shaft} = -71\,\frac{ft\cdot lb}{slug}\,\frac{1}{\left(32.2\,\frac{lbm}{slug}\right)} = -2.2\,\frac{ft\cdot lb}{lbm}$$

12.14

12.14 Sketch how you would arrange four 3-in.-wide by 12-in.-long thin but rigid strips of sheet metal on a hub to create a windmill like the one shown in **Video V12.1.** Discuss, with the help of velocity triangles, how you would arrange each blade on the hub and how you would orient your windmill in the wind.



12.15 Sketched in Fig. P12.15 are the upstream [section (1)] and downstream [section (2)] velocity triangles at the arithmetic mean radius for flow through an axial-flow turbomachine rotor. The axial component of velocity is 50 ft/s at sections (1) and (2). (a) Label each velocity vector appropriately. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. (b) Are you dealing with a turbine or a fan? (c) Calculate the work per unit mass involved. (d) Sketch a reasonable blade section. Do you think the actual blade exit angle will need to be less or greater than 15°? Why?

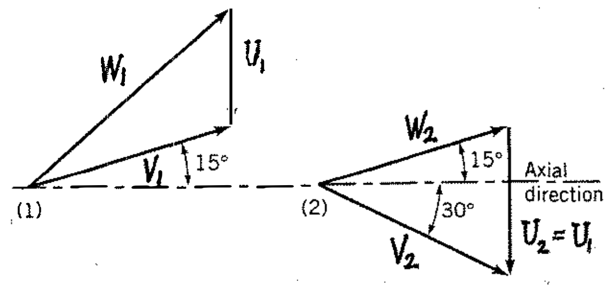


FIGURE P. 12.15

(a) See figure above.

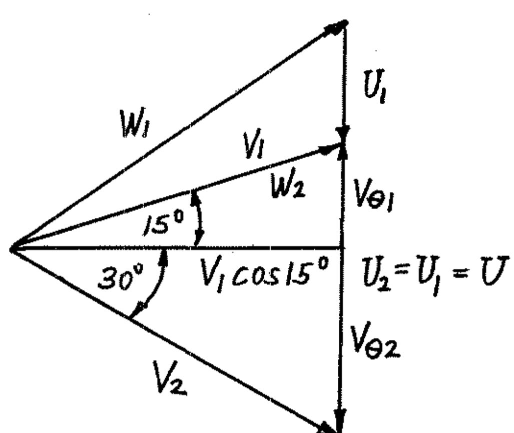
(b) $T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = \dot{m}\, r_{mean}(V_{\theta 2} - V_{\theta 1})$

where $V_{\theta 2} > 0$ and $V_{\theta 1} < 0$ (see figure above)

Thus, $T > 0$. The machine is a <u>fan.</u>

(c) $w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = U(V_{\theta 2} - V_{\theta 1})$ where $U = U_1 = U_2$

Since $V_{x1} = V_{x2} = 50 \frac{ft}{s}$, it follows from the figure that:



$V_1 \cos 15° = 50 \frac{ft}{s}$

or $V_1 = 51.8 \frac{ft}{s}$

and

$V_2 \cos 30° = 50 \frac{ft}{s}$ or $V_2 = 57.7 \frac{ft}{s}$

so that

$V_{\theta 1} = -V_1 \sin 15° = -51.8 \sin 15° = -13.4 \frac{ft}{s}$

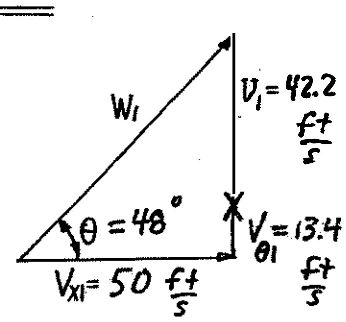
and

$V_{\theta 2} = V_2 \sin 30° = 28.8 \frac{ft}{s}$

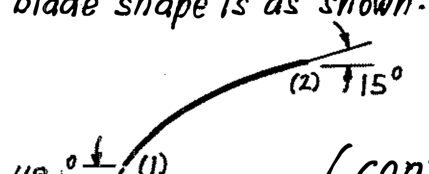
Also, $U = |V_{\theta 1}| + |V_{\theta 2}| = 42.2 \frac{ft}{s}$

Hence, $w_{shaft} = 42.2 \frac{ft}{s}\left(28.8 \frac{ft}{s} - \left(-13.4 \frac{ft}{s}\right)\right) = \underline{1780 \frac{ft^2}{s^2}}$

(d) From the figure $\tan\theta = \dfrac{42.2 + 13.4}{50}$, or $\theta = 48°$



Thus, the blade shape is as shown:



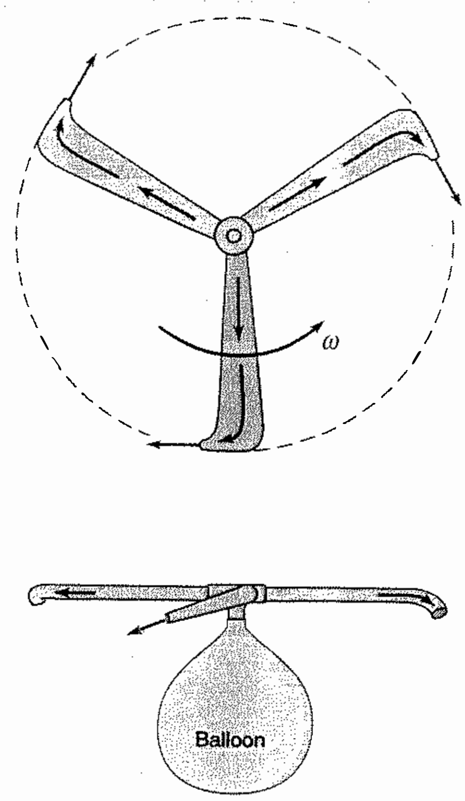
(con't)

12.15 (con't)

The actual blade angle will need to be less than 15°
to achieve a 15° flow angle at the blade exit.
Because of boundary layer development on both surfaces of
the blade, the fluid angle will be different from the
blade angle. Less turning than expected will be actually
achieved.

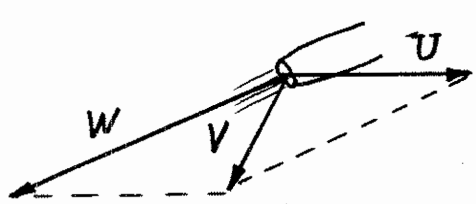
12.16

12.16 Shown in Fig. P12.16 is a toy "helicopter" powered by air escaping from a balloon. The air from the balloon flows radially through each of the three propeller blades and out small nozzles at the tips of the blades. The nozzles (along with the rotating propeller blades) are tilted at a small angle as indicated. Sketch the velocity triangle (i.e., blade, absolute, and relative velocities) for the flow from the nozzles. Explain why this toy tends to move upward. Is this a turbine? Pump?



■ FIGURE P12.16

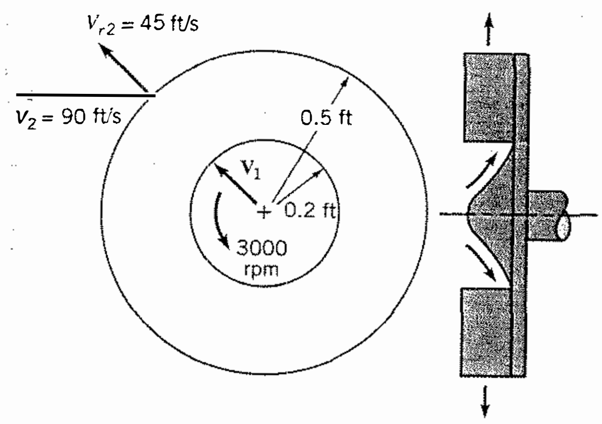
If we assume the helicopter is stationary, then the blade speed is ωR in the horizontal plane as shown in the side view below. The relative velocity, $\vec{W}$, is directed along the nozzle, and the absolute velocity, $\vec{V} = \vec{W} + \vec{U}$, is as indicated.



The toy tends to move upward because the flow over the blades push up on them. The air from the balloon forces the blades to rotate like a turbine. However, the blades act on the ambient air as a pump.

12.18

12.18 The radial component of velocity of water leaving the centrifugal pump sketched in Fig. P12.18 is 45 ft/s. The magnitude of the absolute velocity at the pump exit is 90 ft/s. The fluid enters the pump rotor radially. Calculate the shaft work required per unit mass flowing through the pump.



■ FIGURE P12.18

$$w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} \qquad (Eq.\ 12.5)$$

$$w_{shaft} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (V_{r2}^2 - V_{r1}^2)}{2} \qquad (Eq.\ 12.8)$$

Since the fluid enters radially, $V_{\theta 1} = 0$ so that Eq. 12.5 becomes

$$w_{shaft} = U_2 V_{\theta 2} \qquad (1)$$

with

$$U_2 = r_2\omega = (0.5\,ft)\left[\frac{\left(3000\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{\left(60\,\frac{s}{min}\right)}\right] = 157\,\frac{ft}{s}$$

From Fig. 12.8c

$$V_{\theta 2} = \left(V_2^2 - V_{r2}^2\right)^{1/2}$$

$$= \left[\left(90\,\frac{ft}{s}\right)^2 - \left(45\,\frac{ft}{s}\right)^2\right]^{1/2} = 77.9\,\frac{ft}{s}$$

Thus, from Eq. (1)

$$w_{shaft} = \left(157\,\frac{ft}{s}\right)\left(77.9\,\frac{ft}{s}\right)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$= \underline{\underline{1.22\times10^4\,\frac{ft\cdot lb}{slug}}} = \frac{1.22\times10^4\,\frac{ft\cdot lb}{slug}}{32.174\,\frac{lbm}{slug}}$$

$$= \underline{\underline{379}}\,\frac{ft\cdot lb}{lbm}$$

(con't)

12.18 (con't)

We proceed to calculate the component velocities of Eq. 12.8

$$U_1 = r_1\omega = (0.2\ ft)\left[\frac{\left(3000\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)}{60\ \frac{s}{min}}\right] = 62.8\ \frac{ft}{s}$$

From conservation of mass

$$V_1 A_1 = V_{r2} A_2$$

or

$$V_1 2\pi r_1 b = V_{r2}\, 2\pi r_2 b$$

and

$$V_1 = V_{r2}\frac{r_2}{r_1} = 45\ \frac{ft}{s}\left(\frac{0.5\ ft}{0.2\ ft}\right) = 112\ \frac{ft}{s}$$

For the entering flow

$$V_{r_1}^2 = U_1^2 + V_1^2 = \left(62.8\ \frac{ft}{s}\right)^2 + \left(112\ \frac{ft}{s}\right)^2$$

so

$$V_{r_1} = 128\ \frac{ft}{s}$$

Then from Eq. 12.8

$$w_{shaft} = \frac{\left(90\ \frac{ft}{s}\right)^2 - \left(112\ \frac{ft}{s}\right)^2 + \left(157\ \frac{ft}{s}\right)^2 - \left(62.8\ \frac{ft}{s}\right)^2 - \left[\left(45\ \frac{ft}{s}\right)^2 - \left(128\ \frac{ft}{s}\right)^2\right]}{2}$$

$$= 1.53 \times 10^4\ \frac{ft^2}{s^2}\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{1.53 \times 10^4\ \frac{ft \cdot lb}{slug}}} = \frac{1.53 \times 10^4\ \frac{ft \cdot lb}{slug}}{32.174\ \frac{lbm}{slug}}$$

$$= \underline{\underline{476\ \frac{ft \cdot lb}{lbm}}}$$

Eq. 12.8 invites round off error because of the differences of velocity squared involved

12.19

12.19 A centrifugal water pump having an impeller diameter of 0.5 m operates at 900 rpm. The water enters the pump parallel to the pump shaft. If the exit blade angle, β_2 (see Fig. 12.8), is 25°, determine the shaft power required to turn the impeller when the flow through the pump is 0.16 m^3/s. The uniform blade height is 50 mm.

$$\dot{W}_{shaft} = T_{shaft}\,\omega = T_{shaft}\,2\pi N$$

$$T_{shaft} = \rho Q\,(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \qquad (\text{Eq. } 12.10)$$

With $V_{\theta 1} = 0$

$$T_{shaft} = \rho Q r_2 V_{\theta 2} \qquad (1)$$

From Fig. 12.8c

$$\cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that

$$V_{\theta 2} = U_2 - V_{r2}\cot\beta_2 \qquad (2)$$

For $r_2 = \frac{0.5\,m}{2} = 0.25\,m$ with $\omega = \frac{\left(900\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{60\,\frac{s}{min}} = 94.2\,\frac{rad}{s}$

then

$$U_2 = r_2\omega = (0.25\,m)\left(94.2\,\frac{rad}{s}\right) = 23.6\,\frac{m}{s}$$

Since the flowrate is given, it follows that

$$Q = 2\pi r_2 b_2 V_{r2}$$

or

$$V_{r2} = \frac{Q}{2\pi r_2 b_2} = \frac{\left(0.16\,\frac{m^3}{s}\right)}{(2\pi)(0.25\,m)(0.05\,m)} = 2.04\,\frac{m}{s}$$

Thus, from Eq. (2)

$$V_{\theta 2} = (23.6 - 2.04\cot 25^\circ)\,\frac{m}{s} = 19.2\,\frac{m}{s}$$

and from Eq. (1)

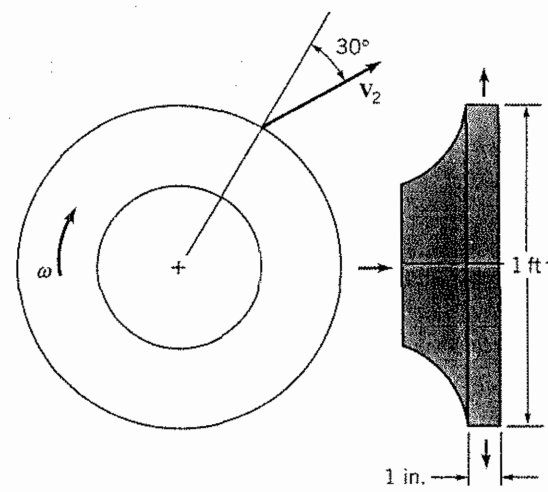
$$T_{shaft} = \left(999\,\frac{kg}{m^3}\right)\left(0.16\,\frac{m^3}{s}\right)(0.25\,m)\left(19.2\,\frac{m}{s}\right) = 768\,N\cdot m$$

so,

$$\dot{W}_{shaft} = (768\,N.m)\left(2\pi\,\frac{rad}{rev}\right)\left(900\,\frac{rev}{min}\right)\frac{1}{\left(60\,\frac{s}{min}\right)}\left(\frac{1}{1000\,\frac{N.m}{s.\,kW}}\right) = \underline{\underline{0.08\,kW}}$$

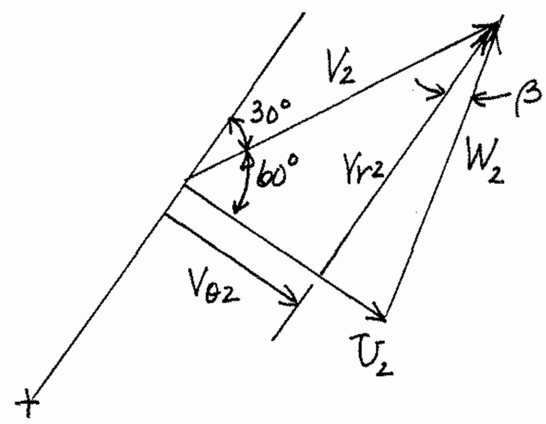
12.20

12.20 A centrifugal pump impeller is rotating at 1200 rpm in the direction shown in Fig. P12.20. The flow enters parallel to the axis of rotation and leaves at an angle of 30° to the radial direction. The absolute exit velocity, V_2, is 90 ft/s. **(a)** Draw the velocity triangle for the impeller exit flow. **(b)** Estimate the torque necessary to turn the impeller if the fluid is water. What will the impeller rotation speed become if the shaft breaks?



■ FIGURE P12.20

(a) The exit flow velocity triangle can be constructed graphically as indicated below,



$$\text{with} \quad U_2 = r_2\omega = (0.5\,ft)\frac{\left(1200\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{\left(60\,\frac{s}{min}\right)} = 62.8\,\frac{ft}{s}$$

From the velocity triangle

$$\tan\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

Since $V_{\theta 2} = V_2 \sin 30°$ and $V_{r2} = V_2 \cos 30°$ it follows that

$$\beta_2 = \tan^{-1}\left[\frac{U_2 - V_2 \sin 30°}{V_2 \cos 30°}\right]$$

$$= \tan^{-1}\left[\frac{62.8\,\frac{ft}{s} - \left(90\,\frac{ft}{s}\right)\sin 30°}{\left(90\,\frac{ft}{s}\right)\cos 30°}\right] = 12.9°$$

(con't)

12.20 (con't)

Thus, from the velocity triangle

$$W_2 = \frac{V_{r2}}{\cos 12.9°} = \frac{V_2 \cos 30°}{\cos 12.9°} = \frac{(90\,\frac{ft}{s})\cos 30°}{\cos 12.9°}$$

$$= 80.0\ \frac{ft}{s}$$

With β_2 and W known, the velocity triangle is completely specified.

(b) From Eq. 12.9 with $V_{\theta 1} = 0$

$$T_{shaft} = \dot{m}\, r_2\, V_{\theta 2} \qquad (1)$$

Since

$$\dot{m} = \rho\, 2\pi\, r_2\, b_2\, V_{r2} \quad \text{and } \rho \text{ for water from Table 1.5 is } 1.94\ \frac{slugs}{ft^3}$$

$$= \left(1.94\ \frac{slugs}{ft^3}\right)(2\pi)(0.5\,ft)\left(\frac{1}{12}\,ft\right)\left(90\,\frac{ft}{s}\right)\cos 30°$$

$$= 39.6\ \frac{slugs}{s}$$

So that from Eq. (1)

$$T_{shaft} = \left(39.6\ \frac{slugs}{s}\right)(0.5\,ft)\left(90\,\frac{ft}{s}\right)\sin 30°$$

$$= \underline{\underline{891\ ft \cdot lb}}$$

A positive torque is in the same direction as the rotation.

When the shaft breaks, the torque becomes zero and the impeller eventually stops because there is no longer a driving torque to force it to rotate. In a pump, the shaft torque drives the impeller and the impeller moves fluid. On the other hand, in a turbine, the moving fluid drives the impeller.

12.21

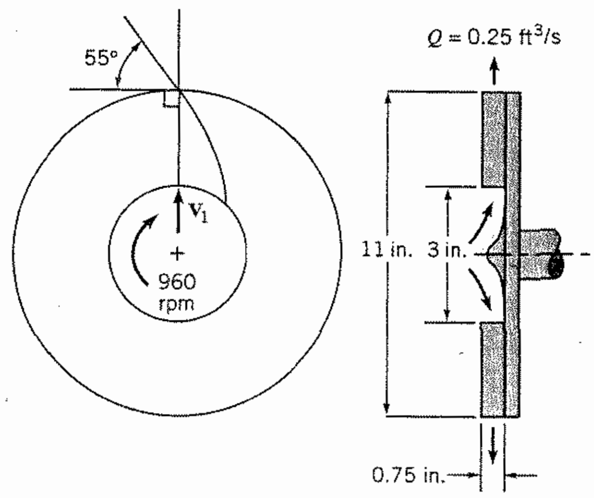
12.21 Discuss the main simplifying assumptions associated with **Eq. 12.13** and explain why actual head rise is always less than ideal head rise. Discuss how ideal head rise is head "added" to the fluid and actual head rise is head "gained" by the fluid. Can Eq. 12.13 be used for a turbine? Explain in terms of actual and ideal changes in head.

Eq. 12.13 is obtained assuming that no loss of available energy occurs in the flow through the pump impeller. The actual head rise across the pump is thus equal to the ideal head rise across the pump minus the loss of available energy suffered by the flowing fluid because of friction. The blades add the ideal head rise amount to the flowing fluid, however, the fluid flow loss results in the actual head rise realized by the flowing fluid being less than the ideal amount by the loss.

Eq. 12.13 may also be used for flow across a turbine rotor, however the change in head will now be negative or in other words the flowing fluid head will drop across the rotor. Further, this head drop across the turbine rotor is the ideal amount, or the amount in the absence of any loss of available energy suffered by the flowing fluid because of viscosity. The actual head drop is larger than the ideal head drop the difference due to losses.

12.22

12.22 A centrifugal radial water pump has the dimensions shown in Fig. P12.22. The volume rate of flow is 0.25 ft³/s, and the absolute inlet velocity is directed radially outward. The angular velocity of the impeller is 960 rpm. The exit velocity as seen from a coordinate system attached to the impeller can be assumed to be tangent to the vane at its trailing edge. Calculate the power required to drive the pump.



■ FIGURE P12.22

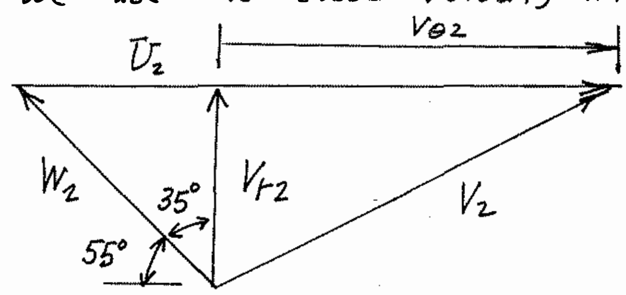
From Eq. 12.11, with $V_{\theta 1}=0$,

$$\dot{W}_{shaft} = \rho Q U_2 V_{\theta 2} \qquad (1)$$

To determine U_2 we use

$$U_2 = r_2 \omega = \left(\frac{5.5 \text{ in.}}{12 \text{ in./ft}}\right)\left(960 \frac{\text{rev}}{\text{min}}\right)\left(2\pi \frac{\text{rad}}{\text{rev}}\right)\left(\frac{1}{60 \frac{\text{s}}{\text{min}}}\right) = 46.1 \frac{\text{ft}}{\text{s}}$$

To obtain $V_{\theta 2}$ we use the exit velocity triangle shown below.



Since

$$V_{\theta 2} = U_2 - V_{r2} \tan 35°$$

and

$$V_{r2} = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 b_2} = \frac{\left(0.25 \frac{\text{ft}^3}{\text{s}}\right)\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)}{(2\pi)(5.5 \text{ in.})(0.75 \text{ in.})} = 1.39 \frac{\text{ft}}{\text{s}}$$

it follows that

$$V_{\theta 2} = \left(46.1 - 1.39 \tan 35°\right) \frac{\text{ft}}{\text{s}} = 45.1 \frac{\text{ft}}{\text{s}}$$

Thus, from Eq. (1)

$$\dot{W}_{shaft} = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)\left(0.25 \frac{\text{ft}^3}{\text{s}}\right)\left(46.1 \frac{\text{ft}}{\text{s}}\right)\left(45.1 \frac{\text{ft}}{\text{s}}\right)$$

$$= 1010 \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

or

$$\dot{W}_{shaft} = \frac{1010 \frac{\text{ft}\cdot\text{lb}}{\text{s}}}{550 \frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}} = \underline{\underline{1.84 \text{ hp}}}$$

12.23

12.23 Water is pumped with a centrifugal pump, and measurements made on the pump indicate that for a flowrate of 240 gpm the required input power is 6 hp. For a pump efficiency of 62%, what is the actual head rise of the water being pumped?

From Eq. 12.23 the pump efficiency is given by the equation

$$\eta = \frac{\gamma Q h_a/550}{bhp}$$

so that

$$h_a = \frac{(\eta)(bhp)(550)}{\gamma Q}$$

$$= \frac{(0.62)(6\,hp)\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}{\left(62.4\,\frac{lb}{ft^3}\right)\left[\left(240\,\frac{gal}{min}\right)\Big/\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)\right]}$$

$$= \underline{\underline{61.3\,ft}}$$

12.24

12.24 The performance characteristics of a certain centrifugal pump are determined from an experimental setup similar to that shown in Fig. 12.10. When the flowrate of a liquid ($SG = 0.9$) through the pump is 120 gpm, the pressure gage at (1) indicates a vacuum of 95 mm of mercury and the pressure gage at (2) indicates a pressure of 80 kPa. The diameter of the pipe at the inlet is 110 mm and at the exit it is 55 mm. If $z_2 - z_1 = 0.5$ m, what is the actual head rise across the pump? Explain how you would estimate the pump motor power requirement.

From Eq. 12.19

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g} \qquad (1)$$

Since

$$V_1 = \frac{Q}{A_1} = \frac{(120\, gpm)(6.309 \times 10^{-5} \frac{m^3/s}{gpm})}{\frac{\pi}{4}(0.110 m)^2} = 0.797 \frac{m}{s}$$

and

$$V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 \left(\frac{110 mm}{55 mm}\right)^2 = (0.797 \tfrac{m}{s})(2)^2 = 3.19 \tfrac{m}{s}$$

Thus, from Eq. (1), with $p_1 = -(h_{Hg})(\gamma_{Hg}) = -(0.095 m)(133 \times 10^3 \frac{N}{m^3})$

and $p_2 = 80 \times 10^3\ N/m^2$,

$$h_a = \frac{80 \times 10^3 \frac{N}{m^2} + (0.095)(133 \times 10^3)\frac{N}{m^2}}{(0.9)(9.80 \times 10^3)\frac{N}{m^3}} + 0.5 m + \frac{(3.19 \frac{m}{s})^2 - (0.797 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})}$$

$$h_a = \underline{\underline{11.5 m}}$$

To estimate the pump motor power requirement use Eq. 12.23

$$\eta = \frac{\gamma Q h_a}{bhp (550)}$$

to get

$$bhp = \frac{\gamma Q h_a}{\eta (550)}$$

For differing values of η, a corresponding bhp can be calculated.

12.25

12.25 The performance characteristics of a certain centrifugal pump having a 9-in.-diameter impeller and operating at 1750 rpm are determined using an experimental setup similar to that shown in Fig. 12.10. The following data were obtained during a series of tests in which $z_2 - z_1 = 0$, $V_2 = V_1$, and the fluid was water.

Q (gpm)	20	40	60	80	100	120	140
$p_2 - p_1$ (psi)	40.2	40.1	38.1	36.2	33.5	30.1	25.8
Power input (hp)	1.58	2.27	2.67	2.95	3.19	3.49	4.00

Based on these data, show or plot how the actual head rise, h_a, and the pump efficiency, η, vary with the flowrate. What is the design flowrate for this pump?

From Eq. 12.19 with $z_1 = z_2$ and $V_1 = V_2$

$$h_a = \frac{p_2 - p_1}{\gamma}$$

Thus, for the first set of data in the table

$$h_a = \frac{\left(40.2 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{62.4 \frac{lb}{ft^3}} = 92.8 ft$$

From Eq. 12.23

$$\eta = \frac{\gamma Q h_a / 550}{bhp}$$

and for the first set of data in the table

$$\eta = \frac{\left(62.4 \frac{lb}{ft^3}\right)\left[(20\ gpm)/\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\right](92.8 ft)}{(1.58\ hp)\left(550 \frac{ft \cdot lb}{s \cdot hp}\right)}$$

$$= 0.297$$

or

$$\eta = 29.7\%$$

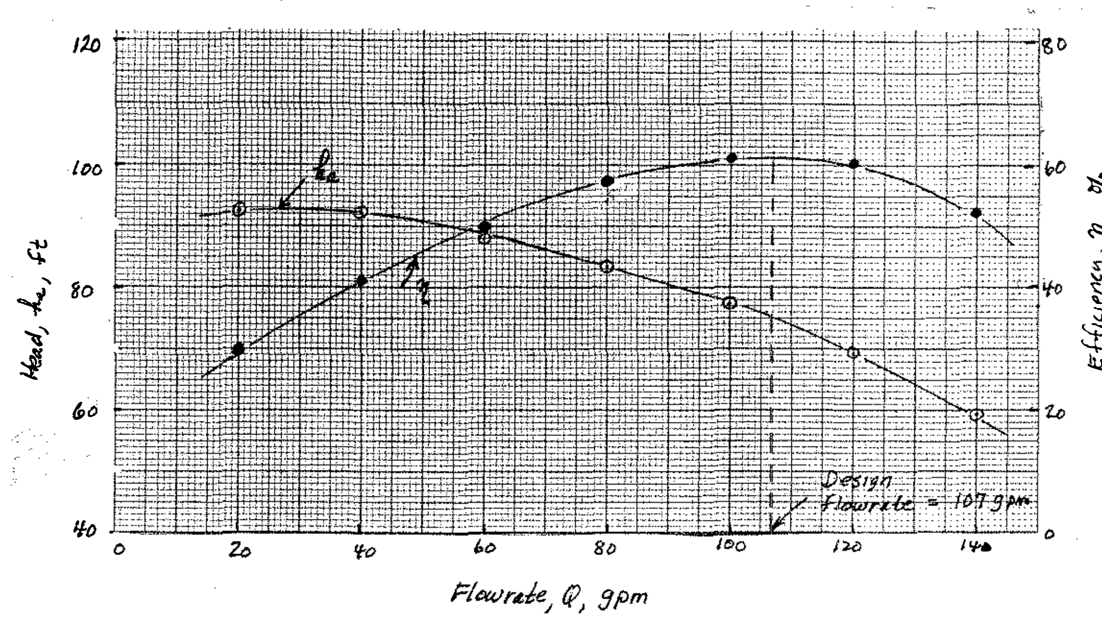
Remaining values for h_a and η can be calculated in a similar manner, and all values are tabulated in the table below.

Q (gpm)	20	40	60	80	100	120	140
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
η (%)	29.7	41.2	49.9	57.5	61.3	60.4	52.6

(con't)

12.25 (con't)

A plot of the data is shown below. The design flowrate occurs at peak efficiency and is 107 gpm.



12.26

12.26 It is sometimes useful to have $h_a - Q$ pump performance curves expressed in the form of an equation. Fit the $h_a - Q$ data given in Problem 12.25 to an equation of the form $h_a = h_o - kQ^2$ and compare the values of h_a determined from the equation with the experimentally determined values. (*Hint*: Plot h_a versus Q^2 and use the method of least squares to fit the data to the equation.)

Based on the data from Problem 12.15, the following table can be created and from a standard, linear regression curve fitting program the following results are obtained.

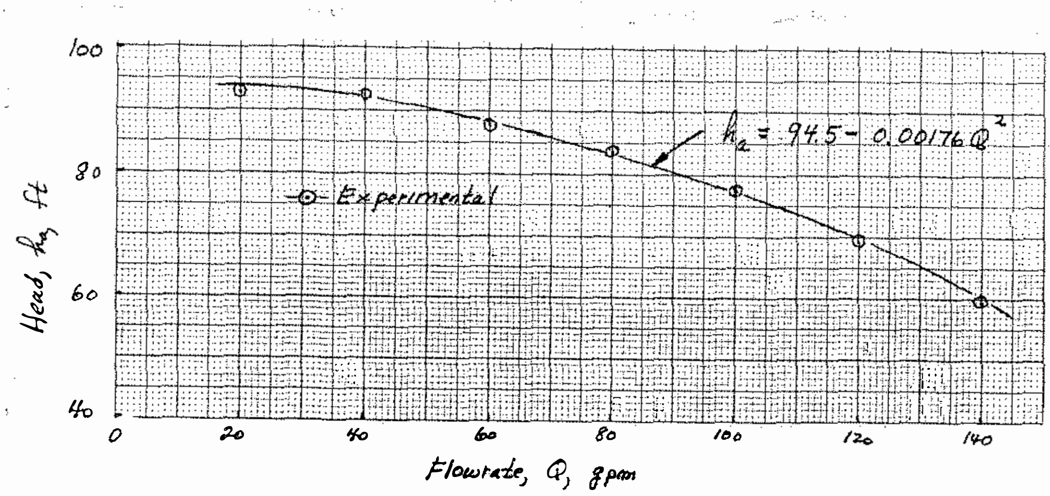
Q (gpm)	20	40	60	80	100	120	140
$[Q\text{ (gpm)}]^2$	4×10^2	16×10^2	36×10^2	64×10^2	100×10^2	144×10^2	196×10^2
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
Δh_a (ft)*	−1.00	0.81	−0.27	0.26	0.39	0.33	−0.52

$^*\Delta h_a = h_a\text{ (experimental)} - h_a\text{ (predicted)}$

The equation obtained from the data using linear regression is

$$h_a = 94.5 - 0.00176\,Q^2 \qquad (1)$$

where h_a is in ft with Q in gpm. A plot showing the comparison between the experimental data and the predicted results (from Eq. 1) is shown below.



12.28

12.28 In **Example 12.3**, how will the maximum height, z_1, that the pump can be located above the water surface change if the water temperature is decreased to 40 °F?

From Table B.1 for 40°F water, vapor pressure is 0.1217 psia and $\gamma = 62.43\ lb/ft^3$. Thus, with this change in Eq. (2) in Example 12.3

$$(z_1)_{max} = \frac{(14.7\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{62.43\ \frac{lb}{ft^3}} - 10.2\ ft - \frac{(0.1217\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{62.43\ \frac{lb}{ft^3}} - 15\ ft$$

so that

$$(z_1)_{max} = \underline{\underline{8.43\ ft}}$$

Thus, there is an increase in height from 7.65 ft to 8.43 ft with decrease in water temperature from 80°F to 40°F.

12.29

12.29 A centrifugal pump with a 7-in.-diameter impeller has the performance characteristics shown in Fig. 12.12. The pump is used to pump water at 100 °F, and the pump inlet is located 12 ft above the open water surface. When the flowrate is 200 gpm the head loss between the water surface and the pump inlet is 6 ft of water. Would you expect cavitation in the pump to be a problem? Assume standard atmospheric pressure. Explain how you arrived at your answer.

From Eq. 12.25

$$NPSH_A = \frac{p_{atm}}{\gamma} - z_1 - \Sigma h_L - \frac{p_v}{\gamma} \qquad (1)$$

From Table B.1 the water vapor pressure at 100°F is 0.9493 psia and $\gamma = 62.00 \frac{lb}{ft^3}$. Thus, with $p_{atm} = 14.7$ psia, $z_1 = 12$ ft, and $\Sigma h_L = 6$ ft, Eq. (1) yields

$$NPSH_A = \frac{(14.7 \frac{lb}{in.^2})(144 \frac{in.^2}{ft^2})}{62.00 \frac{lb}{ft^3}} - 12\ ft - 6\ ft - \frac{(0.9493 \frac{lb}{in.^2})(144 \frac{in.^2}{ft^2})}{62.00 \frac{lb}{ft^3}}$$

$$= 13.9\ ft$$

From Fig. 12.12 at 200 gpm

$$NPSH_R = \sim 12\ ft$$

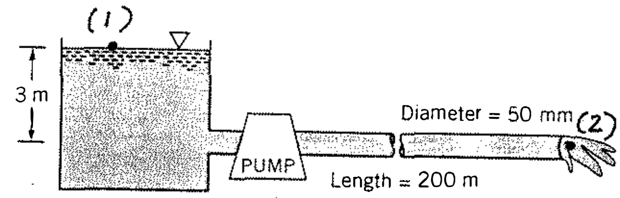
For proper pump operation

$$NPSH_A \geq NPSH_R$$

Since this is true in this case, we expect that cavitation in the pump would not be a problem. <u>No</u>.

12.30

12.30 Water at 40 °C is pumped from an open tank through 200 m of 50-mm-diameter smooth horizontal pipe as shown in Fig. P12.30 and discharges into the atmosphere with a velocity of 3 m/s. Minor losses are negligible. **(a)** If the efficiency of the pump is 70%, how much power is being supplied to the pump? **(b)** What is the NPSH$_A$ at the pump inlet? Neglect losses in the short section of pipe connecting the pump to the tank. Assume standard atmospheric pressure.



■ FIGURE P12.30

(a)
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V_2^2}{2g} \qquad (1)$$

where $p_1 = p_2 = 0$, $V_1 = 0$, $V_2 = 3\ m/s$, $z_1 = 3m$, and $z_2 = 0$. Thus, Eq. (1) becomes

$$z_1 + h_p = \frac{V_2^2}{2g}\left(1 + f\frac{\ell}{D}\right) \qquad (2)$$

Also,
$$Re = \frac{VD}{\nu} = \frac{(3\frac{m}{s})(0.05m)}{(6.580 \times 10^{-7}\frac{m^2}{s})} = 2.28 \times 10^5$$

and from Fig. 8.23 for smooth pipe $f = 0.0152$. Thus, from Eq. (2)

$$h_p = \frac{(3\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}\left[1 + 0.0152\left(\frac{200\ m}{0.05m}\right)\right] - 3m = 25.3\ m$$

Hence,

$$\text{Power gained by fluid} = \gamma Q h_p$$
$$= (9.731 \times 10^3 \tfrac{N}{m^3})(\tfrac{\pi}{4})(0.05m)^2(3\tfrac{m}{s})(25.3m)$$
$$= 1.45 \times 10^3 \tfrac{N \cdot m}{s} = 1.45\ kW$$

and

$$\text{Power supplied to pump} = \frac{\text{Power gained by fluid}}{\text{Efficiency}}$$

$$= \frac{1.45\ kW}{0.7} = \underline{\underline{2.07\ kW}}$$

(b) From Eq. 12.24

$$NPSH = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma} \qquad (3)$$

where p_s and V_s refer to the pressure and velocity at the pump inlet, respectively. Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s + h_L$$

so that with $p_1 = p_{atm}$, $V_1 = 0$, $z_s = 0$, and $h_L = 0$

(con't)

12.30 (Con't)

$$\frac{p_{atm}}{\gamma} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g}$$

and therefore from Eq.(3) the available NPSH is

$$NPSH_A = \frac{p_{atm}}{\gamma} + z_1 - \frac{p_v}{\gamma} \qquad (4)$$

Note that this result corresponds to Eq. 12.25 with z_1 positive (since pump is below reservoir) and $\Sigma h_L = 0$.

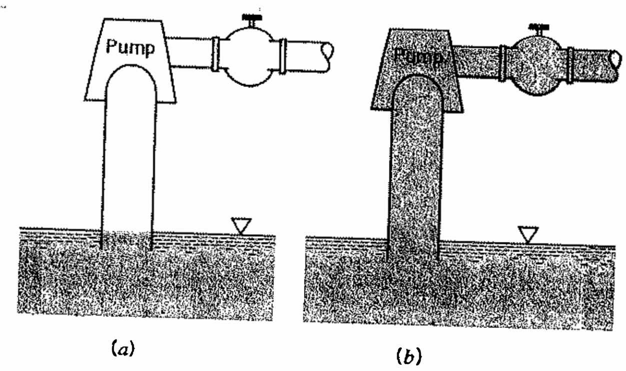
From Table B.2 the water vapor pressure at 40°C is $7.376 \times 10^3 \ N/m^2$ (abs) and $\gamma = 9.731 \times 10^3 \ N/m^3$. Thus, from Eq.(4) with $p_{atm} = 101 \ kPa$

$$NPSH_A = \frac{(101 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})} + 3m - \frac{(7.376 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})}$$

$$= \underline{\underline{12.6 \ m}}$$

12.31

12.31 The centrifugal pump shown in Fig. P12.31 is not self-priming. That is, if the water is drained from the pump and pipe as shown in Fig. P12.31(*a*), the pump will not draw the water into the pump and start pumping when the pump is turned on. However, if the pump is primed [i.e., filled with water as in Fig. P12.31(*b*)], the pump does start pumping water when turned on. Explain this behavior.



■ **FIGURE P12.31**

The head-flowrate characteristics for a typical centrifugal pump are shown in Fig. 12.11. The maximum head that the pump can add occurs when when $Q \approx 0$ (i.e., at start up for example). This head is in terms of the fluid in the pump. Neglecting losses and the velocity head (and cavitation effects) the pump can lift the fluid a height H equal to the head added by the pump. However, if the fluid in the pump is air (i.e., not primed) the head added is in terms of ft or m of air. For example, if $h_a = 30\,ft$ the pump could raise water that high if it is primed (filled with water). If the pump is not primed (filled with air) then the pump can only raise water up to a distance

$$H = 30\,ft\ \frac{\gamma_{air}}{\gamma_{water}} = 30\,ft\ \frac{(0.0765\ \frac{lb}{ft^3})}{(62.4\ \frac{lb}{ft^3})} = 0.0368\,ft$$

Hence the water will not get into the pump.

12.33

12.33 Owing to fouling of the pipe wall, the friction factor for the pipe of Example 12.4 increases from 0.02 to 0.03. Determine the new flowrate, assuming all other conditions remain the same. What is the pump efficiency at this new flowrate? Explain how a line valve could be used to vary the flowrate through the pipe of Example 12.4. Would it be better to place the valve upstream or downstream of the pump? Why?

With $f = 0.03$, Eq. (2) in Example 12.4 becomes

$$h_p = 10\text{ ft} + \left[0.03\,\frac{(200\text{ ft})}{(\frac{6}{12}\text{ ft})} + (0.5 + 1.5 + 1.0)\right]\frac{V^2}{2(32.2\,\frac{\text{ft}}{\text{s}^2})} \quad (1)$$

Since,

$$V = \frac{Q}{A} = \frac{Q\,(\frac{\text{ft}^3}{\text{s}})}{(\frac{\pi}{4})(\frac{6}{12}\text{ ft})^2}$$

Eq. (1) can be written as

$$h_p = 10 + 6.04\,Q^2$$

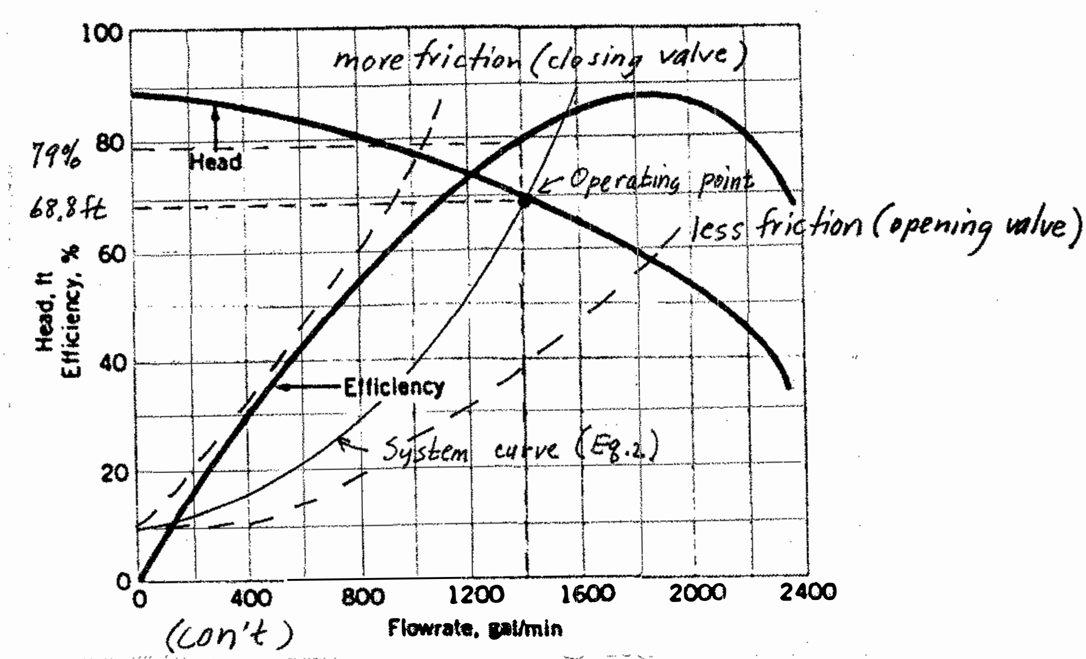
or with Q in gal/min

$$h_p = 10 + 3.00 \times 10^{-5}\left[Q\,(\text{gal/min})\right]^2 \quad (2)$$

The intersection of Eq. (2) (the system equation) with the performance curve for the pump, as shown below, indicates that the new flowrate is

$$Q = 1400\ \frac{\text{gal}}{\text{min}}$$

and the efficiency at this flowrate is approximately $\underline{79.0\%}$.



(con't)

12.33 (Con't)

A line valve acts as a variable frictional resistance to the flow. Closing the valve is equivalent to adding friction and moving the system curve to the left intersecting the head curve at an operational point involving less flowrate than with a more open valve setting. This system curve is sketched in the figure on the previous page and labeled "more friction (closing valve)." Opening the valve is similar to removing friction and moving the system curve to the right intersecting the head curve at an operating point involving more flowrate than with a less open valve setting. This system curve is sketched on the previous page and labeled "less friction (opening valve)."

It would be generally better to place the valve downstream of the pump to avoid the low suction pressure and cavitation possible with upstream placement of the valve.

12.34

12.34 A centrifugal pump having a head-capacity relationship given by the equation $h_a = 180 - 6.10 \times 10^{-4}Q^2$, with h_a in feet when Q is in gpm, is to be used with a system similar to that shown in Fig. 12.14. For $z_2 - z_1 = 50$ ft, what is the expected flowrate if the total length of constant-diameter pipe is 600 ft and the fluid is water? Assume the pipe diameter to be 4 in. and the friction factor to be equal to 0.02. Neglect all minor losses.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_2 - z_1 = 50\text{ ft}$, $f = 0.02$, $D = 4/12\text{ ft}$, and $\ell = 600\text{ ft}$, Eq. (1) becomes

$$h_p = 50\text{ ft} + 0.02\frac{(600\text{ ft})}{\left(\frac{4}{12}\text{ ft}\right)}\frac{V^2}{(2)(32.2\,\frac{\text{ft}}{\text{s}^2})} \qquad (2)$$

Since

$$V = \frac{Q}{A} = \frac{Q\left(\frac{\text{ft}^3}{\text{s}}\right)}{\left(\frac{\pi}{4}\right)\left(\frac{4}{12}\text{ ft}\right)^2}$$

Eq. (2) can be written as

$$h_p = 50 + 73.4\,Q^2$$

or with Q in gal/min

$$h_p = 50 + 3.64 \times 10^{-4}\left[Q\,(\text{gal/min})\right]^2 \qquad (3)$$

The pump head-capacity relationship is

$$h_a = 180 - 6.10 \times 10^{-4}\left[Q\,(\text{gal/min})\right]^2 \qquad (4)$$

Thus, the operating point will occur at the flowrate where $h_a = h_p$, or

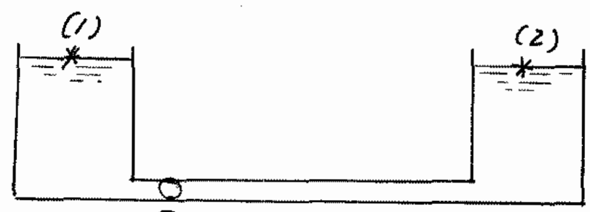
$$180 - 6.10 \times 10^{-4}\,Q^2 = 50 + 3.64 \times 10^{-4}\,Q^2$$

so that

$$Q = \underline{\underline{365\text{ gpm}}}$$

12.35

12.35 A centrifugal pump having a 6-in.-diameter impeller and the characteristics shown in Fig. 12.12 is to be used to pump gasoline through 4000 ft of commercial steel 3-in.-diameter pipe. The pipe connects two reservoirs having open surfaces at the same elevation. Determine the flowrate. Do you think this pump is a good choice? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = z_2 = 0$, $\ell = 4000\,ft$, and $D = 3/12\,ft$ (neglecting minor losses), Eq. (1) becomes

$$h_p = f\,\frac{(4000\,ft)}{(3/12\,ft)}\,\frac{V^2}{(2)(32.2\,\frac{ft}{s^2})} \qquad (2)$$

Since
$$V = \frac{Q}{A} = \frac{Q\,(ft^3/s)}{(\frac{\pi}{4})(\frac{3}{12}\,ft)^2}$$

Eq. (2) can be written as

$$h_p = 1.03 \times 10^5 f\left[Q(ft^3/s)\right]^2 \qquad (3)$$

The friction factor depends on $Re = VD/\nu = 4Q/\pi D\nu$ and with $\nu = 4.9 \times 10^{-6}\,ft^2/s$ for gasoline

$$Re = \frac{4\,Q\,(ft^3/s)}{(\pi)(3/12\,ft)(4.9 \times 10^{-6}\,\frac{ft^2}{s})} = 1.04 \times 10^6\,Q\,(ft^3/s)$$

For commercial steel 3-in. diameter pipe (from Fig. 8.22)

$$\frac{\varepsilon}{D} = 5.8 \times 10^{-4}$$

Thus, for a given Q, f can be obtained from the Moody chart, or the Colebrook equation (Eq. 8.35), and h_p determined from Eq. (3). Tabulated values are given in the following table.

(con't)

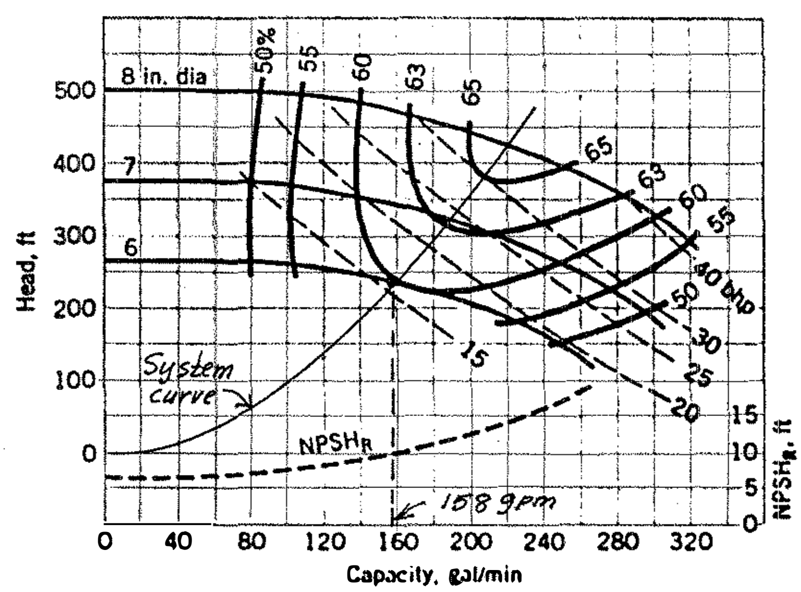
12.35 (Con't)

$Q\left(\frac{gal}{min}\right)$	$Q\left(\frac{ft^3}{s}\right)$	Re	f	h_p (ft)
40	0.0891	9.27×10^4	0.0208	17.0
80	0.178	1.85×10^5	0.0193	63.0
120	0.267	2.78×10^5	0.0187	137
160	0.357	3.71×10^5	0.0184	242
200	0.446	4.64×10^5	0.0182	373
240	0.535	5.56×10^5	0.0181	534

These data (h_p vs. Q) are plotted on Fig. 12.12 (reproduced below), and the flowrate at the intersection of the system curve and the pump curve is

$$Q = 158\ \frac{gal}{min}$$

Since at this flowrate the pump operates near peak efficiency this type of pump would appear to be a good choice if the 158 gal/min flowrate is at or near the desired flowrate.



12.36

12.36 Determine the new flowrate for the system described in Problem 12.35 if the pipe diameter is increased from 3 in. to 4 in. Is this pump still a good choice? Explain.

Refer to solution to Problem 12.23. With $D = 4/12$ ft Eq. (2) becomes

$$h_p = f \frac{(4000\ ft)}{(\frac{4}{12}\ ft)} \frac{V^2}{(2)(32.2\ \frac{ft}{s^2})} \qquad (2)$$

and

$$V = \frac{Q}{A} = \frac{Q\ (ft^3/s)}{(\frac{\pi}{4})(\frac{4}{12}\ ft)^2}$$

so that

$$h_p = 2.45 \times 10^4\ f\ [Q(ft^3/s)]^2 \qquad (3)$$

The Reynolds number becomes

$$Re = \frac{4Q}{\pi D \nu} = \frac{4\ Q\ (ft^3/s)}{(\pi)(4/12\ ft)(4.9 \times 10^{-6}\ \frac{ft^2}{s})} = 7.80 \times 10^5\ Q\ (ft^3/s)$$

For commercial steel 4-in. diameter pipe (from Fig. 8.22), $\frac{\varepsilon}{D} = 4.3 \times 10^{-4}$. Thus, for a given Q, f can be obtained from the Moody chart, or the Colebrook equation (Eq. 8.35), and h_p determined from Eq. (3). Tabulated values are given in the following table.

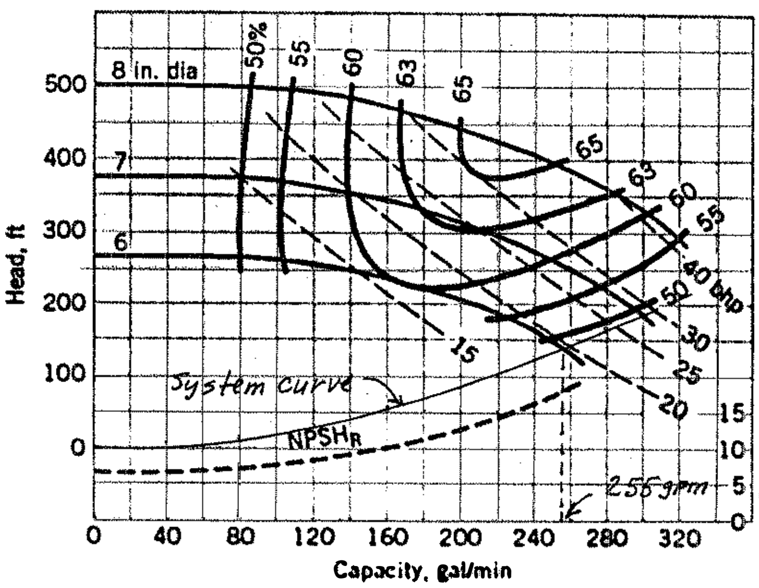
$Q\ (\frac{gal}{min})$	$Q\ (\frac{ft^3}{s})$	Re	f	$h_p\ (ft)$
40	0.0891	6.95×10^4	0.0211	4.1
80	0.178	1.39×10^5	0.0192	14.9
120	0.267	2.08×10^5	0.0183	32.0
160	0.357	2.78×10^5	0.0179	55.9
200	0.446	3.48×10^5	0.0176	85.8
240	0.535	4.17×10^5	0.0174	122
280	0.624	4.87×10^5	0.0172	164
320	0.713	5.56×10^5	0.0170	212

These data (h_p vs. Q) are plotted on Fig. 12.12 (reproduced on the following page), and the flowrate at the intersection of the system curve and the pump curve is

$$Q = 255\ \frac{gal}{min}$$

(con't)

12.36 (Con't)



Since at this flowrate the pump efficiency is fairly low (~49%), this pump is no longer a good choice.

12.37

12.37 A centrifugal pump having the characteristics shown in Example 12.4 is used to pump water between two large open tanks through 100 ft of 8-in.-diameter pipe. The pipeline contains 4 regular flanged 90° elbows, a check valve, and a fully open globe valve. Other minor losses are negligible. Assume the friction factor $f = 0.02$ for the 100-ft section of pipe. If the static head (difference in height of fluid surfaces in the two tanks) is 30 ft, what is the expected flowrate? Do you think this pump is a good choice? Explain.

Application of the energy equation between the two free surfaces, points (1) and (2), gives

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_2 - z_1 = 30\text{ ft}$, Eq. (1) becomes

$$h_p = 30\text{ ft} + \Sigma h_L \qquad (2)$$

The head loss term can be expressed as

$$\Sigma h_L = \left[4(0.3) + 10 + 2 + 0.02\frac{(100\text{ ft})}{(\frac{8}{12}\text{ ft})}\right]\frac{V^2}{2(32.2\frac{ft}{s^2})}$$

with the minor loss coefficients obtained from Table 8.3. Also,

$$V = \frac{Q}{A} = \frac{Q\,(ft^3/s)}{(\frac{\pi}{4})(\frac{8}{12}\text{ ft})^2}$$

and Eq. (2) becomes

$$h_p = 30 + 2.06\,[Q(ft^3/s)]^2$$

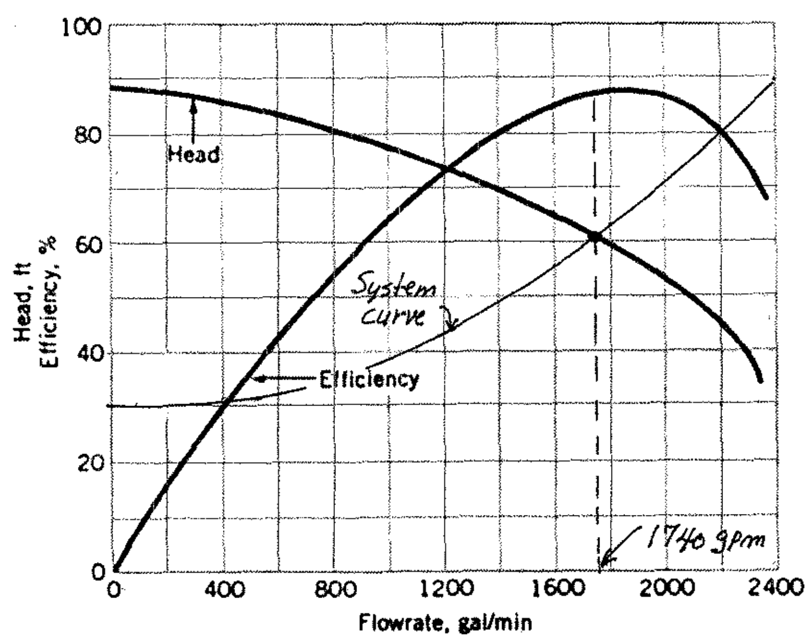
or the system equation can be written as

$$h_p = 30 + 1.02 \times 10^{-5}\left[Q\left(\frac{gal}{min}\right)\right]^2 \qquad (3)$$

The intersection of the system curve (Eq. 3) with the pump curve, as shown on the figure, indicates that

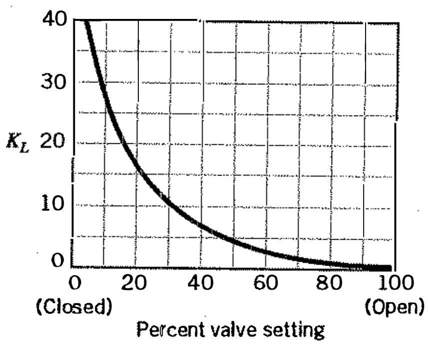
$$\underline{Q = 1740\ \frac{gal}{min}}$$

Since the efficiency at this flowrate is near peak efficiency, as shown on the figure, this pump would be satisfactory.

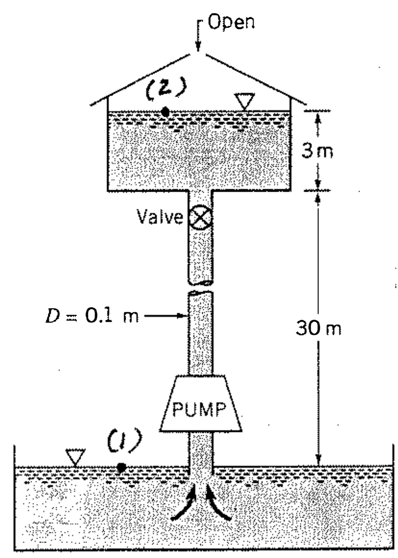


12.38

12.38 In a chemical processing plant a liquid is pumped from an open tank, through a 0.1-m-diameter vertical pipe, and into another open tank as shown in Fig. P12.38(a). A valve is located in the pipe, and the minor loss coefficient for the valve as a function of the valve setting is shown in Fig. P12.38(b). The pump head-capacity relationship is given by the equation $h_a = 52.0 - 1.01 \times 10^3 Q^2$ with h_a in meters when Q is in m³/s. Assume the friction



(b)



(a)

■ **FIGURE P12.38**

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \Sigma h_L \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_2 - z_1 = 33\,m$, Eq. (1) becomes

$$h_p = 33\,m + \Sigma h_L \qquad (2)$$

The head loss term can be expressed as

$$\Sigma h_L = \left(K_L + f\frac{\ell}{D}\right)\frac{V^2}{2g}$$

(a) With the valve open $K_L \approx 1.0$ (from Fig. P12.29 b) so that with $f = 0.02$, $\ell = 30\,m$, and $D = 0.1\,m$, Eq. (2) can be written as

$$h_p = 33\,m + \left[1.0 + 0.02\frac{(30\,m)}{(0.1\,m)}\right]\frac{V^2}{2(9.81\frac{m}{s^2})} \qquad (3)$$

and with

$$V = \frac{Q}{A} = \frac{Q(\frac{m^3}{s})}{(\frac{\pi}{4})(0.1\,m)^2}$$

Eq. (3) becomes

$$h_p = 33\,m + [1.0 + 6.0](826)\left[Q(m^3/s)\right]^2 \qquad (4)$$

or

$$h_p = 33 + 5.78 \times 10^3\left[Q(\tfrac{m^3}{s})\right]^2 \qquad (5)$$

(con't)

12.38 (con't)

Since the pump equation is

$$h_p = 52.0 - 1.01 \times 10^3 \left[Q\left(\frac{m^3}{s}\right) \right]^2 \qquad (6)$$

Eq. (5) and Eq. (6) can be equated to determine the flowrate. Thus,

$$33 + 5.78 \times 10^3 \, Q^2 = 52.0 - 1.01 \times 10^3 \, Q^2$$

and

$$Q = \underline{\underline{0.0529 \ \frac{m^3}{s}}}$$

(b) If the flowrate is to be cut in half so that $Q = 0.0529/2 = 0.0265 \ m^3/s$, the head added by the pump is

$$h_p = 52.0 - 1.01 \times 10^3 \left(0.0265 \frac{m^3}{s}\right)^2$$

$$= 50.6 \, m$$

From Eq. (4) with K_L unknown

$$50.6 \, m = 33 m + (K_L + 6.0)(826)\left(0.0265 \frac{m^3}{s}\right)^2$$

So that

$$K_L = 243$$

From Fig. 12.29 (b) the valve would be $\underline{\underline{13\% \text{ open}}}$ to obtain this K_L

12.41

12.41 What is the rationale for operating two geometrically similar pumps differing in feature size at the same flow coefficient?

If the pumps are similar in geometry and other important ways, operating both of them at a flow coefficient associated with high efficiency would make sense.

12.42

12.42 A centrifugal pump having an impeller diameter of 1 m is to be constructed so that it will supply a head rise of 200 m at a flowrate of 4.1 m^3/s of water when operating at a speed of 1200 rpm. To study the characteristics of this pump, a 1/5 scale, geometrically similar model operated at the same speed is to be tested in the laboratory. Determine the required model discharge and head rise. Assume both model and prototype operate with the same efficiency (and therefore the same flow coefficient).

For similarity the model pump must operate at the same flow coefficient, Eq. 12.32, so that

$$\left(\frac{Q}{\omega D^3}\right)_m = \left(\frac{Q}{\omega D^3}\right)_p$$

where the subscript (m) refers to the model and (p) to the prototype. Thus,

$$Q_m = \frac{\omega_m}{\omega_p}\left(\frac{D_m}{D_p}\right)^3 Q_p$$

and with $\omega_m = \omega_p$, $D_m/D_p = 1/5$, and $Q_p = 4.1\,m^3/s$, then

$$Q_m = (1)\left(\frac{1}{5}\right)^3\left(4.1\,\frac{m^3}{s}\right) = \underline{\underline{0.0328\,\frac{m^3}{s}}}$$

From Eq. 12.33

$$\left(\frac{g h_a}{\omega^2 D^2}\right)_m = \left(\frac{g h_a}{\omega^2 D^2}\right)_p$$

so that

$$h_{a_m} = \frac{g_p}{g_m}\left(\frac{\omega_m}{\omega_p}\right)^2\left(\frac{D_m}{D_p}\right)^2 h_{a_p}$$

and with $g_p = g_m$, $\omega_m = \omega_p$, $D_m/D_p = 1/5$, and $h_{a_p} = 200m$, then

$$h_{a_m} = (1)(1)^2\left(\frac{1}{5}\right)^2(200\,m) = \underline{\underline{8.00\,m}}$$

12.43

12.43 A centrifugal pump with a 12-in.-diameter impeller requires a power input of 60 hp when the flowrate is 3200 gpm against a 60-ft head. The impeller is changed to one with a 10-in. diameter. Determine the expected flowrate, head, and input power if the pump speed remains the same.

For geometrically similar pumps operating at the same speed the effect of a change in impeller diameter is given by Eqs. 12.39, 12.40, 12.41. Thus,

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \qquad \text{(Eq. 12.39)}$$

and with $Q_1 = 3200\text{ gpm}$, $D_1 = 12\text{ in.}$, and $D_2 = 10\text{ in.}$

$$Q_2 = \left(\frac{D_2}{D_1}\right)^3 Q_1 = \left(\frac{10\text{ in.}}{12\text{ in.}}\right)^3 (3200\text{ gpm}) = \underline{\underline{1850\text{ gpm}}}$$

From Eq. 12.40

$$\frac{h_{a_1}}{h_{a_2}} = \frac{D_1^2}{D_2^2} \qquad \text{(Eq. 12.40)}$$

so that with $h_{a_1} = 60\text{ ft}$

$$h_{a_2} = \left(\frac{D_2}{D_1}\right)^2 h_{a_1} = \left(\frac{10\text{ in.}}{12\text{ in.}}\right)^2 (60\text{ ft}) = \underline{\underline{41.7\text{ ft}}}$$

Similarly from Eq. 12.41

$$\frac{\dot{W}_{\text{shaft}_1}}{\dot{W}_{\text{shaft}_2}} = \frac{D_1^5}{D_2^5} \qquad \text{(Eq. 12.41)}$$

and with $\dot{W}_{\text{shaft}_1} = 60\text{ hp}$

$$\dot{W}_{\text{shaft}_2} = \left(\frac{D_2}{D_1}\right)^5 \dot{W}_{\text{shaft}_1} = \left(\frac{10\text{ in.}}{12\text{ in.}}\right)^5 (60\text{ hp}) = \underline{\underline{24.1\text{ hp}}}$$

12.44

12.44 Do the head-flowrate data shown in Fig. 12.12 appear to follow the similarity laws as expressed by Eqs. 12.39 and 12.40? Explain.

The data in Fig. 12.12 show the effect of changing impeller diameter on head-flowrate characteristics. According to the similarity laws expressed by Eq. 12.39 and Eq. 12.40

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \qquad (Eq. 12.39)$$

$$\frac{h_{a1}}{h_{a2}} = \frac{D_1^2}{D_2^2} \qquad (Eq. 12.40)$$

Thus, as the diameter is increased from 6 in. to 7 in. to 8 in. the flowrate increases according to Eq. 12.39 as

(from 6 in. to 7 in.) $\quad Q_2 = \left(\frac{D_2}{D_1}\right)^3 Q_1 = \left(\frac{7\text{ in.}}{6\text{ in.}}\right)^3 Q_1 = 1.59\, Q_1$

and

(from 6 in. to 8 in.) $\quad Q_2 = \left(\frac{8\text{ in.}}{6\text{ in.}}\right)^3 Q_1 = 2.37\, Q_1$

Similarly, from Eq. 12.40

(from 6 in. to 7 in.) $\quad h_{a_2} = \left(\frac{D_2}{D_1}\right)^2 h_{a_1} = \left(\frac{7\text{ in.}}{6\text{ in.}}\right)^2 h_{a_1} = 1.36\, h_{a_1}$

and

(from 6 in. to 8 in.) $\quad h_{a_2} = \left(\frac{8\text{ in.}}{6\text{ in.}}\right)^2 h_{a_1} = 1.78\, h_{a_1}$

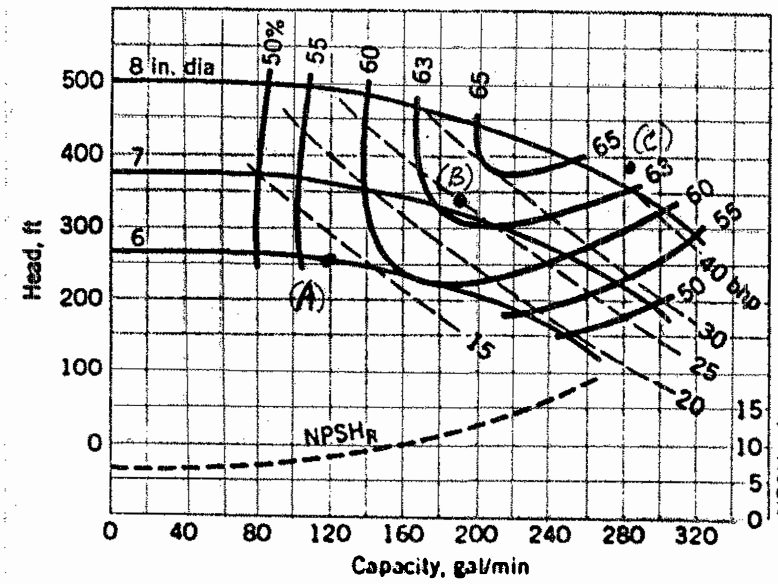
Thus, for any given point, such as (A) where $Q = 120$ gpm and $h_a = 250$ ft (see Fig. 12.12 on following page) for the 6-in. diameter impeller, the corresponding predicted point would be at (B) where

$$Q_2 = (1.59)(120\text{ gpm}) = 191\text{ gpm}$$

$$h_{a_2} = (1.36)(250\text{ ft}) = 340\text{ ft}$$

(Con't)

12.44 (Con't)



Similarly, for the 8-in. diameter impeller the predicted point, point (C), would be at

$$Q_2 = (2.37)(120\ gpm) = 284\ gpm$$

and

$$h_{a_2} = (1.78)(250\ ft) = 445\ ft$$

Points (B) and (C) fall near the corresponding curves in Fig. 12.12 thereby demonstrating that they do appear to follow the similarity laws. Yes.

Note that according to the similarity laws the 6-in. diameter curve is simply translated to the right and upward to obtain the corresponding head-flowrate curves for the 7-in. and 8-in. diameter pumps. It is clear from Fig. 12.12 that this is generally how the three curves are related.

12.45

12.45 A centrifugal pump has the performance characteristics of the pump with the 6-in.-diameter impeller described in Fig. 12.12. Note that the pump in this figure is operating at 3500 rpm. What is the expected head gained if the speed of this pump is reduced to 2800 rpm while operating at peak efficiency?

From Fig. 12.12 for the 6-in. diameter impeller operating at 3500 rpm, $Q = 170$ gpm and $h_a = 230$ ft when operating at peak efficiency (see figure below). Thus, if the pump is still operated at peak efficiency with the speed reduced to 2800 rpm then from Eq. 12.36

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \qquad \text{(Eq. 12.36)}$$

so that

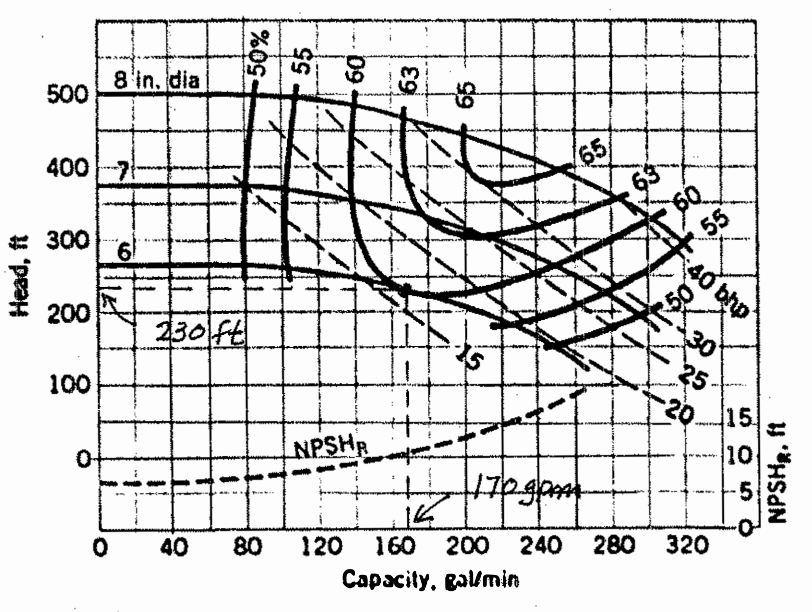
$$Q_2 = \frac{\omega_2}{\omega_1} Q_1 = \left(\frac{2800\ rpm}{3500\ rpm}\right)(170\ gpm) = \underline{\underline{136\ gpm}}$$

From Eq. 12.37

$$\frac{h_{a1}}{h_{a2}} = \frac{\omega_1^2}{\omega_2^2} \qquad \text{(Eq. 12.37)}$$

so that

$$h_{a_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 h_{a_1} = \left(\frac{2800\ rpm}{3500\ rpm}\right)^2 (230\ ft) = \underline{\underline{147\ ft}}$$



12.46

12.46 A centrifugal pump provides a flowrate of 500 gpm when operating at 1750 rpm against a 200-ft head. Determine the pump's flowrate and developed head if the pump speed is increased to 3500 rpm.

For a given pump the effect of a change in speed on Q and h_a is given by Eqs. 12.36 and 12.37. Thus,

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \qquad \text{(Eq. 12.36)}$$

and with $Q_1 = 500\ gpm$, $\omega_1 = 1750\ rpm$, and $\omega_2 = 3500\ rpm$, then

$$Q_2 = \frac{\omega_2}{\omega_1} Q_1 = \frac{(3500\ rpm)}{(1750\ rpm)}(500\ gpm)$$

$$= \underline{\underline{1000\ gpm}}$$

Similarly,

$$\frac{h_{a_1}}{h_{a_2}} = \frac{\omega_1^2}{\omega_2^2} \qquad \text{(Eq. 12.37)}$$

so that with $h_{a_1} = 200\ ft$

$$h_{a_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 h_{a_1} = \left(\frac{3500\ rpm}{1750\ rpm}\right)^2 (200\ ft)$$

$$= \underline{\underline{800\ ft}}$$

12.47

12.47 Explain how Fig. 12.18 was constructed from test data. Why is this use of specific speed important? Illustrate with a specific example.

A variety of pump configurations like the ones shown in Fig. 12.18 were tested over a range of flow rates. Performance data like those shown in Fig. 12.17 were acquired. For each pump configuration, the operation at maximum efficiency was noted and the specific speed, N_s, (Eq. 12. 43) was calculated for that condition of flow. These specific speed values calculated at maximum efficiency operation were then used to distribute the different pump configurations as shown in Fig. 12.18.

Specific speed is important because from desired design operational data (ω, Q, and h_a) a specific speed value can be determined. With that value of specific speed and Fig. 12.18 the designer can decide what kind of pump configuration to use for maximum efficiency operation. For example, at lower values of specific speed, a centrifugal pump is generally best. At higher values of specific speed, an axial-flow pump may be best. In between values of specific speed may suggest that a mixed-flow pump would serve most efficiently.

12.48

12.48 Use the data given in Problem 12.25 and plot the dimensionless coefficients C_H, $C_{\mathcal{P}}$, η versus C_Q for this pump. Calculate a meaningful value of specific speed, discuss its usefulness, and compare the result with data of Fig. 12.18.

From Problem 12.25 the following data were obtained:

Q (gpm)	20	40	60	80	100	120	140
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
η (%)	29.7	41.2	49.9	57.5	61.3	60.4	52.6
Power input (hp)	1.58	2.27	2.67	2.95	3.19	3.49	4.00

For $\omega = \left(1750\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{1}{60\frac{s}{min}}\right) = 183.3\,\frac{rad}{s}$ and $D = \frac{9}{12}$ ft, it follows that

$$C_Q = \frac{Q}{\omega D^3} = \frac{Q(gpm)/\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)}{\left(183.3\,\frac{rad}{s}\right)\left(\frac{9}{12}\,ft\right)^3}$$

$$= 2.88 \times 10^{-5}\, Q\,(gpm)$$

$$C_H = \frac{g h_a}{\omega^2 D^2} = \frac{\left(32.2\,\frac{ft}{s^2}\right) h_a(ft)}{\left(183.3\,\frac{rad}{s}\right)^2\left(\frac{9}{12}\,ft\right)^2}$$

$$= 1.70 \times 10^{-3}\, h_a(ft)$$

$$C_{\mathcal{P}} = \frac{\dot{W}_{shaft}}{\rho\omega^3 D^5} = \frac{\dot{W}_{shaft}(hp)\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(183.3\,\frac{rad}{s}\right)^3\left(\frac{9}{12}\,ft\right)^5}$$

$$= 1.94 \times 10^{-4}\, \dot{W}_{shaft}\,(hp)$$

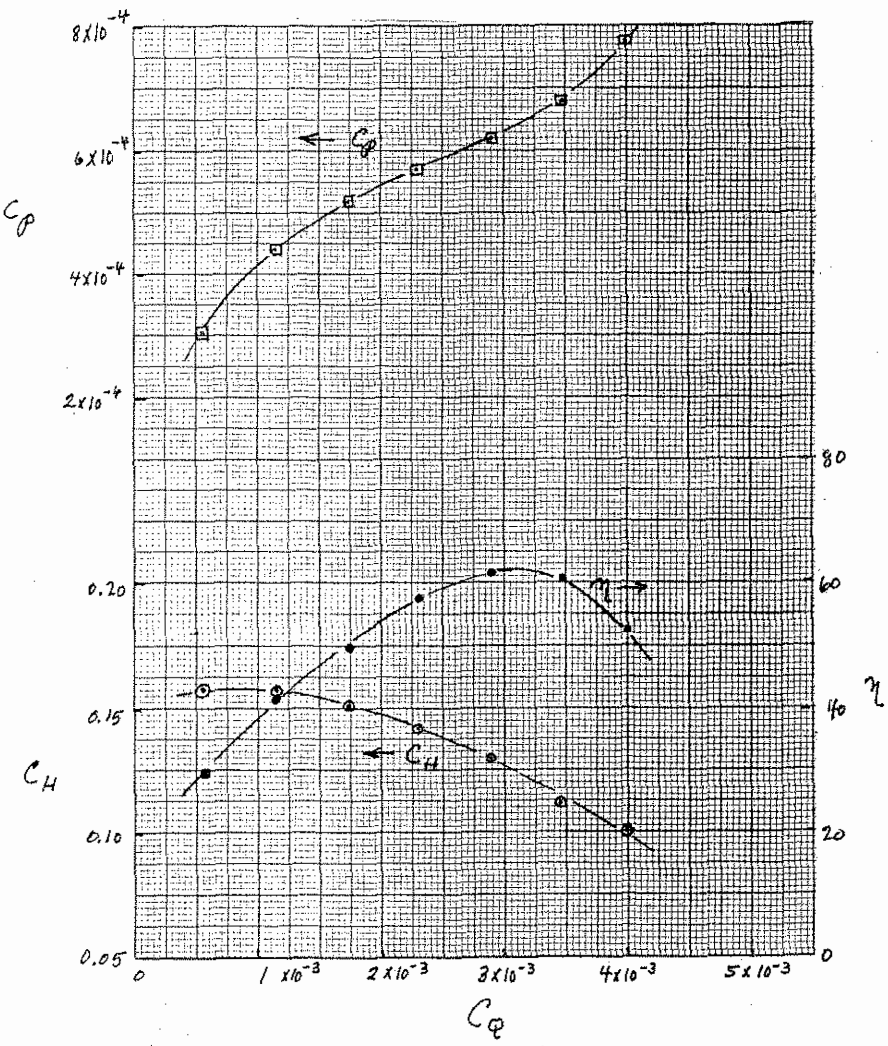
Based on the data above:

Q (gpm)	20	40	60	80	100	120	140
C_Q	5.76×10^{-4}	1.15×10^{-3}	1.73×10^{-3}	2.30×10^{-3}	2.88×10^{-3}	3.46×10^{-3}	4.03×10^{-3}
C_H	0.1581	0.1576	0.1498	0.1423	0.1317	0.1184	0.104
$C_{\mathcal{P}}$	3.07×10^{-4}	4.40×10^{-4}	5.18×10^{-4}	5.72×10^{-4}	6.19×10^{-4}	6.77×10^{-4}	7.76×10^{-4}
η	29.7	41.2	49.9	57.7	61.3	60.4	52.6

(con't)

12.48 (con't)

The plot of C_H, C_P, η versus C_Q is shown below.



$$N_{sd} = \frac{\omega(rpm)\sqrt{Q(gpm)}}{[h_a(ft)]^{3/4}}$$

so for $Q = 100$ gpm at $\eta_{max} = 61.3\%$

$$N_{sd} = \frac{(1750\ rpm)\sqrt{(100\ gpm)}}{[(77.3\ ft)]^{3/4}} = 671$$

which is within the range of N_{sd} values for radial flow pumps in Fig. 12.18

12.49

12.49 In a certain application a pump is required to deliver 5000 gpm against a 300-ft head when operating at 1200 rpm. What type of pump would you recommend?

For $Q = 5000$ gpm, $h_a = 300$ ft, and $\omega = 1200$ rpm, the specific speed is

$$N_{sd} = \frac{\omega\,(\text{rpm})\sqrt{Q\,(\text{gpm})}}{\left[h_a\,(\text{ft})\right]^{3/4}}$$

$$= \frac{(1200\text{ rpm})\sqrt{5000\text{ gpm}}}{(300\text{ ft})^{3/4}}$$

$$= \underline{\underline{1180}}$$

From Fig. 12.18, at this specific speed a radial flow pump (<u>centrifugal pump</u>) would be recommended.

12.53

12.53 A certain axial-flow pump has a specific speed of $N_S = 5.0$. If the pump is expected to deliver 3000 gpm when operating against a 15-ft head, at what speed (rpm) should the pump be run?

Since

$$N_s = \frac{\omega\,(rad/s)\sqrt{Q\,(ft^3/s)}}{\left[g\,(ft/s^2)\; h_a\,(ft)\right]^{3/4}}$$

for $N_s = 5.0$, $g = 32.2\ ft/s^2$, $h_a = 15\ ft$, and with

$$Q = \frac{3000\ \frac{gal}{min}}{\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)} = 6.68\ \frac{ft^3}{s}$$

it follows that

$$\omega\,(rad/s) = \frac{(5.0)\left[\left(32.2\ \frac{ft}{s^2}\right)(15\ ft)\right]^{3/4}}{\sqrt{6.68\ \frac{ft^3}{s}}}$$

$$= 199\ \frac{rad}{s}$$

Hence

$$\omega\,(rpm) = \frac{\left(199\ \frac{rad}{s}\right)\left(60\ \frac{s}{min}\right)}{2\pi\ \frac{rad}{rev}}$$

$$= \underline{\underline{1900\ rpm}}$$

12.54

12.54 A certain pump is known to have a capacity of 3 m³/s when operating at a speed of 60 rad/s against a head of 20 m. Based on the information in Fig. 12.18, would you recommend a radial-flow, mixed-flow, or axial-flow pump?

Since

$$N_s = \frac{\omega\,(\text{rad/s})\sqrt{Q\,(\text{m}^3/\text{s})}}{\left[g\,(\text{m/s}^2)\ h_a\,(\text{m})\right]^{3/4}}$$

for $\omega = 60\ \text{rad/s}$, $Q = 3\ \text{m}^3/\text{s}$, $g = 9.81\ \text{m/s}^2$, and $h_a = 20\text{m}$

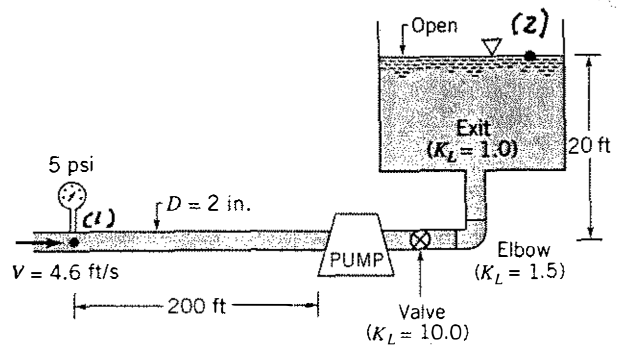
$$N_s = \frac{(60\ \text{rad/s})\sqrt{3\ \text{m}^3/\text{s}}}{\left[(9.81\ \text{m/s}^2)(20\ \text{m})\right]^{3/4}}$$

$$= \underline{\underline{1.98}}$$

From Fig. 12.18 with $N_s = 1.98$ the pump is a <u>mixed-flow pump.</u>

12.55

12.55 Fuel oil (sp. wt = 48.0 lb/ft³, viscosity = 2.0 × 10^{-5} lb·s/ft²) is pumped through the piping system of Fig. P12. with a velocity of 4.6 ft/s. The pressure 200 ft upstream from the pump is 5 psi. Pipe losses downstream from the pump are negligible, but minor losses are not (minor loss coefficients are given on the figure). (a) For a pipe diameter of 2 in. with a relative roughness $\varepsilon/D = 0.001$, determine the head that must be added by the pump. (b) For a pump operating speed of 1750 rpm, what type of pump (radial-flow, mixed-flow, or axial-flow) would you recommend for this application?



■ FIGURE P12.55

(a) $$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \qquad (1)$$

With $\gamma = 48.0\ lb/ft^3$, $p_1 = 5\,psi$, $p_2 = 0$, $V_1 = 4.6\ ft/s$, $V_2 = 0$, and $z_2 - z_1 = 20\ ft$, Eq. (1) becomes

$$\frac{(5\frac{lb}{in^2})(144\frac{in^2}{ft^2})}{48.0\frac{lb}{ft^3}} + \frac{(4.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + h_p = 20\,ft + \sum h_L \qquad (2)$$

The head loss term can be expressed as

$$\sum h_L = \Big[\underbrace{10.0}_{valve} + \underbrace{1.5}_{elbow} + \underbrace{1.0}_{exit} + f\,\frac{200\,ft}{\frac{2}{12}\,ft}\Big]\frac{(4.6\frac{ft}{s})^2}{(2)(32.2\frac{ft}{s^2})}$$

The Reynolds number is

$$Re = \frac{\rho V D}{\mu} = \frac{\left(\frac{48.0\frac{lb}{ft^3}}{32.2\frac{ft}{s^2}}\right)(4.6\frac{ft}{s})(\frac{2}{12}\,ft)}{2.0\times10^{-5}\ \frac{lb\cdot s}{ft^2}} = 5.71\times10^4$$

and with $\varepsilon/D = 0.001$ $\quad f = 0.024$ (from Fig. 8.23).

Thus, $h_L = 13.6\ ft$ and from Eq. (2)

$$\underline{\underline{h_p = 18.3\ ft}}$$

(b) Since

$$Q = VA = (4.6\frac{ft}{s})(\frac{\pi}{4})(\frac{2}{12}\,ft)^2 = 0.100\ \frac{ft^3}{s}$$

or

$$Q = (0.100\frac{ft^3}{s})(7.48\frac{gal}{ft^3})(60\frac{s}{min}) = 45.0\ gpm$$

the specific speed at 1750 rpm is

$$N_{sd} = \frac{\omega\,(rpm)\sqrt{Q\,(gpm)}}{[h_a\,(ft)]^{3/4}} = \frac{(1750\,rpm)\sqrt{45.0\,gpm}}{[18.3\,ft]^{3/4}} = 1330$$

For this specific speed a <u>radial-flow pump</u> would be recommended for this application (see Fig. 12.18).

Master Sys...
10% Rec...
8 1/2 x ...

12.56

12.56 The axial-flow pump shown in Fig. 12.19 is designed to move 5000 gal/min of water over a head rise of 5 ft of water. Estimate the motor power requirement and the $U_2V_{\theta 2}$ needed to achieve this flowrate on a continuous basis. Comment on any cautions associated with where the pump is placed vertically in the pipe.

From Eq. 12.21 we get the power equivalent to the head rise and flowrate involved. This is the minimum power required to achieve the performance specified.

$$P = \gamma Q h_a$$

$$P = \left(62.4\,\frac{lb}{ft^3}\right)\left(5000\,\frac{gal}{min}\right)\frac{1}{\left(7.48\,\frac{gal}{ft^3}\right)}\frac{1}{\left(60\,\frac{s}{min}\right)}(4\,ft)\frac{1}{\left(550\,\frac{ft.lb}{hp.s}\right)}$$

$$P = 5.1\ hp$$

To estimate the shaft or motor power requirement, we need to assume the efficiency of the conversion of shaft or motor power into the pump performance specified.

$$P_{shaft} = \frac{P}{\eta} \quad \text{or for } 80\% \text{ efficiency}$$

$$P_{shaft} = \frac{5.1\ hp}{0.8} = \underline{\underline{6.4\ hp}}$$

$U_2 V_{\theta 2}$ and P_{shaft} are related in Eq. 12.4 through

$$P_{shaft} = \dot{m}\, U_2 V_{\theta 2} = \rho A V U_2 V_{\theta 2} = \rho Q U_2 V_{\theta 2}$$

so

$$U_2 V_{\theta 2} = \frac{P_{shaft}}{\rho Q} = \frac{(6.4\,hp)\left(32.2\,\frac{lbm.s^2}{lbf.ft}\right)\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)\left(550\,\frac{ft.lb}{hp.s}\right)}{\left(62.4\,\frac{lbm}{ft^3}\right)\left(5000\,\frac{gal}{min}\right)}$$

$$U_2 V_{\theta 2} = \underline{\underline{163}}\ \frac{ft^2}{s^2}$$

(con't)

12.56 (con't)

The main caution in placing the pump vertically in the intake pipe is to do so in a way to avoid cavitation in the pump. The collapse of cavitation bubbles in the pump can erode pump blade and other wetted surfaces. Applying the energy equation, Eq. 5.84, between the free surface (1) and the pump entrance (2) we get

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L$$

So

$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_2 - \frac{V_2^2}{2g} - h_L$$

and to maximize $\frac{p_2}{\gamma}$, we minimize $z_1 - z_2$. To achieve this we place the pump high vertically in the intake pipe. This will tend to keep p_2 high enough to avoid cavitation which occurs when p_2 and/or related pressures in the pump become less than the vapor pressure of the fluid.

Master Typ...
10% Red...
8 1/2 x ...

12.61

12.61 Consider the Pelton wheel turbine illustrated in Figs. 12.24, 12.25, 12.26, and 12.27. This kind of turbine is used to drive the oscillating sprinkler shown in Video V12.3 Explain how this kind of sprinkler is started, and subsequently operated at constant oscillating speed. What is the physical significance of the zero torque condition with the Pelton wheel rotating?

As shown on page 795 below Eq. 12.50

$$T_{shaft} = \dot{m} r_m (U - V_1)(1 - \cos\beta)$$

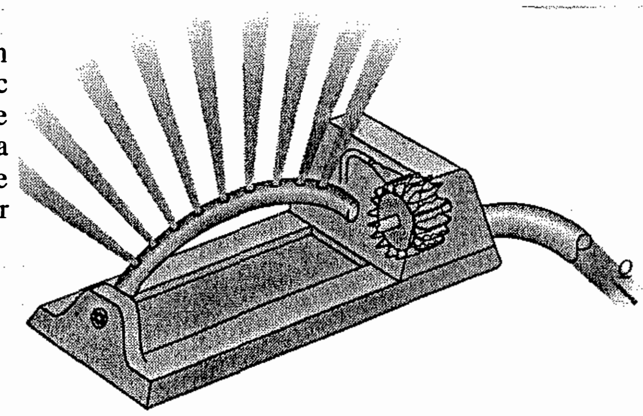
So for no rotation of the wheel or $U = 0$, the variation of T_{shaft} with changing $\dot{m}$ is linear. When T_{shaft} is just larger than the resisting torque provided by the sprinkler, the Pelton wheel rotates and drives the oscillation of the sprinkler. After wheel rotation and sprinkler oscillation begins, any constant value of $\dot{m}$ and T_{shaft} results in a constant value of U and thus rotation speed and also oscillation period.

If the shaft connecting the oscillating sprinkler to the Pelton wheel breaks during operation, the sprinkler will cease oscillating and the Pelton wheel will run at constant rotation speed corresponding to $U = V_1$.

Master Typing ...
10% Reduction
8 1/2 x 11 ...

12.62

12.62 A small Pelton wheel is used to power an oscillating lawn sprinkler as shown in **Video V12.3** and Fig. P12.62. The arithmetic mean radius of the turbine is 1 in., and the exit angle of the blade is 135° relative to the blade motion. Water is supplied through a single 0.20-in.-diameter nozzle at a speed of 50 ft/s. Determine the flowrate, the maximum torque developed, and the maximum power developed by this turbine.



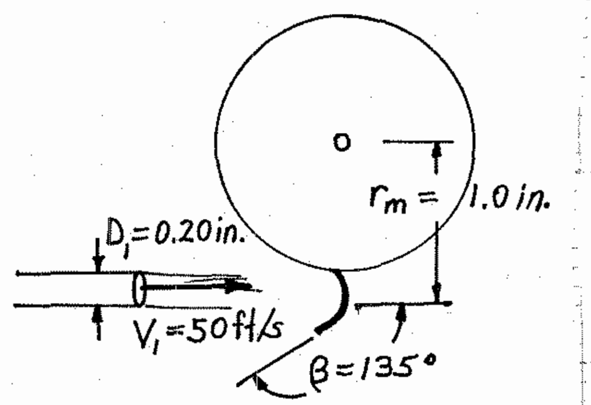
■ FIGURE P12.62

For the Pelton wheel shown

$$Q = A_1 V_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4}\left(\frac{0.20}{12}\,ft\right)^2 \left(50\,\frac{ft}{s}\right)$$

or

$$Q = \underline{\underline{0.0109\,\frac{ft^3}{s}}}$$



From Fig. 11.22

$$T_{shaft_{max}} = \dot{m}\, r_m V_1 (1 - \cos\beta)$$

and

$$\dot{W}_{shaft_{max}} = 0.25\, \dot{m} V_1^2 (1 - \cos\beta)$$

where $\dot{m} = \rho Q = 1.94\,\frac{slugs}{ft^3}\left(0.0109\,\frac{ft^3}{s}\right) = 0.0211\,\frac{slugs}{s}$

Thus,

$$T_{shaft_{max}} = 0.0211\,\frac{slugs}{s}\left(\frac{1}{12}\,ft\right)\left(50\,\frac{ft}{s}\right)(1 - \cos 135°) = 0.150\,\frac{slugs \cdot ft^2}{s^2}$$

$$= \underline{\underline{0.150\;ft \cdot lb}}$$

and

$$\dot{W}_{shaft_{max}} = 0.25\left(0.0211\,\frac{slugs}{s}\right)\left(50\,\frac{ft}{s}\right)^2(1 - \cos 135°) = 22.5\,\frac{slugs \cdot ft^2}{s^3}$$

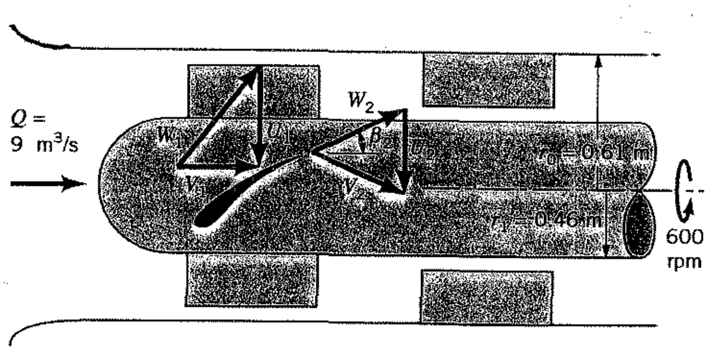
$$= 22.5\,\frac{ft \cdot lb}{s}$$

or

$$\dot{W}_{shaft_{max}} = 22.5\,\frac{ft \cdot lb}{s}\;\frac{1\,hp}{550\,\frac{ft \cdot lb}{s}} = \underline{\underline{0.0409\,hp}}$$

12.63

12.63 The single-stage, axial-flow turbomachine shown in Fig. P12.63 involves water flow at a volumetric flowrate of 9 m^3/s. The rotor revolves at 600 rpm. The inner and outer radii of the annular flow path through the stage are 0.46 and 0.61 m, and $\beta_2 = 60°$. The flow entering the rotor row and leaving the stator row is axial when viewed from the stationary casing. Is this device a turbine or a pump? Estimate the amount of power transferred to or from the fluid.



■ **FIGURE P12.63**

$$\dot{W}_{shaft} = \dot{m}\,(U_2 V_{\theta 2} - U_1 V_{\theta 1}) \quad \text{where } V_{\theta 1} = 0 \tag{1}$$

and

$$U_2 = \omega r_{mean} = \omega \frac{(r_i + r_o)}{2} \text{. Thus, with } \omega = \left(600\,\frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{rev}\right) = 62.8\,\frac{rad}{s}$$

this gives

$$U_2 = \left(62.8\,\frac{rad}{s}\right)\left(\frac{0.46\,m + 0.61\,m}{2}\right) = 33.6\,\frac{m}{s}$$

Also,

$$\dot{m} = \rho Q = 999\,\frac{kg}{m^3}\left(9\,\frac{m^3}{s}\right) = 8991\,\frac{kg}{s}$$

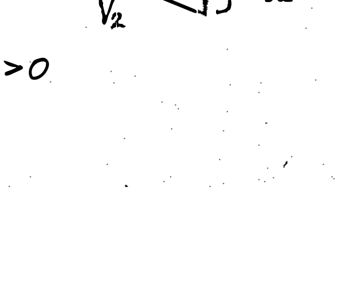
But $Q = W_2 \cos 60°\, A_2$ or since $A_2 = \pi(r_o^2 - r_i^2)$

$$W_2 = \frac{9\,\frac{m^3}{s}}{\pi \cos 60° (0.61^2 - 0.46^2)\,m^2} = 35.7\,\frac{m}{s}$$

Hence, from the velocity triangle,

$$V_{\theta 2} = -W_2 \sin 60° + U_2$$

$$= -35.7 \sin 60° + 33.6 = 2.70\,\frac{m}{s}$$



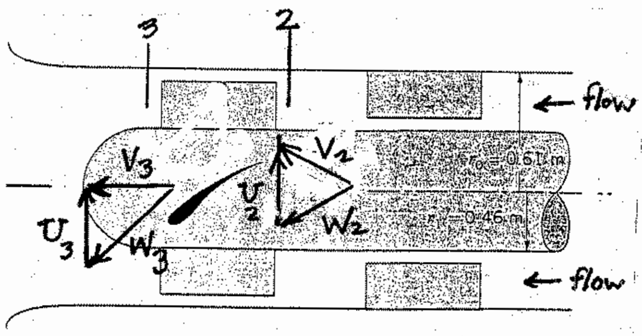
From Eq. (1):

$$\dot{W}_{shaft} = \left(8991\,\frac{kg}{s}\right)\left(33.6\,\frac{m}{s}\right)\left(2.70\,\frac{m}{s}\right) = 8.16 \times 10^5 > 0$$

The device is an <u>816 kW pump</u>

12.64

12.64 Describes what will happen when the flow through the turbomachine of Fig. P12.63 is in the opposite direction (right to left) and the shaft is freed up to rotate in response to the reversed flow.



When flow is reversed as shown in the sketch above, V_2, the velocity of the flow out of the stationary blade row (now a nozzle) will leave at approximately the blade exit angle. The magnitude of V_2 will depend on the magnitude of the flowrate Q. From the velocity triangles sketched above we conclude that the rotor will now move in a direction opposite to the one of problem 12.44. The rotor speed will depend on values of Q and the restraining shaft torque, T. From the velocity triangles we also conclude that the fluid forces on the moving blade sections are in the same direction as blade motion so the fluid is doing work on the rotor. The device is now acting as a turbine. $\dot{W}_{shaft} = \dot{m}(U_3 V_{\theta 3} - U_2 V_{\theta 2})$ may be used to determine shaft power.

12.65

12.65 For an air turbine of a dentist's drill like the one shown in Fig. E12.8 and **Video V12.4** calculate the average blade speed associated with a rotational speed of 350,000 rpm. Estimate the air pressure needed to run this turbine.

We calculate the average blade speed, U, from

$$U = r_m \omega = \left(\frac{r_i + r_o}{2}\right)\omega = \frac{(0.133 + 0.168)\,\text{in}\left(350{,}000\,\frac{\text{rev}}{\text{min}}\right)\left(2\pi\,\frac{\text{rad}}{\text{s}}\right)}{(2)\left(12\,\frac{\text{in}}{\text{ft}}\right)}\frac{1}{\left(60\,\frac{\text{s}}{\text{min}}\right)}$$

$$U = 459\,\frac{\text{ft}}{\text{s}}$$

To estimate the air pressure, P_0, needed to run this turbine, we estimate that the nozzle exit velocity is about twice as large as the average blade velocity or

$$V = 2U = 918\ \text{ft/s}$$

So, the corresponding Mach number, M, is approximately

$$M = \frac{V}{c} = \frac{918\ \text{ft/s}}{1100\ \text{ft/s}}$$ with c estimated to be about $1100\,\frac{\text{ft}}{\text{s}}$

$$M = 0.83$$

Then from Fig. D.1 the value of $\frac{P}{P_0}$ corresponding to $M = 0.83$ is

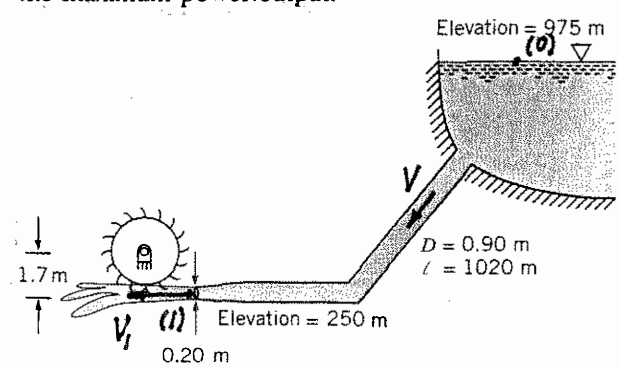
$$\frac{P}{P_0} = 0.1 \quad \text{and} \quad P_0 = \frac{P}{0.1} = 10P$$

So

$$P_0 \approx 10(14.7\,\text{psia}) = \underline{\underline{147\,\text{psia}}}$$

12.66

12.66 Water for a Pelton wheel turbine flows from the headwater and through the penstock as shown in Fig. P12.66. The effective friction factor for the penstock, control valves, and the like is 0.032 and the diameter of the jet is 0.20 m. Determine the maximum power output.



■ FIGURE P12.66

$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta)$ or for maximum power $\beta = 180°$, $U = \frac{V_1}{2}$

Thus,

$$\dot{W}_{shaft\,max} = -\rho Q \frac{V_1^2}{2} \qquad (1)$$

But $\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + f\frac{\ell}{D}\frac{V^2}{2g}$ where $p_0 = p_1 = 0$, $z_0 = 975\,m$, $z_1 = 250\,m$, and $V_0 = 0$

Hence,

$$z_0 = z_1 + \frac{V_1^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{where } A_1 V_1 = AV \qquad (2)$$

or $\frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} D^2 V$. That is $V = \left(\frac{d_1}{D}\right)^2 V_1 = \left(\frac{0.2\,m}{0.9\,m}\right)^2 V_1 = 0.0494\,V_1$

so that Eq. (2) becomes:

$$975\,m = 250\,m + \frac{V_1^2}{2(9.81\,\frac{m}{s^2})}\left[1 + 0.032\left(\frac{1020\,m}{0.9\,m}\right)(0.0494)^2\right] \quad \text{where } V_1 \sim \frac{m}{s}$$

or $V_1 = 114.3\,\frac{m}{s}$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.2\,m)^2 (114.3\,\tfrac{m}{s}) = 3.56\,\frac{m^3}{s}$$

Therefore, from Eq. (1):

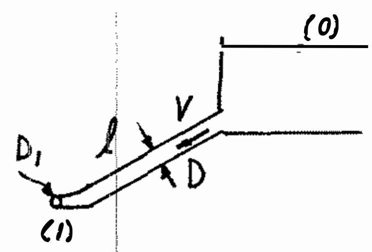
$$\dot{W}_{shaft\,max} = -(999\,\tfrac{kg}{m^3})(3.56\,\tfrac{m^3}{s})\frac{(114.3\,\frac{m}{s})^2}{2} = 23.2 \times 10^6\,\frac{N\cdot m}{s} = \underline{\underline{23{,}200\,kW}}$$

12.67

12.67 Water to run a Pelton wheel is supplied by a penstock of length ℓ and diameter D with a friction factor f. If the only losses associated with the flow in the penstock are due to pipe friction, shown that the maximum power output of the turbine occurs when the nozzle diameter, D_1, is given by $D_1 = D/(2f\ell/D)^{1/4}$.

$\dot{W}_{shaft} = \rho Q U(U-V_1)(1-\cos\beta)$ so the maximum power output occurs with $\beta = 180°$ and $U = \frac{V_1}{2}$. Thus,

$$\dot{W}_{shaft} = \rho Q \frac{V_1^2}{2} \quad \text{where} \qquad (1)$$



$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + f\frac{\ell}{D}\frac{V^2}{2g}$$

But $p_0 = p_1 = 0$, $V_0 = 0$, and $z_0 - z_1 = h$. Thus,

$h = \frac{V_1^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g}$ where since $A_1V_1 = AV$ or $\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D^2V$ we have

$$V_1 = \left(\frac{D}{D_1}\right)^2 V$$

Therefore, $h = \frac{V_1^2}{2g}\left[1 + f\frac{\ell}{D}\frac{D_1^4}{D^4}\right]$ or $\frac{V_1^2}{2g} = \frac{h}{\left[1 + f\ell\frac{D_1^4}{D^5}\right]}$ and Eq. (1) gives

$$\dot{W}_{shaft} = \frac{\rho Q h}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)} = \frac{\rho\frac{\pi}{4}D_1^2V_1 h}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)}, \text{ but } V_1 = \frac{\sqrt{2gh}}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)^{\frac{1}{2}}} \qquad (2), (3)$$

For this problem f, ℓ, D and h are constants; D_1 is variable.
Thus, from Eqs. (2) and (3):

$$\dot{W}_{shaft} = \frac{KD_1^2}{(1 + cD_1^4)^{3/2}} \quad \text{where } K = \text{const., and } c = \text{const.} = f\frac{\ell}{D^5}$$

Note: $\dot{W}_{shaft} \to 0$ as $D_1 \to 0$ and as $D_1 \to \infty$. To find the D_1 that gives maximum power over all, set $\frac{d\dot{W}_{shaft}}{dD_1} = 0$

$$\frac{d\dot{W}_{shaft}}{dD} = \frac{2KD_1}{(1+cD_1^4)^{3/2}} + \frac{(-\frac{3}{2})KD_1^2}{(1+cD_1^4)^{5/2}}(c)4D_1^3 = 0$$

or $\frac{2KD_1}{(1+cD_1^4)^{3/2}}\left[1 - \frac{3cD_1^4}{(1+cD_1^4)}\right] = 0$, or $1 + cD_1^4 = 3cD_1^4$, or $D_1^4 = \frac{1}{2c}$

Thus, $D_1 = \frac{1}{\left(2f\frac{\ell}{D^5}\right)^{1/4}} = \underline{\underline{\frac{D}{\left(2f\frac{\ell}{D}\right)^{1/4}}}}$

12.68

12.68 A hydraulic turbine operating at 180 rpm with a head of 100 feet develops 20,000 horsepower. Estimate the power if the same turbine were to operate under a head of 50 ft.

Since hydraulic turbine flow is incompressible, we use the dimensionless parameters developed for hydraulic pumps, namely, flow, head and power coefficients. For this situation we specify operation at the same efficiency and thus flow coefficient with one half the head. Thus, head coefficient remains constant and

$$\left.\frac{g h_T}{\omega^2 D^2}\right)_1 = \left.\frac{g h_T}{\omega^2 D^2}\right)_2$$

so with $D_1 = D_2$ and $g_1 = g_2$:

$$\frac{100}{(180)^2} = \frac{50}{\omega_2^2} \quad \text{or} \quad \omega_2 = \underline{\underline{127 \text{ rpm}}}$$

Also power coefficient is the same so

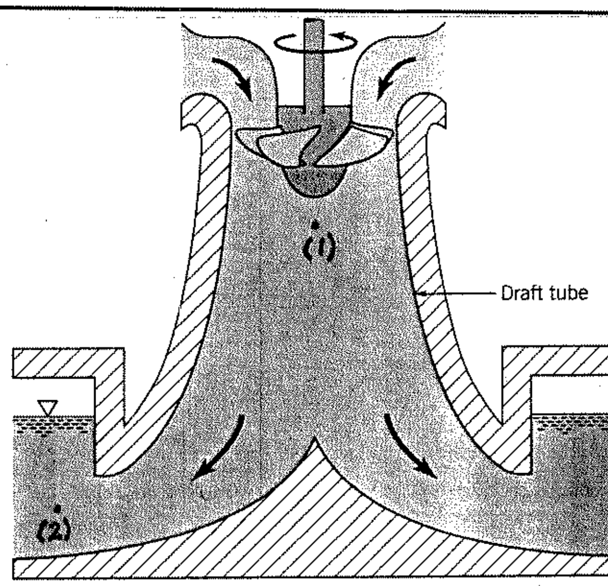
$$\left.\frac{\dot{W}_{shaft}}{\rho \omega^3 D^5}\right)_1 = \left.\frac{\dot{W}_{shaft}}{\rho \omega^3 D^5}\right)_2$$

so with $D_1 = D_2$ and $\rho_1 = \rho_2$:

$$\frac{20{,}000}{(100)^3} = \frac{\dot{W}_{shaft2}}{(127)^3} \quad \text{or} \quad \dot{W}_{shaft2} = \underline{\underline{41{,}000 \text{ hp}}}$$

12.69

12.69 Draft tubes as shown in Fig. P12.69 are often installed at the exit of Kaplan and Francis turbines. Explain why such draft tubes are advantageous.



■ FIGURE P12.69

Without the draft tube there would be a relatively high speed exit jet (speed V_1, pressure $p_1 = 0$). With the draft tube (which acts as a diffuser) the exit speed is much smaller ($V_2 \approx 0$, $p_2 \approx 0$). From Bernoulli equation it follows that $p_1 < 0$ (with the draft tube). Hence there is a larger head available to the turbine. More energy can be removed from the fluid.

12.70

12.70 Turbines are to be designed to develop 30,000 horsepower while operating under a head of 70 ft and an angular velocity of 60 rpm. What type of turbines is best suited for this purpose? Estimate the flowrate needed.

$\dot{W}_{shaft} = 30{,}000\ hp$; $h_T = 70 ft$; and $\omega = 60\,rpm$ so that

$$N'_{sd} = \frac{\omega\sqrt{\dot{W}_{shaft}}}{(h_T)^{5/4}} = \frac{60\sqrt{3\times10^4}}{(70)^{5/4}} = 51.3$$

For this value a Francis turbine would be appropiate.

Also, since $\dot{W}_{shaft} = \gamma Q h_T$ it follows that

$$Q = \frac{\dot{W}_{shaft}}{\gamma h_T} = \frac{(30{,}000\ hp)(550\ \frac{ft\,lb}{s}/hp)}{(62.4\ \frac{lb}{ft^3})(70 ft)} = \underline{\underline{378\ \frac{ft^3}{s}}}$$

12.71

12.71 Show how you would estimate the relationship between feature size and power production for a wind turbine like the one shown in **Video V12.1.**

To estimate the relationship between feature size and power production for a wind turbine we use the dimensionless pi terms of Eqs. 12.29 and 12.30 which are applicable for this incompressible flow. For similar turbines and operating conditions

$$\frac{\dot{W}_{shaft\,1}}{\rho_1 \omega_1^3 D_1^5} = \frac{\dot{W}_{shaft\,2}}{\rho_2 \omega_2^3 D_2^5}$$

and

$$\frac{g h_{a1}}{\omega_1^2 D_1^2} = \frac{g h_{a2}}{\omega_2^2 D_2^2}$$

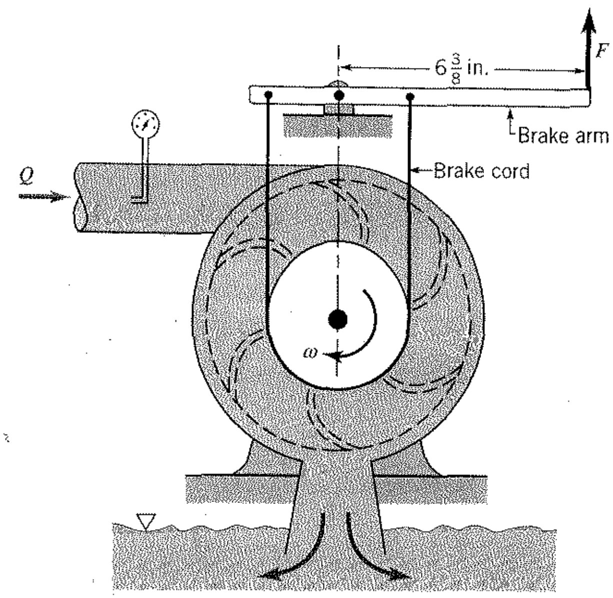
Since $\rho_1 = \rho_2$ and $h_{a1} = h_{a2}$, we combine and get

$$\frac{\dot{W}_{shaft\,1}}{\dot{W}_{shaft\,2}} = \frac{D_1^2}{D_2^2}$$

Or power varies with feature size squared.

12.72

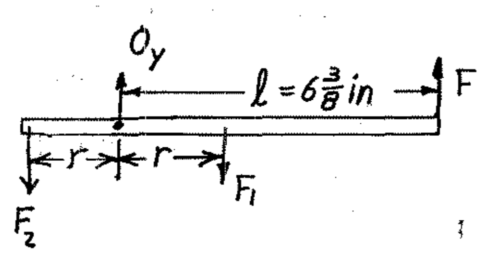
12.72 Test data for the small Francis turbine shown in Fig. P12.72 is given in the table below. The test was run at a constant 32.8 ft head just upstream of the turbine. The Prony brake on the turbine output shaft was adjusted to give various angular velocities, and the force on the brake arm, F, was recorded. Use the given data to plot curves of torque as a function of angular velocity and turbine efficiency as a function of angular velocity.



ω (rpm)	Q (ft^3/s)	F (lb)
0	0.129	2.63
1000	0.129	2.40
1500	0.129	2.22
1870	0.124	1.91
2170	0.118	1.49
2350	0.0942	0.876
2580	0.0766	0.337
2710	0.068	0.089

■ FIGURE P12.72

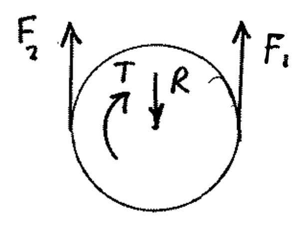
Since $\Sigma M_0 = 0$ for the brake arm it follows that $F\ell = F_1 r - F_2 r$



Also, the torque on the turbine is $T = F_1 r - F_2 r$

or
$$T = F\ell = \left(\frac{6\frac{3}{8}}{12}\,\text{ft}\right)F = 0.531\,F \text{ ft·lb} \quad \text{where } F \sim \text{lb} \qquad (1)$$

Also, $$\eta = \frac{T\omega}{\gamma Q h_T} \quad \text{where } h_T = 32.8 \text{ ft}$$



Thus,
$$\eta = \frac{(T \text{ ft·lb})(\omega \frac{\text{rev}}{\text{min}})\left(\frac{1 \text{min}}{60 \text{s}}\right)\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)}{(62.4 \frac{\text{lb}}{\text{ft}^3})(Q \frac{\text{ft}^3}{\text{s}})(32.8 \text{ ft})}$$

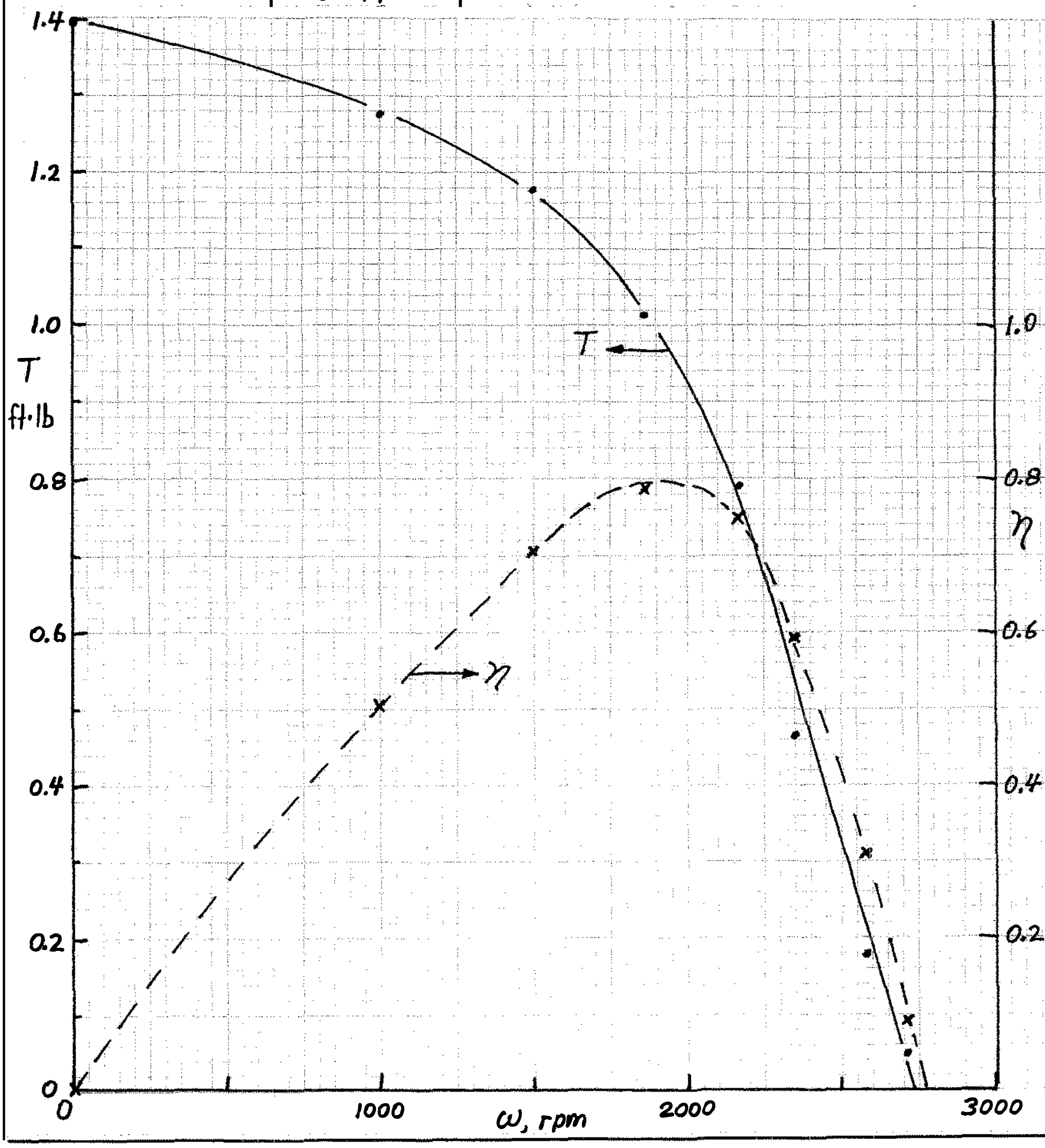
or
$$\eta = 5.116 \times 10^{-5} \frac{T\omega}{Q} \quad \text{where } T \sim \text{ft·lb},\ \omega \sim \text{rpm},\ Q \sim \text{cfs} \qquad (2)$$

Values of T and η are given in the table below and plotted in the graphs shown.

(con't)

12.72 (con't)

ω, rpm	T, ft·lb	η
0	1.397	0
1000	1.275	0.506
1500	1.179	0.701
1870	1.015	0.783
2170	0.792	0.745
2350	0.465	0.593
2580	0.179	0.308
2710	0.047	0.096

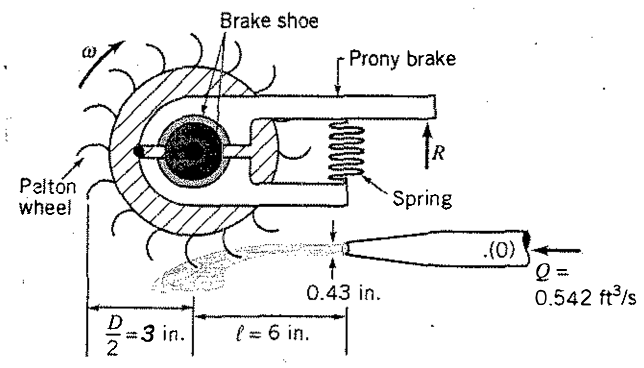


12.74

12.74 The device shown in Fig. P12.74 is used to investigate the power produced by a Pelton wheel turbine. Water supplied at a constant flowrate issues from a nozzle and strikes the turbine buckets as indicated. The angular velocity, ω, of the turbine wheel is varied by adjusting the tension on the Prony brake spring, thereby varying the torque, T_{shaft}, applied to the output shaft. This torque can be determined from the measured force, R, needed to keep the brake arm stationary as $T_{shaft} = R\ell$, where ℓ is the moment arm of the brake force.

Experimentally determined values of ω and R are shown in the following table. Use these results to plot a graph of torque as a function of the angular velocity. On another graph plot the power output, $\dot{W}_{shaft} = T_{shaft}\,\omega$, as a function of the angular velocity. On each of these graphs plot the theoretical curves for this turbine, assuming 100 percent efficiency.

Compare the experimental and theoretical results and discuss some possible reasons for any differences between them.



■ FIGURE P12.74

ω (rpm)	R (lb)
0	2.47
360	1.91
450	1.84
600	1.69
700	1.55
940	1.17
1120	0.89
1480	0.16

(a) Experimental: $T = R\ell = (0.5\,ft)\,R$ or $T = 0.5R$ ft·lb, where $R \sim lb$ (1)

and $\dot{W}_{shaft} = T\omega = T\left(\omega \frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{rev}\right)$

or $\dot{W}_{shaft} = 0.1047\,T\omega\ \frac{ft\cdot lb}{s}$, where $T \sim ft\cdot lb$, $\omega \sim rpm$ (2)

Values of ω, T, and $\dot{W}_{shaft}$ are given in the table and graph below.

(b) Theoretical: $T = \dot{m}r(U - V_1)(1-\cos\beta)$ where assume $\beta = 180°$,

$$V_1 = \frac{Q}{A_1} = \frac{0.542\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.43}{12}\,ft\right)^2} = 53.7\,\frac{ft}{s}, \text{ and}$$

$$\dot{m} = \rho Q = \left(1.94\,\frac{slugs}{ft^3}\right)\left(0.542\,\frac{ft^3}{s}\right) = 0.105\,\frac{slugs}{s}$$

Hence, with $U = \omega\frac{D}{2} = \left(\frac{3}{12}ft\right)\left(\frac{2\pi\omega}{60}\,\frac{rad}{s}\right) = 0.0262\,\omega\,\frac{ft}{s}$, $\omega \sim rpm$

$$T = \left(0.105\,\frac{slugs}{s}\right)\left(\frac{3}{12}\,ft\right)\left[0.0262\,\omega - 53.7\right]\frac{ft}{s}$$

or

$T = 1.41\left[4.88\times10^{-4}\,\omega - 1\right]$ ft·lb, where $\omega \sim rpm$ (3)

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12.74 (con't)

Also, $\dot{W}_{shaft} = T\omega = T(\frac{2\pi}{60}\omega) = 0.1047\,T\omega \ \frac{ft\cdot lb}{s}$, where $T \sim ft\cdot lb$, $\omega \sim rpm$ (4)

Values of T and $\dot{W}_{shaft}$ from Eqs. (3) and (4) are plotted in the graph below.

ω, rpm	experiment T, ft·lb	experiment $\dot{W}_{shaft}$, $\frac{ft\cdot lb}{s}$	theory $-T$, ft·lb	theory $-\dot{W}_{shaft}$, $\frac{ft\cdot lb}{s}$
0	1.235	0	1.41	0
360	0.955	36.0	1.16	43.8
450	0.920	43.3	1.100	51.8
600	0.845	53.1	0.997	62.6
700	0.775	56.8	0.928	68.0
940	0.585	57.6	0.763	75.1
1120	0.445	52.2	0.639	75.0
1480	0.080	12.4	0.392	60.7